Mathematical Exploration of Sections of *Tantraratna* and *Tāladīpikā*: Unraveling *Tippa Gopendra Bhūpāla's* Contribution in Astronomy and Rhythm

A Thesis

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by

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Certificate

This is to certify that this dissertation entitled “Mathematical Exploration of Sections of *Tantraratna* and *Tāladīpikā*: Unraveling *Tippa Gopendra Bhūpāla's* Contribution in Astronomy and Rhythm ” towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Shivani at Indian Institute of Science Education and Research under the supervision of Dr. Venketeswara R. Pai, Associate Professor, Department of Humanities and Social Sciences , during the academic year 2024-2025.

Venketeswara R. Pai **Shivani**

Supervisor

This thesis is dedicated to my cit-raṅga-naṭa

दरोन्मीलत्फालद्युतिमदमृतस्यन्दिशुभिकं
भ्रमन्मीनोष्णीषं पदसरणिपारीणवलयम्।
विराजहंभावव्यतिकरितपुंभावसुभगं
पुरस्तादाविः स्ताद्भुवनपितरौ तन्मम महः॥

-तिप्पगोपेन्द्रभूपालः

The transliteration scheme followed in this thesis to represent Devanāgarī script is given below.

Table of vowels

अ	आ	इ	ई	उ	ऊ	ऋ	ॠ	ऌ	ए	ओ	ऐ	औ	अं	अः
a	*ā*	*i*	*ī*	*u*	*ū*	*ṛ*	*ṝ*	*ḷ*	*e*	*o*	*ai*	*au*	*aṃ*	*aḥ*

Table of consonants

क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
ka	*kha*	*ga*	*gha*	*ṅa*	*ca*	*cha*	*ja*	*jha*	*ña*
ट	ठ	ड	ढ	ण	त	थ	द	ध	न
ṭa	*ṭha*	*ḍa*	*ḍha*	*ṇa*	*ta*	*tha*	*da*	*dha*	*na*
प	फ	ब	भ	म					
pa	*pha*	*ba*	*bha*	*ma*					
य	र	ल	व	श	ष	स	ह	ळ	
ya	*ra*	*la*	*va*	*śa*	*ṣa*	*sa*	*ha*	*ḷa*	

Declaration

I hereby declare that the matter embodied in the report entitled ”Mathematical Exploration of Sections of *Tantraratna* and *Tāladīpikā*: Unraveling *Tippa Gopendra Bhūpāla's* Contribution in Astronomy and Rhythm” are the results of the work carried out by me at the Department of Humanities and Social Sciences, Indian Institute of Science Education and Research, Pune, under the supervision of Dr. Venketeswara R. Pai and the same has not been submitted elsewhere for any other degree.

Venketeswara R. Pai
Supervisor

Shivani

Abstract

The *Vijayanagara Empire* stands as a significant epoch in Indian history. Exerting a profound and lasting influence on the cultural, educational, and economic development of southern India, particularly in the regions corresponding to present-day *Karnātaka*, *Andhra Pradesh*, and *Tamil Nāda*. This period was marked by extraordinary advancements in intellectual and artistic pursuits, fostering an environment that enabled the proliferation of knowledge across multiple disciplines. Beyond the celebrated contributions of eminent scholars and the distinguished *Aṣṭadiggajas*, who played a crucial role in shaping the educational and literary landscape, the rulers of the Vijayanagara Empire were equally remarkable. These kings were not only adept administrators but also patrons of the arts, literature, and scientific inquiry, significantly enriching the intellectual and cultural heritage of this region.

Among the illustrious figures of this era, *Bhūpāla Tippa Gopendra* of the *Sāluva* dynasty remains largely overshadowed despite his significant intellectual contributions. His scholarship spans multiple domains, particularly in musicology and astronomy, where his works exhibit a remarkable synthesis of mathematical precision and artistic insight. This interdisciplinary approach reflects the intellectual traditions of the Vijayanagara period, where scientific inquiry and artistic expressions were not differentiated but deeply interconnected.Among his known works, three stand out: *Tantraratna*, an astronomical treatise; *Tāladīpikā*, a comprehensive study on music systems; and *Kāmadhenu*, a commentary on *Vāmana's Kāvyālaṅkāra Sūtra Vṛitti*. This thesis delves into two of his works, *Tantraratna* and *Tāladīpikā*, through an analysis of primary manuscripts.

The first section of this thesis examines the *Candravākyas* presented in *Tantraratna*. These *vākyas* are decoded, mathematically analyzed, and their underlying principles are explored alongside a comparative study with earlier works by *Mādhava* and *Vararuci*, assessing their methodological continuity and advancements.The second section focuses on the concepts of *tāla*, particularly the *Prastāra* methodology as discussed in *Tāladīpikā*. *Tāla Deśa Prāṇas*, *Māgadeśi* and *Prastāra* systems are studied with references to earlier texts such as the *Nāṭyaśāstra* and *Saṅgīta Ratnākara*.Through this study, the enduring relevance of Tippa Gopendra's intellectual legacy is brought to light, reaffirming his place in the scholarly traditions of medieval India.

Acknowledgments

There are several people without whom this thesis would have been nearly impossible.Firstly, I would like to express my deepest gratitude to my supervisor, Dr.Venkateswara R Pai. I am extremely grateful to God for blessing me with a supervisor of his stature. I fear that my words may not convey the depth of my gratitude. Every effort, every moment of kindness, every gesture of support he has bestowed upon me holds more meaning than I can ever fully convey. His guidance has not only been instrumental in shaping this thesis but has also inspired me to uphold meaningful virtues and ideologies in life.I extend my gratitude to G.V. Pavan Kumar for his insightful suggestions and to Aruna Avadhani for his invaluable help with translations.

I am deeply grateful to my parents and Shammi for their unwavering support and for patiently enduring my chaotic moments. A huge thanks to all my dear friends for their constant encouragement . A special acknowledgment goes to Anagha, Asmi, and Mogha for always lending a listening ear to my endless rants—often without any context, whatsoever .

Contents

0.1 About the author:Tippa-Gopendra-Bhūpāla

Tippa-Gopendra-Bhūpāla (1423–1446 CE) is a multifaceted visionary ruler of the *Sāluva dynasty*. Though not widely known, he made significant contributions to various socio-cultural spheres. Tracing his lineage, Tippa was the grandson of Sāluva Tippa and the brother of *Sāluva Tirumalarāya*, who held the position of viceroy in *Vijayanagara*. Some texts also suggest that he was married to Harimā, the elder sister of Deva Rāya II, and was appointed as a governor under Deva Rāya II before later serving in the viceroyalty of Mulbagal.[1]

Tippa's ruling chronicles have been largely lost in history or remain inadequately studied. However, his contributions across multiple disciplines are highly valued in their respective fields. He is currently credited for authoring *Tantraratna* (an astronomical treatise), *Tāladīpikā* (a musical work), and *Kāmadhenu* (a commentary on *Vāmana's Kāvyālaṅkāra Sūtra Vṛtti*). Apart from these, *Kāmadhenu* suggests the existence of an additional[2]treatise on *Tāla*, alongside *Tāladīpikā*. It also alludes to a work on *Nāṭya*, neither of which has been recovered. Even the extant works remain largely unexplored. This lack of comprehensive study has constrained the potential contributions these texts could offer to both artistic and academic disciplines.

Kāmadhenu (KD), a commentary on Vāmana's *Kāvyālaṅkāra Sūtra Vṛtti*[3], has recently gained recognition, solidifying Tippa's position as a significant figure in Sanskrit *Alaṅkāra* literature. The revival efforts of *Śrī Kṛṣṇa Sūri*[4] have brought this work into scholarly focus, unveiling a literary gem of its kind. *Śrī Kṛṣṇa Sūri* has also published a critical edition of this commentary. Vāmana's treatise is structured into five main sections and Tippa systematically addresses these in twelve of his chapters.These include,*Kāvya-prayojana-parīkṣaṇam* (purpose of poetry), *Adhikārī-vicāraḥ* (who is qualified to compose poetry), *Kāvya-aṅga-kāvya-bheda-pratipādanam* (poetic elements and types of poetry), *Pada-padārtha-doṣa-tattva-vivecanam* (words, meanings, and poetic faults), *Vākya-vākyārtha-doṣa-prapañcanam* (faults in sentences and their meanings), *Guṇa-alaṅkāra-bhedaḥ evaṃ śabda-guṇāḥ* (classification of poetic qualities and ornaments, including qualities of words), *Artha-guṇāḥ* (qualities of meaning) and *Śabda-alaṅkārāḥ* (figures of speech).

This work holds immense significance for contemporary literary scholars and poets, offering valuable insights into the foundational principles of poetry.

Tantraratna (TR) is an astronomical treatise of the *Vākya school*, systematically structured into five sections. A key focus of this text is the treatment of *Candra-vākyas*—a set of 248 coded statements formulated to determine the true longitude of the Moon. These *vākyas* are primarily derived from the parameters of the *Sūrya Siddhānta*, a foundational work in Indian astronomy. This text will be dealt in detail in the first part of this thesis.

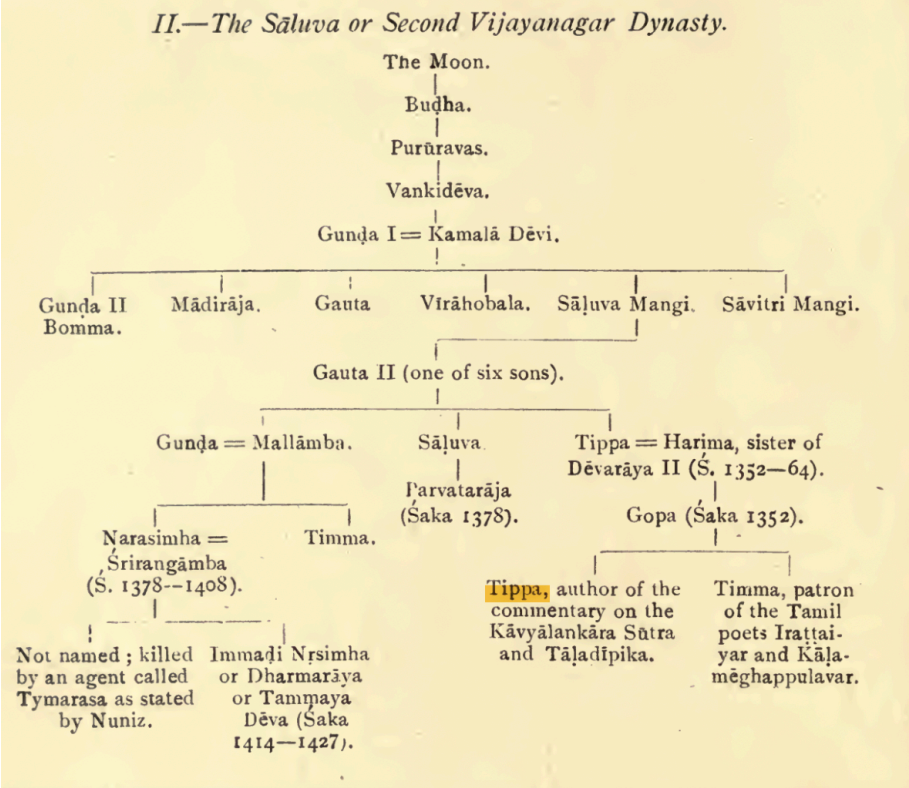
Tāladīpikā (TD) is a comprehensive work on music comprising three chapters: *Mārgatāla Vivekaḥ*, *Deśītāla Vivekaḥ*, and *Mārgadeśītāla Prastārādi Nirūpaṇam*. The first chapter introduces fundamental terminologies of *tāla* and elaborates on the *mārga tālas*. The second focuses on *deśī tālas* and the final chapter explores the *prastāra* system. The concept of '*Tāla-Daśa-Prāṇa*,' which forms the core of the modern Indian rhythmic system, was first documented during the Vijayanagara period, appearing for the first time in *Tāladīpikā* by Sāluva Gōpa Tippēndra. A more in-depth discussion of this text is provided in Section 2 of this thesis. The following extracts from the aforementioned works serve as direct evidence of Tippa's authorship. [1] [2] [3] [4]

प्रबन्धं तालानां भवनुतिमिषेणातनुत यः
शिवाकृष्णाकारा नटनकरणानामपि भिदाः।
स वृत्तेर्व्याख्यानं सरलरचनं वामनकृतेः
विधत्ते गोपेन्द्रत्रिपुरहरभूपालतिलकः ।।
पावनपदविन्यासा समग्ररसदोहशालिनी भजताम्।
घटयति कामितमर्थं काव्यालङ्कारकामधेनुरियम्॥[KD,1.1.7,8]
इति कृतरचनायामिन्दुवंशोद्भहेन त्रिपुरहरधरित्रीमण्डलाखण्डलेन।
ललितवचसि काव्यालङ्क्रियाकामधेनावधिकरणमयासीत्पञ्चमं पूर्तिमेतत्॥
इति श्रीगोपेन्द्रतिप्पभूपालविरचितायां वामनालङ्कारसूत्रवृत्तिव्याख्यायां
काव्यालङ्कारकामधेनौ प्रायोगिकं नाम पञ्चममधिकरणम्।।[KD,5.2]

[1] *Sources of Vijayanagara History*, S. Krishnaswami Ayyangar (1919), pp. 12, 21, 22, 30.
[2] *The Journal of the Madras Music Academy*, Vol. 4 (1933), pp. 51–52.
[3] *Studies in Indian Literary History*, P. K. Gode (1953), p. 210.
[4] *Kāvyālaṅkāra Sūtra Vṛtti*, with the commentary *Kāmadhenu*, edited by Śrī Kṛṣṇa Sūri, 1909.

इति सालुववंशार्णवपूर्णिमाचन्द्रस्य भरतमतपारावारपारीणस्य
श्रीगोपतिप्पभूपालस्य कृतौ तालदीपिकायां
मार्गतालप्रस्तारादिप्रत्ययनिरूपणप्रकरणम्[TD.57]

एतानि वाक्यानि तिप्पभूपाल विरचितानि।
अन्त्ये गोपतिप्पपुन्नाग संयुतिः।[TR]



Tippa Gopendra's[1] lineage tree

Part 1 : Tantraratna

Chapter 1

Tantraratna

The Vākya system is one of the oldest and most prominent methodical marvels in the history of Indian astronomy. A lineage of proficient mathematicians have devised methods to obtain the longitude of the Moon with an accuracy of up to seconds. Beginning with the *Girnāh-śreyādi-vākyas* of *Vararuci* (400 CE) to the recent mastermind, *Saṅgamagrāma Mādhava* (c. 1340 – c. 1425) of the *Kerala School*, the developments in the astronomical realm have been immense. Both the *Vararuci* and *Mādhava vākyas* are popular and have been greatly revived.Tippa Gopendra, a contemporary of *Mādhava*, has formulated his own unique set of *vākyas*. The present section examines his contributions to *candravākyas*, providing a systematic decryption, mathematical analysis, and comparative study with earlier systems.

The *vākyas* numbered 11, 130, 162, 209, and 210, along with the sequence from 93 to 100, are identified as erroneous.Corrected values and alternative *vākya* structures are proposed. A notable trend of similarity is observed between the *Tippa Gopendra vākyas* and the *Mādhava vākyas*. The computational analysis using *Sūrya Siddhāntic* parameters is performed further revealing its characteristics and astronomical significance.

1.1 Introduction

Astronomy has always been a subject of great interest in the Indian subcontinent.Its unequivocal correlation with principles ranging from the measurement of time to astrology has been fundamental in preserving its integrity. Though its foundations lie in the Vedic ethos, it has provided substantial scope for skepticism and advancement.The primary objective of Indian astronomy has been to investigate and understand the motion of planets and planetary

phenomena[SS]. The following topics are elaborately addressed:

• Calculation of longitudes (positions) of planets

• Computation of the elements of the *Pañcāṅga*

• Determination of direction, location, and time

• Calculation and projection of solar and lunar eclipses

• Phenomena related to the Moon, including heliacal rising and setting, lunar phases, the elevation of lunar horns, and the Moon's daily motion

• Phenomena related to planets, such as heliacal rising and setting, planetary conjunctions, and conjunctions with stars

A vast corpus of astronomical literature produced in India since the time of *Āryabhaṭa* (c. 499 CE) is generally classified into *Siddhāntas*, *Tantras*, *Karaṇas*, and *Vākyas*, in decreasing order of theoretical content [3]. Among these, the *Siddhāntik* texts are complex and extensive due to the rigorous computational methods and algorithms employed in determining planetary longitudes and other astronomical quantities. Currently, the most widely followed text in the *Siddhāntas* is the *Sūrya Siddhānta* (early 5th century CE), which presents its principles across fourteen chapters.One might question the credibility of these ancient texts, as most implied models of that period were effectively geocentric. However, it should be noted that *Āryabhaṭa* had already proposed that the Earth rotates on its axis, and *Nīlakaṇṭha* later suggested that the inner planets revolve around the Sun, while their mini solar system still revolved around the Earth [4].Though such differing opinions on planetary motion continued to be debated, astronomers did not restrain their research.

The *Pañcāṅga* is a precise almanac system developed to envision daily time through astronomical parameters. It consists of five (*pañca*) principal elements: 1.*Tithi* (lunar days), 2.*Karaṇa* (half-*Tithis*), 3.*Nakṣatra*, 4.*Yoga*, and 5.*Vāra*(weekday)[SS]. The *Tithis* are 30 subdivisions of the lunar month, measured from one new or no moon to the next. They correspond to successive 12-degree increments of the Moon's motion relative to the Sun. Thus, when the Moon is positioned in the range of 0° to 12° ahead of the Sun, it corresponds to the first *Tithi*,positioned 12° to 24° ahead- corresponds to the second and so on. Determining the *Tithi* at any instant requires the instantaneous true longitudes of both the Moon and the Sun. For these computations the *Siddhāntic* methods were rigorous, and as all calculations were handled

manually, this system in particular, demanded an efficient and accessible alternative. The *Pañcāṅgakāras* (calendar-makers) devised a simpler methodology [5], which led to appraisal *Karaṇa* and *Vākya* systems.

1. **The Vakya System:**

The *Vākya* system represents a school of thought that primarily involves codifying numerical outputs in the form of linguistic arrays or *vākyas*. The term *vākya* literally means a sentence or a string of words. However, in the context of astronomy, the term *vākya* has a very special connotation, referring to a set of phrases or sentences that are primarily meant to convey numerical values associated with various astronomical parameters [2]. These *vākyas* are strategically constructed, even with mathematical ciphering, they remain words of wisdom that evoke moral principles [1](most times). The *vākyas* can be broadly divided into two kinds:

1. Those that present the true longitudes of the planets at certain regular intervals.

2.Those that present certain other numerical values that aid the computation of the true longitudes.

In either case, the numerical values they present are given at regular intervals of time. In fact, the essence of the *vākya* method lies in recognizing the periodicities of certain astronomical phenomena and utilizing them to simplify the computational scheme [2].

The system for determining the true longitude of various planets has been radically simplified with the invention of the *vākya* methodology. Previous procedures, as outlined in various *Siddhāntas*, were more indirect and cumbersome, requiring the determination of the mean longitude followed by corrections using *saṃskāras*. The *vākya* method, however, does not resort to corrections using tedious sine functions [1]. In most cases, encryption follows the *Kaṭapayādi* system, attributed to *Vararuci*[2]. This is a system of encoding numerals as words by substituting each digit with a character. Each of the क, ट, प, य characters is assigned the number 1, and successive characters are assigned the numbers 2 and so on. The complete key is pictorially represented in Fig.2.1.

2.**Works in the vakya system:**

The canonical text of the *Parahita* system, *Grahacaraṇibandhana* by *Haridatta*, a renowned seventh-century astronomical manual from Kerala, introduces *vākyas* for the *manda* and *śīghra* corrections. A more comprehensive and fully developed *vākya* system is outlined in the well-known 13th-century *karaṇa* text, *Vākyakaraṇa*, which presents direct computational methods for determining the true longitudes of the Sun, the Moon, and the planets. Manuscripts of this work are preserved in various repositories across South India, particularly in Tamil Nadu. Scholars *Kuppanna Sastri* and *K. V. Sarma* estimate its composition between 1282 and 1316 CE. Though the author remains unidentified, linguistic and textual evidence suggests that he was likely from the Tamil-speaking regions of South India. This work also has a commentary titled *Laghuprakāśikā*, composed by *Sundararāja* from *Kāñcī* near present-day Chennai. The treatise draws extensively from the *Mahābhāskarīya* and *Laghubhāskarīya* of *Bhāskara I*, who belonged to the *Āryabhaṭa* school, and incorporates principles from the *Parahita* system of *Haridatta*, which was widely followed in Kerala. *Mādhava* of *Saṅgamagrāma* (c. 1340–1425), the pioneer of the Kerala school of mathematics and astronomy, authored two seminal works: *Veṇvāroha* and *Sphuṭacandrapti*. The former is considered an updated version of the latter [madhava]. *Mādhava* also devised an ingenious method wherein *vākyas* could be utilized to compute the true longitudes of the Moon at nine distinct instants throughout a given day, significantly mitigating interpolation errors. Additionally, he introduced the *Agaṇita* system for planetary position computations. Although his work*Agaṇitagrahacāra* remains unedited, manuscripts of this text are extant [6].

