

Studying The Coherence Properties Of Excitonic Bose-Einstein Condensates

A Thesis

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by

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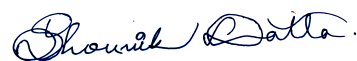
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Certificate

This is to certify that this dissertation entitled "Studying The Coherence Properties Of Excitonic Bose-Einstein Condensates" towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by S.V.U. Vedhanth at Indian Institute of Science Education and Research under the supervision of Dr. Shouvik Datta, Associate Professor, Department of Physics, during the academic year 2023-2024.



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Declaration

I hereby declare that the matter embodied in the report entitled “Studying The Coherence Properties Of Excitonic Bose-Einstein Condensates” are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education and Research (IISER) Pune, under the supervision of Dr. Shouvik Datta, and the same has not been submitted elsewhere for any other degree. Wherever others contribute, every effort is made to indicate this clearly, with due reference to the literature and acknowledgement of collaborative research and discussions.

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This Thesis is dedicated to my beloved parents
S.V. Uthaya Surian and U. Uma Maheswari.

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1 Abstract

An exciton is a bound pair of negatively charged electron and positively charged hole, which is formed by mutual coulomb attraction. In the simplest of approximations, it can be likened to a modified hydrogen atom [1]. Usually, it is expected to be a composite boson. This analogy facilitates the exploration of intriguing phase transitions such as Bose-Einstein Condensation (BEC), which is anticipated at temperatures (few to few 100 Kelvin) [2,3] much higher than those typically observed for bosonic atom BECs which is typically in the nano-Kelvin to micro-Kelvin range [4,5]. Despite numerous efforts to investigate excitonic BEC, results have often been inconsistent or have necessitated extremely low temperatures in the millikelvin range. This challenge stems from the tendency of excitons to undergo radiative and non-radiative recombination, hindering the realization of pure excitonic BEC. Thus, the pursuit of pure excitonic BEC remains ongoing.

In this project, we investigated the presence of Excitonic Bose-Einstein Condensate (BEC) in a 0D-2D Mixed Dimensional III-IV semiconductor. Our approach involved measuring the 2D transverse spatial coherence function and the 2D momentum space distribution of light emitted from exciton recombination, as BEC excitons are expected to exhibit long-range spatial order and occupy a narrow momentum space. To achieve this, we developed a modified Michelson Interferometer [6] capable of measuring the 2D spatial coherence function. Utilizing the Wiener-Khintchine Theorem, we calculated the 2D momentum space distribution of the light. This setup offers advantages over previous methods for measuring spatial coherence, as it employs simple components that are easy to control and provides significant control over the additional phase introduced into the light, which was lacking in many previous attempts. Additionally, we conducted photo-luminescence spectroscopy and photo-capacitance experiments to gain further insight into the physics of our sample. We also explored the feasibility of measuring spatial coherence through photo-capacitance measurements. Such a study can be likened to investigating the matter wave interference of two populations of BEC excitons. Through our various measurements, it became apparent that each method probes different populations of excitons. By simultaneously conducting these measurements, we were able to explore the rich dynamics of different excitonic populations in our sample.

2 Introduction

2.1 Excitons and its BEC

When a light of energy equal to more than the bandgap of a semiconductor, an electron is excited to the conduction band leaving behind a hole in the valence band. The excited electron and hole then are attracted to each other through coulombic force and forms a bound pair called exciton with a certain lifetime. In most semiconductor materials, since the dielectric constant is very high, the binding of electron and hole will be weak and thus loosely bound. Such excitons are called Wannier-Mott excitons. Under simple approximation, the Schrodinger equation for such bound two body system in 3D is given by [1],

$$\left[- \left(\frac{\hbar^2}{2m_e} \right) \nabla_{R_e}^2 - \left(\frac{\hbar^2}{2m_h} \right) \nabla_{R_h}^2 - \frac{e^2}{4\pi\epsilon_0\epsilon_r|R_e - R_h|} \right] \Phi(R_e, R_h) = E\Phi(R_e, R_h) \quad (1)$$

where, m_e/m_h is the effective mass of electron/hole, R_e/R_h is the position coordinate of electron/ hole and ϵ_r is the relative dielectric constant of the material. This equation can be then be decoupled into motion of center-of-mass (CoM) and relative motion due to the Coulomb attraction as,

$$- \left(\frac{\hbar^2}{2\mathbf{M}} \right) \nabla_{\mathbf{R}}^2 \psi(\mathbf{R}) = E_{\mathbf{R}} \psi(\mathbf{R})$$

and

$$\left(- \frac{\hbar^2}{2\mu} \nabla_r^2 - \frac{e^2}{4\pi\epsilon_0\epsilon_r r} \right) \phi(r) = E_r \psi(r)$$

respectively. Solving these similar to a hydrogen atom will yield the wavefunctions and energy as,

$$\phi_{\mathbf{K}}(\mathbf{R}) = \left(\frac{1}{\sqrt{N}} \right) \exp(i\mathbf{K} \cdot \mathbf{R}) \text{ with } E_{\mathbf{R}} = \frac{\hbar^2 \mathbf{K}^2}{2\mathbf{M}} \quad (2)$$

$$\psi_{nlm}(r) = R_{nl}(r) Y_{lm}(\theta, \phi) \text{ with } E_r(n) = E_r(\infty) - \frac{R^*}{n^2} = E_g - \frac{R^*}{n^2} \quad (3)$$

and from Eq. 2 and 3, the combined envelope wavefunction and energy can be written as [1],

$$\Phi_{nlm}(\mathbf{R}, r) = \left(\frac{1}{\sqrt{N}} \right) \exp(i\mathbf{K} \cdot \mathbf{R}) R_{nl}(r) Y_{lm}(\theta, \phi) \quad (4)$$

$$E_{nlm} = E_g + \frac{\hbar^2 \mathbf{K}^2}{2\mathbf{M}} - \frac{R^*}{n^2} \quad (5)$$

where $E_r(\infty)$ is the minimum energy of the continuum states (E_g is the band gap), $R^* = \frac{\mu}{m_0 \epsilon_0^2} \times 13.6 \text{ eV}$ called the Rydberg constant for exciton, m_0 is the rest mass of the electron, E_g is the bandgap, $n/l/m$ are the principal quantum numbers, $\mu = \frac{m_e m_h}{m_e + m_h}$ is the reduced mass of the exciton, $\mathbf{R} = \frac{m_e R_e + m_h R_h}{m_e + m_h}$ is the Center-of-Mass (CoM) coordinate, $r = R_e - R_h$ is the relative coordinate, $\mathbf{K}$ is the exciton wavevector, $\mathbf{M} = m_e + m_h$, $\frac{1}{\sqrt{N}}$ is the normalization constant, $R_{nl}(r)$ is the radial part of wavefunction called Laguerre polynomials and $Y_{lm}(\theta, \psi)$ is the angular part of the wavefunction called spherical harmonic functions. The dispersion relation for an exciton should be represented in a two-body picture as shown in Fig. 1. Such a spherically symmetric parabolic dispersion is sufficient for explaining most of exciton effects in the spectra for most semiconductors [1]. Often in optical excitation, since λ (optical wavelength) is larger than a (lattice constant), the momentum imparted by light $\frac{2\pi}{\lambda}$ will be negligible. Thus, exciton created will not carry much kinetic energy $\frac{\hbar^2 \mathbf{K}^2}{2\mathbf{M}}$ and most excitons are formed around the $k = 0$ point.

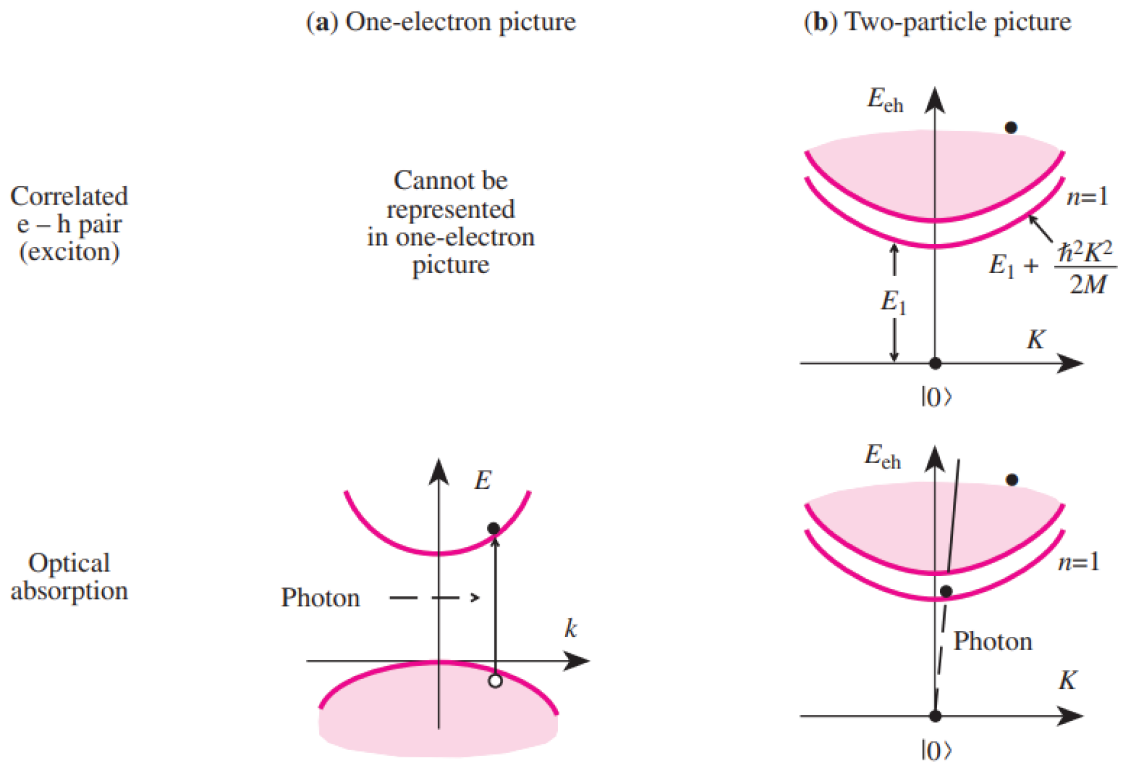


Figure 1: This image shows the correct representation in two-particle picture of the dispersion relation for exciton and how it is represented during optical absorption under the illumination of light.[1]

The binding energy associated with Wannier-Mott excitons typically falls within the range of tens of meV [7]. Given that room temperature inherently possesses an energy level of ap-

proximately 26 meV at 300 K, these direct excitons within bulk materials tend to dissociate at such temperatures, often necessitating the need for cooling. So, to study excitons and its physics people often go to temperatures around few K. One another way to study excitons is to increase its binding energy. This can be achieved by confining exciton to lower dimensions like in quantum well or quantum dot where there is a decrease in the screening of coulomb attraction by material [1]. Such excitons can be studied in temperatures even at room temperature. In contrary to direct excitons where electrons and holes are in same material, one can also create excitons where the individual carriers are in separate layers. These are called spatially indirect excitons [8,9] whose recombination rate will be reduced compared to a direct exciton. The excitons that emit light upon recombination after its lifetime are called bright excitons and those that don't emit light are called dark exciton. This forbidden in emission can be due to spin selection rules of that particular transition [10].

Since these excitons are bosonic quasi particle with fermionic components, one can expect all phase transitions that were studied in case of bosonic atoms like Bose-Einstein Condensation (BEC) and superconductivity. BEC transition is where all the particles occupy same state and their wavefunctions overlap to form a macroscopic coherent state [11,12]. But these phase transitions are studied by increasing the density of particles to invoke many body interactions. Usually, such transitions are observed at very low temperature in orders of nK - μ K [4,5]. But for excitons the critical temperature for observing such BEC is much higher due to its effective mass being less than that of the rest mass of an electron. The critical temperature for BEC transition in case of 3D is given by [11,12],

$$T_c \approx 3.3125 \frac{\hbar^2 n^{2/3}}{mk_b} \quad (6)$$

In Eq. 6 n is the density of particles, m is the mass of the particle and k_b is the Boltzmann constant. In case of excitons when the density is increased, one exciton can screen the other excitons and make it dissociate. Also when density increases the auger recombination, a three body scattering process, thus increasing the probability of non-radiative recombination making it difficult to reach the critical density. But by using indirect excitons one can reduce the recombination rate and try observing such phase transition [8]. Now, to probe such BEC we need to show the long-range order spatial coherence or the narrowing of momentum space

distribution of the excitons. To explain this let us consider the momentum space occupation of particles in case of BEC which is given as [11,12],

$$n(p) = N_0 \delta p + \tilde{n}(p)$$

This is considering the macroscopic occupation of single particle ground state (N_0) during BEC, which in momentum space usually at $p = 0$. The spatial coherence function is given by,

$$G^{(1)}(r, r') = \langle \hat{\psi}^\dagger(r) \hat{\psi}(r') \rangle$$

$$G^{(1)}(r, r') = \frac{1}{V} \int n(p) e^{-ip \cdot s / \hbar} dp \quad (7)$$

upon considering field operator formalism where $s = |r - r'|$. When $s \rightarrow \infty$, the normalized spatial coherence approaches the value N_0/N which is the condensate fraction as shown in Eq. 8.

$$G^{(1)}(s)|_{s \rightarrow \infty} = \frac{N_0}{V}$$

$$g^{(1)}(r, r') = \frac{G^{(1)}(r, r')}{\sqrt{G^{(1)}(r) G^{(1)}(r')}} \text{ and } g^{(1)}(s)|_{s \rightarrow \infty} = \frac{N_0}{N} \quad (8)$$

This says that when there is BEC transition the spatial coherence doesn't approach zero even when they are very far. This is called the emergence of long-range spatial order [11,12]. Such effect can be studied by measuring the transverse spatial coherence of the light emitted from the recombination of these excitons which in turn can tell about the emergence of spatial coherence of those excitons.

2.2 Optical Coherence and Wiener-Khintchine Theorem

Optical coherence [13-15] is the property that describes the phase stability of any light, both in the temporal and spatial domain. The temporal (spatial) coherence is characterized by the coherence time τ_c (with coherence length L_c in the direction transverse to the propagation of light). The coherence time (length) is the duration (distance) over which the phase of the wave remains stable. Let's say that we know the wave's phase at t_1 (r_1), then the phase at the same position (time) but at different time t_2 (position r_2) will be known with a high degree of certainty when time (position) separation lies within the coherence time (length), and with a very low degree when time (position) separation exceeds the coherence time (length). Ordinarily, optical

coherence can be quantified by the First Order Coherence Function, $g^{(1)}(r_1, t_1, r_2, t_2)$ defined as,

$$g^{(1)}(r_1, t_1, r_2, t_2) \equiv g^{(1)}(s, \tau) = \frac{\langle \mathcal{E}^*(r_1, t_1) \mathcal{E}(r_2, t_2) \rangle}{\sqrt{\langle |\mathcal{E}(r_1, t_1)|^2 \rangle \langle |\mathcal{E}(r_2, t_2)|^2 \rangle}} \quad (9)$$

where $s = \delta r = (r_1 - r_2)$ is the spatial separation in the transverse direction and $\tau = \delta t = (t_1 - t_2)$ is the temporal separation along the propagation direction. This (temporal) spatial coherence is calculated from this equation by fixing (position) time at zero. The absolute value of the first order coherence function, $|g^{(1)}(s, \tau)|$ is called the degree of first-order coherence [13-15] and is 1 for a completely coherent, ideal light source, 0 for an completely incoherent light source, and varying in between for any partially coherent source. For interfering electric fields $E_1 = E_{01} \sin(\omega t + \phi_1)$ and $E_2 = E_{02} \sin(\omega t + \phi_2)$, the resultant electric field is given as, $E = E_1 + E_2$ and since intensity $I = |E|^2$, $E_0^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos(\phi_1 - \phi_2)$ where $(\phi_1 - \phi_2)$ represents the phase difference between the fields. Any fluctuation in this phase difference will lead to disappearance of the interference fringes. In complex form of electric field into account and introducing them to Eq. 9 will give $\langle e^{i[\phi(t) - \phi(t+\tau)]} \rangle e^{i\omega\tau} = g^{(1)}(\tau)$ and $\langle e^{i[\phi(t) - \phi(t+\tau)]} \rangle = |g^{(1)}(\tau)|$. So, the interference equation can be written as $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \text{Re}(|g^{(1)}(\tau)|)$ and on calculating the visibility of the fringe pattern can be related to the degree of coherence as

$$\text{Visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2(\langle I_1 \rangle \langle I_2 \rangle)^{1/2}}{\langle I_1 \rangle + \langle I_2 \rangle} |g^{(1)}(\delta r, \tau)| \quad (10)$$

For the interfering sources of equal average intensities, one gets

$$\text{Visibility} = |g^{(1)}(\delta r, \tau)|$$

Interestingly, the coherence function and the spectral distribution are related by the Wiener-Khintchine Theorem of statistical physics [15]. It states that the energy spectrum of the source is the Fourier Transform of the first order temporal coherence as

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g^{(1)}(\tau) e^{i\omega\tau} d\tau \quad (11)$$

where the power spectrum is given by,

$$F(\omega) = \frac{|E(\omega)|^2}{\int_{-\infty}^{\infty} |E(\omega)|^2 d\omega}$$

$$|E(\omega)|^2 = \frac{T}{4\pi} \int_{-\infty}^{\infty} \langle E^*(t)E(t+\tau) \rangle \exp(i\omega\tau) d\tau \text{ and } \int_{-\infty}^{\infty} |E(\omega)|^2 d\omega = \langle E^*(t)E(t) \rangle$$

Similarly, obtaining the first order spatial coherence function as a function of spatial separation can also help us in probing the momentum space distribution.

$$F(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g^{(1)}(\delta r) e^{ik(\delta r)} d(\delta r) \quad (12)$$

2.3 Sample of Interest

The sample under study is a mixed dimensional hybrid heterostructure [16] and synthesized through Molecular Beam Epitaxy (MBE). A disordered array of 0D InAs quantum dots, whose density is nearly 10^{11} cm^{-2} and dots have an extent 1.8 nm in lateral direction and 10 nm in transverse direction, embedded between two AlAs potential barriers. This entire structure is placed within the intrinsic part of p(0.51 μm)-i(60 nm on top of top AlAs barrier and 100 nm on bottom of bottom AlAs barrier)-n(100 nm) GaAs diode structure placed on the top of n^+ GaAs (1 μm). A triangular quantum well forms at the junction of AlAs and i-GaAs which acts as a reservoir for electrons in case of reverse bias. An illustration is shown along with its band diagram in Fig.2. Upon illumination of light, p-GaAs is exposed and the electrons created there will travel towards the triangular quantum well formed at the top of AlAs barrier upon bias. Similarly, the holes from n-GaAs will move towards the QD layer. Upon meeting resonant condition at certain voltage since the electrons are less in mass compared to holes, they will easily tunnel through the QD layer but the holes won't be and most of them will stay in the QDs. So, the electrons confined in the 2D quantum well and the holes stuck in 0D quantum dots forms the Indirect excitons which is of our interest. The sample is studied inside the Closed Cycle Refrigerator (CCR) from Advanced Research Systems (ARS) S204-DMX-20 with DE-204 whose temperature is controlled using the Lakeshore temperature controller model 340. The basic setup for measurement is provided in bellow shown Fig. 3.

