

Superclusters and the Void-High Density Region Correlations in the Sloan Digital Sky Survey

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Doctor of Philosophy

By

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Certificate

Certified that the work incorporated in the thesis entitled “*Superclusters and the Void-High Density Region Correlations in the Sloan Digital Sky Survey*”, submitted by *Shishir Sankhyayan* was carried out by the candidate, under our supervision. The work presented here or any part of it has not been included in any other thesis submitted previously for the award of any degree or diploma from any other University or institution.

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Declaration

I declare that this written submission represents my ideas in my own words and where others' ideas have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

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To
my family & friends

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Abstract

The large-scale structure of the Universe comprising of galaxies and dark matter has a web-like geometry which is called the 'Cosmic Web'. Its main constituents are walls, filaments, clusters, and voids. The galaxy redshift surveys have shown that there are structures present on even larger scales than clusters and these are called the superclusters (clusters of galaxy clusters). Massive superclusters are the largest structures found in the Universe and they are rare. The properties of superclusters can be used to test various cosmological models while there are still unknowns how superclusters form and evolve.

In this thesis, we present the discovery of a massive, 200 Mpc (Megaparsecs) scale supercluster, the Saraswati, at redshift of 0.28. Saraswati is one of the most massive superclusters with at least 43 galaxy clusters and a total mass of $\sim 2 \times 10^{16} M_{\odot}$ (solar masses). We also present a catalog of superclusters, identified using the data on clusters, in the redshift range of $0.05 \leq z \leq 0.42$. This is the largest catalog of superclusters (571 superclusters) in this redshift range having a range of masses and sizes with the most massive supercluster being at $z \sim 0.12$ containing 59 clusters within ~ 170 Mpc. Our new catalog of superclusters is useful for the statistical studies of their properties. With a goal to understand the relation between overdensities (dominated by dark matter) and under-densities (dominated by dark energy), we have extracted voids and high-density regions from the Sloan Digital Sky Survey. Using the Watershed and Voronoi tessellation methods, we measured their two-point correlation functions. This study has led to the understanding of the high correlation scales among voids and high-density regions and their physical inter-relationship.

List of Publications

In Journals :

Thesis related

- Saraswati: An Extremely Massive ~ 200 Megaparsec Scale Supercluster
Bagchi, J.; [Sankhyayan, S.](#); Sarkar, P.; Raychaudhary, S.; Jacob, J.; Dabhade, P. 2017, ApJ, 844, 25

Other research

- Discovery of Giant Radio Galaxies from NVSS: Radio & Infrared Properties
Dabhade, P.; Gaikwad, M.; Bagchi, J.; Pandey-Pommier, M.; [Sankhyayan, S.](#); Raychaudhury, S. 2017, MNRAS, 469, 2886

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Chapter 1

Introduction

In the night sky, with naked eyes, we mostly see distinct stars filling up the celestial sphere. These stars are inside our home galaxy - Milky Way. When we use a powerful optical telescope to see outside Milky Way, we see that the celestial sphere is filled with distinct galaxies. These galaxies are gravitationally bound systems of stars, gas, dust, and dark matter. The properties of these galaxies are inferred by collecting photons coming from these galaxies. The entire astronomy is based on the information gathered from the electromagnetic waves emitted by astronomical objects. The exception is the recently detected gravitational waves ([Abbott et al., 2016](#)), a new window to our Universe.

In 1920s, from the redshifted spectral lines' measurements of galaxies, it was discovered that our Universe is expanding. This suggested that the Universe, in its very early stage, was very dense and hot. Later on, further observations in the last 90 years, like cosmic microwave background radiation (CMBR), supernovae, baryon acoustic oscillation (BAO), etc., supported this model of the expanding Universe. CMBR's temperature fluctuations, of the order of 10^{-5} , gave us the picture of the Universe when it was only $\sim 380,000$ years old. The tiny fluctuations in the density distribution of matter show that the Universe was highly homogeneous and isotropic in its early stages. The long-range gravitational force acted to make those tiny fluctuations grow into stars, galaxies, planets, etc. that we see today. In this chapter, we introduce some basic concepts and theoretical background that will be useful in better understanding of the thesis. We start our discussion with the galaxy redshift survey.

1.1 Galaxy Redshift Surveys

The large scale structure of the Universe, i.e., the distribution of dark matter in the Universe, as a function of cosmological time gives constraints on the cosmological parameters and check the validity of different cosmological models. The baryonic matter is the biased tracer of dark matter distribution. This means that the distribution of galaxies follow the distribution of dark matter but the densities of dark matter and baryonic matter are not equal. A rough analogy would be looking a highway road from an aeroplane in night. From the aeroplane, one could only see the luminous and discrete street lamps. The concrete road corresponds to the dark matter and the lamps to galaxies. The road is continuous but the lamps have discrete distribution. The fact that dark matter distribution is hard to detect, the distribution of galaxies is observed and used for cosmological inferences.

Till 1970s, it was believed that the distribution of galaxies in the Universe, above the length scales of galaxy clusters (few Mpc¹), is a uniform random distribution. Although, the indication of clustering above these scales in the local Universe (de Vaucouleurs, 1953) and in the distribution of galaxies on the sky plane from the Palomar Sky Survey were also present (Abell, 1961). Like galaxies form groups and clusters under the influence of gravitational potential, groups and clusters also form clusters of clusters called superclusters. Abell described this clustering as second order clustering. A three-dimensional map of galaxies is required to have estimates of galaxy clustering on different scales and in turn, the clustering of dark matter.

The photometric galaxy surveys that observe a portion of the sky in few bands are able to give the angular position and apparent magnitude of galaxies, but to find the radial distances to galaxies, spectrum information is needed so that their redshifts can be calculated. The redshift gives the recessional velocity V of the galaxy which is used to calculate the distance to the galaxy from Hubble's law,

$$V = H D$$

where, H is Hubble parameter and D is the distance to the galaxy. The surveys of galaxies dedicated to obtain spectra of galaxies are known as 'Galaxy Redshift Surveys'.

The problem of taking the spectrum is that it is more time consuming than taking the images of the sky. The field of obtaining spectra of galaxies in bulk was revolutionized by the mutli-fiber spectroscopy programmes. It started with the Las Campanas Redshift survey (LCRS; Shectman et al. (1996)). It measured 112 spectra of galaxies simultaneously within 1.5 degrees field of view (FOV) from Las Campanas 2.5 m du Pont telescope in

¹1Mpc = 1Megaparsec = 3.086×10^{22} m

Chile. It gathered spectra of about 26,000 galaxies, with the median redshift $z = 0.1$, over an area of 700 deg^2 . These observations were taken from 1988 to 1994. Then came the two degree field galaxy redshift survey (2dFGRS; Colless et al. (2001)). The 2dFGRS used the 2 degrees FOV, multifibre spectrograph on the Anglo-Australian Telescope which observed the spectra of 400 astronomical objects simultaneously. This survey cover 2000 deg^2 with median redshift of galaxies $z = 0.11$. It has reliable spectroscopy data on around 220,000 galaxies covering around 1500 deg^2 of the sky. The most successful and influential galaxy redshift survey till date is the Sloan Digital Sky Survey (SDSS) (York et al., 2000). The spectroscopic data used in this thesis is taken from SDSS. Therefore, let us describe this survey in sufficient detail.

1.1.1 Sloan Digital Sky Survey (SDSS)

The Sloan Digital Sky Survey (SDSS) has produced the most detailed three-dimensional maps of the Universe ever made, with deep multi-color images of one third of the sky, and spectra for more than three million astronomical objects (see Figure 1.1²). The SDSS began regular survey operations in 2000. It has progressed through several phases, SDSS-I (2000-2005), SDSS-II (2005-2008), SDSS-III (2008-2014), and SDSS-IV (2014-2020). It mostly uses a dedicated 2.5 m altitude-azimuth telescope (Figure 1.2a³) located at Apache Point Observatory, New Mexico, USA (Gunn et al., 2006). A 1.08 m secondary mirror and two corrector lenses result in a 3° distortion-free field of view. The imaging camera (Gunn et al., 1998) collects photometric data using an array of thirty 2048 by 2048 pixel charge-coupled devices (CCDs) arranged in six columns of five CCDs each, aligned with the pixel columns of the CCDs themselves. The survey observes in five filter bands – u, g, r, i, and z whose central wavelengths are $3551 \text{ \AA}$, $4686 \text{ \AA}$, $6166 \text{ \AA}$, $7480 \text{ \AA}$, and $8932 \text{ \AA}$, respectively. These filters cover the respective rows of the array. The survey operates the instrument in a drift scan mode: the camera slowly reads the CCDs as the data is being collected, while the telescope moves along great circles on the sky so that images of objects move along the columns of the CCDs at the same rate the CCDs are being read. As an image of an object moves along the column of the CCDs, a CCD in each row collects data on that object. Therefore, the camera produces five images of a given object, all from the same column of CCDs, one from each CCD in that column.

SDSS has two spectrographs and these were rebuilt to be used in SDSS-III and SDSS-IV. In the early configuration, 640 spectra could be taken in

²<https://www.sdss3.org/science/>

³<https://www.sdss.org/instruments/>

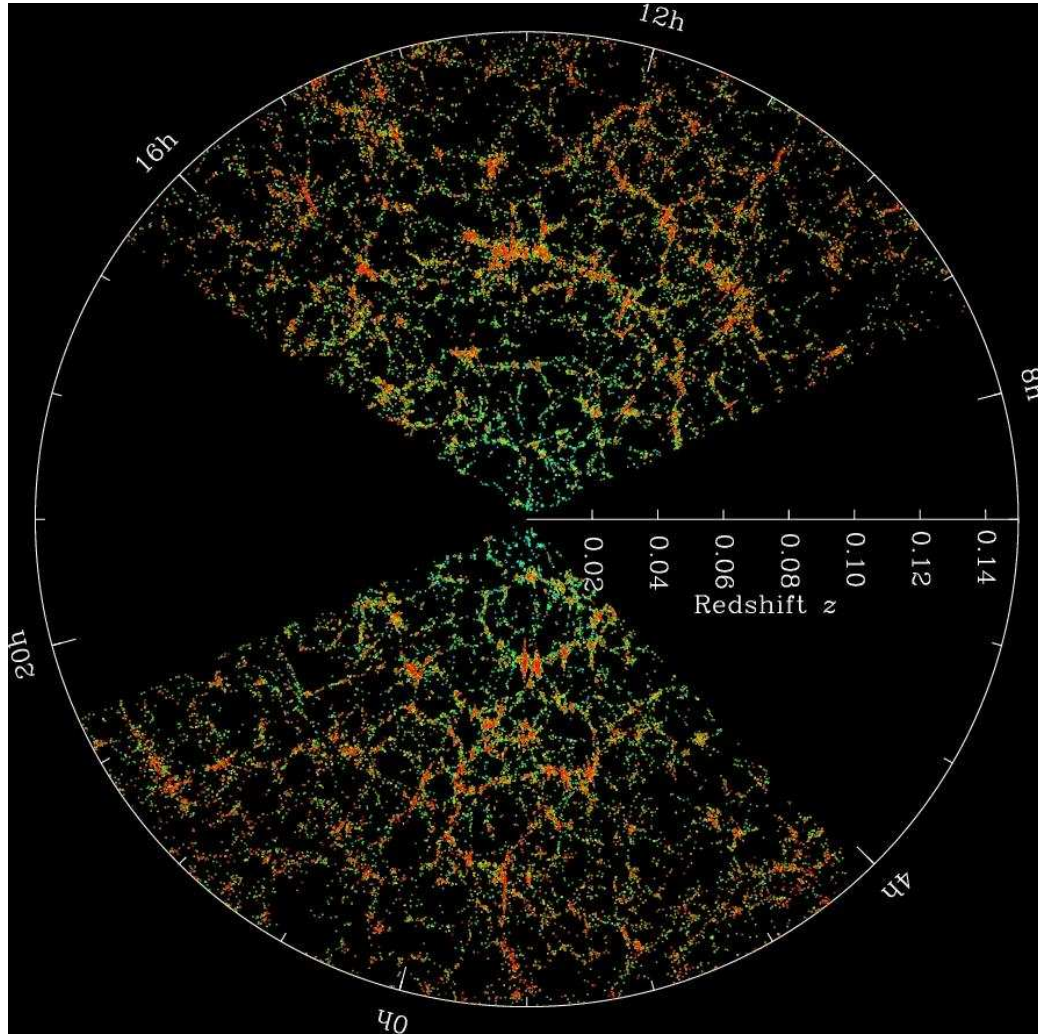


Figure 1.1: An equatorial slice through the SDSS three-dimensional distribution of galaxies. Observer is at the center of the circle. Each point represents a galaxy. Galaxies are colored according to the ages of their stars, with the redder, more strongly clustered points showing galaxies that are made of older stars. The distribution of galaxies is not filling the full circle because of the obscuration of sky by dust present in the Milky Way plane. The declination range limits are -1.25 and 1.25 degrees (suppressed in this figure) Credit: Michael R. Blanton and the SDSS collaboration.

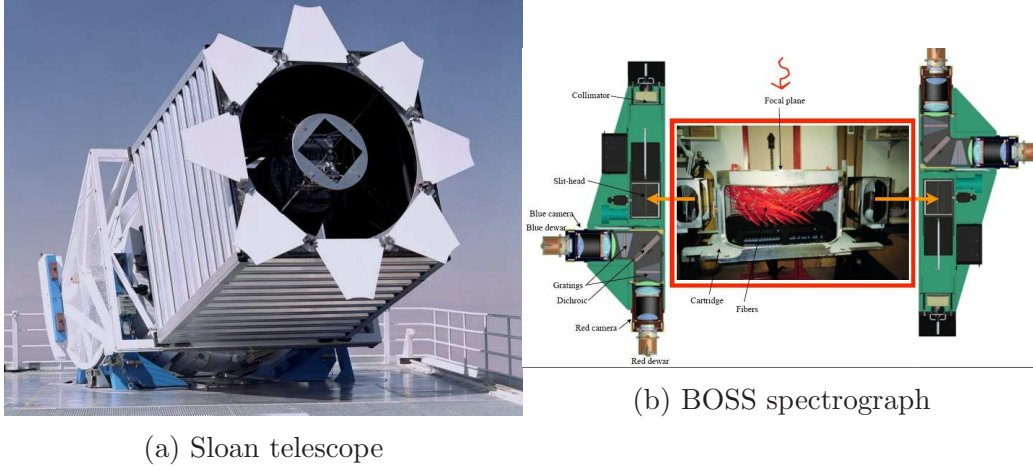


Figure 1.2: (a) 2.5 m Sloan foundation telescope. (b) An illustration of the SDSS's BOSS spectrograph.

one spectroscopic exposure using the optical fibers. An aluminum plate with holes drilled at the desired locations fixes the points from which spectra is gathered. Optical fibers of 3" diameter were plugged into the holes in the plate carry light from the focal plane to slit-heads. The spectrograph covers a wavelength range of $3800 \text{ \AA}$ to $9200 \text{ \AA}$. The latest configuration (Figure 1.2b⁴) can take 1000 spectra at a time. 1,000 holes are drilled in an aluminum plate (a plug plate, which subtends 3 degrees on the sky), each hole corresponding to an astronomical object such as a quasar, a galaxy, a standard star, or a random blank area on the sky (to measure and subtract the foreground sky emission). Optical fibers with 2" diameter are plugged into the holes. The wavelength range of this spectrograph is $3650 \text{ \AA}$ to $10,400 \text{ \AA}$.

1.1.2 Baryon acoustic Oscillation Spectroscopic Survey (BOSS) of SDSS

In chapters 3 and 5, spectroscopic galaxy data from the Baryon acoustic Oscillation Spectroscopic Survey (BOSS; Dawson et al. 2013) of SDSS's data release 12 (DR12; Alam et al. 2015) is used. The BOSS galaxy sample is designed to measure the BAO signature in the two-point galaxy correlation function and to reduce the errors in the measurement of the angular diameter distance and Hubble parameter at $z = 0.35$ and 0.6 . A survey with an area of approximately $10,000 \text{ deg}^2$ and a comoving number density of galaxies around $3 \times 10^{-4} h^3 \text{ Mpc}^{-3}$ within $0.1 < z < 0.6$ is sufficient for these measurements. This density is close to optimal for large-scale cosmological studies (Kaiser,

⁴https://www.sdss.org/instruments/boss_spectrograph/

1986). Therefore, BOSS has obtained spectra of 1,372,737 galaxies over 9376 deg² to fulfill the above requirements.

BOSS has two primary spectroscopic galaxy samples, LOWZ ($z \lesssim 0.4$) and CMASS ($0.4 \lesssim z \lesssim 0.7$). The LOWZ (low-redshift) galaxies are selected using few color-magnitude cuts. These cuts follow the predicted evolution of a passively evolving stellar population with redshift. The selected galaxies are the brightest and reddest of the low-redshift galaxy population. The LOWZ sample extends the SDSS-I/II Luminous Red Galaxies (LRGs) by selecting fainter galaxies, thereby increasing the number density. The apparent magnitude limits of LOWZ are $16 < r < 19.6$.

The CMASS (constant mass) galaxies are also selected using color-magnitude cuts. The CMASS selection is done using (g-r) and (r-i) colors to isolate high redshift galaxies. The CMASS selection algorithm introduces a sliding cut in color vs. magnitude specifically tuned to select the more massive objects in as uniform way as possible as a function of redshift. The apparent magnitude limits are $17.5 < i < 19.9$. The BOSS sample is deeper than the previous galaxy samples of SDSS.

Some of the important advantages of galaxy redshift surveys over other surveys are :

1. It provides three dimensional positions and various properties , like star formation rate, colour, luminosity and morphology, of a large number of galaxies which enable us to do precise statistical analysis.
2. It is used to study the effects of local density environments on the properties and evolution of galaxies.
3. Redshift Space Distortion (RSD), power spectrum, large scale motions, growth rate are calculated from redshift galaxy survey data which gives independent constraints on the cosmological parameter.
4. These, wide angle and densely sampled, surveys are ideal for comparing cosmological models with the N-body simulation data.
5. The CMB observations describe the state of the Universe at redshift $z \approx 1100$. To understand the complete evolution of the Universe, one should also know the distribution of galaxy at $z = 0$, which is given by redshift surveys. Also such survey can be used locally to recover the underlying matter distribution from measurement of peculiar velocity field of galaxies.
6. It provides us such a huge data that one can look and identify large-scale features of the matter distribution in the Universe.

1.2 Homogeneous Universe

The Cosmological Principle states that on sufficiently large-scales the Universe is both homogeneous and isotropic. Homogeneity means that the same observational evidence is available to observers at different locations. Isotropy means you could turn in any direction and the Universe will look exactly the same. This principle was introduced by Einstein and subsequently used by relativistic cosmologists without any observational justifications. There are various probes which test the Cosmological Principle. The main probe for the isotropy of the Universe is the observations of Cosmic Microwave Background Radiation (CMBR) (Hinshaw et al., 2009; Komatsu et al., 2009). The CMBR is very close to isotropic, the anisotropy is one part in 10^5 (Peebles, 1993). The other observational probe for isotropy is the observation of the celestial distribution of radio galaxies which are randomly distributed across the entire sky (Condon, 1999). For recent studies on the cosmic isotropy and the scale of isotropy, please see Marinoni et al. (2012); Gibelyou and Huterer (2012) and Sarkar et al. (2019). The fluctuations in amplitude, observed by HEAO1 (Scharf et al., 2000) in comparison to that of the predicted multipoles of the X-ray background is consistent with the homogeneous Universe (Lahav, 2002). The absence of big voids in the distribution of Lyman- α absorbers is also consistent with the Universe being homogeneous on large scales (Nusser and Lahav, 2000). Other studies using SDSS and other surveys mostly give the homogeneity scale of the Universe in the range from $60 \text{ h}^{-1}\text{Mpc}$ to $150 \text{ h}^{-1}\text{Mpc}$ (but very few do not report any transition to homogeneity) (Coleman and Pietronero, 1992; Bharadwaj et al., 1999; Best, 2000; Hogg et al., 2005; Yadav et al., 2005; Lukács and Fáy, 2008; Bagla et al., 2008; Sarkar et al., 2009; Sylos Labini et al., 2009a,b; Sylos Labini and Baryshev, 2010; Scrimgeour et al., 2012; Nadathur, 2013; Pandey and Sarkar, 2015; Ntelis, 2016; Pandey and Sarkar, 2016; Sarkar et al., 2016; Ntelis et al., 2017; Avila et al., 2018).

If the condition of homogeneity and isotropy is met, then the Universe is described by the Robertson-Walker metric. It is given by :

$$ds^2 = c^2 dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \quad (1.1)$$

Here spherical comoving coordinates $(r; \theta; \phi)$ are used to describe the spatial positions. The curvature k parametrizes the global geometry of the Universe, commonly referred to as closed ($k > 0$), flat ($k = 0$) and open ($k < 0$). The dimensionless scale factor $a(t)$ describes the expansion (or contraction) of the Universe and is normalized with respect to the present-day value, i.e. $a(t_0) = 1$.

Considering a general homogeneous and isotropic Universe model, Friedman in 1922 solved the Einstein equations and derived time-dependent equa-

tions of scale factor, known as Friedman-Robertson-Walker (FRW) equations,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \quad (1.2)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2} \right) + \frac{\Lambda}{3} \quad (1.3)$$

where, G is the Newton's gravitational constant, ρ is the mass density, P is the pressure and Λ cosmological constant that represents the constant dark energy density.

The equation of state for the fluid in the form of $P \equiv P(\rho)$ is given by

$$P = \omega\rho c^2 \quad (1.4)$$

with $\omega = 1/3$ for radiation, $\omega = 0$ for non-baryonic dark matter and $\omega < -1/3$ for dark energy. For dark energy, the observations give the value of $\omega = -1$.

Equation 1.2 gives the expansion rate of the homogeneous Universe at a given epoch and for observational purposes better represented as

$$H(z) = H_0 E(z) \quad (1.5)$$

Here, $H(z) = \frac{\dot{a}}{a}$ is the Hubble parameter and its value at present epoch is H_0 . $E(z)$ depends on the cosmological models and, neglecting the radiation contribution in the total energy density as it is negligible for the redshift ranges concerned in the thesis, is given by

$$E(z) = [\Omega_M(1+z)^3 + (1 - \Omega_M - \Omega_\Lambda)(1+z)^2 + \Omega_\Lambda]^{\frac{1}{2}} \quad (1.6)$$

where, Ω_M and Ω_Λ are the dark matter and dark energy densities relative to the critical density of the Universe at the present epoch. With this, the comoving radial distance d_c , which is the distance to a galaxy at redshift z when the expansion factor of the Universe is taken out, is given by

$$d_c = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')} \quad (1.7)$$

1.3 Inhomogeneous Universe

The above section discussed the understanding of homogeneous and isotropic Universe. However, at small scales, the Universe is far from homogeneity. Figure 1.1 shows the richness of structures from galaxies to groups and clusters of galaxies, to huge superclusters of galaxies. The main driving force

for the dynamics and evolution of galaxies, and the formation of large-scale structure in the Universe is gravity. The small scale density fluctuations, generated in the early Universe, are amplified by gravitational instability. This scenario lead to the formation of large-scale structures that are visible in the present day Universe. The fluctuations in the density field can be conveniently expressed as perturbation in the uniform and isotropic background,

$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}} \quad (1.8)$$

Here, $\rho(x)$ is the density at a given position x and $\bar{\rho}$ is the mean background density.

1.4 Dynamics of Structure Formation

The matter in the Universe is approximated as pressureless and self gravitating fluid. This implies that the mean free path between particle collision is much less than the scale of interest. To describe the dynamics of fluid elements, we require the equation of state. The equations that describe its evolutions are the continuity equation for mass conservation, Euler's equations for motion and Poisson's equations that accounts for the Newtonian gravity. Its better to convert these equations in terms of density fluctuations δ and peculiar velocity $\mathbf{v}$,

$$\begin{aligned} \frac{\partial \delta}{\partial t} + \nabla \cdot \mathbf{v} + \nabla \cdot (\delta \mathbf{v}) &= 0 \\ \frac{\partial \mathbf{v}}{\partial t} + 2H\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} &= -\frac{\nabla \phi}{a^2} \\ \nabla^2 \phi &= 4\pi G \bar{\rho} a^2 \delta \end{aligned} \quad (1.9)$$

where $\mathbf{v}$ is,

$$\mathbf{v} = a\dot{\mathbf{x}}$$

and, ∇ is the spatial comoving derivative, ϕ the potential fluctuation and $\mathbf{x}$ the comoving coordinate. Combining the three fluid equations, we get

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = \frac{1}{a^2}\nabla \cdot (1 + \delta)\nabla \phi + \frac{1}{a^2}\frac{\partial^2}{\partial x^\alpha \partial x^\beta} [(1 + \delta)v^\alpha v^\beta] \quad (1.10)$$

The solution of the above equation will give the complete picture of the evolution of large scale structure in the Universe.

1.4.1 The Linear Approximation

This is a simple case which deals with small inhomogeneities. In this regime, $\delta \ll 1$ and $vt/d \ll 1$, where d is coherence length of the fluctuation field and

$t \sim 1/\sqrt{G\bar{\rho}}$. All the non-linear terms become negligible and Equation 1.10 becomes,

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = 4\pi G\bar{\rho}\delta \quad (1.11)$$

The general solution of this differential equation contains two independently evolving modes, one of which is growing and the other one decays quickly. The growing mode is called the linear growth factor $D_1(t)$, since the density perturbation amplitude changes as the initial density $\delta(x)$ multiplied by the time-dependent coefficient :

$$\delta(x, t) = D_1(t)\delta(x) \quad (1.12)$$

The linear growth rate depends on the cosmological models and as a function of redshift can be expressed as,

$$D_1(z) = E(z) \int_z^\infty \frac{(1+z')dz'}{E^3(z')} \quad (1.13)$$

As the perturbations grow and become comparable to unity, $\delta \sim 1$, the governing equations cannot be linearized anymore. The evolution of the structure moves on to what is called the non-linear regime where the higher order effects cannot be neglected. In general, the non-linear equations are too complex to be solved analytically. However, there still exists empirical models that are capable of describing the non-linear evolution. Zel'dovich (1970) considered to solve the non-linear density fluctuation equation in a Lagrangian co-ordinate system on large scales.

1.4.2 The Zel'dovich Approximation

Zel'dovich (1970) prescribed a simple and elegant approximation to describe the non-linear stage of gravitational evolution. This clever approximation actually predicts the density of caustics to be infinity, but the gravitational acceleration caused by this region remain finite. He considered the density fluctuation equations δ in Lagrangian coordinate instead of Eulerian.

If we consider a homogeneous medium in the presence of gravity then the actual position $\mathbf{r}$ of the particle in Lagrangian coordinate $\mathbf{q}$ and time t is given by,

$$\mathbf{r} = a(t)\mathbf{q} + b(t)\mathbf{s}(\mathbf{q}) \quad (1.14)$$

Here $a(t)$ is the scale factor, $b(t)$ is the product of scale factor and growing mode of the linear fluctuations and $\mathbf{s}(\mathbf{q})$ is the gradient of a potential representing the spatial perturbation.

Since at $t > 0$ density inhomogeneities are created, mass conservation requires that $\rho(\mathbf{r}, t)d\mathbf{r} = \rho_0 d\mathbf{q}$ and hence, density field is given by

$$\rho(\mathbf{r}, t) = \rho_0 \left| \frac{\partial \mathbf{r}}{\partial \mathbf{q}} \right|^{-1} \quad (1.15)$$

The deformation tensor D_{ij} is given by

$$D_{ij} = \frac{\partial r_i}{\partial q_j} = a(t)\delta_{ij} + b(t)\frac{\partial s_i}{\partial q_j}$$

$$D = \begin{pmatrix} a(t) - \alpha b(t) & 0 & 0 \\ 0 & a(t) - \beta b(t) & 0 \\ 0 & 0 & a(t) - \gamma b(t) \end{pmatrix} \quad (1.16)$$

where the eigenvalues $-\alpha$, $-\beta$ and $-\gamma$ are the function of $\mathbf{q}$. Negative eigenvalues correspond to the growing mode. If we consider the mass conservation then,

$$\bar{\rho}a^3 = \rho[a(t) - \alpha b(t)][a(t) - \beta b(t)][a(t) - \gamma b(t)],$$

$$\rho = \bar{\rho}(1 + \delta) = \frac{1}{[1 - \alpha D_+(t)][1 - \beta D_+(t)][1 - \gamma D_+(t)]} \quad (1.17)$$

where $D_+(t)$ is the growing mode.

For simplicity, let's assume that $\alpha > \beta > \gamma$. Since $D_+(t)$ increases, therefore, after some time the first singularity occurs. This singularity is along the principal axis and gives rise to a sheet like structures. Zel'dovich named these objects as pancakes. Other structures like filaments and clusters form by the simultaneous contraction of second and third axis.

The Zel'dovich approximation is, therefore, simply a linear approximation with respect to particles displacement rather than the density. This approximation predicts good result upto the moment of shell crossing. After that this approximation breaks down completely. The reason is that after the formation of pancake the particles entering from either side, merely sailed through and finally passed out to the opposite side. Thus, pancakes appear and instantaneously smeared out. In the real case, the matter in the caustic would feel strong gravitational field and will be pulled back before it escapes.

1.4.3 Non-Linear Evolution and Numerical Simulations

On smaller scales, the structure formation can be described by spherical collapse model. In the "cold" dark matter models, structure formation follows a "bottom-up" scenario - smaller density enhancements of matter collapse and virialize earlier, providing progenitors for larger structures. This process

is called hierarchical clustering ([Peebles, 1971](#)). On the one hand, sizes of galaxy superclusters (tens to a few hundred of megaparsecs) place them in the linear regime of structure formation (e.g., [Zeldovich et al. \(1982\)](#)). However, due to their asymmetrical shapes, other dimensions of supercluster can be considerably smaller and thus be influenced by non-linear processes.

Because of the limitations of the non-linear analytical solutions of the structure formation equations, numerical computer simulations are the powerful tool to study the large-scale structures and their evolution. Numerical simulations show the full view of cosmic web as a complex and intricate network of sheets, filaments, and roughly spherical small density peaks (knots), framing large under-dense void regions ([Doroshkevich et al., 1980](#); [Klypin and Shandarin, 1983](#); [Davis et al., 1985](#); [Sheth and van de Weygaert, 2004](#)). Within this complex network of the cosmic web, matter flows along well-defined paths - with voids feeding sheets, sheets feeding filaments, and filaments finally channeling everything towards knots (see [Cautun et al. \(2014\)](#) for an overview). Computer simulations are also very useful in comparing the outcome of different cosmological models. For example, with a particular cosmological model, one can define a problem and attempt to combine all the relevant knowledge - different types of particles (dark matter, baryons), and the known physical processes (gravity, gas dynamics), to solve it. These outcomes then be compared with the observational data. Figure 1.3 shows four snapshots of a N-body simulation from [Vogelsberger et al. \(2014\)](#). It displays the important characteristics of the cosmic web.

1.5 Cosmic Web : Clusters, Filaments, Walls, and Voids

Galaxy redshift surveys and N-body cosmological simulations revealed that the large scale distribution of matter in the Universe has an intricate, rich, and complex spatial structure. It looks like a web-network of galaxies (Figure 1.1). Structures in this network has sizes from a few Megaparsecs up to tens and even hundreds of Megaparsecs. The prominent constituents of the cosmic web are one-dimensional elongated filaments, two-dimensional sheet-like walls, dense compact spherical/elliptical clusters, and large almost empty regions called voids. This network of galaxies has been called by various names, cellular structure ([Joeveer and Einasto, 1978](#)), supercluster-void network ([Einasto et al., 1980](#)), etc., but it is best known and called as the cosmic web ([Bond et al., 1996](#)). This structure and its evolution is governed by the amount of dark matter and dark energy, and the initial conditions of the Universe. The cosmic web is an excellent tool to constrain the cosmological models as different models predict different structures of the cosmic

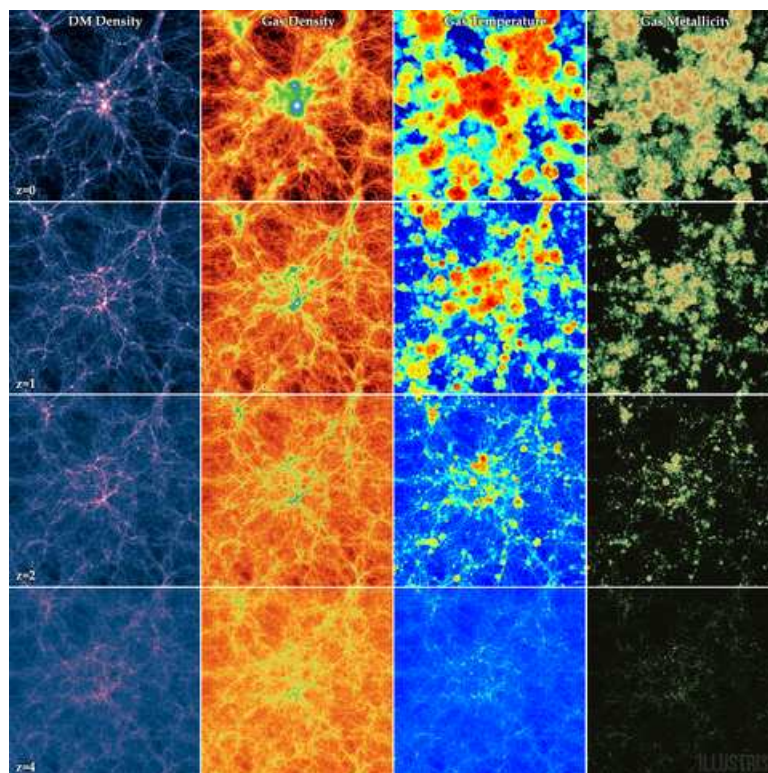


Figure 1.3: Redshift evolution of a box slice at $z=4$, $z=2$, $z=1$, and $z=0$, showing four projections : dark matter density, gas density, gas temperature, and gas metallicity. Credit : Illustris-Project.

web.

1.5.1 Groups and Clusters of Galaxies

Galaxies prefer to be found in groups or clusters instead of isolation, which typically form parts of still larger structure of the cosmic web. Our own Milky Way is a part of a group of galaxies called the Local Group. This is a gravitationally bound system with about 30 member galaxies. Three spiral galaxies dominate the Local Group; the Andromeda galaxy, M33, and the Milky Way. Nearly all the galaxies within the Local Group lie within a sphere of radius 1 Mpc. In general, groups of galaxies have fewer than 50 members, mostly spirals, except in very dense and compact groups in which a good fraction of ellipticals can also be found. They are systems with masses $\leq 10^{14} M_{\odot}$.

Clusters of galaxies are larger bound systems containing from 50 up to thousands of members within a sphere of radius up to 4 Mpc. They contain galaxies of all types. There are clusters with a well-defined spherical or elliptical shape, but others show a more irregular and open structure. It is

also possible to detect substructure within a large cluster. Their typical mass exceeds $10^{14} M_{\odot}$.

The number density of clusters and groups as a function of redshift depends on the cosmological parameters. Therefore, these are used to compare different cosmological models.

1.5.2 Superclusters

Only few galaxies are visible to the naked eye, which made the discovery and observations of galaxies in greater numbers possible only after the proliferation and technical advancement of telescopes. The well-known Catalogue of Nebulae and Star Clusters published by C. Messier in the 18th century contained 40 objects now known to be galaxies. The majority of over thirteen thousand objects in the New General Catalogue (NGC) and Index Catalogue (IC) by J.L.E Dreyer (1888-1907) are actually galaxies. However, the extragalactic nature of these “nebulae” was not determined until the 1920s (e.g., [Opik \(1922\)](#)).

As mentioned above, the inhomogeneous and clustering features of the galaxy distribution on Megaparsec scales became apparent with the first larger galaxy catalogues ([Abell \(1965\)](#), [Oort \(1983\)](#), and references therein). Notably, the nearby massive Virgo cluster was already evident in the earliest surveys of nebulae by W. and C. Herschel. Based on the available observations at the time, [Zwicky \(1938\)](#) reached on the conclusion that almost all galaxies probably belong to some aggregations. From the study of the distribution of sky positions of about 2700 rich galaxy clusters, [Abell \(1958\)](#) suggested that the distribution of clusters indeed shows a tendency toward “second-order” clustering. This “second-order” clustering of very rich clusters, situated in the larger structure of the cosmic web is known as superclusters - clustering of clusters on megaparsec scales.

The first supercluster identified was the Local supercluster (also called the Virgo supercluster) described by [de Vaucouleurs \(1953\)](#), who suggested that the Virgo cluster together with the Local Group, the Ursa Major cluster, and other smaller groups and clusters comprise a large system with a size of tens of megaparsecs. About that time the term “supercluster” for clusters of galaxy clusters was coined, replacing “supergalaxy” which was in use before. Later, with the redshift information accumulated by [Fisher and Tully \(1981\)](#), the galaxy distribution in the Virgo supercluster and its surroundings was mapped in greater detail. Subsequently, several other superclusters were discovered, most notably the Coma supercluster ([Gregory and Thompson, 1978](#)) and the Hercules supercluster ([Tarenghi et al., 1978](#)), with some hints of the existence of a matter-bridge connecting the two ([Oort,](#)

1983). The Perseus-Pisces supercluster (Jöeveer et al., 1978) was identified using the cluster catalogue of Zwicky. The massive Hydra-Centaurus supercluster (Chincarini and Rood, 1979), neighbouring the Virgo Supercluster and located opposite to the Perseus-Pisces supercluster, was pointed out by Lynden-Bell et al. (1988) as the approximate direction of the local galaxy bulk flows - the “Great Attractor”. The extremely massive Shapley supercluster, discovered by Raychaudhury (1989), also contributes to the local bulk motion. It lies roughly in the same direction as that of the Great Attractor, but is situated further away.

Before the advent of deep galaxy redshift surveys in the late 1990s, a convenient way to probe large-scale structures further away was to use galaxy clusters as the basis of clustering analysis. Cluster samples, while being sparser than those of galaxies, can trace the cosmic web to far greater distances. For example, Abell catalogue of rich clusters has been the basis for creation of many supercluster catalogues by Abell (1961) and Einasto et al. (1994, 1997, 2001), in the latter case together with X-ray clusters. In the last decade, X-ray cluster samples have grown large enough to be used to create supercluster catalogues (e.g., Chon et al. (2014)). For example, six groupings of X-ray galaxy clusters were recently identified as superclusters (Koulouridis et al., 2016). The galaxy clusters identified using optical photometric data has even outgrown the X-ray cluster catalogs, making them excellent choice to identify superclusters (Wen et al., 2012). Another class of rare and bright objects, visible to large distances, are quasars. The use of quasars in characterizing the large-scale structures had already been suggested (Oort, 1983). However, the structures extracted from SDSS quasar sample by Clowes et al. (2013) and Einasto et al. (2014) appear to be considerably large than what is commonly regarded as superclusters.