While *Vākyakaraṇa* and the works of *Mādhava* primarily catalog *vākyas* and outline computational techniques for planetary longitudes, it is the *Karaṇapaddhati* of *Putumana Somayāji* (16th–17th century) that elucidates the rationale behind the construction of *vākyas* and provides systematic methodologies for deriving precise longitudes of the Sun, Moon, and planets. Notably, the *Karaṇapaddhati* details the application of continued fractions to approximate planetary mean motions and their anomalies, there by refining the accuracy of calculations. Subsequent astronomical texts introduced further innovations, with the 19th-century treatise *Sadratnamālā* by *Śaṅkaravarman* documenting several of these advancements.[1]

The present work under consideration is the *Tantraratna* of *Bhūpāla Tippa* (1423–1446 CE) of the *Sāluva* dynasty. The manuscript has been sourced from the Kerala Manuscript Library [TR].The present study is based on the Sanskrit transliteration of the manuscript, originally in Malayalam, as undertaken by Dr Venketeswara R. Pai. The *Candravākya* section of the *Tantraratna* consists of two parts: one enumerating the 248 *Candravākyas* and the other outlining their characteristics, referencing previous works, and providing the author's concluding remarks. The author cites the *Sūrya Siddhānta* and *Vararuci's Gīrṇaḥ-śreyādi-vākyas*.[TR] The following sections present a detailed analysis of his work.

	Consonant									
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***0***
	Devanagari									
	क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
	ट	ठ	ड	ढ	ण	त	थ	द	ध	न
	प	फ	ब	भ	म					
	य	र	ल	व	श	ष	स	ह		
	Transliteration - English									
	Ka	***Kha***	***Ga***	***Gha***	***Nga***	***Ca***	***Cha***	***Ja***	***Jha***	***Nya***
	Ṭa	***Ṭha***	***Ḍa***	***Ḍha***	***Ṇa***	***Ta***	***Tha***	***Da***	***Dha***	***Na***
	Pa	***Pha***	***Ba***	***Bha***	***Ma***					
	Ya	***Ra***	***La***	***Va***	***Śa***	***Ṣa***	***Sa***	***Ha***		

Figure 1.1: Katapayadi table: Mapping consonants to numerals

1.2 Terminologies

Candrasphuṭa (चन्द्रस्फुटः) : is the true position of the Moon (true longitude). The mean longitude is calculated under the assumption of uniform motion, whereas the true longitude is determined considering the actual non-uniform motion of the Moon.

Ucca(Apsis) : The term *ucca* is best translated as "apex," and it applies to both *mandocca*(मन्दोच्चः)and *śighrocca*(शीघ्रोच्चः). In case of the sun and the moon ,this is the apogee, the aphelion,though not recognised as such in the of other planets. The śighrocca is best reffered to as the conjunction.

Kendra (केन्द्रः) :(Anomaly), is the angular distance of a celestial body from either of its apices or centers of perturbation. The distance from *mandocca* is called *manda kendra* (मन्दकेन्द्रः), whereas the distance from *śighrocca* is *śighra kendra* (शीघ्रकेन्द्रः). Mean and true anomalies differ based on whether uniform or non-uniform motion is considered.

Ahargaṇa(अहर्गणः) :(A), The total number of days elapsed from a given epoch to the day on which the true longitude of the Moon is to be calculated.

Khanda (खण्डः): (K)*Kaliṇḍasaṅkhyā* Represents a specific number of days elapsed since the beginning of the *Kali Yuga* up to the day when the mean anomaly becomes zero. This day should be close to *Ahargaṇa*.

Khaṇḍaśeṣa (खण्डशेष) : is the difference between *Ahargaṇa* and *Khanda* (i.e., $A-K$). It gives the number of days that have passed since the Moon's anomaly was last zero.

Dhruva (ध्रुवः) :(D), represents the true longitude of the Moon at the end of a *Khaṇḍa*.

Anomalistic revolution : is the revolution of the anomaly corresponding to a pseudo-object having a relative velocity , $v_{\text{planet}} - v_{\text{kendra}}$.

(G/H) : G and H denote the *Guṇākāra* and *Hāraka*) respectively. They are used to represent various periodic cycles and are derived using *Vallūpa-saṁhāra* (continued fraction method).

1.3 Model:

All the calculations in the *Vākya system* are primarily computed within the ecliptic coordinate system. This system is primarily based on an orthogonal spherical reference system, in which two sets of great circles intersect at right angles to determine the position of a point on the celestial sphere. Since this system is defined relative to celestial references, the ecliptic coordinates are, to a first approximation, fixed with respect to the stars. This ensures that the two angular parameters used to identify a celestial body—ecliptic longitude and ecliptic latitude—remain consistent for all terrestrial observers.

For the Earth-Moon System:
The Earth is at the center of the celestial sphere and the moon is revolving around the earth with an orbit positioned on the circumference of the celestial sphere (ecliptic). The moon is believed to have a circular orbit and is orbiting with non-uniform motion. The longitudes and latitudes of the moon refer to the relative angular position of the moon on the celestial sphere ; i.e ecliptic latitude and longitude.Along the celestial sphere there are many starts and constellations which were recognised and set as reference points. The ecliptic is divided into 12 zones */raśhies* to position the movement of earth and correlate it with the time frame of months. Each *raśhi* hones to 30 degrees and is identified based on the relative positioning of the reference constellations/ *nakśhatras*.For more clarity of thought, look into Fig 2.2(a). (*Nakśhataras* are represented as orange markings .)

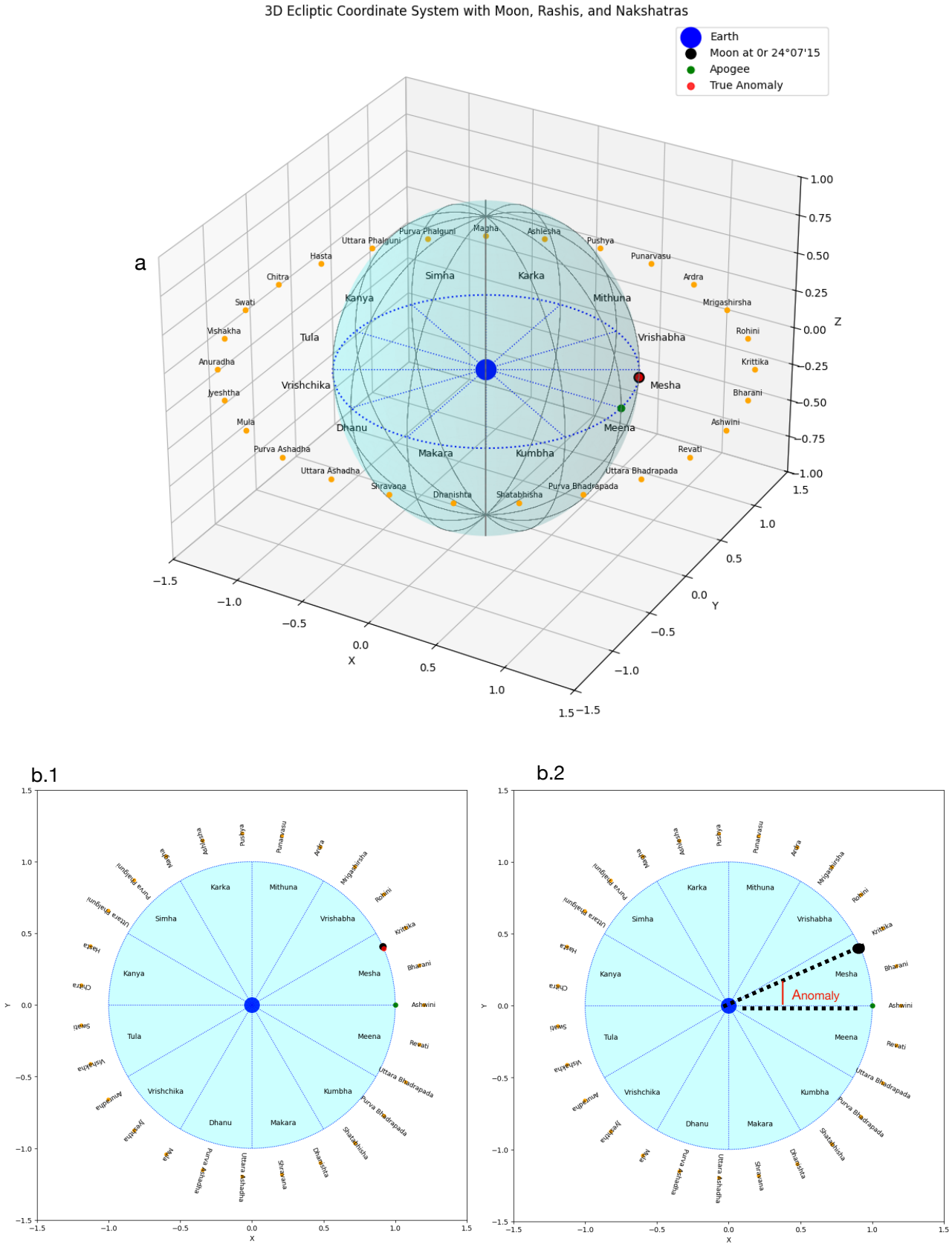


Figure 2.2: (a) A 3D representation of the model depicting the Earth-Moon system, with apogee, anomaly, *rāśis*, and *nakṣatrās* marked. (b) A sectional 2D view: (b.1) Anomaly represented as an object with relative velocity, and (b.2) A depiction of the conventional definition of anomaly.

1.4 The Principle behind the vākya method:

Consider an instance of calculating the true position of the Moon.Accurately determining its true position is a complex problem due to its highly variable motion. A straightforward application of the rule of three, by utilizing its position and instantaneous rate of motion at successive sunrises, often leads to significant errors. Reliable results necessitate higher-order interpolation techniques, such as second- or third-difference methods, which, although accurate, are computationally intensive [3].Simpler techniques are needed. In order to devise easier methods of measurement a deep understanding of the periodicities of celestial bodies is necessary. The analysis of these cyclic periodicities enabled previous astronomers to mathematically compute equations governing the diverse parameters of planetary bodies. *Vākyas* corresponding to the Moon, the Sun, and the planets were structured upon these periodic principles. The foundation for obtaining all the characteristic parameters for a celestial body rests upon the determination of its true longitude. In the case of lunar longitude computation, the periodicity employed pertains to the revolution of the Moon's anomaly. The earliest textual attestation of this approach is encountered in *Gīrṇaḥ-śreyādi-vākyas* by *Vararuci*. Renowned Indologists and historians have opined that *Vararuci* lived around the 4^{th}–5^{th} century CE.[1] This date is arrived by scholars based on the fact that tradition ascribes a text on grammar called *Prākṛta-prakāśa* to *Vararuci*, which is estimated to have been composed by the 5^{th} century[2]. *Vararuci* has encoded his 248 *vākyas* to give the longitudinal positions of the Moon for consecutive days contained in 9 full lunar anomalistic cycles of 27 days, 33*nāḍikās*, $6\frac{24}{55}$ *vināḍikās* each (9 revolutions in 248 days)[5].

Most texts dealing with the *vākya* method for computing the moon's longitude first introduce the *khaṇḍa*, which refers to certain number of civil days elapsed from the beginning of *Kali Yuga*, which is close to the *ahargaṇa* under consideration, such that, at sunrise, the *mandakendra* or the mean anomaly is zero. This is the initial point from which the *vākya*s are constructed.

Consider a day corresponding to the *ahargaṇa* A at mean sunrise. Suppose that "a" full days and "b" part days have passed since the anomaly was last zero. In other words, the current anomalistic cycle started $a + b$ days earlier, where a is an integer and b is a fraction. If we consider the instant A_1, b days earlier than A, the anomalistic cycle would have begun exactly

a days prior to it. Hence, the true longitude corresponding to the instant A_1 is given by the ath *vākya*, to which the longitude of the Moon at the beginning of this anomalistic cycle has to be added. The anomalistic cycle preceding the current one would have begun a days +1 cycle = $a + 27 + \alpha$ days before A_1. Consider an instant A_2 which is α days before A_1. Hence, the true longitude corresponding to the instant A_2 is given by the *(a + 27)*th *vākya*, to which the longitude of the Moon at the beginning of the anomalistic cycle preceding the current one has to be added. Similarly, consider an instant A'_3 such that $A_2 - A'_3 = \alpha$ days. Then $A_1 - A'_3 = 2\alpha$ days. Now, an anomalistic cycle would have begun a days +2 cycles, or $a + 54 + 2\alpha$ days before A_1, or $a + 54$ days before A'_3. Let $A_3 = A'_3 + 1$. Then an anomalistic cycle would have begun $a + 55$ days before A_3. Hence, the true longitude at A_3 is given by the 55th *vākya*, to

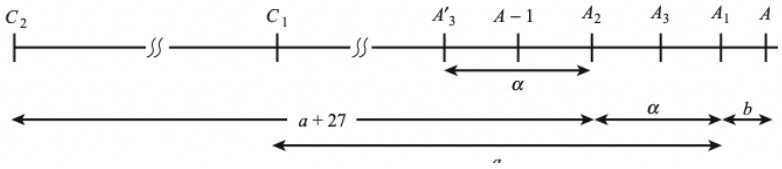


Fig1.4:Method for determining the true longitude of the Moon [3].

which the longitude of the Moon at the beginning of the cycle, which is two cycles prior to the current one, has to be added. Note that A_1, A_2, and A_3 all correspond to the day prior to the instant A. Similarly, true longitudes at six more instants during the previous day can be determined using the above method. Since there are nine anomalistic cycles within a span of 248 days, it follows that the true longitudes can be computed at only nine instants on any given day using this method. Consequently, the true longitudes for nine instants on the subsequent day are obtained by adding one more *vākya* to those corresponding to A_1, A_2, A_3, etc. (as per Madhava's ideology) [3].

The primary limitation of this technique is the approximation of the anomalistic cycle. This issue is resolved by employing the *khaṇḍa-dina* and utilizing the process of *vallūpasamhāra* (continued fraction method) to determine the *guṇakāra*s and *hāraka*s required to compute the anomaly. (For further details, refer to Section: Computing *Candra-vākyas*.)

1.5 Decrypting Tippa Gopendra's *candravākyas*

In continuity with the tradition of the *Vākya* system, Tippa Gopendra devised a distinctive set of 248 *vākya*s, encapsulated within the *Kaṭapayādi* system. His work is structured into two sections: the first enumerating the 248 *vākya*s and the second discussing their characteristics, concluding with reflective verses. The first key objective of this study was to decrypt these 248 *vākya*s from the primary manuscript [TR].

Method of Decryption: An Illustrative Example

The first *vākya* in the *Candra-vākya*s of *Tantraratna* is given as:

"श्री यान्नरस्य"

The process of decryption begins with segmenting the phrase into its constituent characters. For instance:

(श्री = श् + र् + ई).

Using Table (2.1), the numerical equivalents of these characters are determined according to the *Kaṭapayādi* convention. In cases of conjunct consonants, the numeral assigned to the consonant carrying the vowel (*svāra*) is considered:

र् = 2, य् = 1, न् = 0, र् = 2, य् = 1.

This yields the numerical sequence 21021. Reversing it, as per the encoding convention, results in 12012.

The extracted numerical value follows the standard format: *Rāśi*, *Bhāga*, *Kalā*, and *Vikālā*. In this system, there are 12 *rāśi*s (zodiac signs), each subdivided into 30 *bhāga*s (, degrees). Each *bhāga* is further divided into 60 *kalā*s (′, minutes), with each *kalā* contains 60 *vikalā*s (″, seconds).Finally yielding

$$0^r 12^\circ 01' 2'' \tag{1.1}$$

This signifies that the Moon is positioned in the first *rāśi* (zodiac sign), *Meṣa* (Aries), at $12^\circ 01' 2''$. Table (2.5) presents all computed 248 *Tippa Gopendra candrvākyas* alongside *Mādhava-vākyas*. [1]

[1]The Madhava-vākyas are completely based and obtained from previous studies.(refer Pai et al., 2018* Appendix, Pai et al., 2017, Ganitabharati article)

1.6 Table of candravākyas of Tippa Gopendra and Mādhava

vākya number	*Candravākyas* of Tippa-Gopendra			*Candravākyas* of Mādhava		
	in *kaṭapayādi*		in numerals	in *kaṭapayādi*		in numerals
1	श्री यान्नरस्य	*śrī yānnarasya*	12°01′2″	शीलं राज्ञः श्रिये	*śīlaṃ rājñaḥ śriye*	12°02′35″
2	शय्यसने भद्रे	*śayyasane bhadre*	24°07′15″	धिगिदं नश्वरम्	*dhigidaṃ naśvaram*	24°08′39″
3	हेमाधिकस्तान्नृपाः	*hemādhikastānnṛpāḥ*	1^{r}06°19′58″	लोलः पुरुषो नार्याम्	*lolaḥ puruṣo nāryām*	1^{r}06°21′33″
4	धीर्गौरवदायिका	*dhīrgauravadāyikā*	1^{r}18°42′39″	तपस्वी वैदिकः स्यात्	*tapasvī vaidikaḥ syāt*	1^{r}18°44′16″
5	विष्णुः सेव्योऽयं नरैः	*viṣṇuḥ sevyoyaṃ naraiḥ*	2^{r}01°17′54″	सेव्याळका किन्नरैः	*sevyāḷakā kinnaraiḥ*	2^{r}01°19′17″
6	सन्यासी निर्विकारः	*sanyāsī nirvikāraḥ*	2^{r}14°07′17″	दीप्रो दिने भास्करः	*dīpro dine bhāskaraḥ*	2^{r}14°08′28″
7	लक्ष्मीः स्यात् कंसारेः स्त्री	*lakṣmīḥ kaṃsāreḥ strī*	2^{r}27°11′53″	धर्मरम्यं सुराष्ट्रम्	*dharmaramyaṃ surāṣṭram*	2^{r}27°12′59″
8	धनी काले नृपालम्	*dhanī kāle nṛpālam*	3^{r}10°31′09″	स्तनौ लीलानुकूलौ	*stanau līlānukūlau*	3^{r}10°33′06″
9	सीमन्तिनी वराङ्गी	*simantinī varāṅgī*	3^{r}24°06′57″	पुत्रादौ न विरागः	*putrādau na virāgaḥ*	3^{r}24°08′21″
10	कामो मर्मस्थानभित्	*kāmo marmasthānabhit*	4^{r}07°55′51″	शौरिः समर्थ एव	*śauriḥ samartha eva*	4^{r}07°57′25″
11	विभूतिः शङ्करभ्रूः	*vibhūtiḥ śaṅkarabhrūḥ*	2^{r}21°56′44″	दुष्टैर्देशोपद्रवः	*duṣṭairdeśopadravaḥ*	4^{r}21°58′18″
12	मानसोन्नतिर्नृणाम्	*mānasonnatirnṛṇām*	5^{r}06°07′05″	व्यग्रो जनः क्षुन्नाशे	*vyagro janaḥ kṣunnāśe*	5^{r}06°08′21″
13	शिवाङ्गं रत्नैःश्रीमत्	*śivāṅgaṃ raratnaiḥśrīmat*	5^{r}20°23′45″	योगीश्वरो निराशः	*yogīśvaro nirāśaḥ*	5^{r}20°24′31″
14	जलगर्भाघनास्ते	*jalagarbhādyanāste*	6^{r}04°43′33″	शिखण्डी भवनेषु	*śikhaṇḍī bhavaneṣu*	6^{r}04°43′25″
15	श्रीभूर्यज्ञाधिपतिः	*śrībhūryajñādhipatiḥ*	6^{r}19°01′42″	नागो यानाधिपतिः	*nāgo yānādhipatiḥ*	6^{r}19°01′30″

16	मेघास्तटिल्लग्नाः सानौ	*meghāstaṭillagnāḥ sānau*	07^{r}03°16′45″	परिणयेऽङ्गनेच्छा	*pariṇaye'ṅganecchā*	7^{r}03°15′21″
17	हेयाम्बरः सन्यासी	*heyāmbaraḥ sanyāsī*	7^{r}17°23′18″	कविकण्ठस्था कथा	*kavikaṇṭhasthā kathā*	7^{r}17°21′41″
18	घनाधिकेयन्नदी	*ghanādhikeyannadī*	8^{r}01°19′04″	शीलसम्पद्यानन्दः	*śīlasampadyānandaḥ*	8^{r}01°17′35″
19	सूत्ररत्नशय्यार्ह	*sūtraratnaśayyārha*	8^{r}15°02′27″	श्रीर्विना न मुकुन्दात्	*śrīrvinā na mukundāt*	8^{r}15°00′42″
20	सद्रत्नगेहो राजा	*sadratnageho rājā*	8^{r}28°30′27″	निराधारोऽहिराजः	*nirādhāro'hirājaḥ*	8^{r}28°29′27″
21	गाढाम्बुभृत् पयोधिः	*gāḍhāmbubhṛt payodhiḥ*	9^{r}11°43′43″	कुबेरो विकटधीः	*kubero vikaṭadhīḥ*	9^{r}11°42′31″
22	करिकुम्भे विरोधी	*karikumbhe virodhī*	9^{r}24°41′21″	स्तेनानां श्वा विरोद्धा	*stenānāṃ śvā viroddhā*	9^{r}24°40′06″
23	करिवरासीनो नृपः	*karivarāsīno nṛpaḥ*	10^{r}07°24′21″	दीर्घरिरंसुर्ना नाके	*dīrghariraṃsurnā nāke*	10^{r}07°22′48″
24	दण्डगुणाधिको नृपः	*daṇḍaguṇādhiko nṛpaḥ*	10^{r}19°53′38″	वानरो मधुपानाढ्यः	*vānaro madhupānāḍhyaḥ*	10^{r}19°52′04″
25	धीश्लाध्या परनायिका	*dhīślāṃghya paranāyikā*	11^{r}02°11′39″	घननिकरो निर्ययौ	*ghananikaro niryayau*	11^{r}02°10′04″
26	भुवने रविः कृपाढ्यः	*bhuvane raviḥ kṛpāḍhyaḥ*	11^{r}14°20′44″	रोगे धैर्यविपर्ययः	*roge dhairyaviparyayaḥ*	11^{r}14°19′32″
27	कन्यावराप्ता श्रीयाढ्या	*kanyāvarāptā śrīyāḍhyā*	11^{r}26°24′11″	स्थूलो गिरिश्चित्रकूटः	*sthūlo giriścitrakūṭaḥ*	11^{r}26°23′37″
28	वन्यो मुरजित्	*vanyo murajit*	8°25′14″	स्तम्भमात्रो हि	*stambhamātro hi*	8°25′46″
29	स्तुत्यो हरिर्नरैः	*stutyo harirnaraiḥ*	20°28′16″	धीरधीस्त्रिनेत्रः	*dhīradhīstrinetraḥ*	20°29′29″
30	प्रभूतगोत्रो नृपः	*prabhūtagotro nṛpaḥ*	1^{r}02°36′42″	प्रपदौ गुरोर्नम्यौ	*prapadau gurornamyau*	1^{r}02°38′12″
31	अबला शुभोल्कटा	*abalā śubholkaṭā*	1^{r}14°53′30″	छन्नो माणवकः किम्	*channo māṇavakaḥ kim*	1^{r}14°55′07″
32	चण्डकरः सखायः	*caṇḍakaraḥ sakhāyaḥ*	1^{r}27°21′36″	गानगोष्ठी सुखाय	*gānagoṣṭhī sukhāya*	1^{r}27°23′03″
33	अमरा नानुकारः	*amarā nānukāraḥ*	2^{r}10°02′50″	काकुध्वनिर्नकारात्	*kākudhvanirnakārāt*	2^{r}10°04′11″
34	धनधीर्मन्त्रश्रेष्ठे	*dhanadhīrmantraśreṣṭhe*	2^{r}22°59′09″	तनुर्न नगरे श्रीः	*tanurna nagare śrīḥ*	2^{r}23°00′06″
35	रुद्रो नटति नगे	*rudro naṭati nage*	3^{r}06°10′22″	शैलाः पुष्पितनगाः	*śailāḥ puṣpitanagāḥ*	3^{r}06°11′35″
36	शय्या सुगन्धाकुला	*śayyā sugandhākulā*	3^{r}19°37′15″	लोलो जलधिः किल	*lolo jaladhiḥ kila*	3^{r}19°38′33″