3 Optical Spatial Coherence Measurement

For measurement of the coherence function, several investigations have utilized adaptations of Young's double slit experiment [17-19], Michelson Interferometer [6,20,21] and Mach-Zehnder Interferometer [22] to explore such phenomena. Attempts have also been made to measure both

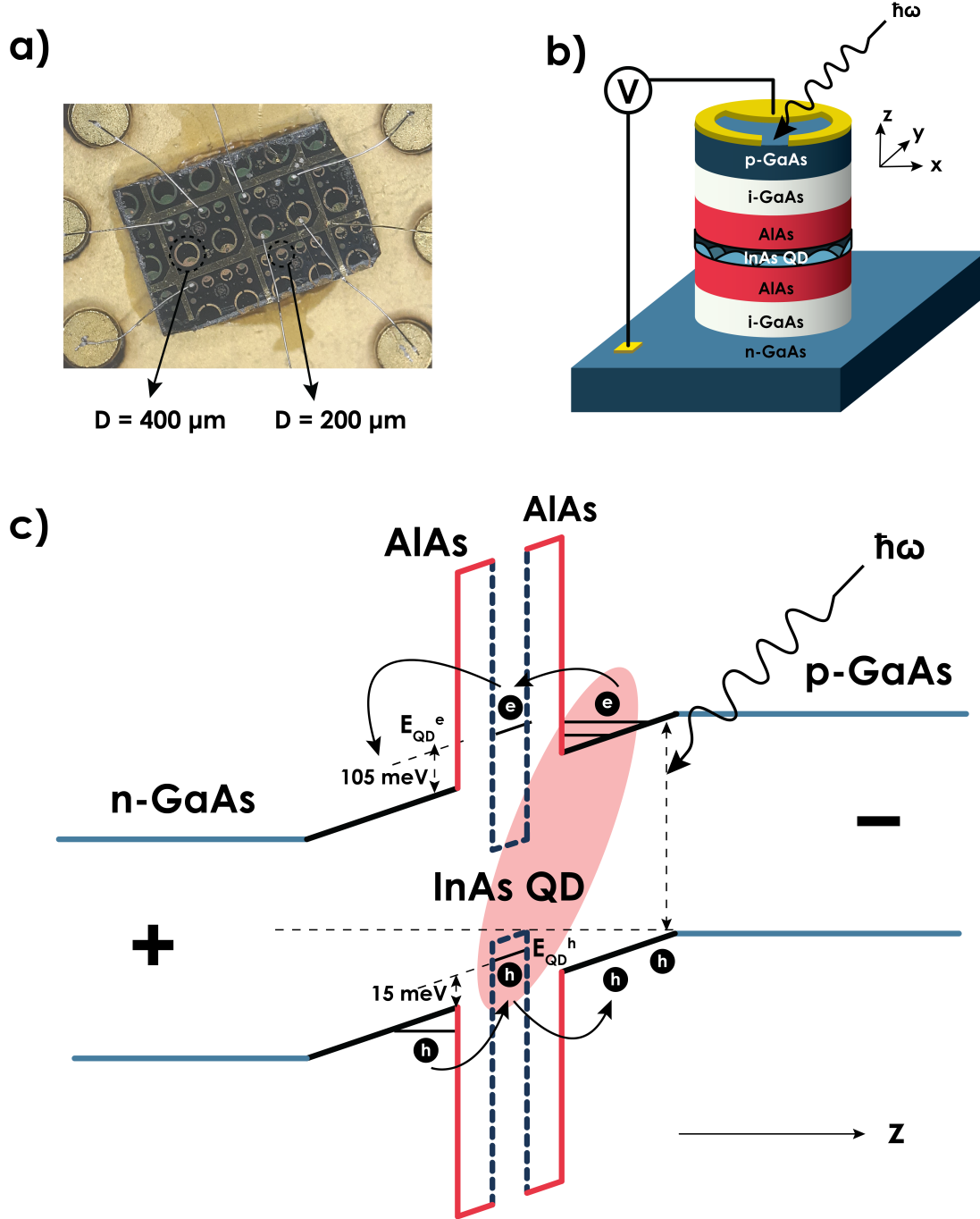


Figure 2: (a) shows the entire sample under the microscope, where the ring shaped electrodes show where exactly the sample we measure is present. This image depicts the presence of samples with two diameters of $200 \mu\text{m}$ and $400 \mu\text{m}$. An illustration how the sample looks in 3D is shown in (b). The band structure of such a heterostructure is given in (c).[16]

spatial and temporal coherence using these setups [21]. As mentioned in the introduction, in the study of BEC, one another approach is to examine these processes by monitoring the system's momentum space or k-space to observe the emergence of spatially long-range order and con-

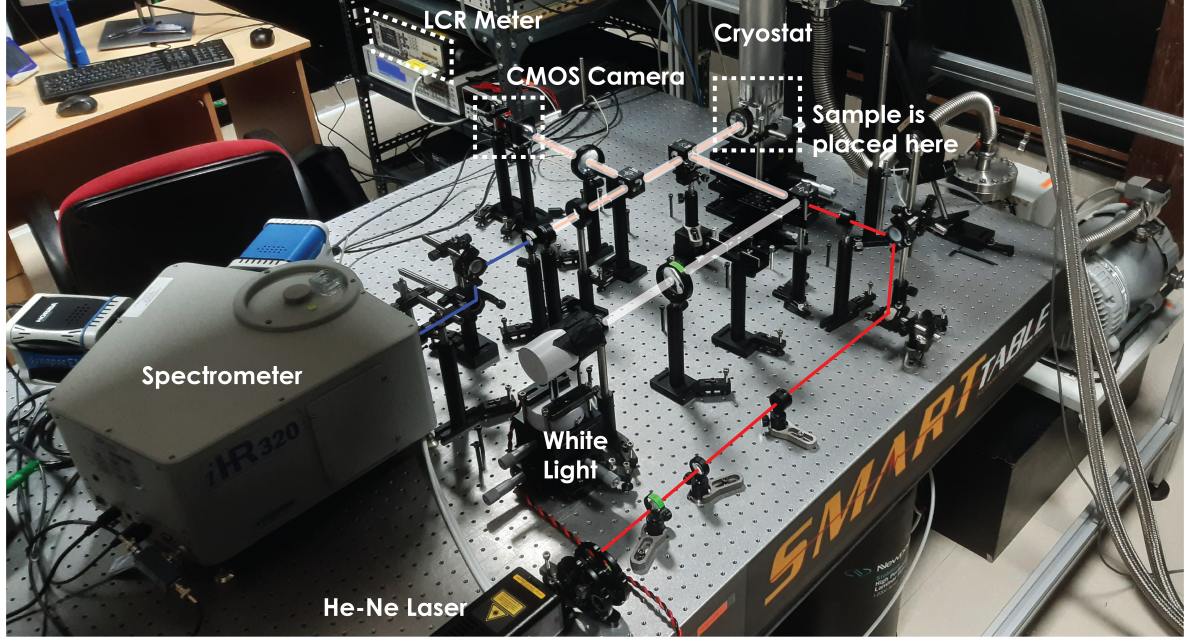


Figure 3: This image shows the optical setup used for measurements regarding our sample.

sequent narrowing in momentum space distribution. The 2D momentum space measurements are often conducted by scanning the Back Focal Plane (BFP) image of a lens [23-25]. Now, for most systems like excitonic BEC, the samples generally need to be kept in a cryostat to go below the critical temperature to observe a BEC. In such cases, performing Young's double slit experiment with varying slit widths and adjustable slit separations kept within the cryostat to measure 1st order spatial correlation to estimate the coherence function can be somewhat challenging regarding instrumentation. BFP imaging also requires placing a high numerical aperture lens close to the sample. All of these actually increase experimental complications for low temperature samples placed inside a cryostat. These include additional heat loads on the sample chamber and/or to the sample itself due to its proximity with various optical elements, some of which are at ambient temperature. To mitigate these challenges, experiments have been conducted outside the cryostat with a modified optical interferometry setup maintained at ambient temperature and pressure, and a distance from the sample.

Previous studies have attempted to measure the spatial coherence function using Mach-Zehnder interferometers [22] or other interferometry techniques [20], but these approaches were limited to one spatial dimension or suffered from optical aberrations and phase differences introduced by curved retroreflectors [26-28] or manufacturing imperfections in dove prisms [29]. There-

fore, there is a need for precise control over these optical phases. In our experiments, we aim to eliminate unintentional optical phases introduced by measurement setups. To achieve this, we sought to measure and generate a 2D map of the coherence function using modified Michelson Interferometry techniques [6]. Subsequently, we calculated the 2D momentum space for a laser diode as a function of bias current. This allowed us to modify the coherence function of emitted light from LED mode, where it is incoherent, to lasing mode, where it becomes coherent, by adjusting the bias current beyond the lasing threshold.

3.1 Experimental Setup

The schematic representation of our setup [6] is shown in Fig. 4. This setup is a modified Michelson interferometer where the usual mirrors in the setup are replaced with triangular retroreflectors (RR1 and RR2) with planar surfaces. The 50:50 beam splitter BS1 is CCM1-BS013/M from Thorlabs. The mirrors M1, M2 are silver protected mirrors PF10-03-P01 also from Thorlabs. In our experimental set up, the plane of reflections from the retroreflectors are not coplanar as earlier and reflected beams from RR1 and RR2 are at different heights. As a result, we needed these two additional mirrors M1 and M2. In general, these retroreflectors can prevent any back reflection from the source. Use of two such retroreflectors can also provide this optical setup with certain tilt resistance capabilities. The light source used is HL6358MG (Thorlabs), an AlGaInP based multi-quantum well commercial single-mode laser diode with the peak wavelength at 645 nm and a threshold for lasing at 25 mA. It is mounted on TCLDM9 mount, which helps to control the diode's operating temperature. It is ran using the ITC4001 controller from Thorlabs. The retroreflector RR2 is attached to an xyz-stage through a piezo actuator so that it can be moved longitudinally to introduce temporal phase difference. The light from the diode is collimated using an objective lens of magnification of $10\times$. The interference is detected using a CCD camera based BC106NVIS/M beam profiler from Thorlabs. The sensor has 1360×1024 pixels, and each pixel is of size $\approx 6.45 \mu\text{m} \times 6.45 \mu\text{m}$. This CCD is used along with Neutral Density (ND) filters of 20 dB or 40 dB based on our requirement (here, these dB values represent the attenuation of these filters).

Initially, the input beam gets split equally by BS1. The retroreflectors of both arms are kept perpendicular to each other. RR1 flips the beam profile from $r(x, y)$ to $r(x, -y)$ and RR2 will flip it from $r(x, y)$ to $r(-x, y)$. These two flipped beams are made to overlap using the mir-

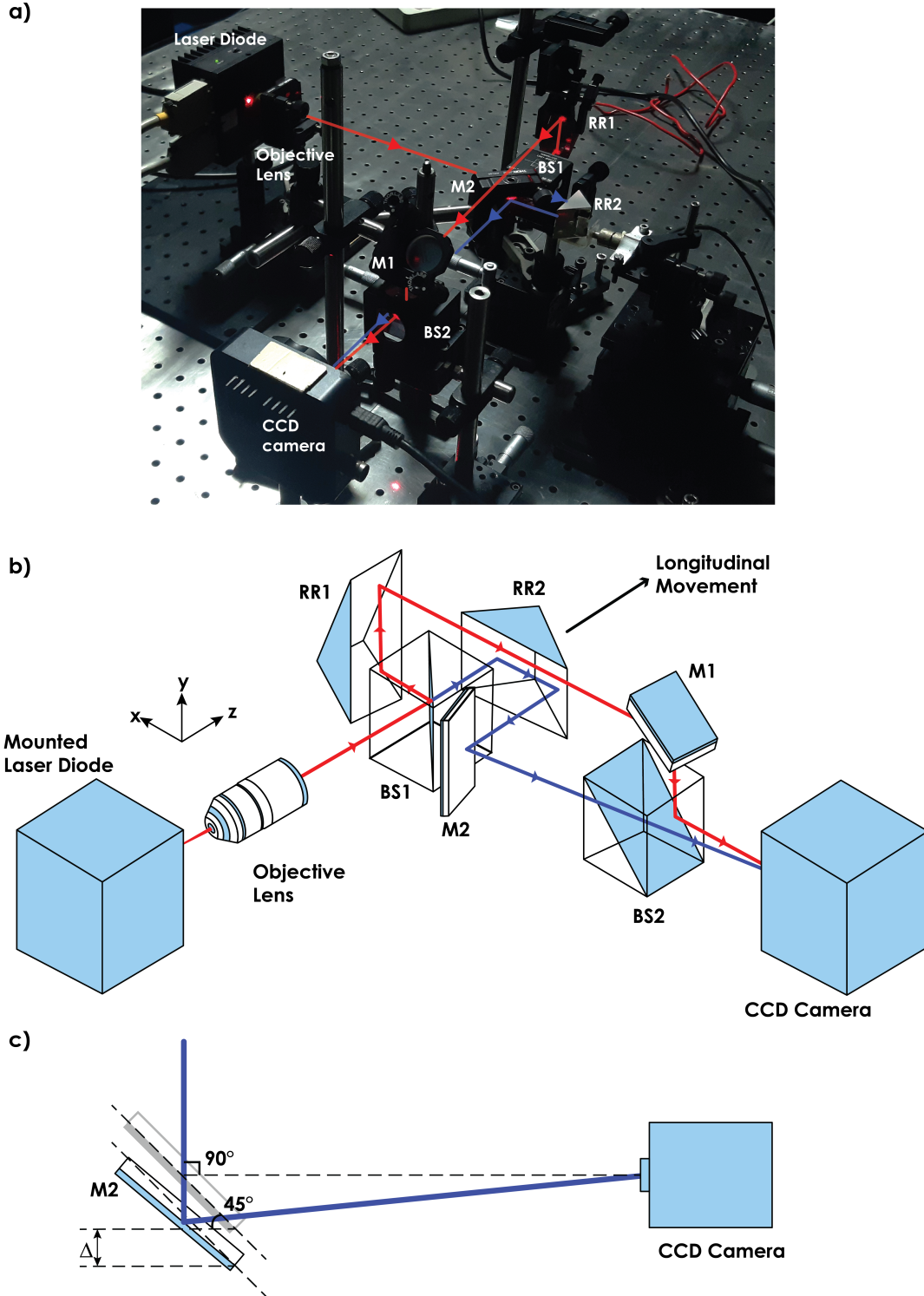


Figure 4: The figure (a) shows the experimental setup and (b) shows the schematic representation of the same to measure 2D spatial coherence. The arrows indicate the direction of propagation of the light beam. The lower image shows how the tilt is introduced into the beam to introduce phase difference for observing the interference pattern. Gray coloured mirror depicts the condition of no tilt.

ror arrangements, shown in Fig. 4. After BS2 the combined beams are incident on the CCD camera. Then in the image plane of CCD, each pixel will correspond to the spatial correlation between the points $r(x,y)$ of the original beam and $r(-x,-y)$ after being reflected from both RR1 and RR2. So each point in the image plane correspond to an effective spatial separation $(\delta x, \delta y) = (2|x|, 2|y|)$. Now, upon overlap, an interference fringe pattern is observed as shown in Fig. 5(a) and 5(b). The fringe pattern also depends on the tilt introduced by M2. This tilt of the M2 mirror is controlled in our case by moving it by a distance Δ and then rotating the mirror handle (lying along x-axis) about y-axis like in Fig. 4(b) to make them interfere on the screen of the CCD. This can also be done by rotating the mirror along any other axes. If we increase the distance Δ , then the fringe width also reduces. Therefore, this control can help us to measure coherence function of light sources/spots having smaller spatial patterns [6]. One can always choose to measure fringe patterns which are at least 2 or 3 pixels wide on the CCD to prevent random fluctuations and pixel blurring in the measurement during the 2D mapping of $|g^{(1)}(\delta x, \delta y)|$.