There is not yet an entirely physical definition for a supercluster. Some use the spherical collapse condition to derive the boundary density threshold, since simulations suggest that while supercluster-sized objects might not be gravitationally bound at the moment, but they will become so in the future (Araya-Melo et al., 2009; Chon et al., 2015). However, this approach has limitations and is not completely accurate in all cases as superclusters are highly asymmetric at the current epoch. Proper investigation of the dynamical state of the large-scale structures using ultra-deep spectroscopic observations of some very massive supercluster regions is needed in order to determine the gravitational potential and galaxy velocities with more precision, i.e. how much region is actually bound. At present, the observational precision required to adequately measure the peculiar motions of galaxies is limited to our close neighbourhood. Tully et al. (2014) studied this aspect in the nearby large-scale structure using Cosmicflows-2 galaxy catalogue (a catalog of peculiar velocities for over 8000 nearby galaxies) with Wiener filter applied on both the galaxy density and velocity fields. In this study, they

mapped the galaxy density and their local motions up to a distance of ~ 160 Mpc by delineating the so-called basins of attraction - regions of space where all galaxies flow inside and tend to converge. Therefore, the natural boundaries of superclusters are the surfaces that diverges the matter flow into separate potential depressions. This gave a lot more clear picture and interpretation of the large-scale structures in the local Universe. As a result, Virgo Supercluster together with Hydra-Centaurus supercluster and several other smaller nearby superclusters form an extensive structure named the Laniakea supercluster, with the “Great Attractor” region being the convergence point for the matter flow. Milky Way is a part of Laniakea supercluster (Figure 1.4).

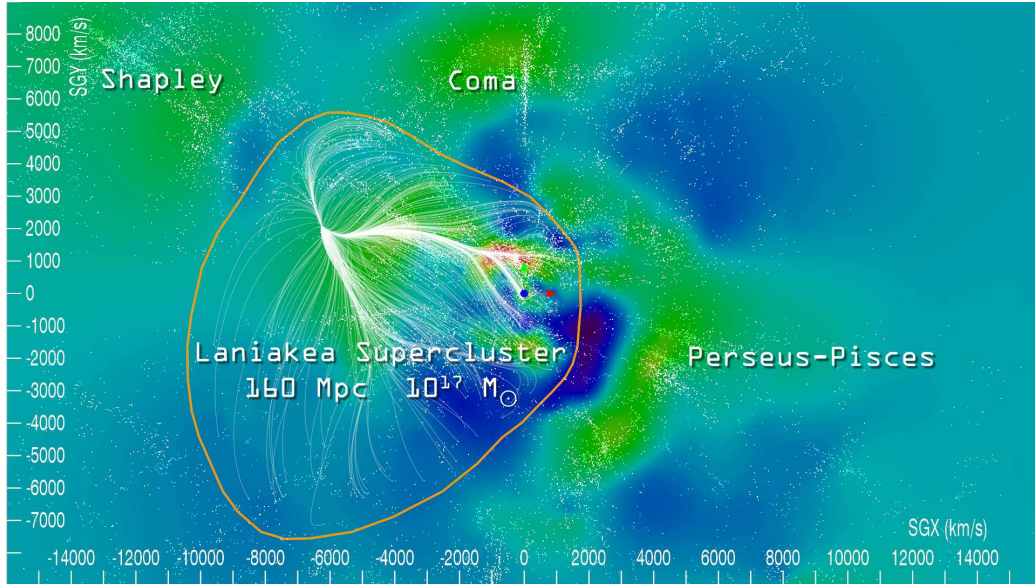


Figure 1.4: A slice of the Laniakea supercluster in the supergalactic equatorial plane. Shaded contours represent densities within the equatorial slice with red at high densities and blue in voids. Galaxies with available redshift information are shown as white dots. Peculiar velocity flow streams within the Laniakea basin of attraction are shown in white. The orange contour, which is the natural boundary of this supercluster, encloses the outer limits of these streams. Laniakea has an extent of ~ 160 Mpc ($\sim 12,000$ km/s) and encloses a total mass $\sim 10^{17} M_{\odot}$. The blue dot shows the location of the Milky Way galaxy. Credit : [Tully et al. \(2014\)](#).

Like number density of clusters, number and mass density of superclusters is an important feature of the cosmic web.

1.5.3 Voids

Voids are the prominent features of the distribution of galaxies and matter on Megaparsec scales (Chincarini and Rood, 1975; Gregory and Thompson, 1978; Einasto et al., 1980; Kirshner et al., 1981, 1987; de Lapparent et al., 1986; Colless et al., 2003; Tegmark et al., 2004; Guzzo and The Vipers Team, 2013; Guzzo et al., 2014). These are huge regions with sizes ranging 10 - 200 h^{-1} Mpc in diameter that have almost no galaxy. They occupy major share of space in the Universe and are usually roundish in shape. Voids are instrumental in the spatial organization of the Cosmic Web (Icke, 1984; Sahni et al., 1994; Sheth and van de Weygaert, 2004; Einasto et al., 2011). Surrounded by filaments, walls, and clusters, they weave the salient web-like pattern of galaxies and matter pervading the observable Universe.

Several recent studies suggest that voids represent a key constituent of the cosmic mass distribution and are one of the cleanest probes and measures of the global cosmology. Their structure, morphology and dynamics depends on the nature of dark energy, dark matter and possibly on non-Gaussian nature of the primordial perturbation field. Filaments, walls, and clusters have dense, chaotic, accreting and merging environments. The pristine environment of voids makes them an ideal and pure setting for the study of galaxy formation and the influence of cosmic environment on the evolution of galaxies. Voids are also the principal regions along which the ionizing radiation produced by the first stars propagates during the reionization process of the Universe.

In a void-based description of the evolution of the cosmic web, voids mark the transition scales at which density perturbations decouples from the Hubble expansion flow and contracts into identifiable structural features. At a given cosmic epoch, the voids that dominate the spatial matter distribution are a manifestation of the structure formation process reaching a non-linear evolutionary stage. On the basis of theoretical models of void formation and evolution, it can be inferred that void-expansion may act as the key organizing aspect for arranging matter concentrations into an all-pervasive cosmic web network (Icke, 1984; Regos and Geller, 1991; Sheth and van de Weygaert, 2004; Aragon-Calvo and Szalay, 2013). The expansion of voids squeeze matter in between them, and forms walls/sheets and filaments at the boundaries of voids. This scenario is supported by numerical studies and computer simulations of the gravitational evolution of voids in more complex and realistic configurations (Martel and Wasserman, 1990; Regos and Geller, 1991; Dubinski et al., 1993; van de Weygaert and van Kampen, 1993; Goldberg and Vogeley, 2004; Colberg et al., 2005; Padilla et al., 2005; Aragon-Calvo and Szalay, 2013; Sutter et al., 2014; Wojtak et al., 2016). An example of the evolution of a typical large and deep void in a numerical simulation of Λ CDM Universe is given by the time sequence of six frames

in Figure 1.5. Notice the enhancement in the densities of structures around the void and depletion of matter from inside the void as time progresses. Basically, structure formation is governed not only by gravitational instability but also by void expansion.

At the center of a typical void, the density contrast $\delta = -0.8$ (see chapter 5 and Colberg et al. (2005)). That is,

$$\delta = \frac{\rho_m}{\overline{\rho_m}} - 1 = -0.8 \quad (1.18)$$

where, ρ_m and $\overline{\rho_m}$ are matter densities at the center of the void and in the background Universe, respectively. This implies that,

$$\begin{aligned} \frac{\rho_m}{\overline{\rho_m}} &= 0.2 \\ \frac{\overline{\rho_m}}{\rho_m} &= 5 \\ \frac{\overline{\rho_m}}{\rho_\Lambda} \times \frac{\rho_\Lambda}{\rho_m} &= 5 \\ \frac{\rho_\Lambda}{\rho_m} &= 5 \times \frac{\Omega_\Lambda}{\Omega_m} \end{aligned}$$

For the values of $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$,

$$\frac{\rho_\Lambda}{\rho_m} = 11.67 \quad (1.19)$$

This shows that the dark energy density is ~ 12 times higher than the dark matter density inside the voids. Therefore, the expansion rate of a large void is higher than the background Hubble expansion and the dynamics of matter inside voids is mainly governed by the dark energy.

The relatively simple structural and dynamical evolution of voids remains strongly influenced by the evolving large scale environment, through dominant tidal influences and direct contact between neighbouring voids. It is this aspect that has been recognized in a surge of recent studies for its considerable potential for measuring the value of cosmological parameters. The evolution of their structure and shape appear to be a direct manifestation of the nature of dark energy, while their dynamics reflects the nature of dark matter or that of the possibly non-standard nature of gravity. Their correlation with the high density regions is the quantity which can be used to verify a cosmological model.

1.6 Motivation of the Work

Superclusters are the largest overdense structures of the cosmic web. The information of the initial conditions are lost in the highly non-linear evolution

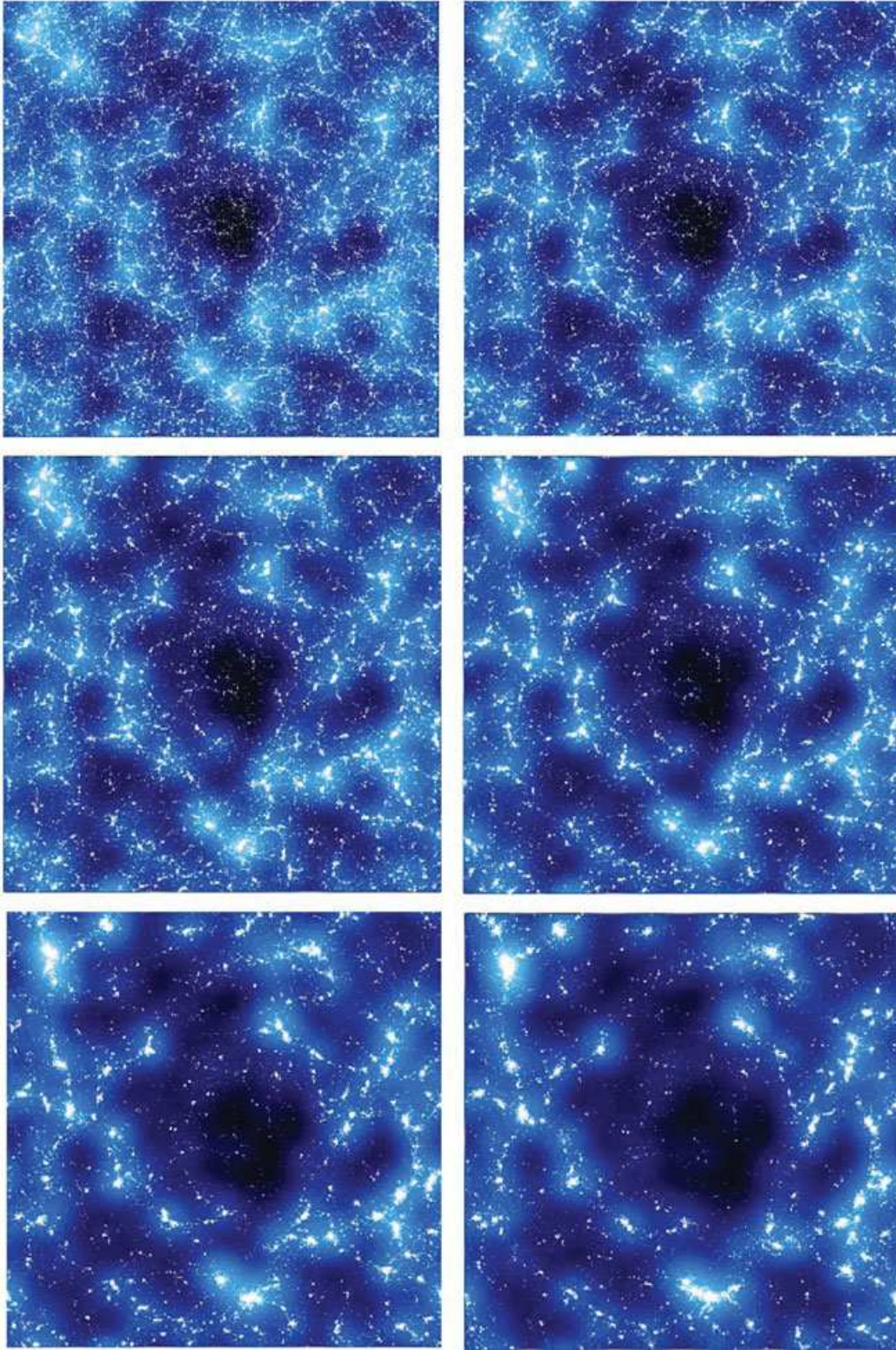


Figure 1.5: Evolution of a void in Λ CDM Universe : The slice is $50 \text{ h}^{-1}\text{Mpc}$ wide and $10 \text{ h}^{-1}\text{Mpc}$ thick taken from a numerical simulation. The figure shows particles and smoothed density field at six different time-steps : $a(t)=0.05, 0.15, 0.35, 0.55, 0.75$ and 1.0 . The smoothing is done on a scale of $4 \text{ h}^{-1}\text{Mpc}$. Image credit : Erwin Platen ([van de Weygaert, 2016](#)).

of galaxy clusters and groups but superclusters are still evolving and have not reached the state of high non-linearity. Therefore, superclusters retain the information about its past. But superclusters are rare structures of the cosmic web, especially massive and large superclusters. While studying the optical, X-ray, and radio properties of the galaxy cluster ZwCl 2341.1+0000, [Bagchi et al. \(2002\)](#) suggested that the morphology and the dynamical state of the cluster would be explained if ZwCl 2341.1+0000 resides in a supercluster environment whose size may go upto 100 Mpc. Neither deep wide-field optical images nor extensive redshift measurements were available at that time in order to explore and analyse a 100 Mpc region around this cluster. But now, with the rich data set of spectroscopic galaxies of SDSS, this region can be studied in detail to confirm if ZwCl 2341.1+0000 actually resides in a supercluster environment or not.

In the clustering analysis using spectroscopic galaxies, a large overdense region may or may not be a supercluster. Since superclusters are clusters of clusters, it should be checked if the overdense region of galaxies actually contains an overdensity of clusters. The superclusters have been cataloged and studied from the data for redshifts $z < 0.1$, but a catalog of superclusters is also required for higher redshifts $z > 0.1$. This catalog would be useful in comparing the properties of superclusters with those in the nearby Universe. [Wen et al. \(2012\)](#) produced the largest catalog of 132,684 clusters and groups of galaxies within $0.05 \leq z < 0.8$ from the photometric data of SDSS. Like clusters have been used to produce supercluster catalogs for lower redshifts ([Einasto et al., 1994, 1997, 2001](#); [Chow-Martínez et al., 2014](#)), [Wen et al. \(2012\)](#) cluster catalog can be used as basis for the identification and construction of a catalog of superclusters within $0.05 \leq z \leq 0.42$.

During the evolution of the cosmic web, matter flows from under-dense voids to high density peaks ([Sheth and van de Weygaert, 2004](#)). This creates a spatial correlation between the high density regions (HDRs) and low density regions (voids) in the matter distribution. A very high density region, maybe a cluster or supercluster, should be accompanied by a very large void. This correlation is another tool to verify different cosmological models. Using the parameter-free Voronoi tessellation techniques, local density at the site of every galaxy can be estimated. This re-constructed density field of the cosmic web can then be used to identify and localize voids and HDRs, and calculate their high spatial correlation lengths.

1.7 Outline of the Thesis

This thesis reports our effort to identify large components of the cosmic web from the Sloan Digital Sky Survey and find the correlation between them.

Below is a brief summary of all the chapters :

1. Chapter 2 : Methods and Techniques

This chapter explains the methods and techniques used for the analysis in chapters 3, 4, and 5. The selection effects associated with galaxy redshifts surveys and the associated modifications in the analysis techniques are explained. The clustering analysis using Friends of Friends algorithm and the theory of spherical collapse model to find the gravitationally bound structures are introduced. The parameter-free geometrical techniques of Voronoi tessellation and Delaunay triangulation to reduce computation time on clustering analysis and to construct density fields from the discrete data set of galaxies are described. The watershed algorithm to bifurcate the density field into over- and under-dense regions of the cosmic web is given. The two-point correlation function and its estimator are also described in detail.

2. Chapter 3 : Discovery of Saraswati Supercluster

This chapter discusses the discovery of very large and massive Saraswati supercluster at redshift $z = 0.28$. The SDSS data of spectroscopic galaxies in Stripe 82 is used to find the overdensity of galaxies and clusters in a ~ 200 Mpc region. The luminosity density of galaxies is used to remove the radial selection biases of SDSS. The robustness of the finding against redshift space distortions is shown. The correlation of this overdensity with the position of known clusters and voids is discussed. The properties of the supercluster - total mass, volume, density contrast, and turn-around radius are calculated. The comparison of the expected number density of clusters within the supercluster volume with Λ CDM cosmology using halo mass function is also done. The properties of this supercluster put it into the list of few most massive superclusters ever discovered.

3. Chapter 4 : Identification of Superclusters from the WHL Clusters

This chapter presents the identification of superclusters to make a new catalog of superclusters using Delaunay tessellation and Friends of Friends algorithm. A sample of clusters, instead of a sample of galaxies, is used to construct this catalog. The incompleteness of the clusters is taken into account while extracting the superclusters. Its comparison with some known superclusters is also discussed.

4. Chapter 5 : Correlation of Voids and High Density Regions

This chapter describes the construction of density troughs (voids) and density crests (high density regions) catalog from the BOSS data set of spectroscopic galaxies of SDSS. The angular and radial selection effects are handled by placing guard buffer particles around survey boundaries

and inside survey holes. The properties of voids are estimated from the void catalog. The high correlation lengths among voids and HDRs are calculated.

5. **Chapter 6 : Conclusions and Future Scope of the Work**

This chapter summarizes the results of our analysis presented in the earlier chapters. We also discussed open issues and directions of future work.

Chapter 2

Methods and Techniques

2.1 Selection Effects and Biases

Galaxy surveys, both photometric and spectroscopic, do not include all existing galaxies in the Universe. There are various reasons for why galaxies are not observed. Because of this, different regions of these surveys have different number of galaxies. This observational bias is known as the selection bias of a survey. To do a statistical analysis on the surveyed galaxies, these selection biases must be taken into account, in order to obtain reliable results.

2.1.1 Malmquist Bias and the Radial Selection Function

Most galaxy surveys are apparent magnitude-limited surveys. This means that there is always a minimum flux limit (maximum apparent magnitude limit) below which no galaxy can be observed. Depending on the targeted science goals, the detector's sensitivity and time given to a particular survey, this magnitude limit is decided. Due to this minimum flux limit, we see more nearby galaxies and less distant galaxies. The number density of observed galaxies decreases as a function of redshift (or radial distance). Only the most luminous galaxies at large distances can be seen. Faint galaxy fluxes do not reach the required flux-limit and hence are not observed. This observational bias in the number density of galaxies is known as the Malmquist bias.

To account for the Malmquist bias, one can make a volume-limited sample or assign weights to galaxies according to their number density as a function of redshift. In a volume-limited sample, two redshift limits are chosen. Between these two redshift limits, only those galaxies are allowed which are luminous enough. That is if a galaxy is placed at the higher end of the redshift limit it must be luminous enough to have observed flux higher than the minimum flux limit of the survey. This removes many faint galaxies from the original galaxy sample and makes the number density of galaxies almost constant with redshift.

To assign weights to galaxies, the radial selection function $\phi(z)$ is used. It is defined as the number density of galaxies $\rho(z)$ as a function of redshift, normalized to the mean galaxy number density $\bar{\rho}$,

$$\phi(z) = \frac{\rho(z)}{\bar{\rho}} \quad (2.1)$$

2.1.2 Angular Selection Functions and Masks

Malmquist bias describes the observational bias in the radial direction, but there are selection biases present in the survey as a function of the angular direction in the sky also. This bias is known as *angular selection function*. The completeness C of a spectroscopic survey, which is defined as the ratio of the observed spectroscopic galaxies and the targeted photometric galaxies for spectroscopy per unit sky area,

$$C = \frac{\text{Number of observed galaxies}}{\text{Total number of targeted galaxies}} \quad , \quad (2.2)$$

decides the value of angular selection function in different directions.

The completeness of a survey is best described by the survey masks. Survey mask also gives the additional information of the sky regions which are outside the observing window, not observed because of the vicinity of a bright stars, ignored due to bad observing conditions, etc. This information is very important to conduct any statistical analysis on the observed galaxy sample. To make the angular completeness masks, division of the sky plane can be done using either equal area and shape preserving pixelization (HEALPix - Hierarchical Equal Area isoLatitude Pixelization: [Górski et al. \(2005\)](#)) or arbitrary shaped spherical polygons (MANGLE: [Hamilton and Tegmark \(2004\)](#); [Swanson et al. \(2008\)](#)). The completeness value is then assigned to each pixel or each spherical polygon. Figure 2.1 shows the HEALPix angular mask of CMASS sample of SDSS.

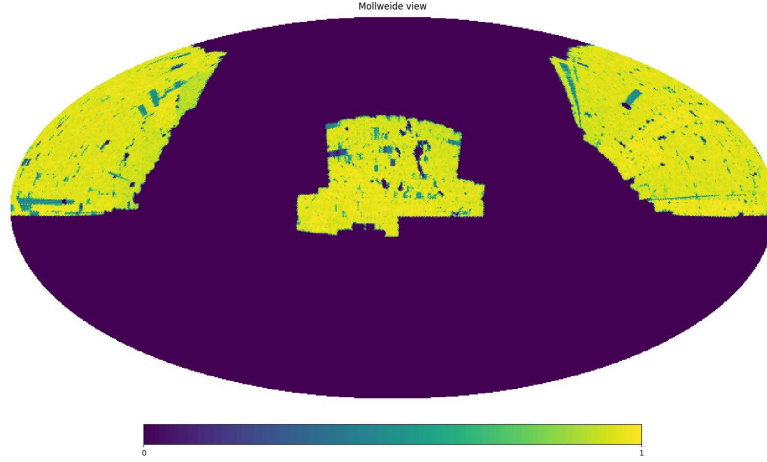


Figure 2.1: HEALPix angular mask, with $N_{side} = 128$, of the CMASS sample of galaxies from BOSS survey of SDSS. The color bar indicates the completeness value.

2.2 K-correction

The expanding Universe with Hubble's law provides the radial distances to galaxies through the measurement of recessional velocities or redshifts. Estimation of the intrinsic luminosity of a galaxy is not a straight forward task even if we know the radial distance to the galaxy because of the incomplete information of the spectral energy distribution.

The absolute magnitude M_Q (or luminosity) of a source in the sky observed through bandpass filter Q is given by

$$M_Q = m_Q - DM \quad (2.3)$$

where, m_Q and DM are the apparent magnitude of the source in the bandpass filter Q and distance modulus, defined by

$$DM = 5 \log_{10} \left[\frac{D_L}{10pc} \right],$$

respectively. Here, D_L is the luminosity distance. Equation 2.3 is another form of inverse-square law for radiation and is true for a non-expanding Universe. In an expanding Universe, the wavelength λ_e of light emitted by a source at redshift z increases and is observed as wavelength λ_o ,

$$\lambda_o = (1 + z)\lambda_e$$

Therefore, equation 2.3 can not be used to calculate the absolute magnitude, because now m and M correspond to different bandpass filters. A

correction term, known as 'K-correction' (Hogg et al., 2002) must be applied to equation 2.3 to get the absolute magnitude in the same band.

Consider the apparent flux of the source measured through a finite observed-frame bandpass R and the intrinsic luminosity through a finite emitted-frame bandpass Q . The K-correction is used in relating these two quantities.

If the source is observed to have apparent magnitude m_R in photometric bandpass R , then its absolute magnitude M_Q in the rest-frame bandpass Q is,

$$M_Q = m_R - DM - K_{QR} \quad (2.4)$$

where, K_{QR} is the K-correction, the transformation between observed and rest-frame broad-band photometric measurements. K-correction must be applied to all galaxies to get correct absolute magnitudes.

2.3 Friends of Friends Algorithm

The *friends-of-friends* (FoF) algorithm is widely used to identify overdense structures in the galaxy distribution (Huchra and Geller, 1982; Press and Davis, 1982; Einasto et al., 1984; Berlind et al., 2006; Tempel et al., 2014) and in numerical simulations (Knebe et al., 2011) based on physical proximity. These overdense structures may be virialized galaxy clusters, galaxy groups, halos of dark matter particles, or structures of galaxies/halos larger than clusters. In order to avoid confusion with the conventional galaxy clusters, these overdense structures shall be called 'FoF clusters'.

In FoF algorithm, a threshold distance, called linking length (l), is chosen for the galaxy distribution. If the separation between two galaxies is less than or equal to l , these two galaxies are considered linked and part of the same FoF cluster, otherwise not. This implies that FoF clusters cannot intersect, which means that a galaxy can be assigned uniquely to just one FoF cluster. Defined in this manner, the size of a FoF cluster naturally becomes sensitive to the linking length used to determine it. The choice of l depends on the goals of the specific study. In Chapter 3, the aim is to find as many overdensities of galaxies as possible. Therefore, the l for a given galaxy distribution is chosen such that a maximum number of FoF clusters is obtained, given a predetermined limit to the minimum number of galaxies required in a FoF cluster.

For a volume limited sample of galaxies, where the number density of galaxies remain almost constant with redshift, a constant linking length is appropriate. On the other hand, if the galaxy sample is not volume limited, different linking lengths are used for different galaxy pairs. The linking lengths in these cases vary according to the selection biases in the galaxy

sample and proper weights should be given accordingly. For example, intrinsic luminosities and the number density of galaxies are used in chapter 3 as a measure of weights for linking lengths.

2.4 Boundness of a Spherical Overdensity - Turn Around Radius

An important scale for structures which can tell us whether a given overdensity is collapsing or will ultimately collapse in the future is the turn-around radius of a spherical structure. The turn-around radius is approximately twice the virial radius (the radius of a spherical region within which the density is $\simeq 200$ times the critical density of the Universe) of a spherical overdensity. It is the transition surface inside which the matter is in accreting phase and outside, the matter expands with the global expansion rate of the universe. This can be considered as the boundary of the structure and the acceleration of the shell at the turn-around radius is zero.

Consider a spherically symmetric mass overdensity of radius r with mass M . The density outside is equal to the average density of the Universe. Because of the symmetry, the evolution of this overdensity with a matter density ρ_m can be considered as the evolution of an independent mini-universe. The growth, then, will be governed by the second Friedmann equation,

$$\begin{aligned}\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) \\ &= -\frac{4\pi G}{3}(\rho + 3\omega\rho) \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(1 + 3\omega)\rho\end{aligned}$$

But we have different energy components in the Universe, so the above equation becomes,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (1 + 3\omega_i)\rho_i$$

Here, $\omega_m = 0$ and $\omega_\Lambda = -1$ for dark matter and dark energy components, respectively. So,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m - 2\rho_\Lambda)$$

At the turn-around radius $R = R_{ta}$, $\ddot{a} = 0$. So, the above equation becomes

$$\begin{aligned}
 0 &= -\frac{4\pi G}{3}(\rho_m - 2\rho_\Lambda) \\
 \rho_m &= 2\rho_\Lambda \\
 \frac{M}{\frac{4}{3}\pi R_{ta}^3} &= 2\rho_\Lambda \\
 R_{ta} &= \left(\frac{3M}{8\pi\rho_\Lambda} \right)^{\frac{1}{3}}
 \end{aligned} \tag{2.5}$$

Equation 2.5 can be further simplified to two different forms. The first one is by using the relation between the dark energy density ρ_Λ and the cosmological constant Λ ,

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$$

which gives

$$R_{ta} = \left(\frac{3GM}{\Lambda c^2} \right)^{\frac{1}{3}} \tag{2.6}$$

The second one is obtained by representing R_{ta} as a function of M with the cosmological parameters H_o and $\Omega_{\Lambda o}$. Equation 2.5 can be re-written as

$$\begin{aligned}
 R_{ta} &= \left(\frac{3GM}{8\pi G\rho_\Lambda \times \frac{\rho_{co}}{\rho_{co}}} \right)^{\frac{1}{3}} \\
 R_{ta} &= \left(\frac{GM}{\frac{8\pi G\rho_{co}}{3} \frac{\rho_\Lambda}{\rho_{co}}} \right)^{\frac{1}{3}}
 \end{aligned}$$

And since, $H_o^2 = 8\pi G\rho_{co}/3$ and $\rho_\Lambda = \rho_{\Lambda o}$,

$$R_{ta} = \left(\frac{GM}{H_o^2 \Omega_{\Lambda o}} \right)^{\frac{1}{3}} \tag{2.7}$$

Equations 2.6 and 2.7 represent the physical turn-around radius of a spherical overdensity. The comoving turn-around radius will be,

$$R_{ta,co} = (1+z)R_{ta}$$

2.5 Density Field and Tessellation of Space

The environment of the cosmic web has different components - clusters, filaments, walls, and voids. These are classified according to the density of these regions with respect to the mean density field. From observations and simulations, it has been noticed that the evolution and properties of a galaxy depend on the cosmic web component in which the galaxy resides. Also, the general motion of a galaxy depends on the cosmic web environment - galaxies in voids move towards the walls, galaxies in walls move towards filaments, galaxies in filaments move towards clusters, and the galaxies in a clusters either move towards the center of the cluster or around it depending on whether the cluster has been virialized or not. Therefore, it is important to know the density field of the galaxy distribution.

The construction of a density field from the discrete distribution of galaxies is traditionally done using a uniform three-dimensional density grid. This density grid divides the space into uniform cubes. The number density of galaxies is then calculated by counting the galaxies in those grids/cubes followed by the smoothing of this field using a kernel of some specified size.

The challenge in the estimation of density field using the grid approach is that the resultant density field depends on the resolution of the density grid and the choice of the kernel. For the same distribution of galaxies, the results may vary significantly with different choices of grids.

In the last few decades, a novel and powerful mathematical tool has been increasingly used and gaining popularity to overcome the above mentioned problem. This is a parameter free tool and is called the *Voronoi tessellation*. The dual of Voronoi Tessellation is *Delaunay triangulation*.

2.5.1 Voronoi Tessellation

The Voronoi tessellation ([Voronoi, 1908](#)) is a unique division of space into non-uniform cells (instead of uniform grid of cubes) based on the discrete distribution of galaxies. Each galaxy is associated with a unique Voronoi cell which is defined as the part of space whose all infinite points are closest to the associated galaxy in comparison to any other galaxy. Therefore, the voronoi tessellation only depends on the distribution of galaxies and there is no requirement of any input parameter (unlike the grid size in density grid approach) here. For different degrees of clustering or spatial correlations between galaxies, the corresponding tessellation presents a completely different aspect. Although we have defined Voronoi tessellation for the distribution of galaxies, it can be defined for any distribution of points set. For the two-dimensional distribution of points, let us call them 'seeds' to differ-

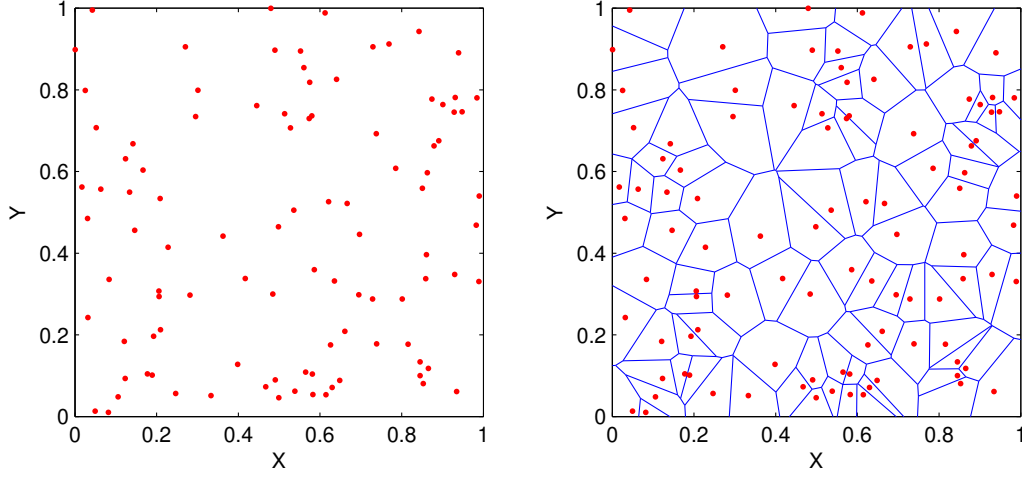


Figure 2.2: Voronoi tessellation for a 2D distribution of random points. *Left* : A distribution of random points. *Right* : Voronoi tessellation of space by Voronoi cells in the shape of polygons.

entiate from other spatial points, the corresponding Voronoi cells are convex polygons. The points on an edge of a polygons will be equidistant from the seeds on either side of the edge and a vertex will be equidistant to all seeds whose cells surround that vertex. Figure 2.2 shows the Voronoi tessellation corresponding to a distribution of random points. For a three-dimensional distribution of points, the Voronoi cells will be polyhedra.

This parameter free division of space, now, can be used to calculate density around each galaxy. The number density of a Voronoi cell will be the inverse of the volume of that cell because each cell contains only one galaxy in it. From this reconstruction of density field, further classification of cosmic web components can be done.

2.5.2 Delaunay Triangulation

For a given two-dimensional distribution of points, a Delaunay triangulation (Delaunay, 1934) is a triangulation such that the circumcircle of each triangle does not enclose any particle in the distribution. The centers of each circumcircle correspond to the Voronoi vertices of the same distribution. Figure 2.3 shows the Delaunay triangulation of a random distribution of points. In three-dimensional distribution of points, triangles are replaced by tetrahedra and circumcircle of triangle is replaced by circumsphere of tetrahedron. In principle, Delaunay triangulation can also be used to construct a parameter free density field (Schaap and van de Weygaert, 2000). In that case, the volume $V_{D,i}$ associated with an i^{th} seed/galaxy is the sum of volumes of all tetrahedra of which the galaxy is a vertex. The density at the i^{th} galaxy

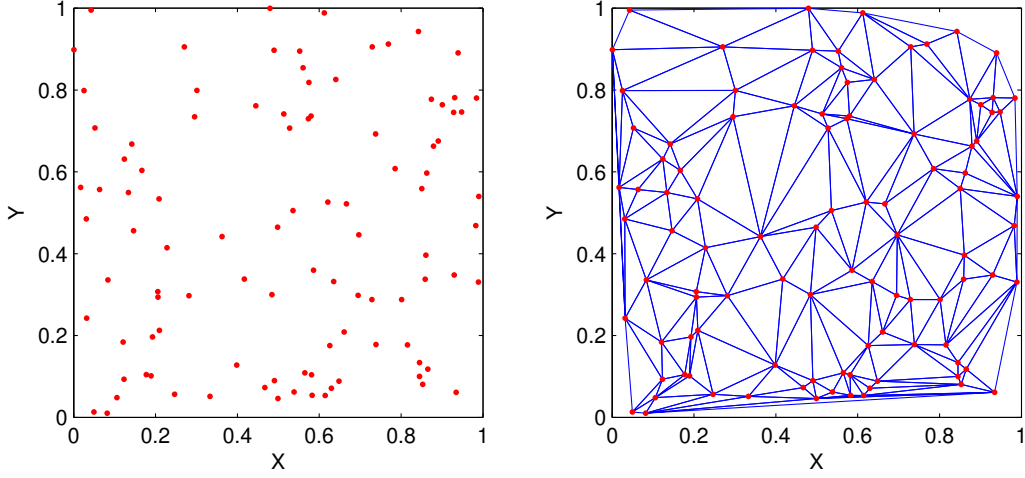


Figure 2.3: Delaunay triangulation for a 2D distribution of random points. *Left* : A distribution of random points. *Right* : Delaunay triangulation of the distribution of random points.

will then be $\rho_i = 4/V_{D,i}$. The factor of 4 comes because each tetrahedron will be used 4 times, once for each tetrahedron vertex, in the density estimation. Because each Delaunay vertex/galaxy is associated with more than one Delaunay cell (tetrahedron), in the estimation of density field, almost every cell will be used more than once. This will increase the computation time of the density estimation and, therefore, Delaunay tessellation density estimation will be slower than the Voronoi tessellation density estimation. On the other hand, the information of Delaunay edges gives us the natural neighbours of every galaxy (adjacent voronoi cells), this way of finding neighbouring Voronoi cells is computationally faster.

Voronoi tessellation and Delaunay triangulation are dual graphs of each other. This means that if we have the information of either of them, the other one can be constructed. Figure 2.4 shows the duality of Voronoi and Delaunay maps.

2.6 Void Finder - Modified ZOBOV

Now, from the reconstructed density field of galaxies using Voronoi tessellation, we extract voids - the underdense regions of the density field. For this, we use ZONES Bordering On Voidness (ZOBOV) algorithm of [Neyrinck \(2008\)](#). The ZOBOV algorithm includes the creation of zones around the local density minima and uses watershed concept to merge zones. This gives a full hierarchy of voids. We have restricted our void finding method to creating zones only. This way, we get maximum number of disjoint voids.