37	धीगो दिव्यगानवित्	*dhīgo divyagānavit*	$4^r 03°18'39''$	धनी नरोऽङ्गनावान्	*dhanī naro'ṅganāvān*	$4^r 03°20'09''$
38	लोकश्लाघ्यः सत्कविः	*lokaślaghyaḥ satkaviḥ*	$4^r 17°13'13''$	सर्वविद् व्यासः कविः	*sarvavid vyāsaḥ kaviḥ*	$4^r 17°14'47''$
39	शैलजा पूज्या नृणाम्	*śailajā pūjyā nṛṇām*	$5^r 01°18'35''$	स्तेनेन द्रव्यनाशः	*stenena dravyanāśaḥ*	$5^r 01°20'06''$
40	नृपेन्द्रो बाणपाणिः	*nṛpendo bāṇapāṇiḥ*	$5^r 15°32'10''$	सूर्यो बलमाकाशे	*sūryo balamākāśe*	$5^r 15°33'17''$
41	शोभनात्मा धरेशः	*śobhanātmā dhareśaḥ*	$5^r 29°50'45''$	मनुष्यो मधुरात्मा	*manuṣyo madhurātmā*	$5^r 29°51'05''$
42	प्रभुर्नम्यो भूपतिः	*prabhurnamyo bhūpatiḥ*	$6^r 14°10'42''$	गानं नेष्टं विपत्तौ	*gānaṃ neṣṭaṃ vipattau*	$6^r 14°10'03''$
43	भूमिस्सराजरक्षा	*bhūmissarājarakṣā*	$6^r 28°27'54''$	पर्वचन्द्रोऽहिग्रस्तः	*parvacandro'higrastaḥ*	$6^r 28°26'41''$
44	सन्नद्धगात्रः पार्थः	*sannaddhagātraḥ pārthaḥ*	$7^r 12°39'07''$	भोगेच्छालं प्रियेऽर्थे	*bhogecchālaṃ priye'rthe*	$7^r 12°37'34''$
45	नटः सङ्गीतरसः	*naṭaḥ saṅgītarasaḥ*	$7^r 26°37'10''$	मागधो गीतरसः	*māgadho gītarasaḥ*	$7^r 26°39'35''$
46	भोगहरोनः पाहि	*bhogaharonaḥ pāhi*	$8^r 10°28'34''$	लीनो नागो निकुञ्जे	*līno nāgo nikuñje*	$8^r 10°30'03''$
47	स्थिरतनुर्वराहः	*sthiratanurvarahaḥ*	$8^r 24°06'27''$	गामुक्षा न विरेजे	*gāmukṣā na vireje*	$8^r 24°06'53''$
48	धृष्टोऽनङ्गः सन्नद्धः	*dhruṣṭonaṅgaḥ sannaddhaḥ*	$9^r 07°30'19''$	रवौ हरेः सन्निधिः	*ravau hareḥ sannidhiḥ*	$9^r 07°28'42''$
49	धर्मोत्तुङ्गो नरधीः	*dharmottuṅgo nardhīḥ*	$9^r 20°36'59''$	वर्णान् वागनुरुन्धे	*varṇān vāganurundhe*	$9^r 20°34'54''$
50	हरिर्धराङ्गना नन्द्यः	*harirdharāṅganā nandyaḥ*	$10^r 03°29'28''$	भावे स्मरोऽङ्गनानां स्यात्	*bhāve smaro'ṅganānāṃ syāt*	$10^r 03°25'44''$
51	गर्भोङ्गनास्तुत्योनित्यम्	*garbhoṅganāstutyonityam*	$10^r 16°03'43''$	गायत्री नास्तिकैर्निन्द्या	*gāyatrī nāstikairnindyā*	$10^r 16°02'13''$
52	देवासुरैर्हरिर्नम्यः	*gevāsurairharirnamyaḥ*	$10^r 28°27'48''$	धनं चोरो हरेन्नित्यम्	*dhanaṃ coro harennityam*	$10^r 28°26'09''$
53	चन्द्रकलिज्ञार्काः पूज्याः	*candrakalijñārkāḥ pūjyāḥ*	$11^r 10°31'26''$	धर्मं धिगनपायस्य	*dharmaṃ dhiganapāyasya*	$11^r 10°39'59''$
54	धीगा सुभद्रा स्त्रीपूज्या	*dhīgā subhadrā strīpūjyā*	$11^r 22°47'39''$	धीगतिर्भद्ररूपेयम्	*dhīgatirbhadrarūpeyam*	$11^r 22°46'39''$
55	भोगीधवस्त्वम्	*bhogīdhavaswatm*	$4°49'34''$	क्षीराब्धौ विभुः	*kṣīrābdhau vibhuḥ*	$4°49'26''$
56	प्राज्ञैः कृष्णः स्तुत्यः	*prājñaiḥ kṛṣṇaḥ stutyaḥ*	$16°51'02''$	यमोऽयमन्तिके	*yamo'yamantike*	$16°51'51''$
57	विनीतो महेन्द्रः	*vinīto mahedrah*	$28°56'04''$	गौरी स्थाणोर्दाराः	*gaurī sthāṇordārāḥ*	$28°57'23''$

58	विभुः सैनिकैः पूज्यः	*vibhuḥ sainikaiḥ pūjyaḥ*	1^{r} 11°07′44″	गरळं नोपयुञ्ज्यात्	*garaḷaṃ nopayuñjyāt*	1^{r} 11°09′23″
59	श्रेष्ठा धरा निरीह्या	*śreṣṭhā dharā nirīhyā*	1^{r} 20°29′22″	गोमानलं गरीयान्	*gomānalaṃ garīyān*	1^{r} 23°30′53″
60	धर्मो रज्ञान्तनुत्रम्	*dharmo rajñāntanutram*	2^{r} 06°02′59″	सुग्रीवोऽनन्तनिष्ठः	*sugrīvo'nantaniṣṭhaḥ*	2^{r} 06°04′27″
61	वामनो महीपरः	*vāmano mahīparaḥ*	2^{r} 18°50′54″	प्राज्ञो रामो दैत्यारिः	*prājño rāmo daityāriḥ*	2^{r} 18°52′02″
62	इभगणोयन्नीलम्	*ibhagaṇoyannīlam*	3^{r} 01°53′40″	अशुभाशया नागाः	*aśubhāśayā nāgāḥ*	3^{r} 01°54′50″
63	दानं प्रायेण काले	*dānaṃ prāyeṇa kāle*	3^{r} 15°12′08″	चपलः कामपालः	*capalaḥ kāmapālaḥ*	3^{r} 15°13′16″
64	श्रीरं शुभाहाराङ्गम्	*śrīraṃ śubhāhārāṅgam*	3^{r} 28°45′22″	वाग्मी तु वादरागी	*vāgmī tu vādarāgī*	3^{r} 28°46′54″
65	प्राज्ञैर्गङ्गा प्राच्यावने	*prājñairgaṅgā prācyāvane*	04^{r} 12°33′02″	गङ्गा भागीरथ्यभूत्	*gaṅgā bhāgīrathyabhūt*	4^{r} 12°34′33″
66	भोगी फल क्षीरभुक्	*bhogī phala kṣīrabhuk*	4^{r} 26°32′34″	तपस्वीगतिरूर्ध्वम्	*tapasvīgatirūrdhvam*	4^{r} 26°34′16″
67	अर्कःप्रभानन्द्यत्मा	*arkaḥprabhānandyatmā*	5^{r} 10°42′10″	जरद्गवोऽनुद्यमः	*jaradgavo'nudyamaḥ*	5^{r} 10°43′28″
68	सेव्यो हिमवान्श्रीमान्	*sevyo himavānśrīmān*	5^{r} 24°58′17″	सूनुर्धामाभरणम्	*sūnurdhāmābharaṇam*	5^{r} 24°59′07″
69	अन्नं देयं धनाप्तैः	*annaṃ deyaṃ dhanāptaiḥ*	6^{r} 09°18′00″	गुणोऽसूया धनिषु	*guṇo'sūyā dhaniṣu*	6^{r} 09°17′53″
70	मान्या सा गङ्गाप्राप्तैः	*mānyā sā gaṅgāprāptaiḥ*	6^{r} 23°37′15″	विकृता गौडरीतिः	*vikṛtā gauḍarītiḥ*	6^{r} 23°36′14″
71	जनो ग्रामस्थानार्थी	*jano grāmasthānārthī*	7^{r} 07°52′08″	लघुर्न मैथुनेच्छा	*laghurna maithunecchā*	7^{r} 07°50′43″
72	अम्बोधिर्मान्यरसः	*ambodhirmānyarasaḥ*	7^{r} 21°59′40″	आनन्दमयो रसः	*ānandamayo rasaḥ*	7^{r} 21°58′00″
73	अवतु मां मुनिजनः	*avatu māṃ munijanaḥ*	08^{r} 05°56′40″	कल्यः शिशुर्मनुजः	*kalyaḥ śiśurmanujaḥ*	8^{r} 05°55′11″
74	शकटो विद्धः पदा	*śakaṭo viddhaḥ padā*	8^{r} 19°41′15″	दृढधीर्लब्धपदः	*dṛḍhadhīrlabdhapadaḥ*	8^{r} 19°39′48″
75	धेनुः कपिला निधिः	*dhenuḥ kapilā nidhiḥ*	9^{r} 03°11′09″	लीना आपोऽम्बुनिधौ	*līnā āpo'mbunidhau*	9^{r} 03°10′03″
76	सेना चरति युद्धे	*senā carati yuddhe*	9^{r} 16°26′07″	क्षामवारिस्तोयधिः	*kṣāmavāristoyadhiḥ*	9^{r} 16°24′56″
77	धरेशश्रीधराधीना	*dhareśaśrīdarādhīnā*	09^{r} 29°25′29″	भाग्यविरोधः क्रोधात्	*bhāgyavirodhaḥ krodhāt*	9^{r} 29°24′14″
78	हेमधनप्रियो नृपः	*hemadhanapriyo nṛpaḥ*	10^{r} 12°09′58″	धरा हीनाश्रया नित्यम्	*dharā hīnāśrayā nityam*	10^{r} 12°08′29″

79	लघु नववस्त्रं नार्यः	*haghu navavastraṃ nāryaḥ*	$10^r\,24°40'43''$	जनोऽन्धो गत्वरो नश्येत्	*jano'ndho gatvaro naśyet*	$10^r\,24°39'08''$
80	सद्बुद्धिमती नायिका	*sadbuddhimatī nāyikā*	$11^r\,06°59'37''$	मुकुन्दान्मोक्ष उपेयः	*mukundānmokṣa upeyaḥ*	$11^r\,06°58'15''$
81	गेर्भूधनधान्यंकार्यम्	*rbhūdhanadhānyaṃkāryam*	$11^r\,19°09'43''$	आगोहीनोऽधिकः पटुः	*āgohīno'dhikaḥ paṭuḥ*	$11^r\,19°08'30''$
82	धीलोला कन्या	*dhīlolā kanyā*	$1°13'39''$	ज्ञानी गार्ग्याय	*jñānī gārgyāya*	$1°13'00''$
83	कृष्णो वन्द्यो गोपैः	*kṛṣṇo vandyo gopaiḥ*	$13°14'51''$	कृपणः कौण्डिन्यः	*kṛpaṇaḥ kauṇḍinyaḥ*	$13°15'11''$
84	सरसः कुमारः	*sarasaḥ kumāraḥ*	$25°17'27''$	लोला दीपशिखा	*lolā dīpaśikhā*	$25°18'33''$
85	भग्नामित्रः सैनिकः	*bhagnāmitraḥ sainikaḥ*	$1^r\,07°25'04''$	स्वर्गस्तु प्रार्थनीयः	*svargastu prārthanīyaḥ*	$1^r\,07°26'34''$
86	गुणज्ञावार्धिकन्या	*guṇajñāvārdhikanyā*	$1^r\,19°40'53''$	सौभ्रात्रं वाधिकं स्यात्	*saubhrātraṃ vādhikaṃ syāt*	$1^r\,19°42'27''$
87	भोगार्थिनी राज्ञः स्त्री	*bhogārthinī rājñaḥ strī*	$2^r\,02°07'34''$	लीनोऽळीनो त्रिनेत्रः	*līno'ḷīno trinetraḥ*	$2^r\,02°09'03''$
88	श्रेष्ठोत्सवा भूपस्त्री	*śreṣṭhotsavā bhūpastrī*	$2^r\,14°47'22''$	धिगाहवविकारः	*dhigāhavavikāraḥ*	$2^r\,14°48'39''$
89	प्रभुर्युवा सुराष्ट्रः	*prabhuryuvā surāṣṭraḥ*	$2^r\,27°41'42''$	शिशिरा वासरश्रीः	*śiśirā vāsaraśrīḥ*	$2^r\,27°42'55''$
90	शीलकामो नकुलः	*śīlakāmo nakulaḥ*	$3^r\,10°51'35''$	अभिरामा नकुली	*abhirāmā nakulī*	$3^r\,10°52'40''$
91	जागर्ति युवा रागी	*jāgarti yuvā rāgī*	$3^r\,24°16'38''$	धर्मोऽर्थः पूर्वरङ्गः	*dharmo'rthaḥ pūrvaraṅgaḥ*	$3^r\,24°17'59''$
92	जगतीशः सेनावान्	*jagatīśaḥ senāvān*	$4^r\,07°56'38''$	भानुजो मिथुनेऽभूत्	*bhānujo mithune'bhūt*	$4^r\,07°58'04''$
93*	धनवंशं मनो मे	*dhanavaṃśaṃ mano me*	$5^r\,05°54'09''$	रौद्रो यमस्यारम्भः	*raudro yamasyārambhaḥ*	$4^r\,21°51'22''$
94*	विशेषज्ञानी रामः	*viśeṣajñānī rāmaḥ*	$5^r\,20°06'54''$	धिगाशामशनेऽस्मिन्	*dhigāśāmaśane'smin*	$5^r\,05°55'39''$
95*	भानुमान् प्रभुनीतिः	*bhānumān prabhunītiḥ*	$6^r\,04°25'04''$	जनार्दनो नरेशः	*janārdano nareśaḥ*	$5^r\,20°08'08''$
96*	नित्यं शम्भुर्जयति	*nityaṃ śambhurjayati*	$6^r\,18°45'10''$	मृगाः शूरा वनान्ते	*mṛgāḥ śūrā vanānte*	$6^r\,04°25'35''$
97*	कुमारो नागनाथः	*kumāro nāganāthaḥ*	$7^r\,03°02'51''$	चण्डो वै भोजपतिः	*caṇḍo vai bhojapatiḥ*	$6^r\,18°44'36''$
98*	शशीवन्द्यः सन्यासी	*śaśīvandyaḥ sanyāsī*	$7^r\,17°14'55''$	धीगम्योऽनङ्ग आसीत्	*dhīgamyo'naṅga āsīt*	$7^r\,03°01'39''$
99*	ज्ञानदायको नन्दः	*jñānadāyako nandaḥ*	$8^r\,01°18'00''$	धैर्यालयः सन्यासी	*dhairyālayaḥ sanyāsī*	$7^r\,17°13'19''$

100*	प्रवृद्धभाग्यः प्रभुः	*spravṛdbhabhāgyaḥ prabhuḥ*	$4^r\,21°44'42''$	चन्द्रात् तापापनोदः	*candrāt tāpāpanodaḥ*	$8^r\,01°16'26''$
101	कामी धेनुर्मान्या हि	*kāmī dhenurmānyā hi*	$8^r\,15°09'51''$	सेव्यो जनैर्मुकुन्दः	*sevyo janairmukundaḥ*	$8^r\,15°08'17''$
102	क्षमा सर्वदा राजनि	*kṣamā sarvadā rājani*	$08^r\,28°47'56''$	अभिषवोऽहरहः	*abhiṣavo'harahaḥ*	$8^r\,28°46'40''$
103	श्रेष्ठा कर्पूरकेळिः	*shreṣṭhā karpūrakelihi*	$9^r\,12°11'22''$	कार्योऽनार्यैरुपधिः	*kāryo'nāryairupadhiḥ*	$9^r\,12°10'11''$
104	लोकाधिकः शास्त्रधीः	*lokādhikaḥ śāstradhīḥ*	$9^r\,25°19'13''$	मानदेयं मुरळी	*mānadeyaṃ muraḷī*	$9^r\,25°18'05''$
105	अन्नप्रियो जनो नित्यम्	*annapriyo jano nityam*	$10^r\,08°12'00''$	ज्वलनो यजने नम्यः	*jvalano yajane namyaḥ*	$10^r\,08°10'34''$
106	कानने मुनीन्द्रा नम्याः	*kānane munīndrā namyāḥ*	$10^r\,20°50'01''$	क्रीडा दृढा नरनार्योः	*krīḍā dṛḍhā naranāryoḥ*	$10^r\,20°48'32''$
107	वरेण कुलीना कन्या	*vareṇa kulīnā kanyā*	$11^r\,03°15'24''$	विश्वं गोपाल एकाकी	*viśvaṃ gopāla ekākī*	$11^r\,03°13'44''$
108	रत्नं नीलिमाढ्यम् पूज्यं	*ratnaṃ nīlimāḍhyam pūjyaṃ*	$11^r\,15°30'02''$	फलाहारो मुख्यकल्पः	*phalāhāro mukhyakalpaḥ*	$11^r\,15°28'32''$
109	वाणी तुङ्गः सुप्रकटः	*vāṇī tuṅgaḥ suprakaṭaḥ*	$11^r\,27°36'54''$	धन्वी शूली सुरैः पूज्यः	*dhanvī śūlī suraiḥ pūjyaḥ*	$11^r\,27°35'49''$
110	दानधीर्बन्धुः	*dānadhīrbandhuḥ*	$9°39'08''$	गोमदो गळी	*gomado gaḷī*	$9°38'53''$
111	क्षीरान्नभुक् पुत्रः	*kṣīrānnabhuk putraḥ*	$21°40'26''$	धनुर्ज्या विपाठा	*dhanurjyā vipāṭhā*	$21°41'09''$
112	क्षमाभृद् भोगी नित्यम्	*kṣamābhṛd bhogī nityam*	$1^r\,03°44'56''$	आढ्यः षड्भागैर्नृपः	*āḍhyaḥ ṣaḍbhāgairnṛpaḥ*	$1^r\,03°46'10''$
113	शिवात्मा शशाङ्कोऽयम्	*śivātmā śaśāṅkosyam*	$1^r\,15°55'45''$	धन्यः स्थाणुमुपेयात्	*dhanyaḥ sthāṇumupeyāt*	$1^r\,15°57'19''$
114	मुनिस्तुत्यो हरिः स्यात्	*nistutyo hariḥ syāt*	$1^r\,28°16'05''$	धिगसौख्यं हिरण्यात्	*dhigasaukhyaṃ hiraṇyāt*	$1^r\,28°17'39''$
115	वराहो वनपरः	*varāho vanaparaḥ*	$2^r\,10°48'24''$	देवो धावन्नैकत्र	*devo dhāvannaikatra*	$2^r\,10°49'48''$
116	कालविद् गर्गश्रेष्ठः	*kālavid gargaśreṣṭhaḥ*	$2^r\,23°34'31''$	तन्वी शीलगरिष्ठा	*tanvī śīlagariṣṭhā*	$2^r\,23°35'46''$
117	ईशः शीलितनागः	*īśaḥ śīlitanāgaḥ*	$3^r\,06°35'50''$	लक्ष्मीस्तुङ्गस्तनाङ्गी	*lakṣmīstuṅgastanāṅgī*	$3^r\,06°36'53''$
118	चारित्रशुद्धो योगी	*cāritraśuddho yogī*	$3^r\,19°52'26''$	चला लक्ष्मीर्धन्यगा	*calā lakṣmīrdhanyagā*	$3^r\,19°53'36''$
119	कूर्यः स्वरगानवित्	*kūryaḥ svaragānavit*	$4^r\,03°24'11''$	धिगशीघ्रगा नावः	*dhigaśīghragā nāvaḥ*	$4^r\,03°25'39''$
120	मान्यो नृपैः सत्कविः	*mānyo nṛpaiḥ satkaviḥ*	$4^r\,17°10'15''$	पूर्णः पयसा कुम्भः	*pūrṇaḥ payasā kumbhaḥ*	$4^r\,17°11'51''$