To measure the degree of spatial coherence, first we vary temporal difference of the interfering light spots by moving the RR2 longitudinally around $\tau = 0$ to eliminate any residual temporal contributions from the spatial coherence measurements using ‘temporal filtering’. This is done using Piezoelectric Actuator PAS005 procured from Thorlabs and then we record the observed interference patterns. The point $\tau = 0$ is fixed by going to a less bias current value of the laser diode with negligible coherence. Here the interference is observed over only small temporal differences. Whenever clear fringes are observed, it is taken as $\tau = 0$ approximately, which implies zero-path difference between the arms of the interferometer for each pixel. The Piezo is controlled using MDT693B 3-Channel open-loop Piezo controller from Thorlabs. The voltage applied is varied from 0.00 to 6.00 V in steps of 0.15 V resulting in a total movement of $\approx 16\mu m$ in Piezo giving a temporal delay of ≈ 108 fs in total around $\tau = 0$. This ‘temporal filtering’ [6,21] is known to eliminate the mixing of unwanted temporal coherence in spatial coherence measurement. The intensity of both beams was also recorded separately by blocking the other to find the values of I_1 and I_2 . From these recorded interference patterns, once the intensity from the same pixel of the CCD was plotted as a function τ , we get $I(\tau)$ vs τ for a particular $(\delta x, \delta y)$ as shown in Fig. 5(d). Using Eq. 10, we then calculate the degree of spatial coherence $|g^{(1)}(\delta x, \delta y)|$ for each pixel corresponding to each $(\delta x, \delta y)$ at $\tau = 0$ condi-

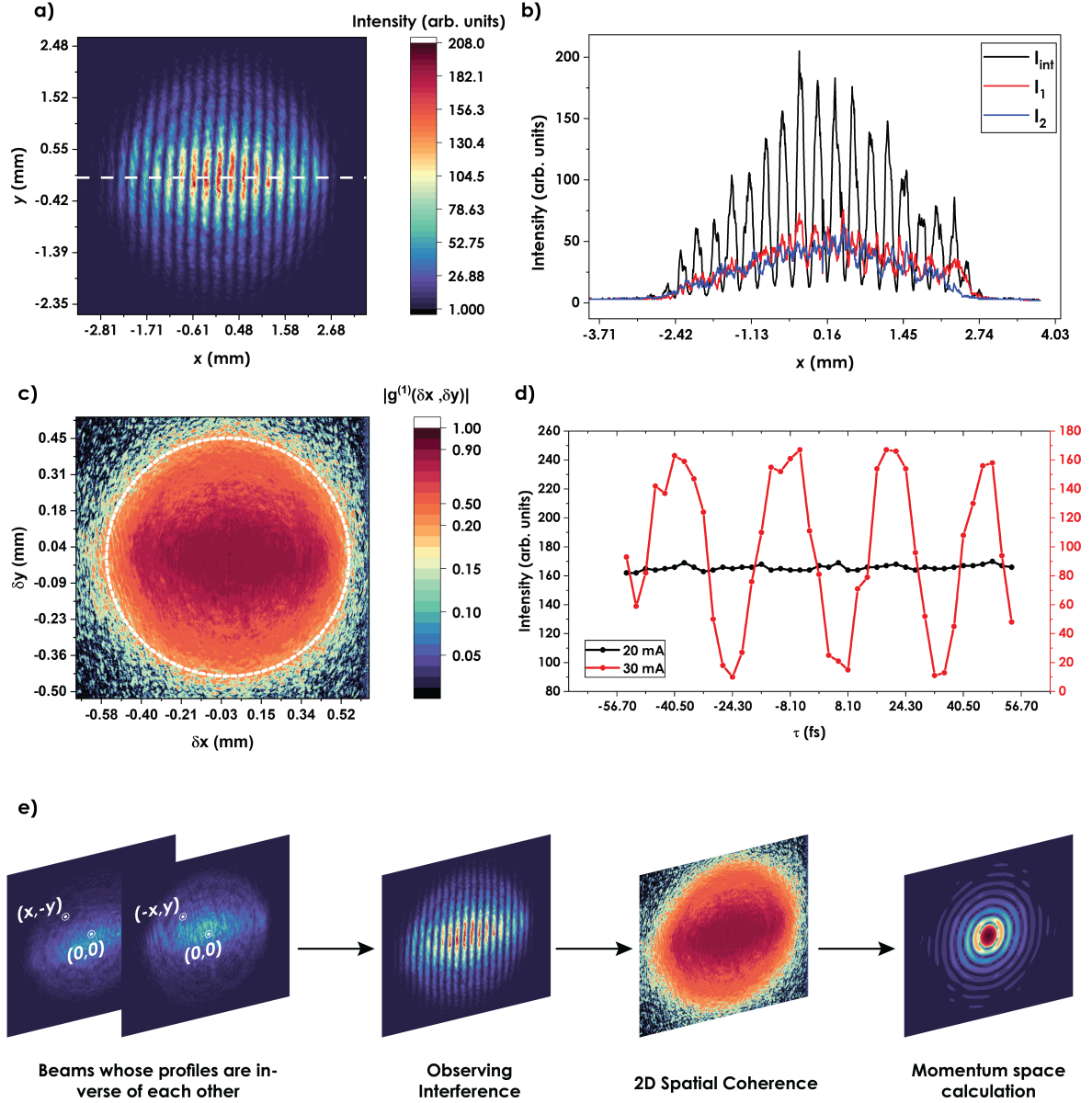


Figure 5: (a) This shows one of the interference patterns observed for 30 mA and (b) shows its profile along x axis. (c) is the 2D coherence function obtained in case of 30 mA. The dotted circle shows the region selected for the k-space calculation. (d) shows the $I(\tau)$ obtained for a pixel near to $\delta x = 0, \delta y = 0$ for the case of lasing (30 mA) and also for LED mode (20 mA). (e) shows the method of obtaining the coherence function and calculating momentum space distribution.

tion. By repeating it over all the pixels, we end up with the 2D map of $|g^{(1)}(\delta x, \delta y)|$. The same measurement is taken five times and the average of it is taken as the final value. The scattered background light outside the beam diameter can affect the Fourier transform, in case the beam is partially coherent at 20 mA bias current. The background noise outside beam spot is removed by keeping only the values in the beam area shown in Fig. 5(c) and then making

all other values be 0. This also helps to avoid any lens vignetting issues around the peripheral regions. The coherence function is calculated for various bias currents and plotted in Fig. 6. In this experiment, we assume the beam centre to be $\delta x = 0$, $\delta y = 0$. Now we take the 2D Fourier transform of this $|g^{(1)}(\delta x, \delta y)|$ while keeping the point $(\delta x = 0, \delta y = 0)$ as the centre to estimate the 2D k-space map of the emitted light. The calculated k-space for varying bias current is shown in Fig. 6. Under Fast Fourier Transform (FFT) for a function $f(cx)$ such that $f(cx) \xrightarrow{\text{FFT}} \frac{1}{|c|} F(\frac{k}{c})$, where $|c| = 2$ is a constant in our experiment as argued above. So we multiply the k-space coordinates with $|c|=2$ to determine the actual $F(k)$ and not $F(\frac{k}{c})$. Only the amplitude of the real part of the obtained Fourier transform $F(k)$ is plotted following Eq. 12.

In our optical coherence method, the Fourier coordinates are determined by the program used for the Fast Fourier Transform (FFT) [6]. The real space information is given as a matrix, say R , whose indices (p, q) run from 0 to $N-1$ and 0 to $M-1$ respectively where N/M is the maximum no. of data points taken in the x or y Cartesian coordinate respectively. The sampling interval (D) is either determined experimentally or on our known, so that each index(p) would correspond to the real space coordinate $p \times D$ and the entire range is $X_{\max} = N \times D$. The 2D FFT performed in python is given as,

$$F_{u,v} = \sum_{p=0}^{N-1} \sum_{q=0}^{M-1} A_{p,q} e^{-i2\pi p \cdot u} e^{-i2\pi q \cdot v} \quad (13)$$

where u and v are indices of Fourier space matrix. The sampling frequency (Δf) is given as $\Delta f = \frac{1}{D} = \frac{N}{X_{\max}}$. The Fourier coordinates or resulting spatial frequencies (U) are given as, $U = \frac{u}{N} \Delta f = \frac{u}{X_{\max}}$ and the corresponding angular spatial frequency (k_x) is given as $k_x = \frac{u \times 2\pi}{X_{\max}}$ and similarly we can write $V = \frac{v}{Y_{\max}}$ thus $k_y = \frac{v \times 2\pi}{Y_{\max}}$. Also if we have used an objective lens of 10X Magnification, taking that into account the above formula becomes, $k_x = \frac{u \times 2\pi}{X_{\max}/10}$ and similarly for k_y . In this method the maximum frequency detectable is $\frac{N}{2} \Delta f$. In our experiment $X_{\max} = \text{pixel size} \times \text{Total no. of pixels taken along } X \times 2 \times 1/\text{Magnification}$. Here the factor of 2 comes due to the fact that in coherence function, $(\delta x, \delta y) = (2x, 2y)$ [6]. From Fig. 6 we can see that on varying the bias current, the coherence of the beam increases. At 20 mA and 22.5 mA the laser diode is in LED mode, and we can see that the measured degree of coherence is very low. At 25 mA, which is the threshold bias current for lasing, there is an onset of coherence, and at 27.5 mA and 30 mA. Here, the coherence function is highly enhanced, which is due

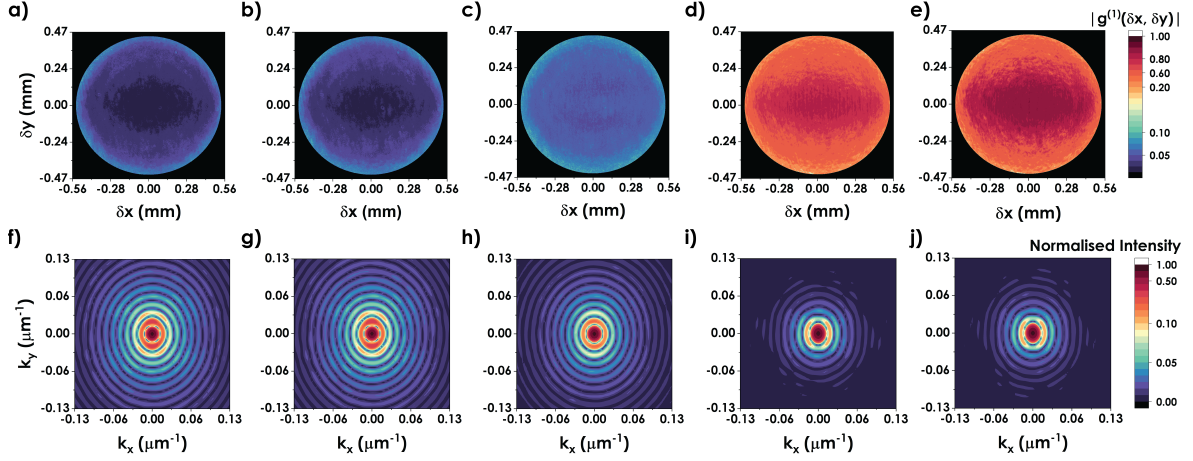


Figure 6: (a)-(e) show the degree of coherence function measured for 20 mA, 22.5 mA, 25 mA, 27.5 mA and 30 mA respectively. (f)-(j) show the calculated 2D in-plane momentum space for the above mentioned bias currents respectively.

to increase in stimulated emission or lasing. The 2D k -space plots, after being normalised as $[M]' = \frac{[M] - \min(M)}{\max(M) - \min(M)}$ where the matrix $[M]$ is the FT of 2D coherence function, for varying bias current are also shown in Fig. 6 respectively. We can see that the occupation of high k -values is decreased and of lower k -values is increased as the bias current increases. It can be said as a result of stimulated emission the light emission from the diode becomes more directional with associated changes in the k -space. While varying bias current, the intensity of the beam also increases, so the exposure time and gain are modified according to the ND filter used. These values are maintained at a constant level during the measurements for the same bias current. Thus, even without considering those parameters, the result should be the same as these will come as a constant multiplied to I_{max} , I_{min} , I_1 and I_2 and this factor would cancel at every place.

3.2 Validation of the setup

Now we have showed that our interferometer can measure the spatial coherence and thus the k -space, but we need to show how valid is our obtained k -space. To show that we used a double slit of slit separation 0.25 mm and slit width 0.2 mm as a source and tried to measure its momentum space using our setup and compare it with the k -space results from the back focal plane imaging and simulation. We have chosen double slit because its k -space distribution is nothing but its diffraction pattern and it is well known.

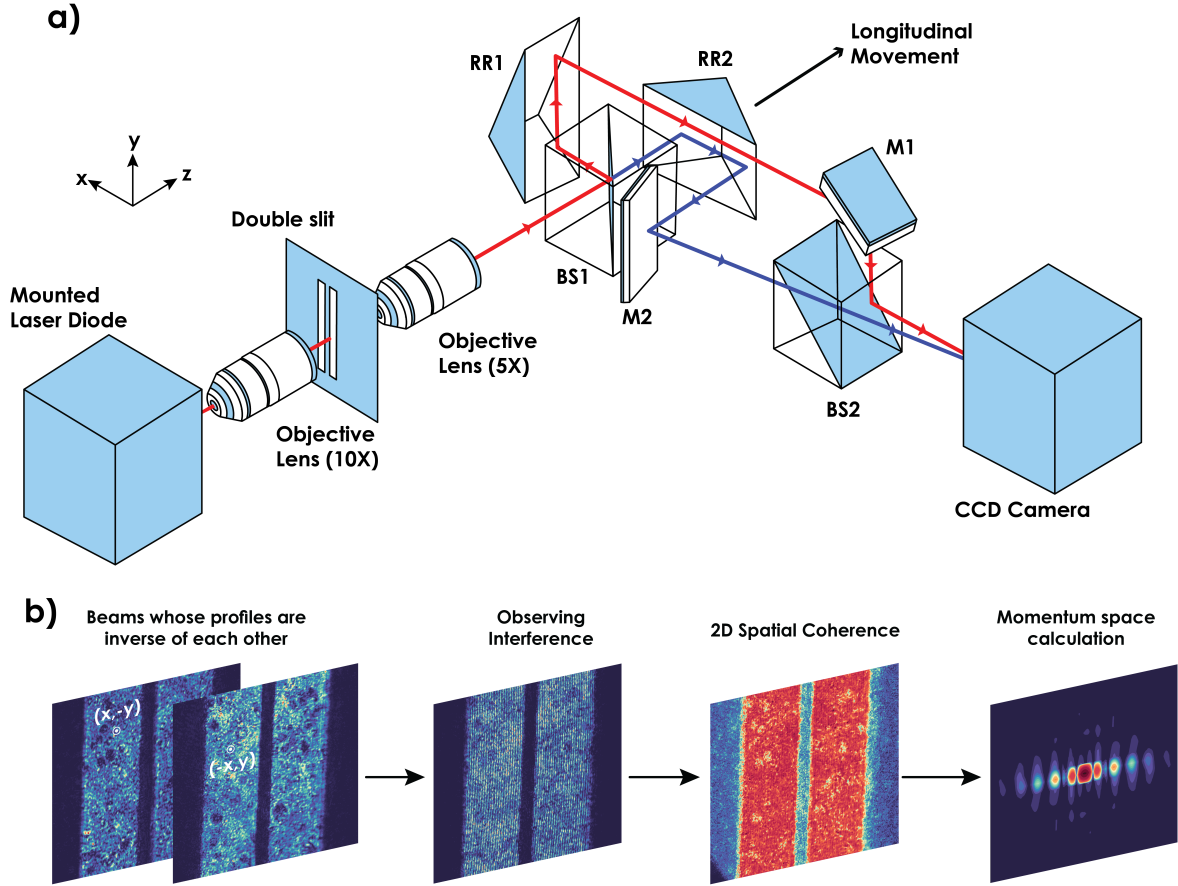


Figure 7: This image shows the modification made to the setup in case of measuring coherence function for double slit. The subplot (b) shows the procedure in obtaining the momentum space distribution

3.2.1 Coherence Measurement Using Double Slit

The light emitted from the double slit manifests as an interference pattern, which diverges as distance from the slit varies and is not collimated, particularly for the Michelson interferometer setup. Therefore, measuring the coherence function of this light directly from the slit does not yield the desired coherence function of the double slit. To address this issue, an additional collimating lens is introduced after the double slit, as depicted in Fig. 7. Placing the double slit at the focal plane of this second objective lens ensures that the image of the double slit is collimated before being sent for coherence function measurement. However, since the double slit acts as an extended source and the beam exhibits some divergence, a correction factor is employed with Fourier coordinates. This factor is calculated as the ratio of the observed slit width at the CCD to the expected slit width based on the known magnification of the objective lens, which in our experimental setup was measured to be approximately 0.46.

It's worth noting that this correction factor is only considered along one axis (y) and only the magnification of the second lens is taken into account, as the source itself is the double slit. The resulting k -space obtained from the coherence function, with background noise removed, is plotted in Fig. 10 alongside the simulation and the back focal plane for comparison. Notably, the Fourier coordinates for the optical coherence method incorporate an additional factor of 2, which will be elaborated upon in the results subsection.

3.2.2 Back Focal Plane Imaging

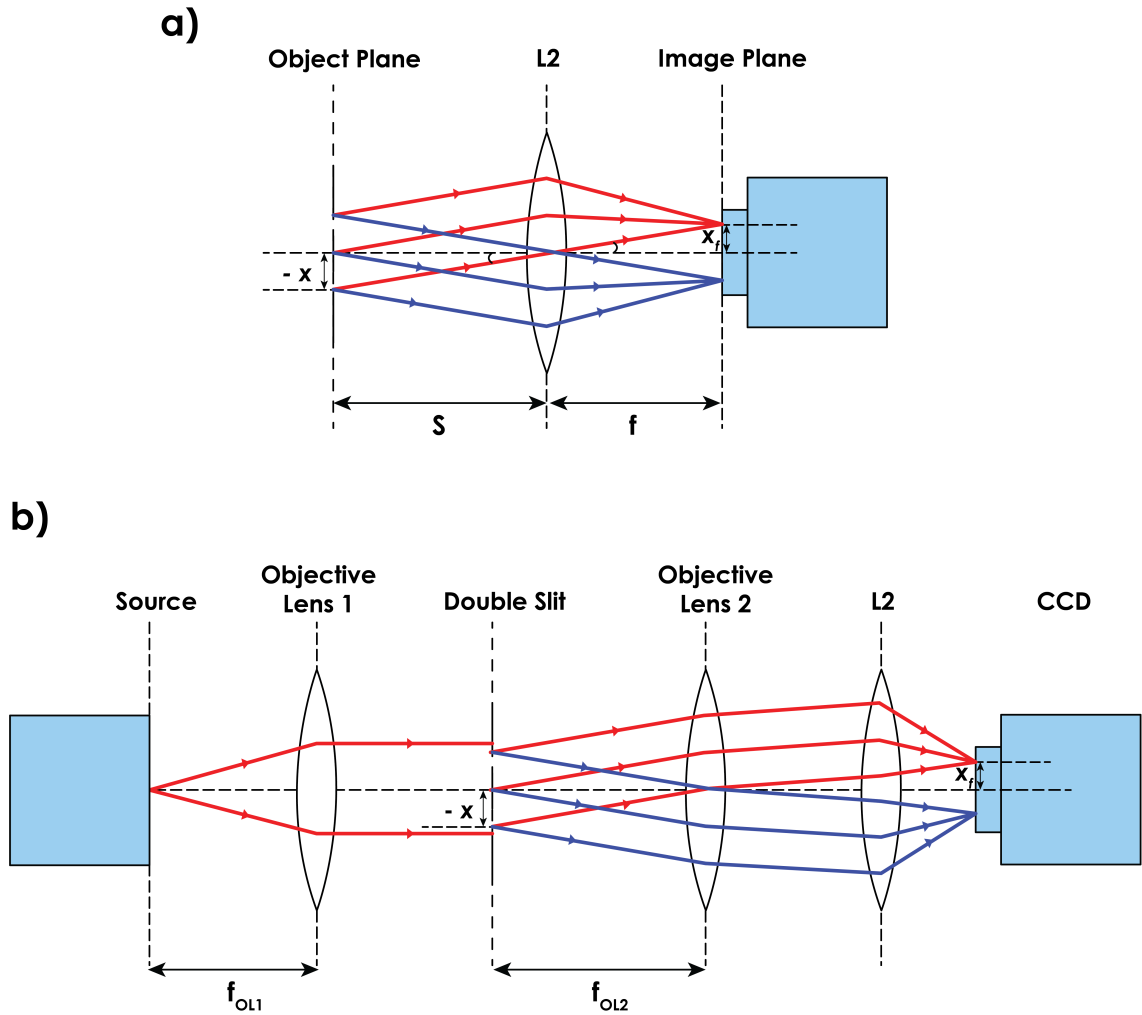


Figure 8: (a) shows the standard setup used for BFP imaging. (b) shows our arrangement for obtaining BFP.

Generally, the back focal plane imaging is done using a lens to do the Fourier transform as

in Fig. 8(a). Here the f is the focal length of the lens L2. From this figure we can tell that [30]

$$\frac{x_f}{f} = \frac{-x}{S}$$

$$x_f = \frac{-x \times f}{S} \quad (14)$$

where $\frac{-x}{S} = \alpha$ is the directional cosine and given as $\alpha = -u \times \lambda$, λ is the wavelength of given light and u is the spatial frequency. So, $x_f = -u \times \lambda \times f$.

$$u = \frac{x_f}{\lambda \times f} \quad (15)$$

The negative sign is because the lens focuses inversely, for our convention we take it to be positive. And therefore, the angular spatial frequency is given as,

$$k_{bfp} = \frac{x_f(2\pi)}{\lambda \times f} \quad (16)$$

Since we are trying to find k-space of double slit which can give rise to interference pattern, we used another objective lens whose focal plane has the double slit as shown in Fig. 8(b). As a result, its magnification is also taken into the coordinates. The double slit is an extended source so the objective lens can't collimate it and we end up with a diverging beam. This results in different slit parameters, at the plane of the lens L2. To rectify this, a correction factor is taken into the Fourier coordinates as the ratio of the observed slit width at the plane of L2 to the expected slit width for an objective lens of known magnification. However, along y direction, not the complete slit is being imaged and since the beam is also not collimated, it is difficult to calculate the expected slit height. As we are only concerned about the x -direction, this correction factor has not been used for y -direction. The obtained k-space is given in Fig. 10.