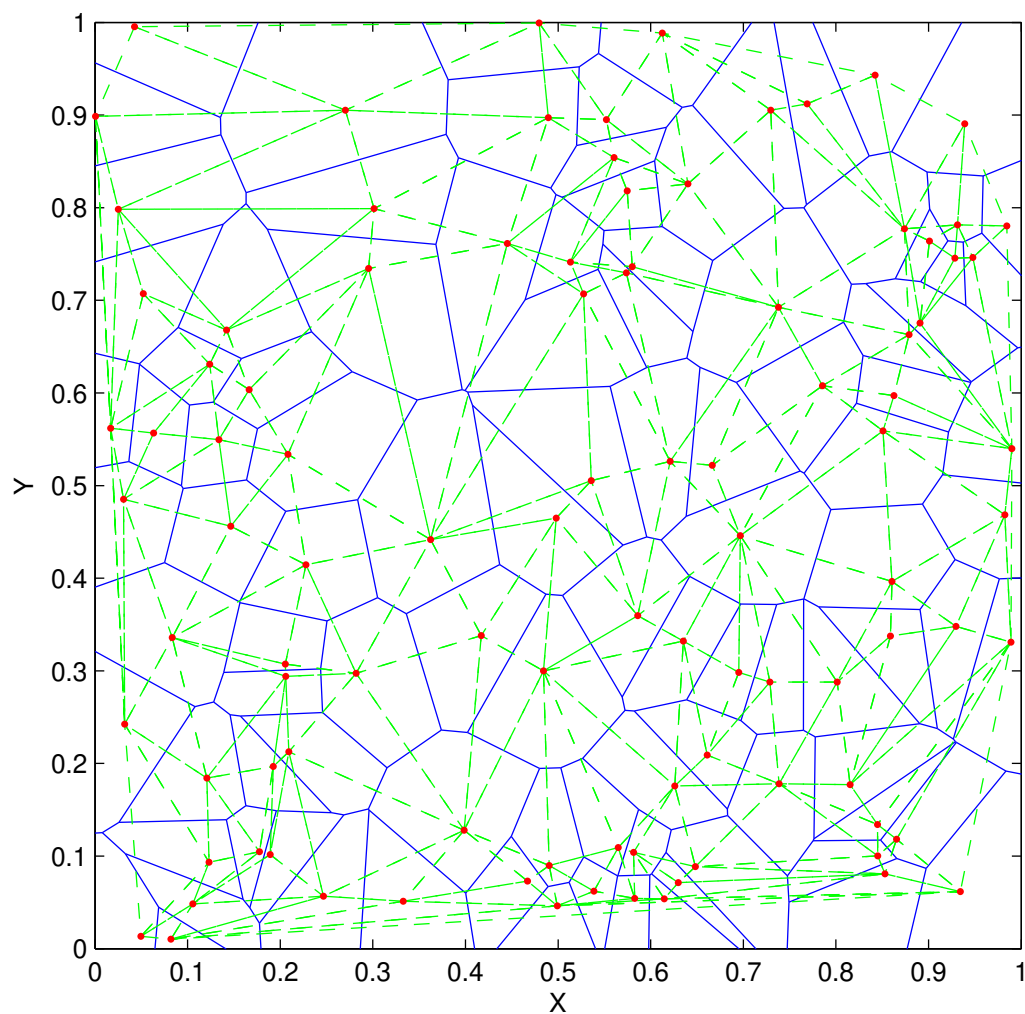


Figure 2.4: Voronoi tessellation (solid blue) and Delaunay triangulation (dashed green) for a 2D distribution of random points (red dots).

2.6.1 Zones

For a given set of positions of galaxies, Voronoi tessellation divides the three-dimensional space into Voronoi cells with exactly one galaxy per cell. The local number density of a cell of i th galaxy is, $\rho_i = 1/V_i$, where V_i is the volume of the Voronoi cell associated with the i th galaxy. The first step is to find the local density minima. The i th galaxy is the site of a local density minimum if the density of its Voronoi cell is less than the densities of the adjoining Voronoi cells. To form zones, all the galaxies will move from high density Voronoi cell to their adjacent low density Voronoi cell with maximum density difference. The galaxy will not move if it is a local density minimum. The set of galaxies which flows to a local density minimum will form a zone. The galaxy with minimum density in the zone is called the *core* of the zone. These zones can be considered as voids or further merging of zones can be done to form bigger voids depending on the density of lowest watershed peak between two zones. In literature, there is not a common set of criteria to merge the voids (e.g., [Neyrinck \(2008\)](#); [Sutter et al. \(2012\)](#); [Nadathur and Hotchkiss \(2014\)](#); [Mao et al. \(2017\)](#)). Generally, the chosen criteria rarely allows zones merging. Therefore, zones themselves are a good representation of voids ([Nadathur, 2016](#)) and we do not further merge the zones. This also retains the inherent topology of the density field. From the voids obtained using this method, the voids with core density greater than the mean density (a small fraction) are ignored, as these voids are spurious.

2.6.2 Modified ZOBOV

The above mentioned method is good enough to find voids in a numerical simulation data with simple geometric and periodic boundaries. However, the void detection in the observational data, having biases and complex survey geometry, is not that straightforward. We implement some modifications in the ZOBOV algorithm, similar to the procedure adopted by [Nadathur \(2016\)](#), to account for the observational biases.

Density Field Modification for Radial Selection Function

Unlike simulations, the number density of galaxies in the galaxy redshift surveys varies as a function of redshift due to Malmquist bias (see section [2.1.1](#)) or because of the imposition of certain color cuts. Overall, the number density decreases as a function of redshift. This means that the Voronoi cells at higher redshifts will have artificially large sizes. Therefore, the density of a Voronoi cell must be modified accordingly.

Consider N galaxies distributed in an observational volume V . The mean number density of the survey will be, $\bar{\rho} = N/V$. The modified local normalized density around i th galaxy will be,

$$\frac{\rho_i}{\bar{\rho}} = \frac{V}{N V_i \phi_i} \quad , \quad (2.8)$$

where, ϕ_i is the value of radial selection function of the survey at the redshift of the i th galaxy.

Density Field Modification for Angular Selection Function

The completeness of a galaxy redshift survey varies with the angular position. If, in a particular direction, the completeness value C is less than 1, the survey had failed to take spectra of some fraction of the targeted galaxies. This produces a non-uniformity in the number density of galaxies on the sky plane, as the targets are chosen so that the angular number density remains uniform. To account for this angular incompleteness, we apply angular completeness weights to the density of Voronoi cells. Therefore, the modified local normalized density around i th galaxy now becomes,

$$\frac{\rho_i}{\bar{\rho}} = \frac{V}{N V_i \phi_i C_i} \quad , \quad (2.9)$$

where, C_i is the value of the completeness of the survey in the direction of the i th galaxy.

Accounting for Voronoi Tessellation Leakage

The survey masks of a galaxy redshift surveys are complicated and contain holes (see Figure 2.1). Also, there are redshift limits of the survey. When we apply Voronoi tessellation on the observational data, the galaxies near the boundaries of the mask, redshift limits, and holes have ambiguous Voronoi cells because of the leakage of Voronoi cells through the edges of the survey.

To stop this leakage in these regions, we put high density buffer particles along the edges. The density of buffer particles is set to be 10 times higher than the mean density of the survey. It is seen that the buffer densities lower than this results in the Voronoi leakage near the edges of the survey (Nadathur, 2016). After adding buffers to the galaxy data, we apply Voronoi tessellation and Delaunay triangulation and get the volumes of the Voronoi cells and their natural neighbours. Even though the leakage of Voronoi cells of the edge galaxies stops, the edge galaxies have artificially high density. All the galaxies which are neighbours to the buffer particles are flagged as edge

galaxies and are ignored before applying the ZOBOV algorithm. Also, all the galaxies which are neighbours to the edge galaxies are flagged. A void is flagged as edge void if it contains any of these galaxies. The true extent of these edge voids are unknown and must not be included in any cosmological analyses.

2.6.3 Void Properties

After applying the modified ZOBOV algorithm, we get a catalogue of voids. We now define some properties of voids.

Center of the Void

The center of the void is defined as the volume weighted barycenter of the galaxies in the void. This places the center more towards the largest Voronoi cell in the void and thus in the lowest density region. The position of the center of the void, therefore, is

$$\mathbf{X}_{center} = \frac{\sum_i \mathbf{x}_i V_i}{\sum_i V_i} \quad (2.10)$$

Effective Radius and Volume

The volume of the void is the sum of Voronoi cells of the galaxies in the void, $V_v = \sum_i V_i$. The effective radius of the void is defined as

$$R_{eff} = \left(\frac{3V_v}{4\pi} \right)^{\frac{1}{3}} \quad (2.11)$$

This is just the representation of the volume/size of the void and doesn't mean that the void has a spherical shape. In fact, the void can have arbitrary shape depending on its surrounding high density environment.

Density Contrast

Apart from their sizes, voids can equally and importantly be characterized by their density contrasts, since they reveal the voids with different depths. We use two measures of the density contrast of voids, core density contrast and average density contrast.

The core density contrast or the minimum density contrast within a void is defined as the density contrast of the core galaxy of the void,

$$\delta_{core} = \frac{\rho_{core}}{\bar{\rho}} - 1, \quad (2.12)$$

where, ρ_{core} is the density of the core galaxy. δ_{core} measures how deep is the void.

The average density contrast of the void is defined as the volume weighted average density contrast,

$$\bar{\delta}_{void} = \frac{1}{\bar{\rho}} \frac{\sum_i \rho_i V_i}{\sum_i V_i} - 1, \quad (2.13)$$

where the sum runs over all Voronoi cells of the void.

2.7 High Density Regions using Modified ZOBOV

In the previous section, we applied the modified ZOBOV to find the voids around density minima. We now apply same modified ZOBOV algorithm to find high density regions (HDR) around density maxima. Here, instead of galaxies moving from high density to the lowest density Voronoi neighbour, they will move from low to high density cells. This will form high density zones around local density maxima. Practically, this can be done with the watershed algorithm running on the inverse of the density of the Voronoi cells. The volume and effective radius of the HDRs is calculated as described in the previous section. The center of the HDR is defined as the galaxy in the HDR with maximum density.

2.8 Two-point Correlation Function

Gravitational instability and dark energy dictates the spatial distribution of galaxies in the Universe and sculpt it in the form of cosmic web, as seen in galaxy redshift surveys and simulations. It is not a uniform random point distribution, at least on scales less than the homogeneity scale of the Universe. In order to compare different cosmological models, the galaxy distribution in various models can be used. The two-point correlation function is the most commonly used estimator of the galaxy distribution in the large scale structure of the Universe (Coil, 2013).

The two-point correlation function $\xi(r)$ is a measure of galaxy clustering amplitude as a function of separation r . It is defined as the excess probability fraction, than what is expected from a uniform random distribution of

galaxies, of finding a galaxy in the volume element dV_1 and another galaxy in the volume element dV_2 at a separation of $(r \pm \delta r/2)$. Therefore, the joint probability dP of finding two galaxies in dV_1 and dV_2 is given by,

$$dP = n^2[1 + \xi(r)]dV_1dV_2 \quad (2.14)$$

where n is the number density of galaxies (Peebles, 1980).

The two-point correlation function is calculated by counting the number of galaxy-galaxy pairs within a certain separation range and then dividing the obtained number counts by expected number counts from a random galaxy distribution. For this, the construction of a random catalog that has the identical 3D coverage as the galaxies in the survey data is required. This random catalog must have the same sky coverage and same shape of the number density as a function of redshift as that of data, but populated by randomly distributed points. Davis and Peebles (1983) gave an early estimator of $\xi(r)$, which remained popular until the estimators introduced by Hamilton (1993) and Landy and Szalay (1993). Among these three popular estimators, Landy-Szalay (LZ) estimator is the most widely used estimator because of its better performance, less sensitivity to the random catalog's size, and better edge corrections. All these factors affect the clustering analysis on large scales (Kerscher et al., 2000).

The LZ estimator is defined as,

$$\xi(r) = \frac{DD(r) - 2DR(r) + RR(r)}{RR(r)} \quad (2.15)$$

where DD , DR , and RR are the normalized data-data, data-random, and random-random galaxy pair counts within the separation $r \pm \delta r/2$. The normalized pair count is the number of pairs within a separation of $r \pm \delta r/2$ divided by the total number of pairs. Suppose there are N_D data points and N_R random points. If $N_{DD}(r)$, $N_{DR}(r)$ and $N_{RR}(r)$ are the numbers of pairs of data-data, data-random and random-random pairs, having separation between $r \pm \delta r/2$, then the normalized pair counts will be

$$DD(r) = \frac{N_{DD}(r)}{(N_D(N_D - 1)/2)}, \quad DR(r) = \frac{N_{DR}(r)}{(N_D N_R)}, \quad RR(r) = \frac{N_{RR}(r)}{(N_R(N_R - 1)/2)}$$

In methods of comparing the data points to a random set, the effect of Poisson noise in the randoms have to be considered. In the LZ estimator, there will be noise in the data-random (DR) and random-random (RR) counts that would go to zero in the limit of infinite numbers of random points. Therefore, the use of many more randoms than data is preferable (Slepian and Eisenstein, 2015).

Instead of galaxies as data points, we can also take centers of voids and HDRs to find the clustering amplitude of void-void and HDR-HDR distri-

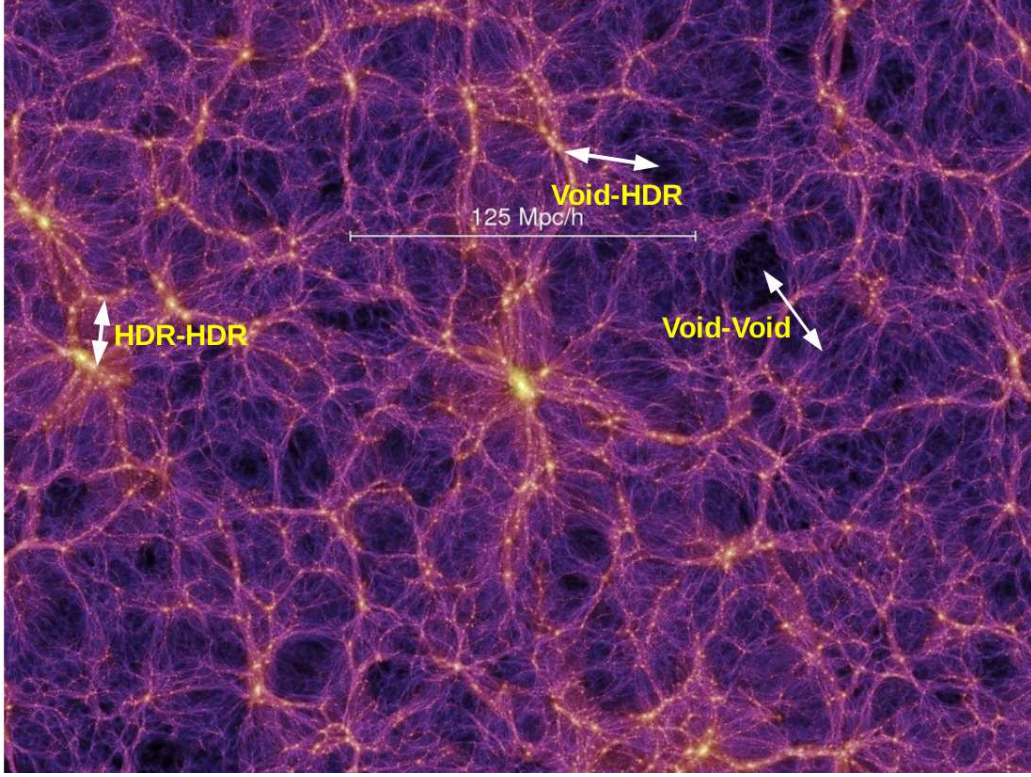


Figure 2.5: The distribution of dark matter in a snapshot of Millenium Simulation (Springel et al., 2005) at $z = 0$. The distances among voids and HDRs are represented by the white arrows. These distances are used to calculate the high correlation lengths in the SDSS data in Chapter 5.

bution. We can also measure the cross-correlation of voids and HDRs (Figure 2.5¹).

The corresponding two-point cross-correlation function will be,

$$\xi_{VH}(r) = \frac{D_V D_H(r) - D_V R_H(r) - D_H R_V + R_V R_H(r)}{R_V R_H(r)} \quad (2.16)$$

where D_V , D_H , R_H , and R_V denote the data void, data HDR, random void, and random HDR distribution. $D_V D_H$, $D_V R_H$, $D_H R_V$, and $R_V R_H$ are the corresponding normalized pair counts within the separation $r \pm \delta r/2$.

¹<https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/>

Chapter 3

Discovery of Saraswati Supercluster

The work in this chapter is derived from Bagchi, J., Sankhyayan, S., Sarkar, P., et al. 2017, ApJ, 844, 25.

3.1 Introduction

Superclusters of galaxies are believed to be the largest concentrations of matter in the Universe whose origin is still being debated (Abell, 1961; de Vaucouleurs, 1956; Jõeveer et al., 1978; Einasto et al., 1994). Occasionally spanning over a hundred megaparsecs (Mpc) superclusters usually display a pronounced filamentary or sheet like morphology, and are surrounded by large almost empty voids (Einasto, 2014). One has been able to associate superclusters with a network of galaxies connected to a few rich clusters systems, as well as with denser and more massive structures characterized by the concentration of several very massive clusters. Contemporary catalogs of superclusters therefore range from relatively smaller systems, such as the Hercules supercluster (Abell, 1961) to the much more massive *Shapley Concentration* containing several major clusters and numerous groups (Raychaudhury, 1989; Bardelli et al., 2000; Proust et al., 2006). More recently, a new supercluster called *Laniakea*, spanning ~ 160 Mpc across, was identified (Tully et al., 2014) which is home to our Milky Way galaxy. Another large, wall-like structure spanning ~ 250 Mpc was discovered from SDSS-BOSS survey (the *BOSS Great Wall*) at redshift 0.47 (Lietzen et al., 2016).

Unlike clusters, superclusters have not yet virialized and so may retain

some memory of their initial conditions which led to their formation. The assembly of the largest superclusters is still a matter of conjecture, with arguments being supporting both, assembly through gravitational clustering (Park et al., 2012) and the more radical point of view that large superclusters should be viewed as rare objects (Sheth and Diaferio, 2011), which might even challenge the widely held Copernican principle (Liddle, 2003). Therefore large scale matter over-densities represented by clusters and superclusters, especially those identified at early cosmological epochs $z \gtrsim 0.3$, (i) are sensitive probes of the presence of dark matter and dark energy at higher redshifts, (ii) shed light on how structure might form in the universe and (iii) provide model-independent tests of General Relativity on the largest scales by extending solar system tests by eight to ten orders of magnitude (Jain and Khoury, 2010).

The primary focus of this chapter is the discovery of *Saraswati*, an unusually massive and large-scale (~ 200 Mpc) supercluster of galaxies at a mean redshift $z = 0.28$ and its properties, identified in the Sloan Digital Sky Survey (SDSS-III) data (Bagchi et al., 2017). This wall-like supercluster is shown to be associated with at least 43 massive galaxy clusters and groups, of which a few extremely massive ones constitute its bound, central core region. Observations show *Saraswati* to be highly unusual in terms of its morphology, mass, richness and in detection of localized non-thermal radio emission, relating to its complex dynamical state. The importance of this finding has been highlighted for ongoing cosmological studies of dark energy and growth of large scale structures in the Universe. Throughout this chapter, the following cosmological parameters, based on 5-year WMAP results (Komatsu et al., 2009), have been used for calculations; $\Omega_{M0} = 0.279$, $\Omega_{\Lambda0} = 0.721$, $\Omega_{R0} = 8.493 \times 10^{-5}$, $\omega_0 = -1$ and $H_0 = 70.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which corresponds to a projected physical size of 4.47 kpc for an object at redshift of 0.3 extending $1''$ on the sky plane.

3.2 Identifying an Exceptionally Massive and Large-scale Supercluster

Figure 3.1 is a redshift cone distribution of SDSS galaxies, between $z = 0$ and 0.4. The visual inspection of the distribution of galaxies within the redshifts of 0.23 and 0.33 (red colored wedge) shows a large concentration of galaxies at $z \approx 0.28$. It will be shown in the subsequent sections that this large concentration of galaxies is a ~ 200 Mpc scale, massive supercluster (*Saraswati*).

Stripe 82 : The galaxy redshift data used in this work is from the central part of the Stripe 82 region of the Sloan Digital Sky Survey (SDSS-

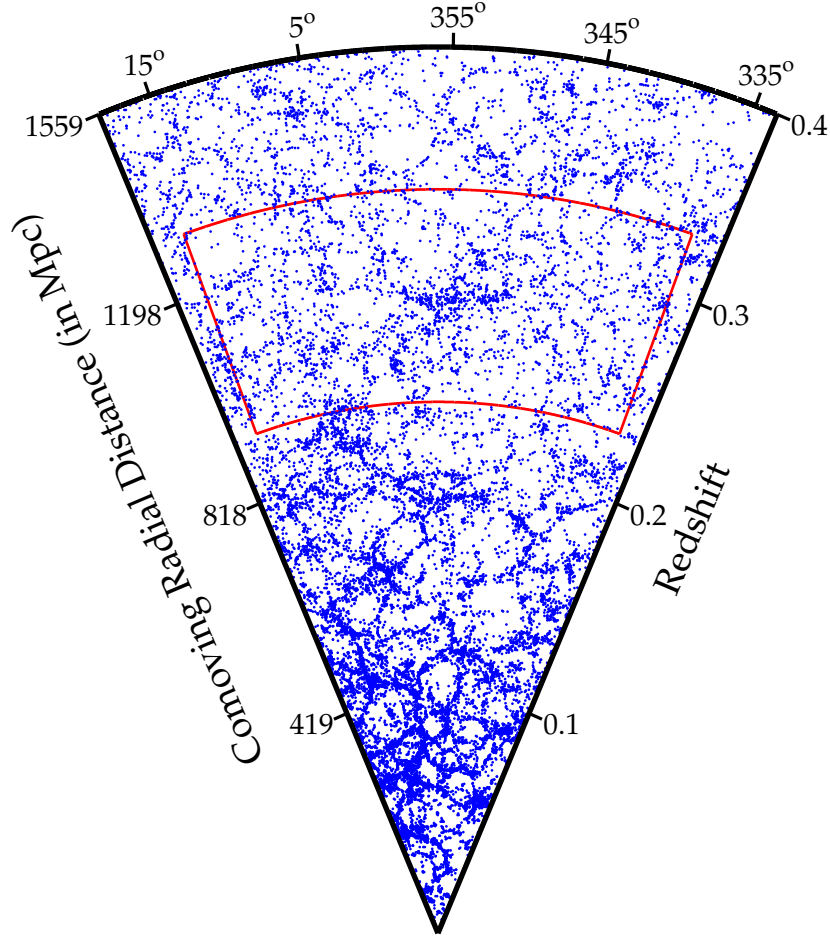


Figure 3.1: A redshift cone distribution of SDSS galaxies is shown (LBS sample; see text), extending from redshift $z = 0$ to 0.4 , $333.5^\circ \leq RA \leq 18.5^\circ$, and $-1.25 \leq Decl. \leq +1.25^\circ$, centered near *Saraswati* supercluster, the prominent large-scale structure within the red box. The small depth in declination direction has been suppressed for clarity.

III DR12) (Stoughton et al., 2002; Alam et al., 2015). Stripe 82 is currently the largest three-dimensional spectroscopic sample of galaxies with a high sampling density, and photometrically up to 2 magnitude deeper than the standard SDSS images. Stripe 82 covers an ~ 270 degree² area along the celestial equator in the Southern Galactic Cap and spans $310^\circ < RA < 59^\circ$ and $-1.25^\circ \leq Dec \leq 1.25^\circ$. A major supercluster was noticed while exploring the large scale distribution of galaxies surrounding an unusual filamentary, merging galaxy cluster ZwCl 2341.1+0000 ($z = 0.27$). This massive cluster, Figure 3.2, also shows large-scale diffuse synchrotron radio emission from structure formation shocks, and thus is a prominent target of several studies (Bagchi et al., 2002; van Weeren et al., 2009; Boschini et al., 2013).

The entire Stripe 82 is not uniformly sampled (spectroscopically), but

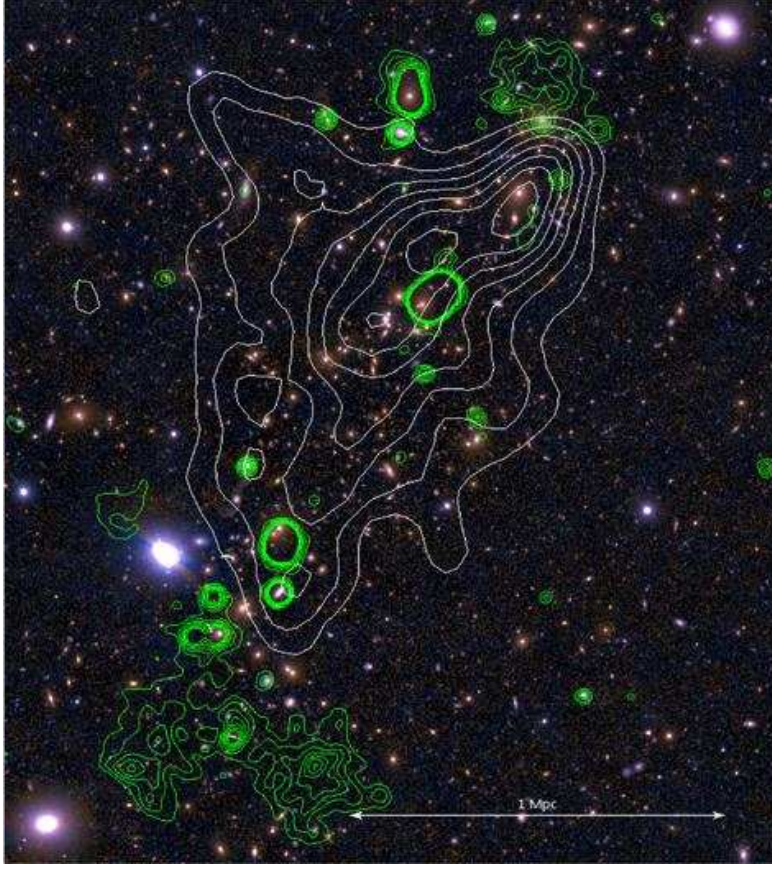


Figure 3.2: Chandra X-ray contours (white) and GMRT radio contours (green) overlaid on Subaru color image of ZwCl 2341.1+0000. The diffuse radio emissions to the northwest and southeast indicate the two radio relics. These relics bracket the elongated and disturbed X-ray emissions, which is typical of a galaxy cluster merger. *Credit: (Benson et al., 2017).*

contains data from several projects. Two largest samples of spectroscopic galaxies are used: the LOWZ sample and the LBS (LEGACY-BOSS-SOUTHERN) sample described below in detail. The absolute magnitudes of galaxies from both samples have been k-corrected using K-CORRECT v4.3 software (Blanton and Roweis, 2007). The K-corrected magnitudes are calculated for rest frame at $z = 0.33$. For the analysis, only those galaxies have been retained which have clean photometry, clean spectra and redshift errors $< 1\%$. Spectra of few galaxies from the samples are shown in Figure 3.3.

The apparent magnitudes are SDSS *cModel* magnitudes corrected for extinction. *cModel* magnitudes¹ are composite model magnitudes, calculated from the best-fitting linear combination of a de Vaucouleurs and an exponential luminosity profile.

¹<https://www.sdss.org/dr12/algorithms/magnitudes>

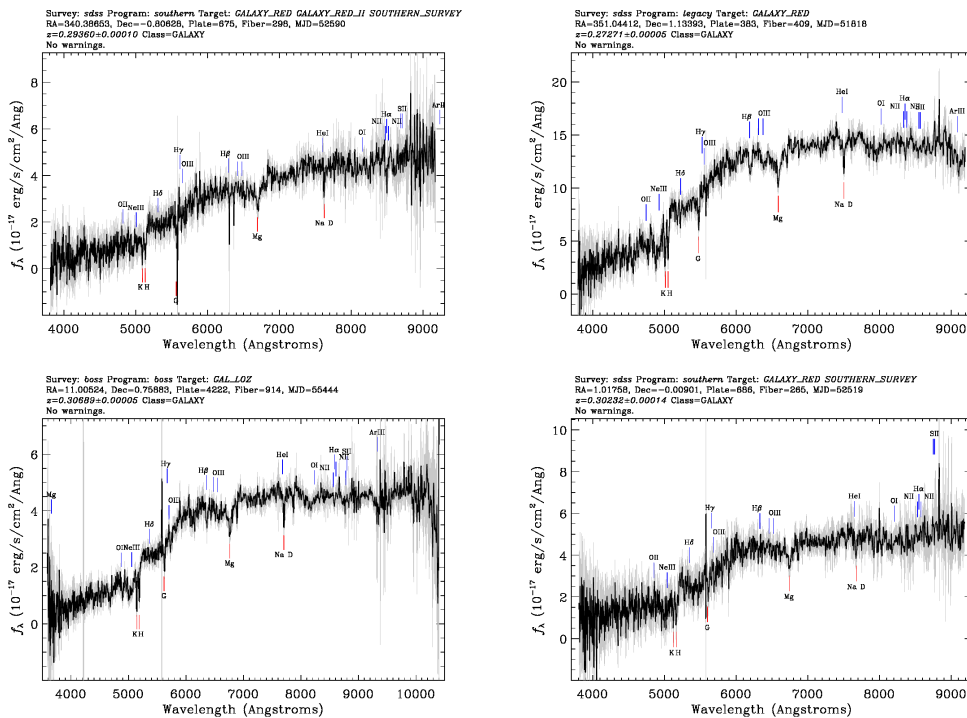


Figure 3.3: Spectra of four galaxies from the samples, taken from SDSS. The header of each spectra gives sky position of the galaxy and its redshift with errors.

3.2.1 LOWZ Spectroscopy Data from SDSS DR12

To evaluate the properties of the new supercluster (at a mean $z = 0.288$) and explore the cosmic-web of galaxies, clusters and voids surrounding it, first, LOWZ data for galaxies is extracted from the Baryon Oscillation Spectroscopic Survey (BOSS) (Ahn et al., 2012) of SDSS DR12, within a wedge of R.A. and Dec. range $336^\circ \leq RA \leq 16^\circ$, $-1.25^\circ \leq Dec. \leq 1.25^\circ$ (limits of survey) and spectroscopic redshift range $0 \leq z \leq 0.6$, within the Stripe 82 central region. Due to a bug in the target selection, LOWZ galaxies were incorrectly targeted during the initial stages of the BOSS survey (affecting all data taken until June 2010). In order to form a uniformly targeted sample of LOWZ galaxies, only those galaxies are used whose TILEID ≥ 10324 (Reid et al., 2016). A volume limited galaxy sample is constructed by restricting the cModel r -band absolute magnitude to $M_r \leq -21.53$ and the redshift to $z \leq 0.33$ (Figure 3.4).

From this volume limited sample a sub-sample is made with $336^\circ \leq RA \leq 16^\circ$, $-1.25^\circ \leq Dec. \leq +1.25^\circ$ and $0.23 \leq z \leq 0.33$ (red wedge in Figure 3.1), which is centered on the *Saraswati* supercluster (hereafter called the ‘analysis region’). This way, a total of 625 spectroscopic galaxies in this sub-sample are obtained. For further analysis, the SDSS angular coordinates (α, δ) and

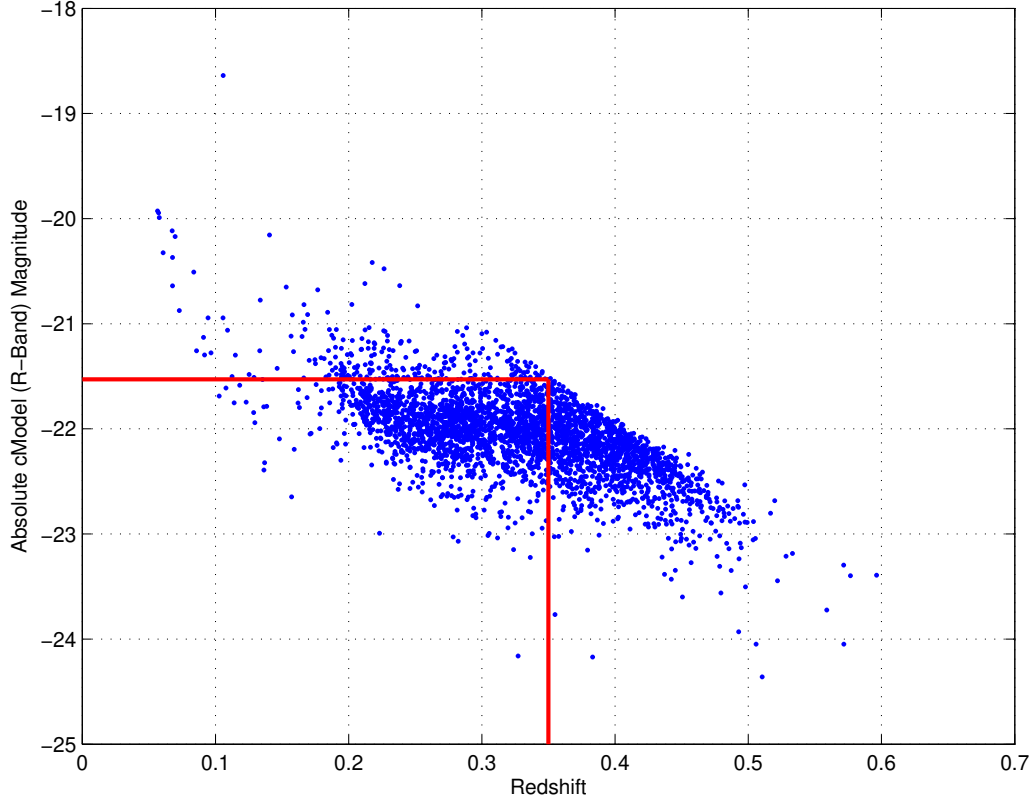


Figure 3.4: Absolute magnitudes and redshifts of LOWZ galaxies (blue dots) upto $Z = 0.6$. A volume-limited sample is made by giving the magnitude and the redshift cuts of $M_r \leq -21.53$ and $z \leq 0.33$.

redshifts (z) of galaxies are converted to comoving Cartesian coordinates (X, Y, Z) according to Equation 3.1. Here, α and δ are RA and Declination, respectively.

$$\begin{aligned} X &= d_c \sin\delta \cos(\alpha + \theta), \\ Y &= d_c \sin\delta \sin(\alpha + \theta), \\ Z &= d_c \cos\delta \end{aligned} \tag{3.1}$$

d_c is the comoving distance to the galaxy and θ is the rotation given in the counter-clockwise direction to align the distribution along the positive y-axis. The two dimensional distribution of these galaxies in comoving coordinates is shown in Figure 3.5.

3.2.2 BOSS-LEGACY-SOUTHERN Data

The LOWZ data is uniformly sampled but it is fairly sparse. In order to obtain higher density of galaxies within the analysis region (as above), all available spectroscopic galaxies with clean spectra and clean photometry taken

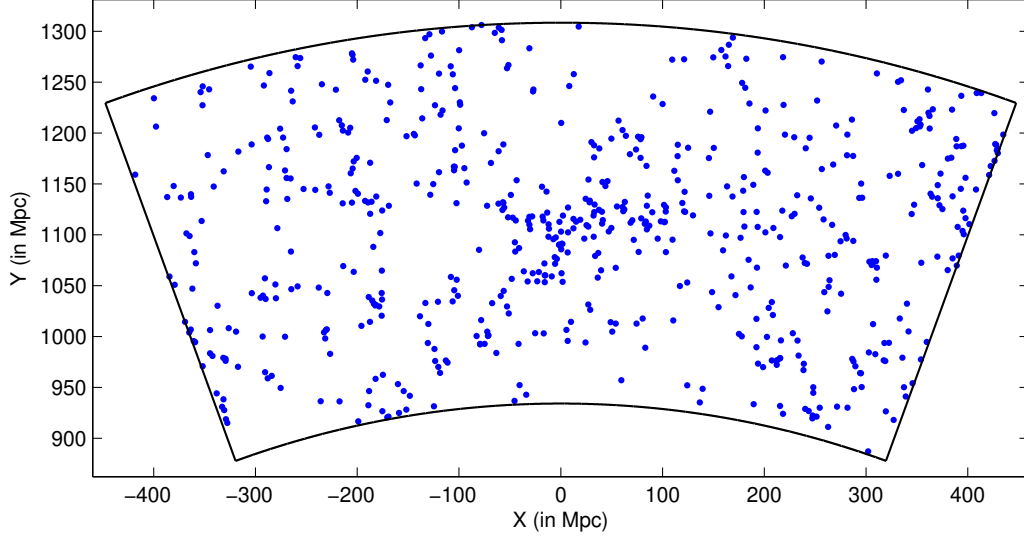


Figure 3.5: Distribution of 625 galaxies in the sub-sample of LOWZ volume limited sample in co-moving coordinates, projected on the 2-dimensional X-Y plane. The small depth in Z-direction (along declination) has been suppressed.

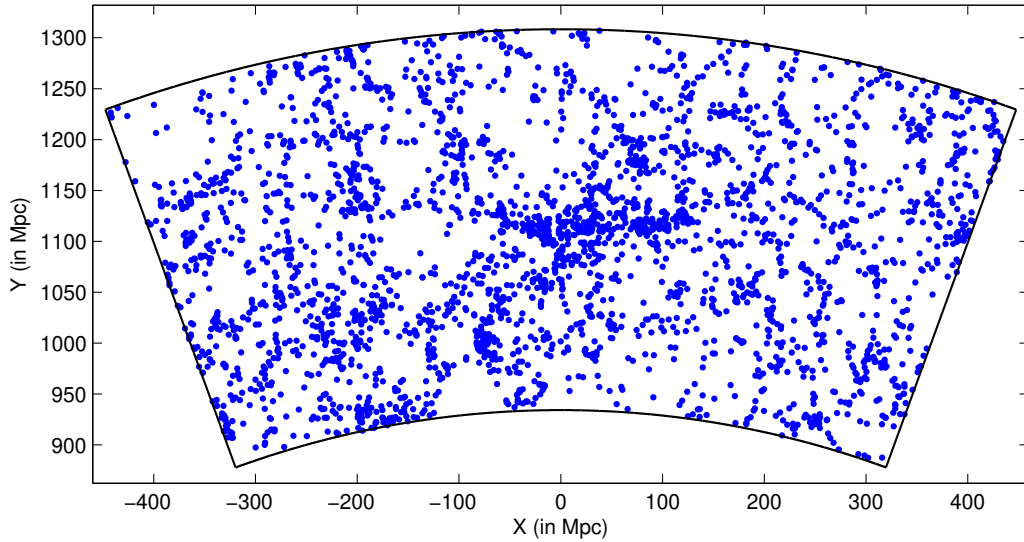


Figure 3.6: The LEGACY-BOSS-SOUTHERN (LBS) sample of 3016 galaxies plotted in comoving coordinates, projected on the 2-dimensional X-Y plane. The small depth in Z-direction (along declination) has been suppressed.

from ‘LEGACY’, ‘BOSS’ and ‘SOUTHERN’ programs of SDSS-III DR12 database are combined. Only these three programs were selected because the galaxies in these programs were deemed to be more or less uniformly distributed within the analysis region. The combined sample of ‘LEGACY’, ‘BOSS’ and ‘SOUTHERN’ programs is also quite uniform in the analysis region. For example, if the region is divided into three equal parts across RA, the mean galaxy counts of these three regions are well within 1 sigma values among each other, making the region quite uniform. This sample gives a total of 3016 galaxies, approximately five times the size of LOWZ sample. Hereafter, this larger sample of 3016 galaxies will be referred as the LBS sample. Figure 3.6 shows the distribution of galaxies from LBS sample within the analysis region in comoving coordinate space.