121	माभ्रजानये नमः	*mābhrajānaye namaḥ*	5^{r}01°08′25″	कठिनोऽयं कीनाशः	*kaṭhino'yaṃ kīnāśaḥ*	5^{r}01°10′21″
122	गोपः सेव्यात्मा कृष्णः	*gopaḥ sevyātmā kṛṣṇaḥ*	5^{r}15°17′13″	धिगहंयुमकस्मात्	*dhigahaṃyumakasmāt*	5^{r}15°18′39″
123	लक्ष्मीरागी धरेशः	*lakṣmīrāgī dhareśaḥ*	5^{r}29°32′53″	षड्भागबन्धुरीशः	*ṣaḍbhāgabandhurīśaḥ*	5^{r}29°33′46″
124	श्रेष्ठा रामा गायते	*śreṣṭhā rāmā gāyate*	6^{r}13°52′22″	कठोरो मृगपतिः	*kaṭhoro mṛgapatiḥ*	6^{r}13°52′21″
125	नमस्कार्यो हारीतः	*anamaskāryo hārītaḥ*	6^{r}28°11′50″	क्षीणो न व्याहरति	*kṣīṇo na vyāharati*	6^{r}28°10′56″
126	गुरुः सुरेन्द्रस्यास्थाने	*guruḥ surendrasyāsthāne*	07^{r}12°27′23″	धर्मशास्त्रं श्रेयसे	*dharmaśāstraṃ śreyase*	7^{r}12°25′59″
127	जीवेशगतिः स्त्री सा	*jīveśagatiḥ strī sā*	7^{r}26°35′48″	लोकोऽभिलाषी रसे	*loko'bhilāṣī rase*	7^{r}26°34′13″
128	मान्यभोगे नृपजनः	*mānyabhoge nṛpajanaḥ*	08^{r}10°34′15″	सागरो गोर्न पदम्	*sāgaro gorna padam*	8^{r}10°32′37″
129	भानुर्नरवरदः	*bhānurnaravaradaḥ*	8^{r}24°20′04″	रविजुष्टं वारिजे	*ravijuṣṭaṃ vārije*	8^{r}24°18′42″
130	वाणी पद्मासनासीना	*vāṇī padmāsanāsīnā*	07^{r}07°51′54″	साम्बोऽनिशं सन्नद्धः	*sāmbo'niśaṃ sannaddhaḥ*	9^{r}07°50′37″
131	दैत्यजनः कराळः	*daityajanaḥ karālaḥ*	9^{r}21°08′18″	प्रत्यासन्नः पुरोधाः	*pratyāsannaḥ purodhāḥ*	9^{r}21°07′12″
132	सुखी धनविज्ञानाढ्यः	*sukhī dhanavijñānāḍhyaḥ*	10^{r}04°09′27″	इष्टिर्दानं विना नेष्टा	*iṣṭirdānaṃ vinā neṣṭā*	10^{r}04°08′10″
133	मन्त्रैः शशी स्तुत्यो नित्यः	*mantraiḥ śaśī stutyo nityaḥ*	10^{r}16°55′25″	नानाभिमतं कनकम्	*nānābhimataṃ kanakam*	10^{r}16°54′00″
134	अम्भस्थः श्रीधरे नित्यम्	*ambhasthaḥ śrīdhare nityam*	10^{r}29°27′40″	श्रीनतः श्रीधरो नित्यम्	*śrīnataḥ śrīdharo nityam*	10^{r}29°26′02″
135	रामः सर्वकार्यपटुः	*rāmaḥ sarvakāryapaṭuḥ*	11^{r}11°47′52″	सुकृतिः स्वयं पाककृत्	*sukṛtiḥ svayaṃ pākakṛt*	11^{r}11°46′17″
136	दुर्गा हिमगिरिकन्या	*durgā himagirikanyā*	11^{r}23°58′38″	काष्ठसमा गात्रयष्टिः	*kāṣṭhasamā gātrayaṣṭiḥ*	11^{r}23°57′21″
137	स्तेनं लुनाति	*stenaṃ lunāti*	6°03′06″	अरिरनाप्तः	*ariranāptaḥ*	6°02′20″
138	वारी घनोदये	*vārī ghanodaye*	18°04′24″	चण्डभानुर्जयी	*caṇḍabhānurjayī*	18°04′36″
139	अवतु ज्ञानी नृपः	*avatu jñānī nṛpaḥ*	1^{r}00°06′40″	व्यवच्छिन्नोऽनुनयः	*vyavacchinno'nunayaḥ*	1^{r}00°07′41″
140	मङ्गलाकारा कन्या	*maṅgalākārā kanyā*	1^{r}12°13′35″	कीनाशो व्याघ्रकल्पः	*kīnāśo vyāghrakalpaḥ*	1^{r}12°15′01″
141	धन्यो हरिर्वरेण्यः	*dhanyo harirvareṇyaḥ*	1^{r}24°28′19″	वाङ्माधुरी वरेण्या	*vāṅmādhurī vareṇyā*	1^{r}24°29′54″

142	गर्भलक्ष्म स्तनाग्रे	*garbhalakṣma stanāgre*	$2^r 06°53'43''$	श्रीकृष्णो मोक्षनिष्ठः	*śrīkṛṣṇo mokṣaniṣṭhaḥ*	$2^r 06°55'12''$
143	देशिको बन्धुपरः	*deśiko bandhuparaḥ*	$2^r 19°31'58''$	हृष्टो लीलाधिकारी	*hṛṣṭo līlādhikārī*	$2^r 19°33'18''$
144	तच्वविच्छ्रेष्ठोऽनङ्गः	*tacvavicchraṣṭhosnaṅgaḥ*	$3^r 02°24'46''$	स्वामी शरीरेऽनिलः	*svāmī śarīre'nilaḥ*	$3^r 02°25'54''$
145	देवो रङ्गे शयालुः	*devo raṅge śayāluḥ*	$3^r 15°32'48''$	स्थाणुर्गङ्गाशयालुः	*sthāṇurgaṅgāśayāluḥ*	$3^r 15°33'57''$
146	स्त्रीरक्षा महत्फलम्	*strīrakṣā mahatphalam*	$3^r 28°56'22''$	तैलार्थी मन्दरोगी	*tailārthī mandarogī*	$3^r 28°57'36''$
147	दुर्गं वलरिपुवनम्	*durgaṃ valaripuvanam*	$04^r 12°34'38''$	स्थानाच्चला रिपवः	*sthānāccalā ripavaḥ*	$4^r 12°36'07''$
148	योगी चराचरवित्	*yogī carācaravit*	$4^r 26°26'31''$	विनोदरुचिः प्रभुः	*vinodaruciḥ prabhuḥ*	$4^r 26°28'04''$
149	गर्भधरान्नकामा	*garbhadharānnakāmā*	$5^r 10°29'43''$	साध्यो योगो नियमात्	*sādhyo yogo niyamāt*	$5^r 10°31'17''$
150	प्रमेयं विवरणे	*prameyaṃ vivaraṇe*	$5^r 24°41'52''$	पत्नी गर्भाभरणा	*patnī garbhābharaṇā*	$5^r 24°43'01''$
151	लोलधीर्मदनार्तः	*lolādhirmadanārtaḥ*	$6^r 08°59'33''$	स्तेनो न निर्धनैषी	*steno na nirdhanaiṣī*	$6^r 09°00'06''$
152	भोगधिकोऽम्बरीषः	*dūṣyaṃ gotraṃ te*	$6^r 23°19'34''$	सेनाधिकाङ्गरक्षा	*senādhikāṅgarakṣā*	$6^r 23°19'07''$
153	विभुः साङ्गं सेनार्थी	*vibhuḥ sāṅgaṃ senārthī*	$7^r 07°37'44''$	श्रीर्गीतालसा नासीत्	*śrīrgītālasā nāsīt*	$7^r 07°36'32''$
154	हरिनामपरोऽसौ	*harināmaparossau*	$7^r 21°50'28''$	धर्माज्जीवेत् परासुः	*dharmājjīvet parāsuḥ*	$7^r 21°48'59''$
155	ईशविष्णू मानार्हौ	*īśaviṣṇū mānārhau*	$8^r 05°54'50$	धनी गुणी मनुजः	*dhanī guṇī manujaḥ*	$8^r 05°53'09''$
156	देवः सर्वाधिकोहि	*devaḥ sarvādhikohi*	$8^r 19°47'48''$	परिषत्स्वधिकेहा	*pariṣatsvadhikehā*	$8^r 19°46'21''$
157	प्रभुसुखं गानाधीनम्	*prabhusukhaṃ gānādhīnam*	$09^r 03°27'42''$	दिव्यः क्षीराम्बुनिधिः	*divyaḥ kṣīrāmbunidhiḥ*	$9^r 03°26'18''$
158	सगरश्शितायुधः	*sagaraśśitāyudhaḥ*	$9^r 16°52'37''$	धीरः कर्णस्तु योद्धा	*dhīraḥ karṇastu yoddhā*	$9^r 16°51'29''$
159	गुरुरनूनज्ञानाढ्यः	*gururanūnajñānāḍhyaḥ*	$10^r 00°02'23''$	तनयो ज्ञानिनां नम्यः	*tanayo jñāninām namyaḥ*	$10^r 00°01'06''$
160	फलति मन्त्रः कनकम्	*phalati mantraḥ kanakam*	$10^r 12°56'32''$	गोकर्णमित्रं पिनाकी	*gokarṇamitraṃ pinākī*	$10^r 12°55'13''$
161	जयोत्तुङ्गः शूरोनृपः	*jayottuṅgaḥ śūronṛpaḥ*	$10^r 25°36'18''$	प्रभवो गुणरत्नाढ्याः	*prabhavo guṇaratnāḍhyāḥ*	$10^r 25°34'42''$
162	धीवरैर्नदी कृष्णा	*dhīvarairnadī kṛṣṇā*	$5^r 18°02'49''$	प्रकृत्याऽऽनन्द उत्पाद्यः	*prakr tyā"nanda utpādyaḥ*	$11^r 08°01'12''$

163	गङ्गा दिव्यनारीपूज्या	*gaṅgā divyanārīpūjyā*	$11^r 20°18'33''$	अनसूया निरपाया	*anasūyā nirapāyā*	$11^r 20°17'00''$
164	वने तरुश्रीः	*vane taruśrīḥ*	$2°26'04''$	जिष्णुर्वरिष्ठः	*jiṣṇurvariṣṭhaḥ*	$2°24'58''$
165	युवो दाराभाग्यः	*yuvo dārābhāgyaḥ*	$14°28''41''$	दिश्यादिन्द्रो भाग्यम्	*diśyādindro bhāgyam*	$14°28'18''$
166	वशी धीरतरः	*vaśī dhīrataraḥ*	$0^r 26°29'54''$	नगो नगोऽचरत्	*nago nago'carat*	$26°30'30''$
167	कमालाम्बुजे नित्यम्	*kamālāmbuje nityam*	$1^r 08°33'51''$	गानशीलो जनोऽयम्	*gānaśīlo jano'yam*	$1^r 08°35'03''$
168	भगवान् भानुप्राप्यः	*bhagavān bhānuprāpyaḥ*	$1^r 20°44'34''$	पुराणो भानुरीड्यः	*purāṇo bhānurīḍyaḥ*	$1^r 20°45'21''$
169	नानागानलग्ना स्त्री	*mnānāgānalagnā strī*	$2^r 03°03'00''$	बालोऽभून्नीलनेत्रः	*bālo'bhūnnīlanetraḥ*	$2^r 03°04'33''$
170	वामालोलः शङ्करः	*vāmālolaḥ śaṅkaraḥ*	$2^r 15°33'54''$	जटी शूली शङ्करः	*jaṭī śūlī śaṅkaraḥ*	$2^r 15°35'18''$
171	धीरदैत्यहारुद्रः	*ndhīradaityahārudraḥ*	$2^r 28°18'29''$	प्रवृद्धोऽयं जाठरः	*pravṛddho'yaṃ jāṭharaḥ*	$2^r 28°19'42''$
172	अन्नं देयं कृपालुना	*annaṃ deyaṃ kṛpālunā*	$03^r 11°18'00''$	सन्निधौ स्यात् कपाली	*sannidhau syāt kapālī*	$3^r 11°19'07''$
173	नानालिला वराङ्गी	*nānālilā varāṅgī*	$3^r 24°33'00''$	दीनेष्वीडा विफला	*dīneṣvīḍā viphalā*	$3^r 24°34'08''$
174	नृपो गानदानवित्	*nṛpo gānadānavit*	$4^r 08°03'10''$	खाण्डवघ्नो जनो वै	*khāṇḍavaghno jano vai*	$4^r 08°04'32''$
175	तच्वार्थी विद्यारुढः	*tacvārthī vidyāruḍhaḥ*	$4^r 21°47'46''$	सौम्यधीः स्वयंप्रभुः	*saumyadhīḥ svayaṃprabhuḥ*	$4^r 21°49'17''$
176	श्रीणाघो वामनमनाः	*śrīṇāgho vāmanamanāḥ*	$05^r 05°44'52''$	बालास्तु वाग्मिनोऽमी	*bālāstu vāgmino'mī*	$5^r 05°46'33''$
177	देवस्त्री मधुकामा	*devastrī madhukāmā*	$5^r 19°52'48''$	शशलक्ष्माधिकांशुः	*śaśalakṣmādhikāṃśuḥ*	$5^r 19°53'55''$
178	सरसज्ञा वनिता	*sarasajñā vanitā*	$6^r 04°07'27''$	सुखदं नवनीतम्	*sukhadaṃ navanītam*	$6^r 04°08'27''$
179	गूढचारो जयति	*gūḍhacāro jayati*	$6^r 18°26'43''$	धन्वन्तरिर्जयति	*dhanvantarirjayati*	$6^r 18°26'49''$
180	मारुतिर्वीरनाथः	*mārutirvīranāthaḥ*	$7^r 02°46'25''$	वागीशो वारनाथः	*vāgīśo vāranāthaḥ*	$7^r 02°45'34''$
181	धरित्रीनाथः पार्थः	*dharikrīnāthaḥ pārthaḥ*	$7^r 17°02'29''$	पयस्यनिच्छा कथम्	*payasyanicchā katham*	$7^r 17°01'11''$
182	समीये पूज्यम् नदी	*samīye pūjyam nadī*	$8^r 01°11'57''$	धान्ये न कस्यानन्दः	*dhānye na kasyānandaḥ*	$8^r 01°10'19''$
183	दारुकः कृष्णपदे	*dārakaḥ kṛṣṇapade*	$8^r 15°11'28''$	शमधनाः शापदाः	*śamadhanāḥ śāpadāḥ*	$8^r 15°09'55''$

184	सामजे मदराजिः	*sāmaje madarājiḥ*	$8^r\,28°58'57''$	चारार्थी महाराजः	*cārārthī mahārājaḥ*	$8^r\,28°57'26''$
185	दानरागो राज्याधीनः	*dānarāgo rājyādhīnaḥ*	$09^r\,12°32'08''$	सोमोऽनङ्गारिर्व्याधः	*somo'naṅgārirvyādhaḥ*	$9^r\,12°30'57''$
186	दारा निशि मारान्धाः	*ddārā niśi mārāndhāḥ*	$9^r\,25°50'28''$	मर्त्योऽधन्वा शरधीः	*martyo'dhanvā śaradhīḥ*	$9^r\,25°49'15''$
187	मुनिगणो जननम्यः	*munigaṇo jananamyaḥ*	$10^r\,08°53'05''$	शशी कुमुदिनीनम्यः	*śaśī kumudinīnamyaḥ*	$10^r\,08°51'55''$
188	वामनो विद्याप्रज्ञाढ्यः	*vāmano vidyāprajñāḍhyaḥ*	$10^r\,21°40'54''$	रुद्रो धीगम्यः प्राज्ञैः स्यात्	*rudro dhīgamyaḥ prājñaiḥ syāt*	$10^r\,21°39'22''$
189	गुरुविद्या विनयाय	*guruvidyā vinayāya*	$11^r\,04°14'23''$	ईशप्रियो विनायकः	*īśapriyo vināyakaḥ*	$11^r\,04°12'50''$
190	मणिशालि तत्कटकम्	*maṇiśāli tatkaṭakam*	$11^r\,16°35'55''$	विद्योज्वला तार्किकस्य	*vidyojjvalā tārkikasya*	$11^r\,16°34'14''$
191	हरिः सर्वदा श्रीयाढ्यः	*hariḥ sarvadā śrīyāḍhyaḥ*	$11^r\,28°47'28''$	धनाप्ताभूदद्रिकन्या	*dhanāptābhūdadrikanyā*	$11^r\,28°46'09''$
192	रङ्गराड् मुनीड्यः	*raṅgarāḍ munīḍyaḥ*	$10°52'32''$	दुर्गेयमनिन्द्या	*durgeyamanindyā*	$10°51'38''$
193	ज्ञानवान्मुरारिः	*jñānavānmurāriḥ*	$22°54'00''$	प्राज्ञौ विष्णुरुद्रौ	*prājñau viṣṇurudrau*	$22°54'02''$
194	लक्ष्मणः शोभनकृत्	*lakṣmaṇaḥ śobhanakṛt*	$1^r\,04°55'53''$	पद्माक्षी शोभनास्या	*padmākṣī śobhanāsyā*	$1^r\,04°56'51''$
195	श्रीकरज्ञः सात्यकिः	*śrīkarajñaḥ sātyakiḥ*	$1^r\,17°02'12''$	भर्गो गोनाथः पूज्यः	*bhargo gonāthaḥ pūjyaḥ*	$1^r\,17°03'34''$
196	रमणी केळिप्रया	*ramaṇī keliprayā*	$1^r\,29°15'52''$	हरिः सेव्यो धरया	*hariḥ sevyo dharayā*	$1^r\,29°17'28''$
197	रग्नानि भूयकरे	*ragnāni bhūyakare*	$2^r\,11°40'02''$	पौलस्त्यो भयङ्करः	*paulastyo bhayaṅkaraḥ*	$2^r\,11°41'31''$
198	नम्यस्तपसि श्रेष्ठः	*namyastapasi śreṣṭhaḥ*	$2^r\,27°16'10''$	स्थाने जयो वरिष्ठः	*sthāne jayo variṣṭhaḥ*	$2^r\,24°18'07''$
199	हेमासनासीनाम्बा	*hemāsanāsīnāmbā*	$3^r\,07°07'58''$	मानधनः सानुगः	*mānadhanaḥ sānugaḥ*	$3^r\,07°09'05''$
200	धन्यम्मन्यो नारीलग्नः	*dhanyammanyo nārīlagnaḥ*	$03^r\,20°15'19''$	तरुणः को न रागी	*taruṇaḥ ko na rāgī*	$3^r\,20°15'26''$
201	शशी तुलालग्नभाक्	*śaśī tulālagnabhāk*	$4^r\,03°36'55''$	गौरी सलीला न वा	*gaurī salīlā na vā*	$4^r\,03°37'23''$
202	वाणीप्रियः सत्कविः	*vāṇīpriyaḥ satkaviḥ*	$4^r\,17°12'54''$	करीभव्योऽसौ युवा	*karībhavyo'sau yuvā*	$4^r\,17°14'21''$
203	कुरङ्गनयनामा	*kuraṅganayanāmā*	$5^r\,01°03'21''$	तमस्विनीयं निशा	*tamasvinīyaṃ niśā*	$5^r\,01°04'56''$
204	हरीशौ नमस्कुर्मः	*harīśau namaskurmaḥ*	$5^r\,15°05'28''$	रत्नासनमुपेमः	*ratnāsanamupemaḥ*	$5^r\,15°07'02''$