3.2.3 Simulation

We know that in case of double slit, the diffraction pattern in the Fraunhofer limit is its momentum space distribution as [13],

$$\tilde{E} = \frac{E_A e^{i(\omega t - kR)}}{R} \iint_{aperture} e^{\frac{ik(Yy + Zz)}{R}} dS \quad (17)$$

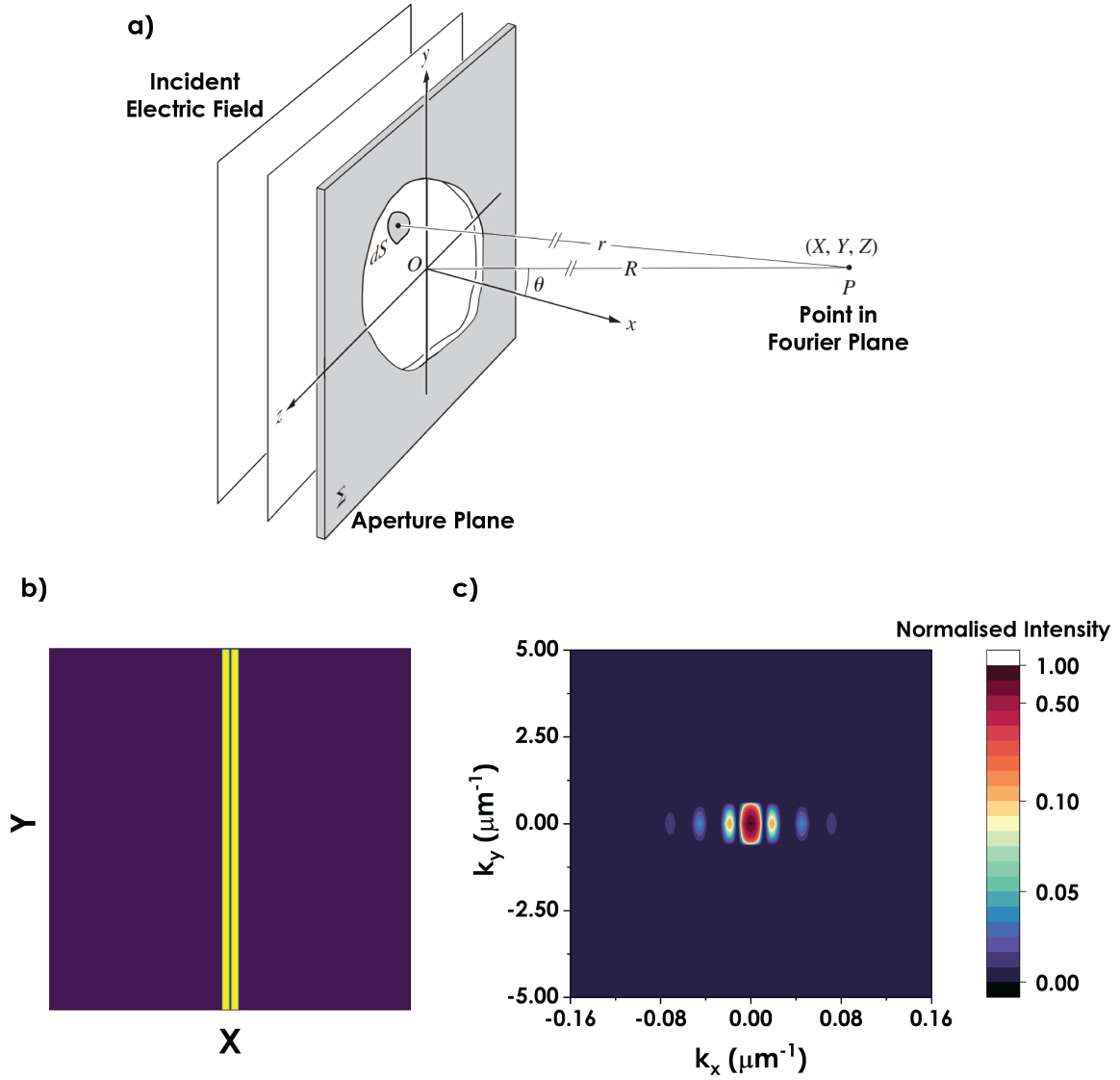


Figure 9: The image (a) shows the geometry considered for the derivation of diffraction equation Eq. 17 [13]. (b) shows the simulated double slit that is used with slit separation of 0.25mm and slit width of 0.2 mm. (c) shows the calculated momentum space distribution for the simulated slit.

where $\tilde{E}$ is the field arriving at the Fourier plane from the double slit and $\frac{E_A e^{i(\omega t - k \cdot r)}}{r}$ is the electric field incident on the slit in which $r = R \left[1 - \frac{(Yy + Zz)}{R^2} \right]$ according to the geometry given in fig 9(a). This Eq. 17 is nothing but the Fourier transform of the aperture function which in our case is a double slit. So, by measuring the Fourier transform of the double slit one can estimate its momentum space distribution. The double slit that we used is simulated in python and by FFT program its momentum space is calculated. In this case we get electric field, thus

the momentum space is calculated as,

$$F(k) = |\tilde{E}|^2$$

The simulated slit and its corresponding calculated momentum space distribution is given in Fig. 9.

3.3 Results and Discussion

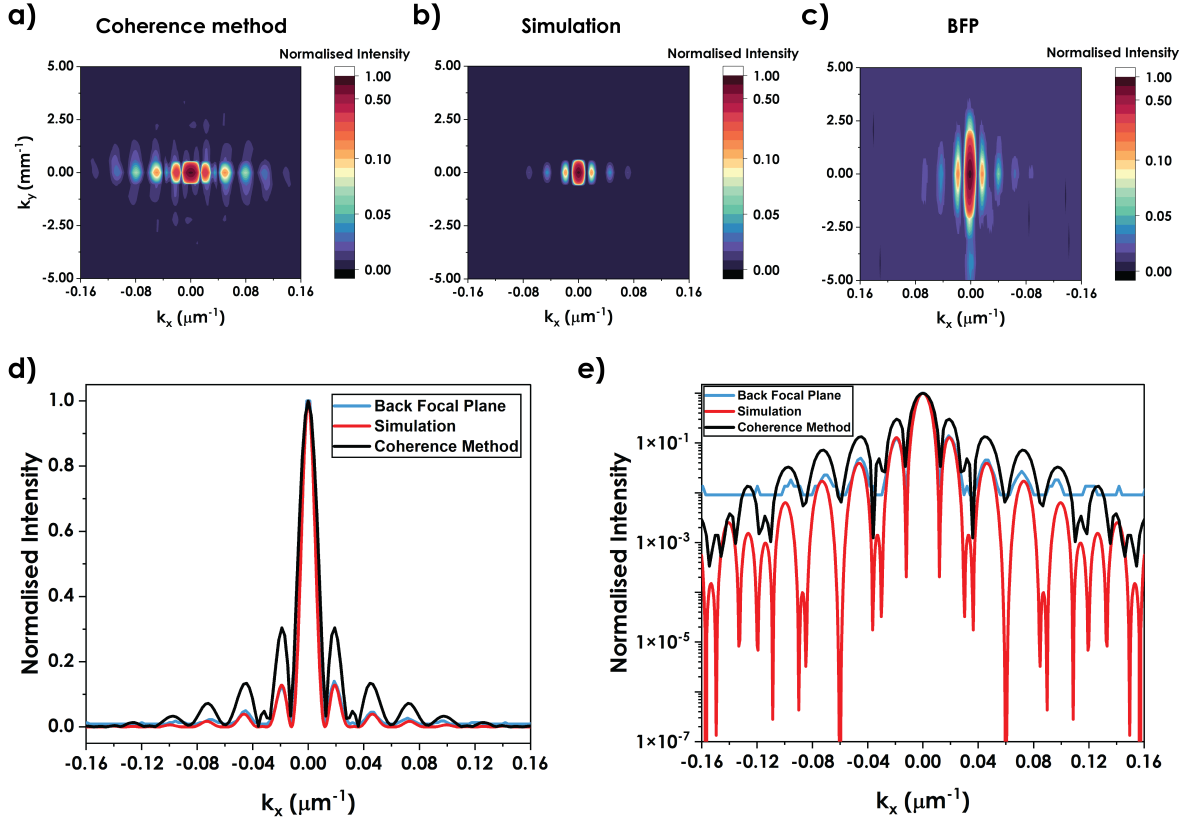


Figure 10: (a), (b), (c) shows the momentum space obtained from our coherence method, simulation and the BFP imaging respectively. (d) shows the combined plot of k_x profiles of all of the above plots for comparison. In the momentum coordinates for coherence method, we have taken a factor of 2 to match the k-space with that of the simulation.

Fig.10 displays the plot of k-space obtained from simulation, Back Focal Plane (BFP) method, and our coherence method. To compare them accurately, we needed to consider the divergence of the light beam due to the extended nature of the source. As previously mentioned for the coherence method, the coordinates are plotted with an additional factor of 2. This adjustment is made to align the k-space from the coherence function with the simulation.

The factor of 2 likely arises because when we measure the coherence function, the observed slit width is doubled compared to the original slit width, due to our method of pixel mapping where every (x, y) corresponds to $(\delta x, \delta y) = (2x, 2y)$. Thus, including this factor of 2 in the k-space coordinate cancels out the same factor in X_{max} .

Furthermore, considering beam divergence helps achieve a good match with the simulation. The BFP method also aligns well with the simulation and our coherence method after accounting for beam divergence. However, some smaller peaks in our simulation are not visible in the BFP method. This discrepancy may be attributed to the incomplete focusing of the diverging light beam on the BFP plane due to optical losses or spherical aberrations. In all three methods, the finite size along the y-axis (slit height) is not factored in, resulting in a thin pattern along k_x . However, in the BFP method, we observe a slightly extended pattern along k_y . This is because the slit image obtained in the experiment is finite for the BFP method, meaning the complete slit height is passed through the optics without loss. This is not the case for the coherence method or simulation. If this factor were included, all three methods could exhibit similar extended profiles along k_y .

The optical elements utilized in our setup are simpler compared to previously proposed setups and can be aligned more easily. The retroreflectors in our setup have no discernible obliquity other than the right angle maintained between two internal reflecting surfaces, preventing polarization-dependent phase shifts in the light beam. Additionally, our system does not employ spherical surfaces that could introduce asymmetric or non-uniform path lengths or phase differences. We have control over the additional phase introduced to the beam, which aids in imaging smaller micro/nano structures. However, the width of interference patterns needs adjustment to accommodate a significant number of fringes within the observed image to measure proper coherence function. We can also adjust the direction in which fringes are formed by manipulating either M1 or M2, optimizing them easily for further measurements of interest. The k-space calculated from the coherence function is confirmed to be the actual momentum space by comparing the results from measuring the k-space of a double slit and simulating the k-space of a double slit. We also compared the 2D k-space measured using this coherence method with the one obtained from the usual BFP imaging method using a common simulation.

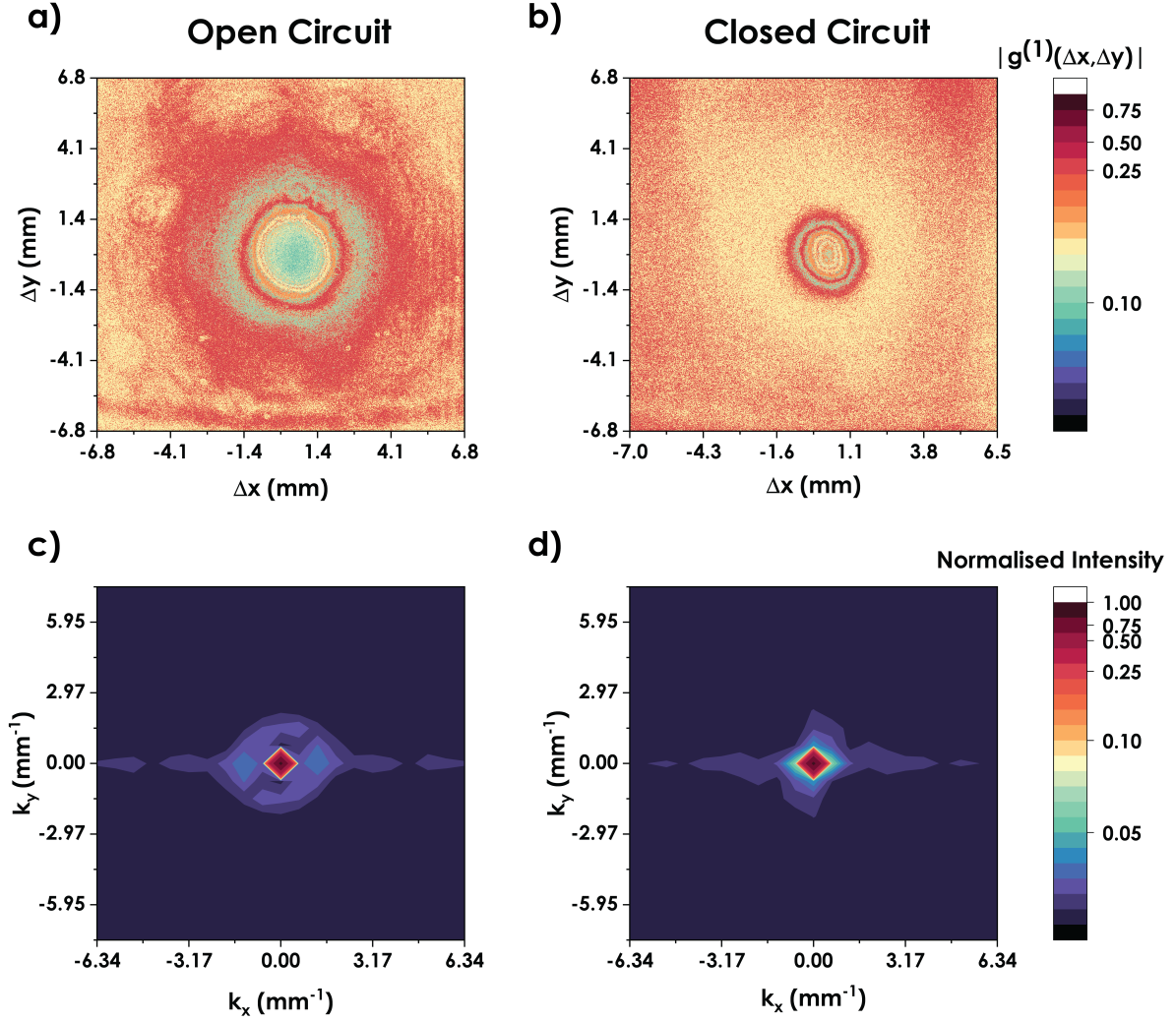


Figure 11: (a) and (b) show the obtained 2D spatial coherence for the PL and, (c) and (d) show its corresponding calculated momentum space.

Once we are confident on our interferometer that we can measure the 2D spatial coherence function and calculate the 2D momentum space distribution of a light, we then used our setup to study the PL coming out the sample. The measured coherence function and the corresponding momentum space distribution is provided in the below Fig. 11. This is measured for both the case of open and closed circuit at higher excitation intensity, since initial measurements were done by using the Thorlabs CCD and it has its limit of minimum intensity detectable. A significance difference between two cases is seen and it is further studied through analysis of the PL in the next section. As we can see the value of coherence is less for both cases. But to confirm this we studied the same with the use of Synapse EMCCD even in low excitation intensity and we didn't see any significance interference. One immediate reason is that as seen in spectra in previous section the PL is broad and ranges from 770-880 nm with the main peak has

FWHM of 10 nm and more prominent than the background as shown in Fig. 14. So, we tried two approaches – one to use a monochromator to separate the PL and measure the coherence function and the other is to resolve the obtained interferogram in the energy scale and measure the coherence function. Through both measurements we didn't see any interference and thus no significant coherence.

There could be three major reasons. One reason for this would be that even though we resolve the PL spectrally there is background contribution even at the position of the peak and the incoherent property of that can affect the coherence measurement of the peak. Another reason would be is that these interference experiments are very sensitive to vibrations. The sample holder in the cryostat, even though is nearly vibration isolated there still be small vibrations of that can affect the measurement. This problem of vibrations can be cryostat can be easily overcome by using advanced cryostat in which we are limited by. The third reason is that the PL may not be from the excitons that undergo BEC as expected. This is explored further in the next section. One important fact to note is that in III-V semiconductors the ground state is optically dark and thus the excitonic BEC we are talking about may indeed be dark and doesn't contribute to PL. Owing to the above mentioned reasons we need to find way to measure the spatial coherence functions where these problems don't pose a threat. So, we tried to use photo-capacitance measurement to see whether any matter wave interference experiments can be performed and presented in detail in the section 6.