In both samples, a prominent, large-scale (~ 200 Mpc) density enhancement of galaxies at mean $z \sim 0.28$ and $X \approx 0$ Mpc, and $Y \approx 1100$ Mpc is clearly visible, which identifies the *Saraswati* supercluster, the focus of the present study.

3.3 Results

3.3.1 Identifying Clusters and Superclusters in SDSS Samples

Superclusters are the primary large scale galactic structures made of concentration of smaller galaxy groups and clusters. The identification of superclusters requires extensive spectroscopic redshift data taken over vast areas of the sky, targeting galaxies in both high and low density regions of the cosmic web. The *Friends-of-Friends* (FoF) algorithm has been adopted, which required only one parameter, the linking length l , to identify over-dense structures in the SDSS galaxy samples. These over-dense structures may or may not be virialized structures. For simplicity, these will be called ‘clusters’ in this sub-section and ‘FoF clusters’ elsewhere, but they must not be confused with the physical cluster of galaxies which are virialized.

Clustering Analysis - LOWZ Galaxies

The clustering analysis (finding clusters) is performed on the sub-sample of 625 galaxies from LOWZ data. To classify clusters, the FoF algorithm is applied. The maximum number of 46 clusters, with at least 4 galaxies in a cluster, is obtained at comoving linking length $l \approx 19$ Mpc (Figure 3.7). Hence, 19 Mpc is used as the linking length for the cluster analysis. This reflects the sparse sampling of LOWZ data. The largest cluster found this way is the *Saraswati* supercluster with 100 galaxies in it. Figure 3.8 shows all

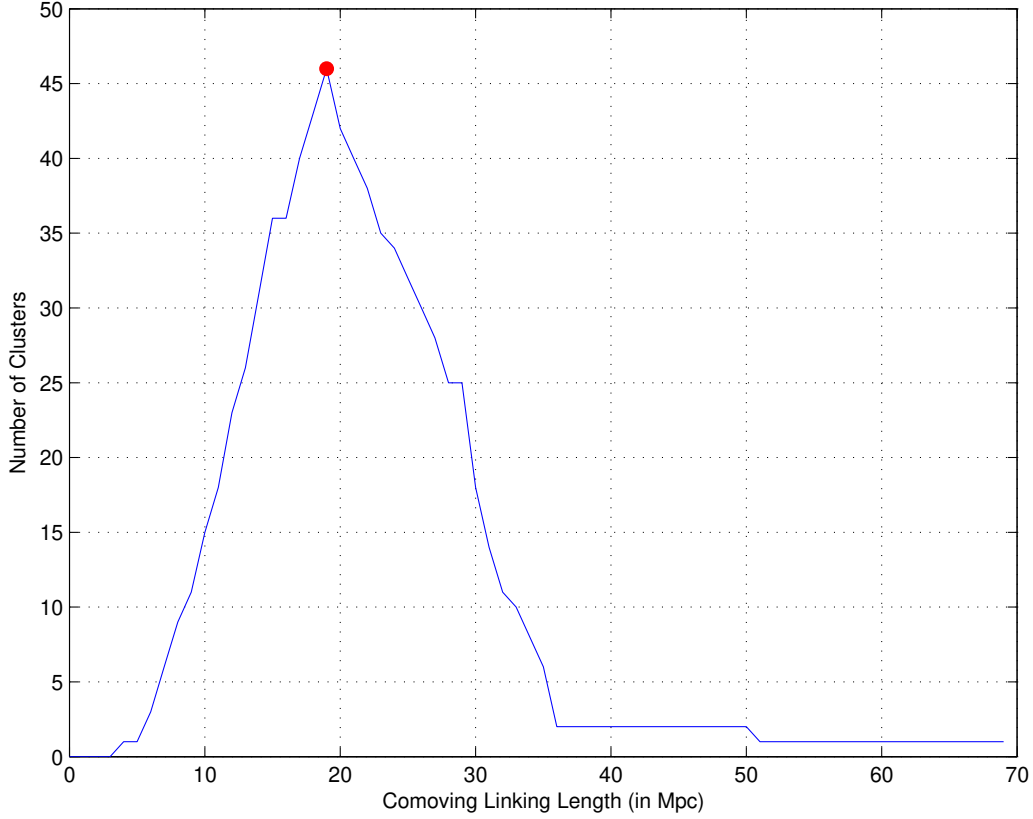


Figure 3.7: Number of FoF clusters obtained for different comoving linking lengths for LOWZ sample with at least 4 member galaxies in a cluster. A maximum number of 46 clusters are obtained at $l \approx 19$ Mpc (red circle). Notice that at the linking length of ≈ 51 Mpc all the galaxies get connected (or percolated).

46 FoF clusters, and the largest supercluster *Saraswati* is shown in blue color at the center of the plot. *Saraswati* spans ~ 200 Mpc of comoving length across our line of sight.

Clustering Analysis - LBS Galaxies

Next, the FoF algorithm is applied on the LBS sample of 3016 galaxies to find clusters. This sample is not strictly uniformly sampled in redshift and there is no fixed upper limit on the observed apparent magnitude (unlike LOWZ sample which has an apparent magnitude limit of $i < 19.9$). The local luminosity function (LF), i.e. the comoving number density per unit absolute magnitude, of the galaxies in LBS sample is used to give proper weights to galaxies so that unbiased clustering analysis can be performed using FoF algorithm. The analysis region is first divided into five constant comoving distance bins and the local luminosity functions of galaxies in each bin are calculated (see Figure 3.9). Then each galaxy (say i^{th} galaxy) is assigned, having absolute magnitude M , a weight w_i according to the values

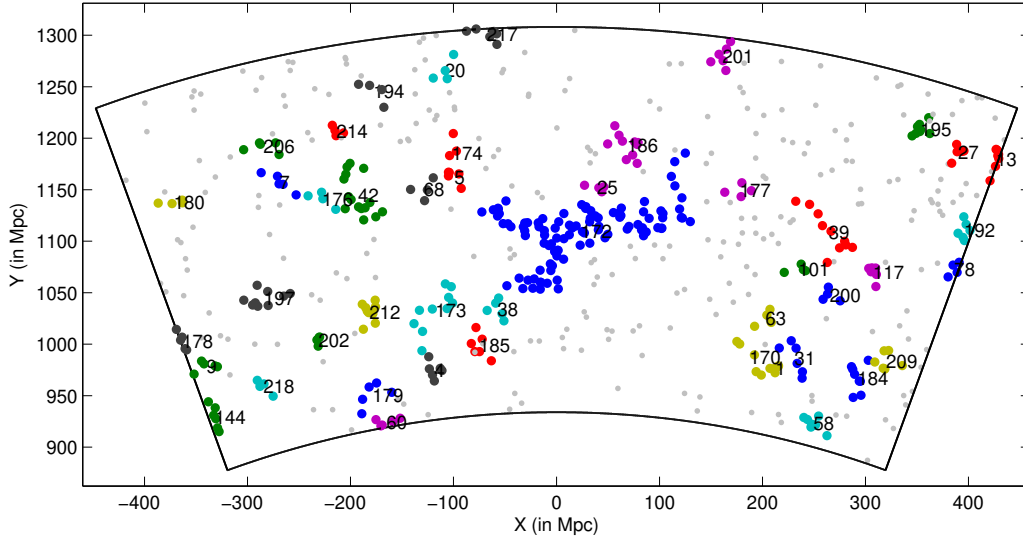


Figure 3.8: Clusters found in the LOWZ subsample by FoF algorithm for comoving linking length of $l = 19$ Mpc. The minimum number of galaxies in a cluster is 4. The clusters are shown in colors and marked with index numbers. In spite of sparse sampling, the largest cluster identified is the *Saraswati* supercluster (index number 172, near the center) shown in blue color, containing ≈ 100 galaxies. Grey dots are galaxies which are not part of any FoF cluster.

of luminosity functions at M ,

$$w_i = \left[\frac{LF_{1,M}}{LF_{n,M}} \right]_i \quad (3.2)$$

where $LF_{1,M}$ and $LF_{n,M}$ are the values of local luminosity functions of the lowest and the n^{th} (the bin in which this i^{th} galaxy is located) comoving distance bins at magnitude M , respectively. $n = 1, 2, 3, 4, 5$ with $n = 1$ for the lowest redshift bin and $n = 5$ for the highest redshift bin. Thus, the weight depends on both, the absolute magnitude and the number density of the galaxy in the comoving distance bin. This will assign a weight $w = 1$ to all galaxies in the lowest bin (Figure 3.10). Next, proper weights are applied on the linking length between two galaxies being linked. If l_o is the selected linking length between galaxy i and galaxy j then the weighted linking length l_{ij} will be

$$l_{ij} = w_{ij} l_o \quad (3.3)$$

where w_{ij} is the weight between i^{th} and j^{th} galaxies,

$$w_{ij} = \frac{w_i^{\frac{1}{3}} + w_j^{\frac{1}{3}}}{2} \quad (3.4)$$

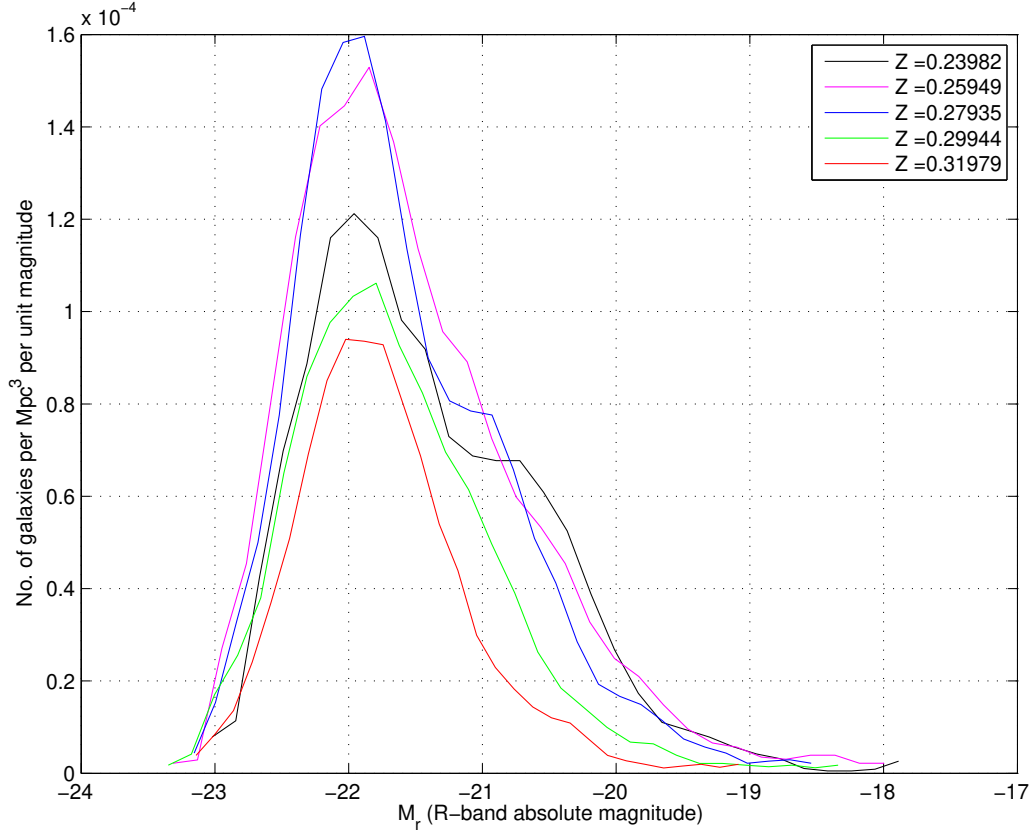


Figure 3.9: Local luminosity functions of LBS sample in five different redshift/comoving distance bins. Different color plots show the comoving number densities of galaxies per absolute cModel r-band magnitude. The mean redshift for each luminosity function plot is shown on top right corner.

where w_i and w_j are the weights associated with i^{th} and j^{th} galaxies according to the local luminosity functions (Equation 3.2 above).

Using this method, the largest number of clusters are obtained at $l_o = 12$ Mpc with minimum number of galaxies in a cluster equal to 10 (Figure 3.11). Using these parameters, a total of 60 FoF galaxy clusters are identified within the LBS analysis region. Once again, the largest cluster with maximum number of linked galaxies, naturally comes out to be the *Saraswati* supercluster with ~ 400 galaxies in it. In Figure 3.12, all the identified clusters are shown with various colors and different index numbers. The *Saraswati* supercluster spanning ≈ 200 Mpc across the line of sight ($\approx 9^\circ$) stands out prominently in the middle of the plot. Moreover, even in limited sampling one gets a clear visual impression of the rich tapestry of filaments, galaxy clusters and large voids of $\approx 40 - 170$ Mpc diameter surrounding the supercluster.

Figure 3.13 shows the FoF clusters using fixed linking length of 12 Mpc

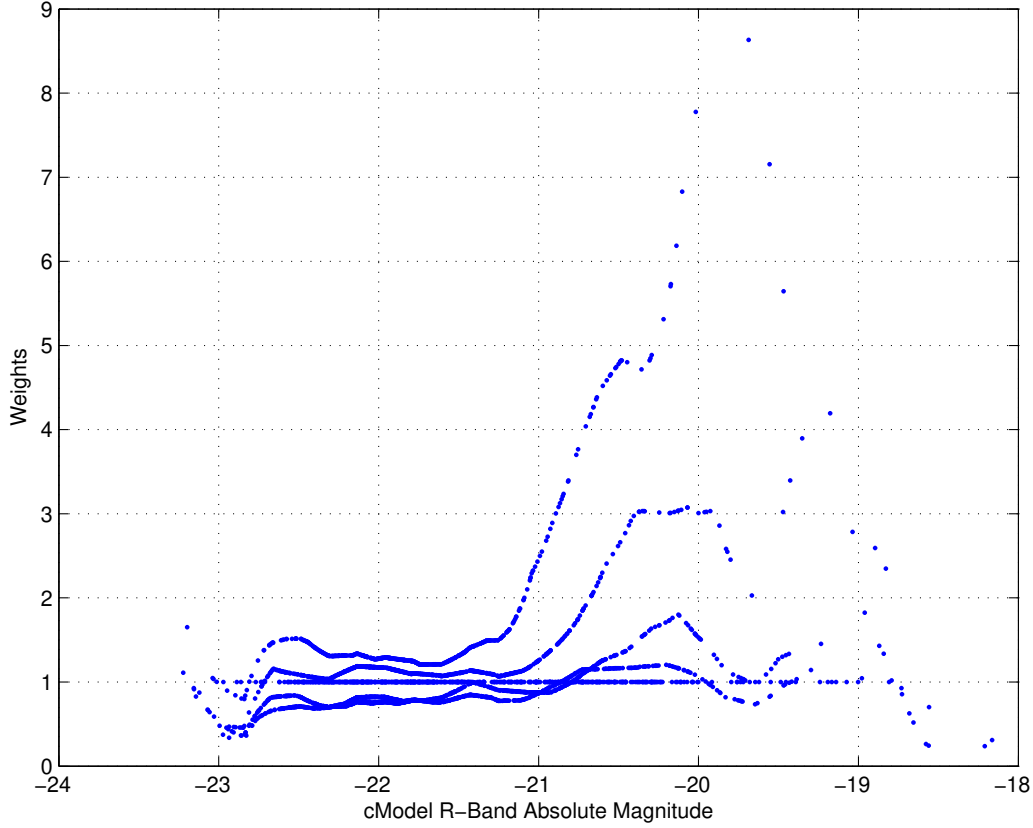


Figure 3.10: Weights of all 3016 galaxies in the LBS sample.

and with all weights set equal to 1. The figure indicates that unweighted linking length further increases the extent of *Saraswati* downwards by linking it with two nearby large galaxy filaments. Whereas the clustering analysis with adaptive weighting scheme detects them as separate structures.

Therefore, based on this study, and with the firm identification of several massive galaxy clusters and voids near the *Saraswati* supercluster, described in following sections, there is very little scope for doubt left that it is a real physical structure formed via cosmological processes, and not an artifact of chance alignment and projection effects.

3.3.2 The Effect of Peculiar Velocities on Clustering Analysis

As mentioned in section 3.2, the spectroscopic redshift errors are less than 1% for all galaxies in the samples. The small redshift errors have a negligible effect on the clustering analysis. Another important factor is the Redshift Space Distortion (RSD), which is more significant for galaxies within the

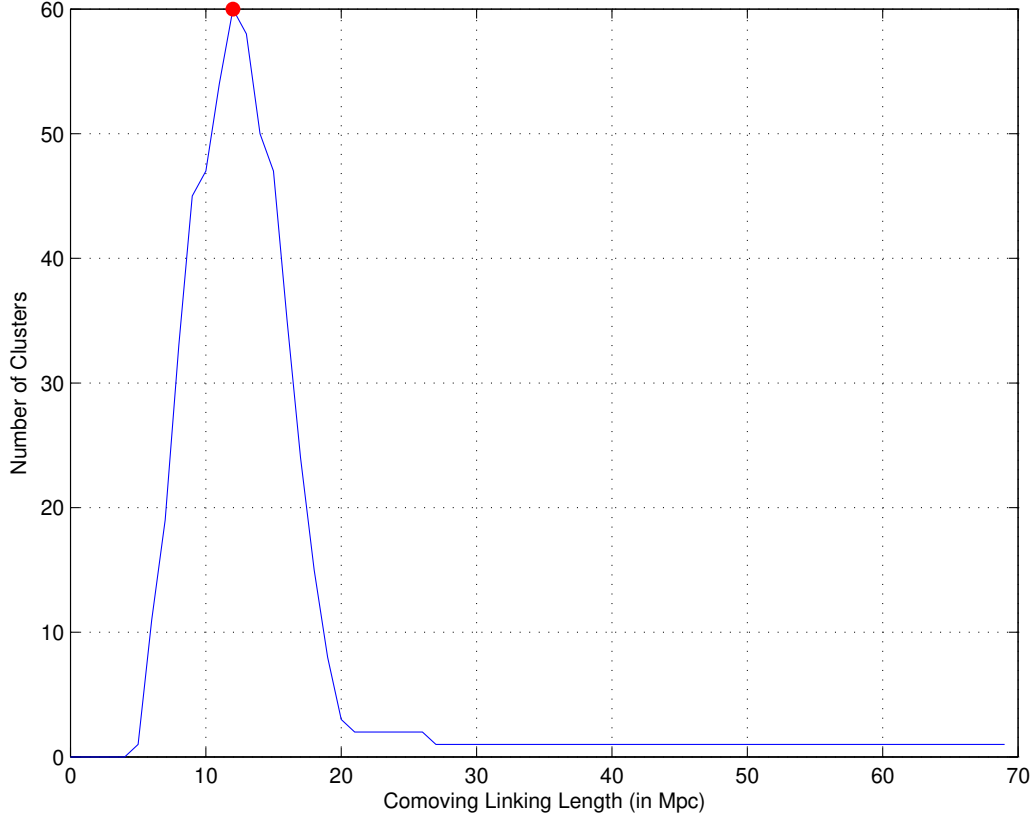
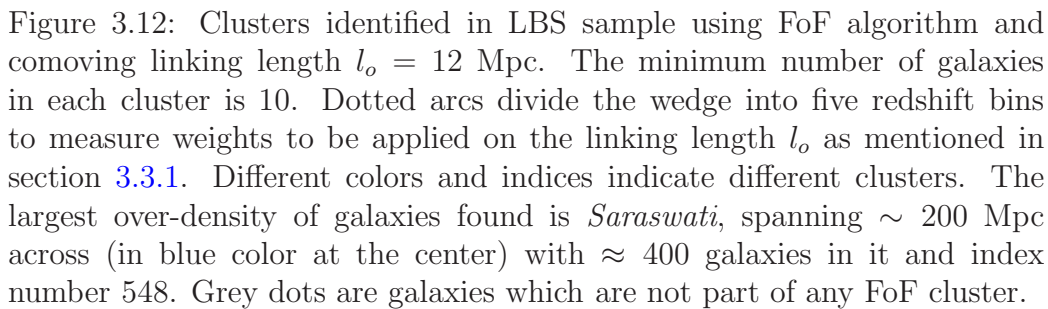


Figure 3.11: Number of FoF clusters obtained for different comoving linking lengths for LBS sample with atleast 10 member galaxies in a cluster. A maximum number of 60 clusters are obtained at $l \approx 12$ Mpc (red circle). Notice that at the linking length of ≈ 27 Mpc all the galaxies get connected (or percolated).

virial radius r_{200} of a cluster. To take into account the effect of RSD, the calculation of the RSD induced by these clusters in radial direction on the surrounding galaxies upto distances r_{200} , for the LBS sample, using the virial masses M_{200} and positions of WHL clusters (next section), is done. The corrections are made to the comoving distances for all those galaxies which are within angular virial radius of the cluster and within a distance of (v_{200}/H_o) from BCG (brightest cluster galaxy) in radial direction using the following equation,

$$d_{gal} = d_{BCG} + (d_{gal}^* - d_{BCG}) \frac{r_{200}}{v_{200}/H_o} \quad (3.5)$$

where d_{gal} is the new comoving distance of galaxy, d_{gal}^* is the old comoving distance of galaxy, d_{BCG} is the comoving distance of BCG, r_{200} is the virial radius of the cluster, v_{200} is the radial peculiar velocity component at r_{200} due to mass M_{200} ($v_{200} = \sqrt{GM_{200}/3r_{200}}$), and H_o is the Hubble parameter



Equation 3.5 is similar to equation (1) of Liivamägi et al. (2012). Since the spectroscopic sample is not dense enough to give high number density of member galaxies in a cluster, only 224 galaxies are found for RSD corrections. The mean, median and standard deviation of correction Δd to comoving distances of these 224 galaxies are 1.29 Mpc, 0.27 Mpc and 1.73 Mpc. After applying these corrections, we again performed the clustering analysis with weights and found that the major results are not changed significantly. Again, 60 FoF clusters in all are obtained, with *Saraswati* being the largest cluster having 386 galaxies in it, exactly the same as before. Re-analysis of RSD corrected LOWZ sample also concludes that RSD has negligible effect on the clustering analysis.

This shows that the detection of supercluster and other clusters is robust to peculiar motions in the data.

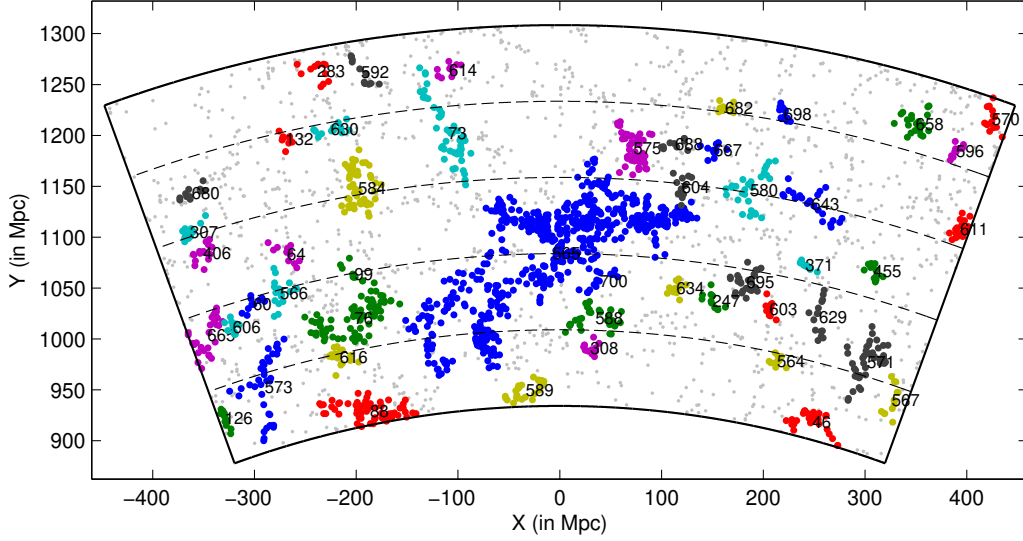


Figure 3.13: Clusters in LBS sample using FoF algorithm and a fixed (unweighted) linking length of 12 Mpc. The minimum number of galaxies in each cluster is 10. Different colors and indices indicate different clusters. This analysis extends the *Saraswati* supercluster further towards lower redshifts by linking it with two nearby large galaxy filaments (in blue color at center).

3.4 Distribution of Known Virialized Clusters and Voids Near the Supercluster

In Λ CDM paradigm, on megaparsec scales, virialized clusters and groups of galaxies trace the local extrema in the underlying density distribution of both dark and luminous matter. As the *Saraswati* supercluster occupies a vast and overdense region of galaxies, some clusters or groups of galaxies should be found at the highest density peaks.

Moreover, in the cosmic-web, overdense regions are surrounded by almost empty voids. Highly dense and large overdensities create larger and deeper voids due to gravitational instability. For quantifying more objectively, the galaxy distribution in overdense (clusters/groups) and the underdense (voids) regions of the cosmic-web around *Saraswati* supercluster, the galaxy distribution with the published catalogs of virialized clusters and voids derived from SDSS data are cross matched.

Let us now compare the distribution of galaxies in the analysis region with the clusters listed in SDSS-III cluster catalog of [Wen et al. \(2012\)](#) (hereafter WHL catalog). In WHL catalog, the photometric redshifts of BCGs of each cluster are given. The spectroscopic redshifts are also listed but only if spectra are available in SDSS. All available spectroscopic redshifts in SDSS-III

have been used for BCGs of WHL galaxy clusters. For other clusters (small in number) whose BCG spectra are not available, photometric redshifts are used. In this way, a total of 238 WHL clusters are identified within the analysis region. The WHL cluster catalog is $\approx 75\%$ complete for clusters of $M_{200} > 0.6 \times 10^{14} M_{\odot}$ and redshifts $z < 0.42$, and $\approx 100\%$ complete for clusters with $M_{200} > 2 \times 10^{14} M_{\odot}$ and $z < 0.5$ (Wen et al., 2012).

Out of the 238 WHL clusters within the analysis region, there is a major concentration of 48 clusters within 90 Mpc comoving distance from the center of *Saraswati* (which is Abell 2631 cluster of galaxies), and 43 of these are associated with the filament/wall-like main structure of *Saraswati* supercluster (Figures 3.18 and 3.23). The data on these clusters are given in Table 3.1, listed in decreasing order of their masses (M_{200}).

Table 3.1: WHL clusters in and around *Saraswati* supercluster

SrNo	RA (deg)	Dec (deg)	Redshift	rmag	R_{200} (Mpc)	N_{200}	M_{200} ($10^{14} M_{\odot}$)	Conv. Dist. (Mpc)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
†1	354.41553	0.27137	0.2772	17.23	1.93	103	10.4544	0.000
2	355.89862	0.33093	0.2694	17.93	1.62	79	7.4352	40.838
3	356.59955	0.74942	0.2746	17.42	1.60	51	4.6789	44.331
†4	355.27872	0.30925	0.2768	17.21	1.36	32	2.9718	16.843
5	354.40656	-0.67781	0.2876	17.51	1.39	31	2.5886	43.159
6	356.86499	-0.15381	0.2639	17.91	1.33	37	2.4723	68.773
7	350.62363	-0.37294	0.2728	18.33	1.32	34	2.4673	75.977
8	351.68036	1.13423	0.2774	18.28	1.37	30	2.4354	55.740
9	351.88278	0.94281	0.2788	17.14	1.27	28	2.3992	51.391
10	356.51950	-0.18573	0.2665	17.62	1.28	27	2.1499	57.477
11	357.56891	0.88373	0.2767	17.79	1.28	28	2.1138	62.379
12	352.86020	0.61525	0.2739	17.84	1.20	26	1.7984	33.171
13	357.83884	0.61691	0.2775	17.67	1.19	23	1.6145	66.884
14	358.78690	0.75341	0.2789	17.87	1.07	24	1.4692	85.906
15	354.03653	-1.18347	0.2650	17.75	1.10	26	1.4580	54.053
16	352.03851	0.18593	0.2768	17.38	1.12	16	1.3240	46.205
17	356.45068	-1.12771	0.2797	17.67	1.08	19	1.2245	49.083
18	356.19455	-0.08880	0.2674	17.98	1.05	17	1.1919	50.579
19	354.56683	0.11680	0.2711	17.60	1.07	24	1.1810	23.271
20	358.25555	-0.23089	0.2779	17.86	1.01	10	1.0581	75.367
21	356.01273	0.22644	0.2714	18.34	1.00	18	1.0368	37.667
22	356.86264	0.52773	0.2729	17.74	1.00	12	1.0255	50.122
23	353.99393	-0.48434	0.2700	17.79	0.99	15	1.0180	31.727
24	356.32980	-0.05331	0.2665	18.29	0.98	19	1.0080	54.668
†25	355.10202	-0.09299	0.2761	18.09	0.98	14	0.9881	15.627
26	358.93271	-0.00393	0.2752	17.95	0.98	16	0.9868	87.932
27	353.62631	-0.98501	0.2772	17.79	0.96	9	0.9502	28.827
28	355.43375	-0.67539	0.2680	17.78	0.94	12	0.9107	43.606

Table 3.1 : Continued

SrNo	RA	Dec	Redshift	rmag	R ₂₀₀	N ₂₀₀	M ₂₀₀	Comv. Dist.
	(deg)	(deg)			(Mpc)		(10 ¹⁴ M _⊙)	(Mpc)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
29	357.93292	0.56078	0.2703	18.01	0.95	17	0.9083	72.542
30	357.23520	-0.89120	0.2793	17.72	0.93	9	0.9034	59.979
†31	354.97913	-0.43282	0.2769	17.53	0.94	13	0.8991	17.552
32	351.02548	0.36311	0.2780	17.90	0.93	12	0.8825	66.039
33	352.66733	0.60940	0.2757	17.66	0.92	14	0.8295	34.962
34	356.67987	0.83429	0.2637	17.89	0.93	15	0.8265	67.370
35	351.58939	0.16173	0.2870	18.15	0.88	12	0.7956	66.786
36	353.87994	-0.88882	0.2974	17.95	0.93	11	0.7842	79.549
37	352.81964	0.72678	0.2812	18.34	0.93	11	0.7782	35.747
38	352.17755	0.82753	0.2784	17.42	0.92	9	0.7584	45.116
†39	353.87866	-0.53116	0.2764	18.27	0.84	10	0.7452	18.973
40	356.76236	-0.08544	0.2655	17.61	0.88	11	0.7327	63.065
41	358.12292	0.60623	0.2698	17.91	0.88	12	0.7018	76.620
42	355.63776	-0.28874	0.2770	18.30	0.86	11	0.6924	26.124
43	358.60349	0.81066	0.2805	18.30	0.76	12	0.6835	83.396
44	351.32236	0.27741	0.2767	18.33	0.87	9	0.6553	60.071
45	351.95999	-0.64258	0.2856	18.08	0.85	8	0.6424	60.422
46	358.44800	0.81641	0.2787	17.80	0.81	8	0.6040	79.441
47	355.18701	0.31234	0.2982	19.15	0.86	13	0.5994	79.790
48	357.44406	0.64598	0.2705	17.84	0.63	8	0.5988	63.779

Note :- Data on 48 galaxy clusters listed in WHL catalog, within 90 Mpc comoving distance from the center of *Saraswati* supercluster. Abell 2631, the most massive cluster (first in the list), is considered as the center of the supercluster. The Columns are as follows: (1) Cluster number; (2,3) R.A. and Dec. of cluster center (BCG); (4) Redshift of the BCG; (5) Apparent r-band magnitude of the BCG; (6) The comoving virial radius of the cluster; (7) Number of observed galaxies within R₂₀₀; (8) Total mass (virial mass) of the cluster within R₂₀₀; (9) Comoving distance between Abell 2631 and the cluster's BCG. Clusters are arranged in decreasing order of their mass and those marked with a † sign are the five clusters within the bound core of supercluster.

The first cluster (the most massive cluster) in the list is Abell 2631 ($z = 0.277$) and the second one (the second most massive cluster) is ZwCl 2341.1+0000 ($z = 0.269$), both well known and studied clusters. The properties of these WHL clusters have been further analyzed below and used to estimate the total mass and overdensity of *Saraswati*.

For the study of voids, the LOWZ void catalog of Nadathur (2016) based on BOSS data of SDSS Data Release 11 (hereafter 'Nadathur voids') is used. Nadathur voids are identified by first making a *Voronoi Tessellation* from the galaxies' three dimensional position data and then implementing the

Watershed algorithm on the local density of galaxies to find the low density regions. These are disjoint voids which are independent underdense regions of space that do not overlap with each other. A total of 24 voids are identified within the analysis region whose effective comoving diameters range from ≈ 40 Mpc to ≈ 172 Mpc (see Table 3.2 and Figure 3.14). Even though voids can have any geometry other than a sphere, their shapes are a good approximation of a sphere/spheroid. This is the reason, effective radius is defined for these voids.

Table 3.2: Voids in the analysis region.

RA (deg)	Dec (deg)	Redshift	R_{eff} (Mpc)
350.6849	0.8486	0.23303	85.72
357.4545	0.5707	0.31232	68.04
344.3417	1.1195	0.30315	72.64
337.4782	-1.0169	0.23256	61.39
14.7674	1.0286	0.23330	68.35
13.8886	0.0757	0.31260	65.47
1.7087	-0.0218	0.27427	56.75
338.2647	1.0409	0.28423	68.06
6.3308	-0.4873	0.24774	46.80
13.6654	-0.5078	0.32148	59.50
2.9050	0.5065	0.30177	64.56
352.2162	0.9479	0.26425	58.27
7.6776	-0.8381	0.27424	44.46
339.1871	-0.8044	0.30564	45.26
339.4053	-0.8466	0.26936	60.67
6.6679	-1.2271	0.31850	46.91
358.3064	-0.7660	0.29395	45.58
344.7811	-0.2459	0.24854	30.74
349.2072	0.0277	0.26926	36.30
9.1117	-0.0983	0.29707	35.97
352.6307	-1.1636	0.28706	36.02
357.7748	0.7036	0.25396	28.62
4.4547	-0.2339	0.23615	27.46
353.1712	0.1797	0.31095	20.66

Note : The properties of 24 voids from Nadathur catalog whose centers are found in the analysis regions. Here, the center of void's right ascension, declination and redshift are given and R_{eff} is the effective comoving radius of the void.

Clusters and Voids - LOWZ Sample : The distribution of WHL clusters in and around *Saraswati* is shown in Figure 3.15, where clusters are plotted with star symbols. This figure shows that number density of galaxy

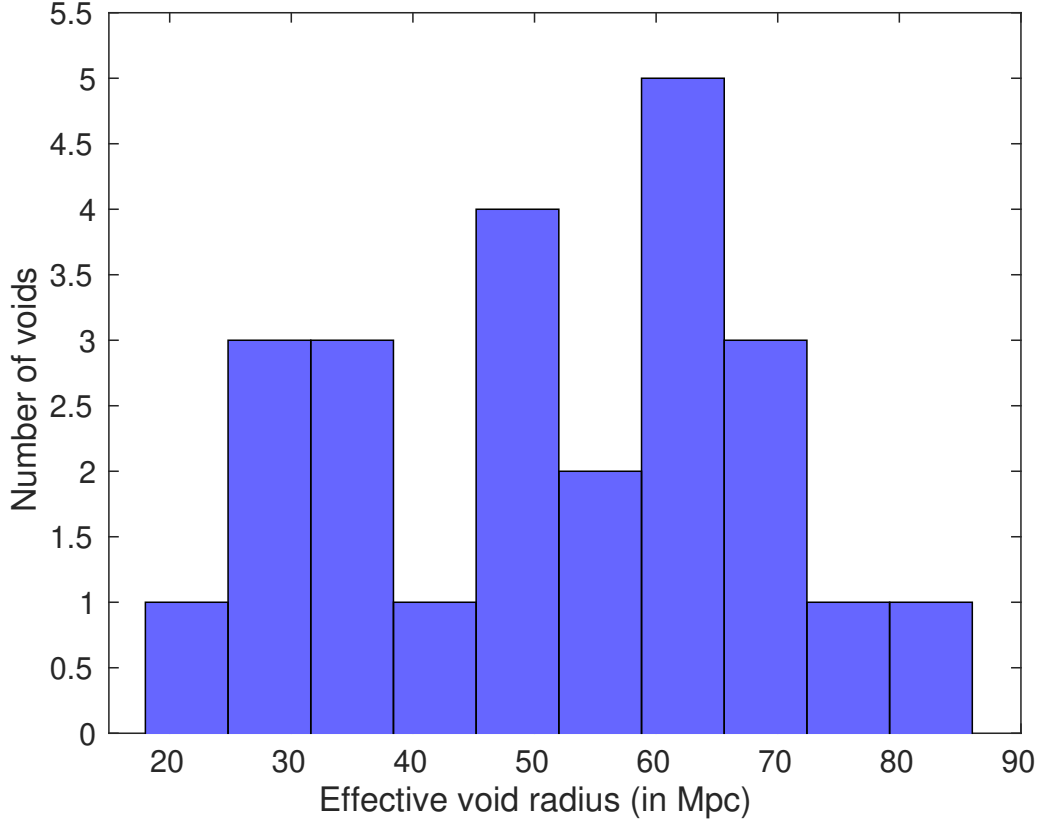


Figure 3.14: Histogram of the size (effective comoving radius) distribution of voids in the analysis region.

clusters within *Saraswati* region is much higher compared to other lower density regions within the analysis region. With the LBS galaxy sample, a similar result is obtained which is discussed below.

Figure 3.16 shows the distribution of voids within the analysis region. The circles (red dashed) show the voids, the black cross markers show the centers of voids and radii of voids are the effective comoving radii as given in the Nadathur catalog. Some voids seem to overlap FoF clusters and other voids because of the projection of these 3D voids on a 2D plane. It can be easily seen that many voids surround the *Saraswati* supercluster, which is expected because superclusters are always surrounded by voids.

Clusters and Voids - LBS Sample : Similarly, the WHL clusters and Nadathur voids are plotted on the LBS galaxy sample, shown in Figure 3.18, Figure 3.17 and Figure 3.19.

The high density of galaxies within *Saraswati* in both galaxy samples (LOWZ and LBS), high density of WHL clusters within *Saraswati*, and the presence of large voids surrounding it strongly support the physical existence of *Saraswati* supercluster.