205	सर्वैस्स्तुत्या धरणी	*sarvaisstutyā dharaṇī*	$5^r29°16'47''$	हेम सम्पद्धारिणाम्	*hema sampaddhāriṇām*	$5^r29°17'58''$
206	रूपवान्बालापतिः	*rūpavānbālapatiḥ*	$6^r13°34'12''$	जडो विडम्बयति	*jaḍo viḍambayati*	$6^r13°34'38''$
207	धर्मलक्ष्मी सत्रेषु	*dharmalakṣmī satreṣu*	$6^r27°53'59''$	सलिलाशा सम्प्रति	*salilāśā samprati*	$6^r27°53'37''$
208	धरित्रीपरः पार्थः	*dhiritrīparaḥ pārthaḥ*	$7^r12°12'29''$	गुरुकार्ये प्रयासः	*gurukārye prayāsaḥ*	$7^r12°11'23''$
209	स्त्रीनिकेतो रतिरसः	*strīniketo ratirasaḥ*	$7^r26°26'102''$	पाण्डवाः प्राप्तरथाः	*pāṇḍavāḥ prāptarathāḥ*	$7^r26°24'31''$
210	नरपालो नयनार्हः	*narapālo nayanārhaḥ*	$8^r01°03'120''$	शिवधीरनापदे	*śivadhīranāpade*	$8^r10°29'45''$
211	धन्वशरवान् राजा	*dhanvaśaravān rājā*	$8^r24°25'49''$	चापी वीरो द्विरदे	*cāpī vīro dvirade*	$8^r24°24'16''$
212	राज्ञां स्थाने दानधीः	*rājñāṃ sthāne dānadhīḥ*	$9^r08°07'02''$	शश्वन्मौनीजनोऽन्धः	*śaśvanmaunījano'ndhaḥ*	$9^r08°05'45''$
213	कमलालय श्रद्धः	*kamalālaya śraddhaḥ*	$9^r21°33'51''$	मूलं फलाढ्यश्राद्धः	*mūlaṃ phalāḍhyaśrāddhaḥ*	$9^r21°32'35''$
214	शैशवं भुवननन्द्यम्	*saisavaṃ bhuvananandyam*	$10^r04°44'55''$	तमालाभो घनो नित्यम्	*tamālābho ghano nityam*	$10^r04°43'56''$
215	दानाय वासंकनकम्	*dānāya vāsaṃkanakam*	$10^r17°41'08''$	प्रभोर्धिगच्छकनकम्	*prabhordhigacchakanakam*	$10^r17°39'42''$
216	गोपराड् राज्ञां नायकः	*goparāḍ rājñāṃ nayakaḥ*	$11^r00°22'13''$	वैश्वानरं नानुयायात्	*vaiśvānaraṃ nānuyāyāt*	$11^r00°20'44''$
217	भर्या नर्मप्रिया पाल्या	*bharyā narmapriyā pālyā*	$11^r12°50'14''$	रागादिवैराग्यं पथ्यम्	*rāgādivairāgyaṃ pathyam*	$11^r12°48'32''$
218	मणितेन मारः पूज्यः	*manitena māraḥ pūjyaḥ*	$11^r25°06'55''$	श्रीरामनाम रम्याग्र्यम्	*śrīrāmanāma ramyāgryam*	$11^r25°05'22''$
219	चापेशः पार्थः	*cāpeśaḥ pārthaḥ*	$7°15'16''$	प्रज्ञावान् पार्थः	*prajñāvān pārthaḥ*	$7°14'02''$
220	राज्यां हेयाधिके	*rājyāṃ heyādhike*	$19°18'12''$	बीभत्सुर्योधोऽयम्	*bībhatsuryodho'yam*	$19°17'43''$
221	नराधिपे कनकम्	*narādipe kanakam*	$1^r01°19'20''$	ग्रामाधिपः कनकी	*grāmādhipaḥ kanakī*	$1^r01°19'52''$
222	श्रीमच्छ्रीरङ्गं पूज्यम्	*śśrīmacchrīraṅgaṃ pūjyam*	$1^r13°22'52''$	धम्मिल्ले फुल्लपुष्पम्	*dhammille phullapuṣpam*	$1^r13°23'59''$
223	जन्मकालांशः श्रेयान्	*janmakālāṃśaḥ śreyān*	$1^r25°31'58''$	कालो बली शरण्यः	*kālo balī śaraṇyaḥ*	$1^r25°33'31''$
224	माननामिच्छेन्नरः	*mānanāmicchennaraḥ*	$2^r07°50'05''$	तुङ्गयशसो नराः	*tuṅgayaśaso narāḥ*	$2^r07°51'36''$
225	प्रबन्धकृन्नरेन्द्रः	*prabandhakṛnnarendraḥ*	$2^r20°19'32''$	धर्मज्ञाः प्राङ्नरेन्द्राः	*dharmajñāḥ prāṅnarendrāḥ*	$2^r20°20'59''$

226	तुम्बुरुनिलीनं गानम्	*tumburunilīnaṃ gānam*	03^{r} 03°02′36″	दीर्घाङ्गो नीलनागः	*dīrghāṅgo nīlanāgaḥ*	3^{r} 03°03′48″
227	श्रेष्ठज्ञानाप्तो योगी	*śreṣṭhajñānāpto yogī*	3^{r} 16°00′22″	फलाढ्यो ऋतुकालः	*phalāḍhyo ṛtukālaḥ*	3^{r} 16°01′32″
228	शिवोऽम्बिकाधाराङ्गः	*śivosmbikādhārāṅgaḥ*	3^{r} 29°13′45″	केशवो योद्धा रङ्गे	*keśavo yoddhā raṅge*	3^{r} 29°14′51″
229	मान्यः प्रवरः कविः	*mānyaḥ pravaraḥ kaviḥ*	4^{r} 12°42′15″	सौबलो वरयुवा	*saubalo varayuvā*	4^{r} 12°43′37″
230	श्रीरामो रक्षाप्रभुः	*śrīrāmo rakṣāprabhuḥ*	4^{r} 26°25′22″	पद्मेषु रन्ता रविः	*padmeṣu rantā raviḥ*	4^{r} 26°26′51″
231	मान्यं करी नृपेण	*mānyaṃ karī nṛpeṇa*	5^{r} 10°21′15″	भ्रमरश्रीर्निकामम्	*bhramaraśrīrnikāmam*	5^{r} 10°22′52″
232	अत्मसुखवान् श्रीमान्	*atmasukhavān śrīmān*	5^{r} 24°27′50″	तपोधराः स्वैरिणः	*tapodharāḥ svairiṇaḥ*	5^{r} 24°29′16″
233	दानं प्रभुर्जानाति	*dānaṃ prabhurjānāti*	6^{r} 08°42′08″	आद्यो गोविन्द एषः	*ādyo govinda eṣaḥ*	6^{r} 08°43′10″
234	स्तनयिन्नुर्लेखातनुः	*astanayipnurlekhātanuḥ*	06^{r} 23°01′06″	सत्काव्यज्ञोऽम्बरीषः	*satkāvyajño'mbarīṣaḥ*	6^{r} 23°01′17″
235	विष्णुना श्री सनाथा	*viṣṇunā śrī sanāthā*	7^{r} 07°20′54″	उद्यानं स्त्रीसनाथम्	*udyānaṃ strīsanātham*	7^{r} 07°20′10″
236	शार्ङ्गी सालोक्यप्रेप्सुः	*śārṅgī sālokyaprepsu*	7^{r} 21°37′35″	दीपतैलं पात्रस्थम्	*dīpatailaṃ pātrastham*	7^{r} 21°36′18″
237	लक्ष्मीः सा वामनार्हा	*lakṣmīḥ sā vāmanārhā*	8^{r} 05°47′53″	सूर्योऽस्तु वो मानदः	*sūryo'stu vo mānadaḥ*	8^{r} 05°46′17″
238	अम्भोजं वेधःपदम्	*ambhojaṃ vedhaḥpadam*	8^{r} 19°48′40″	भानुः सर्वाधिको हि	*bhānuḥ sarvādhiko hi*	8^{r} 19°47′04″
239	चन्द्रः सलिलनिधिः	*candraḥ salilanidhiḥ*	9^{r} 03°37′26″	पीनोत्तुङ्गाङ्गो नळः	*pīnottuṅgāṅgo naḷaḥ*	9^{r} 03°36′01″
240	विरप्रियार्थं युद्धम्	*virapriyārthaṃ yuddham*	9^{r} 17°12′24″	हीनपापः सुयोद्धा	*hīnapāpaḥ suyoddhā*	9^{r} 17°11′08″
241	शङ्करो गानज्ञनटः	*śaṅkaro gānajñanaṭaḥ*	10^{r} 00°32′15″	सूनुः कुलीनोऽनुनेयः	*sūnuḥ kulīno'nuneyaḥ*	10^{r} 00°31′07″
242	शोभते बलढ्यो नृपः	*śaibhate baladhyo nṛpaḥ*	10^{r} 13°36′45″	धरणो बलीयानाढ्यः	*dharaṇo balīyānāḍhyaḥ*	10^{r} 13°35′29″
243	क्षोणीशश्रीश्चारुनया	*kṣoṇīśaśrīścārunayā*	10^{r} 26°25′56″	बालेऽभिरुचिरनल्पा	*bāle'bhiruciranalpā*	10^{r} 26°24′33″
244	सेना यानधनोत्कटा	*senā yānadhanotkaṭā*	11^{r} 09°01′07″	स्थिरधीर्महीनायकः	*sthiradhīrmahīnāyakaḥ*	11^{r} 08°59′27″
245	भवो गौरिष्ठप्रापकः	*bhavo gauriṣṭhaprāpakaḥ*	11^{r} 22°23′44″	स्वर्नारी रम्यरूपाढ्या	*svarnārī ramyarūpāḍhyā*	11^{r} 21°22′04″
246	धन्याक्षी बाला	*dhanyākṣī bālā*	3°36′19″	भीमो वललः	*bhīmo valalaḥ*	3°34′54″

247	कामोऽरं भूमिपानाम्	*kāmosraṃ bhūmipāmām*	15°42′51″	शशी नभोमध्ये	*śaśī nabhomadhye*	15°40′55″
248*	स्याद् बलवान् सुखी	*syad balavān sukhī*	27°43′31″	धीरगीर्भासुरा	*dhīragīrbhāsurā*	27°43′29″

Table 1.1: The *candravākyas* of Tippa-Gopendra and Mādhava.

1.7 Observations and vākyashodana

The analysis of these *vākya*s can be approached in two primary ways—either through their numerical values or from a literary standpoint. This section focuses on the former.

The positional data of the Moon has been systematically recorded across nine of its cycles, yielding expected values for the corresponding days. The computed values from the *Tippa Gopendra vākya*s closely align with those of the *Mādhava vākyas*, with discrepancies only in the range of minutes or seconds. Much like the *Mādhava vākya*s, these values have been documented with a precision extending to seconds, significantly enhancing their astronomical relevance. A preliminary examination reveals inconsistencies in certain *vākya*s—specifically 11, 130, 162, 209, and 210—as well as within the sequence from 93 to 100, which deviate from the expected pattern. Further analysis indicates that the series from 93 to 100 exhibits a cyclic misalignment, wherein *vākya* 100 corresponds to 93, 93 corresponds to 94, and so on. This shift can be corrected systematically. However, the remaining discrepancies require a different analytical approach.

Vākyashodana

It is quite interesting that the *vākya-kāra*s had devised an error-correction methodology,along with documenting or enumerating the data. This technique safeguards and ensures the *vākya*s from corruption over time[1].

Vararuchi's correction is quite simple and is given by:

$$V_i + V_{248-i} = V_{248}, \quad \text{(modulo } 360^\circ\text{)} \tag{1.2}$$

where i represents the *vākya* number, ranging from $i = 1$ to 248. The corrected value for the ith *vākya* (V_i) is obtained as the difference between the value of the 248th *vākya* (V_{248}) and the complementary *vākya* of i (V_{248-i}).This approach implicitly assumes an approximate value of the anomaly as $\frac{9}{248}$, which, in turn, limits the accuracy of the corrected verse.

On the other hand, Madhava employs a more precise and systematically structured methodology.The follwing is the process:

- First, find out the complementary *vākya* V_j, where $j = 248 - i$.
- Then, find the *vākya*s above (V_{j-1}) and below (V_{j+1}) the complementary *vākya* (V_j) and determine their difference:
$$\Delta V_j = V_{j+1} - V_{j-1}.$$
- Subtract twice the first *vākya* (V_1) from ΔV_j, and divide the result by 225. That is, compute:
$$\delta_j = \frac{\Delta V_j - (2 \times V_1)}{225}. \tag{1.3}$$
- Subtract δ_j (in minutes, etc.) from the complementary *vākya*. This gives the transformed complementary *vākya*:
$$V'_j = V_j - \delta_j. \tag{1.4}$$
- The desired *vākya*, if correct, should be equal to the result obtained by subtracting V'_j from the final *vākya* (V_{248}). That is, the desired *vākya* should satisfy:
$$V_i = V_{248} - V'_j. \tag{1.5}$$

Since the Tippa Gopendra *vākya*s closely align numerically with the Madhava *vākya*s and this approach offers greater precision, *vākyaśodhana* has been performed for Tippa Gopendra *vākya* numbers 11, 130, 162, 209, and 210. New *vākya*s have been constructed and proposed based on the corrected values obtained. The following pages present the complete correction process in detail.(The rationale and the process are clearly explained in the Karaṇapaddhati[4])

Correcting the 11^{th} *vākya*

Since the desired *vākya* number $i = 11$, the number of the complementary *vākya* $j = 248 - 11 = 237$. The *vākyas* below (V_{j+1}) and above (V_{j-1}) the complementary *vākya* are:

$$V_{238} = 8^r 19^\circ 48' 40'' \quad \text{and} \quad V_{236} = 7^r 21^\circ 37' 35''.$$

Their difference is given by

$$\Delta V_{237} = 28^\circ 11' 05''.$$

By subtracting twice the first *vākya* ($2 \times V_1 = 0^r 24^\circ 02' 04''$) from the above, we get

$$\begin{aligned} \Delta V_{237} - 2V_1 &= 0^r 28^\circ 11' 05'' - 0^r 24^\circ 02' 04'' \\ &= 4^\circ 09' 01'' \\ &= 14941''. \end{aligned}$$

Dividing the result obtained by 225, we obtain $\delta_{118} = 01' 06''$. Subtracting this from the complementary *vākya* (V_{237}), we have

$$\begin{aligned} V'_{237} &= V_{237} - \delta_{237} \\ &= 8^r 05^\circ 47' 53'' - 01' 06'' \\ &= 8^r 05^\circ 46' 47''. \end{aligned}$$

Now the desired *vākya* V_{11} is given by

$$\begin{aligned} V_{11} &= V_{248} - V'_{237} \\ &= 12^r 27^\circ 43' 31'' - 8^r 05^\circ 46' 47'' \\ &= 04^r 21^\circ 56' 44''. \end{aligned} \tag{1.6}$$

The *vākya* number 11 found in the manuscript of *Tantraratna* is "विभूतिः शङ्करभ्रूः */vibhūtaḥśaṅakarabhrūḥ* ". This when decoded corresponds to the value $2^r 21^\circ 56' 44''$ and the result obtained for V_{11} based on *vākya sodhana* procedure of Mādhava is $04^r 21^\circ 56' 44''$.

Suggested *vākya*

"विभूतिमयः ध्रुवः "/

vibhūtimayaḥ dhruvaḥ

Correcting the 130^{th} *vākya*

Since the desired *vākya* number $i = 130$, the number of the complementary *vākya* $j = 248 - 130 = 118$. The *vākyas* below (V_{j+1}) and above (V_{j-1}) the complementary *vākya* are:

$$V_{119} = 4^r03^\circ24'11'' \quad \text{and} \quad V_{117} = 3^r06^\circ35'50''.$$

Their difference is given by

$$\Delta V_{118} = 26^\circ48'21''.$$

By subtracting twice the first *vākya* ($2 \times V_1 = 0^r24^\circ02'04''$) from the above, we get

$$\begin{aligned} \Delta V_{118} - 2V_1 &= 0^r26^\circ48'21'' - 0^r24^\circ02'04'' \\ &= 2^\circ46'17'' \\ &= 9977''. \end{aligned}$$

Dividing the result obtained by 225, we obtain $\delta_{118} = 44''$. Subtracting this from the complementary *vākya* (V_{118}), we have

$$\begin{aligned} V'_{118} &= V_{118} - \delta_{118} \\ &= 3^r19^\circ52'26'' - 44'' \\ &= 3^r19^\circ51'42''. \end{aligned}$$

Now the desired *vākya* V_{130} is given by

$$\begin{aligned} V_{130} &= V_{248} - V'_{118} \\ &= 12^r27^\circ43'31'' - 3^r19^\circ51'42'' \\ &= 09^r07^\circ51'49''. \end{aligned} \tag{1.7}$$

The *vākya* number 130 found in the manuscript of *Tantraratna* is "वाणी पद्मासनासीना /*vāṇī padmāsanāsīnā*". This when decoded corresponds to the value $7^r07^\circ51'54''$ and the result obtained for V_{130} based on *vākya sodhana* procedure of Mādhava is $09^r07^\circ51'49''$.

Suggested *vākya*

धन्वा पद्मासना धनी/

dhanvā padmāsanā dhanī

Correcting the 162^{th} *vākya*

Since the desired *vākya* number $i = 162$, the number of the complementary *vākya* $j = 248 - 162 = 86$. The *vākyas* below (V_{j+1}) and above (V_{j-1}) the complementary *vākya* are:

$$V_{87} = 2^r02°07'34'' \quad \text{and} \quad V_{85} = 1^r07°25'04''.$$

Their difference is given by

$$\Delta V_{86} = 24°42'30''.$$

By subtracting twice the first *vākya* ($2 \times V_1 = 0^r24°02'04''$) from the above, we get

$$\begin{aligned} \Delta V_{86} - 2V_1 &= 0^r24°42'30'' - 0^r24°02'04'' \\ &= 40'26'' \\ &= 2426''. \end{aligned}$$

Dividing the result obtained by 225, we obtain $\delta_{86} = 11''$. Subtracting this from the complementary *vākya* (V_{86}), we have

$$\begin{aligned} V'_{86} &= V_{86} - \delta_{86} \\ &= 1^r19°40'53'' - 11'' \\ &= 1^r19°40'42''. \end{aligned}$$

Now the desired *vākya* V_{162} is given by

$$\begin{aligned} V_{162} &= V_{248} - V'_{86} \\ &= 12^r27°43'31'' - 1^r19°40'42'' \\ &= 11^r08°02'49''. \end{aligned} \tag{1.8}$$

The *vākya* number 162 found in the manuscript of *Tantraratna* is "धीवरैर्नदी कृष्णा/*dhīvarairnadī kṛṣṇā*". This when decoded corresponds to the value $5^r18°02'49''$ and the result obtained for V_{162} based on *vākya sodhana* procedure of Mādhava is $11^r08°02'49''$.

Suggested *vākya*

धीवरैर्नदी नौकिकः

*d*hīvarairnadī naukikaḥ

Correcting the 209^{th} *vākya*

Since the desired *vākya* number $i = 209$, the number of the complementary *vākya* $j = 248 - 209 = 39$. The *vākyas* below (V_{j+1}) and above (V_{j-1}) the complementary *vākya* are:

$$V_{40} = 5^r 15^\circ 32' 10'' \quad \text{and} \quad V_{38} = 4^r 17^\circ 13' 13''.$$

Their difference is given by

$$\Delta V_{39} = 28^\circ 18' 57''.$$

By subtracting twice the first *vākya* ($2 \times V_1 = 0^r 24^\circ 02' 04''$) from the above, we get

$$\begin{aligned} \Delta V_{39} - 2V_1 &= 0^r 28^\circ 18' 57'' - 0^r 24^\circ 02' 04'' \\ &= 4^\circ 16' 53'' \\ &= 15413''. \end{aligned}$$

Dividing the result obtained by 225, we obtain $\delta_{39} = 1' 08''$. Subtracting this from the complementary *vākya* (V_{39}), we have

$$\begin{aligned} V'_{39} &= V_{39} - \delta_{39} \\ &= 5^r 01^\circ 18' 35'' - 01' 08'' \\ &= 5^r 01^\circ 17' 27''. \end{aligned}$$

Now the desired *vākya* V_{130} is given by

$$\begin{aligned} V_{209} &= V_{248} - V'_{39} \\ &= 12^r 27^\circ 43' 31'' - 5^r 01^\circ 17' 27'' \\ &= 07^r 26^\circ 26' 04''. \end{aligned} \tag{1.9}$$

The *vākya* number 209 found in the manuscript of *Tantraratna* is "स्त्रीनिकेतो रतिरसः / *strīniketo ratirasaḥ*". This when decoded corresponds to the value $72^r 62^\circ 61' 02''$ and the result obtained for V_{209} based on *vākya sodhana* procedure of Mādhava is $07^r 26^\circ 26' 04''$.

Suggested *vākya*

अविज्ञा च रति रसः

avijñā ca rati rasaḥ

Correcting the 210^{th} *vākya*

Since the desired *vākya* number $i = 130$, the number of the complementary *vākya* $j = 248 - 210 = 38$. The *vākyas* below (V_{j+1}) and above (V_{j-1}) the complementary *vākya* are:

$$V_{39} = 5^r01^\circ18'35'' \quad \text{and} \quad V_{37} = 4^r03^\circ18'39''.$$

Their difference is given by

$$\Delta V_{38} = 27^\circ59'56''.$$

By subtracting twice the first *vākya* ($2 \times V_1 = 0^r24^\circ02'04''$) from the above, we get

$$\begin{aligned} \Delta V_{38} - 2V_1 &= 0^r27^\circ59'56'' - 0^r24^\circ02'04'' \\ &= 3^\circ57'52'' \\ &= 14272''. \end{aligned}$$

Dividing the result obtained by 225, we obtain $\delta_{38} = 1'03''$. Subtracting this from the complementary *vākya* (V_{38}), we have

$$\begin{aligned} V'_{38} &= V_{38} - \delta_{38} \\ &= 4^r17^\circ13'13'' - 1'03'' \\ &= 4^r17^\circ12'10''. \end{aligned}$$

Now the desired *vākya* V_{210} is given by

$$\begin{aligned} V_{210} &= V_{248} - V'_{38} \\ &= 12^r27^\circ43'31'' - 4^r17^\circ12'10'' \\ &= 08^r10^\circ31'21''. \end{aligned} \tag{1.10}$$

The *vākya* number 210 found in the manuscript of *Tantraratna* is "नरपालो नयनार्हः / *narapālo nayanārhaḥ*". This when decoded corresponds to the value $80^r10^\circ31'20''$ and the result obtained for V_{210} based on *vākya sodhana* procedure of Mādhava is $08^r10^\circ31'21''$.

Suggested *vākya*

करपालो नृप हनः

karapālo nṛpa hanaḥ

1.8 Literary analysis

The 248 *vākyas* of *Tippa Bhūpāla* meticulously document lunar longitudes with remarkable precision. However, what makes them particularly fascinating is the ingenious manner in which numerical data is encoded within sentences whose literary meaning appears mostly unrelated to the astronomical values they represent. For someone unfamiliar with the *vākya* system, it would not even be apparent that these verses serve as a large dataset documenting various astronomical parameters.

A large portion of *Vararuci's* and *Mādhava's vākyas* primarily convey moral and philosophical reflections. As both were primarily sages and astronomers, their perspectives and ideologies naturally aligned with those themes, and such values were subtly interwoven into their *vākyas*. In contrast, *Tippa Gopendra* was primarily a king, and his *vākyas* reflect a different worldview. The author's *vākyas* are deeply influenced by the his lifestyle, personal inclinations, and immediate surroundings.The 199th *vākya* from the above three sources is presented below. While their numerical values are nearly identical,their literal meanings differ significantly.