4 Photo-Luminescence Studies

4.1 Concept and Experimental Method

The luminescence of a material means that light is emitted by it, in addition to its equilibrium Planck radiation. This is a non-equilibrium process where there is a need for external energy to drive the system to excited state [7]. When the source of that energy is light then the luminescence is called Photo-Luminescence (PL). In semiconductors upon excitation the electrons are excited to the conduction band and they lose some energy during thermalization to the minima of the conduction band. Then they recombine with the hole in the valence band that was left behind upon excitation. Now during this there can be creation of excitons and whose recombination will give peaks in PL spectra with linewidth smaller than the usual free carrier

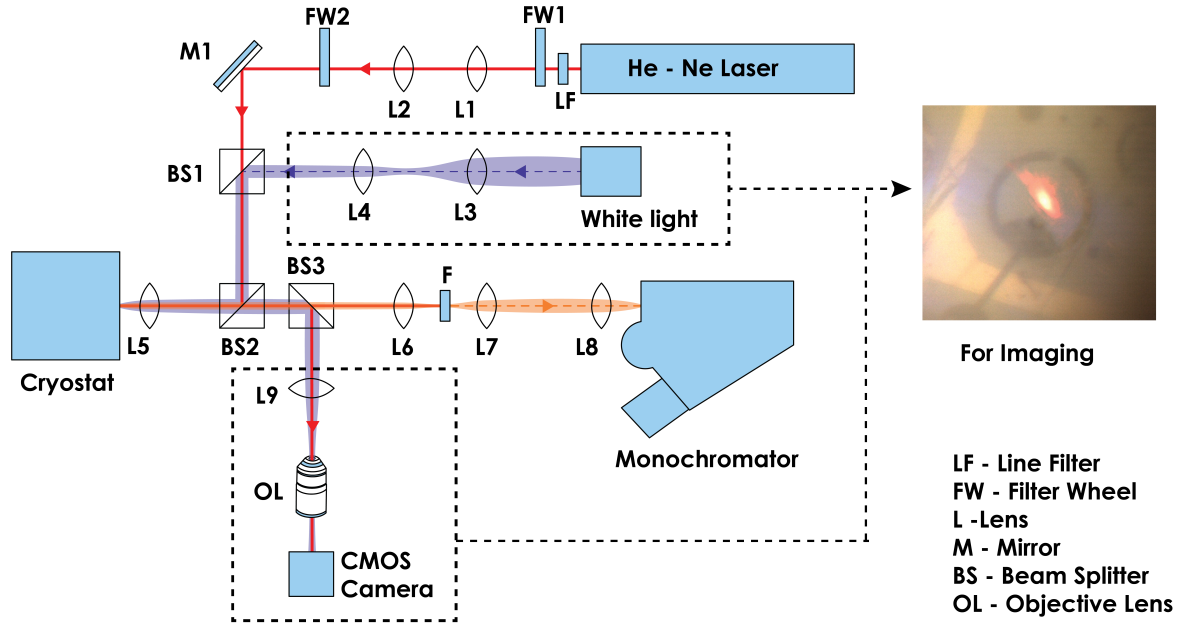


Figure 12: This image shows the schematic illustration of the photo-luminescence spectroscopy setup. The insert image shows the observed sample and the excitation spot using the microscope like setup using white light.

recombination and this linewidth will also decrease with lowering temperature.[7]

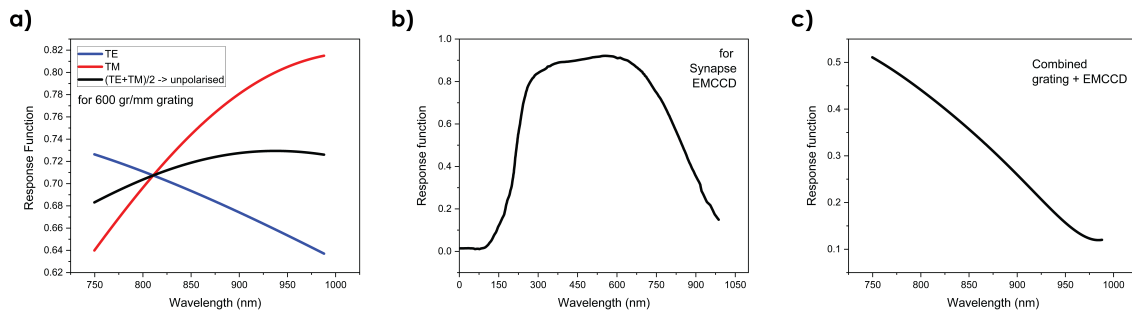


Figure 13: The plots (a) and (b) shows the calibration curves of the 600 gr/mm grating used and Synapse EMCCD respectively. The TE and TM response curves were provided from which the response function for unpolarised light is calculated as the average of the two. (c) shows the combined response function from both grating and EMCCD.

The PL spectra is measured using the Horiba iHR 320 Spectrometer with Synapse EMCCD (Electron Multiplying Charge Coupled Device). In EMCCD the charge signal is amplified before sending it to an amplifier for better sensitivity and fast detection. This helps in bypassing readout noise that is observed in normal CCDs. The arrangement for the PL spectroscopy is given in Fig. 12. The spectra are obtained for different excitation intensity and different tem-

perature. The gain and exposure of the CCD are taken into account as it is varied for different condition. All spectra are taken with Plane Mechanically Ruled Grating of 600 gr/mm with blaze wavelength (for which the efficiency is maximum) of 1500 nm. Every component in the spectrometer is made of certain material which has different response for different region of the spectrum. The function taking into account of this efficiency of a component over a spectral region is called the response function. So, once the spectra are obtained it is calibrated by the response function of the grating and the EMCCD provided by Horiba. The response function of individual components is given in Fig. 13(a) and the combined response is given in Fig. 13(b).

4.2 Results and Discussion

The spectra obtained at 10 K at sufficient intensity is given in Fig. 14(a). As seen in this figure the PL has peak of FWHM nearly 9 nm with a shoulder at lower wavelength. Before analyzing the peak, the origin of shoulder has to be explored. To explore it we moved the excitation spot outside the sample to check the contribution of PL from the substrate. From the analysis provided in Fig. 14(b) we can see that the shoulder part of PL is the contribution from the substrate and the range on the emission also includes the wavelength where the peak is observed. When the excitation spot is on the sample the intensity of peak is orders of magnitude higher than the intensity of the shoulder. So, on further analysis the shoulder part is neglected and only peaks are considered.

The obtained and calibrated spectra for different intensity are given in Fig. 15. The same is obtained for both condition of our device being in closed circuit and open circuit. As shown in Fig. 15 we can see in the condition of open circuit there is an appearance of second peak at higher intensity which vanishes in the cases of the closed circuit. The spectra are fitted with gaussian peaks and the area under the PL is calculated. The plot of integrated PL vs Intensity is fitted with power law function ($I(x) \propto x^\alpha$) [7]. The power exponent of 1 represents the excitonic characteristic of the peak, exponent 2 represent the biexciton nature and exponent of 0.5 shows the PL is from defect states. In our analysis both the peak 1 and the peak 2 is shown to be from exciton recombination as the exponent is nearly one. Even though the exponent of peak 2 is close to 1.5 since it originates in higher energy it is not from any trion or biexciton. It has also been seen that both peak increases linearly with intensity even much higher than the

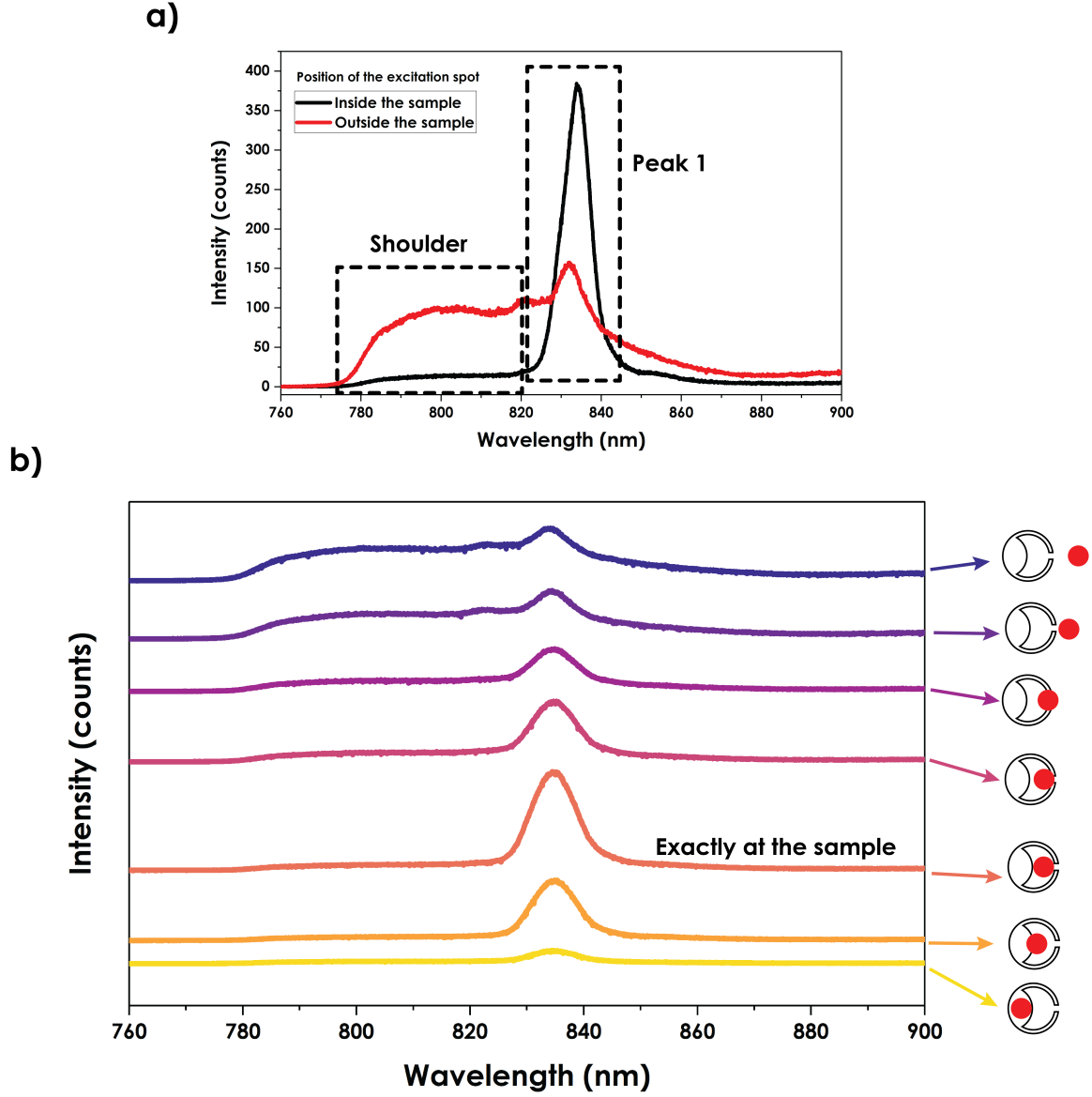


Figure 14: The image (a) shows the obtained PL spectra for excitation spot inside and outside the sample ring. The regions of shoulder and peak are highlighted in this figure. The plot (b) shows the obtained spectra when the excitation spot is moved across the sample.

values mentioned here.

The spectra are also obtained for varying temperature for both case of low and high intensity where two peaks are not seen and seen respectively and the same is given in Fig. 16. Also, the same is observed for both case of open and closed circuit. The integrated PL obtained from fitting gaussian peaks vs temperature for every case is fitted with Arrhenius equation [31,32],

$$I \approx \frac{I_0}{1 + \gamma e^{\frac{\Delta E}{k_B T}}} \quad (18)$$

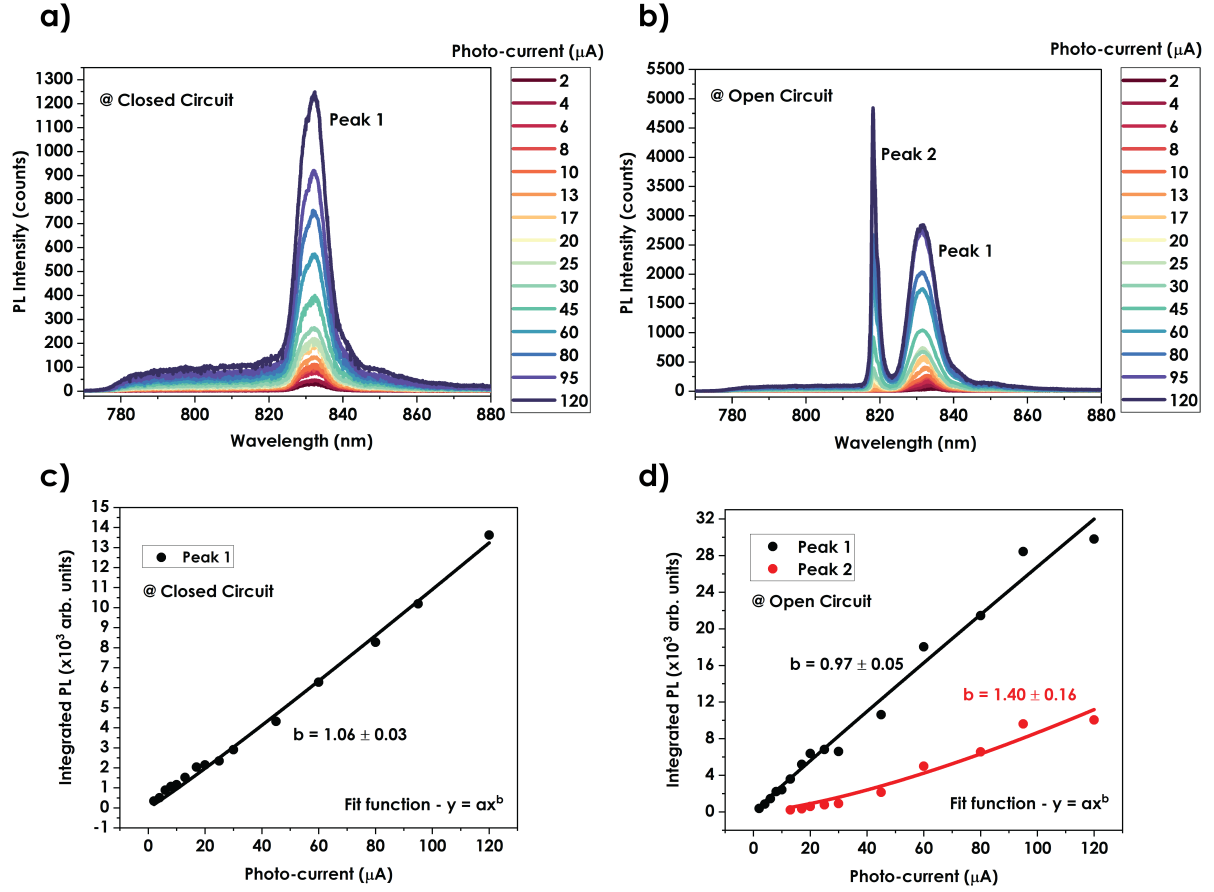


Figure 15: (a) and (b) show the observed photo-luminescence spectra for increasing excitation intensity for case of closed circuit and open circuit respectively. The emergence of Peak 2 at high intensity can be seen in plot (b). (c) and (d) show the power law analysis for both the cases.

where $\gamma = \frac{\tau_r}{\tau_{nr0}}$ represents the ratio of radiative recombination to the non-radiative recombination rate and ΔE represents the barrier height required for undergoing non-radiative recombination, in Fig. 17(c) and (d). This will tell us about the activation energy of any thermally activated process. The peak position is plotted as a function of temperature in Fig. 17(a) and (b) and it is fitted with Varshini equation [33,34] which tells us how bandgap increases with

DC Photo-current	Case	Peak no.	$E_g(0)$ (eV)	$\alpha (\times 10^{-4} \text{ eV/K})$	β (K)
100 μA	Open Circuit	Peak 1	1.493 ± 0.001	$8.86 \times 10^9 \pm 3.54 \times 10^{16}$	$1.41 \times 10^{12} \pm 5.66 \times 10^{18}$
	Closed Circuit	Peak 1	1.5164 ± 0.0004	8.23 ± 1.01	501.81 ± 92.46
5 μA	Open Circuit	Peak 1	1.492 ± 0.001	20.1 ± 23.5	2164.8 ± 2802.3
	Closed Circuit	Peak 1	1.492 ± 0.001	$3.65 \times 10^8 \pm 5.03 \times 10^{10}$	$4.49 \times 10^{10} \pm 6.22 \times 10^{16}$
				175.6 ± 1803.8	19938.95 ± 208125.54

Table 1: Fit parameters from Varshini equation.

Figure 17(a) and (b) show the observed photo-luminescence spectra for increasing excitation intensity for case of closed circuit and open circuit respectively. The emergence of Peak 2 at high intensity can be seen in plot (b). (c) and (d) show the power law analysis for both the cases.

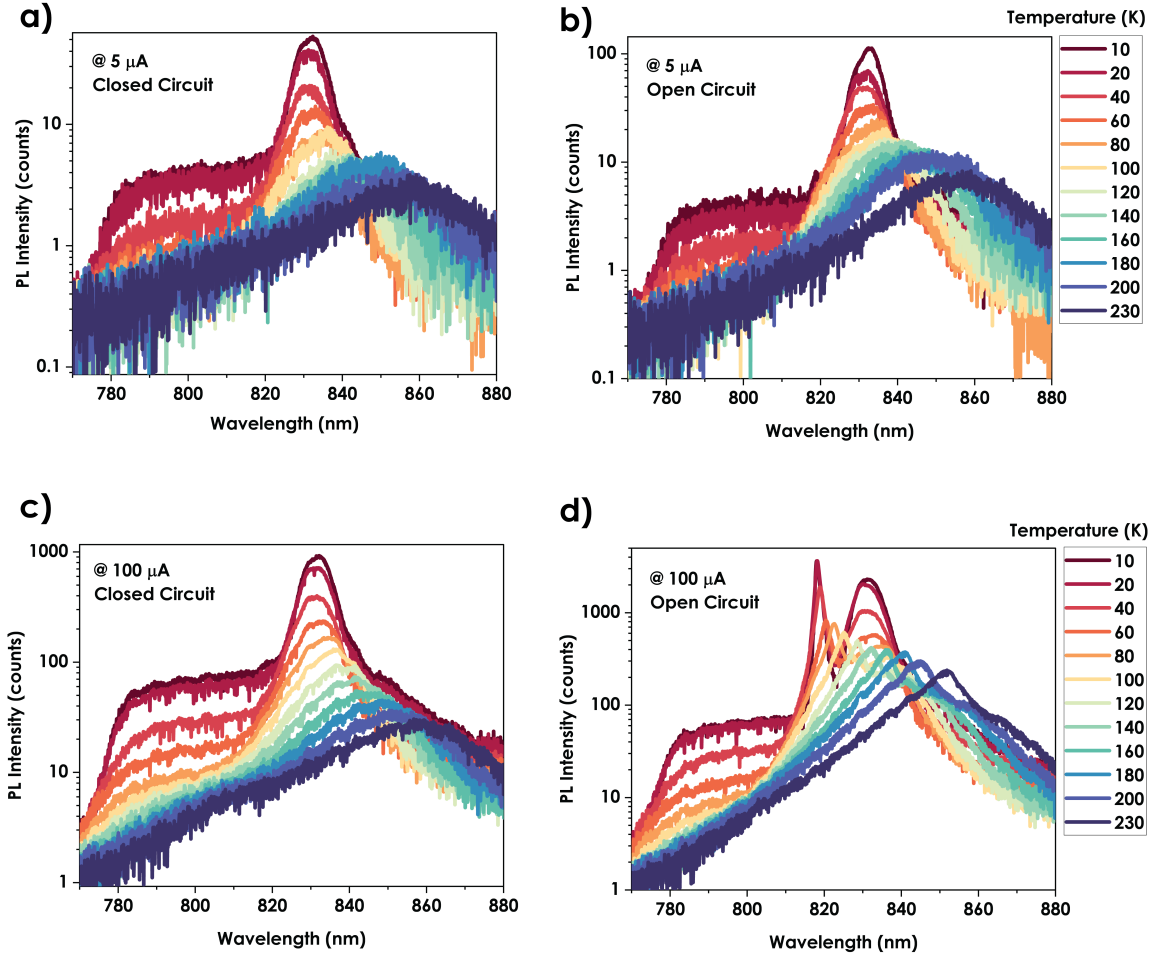


Figure 16: (a) and (b) show the PL spectra observed for the photo-current of $5 \mu\text{A}$ for case of closed and open circuit with temperature. (c) and (d) show the similar spectra for the photo-current of $100 \mu\text{A}$ for case of closed and open circuit with temperature.