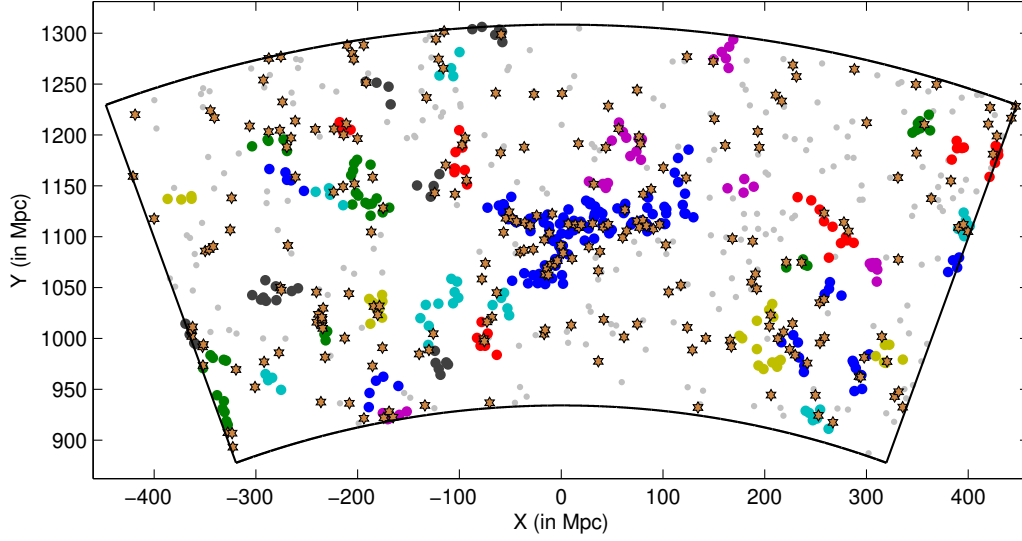


Figure 3.15: The distribution of WHL clusters (denoted by stars) along with FoF clusters identified with LOWZ galaxy data. The high density of clusters near *Saraswati* supercluster is very clear.

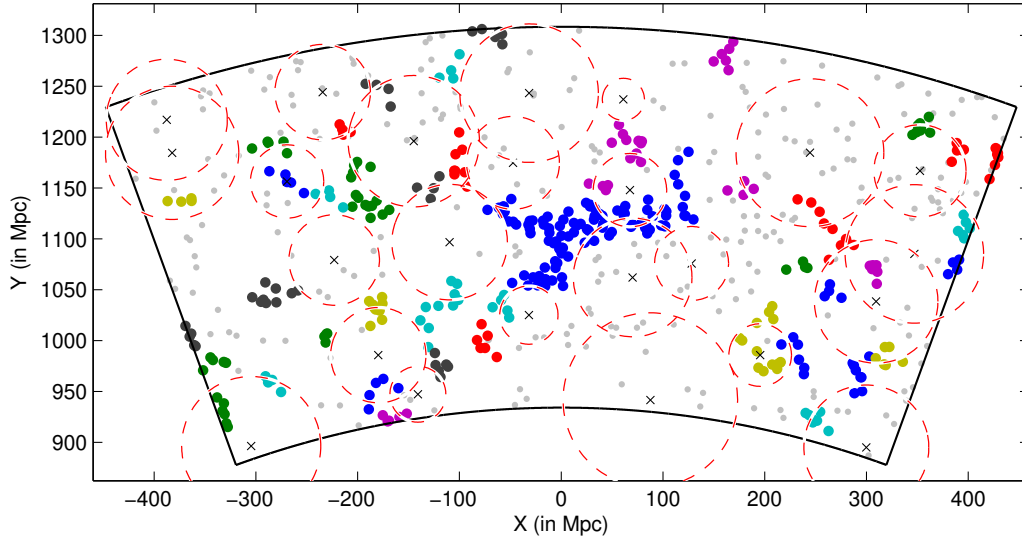


Figure 3.16: The distribution of voids in LOWZ data from Nadathur catalog. The FoF clusters are shown in various colors. Red dashed circles are voids in projection, black crosses are void centers and the radii of circles are equal to the effective radii of voids as given in the Nadathur catalog.

3.5 Properties of *Saraswati* Supercluster

3.5.1 Mass Distribution

Saraswati supercluster is identified as a high density region of galaxies and

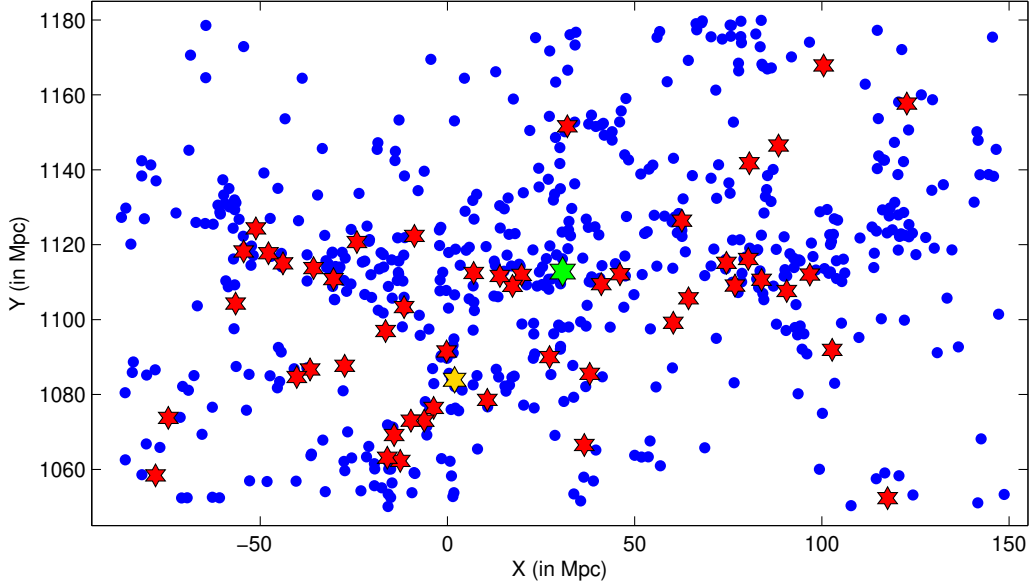


Figure 3.17: In this figure, a zoomed-in view of *Saraswati* supercluster is shown. All cataloged WHL Clusters (red stars) are plotted over the galaxies from LBS sample (blue dots). Location of the most massive cluster Abell 2631 is shown with a green star and the second most massive cluster ZwCL 2341.1+0000 by a yellow star (see Figure 3.20 and Table 3.1). It should also be noted that these two most massive clusters create an apparent low density region between them as both have accreted the matter around them.

clusters. The mean redshift of this supercluster (the mean redshift of all member galaxies in *Saraswati* from LBS sample) is $z \sim 0.28$. It is found, from the Abell cluster catalog (Abell et al., 1989), that *Saraswati* supercluster is centered on an extremely rich (Abell richness class $R = 3$), massive ($M_{200} \approx 10^{15} M_{\odot}$) and hot ($T_e = 8$ KeV) galaxy cluster Abell 2631 at redshift $z = 0.277$. Abell 2631 shows strong detection in X-rays and also a significant Sunyaev-Zel'dovich decrement signal on the cosmic microwave background radiation (Reese et al., 2012). It is shown below that dominant Abell 2631 and a few more nearby massive clusters most likely form the dense, bound core of the *Saraswati* supercluster. Another unusual galaxy cluster ZwCl 2341.1+0000 ($z = 0.269$), a highly filamentary, multiply merging system, is located ~ 45 Mpc to the south in a filamentary spur, joining the main *Saraswati* concentration. This one is the second most massive cluster in the supercluster region. The faint, non-thermal synchrotron radio emission on Mpc-scale discovered in ZwCl 2341.1+0000 cluster (Bagchi et al., 2002; van Weeren et al., 2009; Giovannini et al., 2010) is a clear signpost of the infall and merger dynamics of several galaxy groups during the first phase of the cluster formation process (Boschin et al., 2013). The optical images of clusters Abell 2631 and ZWCl 2341.1+0000, taken from DECam legacy

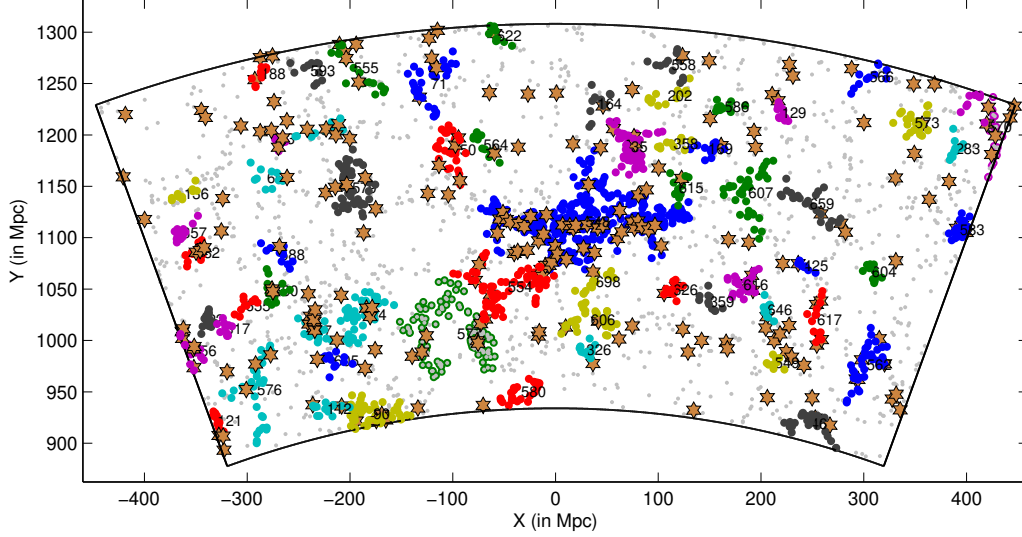


Figure 3.18: The distribution of WHL clusters (denoted by stars) along with FoF clusters identified with LBS galaxy data. Again, the high density of clusters near *Saraswati* supercluster is very clear.

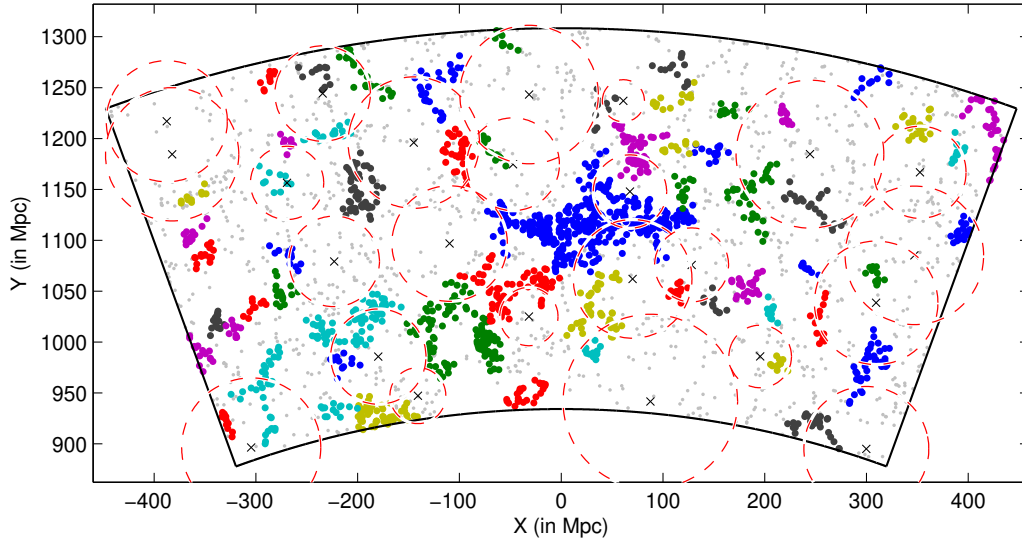


Figure 3.19: In this figure, the distribution of voids in the analysis region is shown. All the 24 voids Nadathur catalog are plotted over the LBS galaxy sample. The FoF clusters are shown in various colors. Red dashed circles are voids in projection, black crosses are void centers and the radii of circles is equal to the effective radii of voids as given in the Nadathur catalog.



Figure 3.20: Optical images of the two most massive clusters of *Saraswati* Supercluster; Relaxed type Abell 2631 ($z = 0.277$) on left and highly filamentary ZWCl 2341.1+0000 ($z = 0.269$) on the right, taken from DECam legacy survey (DECaLS). Each image is $\approx 10' \times 10'$ across, corresponding roughly to 2.5 Mpc on a side.

survey (DECaLS)², are shown in Figure 3.20.

Saraswati supercluster contains at least 43 massive clusters/groups of galaxies identified from the WHL catalog. The total mass of a supercluster mostly resides in its member groups and clusters and is, therefore, indicated by the number of clusters/groups it contains (Chon et al., 2014). The total bound halo mass of these 43 clusters is estimated to be $\approx 2 \times 10^{16} M_{\odot}$ (section 3.5.2). From the known masses of clusters, through X-ray and weak lensing observations of clusters, and their richness, from the photometric data of SDSS, Wen et al. (2012) found a correlation between virial mass M_{200} (mass of the cluster within r_{200} , the virial radius) and cluster richness parameter R_{L*} of their WHL clusters, given by,

$$\log M_{200} = (-1.49 \pm 0.05) + (1.17 \pm 0.03) \log R_{L*}, \quad (3.6)$$

where M_{200} is in units of $10^{14} M_{\odot}$.

Using above relation, the mass of each cluster within r_{200} is estimated (see Figure 3.21 and Table 3.1).

However, significant amount of matter must lie beyond the virial radius of a cluster, which needs to be accounted for. Therefore, this estimate of virial mass M_{200} is further scaled up to calculate the bound halo mass $M_{halo} \approx M_{5.6}$

²<http://legacysurvey.org/>

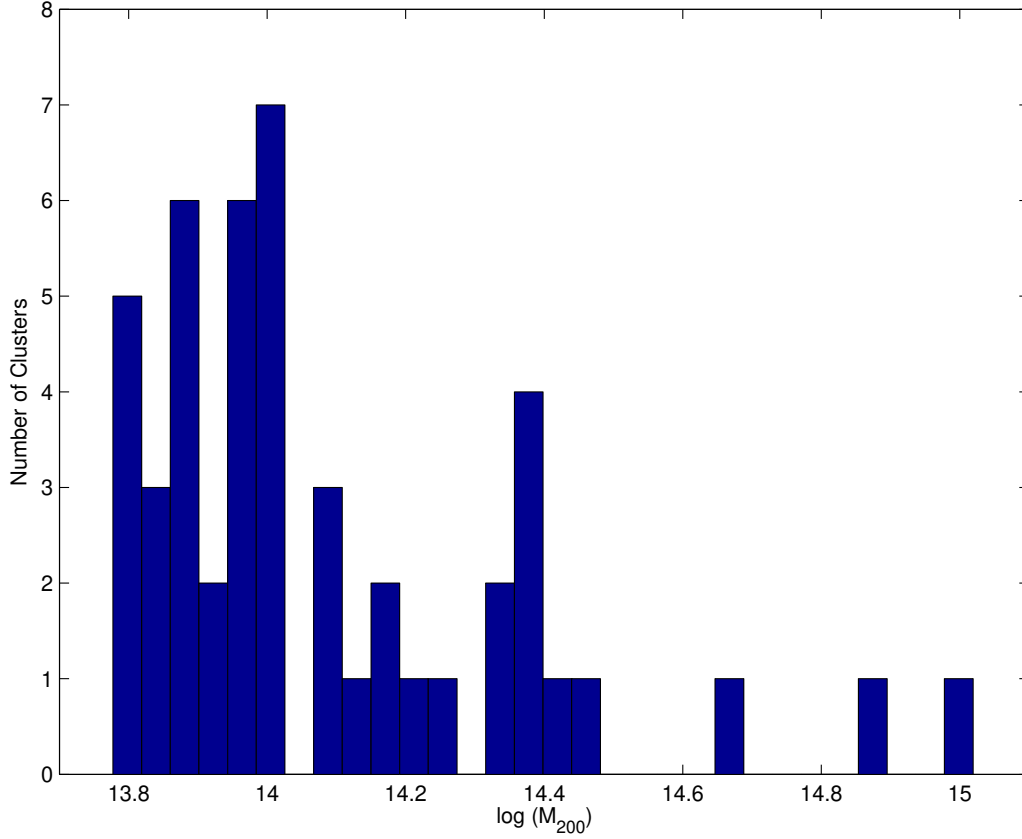


Figure 3.21: Histogram of the virial mass of the 43 clusters in *Saraswati* supercluster.

of each cluster; where $M_{5.6}$ denotes the mass inside $r_{5.6}$, the radius within which the mass over-density is 5.6 times the critical density of the Universe. Several studies find that the bound halo mass of a cluster is close to $M_{5.6} \sim 2.2 \times M_{200}$ (Rines et al., 2013; Busha et al., 2005; Rines and Diaferio, 2006). This gives the total estimated halo mass of 43 main clusters $\sim 2 \times 10^{16} M_{\odot}$. This is strictly a lower limit as the full extent of *Saraswati* supercluster is not known yet, due to the declination limit of SDSS on Stripe 82. In addition, certain fraction of the total mass, which is not bound to the virializing dark matter halos is still unaccounted for.

Figure 3.23 (top) plots the 3-d distribution (X,Y,Z co-moving coordinates) of 43 clusters in *Saraswati* supercluster, spanning ~ 200 Mpc across. It shows that the low mass clusters surround the massive clusters like irregular halos. In this figure, the radius of a sphere (cluster) is proportional to its r_{200} . Colors represent the masses of the clusters. The largest dark red sphere is the most massive Abell 2631 ($M_{200} \approx 10^{15} M_{\odot}$; Reese et al. (2012)) and small blue spheres show the least massive clusters. Significantly, 23 among these 43 clusters are very massive, having virial mass $M_{200} > 1 \times 10^{14} M_{\odot}$ (Table 3.1). This implies an unusually high mass concentration in mas-

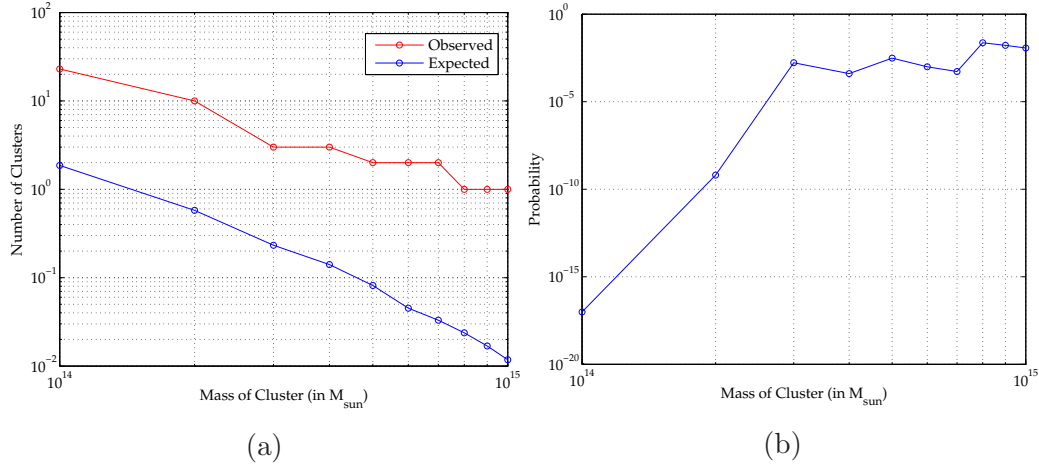


Figure 3.22: (a) The number of expected virialized halos (blue line), from the halo mass function, and the observed WHL clusters (red line) with masses greater than a given virialized mass in the volume of *Saraswati* supercluster. (b) The probability of finding observed number of WHL clusters greater than a given virialized mass in *Saraswati* supercluster's volume according to the halo mass function.

sive clusters within the supercluster region. In Λ CDM cosmology, using halo mass function, the expected number of dark matter halos (Sheth et al., 2001) (with mass $M_{200} > 10^{14} M_{\odot}$) within the *Saraswati* supercluster volume (section 3.5.2), at $z \sim 0.28$, is only ≈ 2 (Figure 3.22). The halo mass function gives the expected number density of virialized halos greater than a given halo mass. The halo mass function calculator *HMFCalc* (Murray et al., 2013) is used to calculate the expected number and probabilities of finding WHL clusters within *Saraswati*. Sheth-Mo-Tormen fitting function $f(\nu)$ (Sheth et al., 2001) is used in the *HMFCalc*,

$$f(\nu) = A \sqrt{\frac{2a}{\pi}} [1 + (a\nu^2)^{-p}] \nu \exp \left[-\frac{a\nu^2}{2} \right]$$

where,

$$\nu = \frac{\delta_c}{\sigma}, \quad \delta_c = 1.686, \quad A = 0.3222, \quad a = 0.707, \quad p = 0.3$$

and σ is the rms variance of mass. Figure 3.22a shows the expected and observed numbers of halos/clusters within the volume of *Saraswati* supercluster. Figure 3.22b plots the probabilities of finding the observed numbers of WHL clusters in any part of the Universe with the same volume as of *Saraswati*.

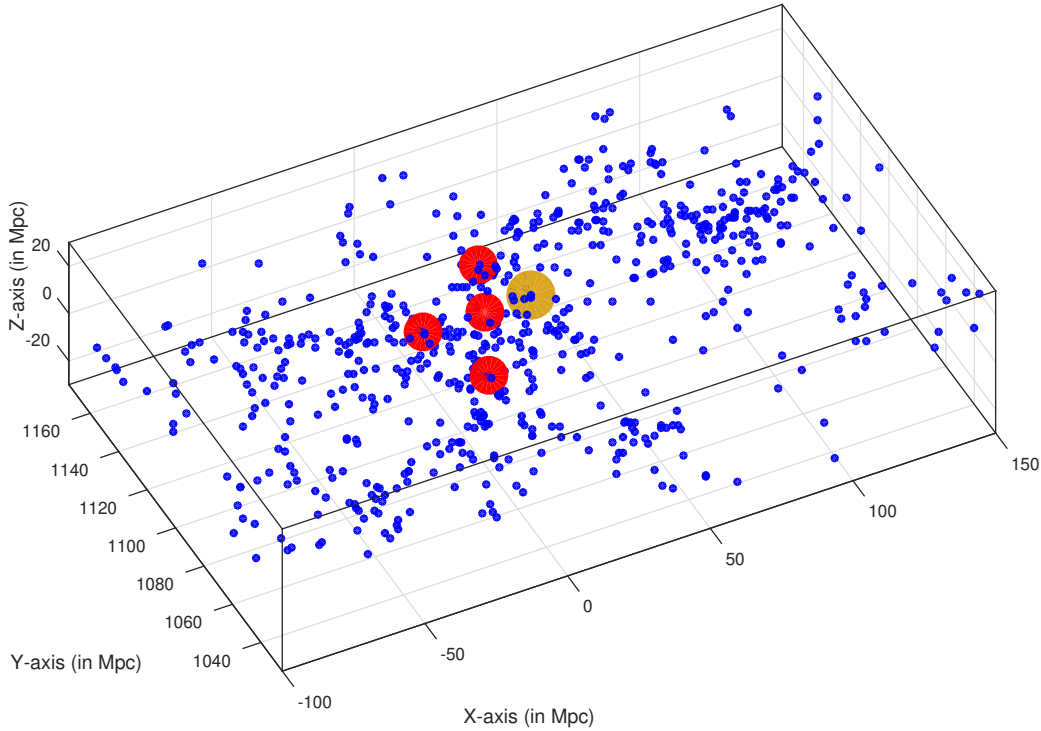
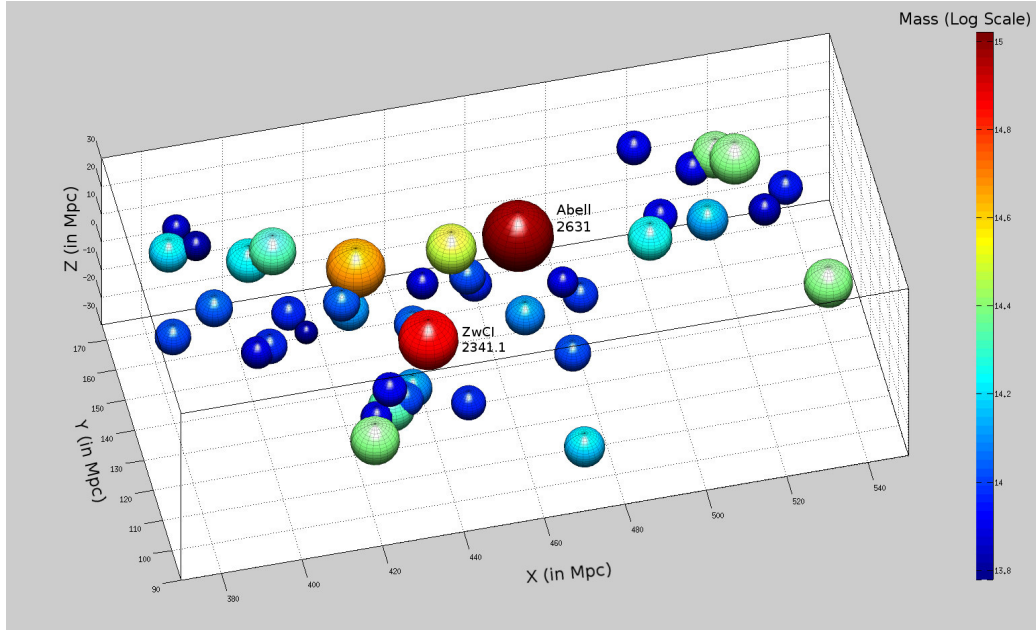


Figure 3.23: *Upper*: This figure shows the 3-D distribution of wall-like structure of *Saraswati* supercluster, mainly comprising of 43 known clusters of galaxies, here represented by spheres. The radius of each cluster is proportional to its virial radius r_{200} . Colors represent the masses of clusters (in $\log(M_{200})$ scale shown with the color bar). The dark red sphere represents the most massive and rich galaxy cluster Abell 2631 and bright red the next massive one, ZwCl 2341.1+0000, and subsequently orange, lemon yellow, green etcetera. Dark blue spheres show the least massive clusters. *Lower*: 3-D distribution of the galaxies and some most massive clusters near Abell 2631 cluster is shown. Small blue circles are the galaxies from LBS sample, large golden circle represents Abell 2631 and other big red circles are the next four most massive clusters of *Saraswati* supercluster.

3.5.2 Bounded Region of Saraswati - Turn Around Radius

While a supercluster of the scale of *Saraswati* is unlikely to be in a state of dynamical equilibrium with well-defined boundaries, one can still ask whether or not it is gravitationally bound? We attempt to estimate the fraction of this supercluster that possibly is gravitationally bound.

Although *Saraswati* is more planar than spherical, nevertheless, the widely used spherical collapse model is applied in which the turnaround radius R_{ta} (the radius of shell separating infalling material from the outwards expansion of the Hubble flow (Peebles, 1980; Padmanabhan, 1993; Busha et al., 2003; Pavlidou and Tomaras, 2014) for a mass M is

$$R_{ta} = \left(\frac{3GM}{\Lambda c^2} \right)^{\frac{1}{3}}, \quad (3.7)$$

or

$$R_{ta} = \left(\frac{3GM}{H_o^2 \Omega_\Lambda} \right)^{\frac{1}{3}}, \quad (3.8)$$

where $\Lambda = (8\pi G\rho_\Lambda/c^2)$ is the cosmological constant and the dark energy density is ρ_Λ . Since the total halo mass of each cluster cannot be independently calculated, the results based on numerical simulations are used. As before, total bound halo mass of a component cluster is taken to be $M_{5.6} \approx 2.2 \times M_{200}$ (Rines et al., 2013; Busha et al., 2005; Rines and Diaferio, 2006), and thus the sum of masses of 43 clusters as the total mass of *Saraswati* is obtained, which is $M \approx 2 \times 10^{16} M_\odot$. This is strictly a lower limit because mass of matter (dark-matter + galaxies) dispersed within the inter-cluster space is not included. We also correct for this extra mass as discussed below.

Using this total mass in equation 3.8, $R_{ta} \approx 30$ Mpc (only about 1/3 of its ≈ 100 Mpc physical radius) is obtained. This suggests that the entire *Saraswati* supercluster can not be a gravitationally bound structure. Further, with Abell 2631 as the center, we integrate the galaxy cluster masses enclosed within spherical shells upto a certain radius R . This is compared to R_{ta} (Figure 3.24). It is inferred that only mass within radius $R \sim 20$ Mpc is gravitationally bound and reached turnaround. Four high mass galaxy clusters, each of mass $M_{200} \sim 10^{14} M_\odot$, and the most massive Abell 2631 ($M_{200} \gtrsim 10^{15} M_\odot$; Reese et al. (2012)), near the center comprise this bound core (Table 3.1), as one expects from the gravitational collapse model. The aggregate mass of this dense, bound core is at least $4 \times 10^{15} M_\odot$, or about 20% of the mass of the entire supercluster. Even neglecting the (unknown) small mass of matter dispersed within the inter-cluster space, this gives a

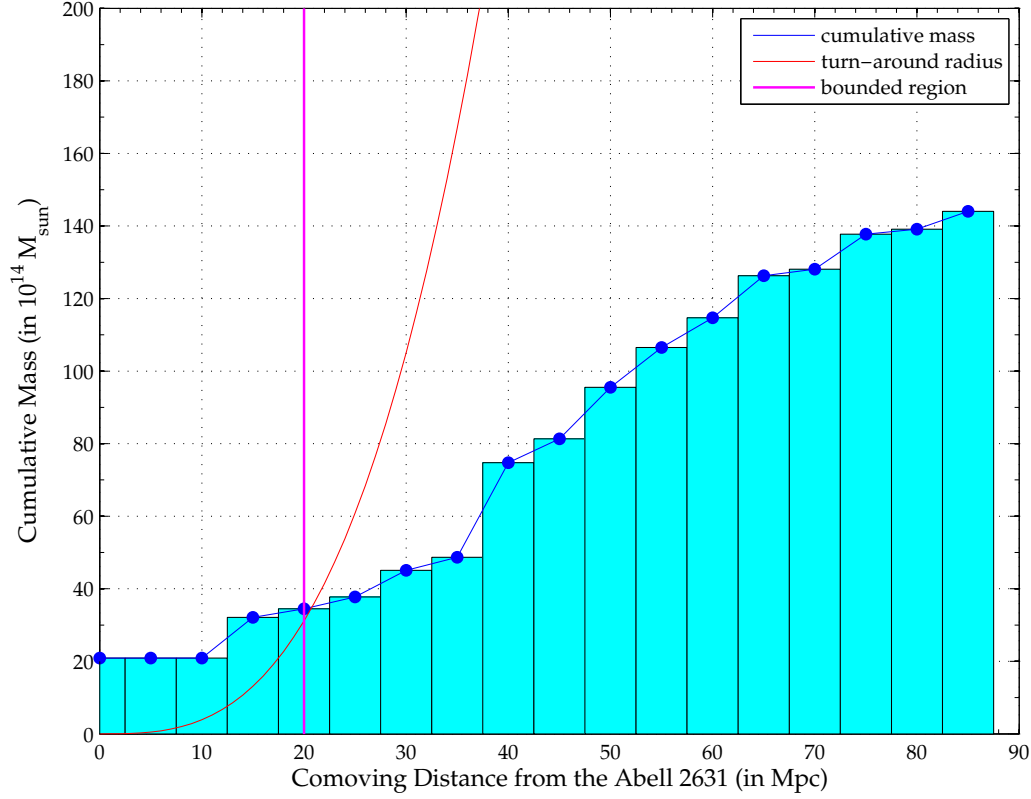


Figure 3.24: The radial mass distribution of WHL clusters in *Saraswati* supercluster upto $R \approx 90$ Mpc radius. The x-axis is the radial distance (R) from Abell 2631 (the most massive galaxy cluster in the core of *Saraswati* supercluster) and y-axis is the cumulative bound halo mass of the WHL clusters within the sphere of radius R (bin size ~ 5 Mpc). Red curve is the theoretical turn-around radius (equation 3.8). Blue curve shows cumulative mass distribution. Five clusters are inside the turn-around curve and 38 are outside it. The clusters within the turn-around radius are altogether bound while the clusters beyond it are unbound. The bounded region has a comoving radius of ≈ 20 Mpc (magenta line).

lower bound to the core over-density of $\rho_{core}/\rho_{cz} = 1.4$ and $\rho_{core}/\rho_m = 3.12$. Here ρ_{core} , ρ_m and ρ_{cz} are the density of the bound core of *Saraswati*, the mean matter density of the Universe at $z = 0.28$ and the critical density of the Universe at $z = 0.28$, respectively. According to the definition of a supercluster proposed by Chon et al. (2015), superclusters are structures defined based on an overdensity criterion that selects only those objects that will collapse in the future, including those that are at a turn-around in the present epoch. Therefore, based on this definition, *Saraswati* indeed qualifies for being a supercluster.

3.5.3 Density Contrast of Supercluster

Here, we calculate the density contrast of *Saraswati* supercluster. As the halo mass information of all the 43 clusters is known, we require the volume (V) estimate of *Saraswati* to calculate its mass density. Since *Saraswati* is a wall-like extended structure a spherical volume is not assumed, because that would largely overestimate V and give a much smaller density contrast. Therefore, the volume of the Convex Hull of the 43 clusters (set of points) within *Saraswati* is calculated. A Convex Hull of a set of points $\mathbf{P}$ in three dimensional Euclidean space is the convex surface (envelope) of minimum possible volume on $\mathbf{P}$. *Qhull* algorithm (Barber et al., 1996) is used to construct the Convex Hull and to calculate its volume. This gives the comoving volume of *Saraswati* $V \approx 2.05 \times 10^5 \text{ Mpc}^3$. Now, the corresponding matter density contrast δ is

$$\delta = \frac{\rho_{SS}}{\rho_m} - 1 = \left(\frac{\rho_{SS}}{\rho_c} \frac{1}{\Omega_M} \right) - 1 \quad (3.9)$$

where ρ_m is the background matter density at redshift $z = 0.28$, ρ_c is the critical density at present epoch and ρ_{SS} is the comoving mass density of *Saraswati* supercluster. The total bound halo mass of 43 clusters, estimated to be $\approx 2 \times 10^{16} M_\odot$, is used to calculate ρ_{SS} . This gives average matter density contrast $\delta = 1.62$ ($\rho_{SS}/\rho_m = 2.62$) and $\rho_{SS}/\rho_c = 1.17$ for the supercluster. Both these numbers fall short of the minimum density $\rho/\rho_c > 2.36$ and $\rho/\rho_m > 7.86$ necessary if the entire supercluster is to remain gravitationally bound at present, and also in distant future (Dünner et al., 2006; Nagamine and Loeb, 2003). At the present epoch, the largest known gravitationally bound, virialized structures are the galaxy clusters spanning a few Mpc across with masses $\sim 10^{14-15} M_\odot$. If the space between clusters of *Saraswati* supercluster is uniformly filled with matter at background density ρ_m , then its total mass will be $\approx 4 \times 10^{16} M_\odot$, giving higher density contrast $\delta \approx 3$ and $\rho_{SS}/\rho_m \approx 4$.

3.6 Galaxy properties in Saraswati Supercluster

In this section, we attempt to explore the properties of the galaxies present in the Saraswati Supercluster. Total number of galaxies in Saraswati supercluster region with spectroscopic information available in LBS sample is 386. This sample does not represent the total galaxy population in the Saraswati supercluster but only the confirmed ones. The larger aim of such

study would be to see if galaxies in supercluster region exhibit any different properties when compared to galaxies in non-supercluster environments.

3.6.1 Quasars in Saraswati region

Using [Pâris et al. \(2018\)](#), we find 5 quasars in the Saraswati region ($350.5^\circ \leq RA \leq 359.5^\circ$, $-1.25^\circ \leq Declination \leq +1.25^\circ$ and $0.2625 \leq Redshift \leq 0.2825$). Visual inspection shows more quasars associated with the Saraswati supercluster but for now, we have given the simplest constraint possible on the coordinates. The details of the five quasars are given below :

RA (deg)	Dec (deg)	Redshift
359.02015	-0.97126	0.28126
358.67310	-0.08019	0.27900
357.38654	-0.61273	0.27880
358.21616	+0.63750	0.27280
358.73788	+0.70553	0.27040

3.6.2 Distribution of Black Hole Mass

The M_{BH} - σ relation is based on a strong correlation between the central galactic black hole mass and effective stellar velocity dispersion in the galactic bulge ([Ferrarese and Merritt, 2000](#); [Gebhardt et al., 2000](#)),

$$\log \left(\frac{M_{BH}}{M_\odot} \right) = \alpha + \beta \log \left(\frac{\sigma}{200 \text{ km s}^{-1}} \right) \quad (3.10)$$

where $\alpha = 8.32 \pm 0.05$ and $\beta = 5.64 \pm 0.32$ ([McConnell and Ma, 2013](#)).

The mass of the black hole of galaxies in Saraswati supercluster is estimated using - the M_{BH} - σ relation, whose distribution is shown in Figure 3.25. The median value of black hole mass of galaxies is $2.73 \times 10^8 M_\odot$ whereas the mean value is $4.86 \times 10^8 M_\odot$.

3.6.3 WISE Galaxy Diagnostics

We use the Wide-field Infrared Survey Explorer (WISE; [Wright et al. \(2010\)](#)) results to study the hosts of Saraswati supercluster galaxies in the mid-infrared (mid-IR) wavelength range. The WISE, which is a space based telescope, carried out an all sky survey in four mid-IR bands [W1 ($3.4\mu\text{m}$),

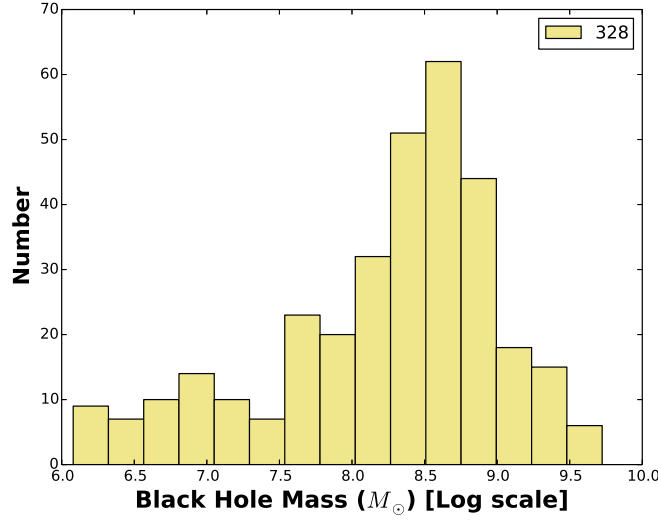


Figure 3.25: Black hole mass distribution of galaxies in Saraswati supercluster.