Vararuchi:

धेनुः सेनाङ्गं—3r 07° 09'

-"The cow is a part of the army"

Madhava:

मानधनः सानुगः—-3r 07° 09'05"

-"A leader who values honor as his true treasure , is followed by devoted companions.

Tippa:

हेमासनासीनाम्बा—-3r 07° 07' 58"

-"The mother is seated on a golden throne".

This divergence highlights the broader spectrum of *vākya* compositions, challenging the prevalent assumption that all ancient *vākyas* were fundamentally intended to moral principles. It also demonstrates that the *vākya* framework is a flexible computational tool capable of accommodating multiple intellectual pursuits. Additionally, it provides the *vākyakāras* with utmost freedom and individuality, allowing them to compose vākyas shaped by their unique ideological,professional and cultural backgrounds.

Given below are themes that the author *Tippa Bhupala* has addressed in his *vakyas:*

- **Kingship and Warfare (50 vākyas)**: Discusses sovereignty, duties, military organization, weaponry, the ethics of war, royal virtues, war elephants , wealth, and dynastic lineage. Selected references: ([TR] vākyas: 19–25, 75, 86, 128, 147, 212, 242, 244).

- **Women and Femininity (30 vākyas)**: Explores themes of feminine grace, the aesthetics of beauty, amorous longing (kāma), conjugal fidelity, maternity, queenship, and the ethical dimensions of womanhood. Selected references: ([TR] vākyas: 82, 88, 149, 186, 196, 217, 246).

- **Vishnu and His Incarnations (20 vākyas)**: Examines the manifestations of Vishnu, particularly his avatāras—Varaha, Vāmana, and Ranganātha—(benediction). Selected references: ([TD] vākyas: 6, 28, 52, 61).

- **Music, Dance, and Aesthetics (10 vākyas)**: particularly in relation to Śiva as the divine patron of the performing arts. Selected references: ([TR] vākyas: 35, 37, 45, 226, 241).

- **Goddess Lakṣmī** : the goddess of wealth and prosperity, in connection with finance and auspiciousness. Selected references: ([TR] vākyas: 9, 124, 235).

- **Nature**: Depicts natural elements, including rivers (e.g., Gaṅgā, Kṛṣṇa), forests, clouds etc. Selected references: ([TR] vākyas: 162, 163, 164, 205).

- **Sages, Knowledge and teachers**: Engages with themes of yoga/yogi, intellectual pursuit, and the guru's role in preserving wisdom. Selected references: ([TR] vākyas: 104, 159, 227).

- **General Category**:addressing various common worldly activities and benedictions.

No.	Tippa vākyas	Meaning
1	श्रीयान्नरस्य	The auspicious path of the man.
2	शय्यासने भद्रे	O auspicious one, on the resting seat.
3	हेमाधिकस्तान् नृपाः	The kings possess an abundance of gold.
4	दीर्घौरवदायिका	One who bestows great honor and respect.
5	विष्णुः सेव्योऽयं नरैः	Lord Vishnu is revered by mankind.
6	संन्यासी निर्विकारः	An ascetic, free from worldly distractions.
7	लक्ष्मीः स्यात् कंसारेः स्त्री	Goddess Lakshmi is the consort of Lord Vishnu, the slayer of Kamsa.
8	धनी काले नृपालम्	A wealthy ruler at the right time.
9	सीमन्तिनी वराङ्गी	A married woman adorned with beauty.
10	कामो मर्मस्थानभित्	Desire strikes at the most vulnerable points.
11	विभूतिः शङ्करभ्रूः	Sacred ash on the brows of Lord Shiva.
12	मानसोनात्तिर् नृणाम्	Mental elevation is esteemed among men.
13	शिवाङ्गं रत्नैः श्रीमत्	The divine body of Lord Shiva is adorned with precious gems.
14	जलगर्भाघनास्ते	The clouds are heavy with water.
15	श्रीभूर्यज्ञाधिपतिः	The Lord of all sacred rites and rituals, the protector of the Earth.
16	मेघास्तडिल्लग्नाः सानौ	Clouds clinging to the mountain peaks with lightning.
17	ह्यायाम्बरः संन्यासी	An ascetic renounces worldly attire.
18	घनाधिकेयन्नदी	A river swells with an abundance of water.
19	सूत्ररत्नशय्यार्हः	One worthy of a luxurious bed adorned with pearls.
20	सद्रत्नगेहो राजा	A king who resides in a palace adorned with jewels.
21	गाढाम्बुभृत् पयोधिः	The ocean holds vast waters.
22	करिकुम्भे विरोधिः	Opposition arises in the presence of great strength.
23	करिवरासीनो नृपः	A king seated upon a noble elephant.
24	दण्डगुणाधिकः नृपः	A ruler excelling in discipline and virtue.
25	धीश्लाघ्या परनायिका	An intellectual woman is greatly admired.
26	भुवने रविः कृपाढ्यः	The sun bestows its radiance upon the world.

No.	Tippa vākyas	Meaning
27	कन्यावाराप्ता श्रियाढ्या	A bride endowed with great prosperity.
28	वन्यो मुरजित्	The conqueror of the wilderness.
29	स्तुत्यो हरिर् नरैः	Lord Vishnu is worshipped by mankind.
30	प्रभूतगोत्रो नृपः	A king belonging to an illustrious lineage.
31	अबला शुभोल्का	A woman likened to an auspicious meteor.
32	चण्डकरः सखायः	A fierce and powerful ally.
33	अमरा नानुकारः	The gods are beyond imitation.
34	धनधीः मन्त्रश्रेष्ठे	The wise and wealthy excel in sacred hymns.
35	रुद्रो नटति नागे	Lord Rudra performs his cosmic dance upon the mountains.
36	शय्या सुगन्धाकुला	A bed fragrant with sweet scents.
37	धिगो दिव्यगानवित्	The wise are well-versed in celestial music.
38	लोकश्लाघ्यः सत्कविः	A great poet is acclaimed by the world.
39	शैलजा पूज्या नृणाम्	The daughter of the mountains (Goddess Parvati) is worshipped by men.
40	नृपेन्द्रो बाणपाणिः	The king wields his arrows in battle.
41	शोभनात्मा धरेशः	The ruler of the earth is resplendent.
42	प्रभुर्नम्यो भूपतिः	The sovereign is worthy of reverence.
43	भूमिः सराजरक्षा	The earth is safeguarded by noble rulers.
44	सन्नद्धगात्रः पार्थः	Arjuna is ever prepared for battle.
45	नटः सङ्गीतसंरसः	The performer is immersed in musical expression.
46	भोगहरो नः पाहि	O Lord! Protect us by dispelling suffering.
47	स्थिरतनुर् वराहः	The Boar incarnation (Varaha) possesses a firm and steadfast form.
48	धृष्टोऽनङ्गः सन्नद्धः	The bold god of love (Kama) stands well-equipped.
49	धर्मोत्तुङ्गो नरधीः	The enlightened one stands exalted in righteousness.
50	हरिर्धराङ्गनानन्द्यः	Lord Hari is the joy of the women of the earth.

Above content strongly suggests that he was either a ruling monarch or closely associated with the royal administration. His references to specific rivers and deities hint his ethnicity and regional geographical location. The river Kṛṣṇā is mentioned at multiple vakyas is most significant in Karnataka, Telangana (briefly), and Andhra Pradesh. *Pālkuriki Sōmanātha*, in his *Paṇḍitārādhya Caritra* (14th century), describes the geography of the Telugu region and explicitly mentions the Kṛṣṇā River. Similarly, *Acyutarāya*, a 16th-century Vijayanagara ruler, refers to the river in his poetic work *Acyutarāya Abhyudayam*, as documented in *Sources of Vijayanagara History* (1919). Several of his *vākyas* alos invoke Viṣṇu in the form of *Varāha*, a deity of considerable regional significance to the people of the Andra region,(only temple devoted to the Varaha form of Vishnu is in Varāhapurī,Andra). Nārāyaṇa Tīrtha, a renowned Telugu poet, composed multiple *Taraṅgams* in praise of Varāha of Varahapuri. These references proves the author's connection to the Vijayanagara dynasty and gives his approximate regional placement.

Analysis of Similarities Between Tippa Vākyas, Vararuci Vākyas, and Mādhava Vākyas

An analysis of the similarities between Tippa's *vākyas* and those of Vararuci and Mādhava reveals that several verses share structural patterns. A good number of Tippa's verses appear to have been constructed with reference to or inspired by Vararuci.In some cases, the second half or the final words of the *vākya* (*uttarārdha*) remain identical. Since the final letters in vakya usually fall into the *rāśi* on numerical decoding,its highly unlikely that they change on employing diffrent methologies. As a result, Tippa, in several instances, retained the second word (*uttara-pada*) from Vararuci while modifying the first part (*pūrva-pada*) to align with his computational value. Table 2.8(1) presents a selection of such similar *vākyas* from Tippa and Vararuci.

Interestingly, some verses in Tippa's corpus also resemble those of Mādhava, independent of Vararuci. While Tippa Gopendra explicitly cites Vararuci's *vākyas* in the second part of *Candravākyas*, there is no direct reference to Mādhava (Table 2.8(2)). Given these similarities, one might hypothesize that Tippa was familiar with Mādhava's *vākyas* as well.Tippa's numerical framework, extending calculations to the second-order level, and his literary compositions exhibit resemblances to Mādhava's structure. However, there is currently no definitive confirmation or citation found.

Table 1.8(1): Tippa Vākyas vs. Vararuchi Vākyas

	Tippa Vākyas	Vararuchi Vākyas
12	मानसोन्नतिर्नृणाम्	दीनास्ते नृणाम्
14	जलगर्भाघनास्ते	भवभग्नास्ते
20	सद्रत्नगेहो राजा	धीरो हि राजा
74	शकटो विद्धः पदा	नाविदधः पादो
75	धेनुः कपिला निधिः	उत्पलं निधिः
76	सेना चरति युद्धे	शूद्रस्तु योद्धा
101	कामी धेनुर्मान्या हि	जनो मान्यो हि
102	क्षमा सर्वदा राजनि	स वादी राजा
111	क्षीरान्नभुक् पुत्रः	कविः पुत्रः
115	वराहो वनपरः	अशनपरः
117	ईशः शीलितनागः	सङ्गतो नागः
118	चारित्रशुद्धो योगी	विशुद्धो योगी
124	श्रेष्ठा रामा गीयते	रमा गीयते
133	मन्त्रैः शशी स्तुत्यो नित्यः	भूमिस्तस्य नित्यम्
153	विभुः साङ्गं सेनार्थी	तुलार्थनोऽर्थी
177	देवस्त्री मधुकामा	गोशुद्धिकामः
198	नम्यस्तपसि श्रेष्ठः	हर्यश्वः श्रेष्ठः
202	वाणीप्रियः सत्कविः	वैद्यः स कविः
203	कुरङ्गनयनामा	मेनका नाम
227	श्रेष्ठज्ञानाप्तो योगी	प्रज्ञातो योगी
235	विष्णुना श्री सनाथा	धन्यः स नाथः
238	अम्भोजं वेधःपदम्	षड्विधं पदम्
248	स्याद् बलवान् सुखी	भवेत् सुखम्

Table 1.8,(2): Tippa Vākyas vs. Madhava Vākyas

	Tippa Vākyas	Madhava Vākyas
15	श्रीभूयर्ज्ञाधिपतिः	नागो यानाधिपतिः
20	सद्रत्नगेहो राजा	निराधारोऽहिराजः
32	चण्डकरः सखायः	गानगोष्ठी सुखाय
38	लोकश्लाघ्यः सत्कविः	सर्वविद् व्यासः कविः
45	नटः सङ्गीतरसः	मागधो गीतरसः
72	अम्बोधिर्मान्यरसः	आनन्दमयो रसः
74	शकटो विद्धः पदा	दृढधीर्लब्धपदः
90	शीलकामो नकुलः	अभिरामा नकुली
134	अम्भस्थः श्रीधरे नित्यम्	श्रीनतः श्रीधरो नित्यम्
145	देवो रङ्गे शयालुः	स्थाणुर्गङ्गाशयालुः
170	वामालोलः शङ्करः	जटी शूली शङ्करः
179	गूढचारो जयति	धन्वन्तरिर्जयति
180	मारुतिर्वीरनाथः	वागीशो वारनाथः
213	कमलालय श्रद्धः	मूलं फलाढ्यश्राद्धः
215	दानाय वासंकनकम्	प्रभोर्धिगच्छकनकम्
219	चापेशः पार्थः	प्रज्ञावान् पार्थः
221	नराधिपे कनकम्	ग्रामाधिपः कनकी
240	विरप्रियार्थं युद्धम्	हीनपापः सुयोद्धा

1.9 Computing *candravākyas*

The *Candravākyas* of *Tantraratna* do not provide an explicit rationale or method for calculating the values of the longitude encoded within the *vākyas*. This section demonstrates that Tippa's *vākyas* are derived through precise computations of the true longitude of the Moon, incorporating the *manda* correction, also known as the equation of the center. The determination of the Moon's true longitude is influenced by the motion rates of the mean Moon and its anomaly, as well as the mean radius of its epicycle[0]. Initially, *Parahita* v parameters and Parahita values were adopted, as documented in the *Karaṇapaddhati*.These parameters has been shown to be valid for Mādhava[4]. Due to the similarity of Tippa's *vākyas* in both values and structure with Mādhava's *vākyas*, these values were initially considered. However, they did not yield accurate results for Tippa Gopendra's *vākyas*. The underlying issue was understood through the second section of the *Candravākyas*, where Tippa explicitly mentions that the calculations are based on*Sūryasiddhāntic*parameters.Even after considering all the parameters from the *Sūryasiddhānta*, the computed values did not align accurately with the *Tippa Vākya* values. However, since *Vākyasodhana* was effective and the present sections also show similar patterns for the values of *G* and *H*, some form of periodicity is evident. Future work is planned to further analyze this model.

The equations used for the computation, adapted for the *equation of center*, are given below.

>The base equation

$$\textbf{TrueLongitude} = \frac{\boldsymbol{360 \cdot i}}{\boldsymbol{Sp}} - \sin^{-1}\left(\frac{\boldsymbol{M}}{\boldsymbol{360}} \cdot \sin(\boldsymbol{A \cdot i \cdot 360})\right) \tag{1.11}$$

The first term corresponds to the mean longitude, where Sp denotes the sidereal period of the Moon. The second term represents the equation of the center, with A being the rate of motion of the anomaly, and $A \times 360$ giving the anomaly at the end of the first day. The parameter M refers to the *Manda-paridhi*.

>Surya siddantik parameters:

The *Sūrya Siddhānta* employs the *Bhūta Saṅkhyā* system of numeration. The following represent the verses and the extracted number.

इन्दोः रसाग्नित्रित्रीषुसप्तभूधरमार्गणा:[1.30]

चन्दोच्चस्यग्निभून्याश्विवसुसर्पार्णवा युगे[1.33]

वसुध्वष्टद्रिरूपाङ्कमप्तद्रितिथयो युगे[1.37]

>Number of Revolutions of the Moon in a Mahāyuga = 57, 753, 336 (1.12)

>Number of Revolutions of the Moon's Mandocca in a Mahāyuga =4, 88, 203 (1.13)

>Number of Civil Days in a Mahāyuga =1, 577, 917, 828 (1.14)

> Value of Manda paradhi(M)=31.667 (1.15)

First the values of the sideral period (Sp) and the anomaly (A) are to be calculated.Using the above values (2.12) and(2.14) the sidereal period of the Moon would be

$$Sp = \frac{157, 7917, 828}{57, 753, 336} \tag{1.16}$$

To determine the rate of motion of the Anomaly, the*gunakaras(G) and haras(H)* of the Moon and its *Mandocca* (Apogee) must be obtained.There parameters ensure that accurate values of the same.The Tippa gopendra's Tantraratna gives the value of Number of Revolutions of the Moon's Mandocca in a Mahāyuga to be 488211.The below section shows the calulation of the Anomay based on both the values of the Apogee. The anomaly (relative) is calculated subtracting the ratio of *gunakaras and haharas of the Apogee* from the corresponding ratio of the *Moon*.

Moon:

$$\frac{G_m}{H_m} = \frac{57, 753, 336}{157, 7917, 828} \tag{1.17}$$

Apogee1:

$$\frac{G_a1}{H_a1} = \frac{488203}{157, 7917, 828} \tag{1.18}$$

Apogee2:

$$\frac{G_a1}{H_a1} = \frac{488211}{157, 7917, 828} \tag{1.19}$$

Anomaly1:

$$A_1 = \frac{G_m - G_a1}{H} = \frac{57, 753, 336 - 4, 88, 203}{157, 7917, 828} = \frac{57, 265, 133}{157, 7917, 828} \tag{1.20}$$

Anomaly2:

$$A_2 = \frac{G_m - G_a2}{H} = \frac{57, 753, 336 - 4, 88, 211}{157, 7917, 828} = \frac{57, 265, 125}{157, 7917, 828} \tag{1.21}$$

Using these values the *Vallupasamhara* is to be performed.The process of *Vallyupasaṃhāra* involves generating sets of *hārakas* (denominators) and *guṇakas* (numerators) using continued fraction expansion. The steps are as follows: The continued fraction expansion of H/G is performed to obtain the successive quotients:

$$q_1, q_2, q_3, \ldots$$

Next, Column I is created by placing all the quotients in sequence. Column II is created by placing the same quotients but omitting the first one (q_1). The value **1** is placed at the top of both columns as an initial value.The entries in Column I (denoted as (H)) generate the set of *hārakas*, while the entries in Column II (denoted as (G)) generate the set of *guṇakas*. These values are computed step by step from the top down, as illustrated in **Table 1.9(1)**. According to the instruction:

"tyājyaṃ tatprathamordhvagam"

The first entry (1) in Column I must be removed before pairing with Column II. Once the first value is removed from Column I, the remaining entries from both columns are paired. The resulting pairs (H_i, G_i) form different sets of *alpahārakas* (denominators) and *alpaguṇakas* (numerators). These computed values are listed in Table 1.9(2).

Using the computed values, Equation (2.11) simplifies to:

$$truelongitude = \frac{360 \cdot i}{\frac{1{,}577{,}917{,}828}{57{,}753{,}336}} - \sin^{-1}\left(\frac{31.66}{360} \cdot \sin(A \cdot i \cdot 360)\right)$$

where A can take any value from the Anomaly G and H table (Table 2.8.2c). This line of thought still persists, but alternative procedures need to be explored to rectify the errors obtained using this method. The difference in the computed values is observed in the range of seconds. With proper computational techniques, this can be improved.