DC Photo-current	Case	Peak no.	γ	ΔE (meV)
$100 \mu\text{A}$	Open Circuit	Peak 1	7.16 ± 6.89	7.51 ± 2.62
		Peak 2	18.05 ± 32.03	13.70 ± 6.87
	Closed Circuit	Peak 1	1.15 ± 0.34	4.14 ± 0.38
$5 \mu\text{A}$	Open Circuit	Peak 1	0.026 ± 0.001	1.40 ± 0.12
	Closed Circuit		3.44 ± 1.44	4.7 ± 0.7

Table 2: Fit parameters from Arrhenius equation.

lowering temperature and is given as,

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{\beta + T} \quad (19)$$

where α and β are Varshni's fitting parameters and $E_g(0)$ is the bandgap at 0 K. Even though our obtained PL will have energy lower than the bandgap, since it is excitonic in origin, we

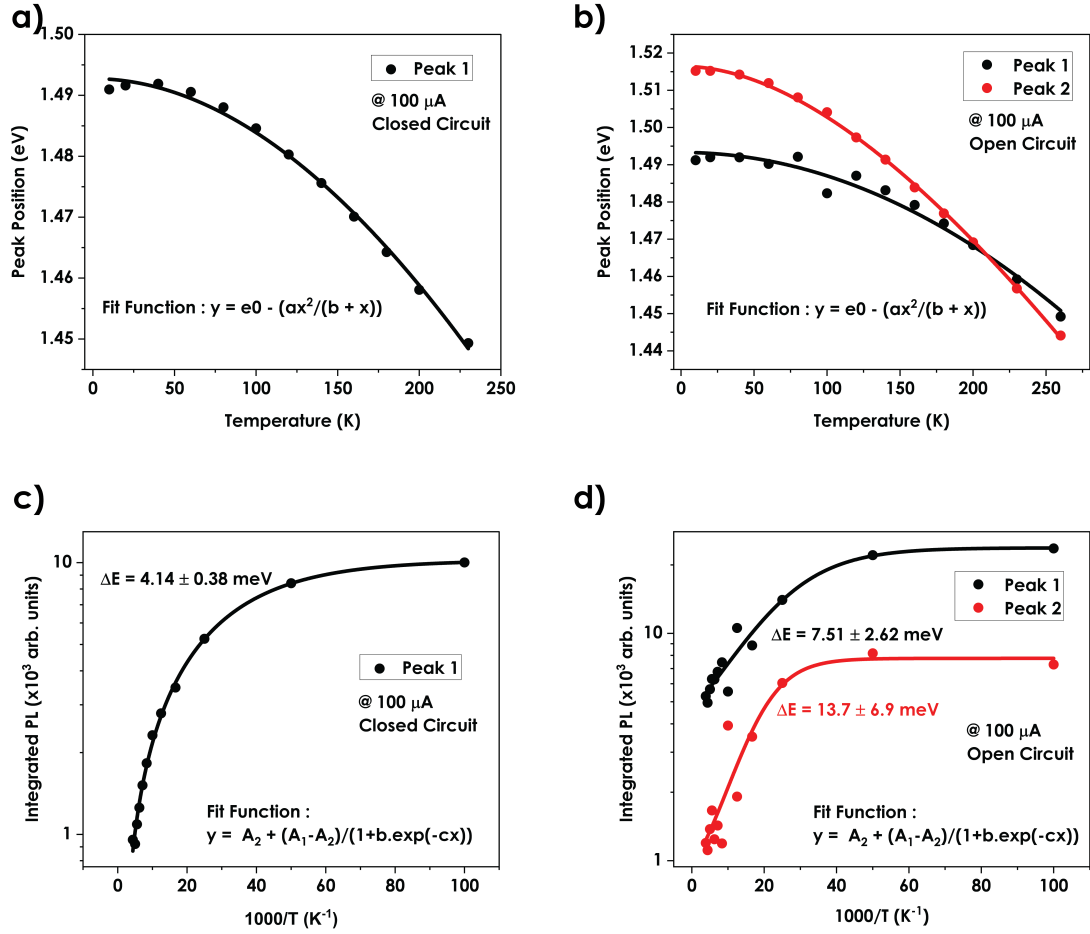


Figure 17: (a) and (b) show the analysis of the peak position with temperature by fitting with Arrhenius equation for high excitation intensity in case of closed and open circuit respectively. (c) and (d) show the analysis of the integrated PL with temperature by fitting with Varshni equation for the same high excitation intensity in case of closed and open circuit respectively.

expect it to be proportional to the bandgap at any temperature. We can see that for both cases the emission energy decreases with temperature and this is general feature for semiconductor. It says that with lowering temperature energy required for excitation will increase and thus increase in the bandgap. The fit parameters of peak 2 match with the previous studied fit parameters of InAs QDs [34]. So, the appearance of second peak at higher intensity is argued as the result of penetration of the light into the InAs QD layer and exciting carriers inside it directly and since its energy barrier, as seen from Table 2, is high and thus the recombination in QD is preferred. Now, for the case of difference between the open and closed circuit, upon looking at the obtained bandgap in relative scale given in Table 1, the bandgap of InAs emission is more than that of the excitonic PL. So, when the device is in closed circuit then there will be a tunneling or conduction pathway for the electrons generated in InAs QDs to flow and

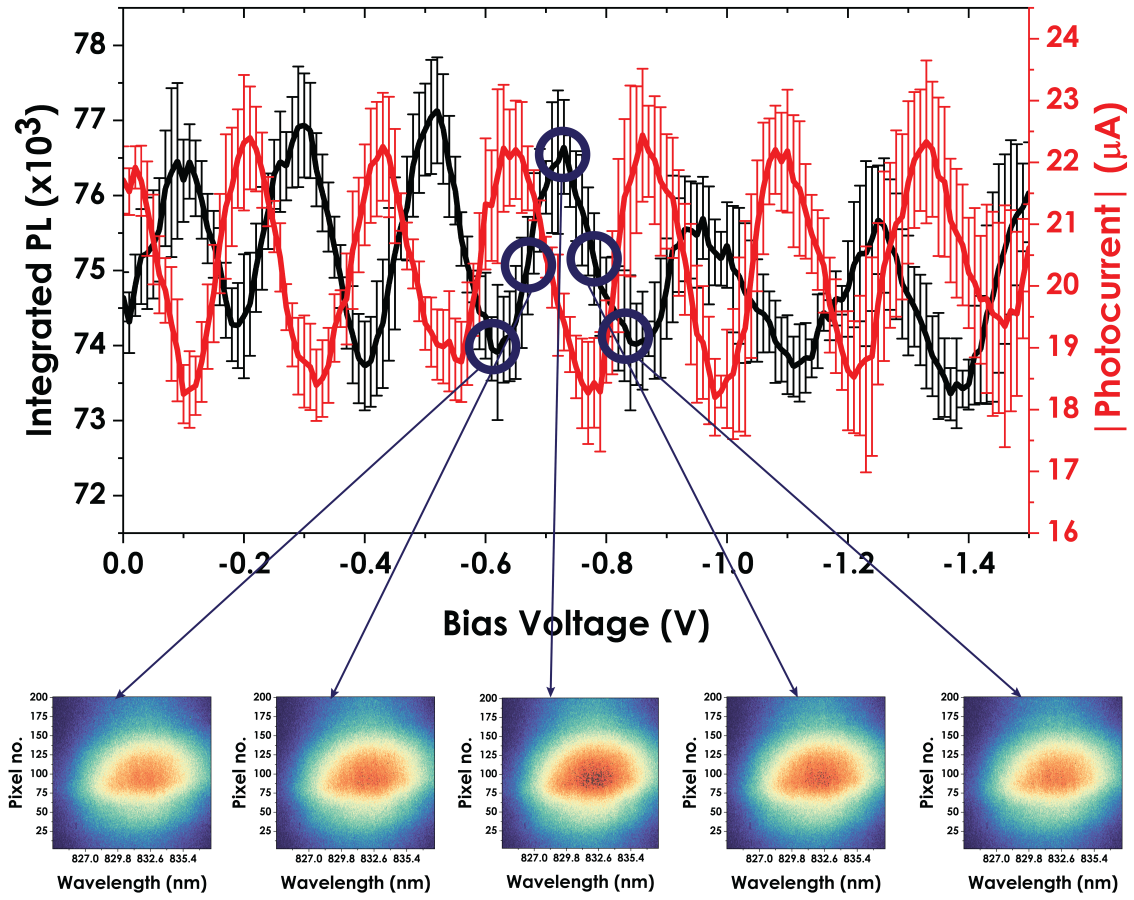


Figure 18: The image shows the observed Integrated PL oscillation with voltage which is observed to be 180° out of phase with the photo-current oscillation. The insert images show the spatial PL spectra observed at different voltages where the oscillation in PL intensity can be seen.

thus quenches the PL completely. Apart from this the PL is also studied with varying bias and shown in Fig. 18. Upon analysis we noticed that the integrated PL oscillates with varying voltage and this oscillation is 180° out of phase with the photo-current oscillation and 90° out of phase with the photo-capacitance oscillation. So, it is inversely correlated with the photo-current and this can be interpreted as the when the photocurrent is high, there will be flow of carriers and there will be less carriers for recombination. Similarly, when there are less carriers flowing there will be more carriers recombining. This is the result of confinement of charge carriers in the triangular quantum wells in our structure. When the resonant condition is met, there will be more carriers tunnelling through and thus less recombination and when it is not then less carriers tunnelling and thus more carriers will recombine. So, all these results suggests that the observed PL is not related to the indirect excitons that undergo BEC transition

and this could result in the observation of low coherence using our interferometer setup. This can also suggest that our indirect excitons forming BEC may be in dark state. One can try making them bright through various methods like use of z-polarized (along the direction of propagation) light for the optical study of spatial coherence. In our setup since our sample is not an isolated single sample and kept inside the cryostat, it will be difficult for the realization of such polarization. So, we tried to use C-V measurements to study our sample and explore the possibility of measuring any spatial coherence.

5 Photo-Capacitance Studies

5.1 Concept and Experimental Method

Capacitance is the measure of the storage of charges between two electrodes where the energy will be stored in form of electric fields. Conventional macroscopic capacitors are made of two parallel plates where the capacitance is given by $C = \frac{\epsilon_0 \epsilon_r A}{D}$ where $\epsilon = \epsilon_0 \epsilon_r$ is the dielectric constant of the material in between the plates, A is the area of the plates and D is the distance between the plates. When the size of such capacitive devices approaches the scale of nm, quantum mechanical process like exchange and correlation effects become more relevant. One such small device is a p-n junction where upon reverse bias the p and n side acts as a plate and charges are stored inside the depletion region. For cases where quantum effects matter, the capacitance is written as [35,36],

$$\frac{1}{C_{tot}} = \frac{1}{C_{geo}} + \frac{1}{C_{kin}} + \frac{1}{C_{ex}} + \frac{1}{C_{cor}} + \frac{1}{C_{el-ncc}} \quad (20)$$

In this Eq. 20 C_{geo} is the usual geometric capacitance in conventional capacitors, C_{kin} is the contribution due to kinetic energy of the carriers, C_{ex} is from exchange effects, C_{cor} is from correlation effects and C_{el-ncc} represents the contribution of the interaction of electron and non-charge carriers like phonons. The correlation contribution to quantum capacitance have been studied for our sample in [16] previously. In this study since we are only interested in studying the excitons that are created upon illumination, we model it as,

$$C_{total} = C_{ph} + C_{dark}$$

$$\Delta C_{ph} = C_{measured} - C_{dark} \quad (21)$$

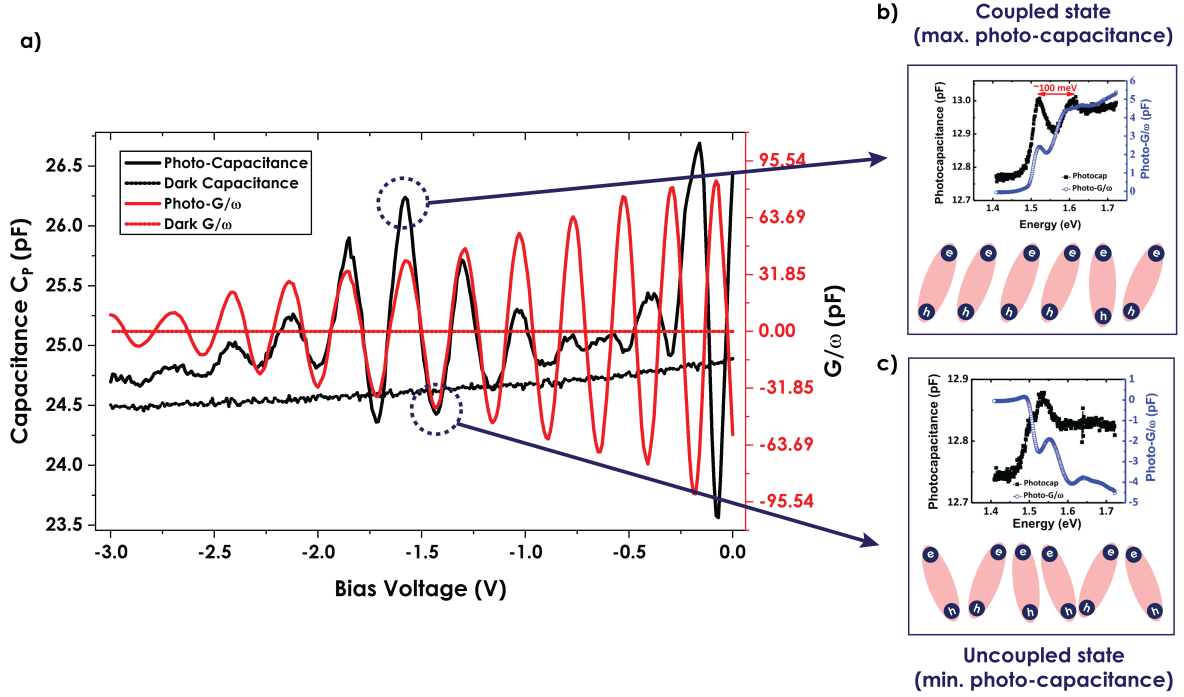


Figure 19: The image (a) shows the oscillation in photo-capacitance measurement with varying voltage for excitation wavelength of 632.8 nm in smaller sample of diameter 200 μm . The insert images (b) and (c) show the previously measured photo-capacitance spectra for the voltage corresponding to maxima and minima respectively.

So, from the measured capacitance we subtract the dark capacitance which usually accounts for any geometric contributions and the resultant capacitance represents only photoinduced effects. We measure this photo-capacitance as a parallel component using LCR Meter E4980A from Agilent. The capacitance is measured as a parallel component because any diode is modelled as a capacitor and a resistor in parallel. The LCR meter measures capacitance by measuring the impedance of the device by automatic bridge balance method. While measuring Photo-capacitance we also measure photo-conductance and photo-current. The photo-conductance(G) gives us $\frac{dI}{dV}$ whereas the photo-current gives us the DC component of the current through our device. From photo-conductance we calculate a quantity G/ω where $\omega = 2\pi\nu$, ν is the AC frequency used to measure. This quantity has units of F and used as a quality check of our capacitance data. If G/ω is more than 10 times the $C_{measured}$ then it means that our carriers are more conductive and the capacitive data will be noisy and also cannot be trusted. There is formation of indirect excitons in our sample and the density of such indirect excitons can be

calculated from the surface charge density created upon illumination ($n \approx \sigma_{ph}$). We know that $Q = CV$ so [16],

$$\sigma_{ph}e = CV = \langle \vec{P} \rangle \cdot \hat{z} \quad (22)$$

where e is electron charge, C is capacitance per unit area and $\langle \vec{P} \rangle$ is the average polarization of the dipoles in the material. But the calculated density will consist of excitons that responded to particular frequency of measurement. Nevertheless, this equation tells us that the photo-capacitance measured can be used to quantify the average orientation of indirect excitonic dipoles. Any enhancement in the capacitance is regarded as the polarization of the dipoles along particular direction and a decrease in capacitance tells us that the average polarization is reduced. Since this capacitance measurement respond only to dipolar orientation it won't matter if these dipoles i.e., excitons are either bright or dark. Thus, is responsive to both the bright and dark excitons almost equally. Also, since the charge carriers created are proportional to the intensity of the excitation light, in the further measurements the photo-current at zero bias is taken as the measure of intensity.