W2 ($4.6\mu\text{m}$), W3 ($12\mu\text{m}$), W4 ($22\mu\text{m}$)] with an angular resolution of $6.1'$, $6.4'$, $6.5'$ and $12'$, respectively. The WISE All-Sky Source Catalog was used to obtain magnitudes of the galaxies in the four mid-IR bands. After applying photometric quality cuts of 3σ , reliable values were obtained for 360/386 galaxies.

We employ the scheme from Mingo et al. (2016), which is based on the earlier work of Wright et al. (2010), Lake et al. (2012) and Gürkan et al. (2014), to classify the host-AGN & galaxies into (Low Excitation Galaxies) LEGs, (High Excitation Galaxies) HEGs, quasars (QSOs), star forming galaxies (SFGs) and Ultra Luminous Infrared Galaxies (ULIRGs). The mid-IR information of galaxies is very useful in gauging the radiative efficiency because the optical-UV radiation from the accretion disk of the AGN is absorbed by the surrounding dusty torus and is re-radiated in mid-IR wavelengths.

Figure 3.26 is a colour-colour plot using four mid-IR bands [W1 ($3.4\mu\text{m}$), W2 ($4.6\mu\text{m}$), W3 ($12\mu\text{m}$), W4 ($22\mu\text{m}$)] which can distinguish not only between LERGs and HERGs but also effectively distinguish AGNs from star-forming and passive galaxies. The figure has been marked into four regions, which signify the following :

1. Region I: HEGs and quasars ($W1 - W2 \geq 0.5$, $W2 - W3 < 4.4$).
2. Region II: LEGs ($W1 - W2 < 0.5$, $0 < W2 - W3 < 1.6$).
3. Region III: LEGs and Star forming galaxies ($W1 - W2 < 0.5$, $1.6 \leq W2 - W3 < 3.4$).

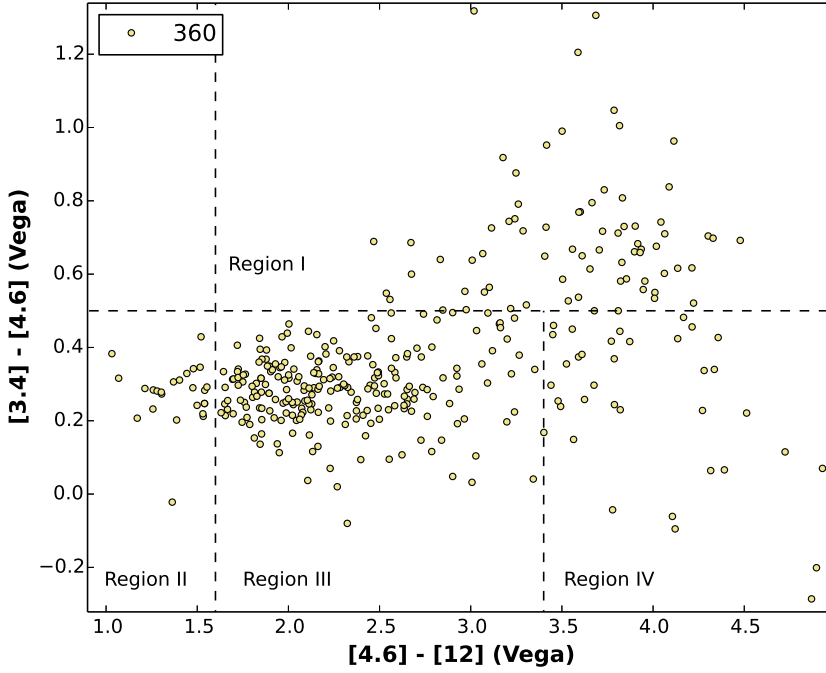


Figure 3.26: The plot indicates the position of galaxies in Saraswati Supercluster on the mid-IR color-color plot using WISE mid-IR magnitudes. W1, W2, W3 and W4 have 3.4, 4.6, 12 and 22 μm Vega magnitudes, respectively.

4. Region IV: ULIRGs ($W1 - W2 < 0.5$, $W2 - W3 \geq 3.4$).

3.7 Summary and Discussion

The significant mass overdensity on ≈ 200 Mpc scale shows *Saraswati* supercluster as a rare and the largest prominent density enhancement discovered at the redshift range $0.15 < z < 0.45$. Only four such massive and large superclusters, other than *Saraswati*, are known at present. Observing such massive, large scale structures at early cosmic aeons has major implications for cosmological studies, as the number of highest density peaks represented by clusters and superclusters and their growth function places strong constraints on the initial conditions of the Universe and the dark energy equation of state, which connects pressure (P) to energy density (ρ_λ); $P = w(z) \rho_\lambda$ (Sahni and Starobinsky, 2000; Sheth and Diaferio, 2011; Park et al., 2012; Frieman et al., 2008). Further, to ascertain if Einstein's General Relativity correctly describes the formation of such large structures, testing it across a wide range of physical scales and mass densities is necessary, and superclusters like *Saraswati* are powerful probes of gravity in its extreme large-scale, weak-field limit (Jain and Khoury, 2010). Numerical simulations are also

very important to test and calibrate the competing cosmological models as applied to the present structures (Park et al., 2012).

Interestingly, the redshift epoch of *Saraswati* complex is near

$$z = \left(\frac{\Omega_\Lambda}{\Omega_m} \right)^{1/3} - 1 \approx 0.37,$$

when global energy density in matter and dark energy became almost equal. The Universe started to accelerate even before $z \sim 0.37$. The redshift, at which this transition of the Universe happened, from decelerated to accelerated expansion, is

$$z = \left(\frac{2\Omega_\Lambda}{\Omega_m} \right)^{1/3} - 1 \approx 0.73.$$

This gives rise to an important question; if major growth of this supercluster had taken place much before $z \sim 0.73$, or that happened fairly recently? A large-scale structure this massive evolves very slowly and therefore it may reflect the whole history of galaxy formation and the primordial initial conditions which have ‘seeded’ it. The mass and spatial scale of the present super structure is comparable to that of Sloan Great Wall (Gott et al., 2005) ($\sim 200 - 400$ Mpc) and the Shapley Concentration (Raychaudhury, 1989; Bardelli et al., 2000; Proust et al., 2006) (~ 200 Mpc), two of the largest and most massive structures found in the nearby universe. Other important questions which arise from this work are; by what process these extremely large and overdense cosmic structures have formed and whether their existence is in accord with a homogeneous, isotropic universe with primordial Gaussian random phase density fluctuations as predicted by the inflation?

The physical mechanism of supercluster formation is still not well understood. The probability of massive *Saraswati* like superclusters arising from tiny perturbations in the primordial Gaussian random density field is extremely small. Therefore, significant galaxy concentrations developing on the scale of ~ 200 Mpc or more are puzzling which might challenge the belief that Universe becomes (statistically) homogeneous and isotropic on scales $\gtrsim 100 h^{-1}$ Mpc (Sarkar et al., 2009). These massive structures provide excellent model-independent tests of General Relativity on the largest scales. For example, Park et al. (2012) have identified only a few such extreme scale superclusters (size $\sim 200 - 400$ Mpc) in their *Horizon Run 2* simulation covering $7.2^3 h^{-3} \text{ Gpc}^3$ volume. They conclude that an initially homogeneous isotropic universe with primordial Gaussian random phase density fluctuations growing in accordance with the General Relativity, can explain the richness and size of the observed large-scale structures in the SDSS. More recently another large, galactic-wall like structure spanning ~ 250 Mpc at $z \sim 0.47$ was identified from BOSS survey (Lietzen et al., 2016). On the

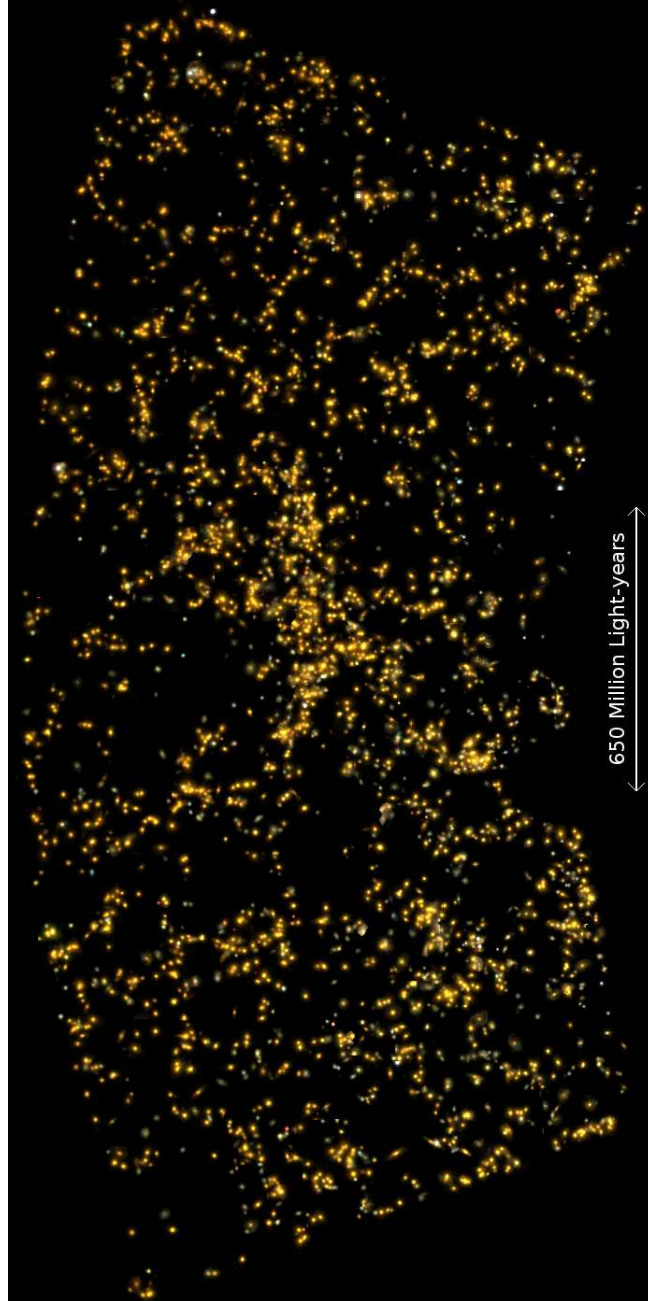
other hand, [Sheth and Diaferio \(2011\)](#) advocate that formation of even a few extremely large and massive structures like the *Sloan Great Wall* in local Universe will result in tension with the Gaussian initial conditions.

Dark energy (either a ‘cosmological constant’ or some other form) appears to dominate the present Universe, that not only alters its expansion rate, but also affects the evolution of perturbations in the density of matter, slowing down the gravitational collapse of material in recent times ([Friedman et al., 2008](#)). A powerful probe of dark energy, and how it affects the time evolution of gravitational potential of superclusters and voids, is provided by the observation of secondary temperature anisotropies in the cosmic microwave background (CMB) imprinted via the late-time integrated Sachs–Wolfe (and the Rees-Sciama effect in non-linear regime) (ISW+RS) effect ([Nishizawa, 2014](#)). Existence of large scale superclusters and voids of $\gtrsim 100$ Mpc size at $z \sim 0.5$ is implied by correlated CMB temperature fluctuations of $|\Delta T| \approx 10 \pm 2 \mu K$ ascribed to late-time ISW+RS effect ([Inoue et al., 2010](#); [Granett et al., 2008](#); [Giannantonio et al., 2012](#)). The significance of such observations is still under scrutiny, but if confirmed, they appear to be in tension with the Λ CDM predictions, e.g., ([Nadathur et al., 2012](#); [Cai et al., 2014](#); [Nadathur and Crittenden, 2016](#)). To get an estimate of *Saraswati* supercluster’s ISW+RS signal, matter density contrast $\delta = 1.62$ and effective comoving radius $r_c = [3V/4\pi]^{1/3} \approx 40$ Mpc are used. In linear growth regime, $\Delta T \approx 1.6 \mu K$ on $\approx 6.5^\circ$ scale is obtained, which is smaller than some present measurements ([Inoue et al., 2010](#); [Granett et al., 2008](#); [Giannantonio et al., 2012](#)). However, the estimation is uncertain at present owing to the lack of detailed knowledge about the shape and depth of gravitational potential of this supercluster and its time evolution. It is to be highlighted that finding some more extremely massive, overdense structures like *Saraswati*, the Shapley Concentration, and Sloan/BOSS Great Wall at higher redshifts ($z \approx 0.3 - 1$), in the dark energy dominated era, and measuring their ISW+RS and Sunyaev-Zel’dovich imprint on the CMBR sky will test the competing cosmological models and provide an alternative window to dark energy.

A possible clue to the formation of this large galactic super-structure is provided by detection of huge voids of $\sim 40 - 170$ Mpc diameter observed around the main wall-like overdense structure of *Saraswati*. In Λ CDM cosmology, the space within voids (typically $\delta(\rho)/\bar{\rho} \sim -0.8$; [Colberg et al. \(2005\)](#)) expands faster than the background Hubble expansion and thus matter inside voids will have an outward component of peculiar velocity away from the void centers ([Padilla et al., 2005](#)). Hence, matter within voids will be swept up into dense sheets or pancakes, which intersect one another, forming long galaxy filaments of cosmic-web separating neighboring voids. The overwhelmingly large negative pressure of dark energy ($P = w \rho_\Lambda$, $w = -1$ with $\rho_\Lambda/\rho_m \sim 10 - 20$ inside void) and gravitational attraction of surrounding

mass distribution together govern the structure and dynamics of large scale voids and possibly superclusters. This effect will be most pronounced when dark energy becomes dominant over matter at late times, around $z \sim 0.37$ for $w = -1$. *This process might have played an important role in evolution of nearby large-scale structures like Saraswati, Shapley Supercluster or Sloan/BOSS Great Wall.* Recently, repulsion from a pronounced void in galaxy distribution (the ‘dipole repeller’) has been attributed to the observed flow of nearby galaxies, converging towards a single attractor — associated with the Shapley Supercluster ([Hoffman et al., 2017](#)), thus providing support to the model.

In future, *Saraswati* supercluster and its environs in Stripe 82 region must be surveyed in greater depth with more galaxy redshifts taken on a wider scale for a better understanding of the dynamics of such enormous cosmic structures in the Universe. It also offers exciting possibilities for gravitational weak lensing mapping of dark matter distributed in clusters, filaments and voids ([Vikram et al., 2015](#)), and for Sunyaev-Zel’dovich imaging of the hot, baryonic matter via its characteristic imprint on the cosmic microwave background photons. This region is also a prime target for future deep surveys in X-rays, from the hot gas trapped in the numerous overdense collapsing structures (clusters) and from the mass accreting super massive black holes (AGNs), and for deep radio observations of non-thermal shocks in the magnetized cosmic-web ([Kale et al., 2016](#)). This supercluster region may contain large amounts of warm-hot intergalactic medium (WHIM), which might be detected via its spectral signatures in soft X-rays and UV. These observations will open a new window on the study of large scale structures in the Universe.



The distribution of actual SDSS images of galaxies from LBS sample with high over-density of galaxies in *Saraswati* supercluster at the center.

Chapter 4

Identification of Superclusters from the WHL Clusters

In the previous chapter, the discovery of Saraswati supercluster was presented. The extraction of the supercluster from the spectroscopic galaxy data of SDSS and its excellent correlation with the excess of WHL clusters and groups in the same region gave motivation to look for superclusters using the WHL catalog.

4.1 Data

Virialized groups and clusters are taken from the WHL cluster catalog of 132,684 clusters ([Wen et al., 2012](#)). This catalog was constructed from the photometric redshift data of galaxies, within the redshift range $0.05 \leq z \leq 0.8$, from the SDSS-III. The galaxies with deblending problems and saturated objects were excluded. The data covers a sky area of 14,000 deg². The clusters were identified. The completeness of the data is defined as the detection rate of the mock clusters using their algorithm and depends on the virial mass M_{200} of the clusters. It was found that the detection rate is $> 95\%$ for clusters with masses $M_{200} > 10^{14} M_{\odot}$ and 100% for $M_{200} > 2 \times 10^{14} M_{\odot}$ for the redshift range, $0.05 \leq z \leq 0.42$. Therefore, the data within this redshift range (82,656 clusters) is considered to extract superclusters (Figure 4.1). The full details of the construction of the cluster catalog can be found in [Wen et al. \(2012\)](#).



Figure 4.1: Blue histogram represents the redshift distribution of the entire sample of 132,684 WHL clusters and red represents the selected sample within $0.05 \leq z \leq 0.42$.

4.2 Supercluster Finding Algorithm

To identify the superclusters and making the supercluster catalog, a modified friends-of-friends (mFoF) algorithm making use of Delaunay triangulation and weighted linking length is used. We describe here, in detail, the steps to build this catalog.

4.2.1 Radial Selection Weights

Because the number density of WHL clusters varies with redshift, the distribution of WHL clusters is not uniform in the radial direction. The number density $\phi(z)$ of WHL clusters is shown in Figure 4.2. The radial selection weight of the i^{th} cluster at redshift z is defined as

$$w_{r,i} = \frac{\bar{\phi}}{\phi(z)} \quad (4.1)$$

where, $\bar{\phi}$ is the mean number density of the survey.

4.2.2 Completeness Weights

As described above, the completeness (or detection rate) depends on the mass of the cluster. The low mass clusters have less completeness and the

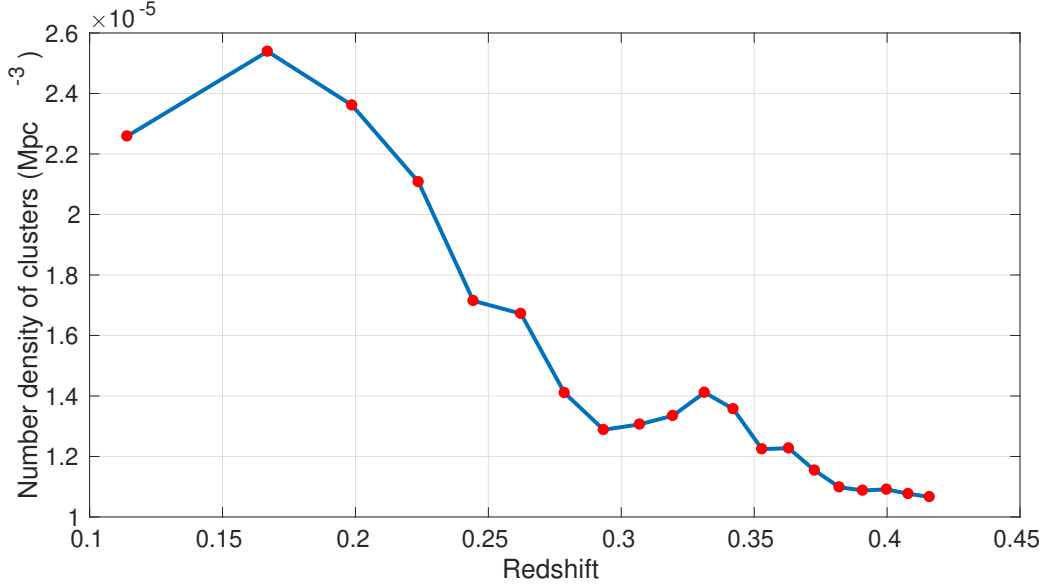


Figure 4.2: Number density of WHL clusters as a function of redshift.

high mass clusters have high completeness. The detection rate of the cluster as a function of cluster mass is shown in Figure 6 of [Wen et al. \(2012\)](#). The completeness weight of the i^{th} cluster is defined as

$$w_{c,i} = \frac{\text{Detection rate of the } i^{th} \text{ cluster}}{100} \quad (4.2)$$

4.2.3 Weighted Linking Length

As describe in the analysis of extracting Saraswati supercluster (section 3.3.1), linking lengths will be different for different pairs of clusters. The radial selection weights are computed through three dimensional density estimations, therefore, to apply these weights on the one dimensional linking length, cube root of the weights are taken. The completeness weights can be applied directly. Thus, the combined weight of each cluster will be

$$w_{rc,i} = w_{c,i} w_{r,i}^{\frac{1}{3}} \quad (4.3)$$

The combined weights of all the clusters is shown in Figure 4.3. Now, if l_o is the selected linking length, then the weighted linking length ll_{ij} between i^{th} and j^{th} clusters will be

$$ll_{ij} = \frac{w_{rc,i} + w_{rc,j}}{2} l_o \quad (4.4)$$

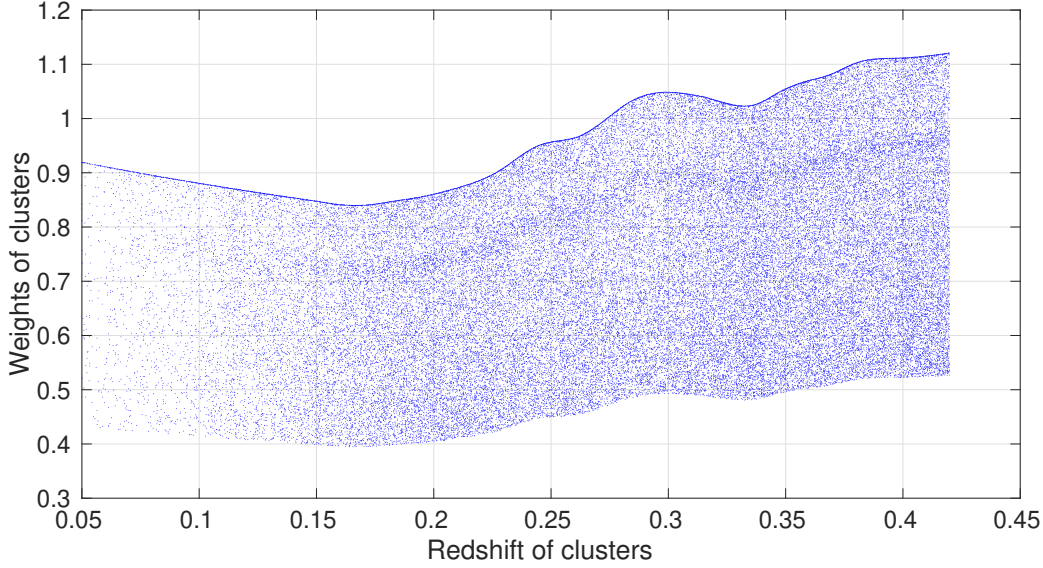


Figure 4.3: Combined weights w_{rc} of WHL clusters.

4.2.4 Distances using Delaunay Triangulation

A naive way to proceed is to calculate $O(N^2)$ distances for a distribution of N clusters and then find l_o for which the number of superclusters is maximum. This method is computationally very expensive for large values of N . To increase the computation efficiency, Delaunay triangulation is used in which a cluster is linked to its natural nearest neighbours and only the distances between the natural neighbours are considered. This reduces the calculation of $O(N^2)$ distances to $O(N \log N)$.

4.2.5 Selecting the Linking Length

The l_o is chosen such that maximum number of superclusters are obtained for a given minimum number N_{min} of clusters in a supercluster. In literature, different FoF algorithms choose different N_{min} for clustering analysis according to the data and the aim of the study. There is no settled criteria on choosing N_{min} and the choice changes the value of linking length and thus the clustering properties. While examining our catalog also, it is realized that due to the observation in contiguous as well as stripe regions, higher number of N_{min} results in high linking length and spurious superclusters. Choosing high N_{min} misses the clustering information in striped regions. Considering this, N_{min} is set equal to 2 so that even the small stripes contribute in the clustering information of the data. With this, the linking length l_o for our sample giving maximum number of superclusters is $27.63 Mpc$ (see Figure 4.4).

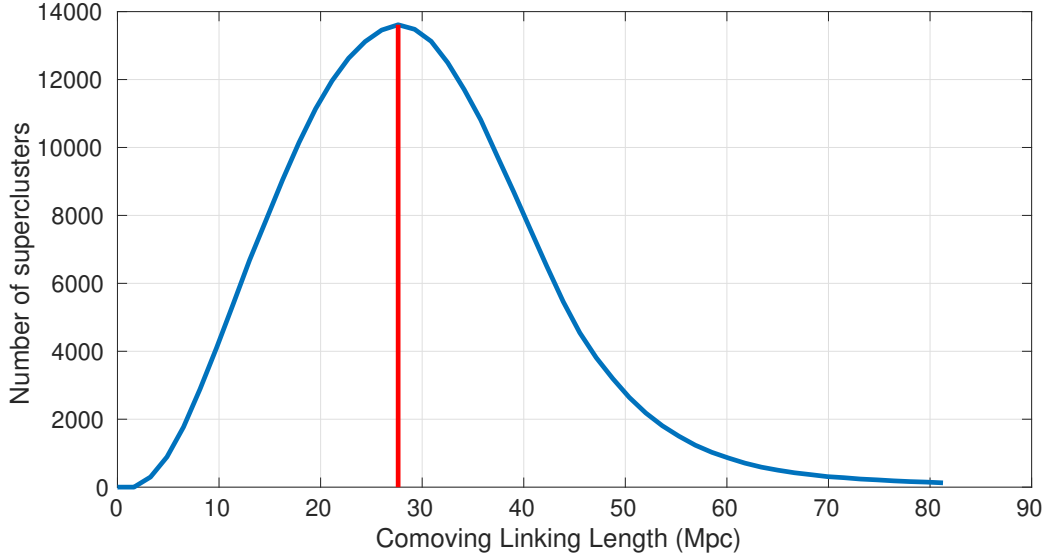


Figure 4.4: Number of candidate superclusters as a function of comoving linking length l_o (in blue). Red line shows the position $l_o = 27.63$ Mpc where the number of candidate superclusters is maximum.

The above method gives a range of masses of superclusters with at least two clusters in a supercluster. These are actually candidate superclusters and only massive candidate superclusters are real superclusters. Therefore, candidate superclusters with at least 10 clusters in them ($\gtrsim 10^{15} M_\odot$) are defined as superclusters.

4.3 Results and Discussion

The mFoF algorithm with at least 10 member clusters gives a total of 571 superclusters in the redshift range of $0.05 \leq z \leq 0.42$. This catalog is the largest supercluster catalog constructed from the data of clusters in this redshift range. Out of 571, 219 superclusters have Abell clusters (Abell et al., 1989) in them. The presence of Abell clusters in many superclusters (for example, Shapley supercluster, Sloan Great Wall, Saraswati supercluster, Laniakea supercluster) increases the robustness of our supercluster catalog. The remaining superclusters with no Abell clusters are due to the difference in the coverage of Abell cluster catalog in sky area and redshift range as compared to SDSS.

The mass of a supercluster is calculated in the same way as done for Saraswati supercluster. The virial masses of each member cluster is multiplied by a factor of 2.2 to get its mass upto turn-around radius and then adding them to get the mass of the entire supercluster. The comoving linear size of the supercluster is calculated by measuring the maximum distance

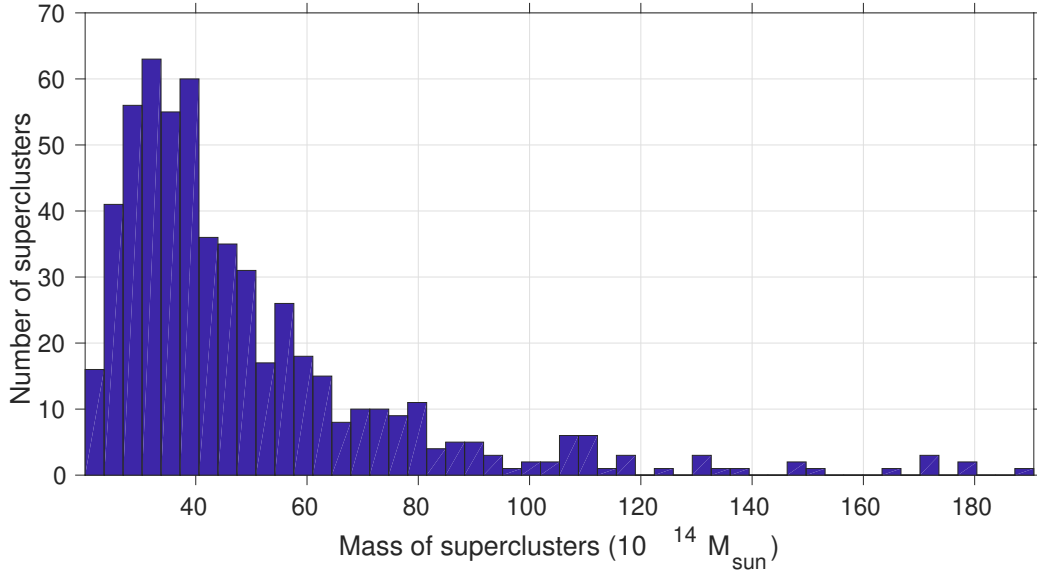


Figure 4.5: The mass distribution of 571 superclusters.

between pairs of clusters in a supercluster. The comoving position $\mathbf{X}$ of the supercluster is taken as the virial mass weighted average of the positions of member clusters,

$$\mathbf{X} = \frac{\sum_i M_{200,i} \mathbf{X}_i}{\sum_i M_{200,i}} \quad (4.5)$$

where, $M_{200,i}$ and $\mathbf{X}_i$ are the virial mass and comoving position of the i th member cluster. The sky coordinates, redshift and properties of the superclusters are listed in Table 4.1. The distribution of superclusters' mass, size, and N_{mem} are shown in Figures 4.5, 4.6, and 4.7.

Apart from few superclusters, which are rediscovery of the previously known superclusters, most of the superclusters are newly discovered. Most of the Saraswati supercluster ($SrNo = 11$) and part of Sloan Great Wall ($SrNo = 17$) are picked by our catalog. The reason for partial detection of these two superclusters is the weights given to the linking length according to number density and virial masses of clusters so that all superclusters in the entire redshift range can be recovered. If the analysis were done in the regions of only these two superclusters separately, the entire extent of superclusters would have been recovered.

The most massive supercluster ($SrNo = 1$) is found at redshift ~ 0.12 with mass $\sim 2 \times 10^{16} M_\odot$ and size $\sim 170 Mpc$. This supercluster is also reported in the Einasto et al. (1997) with the mention of only 10 Abell clusters, but our constructed catalog gives detailed extent and properties of this supercluster with 59 member clusters, including 10 Abell clusters. Figure 4.8 shows the three dimensional distribution of 59 clusters of this supercluster in comoving space. It is worth noticing that this massive supercluster mor-

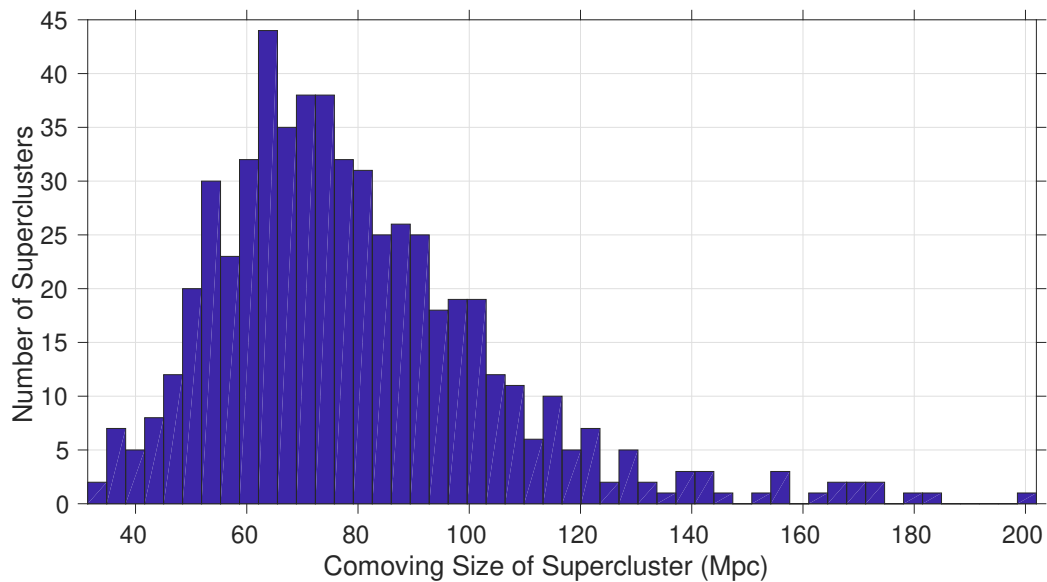


Figure 4.6: The size distribution of 571 superclusters.

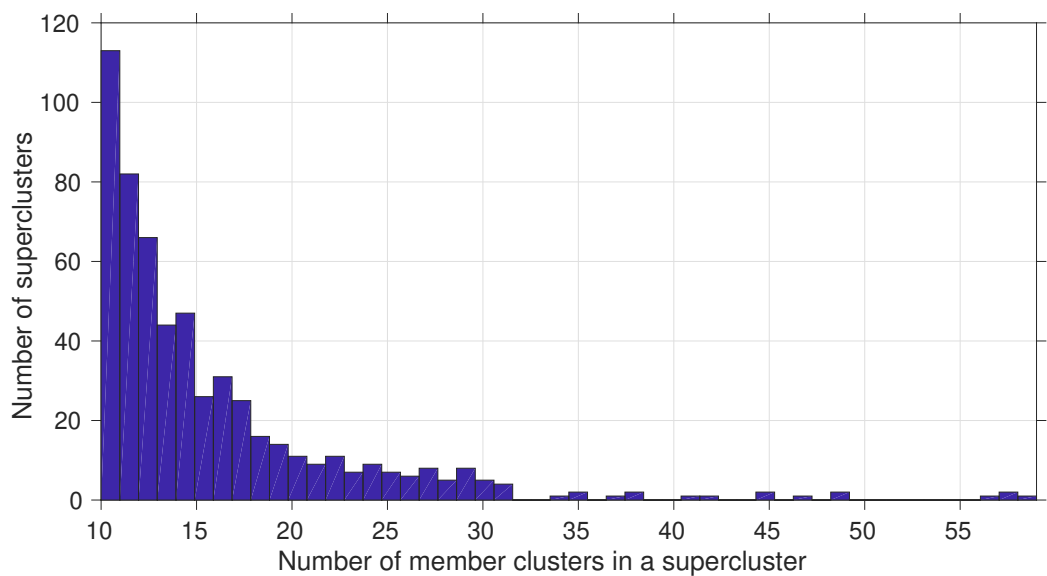


Figure 4.7: Member clusters distribution of 571 superclusters.

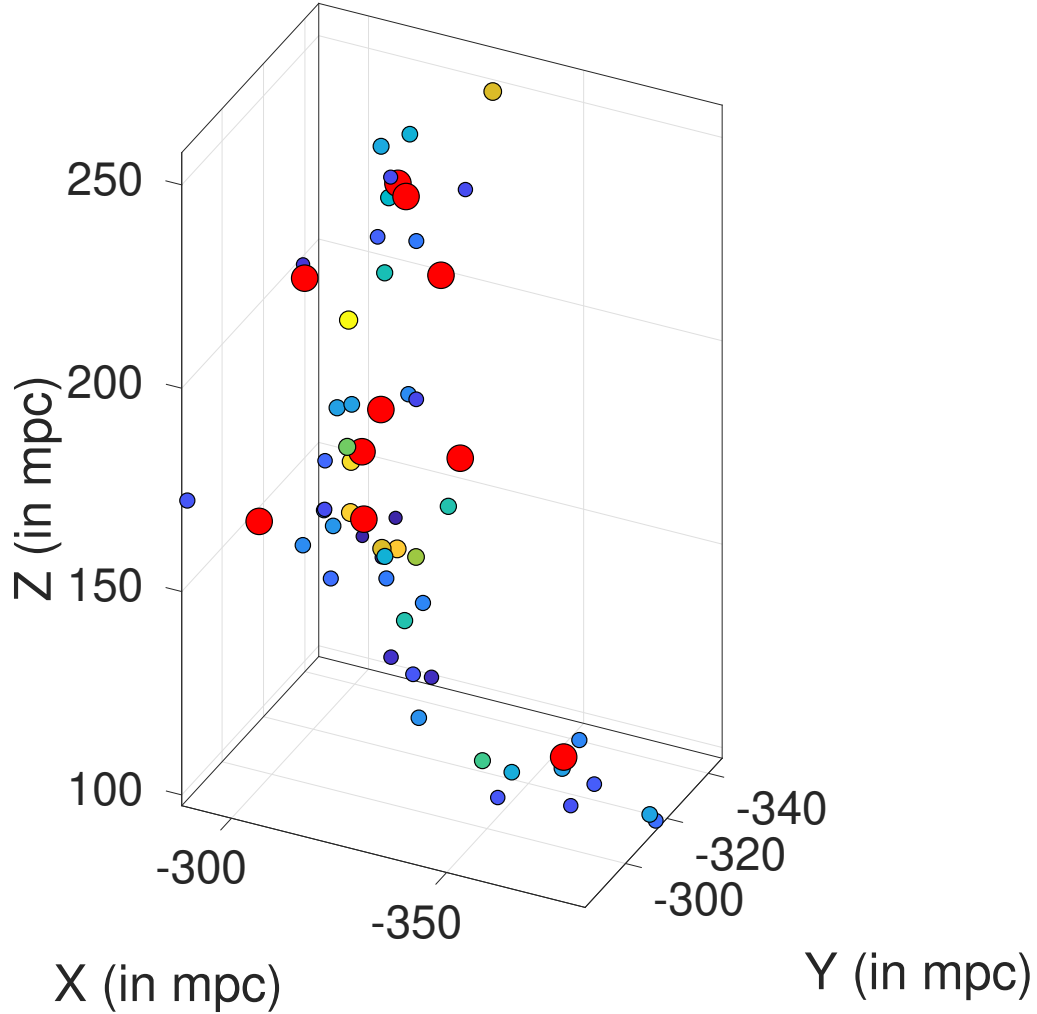


Figure 4.8: The three dimensional distribution of clusters in most massive supercluster. Colors represent the mass of the clusters with blue representing less massive cluster and yellow most massive. Red spheres represent the Abell clusters.

phology resembles the morphology of Saraswati supercluster with main thick filamentary body and some filamentary extensions. Figures 4.9, 4.10, and 4.11 show the optical images from SDSS of the core regions of the these 59 clusters. Few cluster cores indicate signs of merging morphology, another indication of clusters embedded in a supercluster environment.

The average size of the superclusters is ~ 80 Mpc, which is close to the homogeneity scale of the Universe (Sarkar et al., 2009). The mean mass of the superclusters is found to be $5 \times 10^{15} M_{\odot}$. As mentioned above, this catalog gives the minimum size and content of the superclusters because of the selection effects, they might be larger and more massive than what is seen here. To unravel the true extent of them, a very deep and focused spectroscopic survey is needed. This sample will also be useful for comparison

of similar studies using numerical simulations. Also, since the catalog has a relatively wide range of redshift coverage, the study of the evolution of the properties of galaxies in the supercluster environments can be tried.