Table 2.8.1: Vallupasamhara II for moon

Vallupasamhara II for Moon (H)

1	1	1	1	1	1	1	1	1	1	1	1	1	1	
27	27	27	27	27	27	27	27	27	27	27	27	27	27	27
3	82	82	82	82	82	82	82	82	82	82	82	82	82	82
9	9	765	765	765	765	765	765	765	765	765	765	765	765	765
5	5	5	3907	3907	3907	3907	3907	3907	3907	3907	3907	3907	3907	3907
11	11	11	11	43742	43742	43742	43742	43742	43742	43742	43742	43742	43742	43742
1	1	1	1	1	47649	47649	47649	47649	47649	47649	47649	47649	47649	47649
1	1	1	1	1	1	91391	91391	91391	91391	91391	91391	91391	91391	91391
3	3	3	3	3	3	3	321822	321822	321822	321822	321822	321822	321822	321822
1	1	1	1	1	1	1	1	413213	413213	413213	413213	413213	413213	413213
1	1	1	1	1	1	1	1	1	735035	735035	735035	735035	735035	735035
27	27	27	27	27	27	27	27	27	27	20259158	20259158	20259158	20259158	20259158
1	1	1	1	1	1	1	1	1	1	1	20994193	20994193	20994193	20994193
2	2	2	2	2	2	2	2	2	2	2	2	62247544	62247544	62247544
6	6	6	6	6	6	6	6	6	6	6	6	6	394479457	394479457

Column I

Vallupasamhara II for Moon (G)

1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	3	3	3	3	3	3	3	3	3	3	3	3	3
9	28	28	28	28	28	28	28	28	28	28	28	28	28
5	5	143	143	143	143	143	143	143	143	143	143	143	143
11	11	11	1601	1601	1601	1601	1601	1601	1601	1601	1601	1601	1601
1	1	1	1	1744	1744	1744	1744	1744	1744	1744	1744	1744	1744
1	1	1	1	1	3345	3345	3345	3345	3345	3345	3345	3345	3345
3	3	3	3	3	3	11779	11779	11779	11779	11779	11779	11779	11779
1	1	1	1	1	1	1	15124	15124	15124	15124	15124	15124	15124
1	1	1	1	1	1	1	1	26903	26903	26903	26903	26903	26903
27	27	27	27	27	27	27	27	27	741505	741505	741505	741505	741505
1	1	1	1	1	1	1	1	1	1	768408	768408	768408	768408
2	2	2	2	2	2	2	2	2	2	2	2278321	2278321	2278321
6	6	6	6	6	6	6	6	6	6	6	6	14438334	14438334

Column II

a)

gunakāras (G_m)	*hārakas* (H_m)
1	27
3	82
28	765
143	3907
1601	43742
1744	47649
3345	91391
11779	321822
15124	413213
26903	735035
741505	20259158
768408	20994193
2278321	62247544
14438334	394479457
57753336	1577917828

b.1)

gunakāras (G_a1)	*hārakas* (H_a1)
1	3232
10	32321
11	35553
32	103427
395	1276677
427	1380104
822	2656781
6181	19977571
7003	22634352
13184	42611923
20187	65246275
33371	107858198
53558	173104473
86929	280962671
488203	1577917828

b.2)

gunakāras (G_a2)	*hārakas* (H_a2)
1	3232
24	77569
25	80801
49	158370
172	555911
393	1270192
4495	14528023
67818	219190537
140131	452909097
488211	1577917828

Table :1.9(2): *gunakākas* and *hārakas* of the moon(a) , apogee(b.1,b.2)

gunakāras (G_A1)	*hārakas* (H_A1)
1	27
1	28
2	55
9	248
110	3031
119	3279
348	9589
1163	32046
2674	73681
6511	179408
15696	432497
37903	1044402
53599	1476899
91502	2521301
145101	3998200
1977815	54497901
4100731	112994002
26582201	732461913
57265133	1577917828

gunakāras (G_A2)	*hārakas* (H_A2)
1	27
1	28
2	55
9	248
110	3031
119	3279
348	9589
467	12868
19962	550045
20429	562913
60820	1675871
628629	17321623
689449	18997494
2007527	55316611
2696976	74314105
18189383	501201241
57265125	1577917828

Table :1.9(2): gunakākas and hārakas the anomaly (1,2)

1.10 Dhruvās and Periods

Post the 248 vakyas . The author puts forward his remarks,copyright and citations. Given below are the verses.[TD]

एतानि वाक्यानि तिप्पभूपालविरचितानि।।
एतानि देवेन्द्रमितानि सूर्यसिद्धान्तोक्तानि गीर्नः श्रेयप्रमितानि, विहितादीनि।

अनेन चन्द्रस्फुटानयनम् -शुद्धदिनं भृगुशुद्धं अजितहारकैर्विभजेत्।
ततोऽजिताब्जरूपैः ।
ततः कालानङ्गैः। ततो देवरैः। हतशिष्टे पूर्वादिवाक्यानि।

अजितहारकहारस्य ध्रुवमधीग तमिष्ठरोधिनमिति।
जिताब्जरूपहारस्य सोमसूर्याम्बर पटमिति।
कालानङ्गस्य वज्रियागसुनायकम् ।
देवरहारस्य सूनुर्विश्वसुखमिति।
अन्त्ये गोपतिप्पपुन्नाग संयुतिः ।
एभियुक्तः चन्द्रध्रुवः।
ध्रुवः श्रीपूर्वादिवाक्ययोगेन उदयचन्द्रस्फुट:, अहर्गणे भृगुशुद्धे इनोद्यावधि चन्द्रस्फुटः स्यात् ।

सूर्यस्य दिवसगतिः - 365, नाड्यः 15, विनाड्यः 31, गुर्वक्षराणि 31, तदथः स्थाने 24. एषा रवेरेकाब्दगतिः।
नारी रागाद्रामशरेणाचलो नुन्नो शति वार्षकी गति:
अभूद्वेव शिवं कामी नागस्थाने शतद्वयी।
सत्संगात्वर्थ समुग्द्धनित्यानां त्रिशती गतिः ।
विद्यामान्या देवकीद्ध पुत्रज्ञा षट्च्छती गति:।
क्ष्मी नूनं लब्धशान्ते स्वदनेनानुवीरगति: ।
श्रीमयो नीतिदो योगी सुस्थोयं ---- गति: ।

In the first verse, the poet confirms his authorship and follows it by citing his use of the ordained methodology given in the *Sūryasiddhānta*, *Vararuci*'s *Gīrṇaḥ-śreyādi-vākyas*, and other sources. The word देवेन्द्रः suggests the 248 vakyas (katapayadi). He then proceeds to

provide the computation of the true longitude of the Moon (uncorrected). The procedure he puts forward is quite similar to the one given the *Vākyakaraṇa* by Putumana Somayāji > Vakyakarana's *Shuptacandra calculation*:

वेदेब्धज्ञानतुष्टोनदिवसात् रसगैरिकैः।
(16,00,984) (12,372)

कालानलैः देवरैश्चवाक्यं शशाङ्कजम्।
(3031) (248)

गीर्णः श्रेयादिकं ज्ञेयं क्रमाल्लब्धफलैस्त्रिभिः।

नित्यदेवसुखाधीनं यज्ञयोगसुनायकम्
(9–27–48–10) (11–7–31–1)

तनुर्विश्वसुखं सेनानारि नाथनम्
(0–27–44–6) (7–2–0–7)

तद्योगे निक्षिपेच्छुदधं विकलाद्यं ध्रूवं विघोः।।(VK.9,10,11)

Here the commentator elaborates the methodology as follows: The वेदेब्धज्ञानतुष्ट which accounts to 16,00,984 must be subrstracted from the day of interest(no. of kali days.(स्वेष्टकालविदितवारसंवादाहर्गणाद्). The difference divided by 12,372(रसगैरिकैः)and its remainder is divided by 3031(कालानलैः) and further with 248 (देवरः). The corresponding obtained at each step are, For (रसगैरिकैः),9r 27° 48' 10"; (कालानलैः),11r 7°31'1" and 0r 27°44'6" for (देवरः).This process has been clearly explained in[4].Mathematical representation:

- Initially we have to find the difference between the *ahargaṇa* and the *khaṇḍa* which is generally referred to as the *khaṇḍaśeṣa*. That is

$$\text{khaṇḍaśeṣa} = A - K.$$

- This *khaṇḍaśeṣa* is to be divided by 12372. The quotient and the remainder thus obtained are to be stored separately.

$$\frac{(A-K)}{12372} = q_1 + \frac{r_1}{12372}.$$

- Now, we have to divide the remainder (r_1) by 3031.

$$\frac{r_1}{3031} = q_2 + \frac{r_2}{3031}.$$

  Having stored the quotient (q_2) and the remainder (r_2) separately, the process has to be repeated with r_2 and the number 248. Thus we have

$$\frac{r_2}{248} = q_3 + \frac{r_3}{248}.$$

- Let D_0 be the mean longitude of the moon at the end of the *khaṇḍa* or *khaṇḍāntya*, K. This is known as the *dhruva* at the *khaṇḍāntya*. Let D_1, D_2, and D_3 denote the *dhruvas* at the end of 12372, 3031, and 248 days respectively which represent the change in the mean longitude of the moon corresponding to these *hāra*kas. Then the true longitude of moon (θ_A) corresponding to the *ahargaṇa* (A) is given by

$$\theta_A = [D_0 + (D_1 \times q_1) + (D_2 \times q_2) + (D_3 \times q_3)] + Vr_3,$$

In accordance with a well-established method, Tippa's methodology is analyzed in a systematic manner. He begins by stating that *Śuddhadinam Bṛguśuddham* must be divided by *Ajitahāraka*. This process is then iteratively repeated by successively dividing the remainder by *Jitābjārūpa*, *Kālāṅgana*, and *Devarāha*. Notably, the terms *Kālāṅgana* and *Devarāha* are well-documented and are analogous to the approach found in *Vākyakaraṇa*. The corresponding computed values for each of these parameters are systematically provided in the subsequent verses (ref. *Dhenuḥ Śrī*). Tippa also explicitly mentions that this computation is performed for *Udayacandra*. However, there are no explicit references to the subtraction of *Kali* days or the summation of the *Dhruvas* provided in the text. The table below presents the derived numerical values for these parameters in a structured manner. He has codified the values for *Ajitahāraka-hāra*, *Jitābjārūpa-hāra*, *Kālāṅga*, and *Devara*, along with their respective parameters, using the *Kaṭapayādi* system.

hārakas	**value**	***dhruvas***	**value**
अजितहारकः	128680	धीगतमिष्ठरोधिनम्	9r 22° 56’ 39’’
जिताब्जरूपः	12868	सोमसूर्याम्बर पटम्	11r 23° 17’ 57”
कालानङ्गः	3031	वज्रियागसुनायकम्	11r 07 °31’24’’
देवरः	248	सूनुर्विश्वसुखम्	0r 27°44’ 07’’

Table(2.10.1):*harakas* and *dhruvas* given in *Tantraratna*

The final set of verses presents the mean sidereal periods of the Sun across various time intervals, including daily, yearly, 100-year, 200-year, 300-year, and 600-year cycles. These periods are expressed in units of *days*, *nāḍī* (n), *vināḍī* (v), and *guruakṣaras* (g). The computed values are presented in Table(2.10.2)

The sidereal period values documented by Gopendra Tippa Bhupala exhibit a close alignment with those recorded in *Vākyakaraṇa*. The length of the sidereal year is specified as 365 days, 15 *nāḍīs*, and 31 1/4 *vināḍīs* (VK.1(2-3)). While most *Siddhāntas*, with the exception of the *Romaka Siddhānta*, adopt this value for the sidereal year, the more precise duration is calculated as: 365 days, 15 *nāḍīs*, 22 *vināḍīs*, and 52.9 *tatparas*.This minor discrepancy indicates that the first point of *Meṣa*, which these astronomers considered fixed, actually exhibits a progressive motion along the ecliptic at a rate of approximately 8.3° per annum [VK.1(2-3)].

	vākyas	**Days**	**(n)**	**(v)**	***(g)***
रवेरेकाब्दगतिः (1 year)	-	365	15	31	31
शति वार्षकी गति: (100 years)	नारी रागाद्रामशरेणाचलो नुन्न	36525	52	32	20
शतद्वयी (200 years)	अभूद्धने शिवं कामी नागस्थाने	73051	45	4	40
त्रिशती गतिः (300 years)	सत्संगात्वर्थ समुग्द्धनित्यानां	109577	44	77	-
षट्च्छती गति: (600 years)	विद्यामान्या देवकीद्ध पुत्रज्ञा	219148	15	14	-
वीरगति: (2400 years)	क्ष्मी नूनं लब्धशान्तोस्सदनेनानु	876593	0	5	-
—— गति: (4800 years)	श्रीमयो नीतिदो योगी सुस्थोयं	1773186	1	52	-

Table:2.10.2: *suryagati* for different set of periods

1.11 Conclusion

The 248 *Candravākyas* of *Tantraratna* accurately encode the true longitude of the Moon, achieving a precision of up to one second. These values closely align with those documented by *Mādhava*, demonstrating the remarkable accuracy of the *vākya* tradition. However, from a literary perspective, these *vākyas* differ significantly in structure and style from earlier texts, highlighting the flexibility and diversity inherent in the *vākya system*.These *vakyas* bear striking similarities to those of *Vararuci* and also suggest his familiarity with *Mādhava's* works.

The equation of the center, when computed using the *Parahita* system and the parameters of the *Sūryasiddhānta*, slightly deviate from the Tippa vākyas .In a few cases of discrepancies, they were identified and were effectively corrected using the error rectification technique of *Madhava*.The *Harakas, Druvas,* for the true longitude of the moon and mean sidereal periods for the sun enlisted by Tippa are also consistent with the values of *Vākyakaraṇa*. Taken together, these findings reinforce the enduring mathematical sophistication of Indian astronomical traditions and their continued relevance in precise astronomical computations.

1.12 References

[TR]: Tippa Gopendra. *Tantraratna: Chandravākyas* – Vākyas obtained from the Kerala University Manuscript Library, Trivandrum.

[Madhava]: Madhava of Sangamagrama. *Sphuṭa-Candrapti*.

[SS]: Sūryasiddhānta with Commentary of Parameśvara. Edited by Shukla, 1957.

[VK]: Kuppanna Sastri and Sarma, K.V. (eds.). *Vākyakāraṇa-Laghuprakāśikā Vyākhyā*. 1962.

[SS.e]: Burgess, E. (Ed.). *Sūryasiddhānta: A Textbook of Hindu Astronomy*. Reprinted by Calcutta University, 1935.

[1]: Venketeswara Pai, K. Ramasubramanian, M.D. Srinivas, and M.S. Sriram. *Candravākyas of Mādhava*. Gaṇita Bhāratī, Vol. 38, No. 2, pp. 111–139, December 2016.

[2]: Venketeswara Pai. *An Appraisal of Vākyakaraṇa of Parameśvara*. Indian Institute of Science Education and Research, Pune.

[3]: M.S. Sriram. *Vākya System of Astronomy*. University of Madras.

[4]:Venketeswara Pai, K. Ramasubramanian, M.S. Sriram, and M.D. Srinivas. *Karaṇapaddhati of Putumana Somayājī*. Hindustan Book Agency, 2018.

[5]: Pandu Santhoju, Punit Bhalla, Laxmidhar Behera, and R. Venketeswara Pai. *Mathematical Model as per the Algorithm of Sūryasiddhānta for Computation of Location-Specific True Position of the Planets*. J. Astrophys. Astr. (2025) 46:9.

[6]: Venketeswara Pai, K. Ramasubramanian, M.S. Sriram, and M.D. Srinivas. *An Overview of the Vākya Method for Computing the Longitudes of the Sun and the Moon*. ResearchGate2025.

Part 2 : Tāladipika

Chapter 2

Tāladipika

Throughout history, India has endured numerous invasions that have nearly eradicated a significant set of invaluable principles and techniques. Since music was primarily instructed through an oral tradition, many aspects might have naturally faded into obscurity. Moreover, ignorance and deliberate concealment continue to pose significant challenges within the artistic sphere, further complicating efforts to access and revive these traditions.*Bharata's* (500 BCE) *Nāṭyaśāstra* and *Nihśaṅka Śārṅgadeva's Saṅgītaratnākara* (13th century) have served as foundational pillars in preserving ,reviving and reshaping the existing Indian aesthetics. Music is a field that continuously evolves, with concepts that may radically change and be interpreted differently over time. This section of the thesis aims to explore the concepts presented in the *Tāladipika*, a comprehensive musical work by *Bhupala Tippa Gopendra*. The goal is to examine the system of tāla and its development during the Vijayanagara period, providing insight into how musical practices and rhythmic structures evolved during this significant era. Due to the vastness of the subject, the present study focuses on the core concepts and examines them in comparison with earlier texts.

Tāladipika is composed of three main chapters; *(Mārgatāla Vivekaḥ) (Deśītāla Vivekaḥ)*, and *(Mārgatāla Prastārādi Nirūpaṇam)*.The contents of this thesis are also presented in the respective order. Key findings include a detailed description of all the *tāla deśa prāṇas*. A distinct system of notation to represent the *tāla aṅgas*. The author challenges the existing classification system of tālas and offers a new, organized, and systematic ordering of talas in comparison to *Ratnākara*. The work catalogs a total of 223 *deśī talas* comprising a unique set of originally composed talas, incorporating *kakapāda* and *virāma*, . A simpler summary for the *prastāra* method is described along with its construction, and practicality.

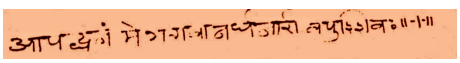
2.1 Manuscript details :

Tālaipka(Mad.Library: Trien.Cat.,1910-11 to 1912-13, R.770; Tanjore Burnell Cat.,p.60)

The *Tāladīpikā* attributed to *Bhupāla Tippa* (mid-15th century), is preserved in multiple archival sources. A copy of this work exists in the Government Oriental Manuscripts Library (GOML), Madras, cataloged as R 770, with 57 folios in Sanskrit script. A xeroxed version of this manuscript, derived from a microfilm (Film-6), was recorded on July 29, 1986. Another reference to *Tāladīpikā* is found in the personal library of Dr. R. Satyanarayana, Mysore, documented in Film 11 on August 11, 1986. The manuscript is also listed in the Mysore Oriental Research Institute (ORI), under the number 105, consisting of 11 folios.

Tāladīpikā remains unpublished, with its Sanskrit verses preserved solely in manuscript form. No existing commentaries offer interpretations, and the absence of prior editions meant that its documentation and translation had to be undertaken independently. The work references *Hanumān, Śārṅgadeva, Śailādri*, and others; however, apart from Śārṅgadeva's *Saṅgīta Ratnākara*, the other cited works have yet to be found. The foundational principles of *Saṅgīta Ratnākara* and *Nāṭyaśāstra* served as guiding frameworks, helping to reconstruct missing links. In cases where manuscript passages were unclear or illegible, parallels from *Saṅgīta Ratnākara* were drawn upon, ensuring that the essence of the work remained intact .

These translations have been undertaken to the best of my knowledge .Any inadvertent errors in the translations are solely my responsibility, and I request the reader's understanding in this regard.Some important section of these verses are provided at the beginning of the discussions.Kindly look into it for better understanding.



2.2 Abbreviations

For Primary Sources

- **[TD]** – Tippa Gopendra's *Tāladīpikā* (Page are numbers corresponding to the manuscript).textbf[TD.2] – *Tāladīpikā*, Page 2.
- **[SR]** – Śārṅgadeva's *Saṅgīta Ratnākara*.
- **[NS]** – Bharata's *Nāṭya Śāstra*.
- **[AB]** – Abhinavagupta's *Abhinavabhāratī*, a commentary on *Nāṭya Śāstra*.

For Works of Akella Mallikarjuna Sarma

- **[AK]** – General reference to his works.
  - **[AK.1]** – *Indian Genius in Tālaprastāra*.
  - **[AK.2]** – *Tālaprastāra of Saṅgīta Ratnākara*.
  - **[AK.3]** – *Systematization of Prastāra Details of Deśī Tālas*.

For Works of N. Ramanathan

- **[RN]** – General reference to his works.
  - **[RN.1]** – *Tāla-Daśa-Prāṇa and the Musicological Problems in the Present-Day Tāla System*.
  - **[RN.2]** – *The Treatment of Tāla in Saṅgīta Ratnākara, Nāṭya Śāstra, and Dattilam*.
  - **[RN.3]** – *Understanding Modern Tālas in the Background of the Ancient System of Tālas*.

Others

- **[GE]** – Girija Easwaran, *Tāla-Daśa-Prāṇa Details in Music Texts*.
- **[CM]** – M.D. Srinivas, *Combinatorial Methods in Indian Music: Pratyayas in Saṅgīta Ratnākara of Śārṅgadeva*.
- **[CK]** – Amba Kulkarni, *Recursion and Combinatorial Mathematics in Chandaśāstra*.
- **[VE]**:Evolution of the Tāla-Daśa-Prāna concept as seen in Musicological Works of the Vijayanagara Empire

2.3 Contents

मार्गतालविवेकः (I)

देशितालविवेकः (II)

मार्गदेशितालप्रस्तारादि निरूपणमम् (III)

2.4 Mārgatālavivekaha

[1] ताले घन इति प्रोक्तः कलापातलयान्वितः।
कालस्तस्य प्रमाणं हि विज्ञेयंतालयोत्कृभिः।।
या लौकिकी कला काष्ठा निमेषश्च स्मृतो बुधैः।
नसा तालकला ज्ञेया ह्यन्यैषा ताल जालका।।
निमेषकालो मात्रा स्यान्मात्रायोगात्कला स्मृता।
निमेषाः पञ्चविज्ञेया: गातकाले कलान्तरम्।।
ततः कलाकालकृतो लय इत्यभिसंज्ञितः।
त्रयोनधास्तु विज्ञेया ध्रुत मध्यविलम्बिता:।।
यस्तत्र मध्ये विलय स्तत् प्रमाणा कला
भवेत्।। कलाकालप्रमाणेन ताल इत्यभिधीयते।
इतिकण्ठत एवोक्तं मुनिना ताललक्षणम्।
अथ शैलादिहतुमच्छाब्ददेवादि संमतम्।
तावन्निरूपयिष्यामस्तालसामान्यलक्षणम्।
लघ्वादिः परिच्छेदं गदया क्रिययान्वितः।
गीतादर्थो मतिं कुर्वन् कालस्तालस्स कथ्यते।
अयं हि मातृकः पातो रागमार्गप्रयोजकः।
शम्यात्वसव्यहस्तस्य तालस्तदितर स्य च।
संनिपातो द्वयोः पञ्चशारवयोः पात उच्यते।
आसां क्रियाणामष्टानामविशिष्टा कलाभिधा।
विशिष्ट सर्वशब्दानां पातसंज्ञावि भाष्यते। [3]
ध्रुवश्चित्रो वार्तिकस्च दक्षिणश्चेति तालगाः।
मार्गाश्चत्वार एवात्र मात्रकेवलाध्रुवे। तेद्वे
चित्रे चतस्रस्ता वार्तिकेष्टो तु दक्षिणे।
पञ्चानां लघुवर्णानामुच्चारणमितात्र तु।
मात्रा स्यादनया कार्या लघ्वादिपरिकल्पना।
अष्टौ मात्राध्रुवे सर्पिण्यपिकृष्याथ पद्मिनी।
विसर्जिताच विक्षिप्ता पताका पतितापि च।
छोटिकाशब्द योगाश्च वामतागगमादपि।
दक्षिणाद्देशगमनादधो भागगमादपि।
बहियांनाज्ञानाद प्यूवेगाने पातनात्।

[2] यतोस्तलप्रतिष्ठामि यामत्यस्मात्प्रत्यये
पन्ति। पुरोधिकारार्थेत् तालशब्दो व्यवस्थितः।
तस्मिन् हि गीतं नृत्तं च वाद्यं चापि प्रतिष्ठितम्।
सोयं मार्गगतो देशी गतश्चेति दिथा मतः।
तत्रादिमस्य तालस्य क्रियाद्वेथानिगद्यते।
निश्शब्दाच सशब्दतदः चेत्याद्यातत्र चतुविधा।
आवापनिष्काम विक्षेप प्रवेशा इति च क्रमात्।
सशब्दापि चतुर्धा स्यात्द्रुवश्शम्याच तालकः।
संनिपातश्चेत्यथैषामष्टानां लक्ष्म चक्ष्महे।।
तत्रावापः करस्योत्वं तालस्याङ्गुलि कुञ्चनम्।
निष्क्रामोन्यत्तलस्यास्य पञ्चशाखाप्रसारणम्।
विक्षेपो दक्षिणस्योर्ध्वतलस्य प्रसृताङ्गुलैः।
क्षेपप्रवेशस्तस्याथस्तलस्याङ्गुलि कुञ्चनम्।
ध्रुवस्तुवामपाणेस्तु च्छष्टिकाध्वनिमेदुरम्।
लक्षितापातेन योज्यातावापकादिषु।
असारितादौ नियत मत्युक्ता। जातिषु।
आसां प्रयोग मष्टानां मात्राणां। मन्यते
बुधाः। कनिष्ठासारित कार्य। ध्रुवपातादूपोह
ने। विशेषपाका चनामार्गे व्यङ्गीकृते। ध्रुवे।
ध्रुवकादिभिरष्टभिर्मुख्योन्ततिः:परे विदुः।
धुवाधुवे स्यादधन्ते चित्रे स्यातामचादि मे।
देवातिकेन्तिमे च द्वे क्रमेणादि पतितांगे।
परम्पाता दिक्षिता त्पपतिताप्लुते। निशब्दे
सपिंणीकृष्या पताकपतिता स्मृता। [4]मुनि
प्रसिद्धान्मार्गां स्त्रीन्कथयामास कण्ठतः।
अगूढवचतुर्थं तु द्रुतेस्तु विरलत्वतः। ध्रुवस्तु
मात्रिकःपात इति केवलमात्रिका:।वदताम्मुनिनां
मार्गस्तादृशः सूचितोः ध्रुव।अथ चित्रतमं