5.2 Results and Discussion

The oscillations seen in C-V measurement is shown in the Fig. 19(a). It has been shown in previous study [16] that the maxima of these oscillations correspond to a coupled state of excitons whereas the minima correspond to an uncoupled state. This has been shown through the study of photo-capacitance spectroscopy where capacitance is measured for varying incident wavelength. The results of such study is shown in Fig. 19(b) and (c). Also, the quantum capacitance measured in such oscillation show that at the maxima of these oscillation there is strong correlation which is absent in minima [16]. These results show that the at maxima the indirect excitons are coupled to one another and they orient themselves more along z direction whereas that is not the case for minima. These oscillations are only possible if there is a 'coherent' resonant tunneling process in our structure since this is a measurement done in a large area and also if not coherent these oscillations will vanish. For a coherent phenomenon, there should be momentum narrowing along the x - y direction. This tells us that on applying voltage these indirect excitons are being driven towards and out of the BEC where momentum narrowing is expected. These oscillations are observed for both samples of diameter 200 μm and 400 μm . The result of oscillations for bigger spot is given in Fig. 20(a). These oscillations were studied

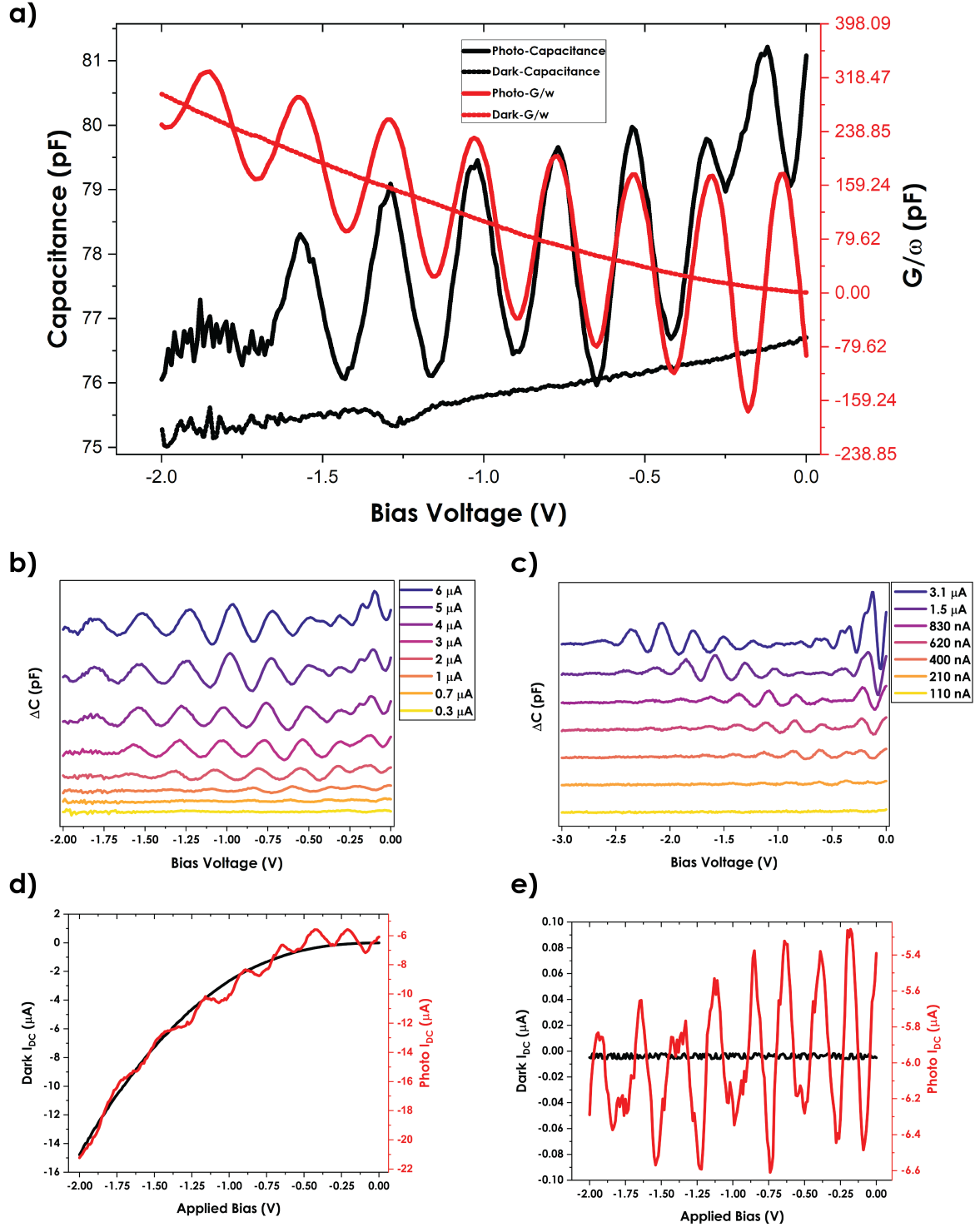


Figure 20: The plot (a) shows the C-V oscillation observed for the bigger sample of diameter 400 μm for the excitation wavelength of 632.8 nm. (b) and (c) show the measured C-V oscillation on varying the intensity of excitation light for bigger sample and smaller sample respectively. (d) and (e) show the I-V measurement done for big and small sample respectively with $I_{DC} = 6 \mu A$ at 0 V

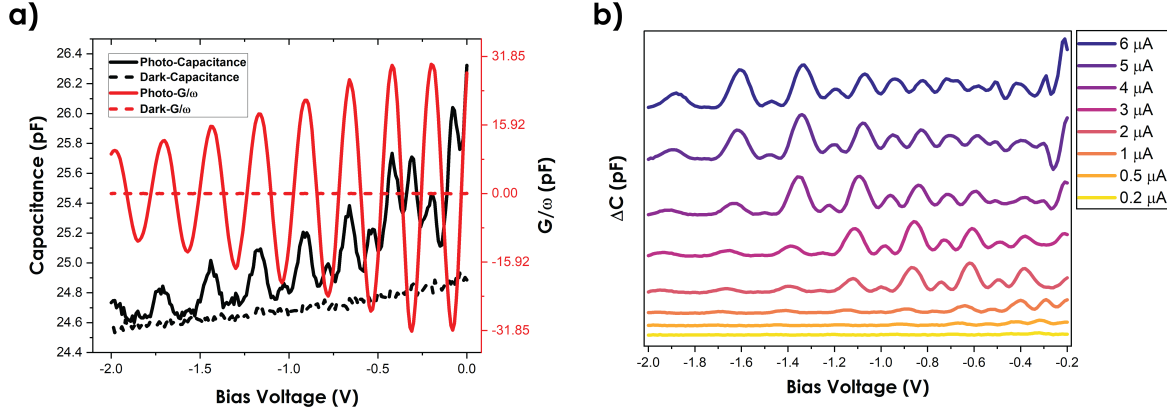


Figure 21: (a) shows the C-V and G/ω oscillation observed for the smaller sample for the excitation wavelength of 488 nm. (b) shows the same observed for varying excitation intensity.

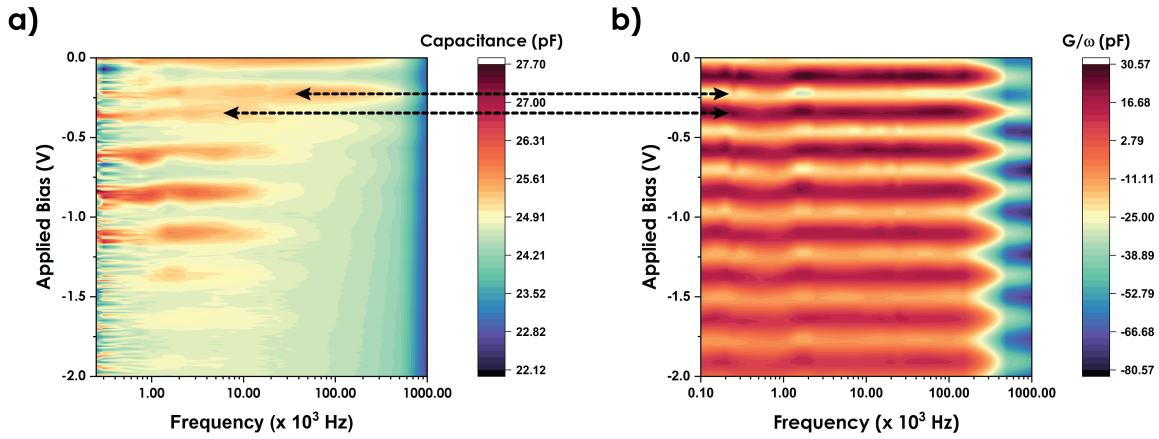


Figure 22: (a) shows the C-V oscillations probed for varying frequency for DC photo-current of 0.5 μA in the smaller sample and (b) shows the G/ω oscillation observed for the same. The capacitance peak seen at the conductance minima is seen for broader frequency range and the peak at the conductance maxima is seen for narrower frequency range.

for different light intensity (given in Fig. 20(b) and (c)) and these show that the observed oscillations are seen above certain intensity and this threshold intensity is more for bigger sample. The bigger the sample leakier it is as seen evidently from Fig. 20(d) and (e), more carriers are required to surpass this loss and thus the threshold is pushed to higher intensity.

All the previous oscillations are studied with excitation wavelength of 632.8 nm from He-Ne Laser. Same studies have been carried out with Ar^+ ion laser with main emission line at 488 nm and one of the measured oscillations is presented in Fig. 21. Here we can see that there is also a peak in capacitance at the minima of these oscillations which were not previously seen. The reason for this has to be explored but this was done to help us with the study done

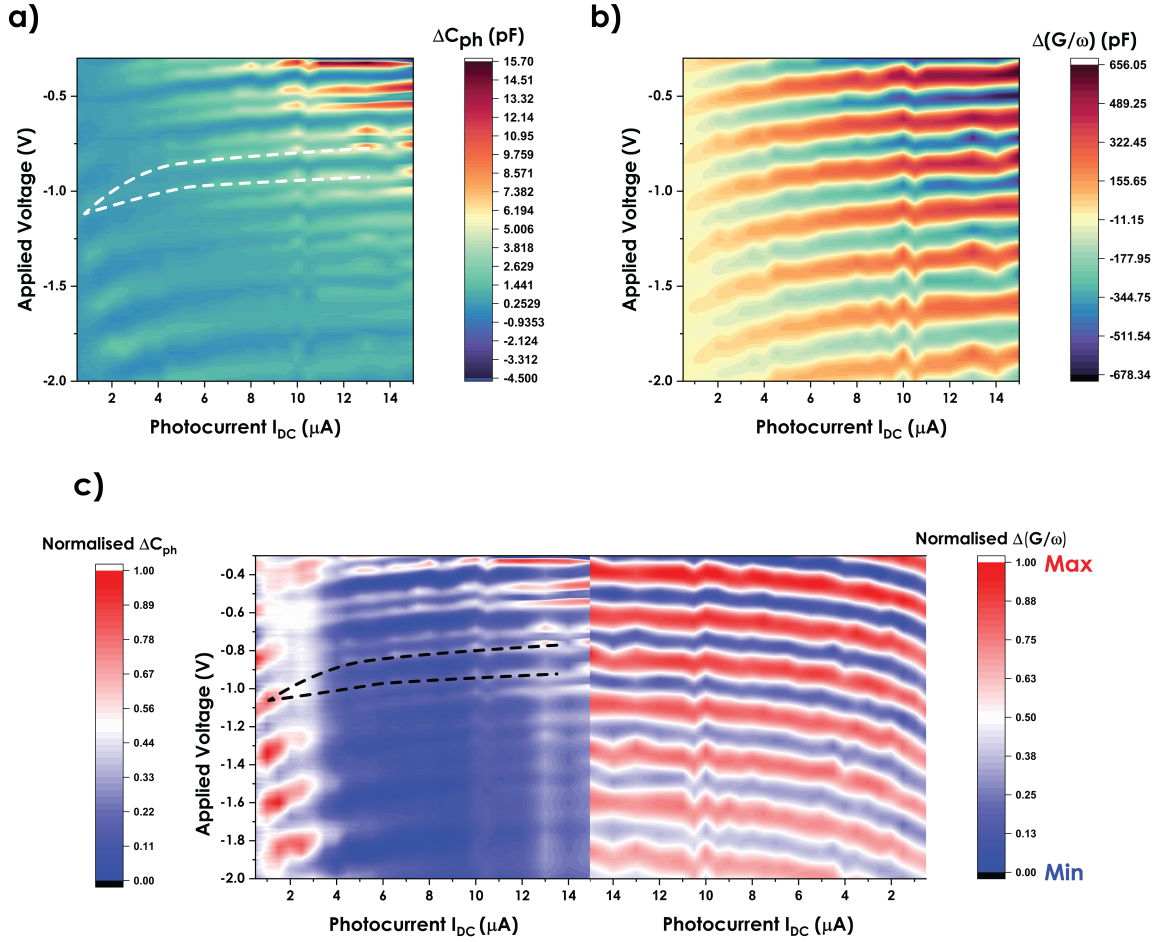


Figure 23: The image (a) shows the observed ΔC_{ph} oscillation with varying intensity to higher photo-currents of 15 μA where the splitting of capacitance peaks about maxima is seen and marked with white dotted lines. (b) shows the observed $\Delta(G/\omega)$ oscillations for the same case but devoid of any splitting. (c) shows the same in the normalized scale for clear visibility of the splitting.

in section 6. Further we have also studied the nature of these capacitance oscillations with varying the frequency given in Fig. 22. As we can observe that the capacitance peak seen at the conductance minima is seen for broader frequency range and the peak at the conductance maxima is seen for narrower frequency range. This if we translate into timescale, we can say that the species responsible for any peak in capacitance at conductance minima are short lived in the order of few μs and the species responsible for peak at conductance maxim are long lived in the order of few hundreds of μs . Such short-lived species of excitons are observed more in case of 488 nm excitation as compared to 632.8 excitation. But such analysis has to be explored further. One another interesting result is that higher intensity there is appearance of split peaks around maxima of capacitance oscillation as shown in Fig. 23. This appearance of two new peaks in capacitance can be thought to be similar to band gap opening observed during

Bardeen–Cooper–Schrieffer (BCS) transition upon studying superconductivity [37]. Such similar observation motivates us further to study our sample in detail and more in dept studies are required to study such BEC-BCS crossover which is not of our interest in this project.

6 Matter-Wave Interference Studies

6.1 Concept and Experimental Method

In our coherence method as stated in section 3 we have difficulties with measuring optical spatial coherence function for our sample. Now to study such spatial coherence we have one another way and that is through matter wave interference [38,39]. Our excitons are involved in the formation of BEC which is a coherent quantum state as discussed in Introduction. Similar to interference of classical electric fields,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Delta\phi)$$

when we have two quantum states ψ_A and ψ_B then they can also interfere resulting in interference.

$$\psi = \psi_A + \psi_B$$

$$|\psi|^2 = (\psi_A + \psi_B) \cdot (\psi_A^* + \psi_B^*)$$

$$|\psi|^2 = |\psi_A|^2 + |\psi_B|^2 + \langle \psi_A \cdot \psi_B^* \rangle + \langle \psi_A^* \cdot \psi_B \rangle$$

$$|\psi|^2 = |\psi_A|^2 + |\psi_B|^2 + 2|\langle \psi_A \cdot \psi_B^* \rangle| \cos(\Delta\phi) \quad (23)$$

Similarly, we tried generating population of excitonic BEC and tried to observe interference effects in C-V measurements. This is done by using two lasers (He-Ne laser (Red excitation spot) and Ar⁺ ion laser (Blue excitation spot)) to create two different excitonic population. The illustration for this setup is given in Fig. 24. We also tried to observe this interference effects with varying the separation between the excitation spots and see whether any equivalent of spatial coherence function can be estimated as shown in Fig. 25(a).

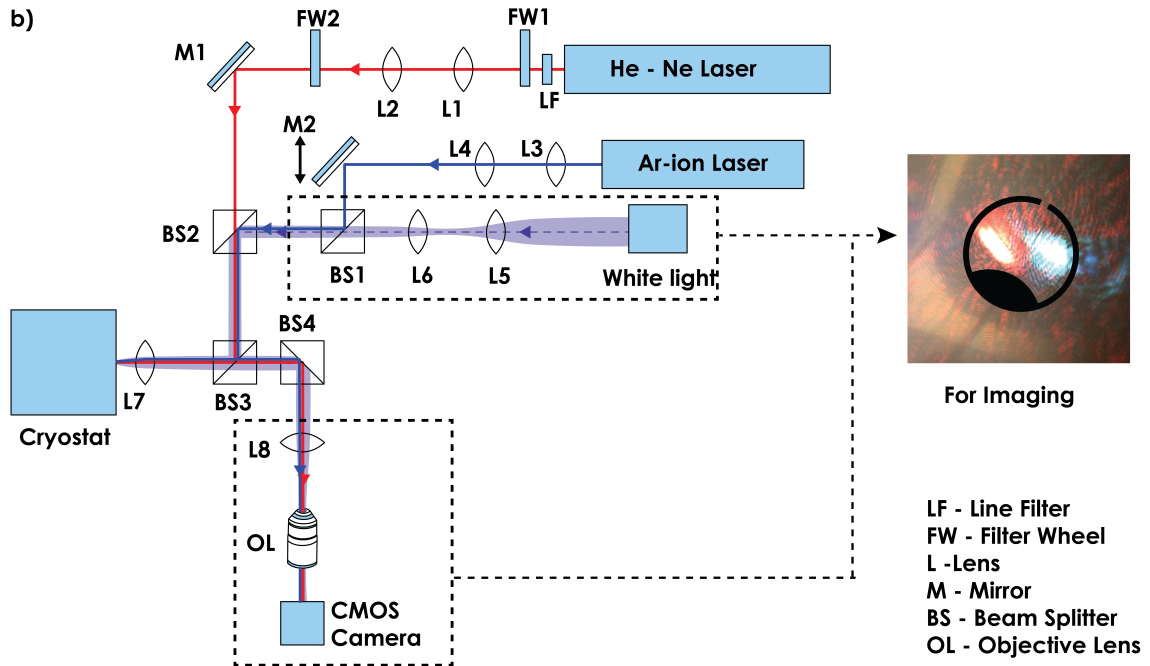
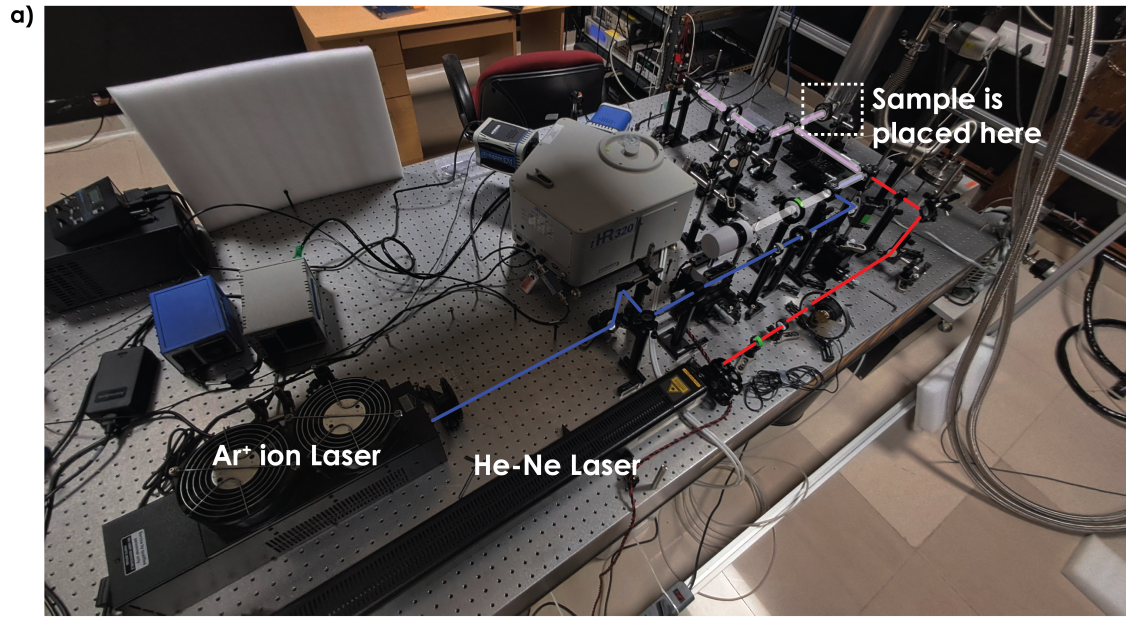


Figure 24: This image shows the schematic illustration of the setup used to measure the matter wave interference using two lasers. An insert image of sample with two excitation spots is also given.