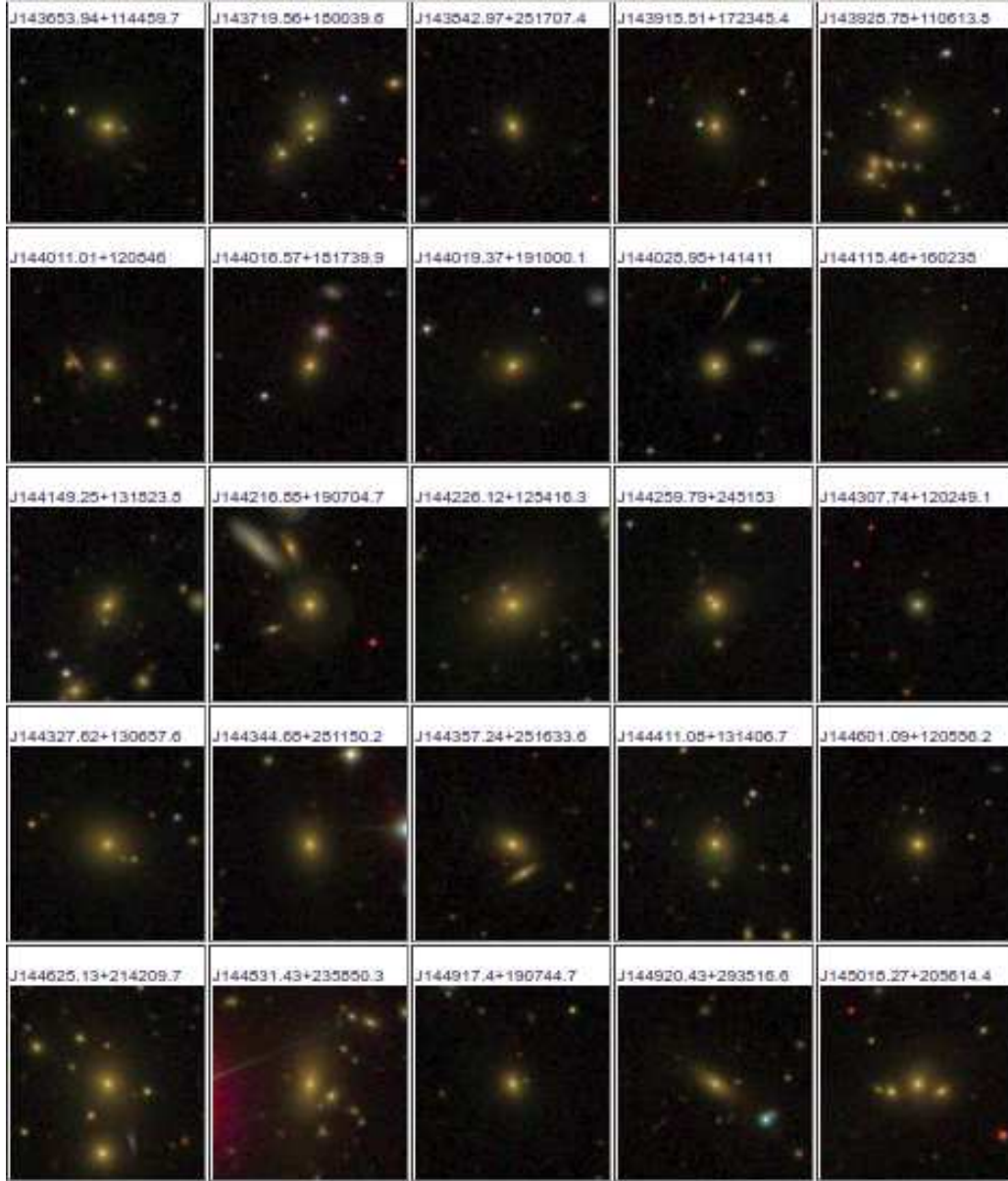


Figure 4.9: Optical images, from SDSS, of the core regions of clusters in the most massive supercluster.

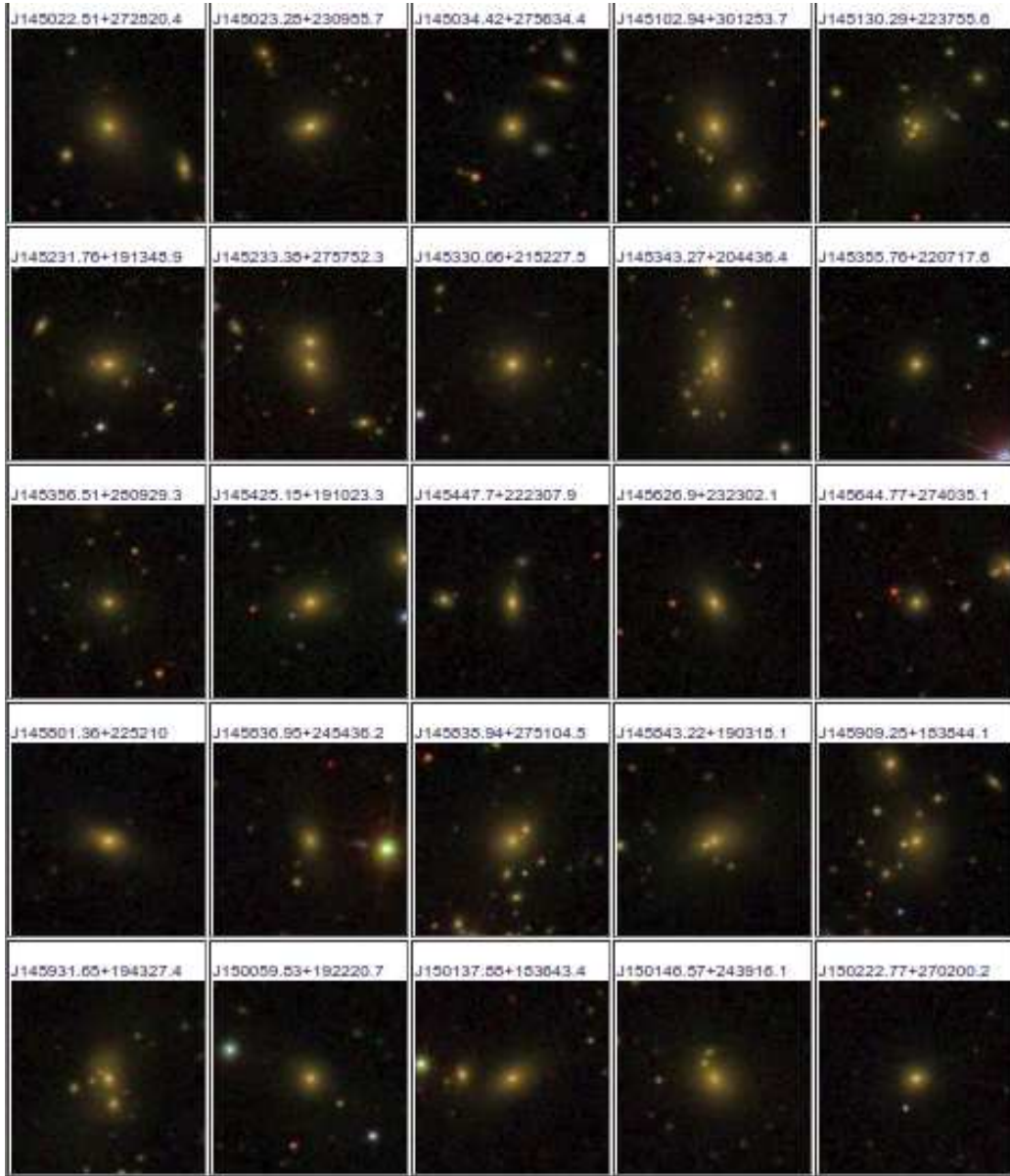


Figure 4.10: Optical images, from SDSS, of the core regions of clusters in the most massive supercluster.

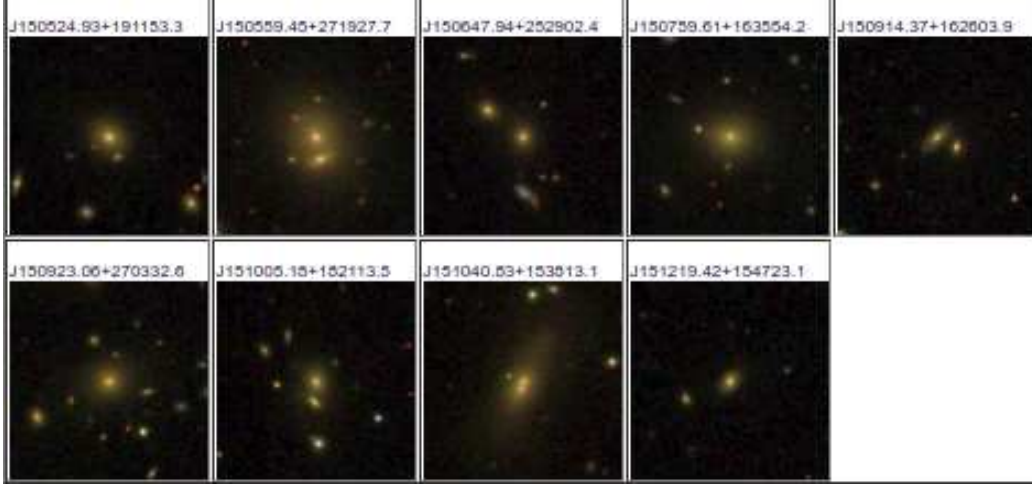


Figure 4.11: Optical images, from SDSS, of the core regions of clusters in the most massive supercluster.

Table 4.1: Properties of 571 superclusters in the decreasing order of their masses. Columns represent : (1) Serial number, (2) Right ascension, (3) Declination, (4) Redshift, (5) Linear comoving size, (6) Total Mass, (7) Number of member clusters, and (8) Number of Abell clusters.

SrNo	RA	Dec	Z	Size	Mass	N_{mem}	N_{Abells}
(1)	(deg)	(deg)	(4)	(Mpc)	($10^{14} M_{\odot}$)	(7)	(8)
1	223.3009	21.1561	0.1186	170.79	190.631	59	10
2	6.9826	-0.5940	0.2152	166.19	178.177	57	0
3	228.2877	4.4089	0.0847	88.41	177.032	38	8
4	222.3067	38.5293	0.3453	166.32	171.206	58	1
5	231.4470	26.0544	0.0762	171.61	170.595	49	6
6	181.4087	28.4453	0.3399	157.20	170.325	58	1
7	209.3122	3.5116	0.2457	129.42	165.914	45	2
8	222.3210	26.5289	0.2352	181.31	149.717	47	1
9	158.5395	16.1535	0.3658	162.19	148.806	49	0
10	234.6543	35.3546	0.2324	120.64	148.004	37	3
11	355.9828	0.1505	0.2727	108.15	136.628	35	1
12	223.9202	8.3245	0.3128	183.07	133.925	41	0
13	112.8668	31.5331	0.1880	99.83	132.329	31	1
14	175.8728	36.4593	0.1745	172.12	131.687	42	3
15	206.9949	12.2167	0.1389	142.33	131.538	45	4
16	27.4360	-1.4696	0.2416	136.78	125.578	25	6
17	198.4762	-1.5244	0.0846	100.00	117.711	30	7
18	182.2427	5.5120	0.1357	89.76	116.728	25	5
19	229.3885	30.9982	0.1139	70.81	116.718	26	6
20	215.7354	37.2563	0.1678	102.57	112.553	27	3

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
21	123.7427	9.1482	0.2636	156.80	111.724	27	0
22	239.4890	27.1226	0.0839	83.87	111.418	27	2
23	171.0377	42.9038	0.1920	114.67	110.987	27	4
24	175.9589	22.6928	0.1763	124.33	109.904	31	4
25	154.9950	49.8233	0.1463	121.31	109.297	29	8
26	169.8472	15.5582	0.1692	98.15	109.012	30	4
27	8.8614	2.6347	0.3667	169.91	107.780	38	0
28	16.4311	0.0629	0.2642	111.02	107.161	29	1
29	150.3284	20.0804	0.3058	156.44	107.064	35	0
30	174.1734	40.0972	0.2912	113.37	106.663	25	5
31	179.9319	32.8117	0.2150	108.15	106.187	34	5
32	249.4275	46.8879	0.2301	65.85	105.617	20	2
33	140.6520	-0.3518	0.3221	115.40	104.785	28	1
34	328.3989	1.5657	0.2227	102.36	103.714	26	2
35	353.3582	25.1037	0.1149	102.58	101.172	30	6
36	195.3753	45.2833	0.2311	101.36	99.432	29	2
37	253.3374	36.7787	0.1636	108.90	96.309	29	2
38	158.2849	14.1610	0.3131	99.87	95.049	23	0
39	156.0816	37.6839	0.1462	106.63	93.932	31	8
40	69.4931	9.9343	0.1164	122.76	93.785	17	2
41	144.5562	6.6684	0.2211	123.05	90.951	27	2
42	180.2901	26.6795	0.1386	100.52	90.250	23	6
43	135.2240	3.1402	0.1970	74.75	90.073	25	1
44	207.4766	47.2023	0.1692	78.76	89.947	28	4
45	170.0449	21.2086	0.1354	70.01	88.427	22	4
46	149.7419	22.5500	0.1977	87.37	87.263	20	2
47	154.2578	61.0163	0.1990	111.38	87.174	27	2
48	7.0718	6.1236	0.2657	114.57	86.198	24	1
49	236.9477	11.7012	0.2202	116.96	85.614	27	1
50	186.7664	7.4265	0.0796	78.83	85.548	23	4
51	145.2178	2.2873	0.3500	120.60	83.506	31	0
52	12.0522	15.5164	0.3539	201.96	83.325	29	0
53	125.6938	56.6990	0.2673	130.40	82.311	28	2
54	197.7637	45.0072	0.1822	84.23	81.683	21	3
55	181.0015	49.8623	0.3472	102.46	80.558	21	1
56	227.5224	1.1481	0.2186	129.54	79.886	28	0
57	121.7534	39.8676	0.3690	90.22	79.719	21	0
58	16.9956	25.8627	0.2032	114.59	79.636	28	1
59	248.1179	21.2299	0.1527	90.80	79.247	23	1
60	252.3888	29.8306	0.0995	89.70	79.186	22	5

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
61	193.5146	7.9463	0.3222	153.71	79.085	25	1
62	153.0202	32.4989	0.2965	93.73	79.051	25	0
63	334.4916	-2.5741	0.0938	109.05	78.837	19	2
64	163.4188	35.2141	0.3783	128.51	78.594	29	0
65	127.3633	66.0708	0.1883	77.22	78.391	13	1
66	163.0215	10.2885	0.2187	57.32	78.003	20	5
67	187.5722	48.6133	0.2618	142.21	77.790	26	0
68	216.0382	22.9644	0.1342	102.50	77.594	22	2
69	144.2997	12.1346	0.1788	63.00	77.526	21	4
70	12.8650	-6.4830	0.1948	95.09	76.016	24	1
71	125.3345	44.4608	0.1445	79.09	75.775	24	5
72	233.4575	37.0225	0.1540	56.02	75.512	20	3
73	346.3695	15.1832	0.2407	98.70	74.857	17	0
74	148.6803	1.8932	0.0897	115.45	74.829	30	2
75	206.1119	39.3133	0.1703	90.13	74.601	26	2
76	202.5988	20.8448	0.1739	72.45	74.446	22	2
77	184.0454	25.8625	0.2873	89.99	73.975	23	1
78	210.5164	26.0370	0.1386	100.77	72.744	29	5
79	188.2756	41.4146	0.2588	139.65	72.490	27	1
80	143.1071	8.2368	0.1327	101.80	72.192	19	5
81	188.5743	15.0939	0.2851	83.68	71.932	19	2
82	140.9393	4.3292	0.2650	67.03	71.692	17	1
83	181.0828	23.6724	0.2604	93.50	71.546	20	2
84	236.1007	34.6105	0.1943	76.89	71.518	24	2
85	228.8226	36.5931	0.2036	64.35	70.992	22	1
86	138.5527	51.7942	0.2177	84.29	70.497	11	3
87	167.0127	33.7973	0.2469	83.29	70.378	24	3
88	173.2878	34.0285	0.2225	70.16	70.188	19	5
89	213.9227	7.1730	0.1117	87.06	69.805	29	3
90	129.7797	5.3162	0.2022	96.79	69.472	30	0
91	209.5298	27.1855	0.0699	99.03	69.212	23	7
92	154.4630	19.6111	0.1141	73.24	68.652	21	4
93	202.0416	53.9441	0.3725	98.97	68.636	17	1
94	132.8480	30.7941	0.2871	119.55	68.347	26	0
95	183.9698	36.2306	0.2420	95.41	67.828	22	0
96	217.1208	11.5895	0.1581	93.96	67.599	22	0
97	135.7997	15.8397	0.3190	112.63	66.795	26	0
98	348.9577	3.9142	0.3367	129.36	66.772	22	0
99	140.8766	12.6862	0.1855	102.88	65.866	18	1
100	235.4045	24.5422	0.2253	84.84	65.681	19	0

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
101	202.7220	-2.1182	0.1799	72.20	65.583	25	0
102	117.1698	17.1787	0.1859	56.77	65.035	15	1
103	224.6228	20.2392	0.3396	99.48	64.398	24	0
104	228.0270	14.8869	0.1841	54.51	64.373	15	3
105	239.9282	50.1349	0.2398	92.22	64.353	16	1
106	194.3721	7.5953	0.3497	146.26	64.312	16	0
107	16.4037	29.6967	0.3644	128.02	64.057	17	0
108	259.4127	29.9982	0.2784	69.01	63.932	18	0
109	125.8413	47.3374	0.1300	67.24	63.776	15	3
110	13.3074	3.5982	0.4131	115.56	62.797	18	0
111	30.1902	-0.0358	0.3746	107.62	62.714	22	0
112	186.0729	2.0501	0.2283	77.15	62.179	16	2
113	190.8705	8.5504	0.2281	109.06	61.921	17	0
114	131.1509	29.2021	0.2003	61.36	61.869	18	1
115	155.3451	14.7382	0.2486	53.02	61.823	10	2
116	21.4296	22.8037	0.2557	116.01	61.793	22	0
117	180.0159	8.6015	0.2913	137.64	61.095	21	0
118	41.7352	-2.2163	0.2336	72.99	60.908	15	1
119	204.2045	50.2970	0.2763	66.47	60.475	12	1
120	203.0387	48.2621	0.3304	89.64	60.432	24	0
121	191.2710	27.8396	0.1966	89.45	60.256	16	5
122	221.1412	19.0527	0.1881	50.14	60.062	19	2
123	250.4782	28.4082	0.1444	70.96	59.949	16	3
124	354.6679	4.8392	0.1935	76.03	59.863	19	1
125	9.6945	8.1353	0.2819	87.64	59.777	19	1
126	185.8980	35.6494	0.3644	139.09	59.411	24	0
127	148.9141	20.7080	0.2120	71.32	59.358	21	0
128	134.6844	47.3573	0.1835	74.41	59.319	24	0
129	2.7710	10.1187	0.1683	87.92	59.223	15	1
130	169.5327	48.9908	0.1128	55.13	58.883	17	3
131	184.6365	-2.5892	0.2935	104.93	58.406	18	0
132	216.6701	49.8428	0.1887	80.13	58.152	22	0
133	183.8857	8.1089	0.2484	105.90	57.997	16	0
134	348.0230	4.1127	0.3091	59.38	57.810	13	1
135	172.2658	-1.0066	0.3266	94.23	57.805	20	0
136	220.8888	30.7589	0.2243	74.22	57.628	18	4
137	325.8066	-7.4820	0.1988	52.69	57.523	11	1
138	8.0775	13.0930	0.4072	89.93	57.509	20	0
139	246.6967	26.6351	0.2223	96.64	57.333	20	0
140	194.5361	66.2011	0.2323	71.68	57.307	19	1

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
141	149.9440	10.7061	0.1623	74.53	56.770	16	1
142	159.1273	49.2885	0.3744	102.39	56.732	23	0
143	32.4474	-1.3743	0.1749	68.74	56.719	14	5
144	253.9476	39.1496	0.1448	86.22	56.554	19	1
145	335.6123	-1.0246	0.3277	96.68	56.463	18	0
146	356.2038	42.2014	0.3034	81.62	56.200	16	0
147	181.5762	4.0526	0.2364	117.98	56.157	16	0
148	223.9841	21.9797	0.2566	64.60	56.133	13	0
149	190.6813	30.5146	0.3632	125.56	55.957	20	0
150	346.8103	14.8470	0.3721	83.40	55.795	19	0
151	111.8706	30.3039	0.4045	122.40	55.786	18	0
152	125.4454	18.8192	0.0941	68.59	55.754	18	3
153	182.4768	-0.9509	0.1793	79.28	55.582	16	0
154	171.9205	20.9964	0.1641	59.30	55.445	14	1
155	174.7461	26.6838	0.2998	103.72	55.378	19	0
156	180.7735	12.6506	0.2527	101.01	55.031	19	1
157	19.3666	-0.5416	0.1795	75.73	55.004	16	1
158	330.1330	11.7575	0.1927	63.07	54.927	16	2
159	223.6942	60.8101	0.2435	104.31	54.799	17	1
160	219.9345	-0.9820	0.2939	112.17	54.761	20	0
161	247.4420	23.5111	0.2044	91.20	54.617	18	0
162	350.3414	6.9276	0.2286	68.36	54.009	17	0
163	208.9086	3.4475	0.2592	63.28	53.836	16	1
164	219.0845	29.1952	0.3642	69.65	53.691	17	0
165	351.4777	18.2243	0.1509	85.68	52.930	13	2
166	158.9339	11.8112	0.3240	64.00	52.677	14	0
167	7.8277	-9.4852	0.3345	109.77	52.546	16	1
168	111.7758	36.1380	0.3028	101.92	52.383	21	0
169	52.7210	0.4677	0.3209	64.70	52.341	14	0
170	260.8928	54.4884	0.2558	88.85	52.303	14	1
171	22.2738	-8.8551	0.3602	116.08	51.845	17	0
172	358.4301	30.6451	0.2773	99.25	51.623	13	0
173	129.0046	36.5494	0.2994	94.81	51.534	14	2
174	350.8409	8.3683	0.2035	89.20	51.445	14	1
175	150.3142	27.6481	0.1848	86.84	51.327	18	0
176	15.0934	6.3606	0.3369	109.09	51.130	15	0
177	249.1441	42.9961	0.2258	55.54	50.908	12	3
178	245.8234	36.5314	0.1874	102.87	50.905	21	0
179	149.8481	56.1532	0.2220	80.98	50.704	16	2
180	162.0140	15.6962	0.2090	86.48	50.644	14	1

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
181	204.3106	33.8150	0.2514	95.23	50.431	14	1
182	184.7719	20.1862	0.2018	46.27	50.415	12	1
183	9.6593	18.9087	0.1585	60.13	50.290	14	4
184	224.8476	23.0241	0.1794	51.63	50.270	14	4
185	16.7853	29.9941	0.3331	98.68	50.034	15	0
186	176.9925	37.2554	0.1639	59.21	49.854	10	3
187	353.5167	7.5038	0.2921	64.29	49.739	12	0
188	178.6300	57.2407	0.0588	66.54	49.608	12	6
189	163.5114	30.1215	0.2486	59.59	49.417	17	1
190	150.2666	2.6056	0.2190	71.59	49.409	17	0
191	125.3521	38.0475	0.1993	81.82	49.380	17	2
192	22.1666	0.0964	0.3910	77.12	49.366	14	0
193	9.2069	2.3686	0.3902	122.49	49.290	17	0
194	211.6334	55.2797	0.2471	78.43	49.156	20	0
195	158.9354	4.5057	0.1538	104.17	49.155	13	3
196	236.5853	9.4711	0.2344	67.19	49.031	16	1
197	141.9248	37.9481	0.2452	54.52	48.976	11	1
198	201.4144	3.9409	0.2265	51.79	48.927	10	2
199	195.0215	52.3356	0.2748	93.00	48.810	17	0
200	142.6942	29.1363	0.3002	44.73	48.482	10	1
201	48.9875	40.5360	0.2213	79.69	48.467	14	0
202	200.4601	4.5767	0.2671	65.79	48.412	14	0
203	241.5006	24.8571	0.2698	104.47	48.389	14	1
204	212.3470	26.6979	0.1626	53.54	48.368	11	4
205	144.5163	64.8933	0.1203	65.66	48.357	17	5
206	147.4695	17.1177	0.3893	79.59	48.300	13	0
207	34.7873	-10.4559	0.1598	63.76	48.233	16	2
208	157.4705	45.7681	0.1857	64.98	48.053	14	1
209	245.1699	9.0521	0.2329	74.99	47.470	14	0
210	190.4023	28.8617	0.2287	66.46	47.369	18	1
211	333.8347	26.0390	0.3542	74.98	47.336	15	0
212	169.1095	49.3269	0.3211	107.67	47.149	13	1
213	201.3776	49.2377	0.2808	66.07	46.736	16	0
214	216.3214	26.4257	0.1749	65.39	46.628	14	1
215	156.8277	38.1909	0.0999	85.44	46.548	17	5
216	342.4831	-1.1036	0.3471	118.71	46.506	16	0
217	183.9200	1.5878	0.3974	142.18	46.419	18	0
218	345.2652	9.0859	0.1710	83.26	46.407	16	1
219	219.6467	4.8675	0.2211	59.91	46.270	10	1
220	222.9747	58.7097	0.2788	76.63	46.243	18	0

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
221	326.6434	-7.8217	0.1277	57.10	46.226	13	1
222	147.1303	6.4030	0.3329	105.81	46.206	19	0
223	150.6430	34.2858	0.3964	92.56	46.107	16	0
224	198.6708	34.3263	0.3012	80.21	45.952	15	2
225	245.0438	42.3731	0.1325	64.35	45.925	16	5
226	3.1222	8.1433	0.3616	75.94	45.906	17	0
227	26.3040	12.7868	0.1579	44.74	45.879	12	0
228	6.2474	7.2233	0.3740	130.90	45.716	15	0
229	1.0994	2.5709	0.0988	81.85	45.678	10	3
230	111.9097	41.8278	0.1181	74.16	45.568	14	2
231	15.7343	25.0595	0.2755	64.61	45.561	12	0
232	206.2182	31.5924	0.3269	86.19	45.505	14	1
233	235.1746	9.5404	0.3355	104.03	45.397	17	0
234	40.4721	-6.5637	0.3514	118.75	45.204	17	0
235	179.3873	67.2917	0.1135	48.86	44.955	10	4
236	175.4502	12.7882	0.0820	76.53	44.868	13	2
237	2.6052	6.7047	0.2397	97.91	44.661	11	1
238	131.2725	29.4148	0.1059	60.01	44.646	12	3
239	123.6864	21.9335	0.3940	105.51	44.537	16	0
240	115.2422	32.1619	0.3243	80.93	44.432	15	0
241	246.7678	13.8493	0.0900	67.39	44.252	13	2
242	22.7374	-9.3484	0.1169	73.47	44.173	15	4
243	355.8446	21.2878	0.3842	91.31	44.031	13	0
244	209.5683	61.8418	0.3258	72.45	44.019	12	0
245	228.0886	22.1660	0.3508	82.94	43.921	14	0
246	30.0850	-2.2527	0.1612	76.25	43.902	14	2
247	20.9880	33.1289	0.2740	92.43	43.822	14	0
248	142.7072	-0.6116	0.3379	79.20	43.576	11	0
249	246.5508	56.2937	0.2557	94.82	43.563	14	1
250	243.9784	6.8687	0.3399	68.69	43.480	12	0
251	175.5046	27.8121	0.1533	80.97	43.440	14	2
252	158.6226	28.8764	0.2862	94.62	43.320	15	0
253	340.6989	26.7873	0.3099	73.44	43.279	11	0
254	139.0291	52.8026	0.1901	35.33	43.234	10	0
255	220.2739	29.5265	0.3181	78.60	43.113	17	0
256	171.6573	52.6952	0.3285	98.90	42.928	14	0
257	250.2535	9.8910	0.3328	56.74	42.888	16	0
258	19.6975	15.1095	0.1278	87.81	42.627	13	0
259	14.8655	1.4700	0.2996	104.20	42.587	16	0
260	209.9503	5.0166	0.0830	86.44	42.586	10	3

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
261	260.5969	30.2579	0.1086	62.21	42.284	12	1
262	132.3288	13.5892	0.1486	72.35	42.187	16	1
263	201.9912	31.2748	0.2401	68.94	42.160	14	0
264	203.0170	58.0004	0.3080	58.16	42.046	14	1
265	228.9943	28.0516	0.2275	78.69	42.007	14	0
266	11.6918	22.8837	0.2660	88.14	41.951	15	0
267	359.0748	24.1959	0.2222	91.96	41.919	14	0
268	3.2686	-8.9082	0.3377	75.86	41.762	10	0
269	12.8053	-2.1837	0.1323	72.65	41.560	15	2
270	219.0598	21.8104	0.2330	65.95	41.479	13	0
271	221.9924	31.4823	0.4064	80.22	41.403	14	0
272	17.8033	4.5377	0.4140	73.88	41.369	11	0
273	350.5193	28.9163	0.4027	79.80	41.324	11	0
274	144.0018	8.2429	0.3771	67.86	41.302	13	0
275	170.8456	21.4131	0.1917	59.17	41.186	10	1
276	179.6987	43.6899	0.2934	74.07	41.158	14	0
277	17.9771	28.7464	0.2580	93.51	41.075	12	0
278	226.6222	34.6164	0.3582	72.07	41.065	10	0
279	18.7730	32.6698	0.1202	63.21	40.854	12	2
280	125.6961	18.2845	0.2955	59.79	40.673	16	0
281	29.8759	-4.8345	0.1295	65.52	40.561	10	2
282	27.1802	14.4965	0.2288	59.03	40.470	11	0
283	136.6765	-0.1499	0.3020	83.23	40.370	14	0
284	25.8930	14.1427	0.3324	70.42	40.337	13	0
285	202.9128	17.0442	0.2066	33.31	40.305	10	0
286	201.9383	-2.1550	0.2902	79.10	40.290	11	0
287	181.3512	37.0867	0.4042	95.08	40.289	12	0
288	195.4708	42.4399	0.3275	78.68	40.254	13	0
289	163.2676	53.3046	0.3156	77.97	40.225	14	0
290	129.7064	54.1336	0.3456	90.78	40.192	15	0
291	232.1374	10.4420	0.2232	59.35	40.058	15	1
292	118.8046	31.4407	0.4037	101.03	39.978	16	0
293	202.9211	4.5879	0.1721	51.63	39.976	13	1
294	348.3292	-5.9877	0.3434	112.22	39.907	12	0
295	166.3469	26.6442	0.3303	71.29	39.828	13	0
296	169.7111	0.5760	0.1024	62.63	39.736	12	2
297	33.8881	0.3717	0.2919	53.49	39.715	12	1
298	196.3461	30.7122	0.1843	69.05	39.594	13	2
299	244.5406	61.7556	0.3329	86.78	39.510	12	0
300	156.6235	31.6156	0.2406	93.41	39.472	13	2

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
301	166.0807	28.4936	0.2123	62.44	39.440	13	0
302	358.5825	-10.0998	0.4004	80.59	39.401	10	0
303	126.0112	23.8358	0.2715	89.35	39.307	11	0
304	38.4025	25.1743	0.3567	91.16	39.266	15	0
305	195.6687	34.2710	0.1869	36.65	39.253	10	1
306	350.8219	17.5451	0.1670	69.09	39.077	14	1
307	129.8545	1.9945	0.3440	64.89	39.043	14	0
308	184.7206	14.7190	0.1582	87.14	38.999	13	1
309	166.4418	12.3698	0.1199	53.07	38.987	14	4
310	146.9139	51.6986	0.2151	49.56	38.916	11	0
311	205.5785	39.9721	0.2545	93.30	38.829	15	1
312	181.5666	37.0619	0.1910	56.36	38.758	12	1
313	177.9706	61.2648	0.3719	83.70	38.718	14	0
314	193.1717	35.4366	0.1615	51.58	38.684	11	1
315	353.9165	16.6451	0.3922	85.60	38.618	12	0
316	255.7731	24.2938	0.1692	52.67	38.594	14	1
317	173.1803	11.5906	0.1724	61.51	38.541	16	0
318	195.6191	19.5781	0.3448	96.37	38.440	16	0
319	337.1808	5.0168	0.1085	80.15	38.400	10	0
320	128.1226	46.4438	0.3685	61.96	38.358	12	0
321	245.8722	48.0162	0.1973	57.85	38.257	11	3
322	253.6259	62.6867	0.3175	56.22	38.200	10	0
323	151.5247	68.7534	0.1958	74.64	38.182	18	0
324	151.4679	30.1043	0.3720	83.93	38.161	13	0
325	150.4333	30.9492	0.3252	70.86	38.069	10	0
326	342.7956	26.5865	0.3666	98.36	37.944	11	0
327	10.3081	0.0143	0.2190	44.48	37.916	10	0
328	19.1814	14.1031	0.0640	65.30	37.764	12	5
329	120.5315	30.1753	0.3205	60.77	37.752	10	0
330	191.7886	29.2330	0.2413	62.69	37.719	13	2
331	179.8435	9.6790	0.2362	84.54	37.629	17	0
332	128.9841	32.1100	0.2529	91.63	37.620	12	0
333	175.8477	10.9042	0.2779	74.28	37.592	12	0
334	180.0369	48.3044	0.2743	69.53	37.561	12	0
335	3.8499	8.8263	0.4112	92.30	37.470	12	0
336	218.5875	18.6387	0.0984	60.24	37.447	14	0
337	357.8954	37.8802	0.3623	93.84	37.436	12	0
338	336.4034	-0.1714	0.4079	91.20	37.407	13	0
339	144.5576	31.4761	0.3876	112.19	37.395	18	0
340	185.6876	19.6357	0.3266	73.69	37.188	12	0

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
341	188.6584	21.6254	0.2537	70.41	37.157	10	0
342	168.6009	31.3419	0.2228	51.99	37.116	11	1
343	187.3462	38.5402	0.3832	65.76	37.101	12	0
344	224.9701	42.4884	0.2901	36.25	37.080	11	0
345	194.3546	24.5545	0.2784	80.32	37.068	14	0
346	34.9308	-0.9845	0.3508	80.14	37.046	10	0
347	42.2333	-0.3535	0.3608	77.50	37.013	12	0
348	324.2459	-5.4870	0.3150	72.73	36.993	10	0
349	182.7127	40.2427	0.2579	116.24	36.988	15	0
350	170.6574	15.2227	0.3396	69.65	36.915	11	0
351	206.9393	15.0778	0.3330	74.21	36.834	12	0
352	342.9682	13.8601	0.2333	48.33	36.816	11	1
353	144.1928	14.2425	0.1325	58.28	36.594	12	1
354	157.9302	20.0364	0.3960	67.15	36.461	10	0
355	169.8938	38.0865	0.1038	56.73	36.375	15	0
356	186.3336	39.7719	0.3107	86.96	36.163	11	0
357	31.8692	17.6923	0.3554	54.47	36.130	12	0
358	7.3086	-6.8013	0.2414	62.79	36.095	10	0
359	176.1676	65.7464	0.3805	68.22	36.091	11	0
360	239.6387	33.6111	0.2880	94.41	35.937	14	0
361	163.3099	9.0608	0.1683	53.61	35.757	10	3
362	220.2377	-0.2826	0.1427	61.82	35.751	10	1
363	7.7968	13.3662	0.1408	71.50	35.735	11	2
364	10.0341	22.1336	0.3912	74.03	35.729	12	0
365	322.2134	4.1220	0.0932	62.58	35.709	12	1
366	149.5033	2.7632	0.3479	75.54	35.614	17	0
367	186.2961	49.6633	0.1870	76.84	35.612	15	0
368	170.4739	11.8070	0.2395	73.93	35.597	15	0
369	194.0070	-2.4124	0.4116	87.91	35.565	14	0
370	223.4027	44.7661	0.2840	59.41	35.287	10	0
371	164.8966	51.1831	0.4021	81.03	35.050	11	0
372	146.9796	28.2415	0.2361	65.15	35.017	13	0
373	239.8608	22.2187	0.3789	50.13	35.016	10	0
374	244.4344	44.2051	0.2167	58.21	34.968	10	0
375	351.4288	26.0662	0.2259	85.64	34.961	11	0
376	27.3974	-0.0638	0.4055	89.35	34.923	13	0
377	330.8397	23.3964	0.2543	38.18	34.855	10	0
378	212.6201	2.0045	0.1792	48.05	34.854	10	0
379	10.1352	10.3159	0.3611	104.06	34.835	13	0
380	229.8975	31.6361	0.3029	80.01	34.780	16	0

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
381	218.2298	20.2600	0.3261	86.28	34.725	13	0
382	176.9759	51.5125	0.1288	40.81	34.718	10	2
383	190.1916	25.1578	0.1734	77.18	34.638	15	0
384	173.7212	9.6918	0.3034	79.33	34.465	11	0
385	191.3066	50.0826	0.2306	81.60	34.312	12	1
386	130.2713	51.6991	0.1233	69.25	34.172	14	2
387	247.3449	46.6278	0.2688	62.98	34.121	11	0
388	182.5635	38.2109	0.3442	54.05	34.073	11	0
389	169.6864	25.1965	0.2556	65.64	34.066	11	0
390	184.4171	24.5546	0.3918	64.84	34.065	11	0
391	116.0891	48.7435	0.2229	54.12	34.043	12	0
392	133.5032	51.7240	0.0972	68.53	33.906	12	0
393	235.4475	51.2841	0.1524	59.26	33.902	10	0
394	327.6260	12.7261	0.3393	98.45	33.852	13	0
395	220.3103	20.5723	0.2497	70.51	33.800	10	0
396	194.6825	48.4290	0.2486	63.76	33.694	10	0
397	138.6324	29.1599	0.1809	64.48	33.564	17	0
398	241.5029	28.5607	0.1708	55.23	33.521	12	0
399	196.6426	6.6214	0.3618	77.93	33.423	13	0
400	343.2137	14.3040	0.3271	100.47	33.399	12	2
401	143.3663	24.1502	0.2100	56.61	33.395	11	0
402	231.6272	37.3892	0.4021	66.62	33.375	10	0
403	112.5968	43.0647	0.2026	88.85	33.192	11	0
404	144.9064	32.9641	0.1293	48.12	33.127	11	0
405	347.0045	-7.5272	0.4036	79.93	33.102	13	0
406	207.7206	35.9862	0.2473	44.16	33.050	15	1
407	188.2001	7.2157	0.4039	52.96	33.023	12	0
408	138.3268	60.1719	0.2945	97.56	32.841	10	0
409	160.0285	47.1814	0.2249	65.24	32.835	10	1
410	213.3589	36.6313	0.3298	61.65	32.819	12	0
411	235.6332	47.1966	0.2009	55.46	32.777	14	0
412	189.3017	39.2517	0.4091	83.26	32.719	11	0
413	250.2755	32.6334	0.3629	80.22	32.695	11	0
414	175.4041	41.9021	0.2534	50.39	32.672	11	0
415	181.3273	44.4015	0.3397	54.50	32.581	10	0
416	28.6672	-7.5640	0.3568	80.37	32.507	14	0
417	41.0265	31.1234	0.3454	92.56	32.456	12	0
418	112.0762	42.2557	0.2102	47.69	32.444	10	1
419	37.3518	23.9273	0.3355	68.48	32.317	11	0
420	159.9795	20.2632	0.3438	73.38	32.312	11	0