6.2 Results and Discussion

The Fig. 26 shows the oscillations from individual excitations and also the oscillation obtained upon simultaneous excitation is also shown. When two lasers are used there is a beat formation

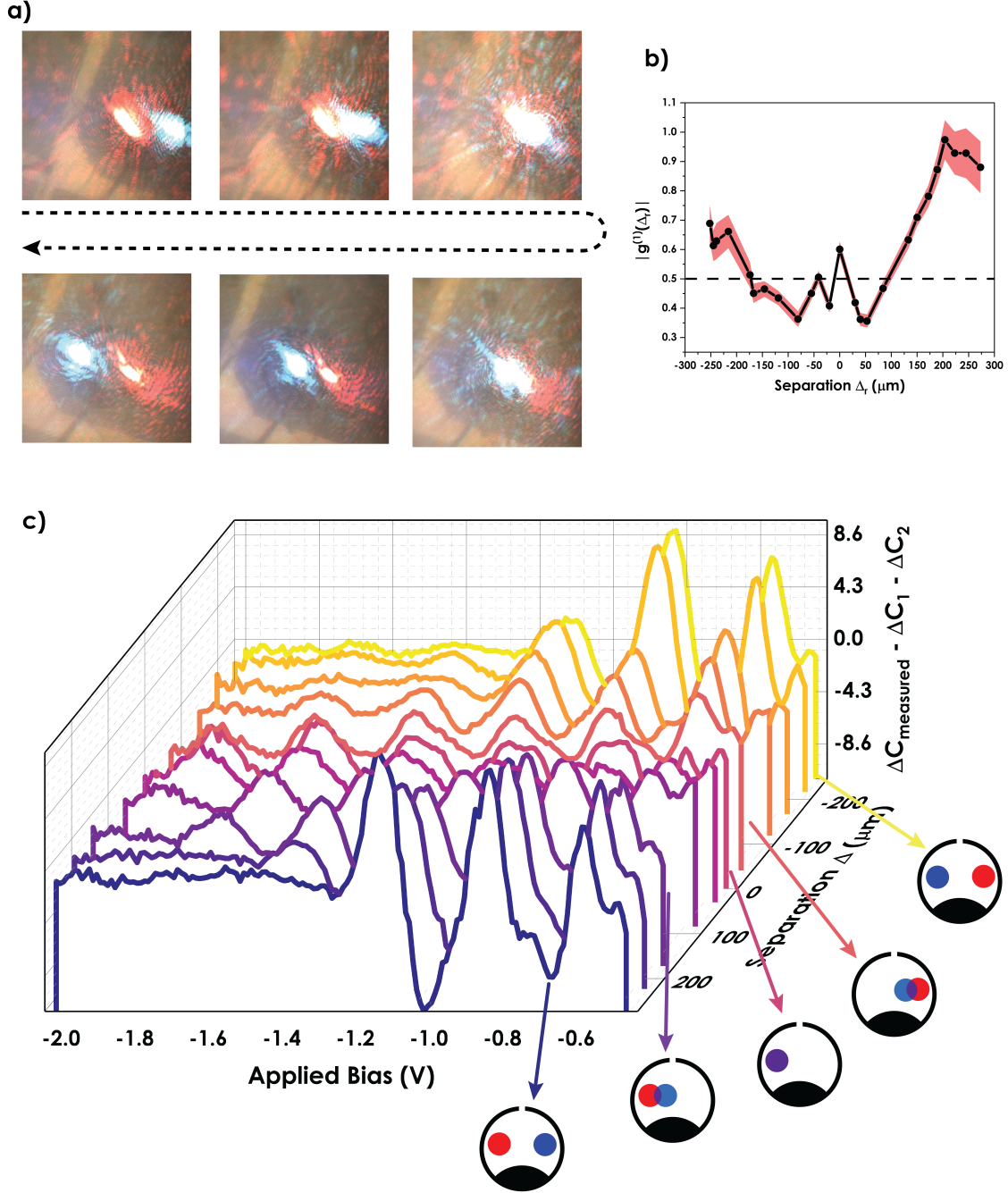


Figure 25: (a) shows how the separation between two spots is varied. (c) shows the observed beat formation in C-V oscillation with different separation in the bigger sample and (b) shows the plot of visibility calculated for varying separation.

in the C-V measurement. As discussed earlier in section 3,

$$\Delta C_{ph} \propto n_{ex} \text{ and } n_{ex} \propto I \Rightarrow \Delta C_{ph} \propto I \quad (24)$$

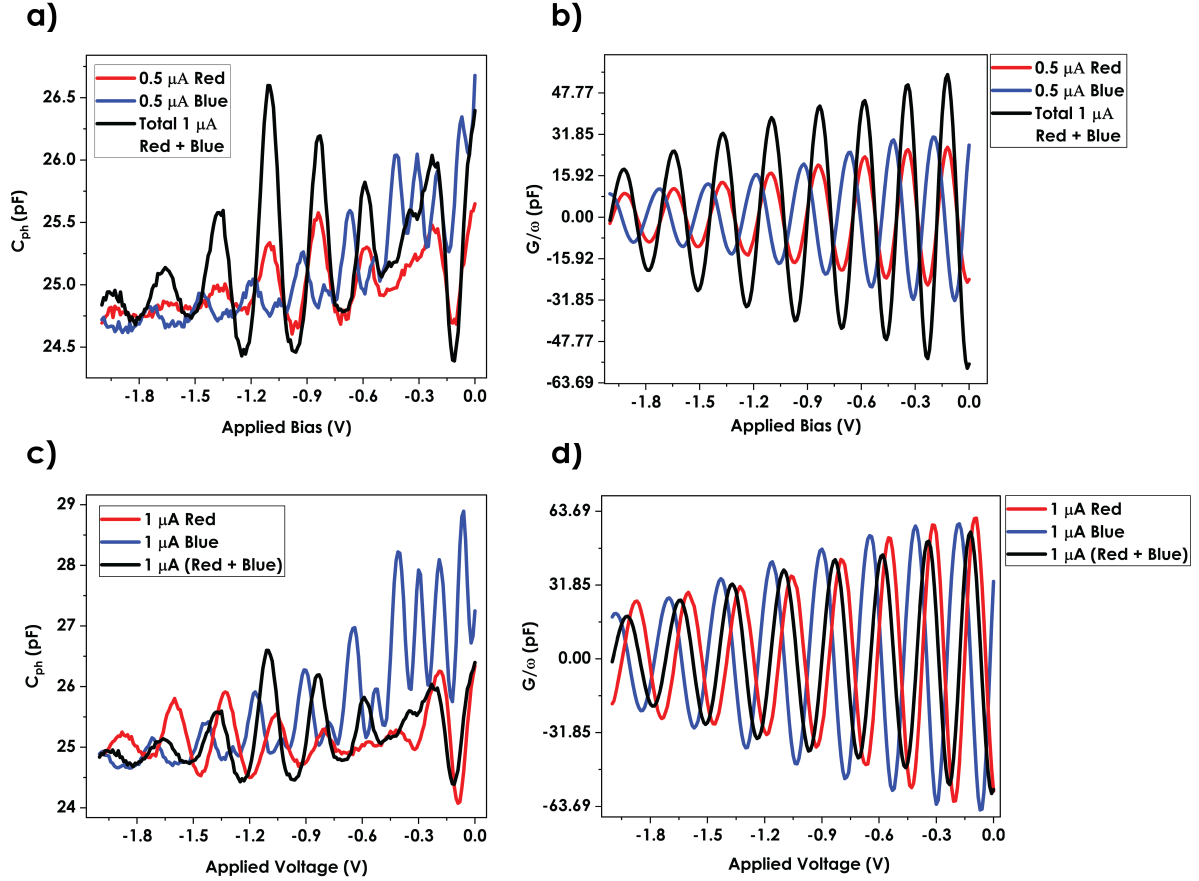


Figure 26: (a) and (b) show the C-V and G/ω oscillation observed for the smaller sample with both excitation wavelengths. (c) and (d) show the same C-V and G/ω oscillation observed for individual excitation with same intensity as combined excitation for comparison.

where n_{ex} is the exciton population density. This tells us that the capacitance measure can be compared with the intensity. Now, in our capacitance measurement,

$$\Delta C_{ph}^{beat} - \Delta C_{ph}^A - \Delta C_{ph}^B \propto I_{interference} - I_1 - I_2$$

and thus, if the value of $\Delta C_{ph}^{beat} - \Delta C_{ph}^A - \Delta C_{ph}^B$ oscillates between negative and positive values then this can be interpreted as interference. The same is observed in Fig. 26. To be sure that what we see is interference and not any intensity based measurement we also measured the oscillation from individual excitation with intensity same as that of the combined intensity. As shown in Fig. 26(c) and (d) we can be sure that it is not any intensity dependent result. Another interesting fact is that, this interference type pattern we see in capacitance measurement is not seen in the conductance measurements and in conductance measurements it looks like it is just intensity based observation oscillation.

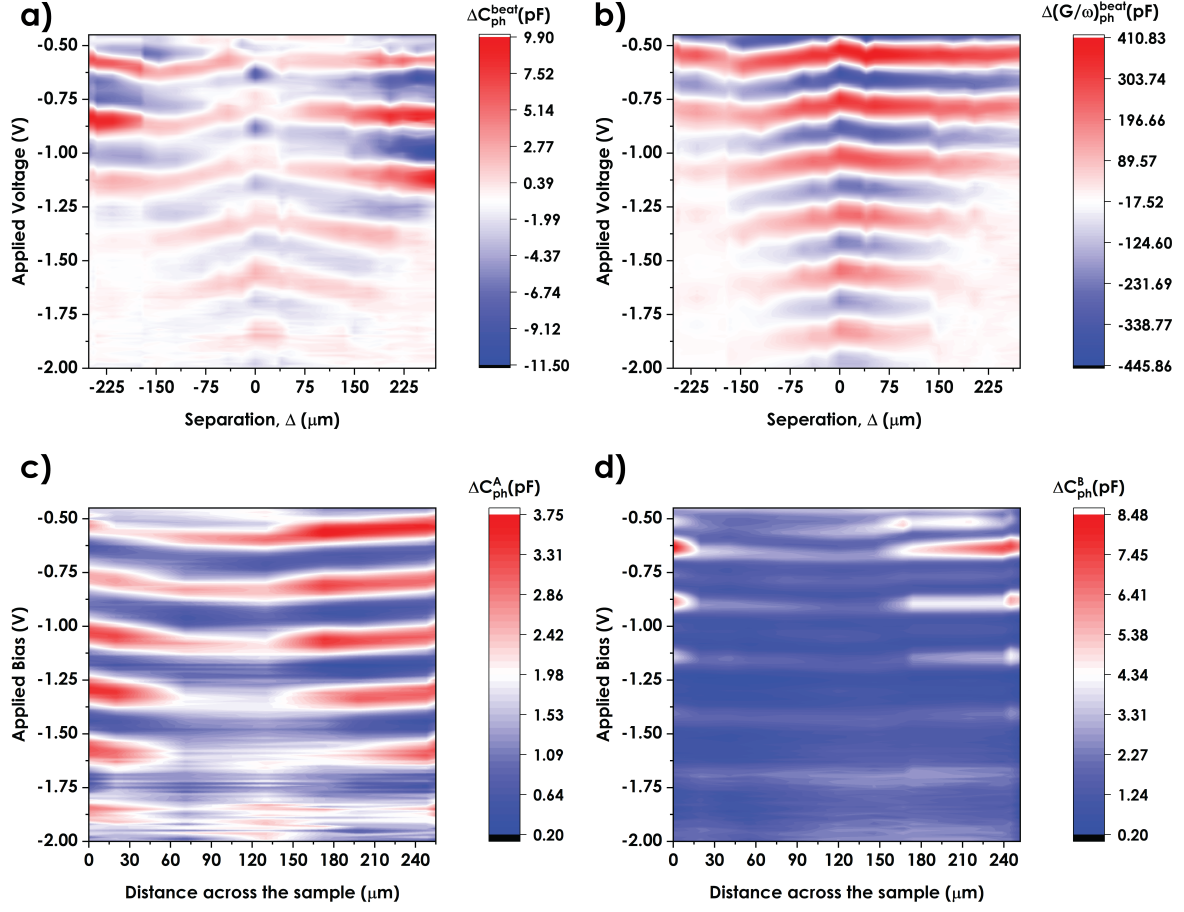


Figure 27: (a) and (b) show the contour of ΔC and $\Delta(G/\omega)$ oscillation observed for the both excitation wavelengths observed with varying separation. (c) and (d) show the capacitance oscillation observed on moving across the sample for blue and red respectively.

Now, this observed beat formation is observed for varying separation between excitation spots and this is plotted in Fig. 25(c). This figure shows that when the spots overlap, they show some visibility in interference whereas when they are separated, initially they don't show any beat but once they are far apart they show clearly visible beat pattern in capacitance oscillation. From the beats observed one can try to calculate the coherence by fitting cosine function and taking its amplitude as the value of coherence since

$$\Delta C_{ph}^{beat} - \Delta C_{ph}^A - \Delta C_{ph}^B \propto 2\sqrt{I_1 I_2} |g^{(1)}| \cos(\Delta\phi)$$

$$\frac{\Delta C_{ph}^{beat} - \Delta C_{ph}^A - \Delta C_{ph}^B}{2\sqrt{\Delta C_{ph}^A \Delta C_{ph}^B}} = |g^{(1)}| \cos(\Delta\phi) \quad (25)$$

Measuring this coherence as a function of separation of spots can give us information about the spatial coherence function and is shown in Fig. 25(b). On seeing the beat formation, the beats are more prominent when the spots are near the ring electrodes. So, on studying the capacitance oscillation of individual excitation spot when it is moved across the sample, it is evident that the spot being close to the sample matters. The contour of the observed $\Delta C_{ph}^{beat} = \Delta C_{ph}^A - \Delta C_{ph}^B$ with varying separation is provided in Fig. 27(a) and the individual ΔC_{ph}^A (Red) and ΔC_{ph}^B (Blue) upon moving across the sample is provided in Fig. 27(c) and (d). The electric field generated by the electrodes will be high near the electrode and carriers experience more potential when excited near the electrode. This modulation in beat formation in principle should have been eliminated when we calculate the coherence value since it takes into account of the difference in individual capacitance as difference in intensity is taken care of in the classical interference. Nevertheless, it is interesting to see that these oscillations are indeed ‘coherent’ in origin leading to formation of beat patterns and spatial separation also matters.

7 Conclusion

We set out to investigate the optical 2D transverse spatial coherence of the light emitted by excitons in our mixed dimensional 0D-2D heterostructure of III-V semiconductor. We made a modified Michelson Interferometer to measure the 2D spatial coherence and the momentum space distribution of the light through Wiener Khintchine Theorem [6]. The interferometer is initially studied with the commercially available laser diode to observe its lasing transition from LED mode to Lasing mode and its corresponding momentum space is calculated. When this setup is then applied on the PL from our sample. Upon calculation we have observed that the PL is not coherent within our measurement. Thus, PL spectroscopy is used to study the source of the PL. From the observed spectra and the PL intensity oscillation with voltage it is evident that the PL is not from the indirect excitons that contribute to formation of BEC. The indirect excitons that form BEC may be in dark. But now to overcome this we used capacitance measurements which can probe dark excitons to measure the spatial coherence through matter wave interference. The capacitance measurement shows oscillation with voltage which is studied to be a ‘coherent’ phenomenon through beat formation in C-V measurement upon two excitation wavelengths. This beat formation is argued as a result of matter wave interference and it is studied with varying separation to measure the spatial coherence. Upon calculation it

is shown that it is being modulated by shape of the electrode but still depicts the dependence of beat formation with spatial separation.

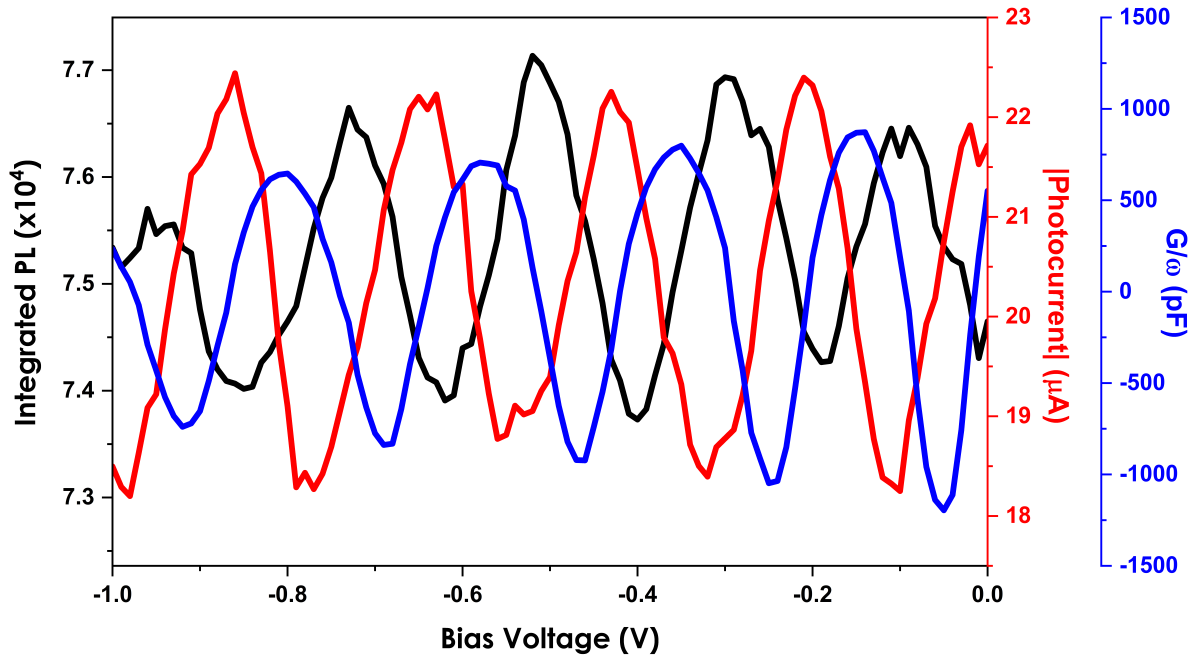


Figure 28: This figure shows the measured oscillations in photo-conductance, photo-current and photo-luminescence experiments. Photo- G/ω oscillation is 90° out of phase with the photo-current and photo-luminescence and photo-current is 180° out of phase with the photo-luminescence oscillation.

While doing all these measurements we observed oscillations in them with varying voltage. These oscillations are shown in Fig. 28. In this figure photo- G/ω correspond to photo-capacitance oscillation which shows the accumulated the carriers or indirect excitons across the barrier. In general photo-current measures the carriers that are flowing through the junction and does not form any exciton or recombine. Whereas photo-luminescence measure the recombination of the free carriers and excitons. The Fig. 28 shows photo- G/ω oscillation is 90° out of phase with the photo-current and photo-luminescence and photo-current is 180° out of phase with the photo-luminescence oscillation. These out of phase oscillations depicts the dynamics between the different population of excitons created upon photo-excitation with varying voltage as,

$$n_{total}^{Excitons} \longrightarrow n_{Photo-capacitance(\omega)}, n_{DC Photo-current}, n_{PL}$$

For the exact analysis of this requires further study along the line but nevertheless it shows evidence for a rich dynamics in this mixed dimensional heterostructure. In previous study [16] it has been shown that this dynamics result in driving the system to and from the BEC state and can be used to realize Rabi oscillation (population oscillation between two states upon changing pulse width) in this system. This realization of Rabi oscillation provides motivation for use of this system towards quantum computation using excitonic BEC states. Through this study we have studied the spatial coherence properties of the excitonic BEC in our sample and have given motivation for study of our system for exploration of dynamics between different exciton populations.

8 References

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