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
421	334.1478	26.5376	0.4032	60.84	32.191	10	0
422	345.7306	17.6405	0.3990	91.61	32.134	11	0
423	165.7411	4.0461	0.3270	54.33	32.116	10	0
424	171.8170	2.2769	0.2365	50.76	32.102	12	0
425	152.7778	57.8647	0.3981	73.51	32.062	11	0
426	200.3442	-0.7424	0.2214	71.34	32.048	11	0
427	231.4684	19.0963	0.2127	60.05	32.036	13	0
428	11.2912	40.4557	0.1203	46.29	31.848	12	0
429	237.3524	12.2577	0.2994	71.50	31.827	11	0
430	156.6293	17.4163	0.2960	82.78	31.740	13	1
431	329.4127	4.6195	0.3963	91.59	31.706	13	0
432	120.8899	32.8075	0.3777	62.55	31.680	11	0
433	16.4015	5.6523	0.3218	63.96	31.673	11	0
434	144.8798	14.7334	0.4009	80.23	31.666	13	0
435	26.3991	17.0413	0.4054	87.96	31.646	12	0
436	226.2651	16.5804	0.1501	43.96	31.600	11	1
437	346.9748	25.7485	0.2815	66.90	31.498	10	0
438	224.6172	12.5914	0.3303	69.54	31.433	10	0
439	167.7225	15.3864	0.2785	51.04	31.428	10	0
440	122.7579	51.4139	0.3842	75.04	31.412	13	0
441	132.0084	31.1319	0.3074	68.48	31.321	12	0
442	150.7486	8.7632	0.2592	64.74	31.269	10	0
443	189.9798	37.5940	0.1907	37.88	31.122	10	0
444	144.2794	23.2097	0.2528	79.05	30.992	10	0
445	209.0306	56.1666	0.0700	74.39	30.946	12	2
446	126.5961	31.6074	0.1723	47.27	30.918	14	2
447	126.0212	38.7054	0.3462	65.13	30.859	10	1
448	232.6896	28.2526	0.2191	64.40	30.825	11	1
449	141.5521	11.6309	0.2610	55.14	30.798	10	0
450	181.1489	29.5590	0.2837	78.62	30.673	13	0
451	53.8856	-13.4180	0.3897	79.71	30.556	10	0
452	247.9898	32.7379	0.2985	67.04	30.502	12	0
453	149.4489	4.6031	0.1626	71.71	30.474	11	0
454	167.6282	59.3451	0.4096	52.32	30.408	10	0
455	169.6603	4.9774	0.3044	66.61	30.405	10	0
456	222.2297	53.9058	0.2852	82.93	30.399	11	0
457	125.5133	49.5155	0.1557	73.21	30.390	12	3
458	145.4582	20.6573	0.2241	52.02	30.387	10	1
459	168.7971	55.0625	0.0701	60.34	30.328	10	2
460	190.3488	40.5300	0.1894	69.76	30.218	15	0

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
461	239.5357	9.7686	0.2009	41.31	30.195	13	0
462	163.5704	17.7695	0.1905	59.01	30.001	12	0
463	222.6995	14.8948	0.3025	72.74	30.000	12	0
464	158.7953	52.9665	0.1351	43.23	29.952	11	1
465	175.6030	55.3784	0.2282	73.96	29.940	12	0
466	127.2516	24.6634	0.3861	106.37	29.928	13	0
467	157.3303	49.8898	0.1954	49.24	29.879	10	0
468	18.3481	27.2584	0.3166	63.94	29.744	11	0
469	186.0455	-0.8908	0.1582	62.24	29.736	10	0
470	200.7446	32.5803	0.2604	79.36	29.717	12	0
471	230.0991	22.7254	0.3167	60.70	29.676	13	0
472	147.8500	15.5387	0.3464	66.94	29.526	12	0
473	182.1764	15.0784	0.2652	50.94	29.421	11	0
474	245.7734	43.1034	0.2694	61.40	29.387	10	1
475	233.8297	42.8052	0.1446	49.47	29.318	10	1
476	260.7642	29.4202	0.1918	78.18	29.281	12	1
477	200.2009	38.0959	0.2376	64.71	29.193	10	0
478	244.6155	28.2327	0.0967	71.24	29.152	11	2
479	151.2266	3.8261	0.1223	49.75	29.147	11	0
480	36.0855	44.8079	0.1704	50.57	29.123	11	0
481	15.7259	14.4078	0.3784	63.28	29.120	11	0
482	158.3022	35.0855	0.2704	70.39	29.113	11	0
483	136.1887	31.9421	0.2700	79.35	29.061	10	0
484	189.7554	16.0185	0.2355	60.82	29.034	12	0
485	196.0482	20.6523	0.4096	71.51	29.030	11	0
486	209.2938	-8.2681	0.4067	98.32	28.796	10	0
487	329.8648	-1.2001	0.1472	56.88	28.772	10	0
488	117.3370	21.7919	0.2627	83.18	28.710	10	0
489	218.7492	-1.1352	0.2176	47.09	28.703	13	0
490	239.0367	18.8282	0.2174	108.67	28.648	12	0
491	348.9182	5.8140	0.3506	74.09	28.643	12	0
492	239.8458	15.4911	0.1324	84.81	28.543	11	0
493	143.2630	24.5444	0.2872	55.74	28.262	11	0
494	148.8969	20.0734	0.1705	43.51	28.207	10	1
495	178.7337	19.9175	0.3285	58.98	28.080	11	0
496	174.5809	23.8662	0.3113	76.70	28.014	11	0
497	126.8565	9.3417	0.3159	77.12	27.889	10	0
498	229.1483	42.4750	0.3474	64.41	27.721	10	0
499	231.1363	2.4320	0.3090	52.00	27.715	11	0
500	218.3284	4.0852	0.1577	39.35	27.683	11	1

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
501	212.0318	57.8546	0.3226	92.20	27.654	11	0
502	183.7027	35.7243	0.1342	69.35	27.487	10	1
503	119.2916	19.5392	0.2600	61.29	27.450	13	0
504	145.2660	5.5065	0.3851	61.89	27.439	11	0
505	195.8287	47.6591	0.1426	31.40	27.435	10	2
506	180.8868	41.7590	0.3292	47.32	27.429	11	0
507	159.4272	16.2729	0.1807	38.20	27.335	10	1
508	122.4996	22.4470	0.2729	68.32	27.306	10	0
509	347.6930	23.8545	0.2969	97.99	27.211	10	0
510	334.4457	11.8618	0.1233	56.03	27.159	11	2
511	31.2781	-12.5635	0.3438	70.01	27.102	10	0
512	337.2999	3.8165	0.1878	53.39	27.021	10	1
513	179.3659	-1.9635	0.2728	88.71	27.008	12	0
514	333.0906	-8.2162	0.2138	41.27	26.979	10	0
515	174.0031	36.6885	0.3470	85.03	26.932	11	0
516	30.3478	-7.0515	0.3172	73.79	26.881	12	0
517	21.9518	15.3178	0.1821	54.48	26.812	11	0
518	216.2763	-0.2237	0.3172	64.93	26.753	11	0
519	132.7338	35.6444	0.3885	97.20	26.747	12	0
520	190.7470	54.5454	0.3429	88.50	26.707	10	0
521	205.4881	38.4914	0.2049	57.70	26.705	10	1
522	208.0254	-6.2226	0.3754	71.57	26.670	12	0
523	185.8081	17.8226	0.3618	77.19	26.457	12	0
524	254.6149	26.7601	0.1885	41.78	26.377	10	0
525	220.8744	50.3617	0.1282	52.23	26.347	10	1
526	231.5149	23.0064	0.2262	78.18	26.320	10	0
527	141.6800	5.2915	0.3367	87.61	26.226	11	0
528	200.3007	8.4598	0.2309	50.88	26.073	10	0
529	314.0762	0.7788	0.4098	47.43	26.033	11	0
530	351.5178	31.0298	0.3200	65.65	26.008	10	0
531	171.5606	62.8343	0.2459	73.84	25.909	10	3
532	332.7019	5.6524	0.3239	49.35	25.907	10	0
533	193.7693	35.9458	0.2193	34.96	25.794	10	0
534	2.5895	6.2603	0.3550	84.26	25.691	11	0
535	226.3826	-0.0167	0.2358	66.64	25.690	11	0
536	177.6136	21.7032	0.1925	53.17	25.370	10	0
537	211.4311	45.2161	0.3879	73.19	25.292	10	0
538	163.3315	19.8085	0.3303	54.85	25.206	10	0
539	133.1166	17.6700	0.1834	49.08	25.160	11	0
540	359.5310	1.7498	0.2668	81.89	25.134	10	0

Table 4.1 – continued

SrNo	RA (deg)	Dec (deg)	Z	Size (Mpc)	Mass ($10^{14} M_{\odot}$)	N_{mem}	N_{Abells}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
541	333.4263	11.3196	0.3937	76.94	25.116	11	0
542	213.5983	0.0782	0.1254	39.69	25.074	10	0
543	22.3432	27.4763	0.4101	72.32	25.070	10	0
544	131.8480	35.9600	0.4058	55.62	24.876	10	0
545	134.1965	36.8940	0.2184	56.73	24.826	11	0
546	147.0089	45.1003	0.3334	62.12	24.817	11	0
547	218.2560	48.4879	0.2551	61.56	24.680	10	0
548	232.5630	46.1745	0.3129	57.92	24.263	10	0
549	130.7000	15.2818	0.3138	92.78	24.203	11	0
550	197.1872	11.3447	0.3742	67.89	24.115	12	0
551	358.6163	10.4825	0.3778	70.47	23.949	10	0
552	245.1822	11.5609	0.2025	68.11	23.914	10	0
553	256.8197	21.2253	0.2115	63.84	23.896	11	0
554	240.4866	7.4866	0.3331	72.87	23.635	11	0
555	344.2509	10.0150	0.1545	58.90	23.624	10	1
556	220.0594	9.2009	0.3210	70.40	23.411	11	0
557	242.4677	14.8100	0.3095	63.63	23.336	10	0
558	220.2519	24.3797	0.1574	65.10	23.177	12	0
559	139.0198	25.3861	0.3241	83.05	23.122	10	0
560	199.4825	11.0943	0.2129	51.24	22.946	10	0
561	241.6706	7.5126	0.3622	70.14	22.905	10	0
562	182.3080	55.5341	0.3326	53.73	22.612	10	0
563	165.4018	22.7482	0.3949	55.02	22.325	10	0
564	250.2542	23.9605	0.2135	48.40	22.291	10	0
565	200.3486	6.7289	0.2847	76.04	22.181	10	0
566	7.9745	1.4290	0.3375	56.47	22.040	10	0
567	229.7661	12.2047	0.3529	63.87	22.018	10	0
568	195.7331	2.0663	0.3576	79.69	21.634	10	0
569	181.5561	1.4680	0.2566	53.08	21.616	10	0
570	232.3803	36.1066	0.4149	48.42	21.220	10	0
571	2.0733	30.0413	0.3566	94.53	20.120	10	0

Chapter 5

Correlation of Voids and High Density Regions

In the large scale structure of the Universe, most of the matter is distributed in the skeleton of the cosmic web, namely filaments, walls, and clusters. These regions have densities above the mean density of the Universe. On the other hand, the voids dissipate matter into this skeleton and in the process become more and more empty. This gives a correlation between positions of voids and high density regions (HDRs) as can also be seen in Figure 3.19.

In this chapter, we construct a catalog of voids and HDRs from LOWZ and CMASS galaxy data of SDSS and then calculate the scales at which they are highly correlated. We have used the following cosmological parameters to calculate physical quantities – $H_o = 100 h \text{ kms}^{-1} \text{ Mpc}^{-1}$, $\Omega_{MO} = 0.308$, and $\Omega_{\Lambda O} = 0.692$.

5.1 Data

The spectroscopic galaxy data used in this work is taken from the Baryon acoustic Oscillation Spectroscopic Survey (BOSS; Alam et al. (2015)) of the data release 12 (DR12) of the third generation of SDSS (SDSS-III). A total of 1.21 million unique galaxies' spectra is observed by BOSS for large scale structure studies which is distributed in two contiguous regions of the sky in the Northern and Southern galactic hemispheres. These regions have a

survey footprint of $10,400 \text{ deg}^2$ and a sky area (excluding regions having bright stars and data that is irrelevant to the survey goals) of 9376 deg^2 . Spectroscopic galaxy targets were selected to create two catalogues for two different redshift ranges – LOWZ ($z < 0.45$) and CMASS ($0.4 < z < 0.7$). The details of selection criteria, data processing techniques, and final galaxy catalogue of LOWZ and CMASS samples are given in (Eisenstein et al., 2011; Dawson et al., 2013; Alam et al., 2015; Reid et al., 2016).

To the LOWZ and CMASS datasets, we give the redshift cuts of $0.15 < z < 0.43$ and $0.43 < z < 0.7$ (Nadathur, 2016). Figure 5.1 shows the angular completeness masks of the two samples. To extract voids and HDRs, the Northern and Southern regions of both, LOWZ and CMASS, samples are used separately. The two samples have different radial selection functions because of different selection criteria imposed on them, which results in the variation of number density $n(z)$ of galaxies as a function of redshift. Appropriate weights are applied on the clustering algorithm to account for the radial and angular selection effects. The summary of the data sets are given in the Table 5.1.

Table 5.1: Data summary of LOWZ and CMASS samples

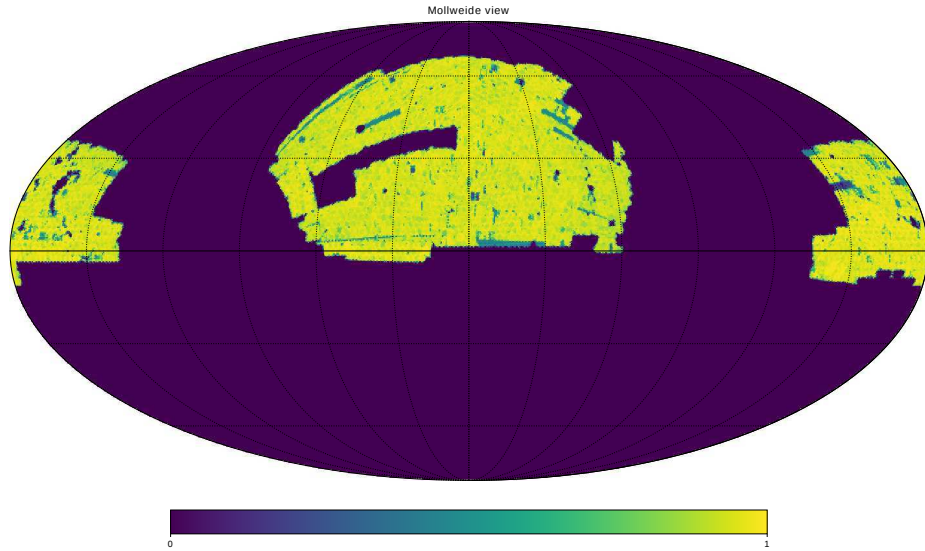
	LOWZ (North)	LOWZ (South)	CMASS (North)	CMASS (South)
Number of galaxies	248,237	113,525	568,776	208,426
Redshift range	0.15–0.43	0.15–0.43	0.43–0.7	0.43–0.7
Effective sky area (deg^2)	464	198	544	200
Volume ($h^{-3} \text{ Gpc}^3$)	0.86	0.37	2.63	0.97

Note : Here, effective sky area is the weighted sky area (sky area weighted by the angular weights).

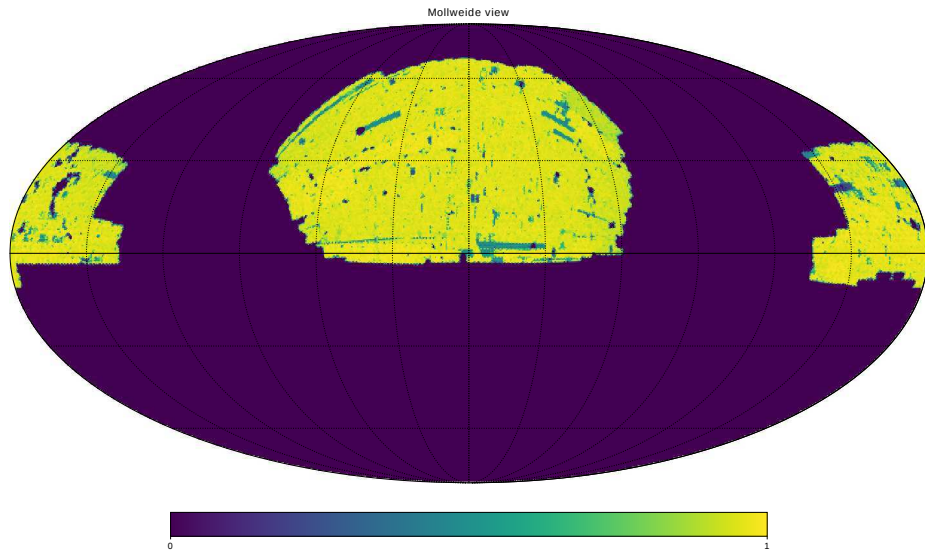
5.2 Methods

5.2.1 Voids and High Density Regions

As described in chapter 2, we used modified ZOBOV algorithm to extract voids in LOWZ (North), LOWZ (South), CMASS (North), and CMASS (South) samples of galaxies. To construct a catalog of voids and high density regions (HDRs), all the selection effects are taken into account. The angular selection functions, function of sky position, are given in Figure 5.1. The radial selection functions, function of redshift, are calculated from the redshift dependence of number density of galaxies (see Figure 5.2). Please refer chap-



(a) LOWZ mask



(b) CMASS mask

Figure 5.1: Angular completeness masks of LOWZ and CMASS galaxy samples in Mollweide projection. The colorbar indicates the completeness values from 0 to 1. The masks are centered at $RA = 180^\circ$ and $Dec = 0^\circ$.

ter 2 for full details of the void and HDR finding algorithm. The number of

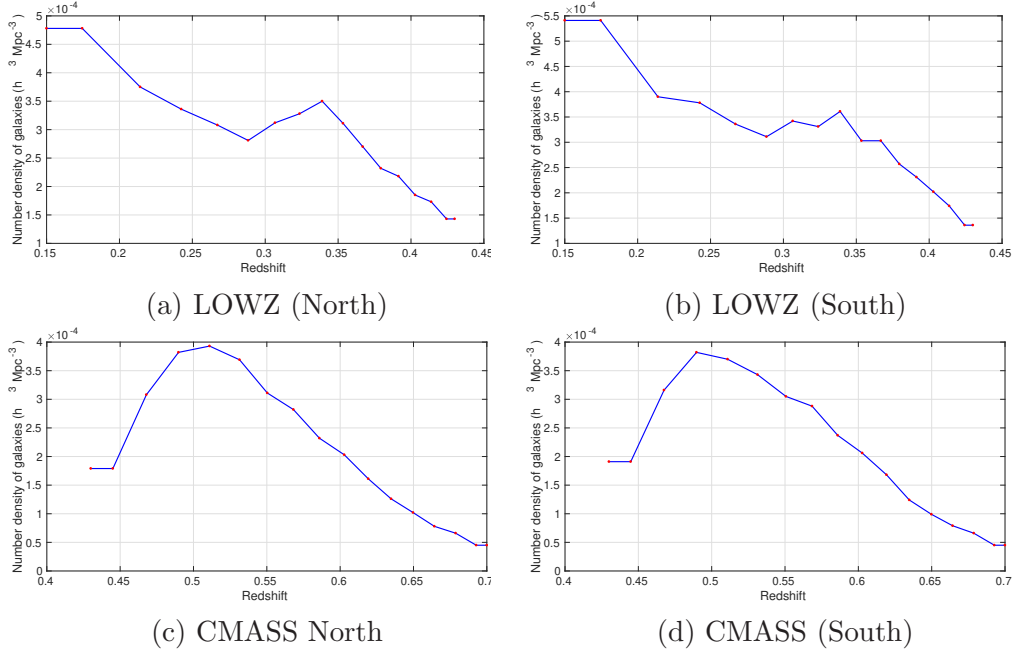


Figure 5.2: Number density of galaxies in LOWZ and CMASS samples.

voids and HDRs obtained in this manner are summarized in Table 5.2. The

Table 5.2: Number of voids and HDRs.

Survey	Voids	HDRs
LOWZ (North)	2122	18022
LOWZ (South)	1007	8088
CMASS (North)	5425	15185
CMASS (South)	1996	8100

large difference in the number of voids and HDRs is because voids occupy large regions of space while HDRs are comparatively smaller.

5.3 Results and Discussion

We now present the results from the void and HDR catalogues in the LOWZ and CMASS datasets of SDSS's DR12. We get 3129 and 7421 voids from LOWZ and CMASS datasets, respectively. The number of HDRs in LOWZ and CMASS datasets are 23285 and 26110, respectively.

5.3.1 Distribution of Voids

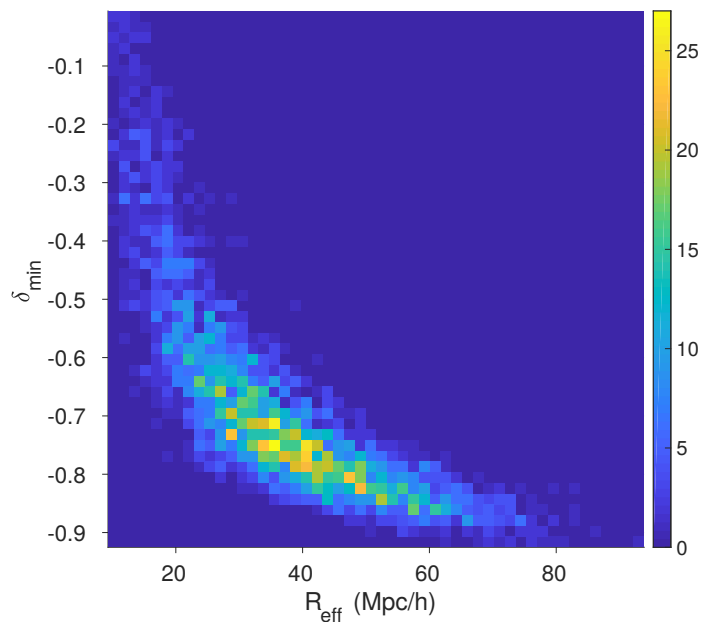
The void catalogue shows that the minimum density δ_{min} in a void is highly correlated with the size (or effective radius R_{eff}) of the void. This can be seen in Figure 5.3 for LOWZ and CMASS data. Deeper voids have larger radii in both LOWZ and CMASS data. This feature is also seen in simulations and does not depend on the type of tracer particle (dark matter or baryonic matter). Voids extracted from other void finding algorithms also show similar behaviour (Cautun et al., 2016; Zhao et al., 2016; Nadathur, 2016). These distributions peak around $\delta_{min} = -0.75$ and $R_{eff} \sim 35 Mpc/h$ for LOWZ sample, and $\delta_{min} = -0.75$ and $R_{eff} \sim 43 Mpc/h$ for CMASS sample. This correlation suggests that the smaller voids are likely to collapse while larger voids will keep on expanding (Sheth and van de Weygaert, 2004). This is because, in general, smaller voids have overall higher densities than the mean density, mostly is the contribution from the surrounding high density boundaries. And larger voids, due to their occupation of larger volumes, have overall less densities than the mean density. It gives the larger voids time to grow even larger.

5.3.2 Radial Density Profile of Voids

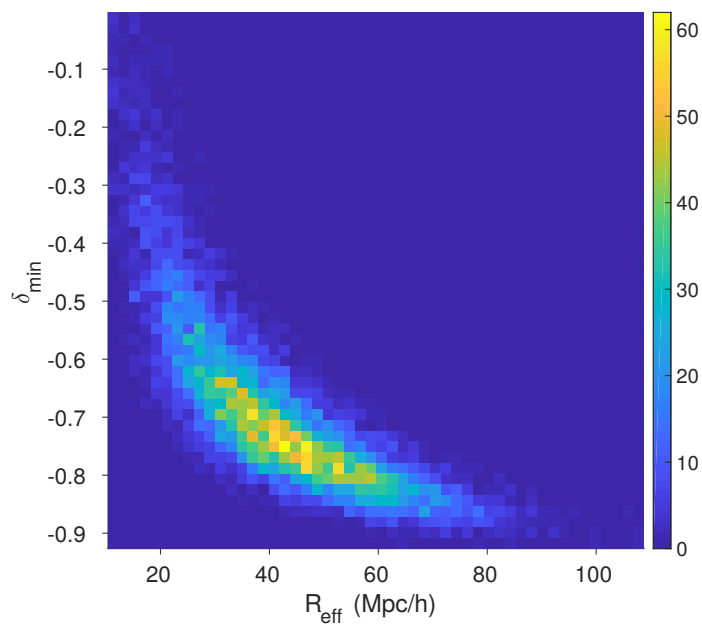
To get the density profile of voids, we stacked the voids by coinciding the centers of all voids and rescaling the size of voids relative to their effective radii. The stacked density profile of the voids is shown in Figure 5.4. This profile shows that the typical density contrast at the center of void is $\delta \sim -0.8$. From the center, the density gradually increases as one move away from the void center. When it reaches around 80% of the effective radius the density crosses the mean density. It then rises upto the effective radius and then decreases slowly and goes towards the mean density. This overdensity around the voids correspond to the HDRs surrounding the voids. The errors are higher near the void center and lower away from the void center because the number of galaxies near the void center is much less than the number of galaxies at the surrounding outer regions.

5.3.3 High Correlation Scales

The void centers and HDR peaks, found from the void and HDR catalogs, is used to estimate the correlation functions of void-void, void-HDR and HDR-HDR. Landy-Szalay (LZ) estimator (see chapter 2) is used to estimate the correlation functions. Figure 5.5 shows the three correlation functions. This shows that the HDRs are highly correlated with each other on a length scale of $\sim 10 Mpc/h$, voids are highly correlated on the length scales of ~ 65



(a) LOWZ



(b) CMASS

Figure 5.3: The distribution of voids as a function of minimum density in a void $\delta_{\min}$ and its effective radius R_{eff} . The colorbar shows the number of voids.

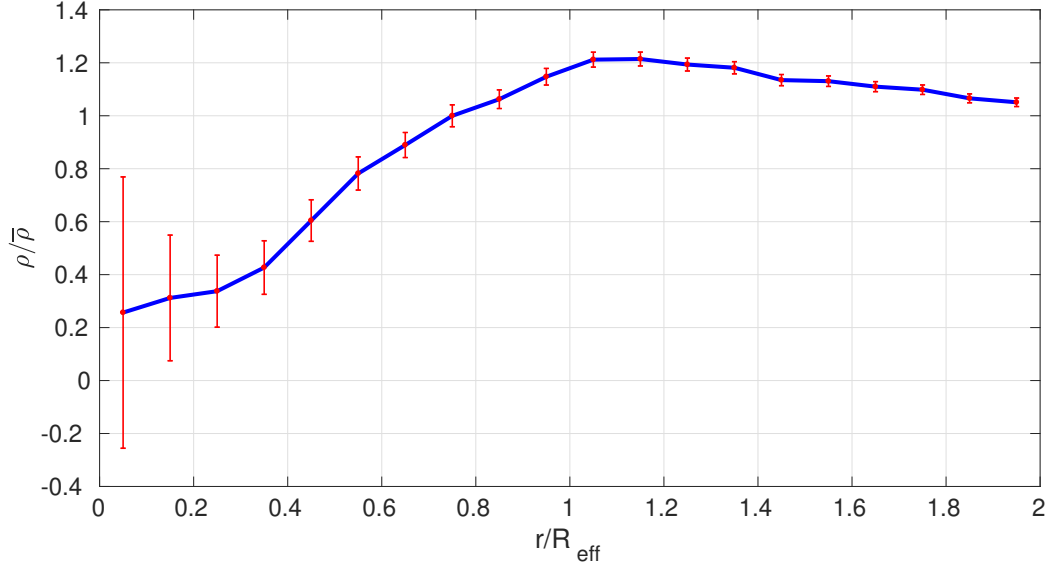


Figure 5.4: The stacked density profile of voids from BOSS data. Errors are enlarged by a factor of 100 for the representation. The y-axis shows the overdensity and x-axis shows the radial distance from void center in units of R_{eff} .

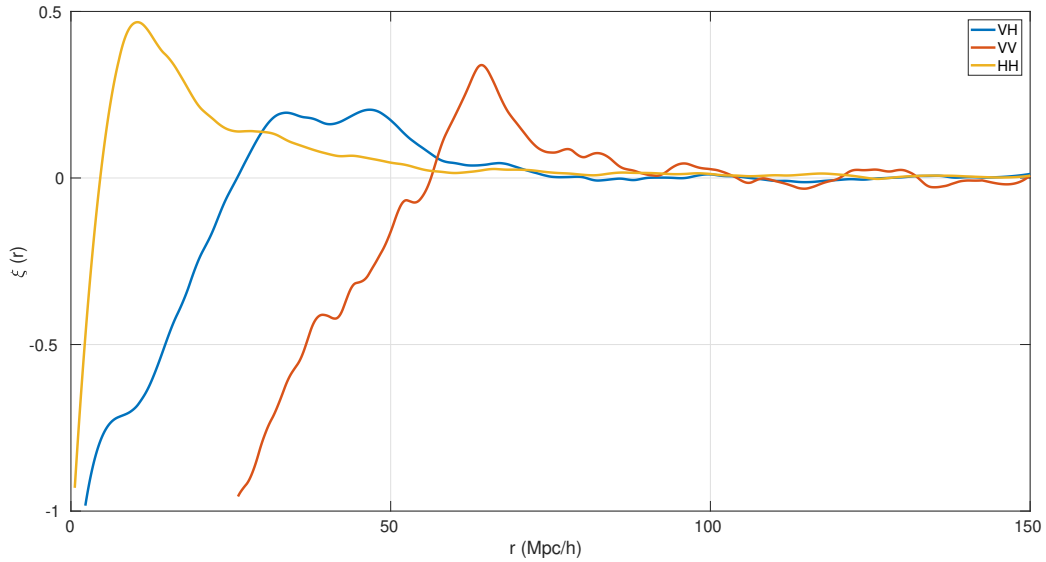


Figure 5.5: Correlation functions of void-void (VV), void-HDR (VH), and HDR-HDR (HH). The Landy-Szalay estimator is used to find these correlations.

Mpc/h, and the voids and HDRs are highly correlated over a range of length scales of 35 – 50 Mpc/h.

We define these length scales, where the correlation functions has highest values, as ‘High Correlation Scales’ of void-void, void-HDR and HDR-HDR correlations. The high correlation scales of HDR-HDR and void-void are approximately twice the effective radii of HDRs and voids suggesting that at these lengths, they are compactly packed. The range of high correlation scale of void-HDR suggests that the distribution of HDRs around voids has a spread which is limited by the next voids. That means the HDRs are sandwiched between voids. Our interpretation of these results is that the HDR-HDR correlation represents the gravitational effect due to dark matter, void-HDR correlation represents the effect of dark energy on the dark matter distribution, and void-void correlation represents the effect of dark energy on a larger scale where dark matter distribution is sandwiched between the voids. The anti-correlation observed on small scales is due to the non-zero and finite sizes of voids and HDRs, i.e., the void centers and the HDR peaks avoid to exist in other voids or HDRs.

This was the first step in order to understand the effects of dark energy, due to void expansion, on the structure formation. Later studies using different galaxy data and numerical simulation data will be done for the better understanding of the process.

Chapter 6

Summary and Future Scope of Study

6.1 Summary of Contributions

In this thesis, we focused on identifying the large-scale structures in the Universe and their correlation. With the help of relatively dense samples of spectroscopic and photometric galaxies of SDSS, we have identified two main large scale features of the cosmic web - superclusters and voids. Taking the first step to see the effects of void dynamics on the matter density enhancement in observations, we have calculated the high spatial correlation lengths among density maxima and density minima.

We have discovered a large and massive supercluster, containing 43 galaxy clusters, at $z = 0.28$ from the spectroscopic galaxy sample of SDSS and named it Saraswati supercluster. With the mass of $\sim 2 \times 10^{16} M_{\odot}$ and size of ~ 200 Mpc, this supercluster is comparable to the nearby superclusters, like Shapley supercluster, Sloan Great Wall, and Laniakea supercluster. We used the luminosity density of galaxies and friends of friends algorithm to extract this supercluster. Although the current density estimate of this supercluster suggest that it is not a bound system, the number of massive clusters found in it are much more than what is expected from Λ CDM cosmology. The future deep spectroscopic observations can shed light on its true extent. This supercluster is also important as there is no other supercluster discovered of this size and mass within the redshift range of $0.2 \leq z \leq 0.4$.

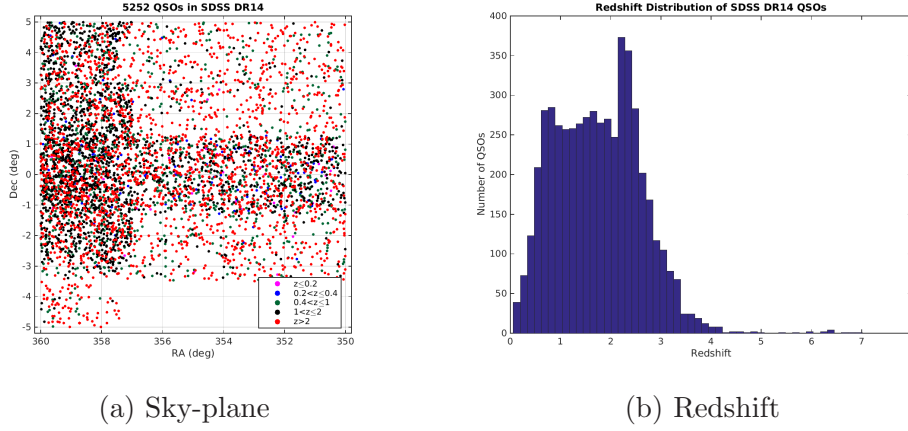


Figure 6.1: Sky-plane and redshift distributions of quasars (QSOs) in a 100 deg^2 region around Saraswati supercluster.

The sky-plane region of Saraswati supercluster has available spectra of *sim* 5,250 quasars in DR14 of SDSS (see Figure 6.1). There are over 2800 background quasars for the Saraswati supercluster. The spectra of these quasars can be used to study the absorption features due to Saraswati supercluster. The distribution of quasars on the sky-plane is not uniform because of varying sky coverage for quasars in different SDSS programmes.

We have identified 571 superclusters using WHL cluster catalog made from photometric galaxy data of SDSS. The minimum clusters in each supercluster is 10. This catalog has revealed that there can be superclusters with more than 50 groups and clusters within them. The techniques of Delaunay triangulation and friends of friends algorithm were implemented for the construction of the catalog. Selection effects were taken into account in the clustering analysis. The most massive supercluster is found at $z = 0.12$ with 59 groups and clusters. Out of these 59 clusters, 10 are massive Abell clusters. These superclusters have mean mass $\sim 5 \times 10^{15} M_{\odot}$ and size $\sim 80 \text{ Mpc}$. The mean size is found to be close to the homogeneity scale of the Universe. This catalog also identifies some previously discovered superclusters, making it a robust catalog. This catalog will provide ground to various study of structure formation and evolutionary properties of galaxies.

From the BOSS data of SDSS, we have identified high density and low density (voids) regions in the galaxy distribution. We applied the parameter-free Delaunay triangulation, Voronoi tessellation and watershed algorithm, along with careful removal of the selection effects in the survey, in the analysis. We found a total of 10,550 voids and 49,395 HDRs. The two-point correlation functions among voids and HDRs were calculated which show that the high correlation lengths between HDR-HDR is $\sim 10 \text{ Mpc/h}$, Void-HDR is $35\text{-}50 \text{ Mpc}$, and void-void is $\sim 65 \text{ Mpc/h}$. This study gives the length scales on which the probability of finding HDRs and voids is higher than the

random distribution.

6.2 Future Scope

Saraswati supercluster region provides a wide range of density environments to galaxies. To understand the properties of Saraswati supercluster in more detail, multi-wavelength study of this supercluster is needed. With the main aims of probing galaxy evolution in supercluster environment and probing diffuse radio emission on group, cluster and supercluster scales to study the cosmic ray acceleration and magnetization, we have already started the observations of Saraswati with the upgraded Giant Meterwave Radio Telescope (uGMRT). Because of the large region covered by Saraswati, these observations will be completed in multiple observational cycles of uGMRT. To understand the dynamical state of this supercluster, a dedicated multi-fibre spectroscopic programme is required which will observe at least 10,000 galaxy spectra within this region. We plan to do that using 4m Anglo-Australian Telescope (AAT). AAT is ideal to observe an object situated at redshift $z \sim 0.3$.

The supercluster finding algorithm can still be improved to find the true extent of the superclusters. Even though the current algorithm gives the minimum extent of superclusters, it has identified so many massive superclusters. We plan to make a more sophisticated algorithm which will be built on the algorithms presented in the thesis. We also plan to do similar study using Horizon Run 4 (HR4) simulations and compare the results.

The correlation between dark energy and dark matter via void expansion is a special feature of the cosmic web. As the selection effects have now been calculated for BOSS galaxy data, numerical simulations can be used to create mock galaxy catalogs with the exact geometry as that of the BOSS survey. We plan to extract voids and HDRs from the galaxy catalog of HR4 simulation and calculate void-void, HDR-HDR, and void-HDR correlation lengths. This will be used to compare the observations with the simulations and see if there is any need to modify the simulation codes for the effects of higher Hubble parameter values in voids.

As the voids and HDRs are not exactly point objects but are extended 3-Dimensional objects, the cross-correlation function will have additional dependency on the sizes and morphologies of voids and HDRs. This will be explored in the future works.

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