षष्ठमिति चित्रतमं ब्रुवन्। अगूढवचतुर्थं तु द्रुतेस्तु विरलत्वतः। ध्रुवस्तु मात्रिकःपात इति केवलमात्रिका। वदताम्मुनिनां मार्गस्तादृशः सूचितोः ध्रुव। अथ चित्रतमं षष्ठमिति चित्रतमं ब्रुवन्। षोढामार्गान्मुनिः प्राह प्राक्सीमोल्लङ्घना पथा। तस्मान्मार्गचतुष्कं तु मुनिना किलमोदितम्। चूडामणेः प्रबन्धे तु मुक्तामार्गध्रुवं ततः। दक्षिणं वार्तिकं चित्रं तथाचित्रतरं पुनः। सपञ्चमे कलां बिन्दुं षष्ठे सार्धं कलां जगौ। नैतच्चमत्कृतिकरं मत्वा पञ्चमषष्ठयोः। मदकादिषुप्रयुक्तो मार्गे तालोपजीविषु। मार्गतालाक्षरोन्नीत्या मार्गेष्वपि कलाविधेः। बिन्दोरिन्दोरिवामायां पञ्चताल्यामदर्शनात्। प्रस्थारस्तायितावश्यं भवाम्मार्गकलात्मनाम्। बिन्दोस्तारे तु निष्पत्तेर्देश्येकविनियोजनात्।

क्वापिकोणेभवान् सद्धिमित्युपेयात्तथापिच। विच्छेदकाकुसुमद्राया एवस्यात्पादघोषणा। नतुमात्रान्तराकारमधिकत्य प्रवतते। यदि मात्रान्तरम् च हि तालाङ्गत्वाद्विशेषतः। अपास्य द्रतमस्यै स्यान्तराञ्चलगामिताः। द्रुतमप्यधिक्रत्यान्यः प्रस्तारस्स्यादधस्तनः। खण्डतालाक्षरेष्वस्य स्वायतन्त्र्यम् स्यात्पृथक स्थितिः। लघव्यञ्जनशेषत्वम् गत्वैवम् श्रुयते यति। [6] तद्विरम्य कचिद्गत्यै दनयोरल्यकालयो। कलिता काकुमुद्देषा बहुकालौ गयौ विना। मार्गे देश्याविवेकेज तालानुक्रमणीकृतः। महंतस्य महाबुद्धेरस्तग बुधमोहनम्। चतुरश्रस्त्र्यश्र इति मार्गतालोमतो द्विधा। आद्यश्च चत्पुटाख्योत्र परश्चाचपुटाभधः। त्रश्र भेदः पुनश्चान्यषड्विता पुत्रकाइचयः। षड्विता पुत्रकस्यैव पर्यायद्वयमन्जसा।

गतोन्निपञ्चमोगर्भस्रावेणेव तथान्तिमः। अध्वद्रुताभिधानायाः कलायाः काप्यदर्शनात्। तस्मात् द्रुवादींश्चतुरो मार्गान्मानयतो मुनेः। [5] शलादराञ्जनेयस्य शाङ्गस्याप मतं स्थितम्। नन्वद्विद्रुतमात्रापि खण्डतालेषु लक्षिता। विरामापर्याया व्याहस्तेन गच्छति। यद्यप्येषा चतर्मेरुप्रस्तारेनैव दृश्यते। तथा दुष्टावैरामप्रस्तारे पादमात्रिका। एतदाचार्यपादीया ननुमन्यन्त एव हि। नाङ्गीकुर्यात्कपीन्द्रो वा तालतन्त्रप्रसाधकः। अपि द्रुतार्ध मिष्टम् चेदतिसूक्ष्मदृशः कवः। ईदं च गणनीयं स्यात्तालाङ्गानुक्रमस्थले। जाजीगणश्च हनमानन्यद्दलगपान्विता। कत्राप्यसिध्धिर्वै रामाकारं प्रस्तार विस्त्रूतेः।

उत्तरः पञ्चपाणिरचेत्यूचिते भरता नुगाः। संपदेष्टाक इत्यन्यषड्विंतापुत्रकोद्भवः। उद्धड्डु इति तालोन्य उच्यते त्र्यश्रसं भवः। पञ्चमे मार्गतालाः स्युः भरतेन परिष्कृता। यथाक्षरः प्राग्द्विकलस्ततः पश्चाच्चतुष्कलः। इति प्रत्येकमेतेषामिष्यते हि विधात्रयम्। यथाक्षरस्त भवेत्तत्तनामगतैर्गलैः। अयमेवैक कला श्ताचांचैरभिधीयते। प्रतिज्ञातं प्लुतंवञ्चत्पुटे त्वान्यमक्षरम्। विधायविहितस्सोयं षपितापुत्रकाभिधः। स्वातंन्त्र्यमस्य तालस्य स्वाधीनप्लुतयोगतः। [7] तत्संख्याननिदेश्यतं कलादेद्विलादिके। यथाक्षरषट् पितापुत्रको यथा - द। टट । द । प्लुतयोरन्तरे लघ्वोर्गुरू कृत्या गुरुत्रये।

निष्पन्ने तूत्तराज्यातस्संपकेष्टाकसंज्ञकः। विधायविहितस्सोयं षपितापुत्रकाभिधः। स्वातंन्त्र्यमस्य तालस्य स्वाधीनप्लुतयोगतः। तत्संख्यानतिदेश्यतं कलादेद्विलादिके। यथाक्षरषट् पितापुत्रको यथा - द। टट । द । प्लुतयोरन्तरे लघ्वोर्गुरू कृत्या गुरुत्रये। निष्पन्ने तूत्तराज्यातस्संपकेष्टाकसंज्ञकः। यथाक्षरसंपदेष्टाको यथा-द टटटद-। गुर्वोर्मध्यजुषस्त्र्यश्रे लद्वयस्य गुरुकृतेः। गुरुत्रयात्मकस्ताल उद्धट्टः उपकल्पितः। यथाक्षरोद्धहो यथा टटट। एतेषु तालेष्वनन्तेषु द्विकलत्वे च कीर्तिते। लपाद्याङ्गौ लघुयाङ्गः प्लुताद्यां च गुरुत्रयम्। संपाद्य सर्वगुरुतां नेयः पूर्वं यथाक्षरः। प्रकृतित्येवान्त उद्धट्टः सर्वगुर्वात्मकः खलु। द्विकलो हिपरिग्रात्यो गुरुरेव कलात्मना।

चतसः ताः करलाः पात भागश्चेत्सचतुष्कलः। यथाक्षरगता मात्राः प्रापितास्ताः कलात्मताम्। चतुर्गुणत्वं गच्छन्ति यत्रसो यंचतुष्कलः। चतुष्कले षोडशाद्ये द्वाशस्युरुपाधके। कलाश्चतुर्विंशतिस्तृतीये योनिदन्त्ययोः। चतुष्कलचञ्चत्पुये यथा टटटट टटटट टटटट.टटटट। चतुष्कला चाचपुटो यथा टटटट टटटट टटटट. चतुष्कलषद्वितापुत्रकोथथा टटटट टटटट- टटटट टटटट टटटट टटटट। चतुष्कलसंपकेष्टाको यथा टटटट टटटट टटटट टटटट टटटट। चतुष्कलोद्धट्टो यथा टटट टटटट टटटट। चतुष्कलोकयः पातभागे गुरुचतुष्टयात्।

ते द्वे कलेपात भागो यत्रासौ द्विकलो मतः। मात्रा यथाक्षरे सिद्धा गमितास्तु कलात्मताम्। भजन्ति यत्र द्वैगुण्यं सोयं द्विकल उच्चते। [8]चञ्चपुटे ष्टौ द्विकले कलाश्चाचपुटे तु षट्। पञ्चवाणौ द्वादश संपतेष्टाकेपियोनिवत्। षडुद्धट्टाभिधे ताले स्वयोन्यनुविशारदः। द्विकलचञ्चत्पुटरो यथा टट. टट. टट. टट। हिकलचाचपुटो यथा टट टट टट। द्विकन षट् पितापुत्र कोयय टट टट-टट. टट. टट. टट। द्विकलसंपकेष्टाको यथा टट. टट टट. टट. टट. टट। द्विकलोद्धट्टो यथा टट. टट. टट. चतुष्कलचिकीर्षायामपि गर्वगुरुकृतिः। कर्तव्या त्रिकालवेन स्वीकृतिः स्यामुरोरपि।

तच्चतुष्कोणमात्रैका मद्रकादिषु गीतिषु। चतुष्कलात्परं चान्यत् त्र्यश्रभेदत्रयं मतम्। द्विगुणद्विगुणलेन यावत् षण्णवतिक्रमात्। गुरवः-24 टटटटटटटटटटटटटटटटटटटटटटटट इति . गुरवः 48। इति. 48. गुरवः 96। अन्यत्रधर्ममन्यास्तु दिगुणद्विगुण क्रमात्। षड् भेदान् संगिरन्ते हि समारभ्य गुरुत्रयम्। अथ चञ्चत्पुटादीनां वच्मः पातकलाविधिम्। विशर्द्धेरध्रुनैःपातकलायोगो यथाक्षरे त्रिभिरेतैस्स। निष्क्राम प्रवेशै द्विकले तुसः। त्रिभिरेतैस्मनिष्क्रामप्रवेशैर्द्विकलेतुसः। क्रिभिस्सशब्दैश्चतुष्कैश्तुर्भिश्च चतुष्कले। [9]

2.4.1 Tāla:

Music is merely a play in time. Given that time is continuous rather than discrete, it must be confined within a defined structure to enable a coherent and perceivable musical experience. *Tāla* is one such a systematic construct that allows for the measurement and organization of time. Certain regular, periodic actions are employed to conceptualize a standard reference of time, which is then used to measure all other irregular or varying periods.Thus the Tāla is primarily based on two components , the time *(kāla)* and actions*(kriyās)*.

Various approaches to concept of *tāla*:

वाहं तु यद् घनं प्रोकं कलावादलयात्मनम्।
कालस्तस्य प्रमाणं हि निन्यं तालनियोगतः।। (NS 31.1)

तालस्तलप्रतिष्ठायामिति धातोः श्रेयांसि स्मृतः।
गीतं वाद्यं तथा नृतं यत्तस्माले प्रतिष्ठितम्।। (SR 5.2)

कालो लघुगुरूत्वमितया क्रियया संमितो निमितम्।
गीतादिव्यवधूतु ताः।। ...।। (SR 5.3 abc)

तस्य प्रमाणं यद्बाला ताले हन्त तस्य विभिच्छेदः।
श्रोत्रसमागतलेख्स्य यद्विकल्पं विभागस्त्र चन्द्रतुर्यादिदिवभागा
एव निमित्तं इति। (AB. Vol. IV, p. 150, l1, 17-18.)

At first notice, one can ascertain that all these definitions use variable terminologies. Terms such as kāla, kalā, kriyā, and other concepts related to instruments, dance, and different artistic mediums are observed. Bharata, in his *Nāṭyaśāstra*, defines the concept of *tāla* in relation to the *ghana-vādya* (instruments such as cymbals, mridangams, etc.). However, it must be understood that *tāla* is not limited to these instruments, but is rather employed as a framework to elucidate the concept itself.He employs the terms *kāla* and *kalā* (which will be dealt with in further sections). It must be noted here that the concept of*kalā* is considered as a representative of time- action and it deviates from present-day understanding.

Tippa Gopendra's introductory verses (up to *kalā bhavath* TD.1) are a direct *extractions* from Bharata's *Nāṭyaśāstra*, indicating the alignment of Tippa's ideologies. Identifying these verses as *excerpts* from Bharata's *Nāṭyaśāstra* has played a crucial role in the interpretation of the text. Sārangadeva, the author of the *Saṅgīta Rātnākara*, employs a rather different approach. He states that *tāla* is the foundation upon which *gīta* (melody), *vādyā* (instrumental music), and *nṛtta* (pure dance) depend. He primarily uses two terms, *kāla* and *kriyā*, to define the structure.

Though various approaches have been employed to instruct the concept of *tāla*, a rigorous comprehension necessitates an understanding of its fundamental components. As discussed, *kriyā*, *kāla*, and *kalā* are some of the components. This section systematically examines the ten primary elements collectively termed the *Tāla Deśa Prāṇas* which define the nature of the tāla.

2.4.2 Tāla Deśa Prāṇas

The term *Tāladaśaprāṇa* literally translates to the "ten vital elements of *Tāla*." The ten elements are: Kāla, Mārga, Kriyā, Aṅga, Graha, Jāti, Kalā, Laya, Yati, and Prastāra.
Arthi Rao, in her in-depth study of musicological developments during the Vijayanagara period, asserts that *Tippa's Tāladīpikā* was the first treatise to comprehensively define all these elements. Some scholars, however, argue that the concept can be traced back to Bharata's *Nāṭyaśāstra* [BR], albeit in an implicit or codified form. It is widely agreed that all these elements were discussed under one name only in the medieval period. Subsequent texts, such as *Tālakalābhī* and *Saṅgītasūryodaya*, both composed during the Vijayanagara era, explicitly employ this terminology. Tippa discusses nearly all of the *Prāṇas* in the first chapter, except for *Prastāra*, which is treated as an independent chapter. However, the author does not explicitly mention the term *Tāla Deśa Prāṇas*.References for the tāla prānas , *Kāla*[TD.3,15], *Mārga*[TD.3,4], *Kriyā*[TD.19], *Aṅga*[TD.3,5,17], *Graha*,[TD.15,16], *Jāti*[TD.6,14], *Kalā*[TD.6,9], *Laya*[TD.15], *Yati*[TD.15]

Note: These terminologies have undergone multiple transformations over time, reflecting the evolving nature of the musical tradition. A significant challenge arises when identical terms are used with vastly different meanings, often leading to misconceptions. The linguistic evolution of these terminologies could be attributed to the continuous development of language itself. From *Bharata* in 500 BCE to *Ratnākara, Bhoja, Tippa,* and contemporary scholars, language has undoubtedly evolved and hence has played a crucial role in shaping and refining the system.The present explanation of the *deśa prāṇas* considers all possible meanings of the terminologies, tracing their evolution.The order followed is not in the sequential list of *prānas*. This approach has been intentionally adopted to better accommodate the interdependent nature of the elements involved.

Kāla, Kriya and Kalā

Kāla, or time, serves as the fundamental parameter of the *Tāla* system. The perception of time within this framework is intrinsically linked to a sequence of ordered actions or events. Rather than being an absolute entity, time is conceptualized as the interval between successive *Kriyās* (actions) [RN]. The measurement and regulation of these temporal intervals necessitate a standardized reference unit, ensuring consistency and coherence in rhythmic execution. There exist two principal schools of thought regarding the standardization of reference time units in the *Tāla* system.

(a) The first approach establishes a fundamental unit called 'Mātra,' which is defined as the time required to articulate five 'Laghu Akṣara-s' . This *Mātra* serves as the standardized *Kāla* reference for measuring temporal intervals.

(b) The second approach, in contrast, does not adhere to a fixed *Mātra* standard, recognizing that the duration of a syllable can vary from person to person. Instead, this model begins with *Kṣaṇa* as the smallest time unit and progresses hierarchically through *Lava*, *Kāṣṭhā*, *Nimēṣa*, *Kalā*, and *Truṭi*, ultimately transitioning into the musical time units such as *Aṇudruta*, *Druta*, *Laghu*, and *Guru*.

The *Tāladīpikā* of Tippa Gopendra aligns with the theoretical framework established by Bharata and Śārṅgadeva. Like these predecessors, he adheres to the first approach, which employs a standardized unit—*Mātrā*—as the fundamental measure for defining *Kāla*.[TD(1,,3)][SR.5].
N. Ramanathan, a pioneering musicologist, in his study *Problems with the Present Tāla System*, highlights the issue of the absence of standardized time units in contemporary musical traditions. The tempo of a musical composition does not necessarily correspond to the speed of the *Tāla*. A song rendered in a fast tempo may not always require or imply a rapidly paced *Tāla*, and vice versa. While the speed of melodic composition is often accounted for, the independent regulation of *Tāla* speed has largely been overlooked in modern practice.Present measures of using instrumentations like metronomes have initiated the revival of conscious specification of the kāla of the tāla. However, this does not imply that ancient musicologists neglected the concept of *Kāla*; on the contrary, it was recognized as one of the most important elements of music. Given that musical traditions were predominantly transmitted through oral instruction, it is highly plausible that precise indications of *kāla* may have been lost over time.
One thing to keep in mind is that the concept of measuring time (*kāla*) primarily requires

kriyā (action). This term is not found in *Nāṭyaśāstra*, *Dattilam*, or *Bṛhaddeśī.* Instead, actions were classified as *kalā* or *pāta* based on whether they produced sound. The term *kriyā* was first introduced by Abhinavagupta, a commentator on *Nāṭyaśāstra* [RN]. Following his work, explicit usage of this terminology is observed in Śārṅgadeva's *Saṅgītaratnākara*.*Kriyās*(actions) are classified into two broad categories:

- **Saśabda Kriyā-s** (actions with sound): Dhruva, Śamyā, Tāla, Sannipāta
- **Niśabda Kriyā-s** (actions without sound): Āvāpa, Niṣkrāma, Vikṣēpa, Pravēśa

In addition to these classes, some *kriyās* are defined for specific applications, including Dhruvakā, Sarpiṇī, Kṛṣṇā, Padminī, Visarjitā, Vikṣiptā, Patāka, and Patitā. *Sāṅgīta Ratnakara* also addresses the dimensions of these kriyas. *Laghu*, *guru*, and *pluta*, which are clubbed under one term, *lagvādi* [RN].

The treatment of actions in *Tāladīpikā* may appear to exhibit textual inconsistency. In its introductory sections, where the text closely follows *Nāṭyaśāstra*, the term *kalā* is employed, whereas in later sections, the term *kriyā* appears. However,this does not necessarily indicate ambiguity. The key distinction lies in understanding that *kriyā* and *kalā* are not synonymous[AK]. When *kriyā* is used, it strictly denotes an action, while *kalā* refers to time-action. Mukund Lath quotes " kalā was thus a unified notion both indicative of the actions which manifested the time measure in tāla and the units forming the time measure itself.The two aspects of forming tala are in fact inseperable.The duration of kalā is equivalent to that of guru.[SR 5,20]. A comparable distinction in the use of these terminologies is also observed in *Saṅgītaratnākara*[SR 5.6c, RN], further reinforcing the need for precise contextual interpretation.

Mārga,Aṅga and Jāti

Greater multiples of this fundamental time kāla unit give scope for various rhythmic variations. Within this framework or the temporal arena, different combinations are systematically created and explored. The different modes or paths of playing in this arena is termed as the Mārga. Consider an action that can be performed over various time periods. If this action(kriyā) follows a path that takes 1 mātrā (standard unit time), it corresponds to one kind of mārga , if it takes 2 mātras it classifies a diffrent mārga and so on.While defining the mārga the time-action reference is used(kalā). Tippa Gopendra Bhūpāla mentions two systems of classification. The first categorization is as follows: *Dakṣiṇa* :*kalā* = 8 *mātras*, *Vārtika*: *kalā* = 4 *mātras*, *Citra* :*kalā* = 2 *mātras*, and *Dhruva* :*kalā* = 1 *mātra*. In the second mode of classification, the divisions are *Dakṣiṇa*, *Vārtika*, *Citra*, *Citratara* (1 *mātra*), *Citratama* (½ *mātra*), and *Aticitratama* (*druta*). In contrast, *Saṅgīta Ratnākara* only mentions the first mode of classification.The concept of *mārga*, once central to rhythm and structure in classical music, has largely faded from contemporary usage. While Sambamurti (1963) describes it as a framework for maintaining rhythmic accuracy and notation consistency, its interpretation has evolved over time[RN].The modern perspective aligns closely with *Gati* or *Naḍai*, emphasizing tempo variations rather than its original structural significance. Additionally, later texts introduce the term *Kaḷai* to define the subdivisions, reflecting a shift in how rhythmic structuring is understood and applied in music practice presently.

Aṅga or 'a part' , refers to the subdivisions within a kāla.Elements such as *druta*, *laghu*, *guru*, *pluta*, *kākapāda*, and *virāma*, are categorized as *aṅgas* in *Tāladīpikā*. While *Saṅgīta Ratnākara* includes most of these rhythmic units, it does not explicitly use the term *aṅga*. In fact, the term *aṅga* is absent in earlier works and appears only in texts that elaborate on the *tāla-daśa-prāṇas*, marking a conceptual additional to the *prānas*.The present-day concept of *Jāti* also emerges only in later texts that discuss the *Deśa Prāṇas*. In *Saṅgīta Ratnākara*, the rhythmic structures *Caturaṣra*, *Tryaṣra*, *Khaṇḍa*, *Miṣra*, and *Saṅkīrṇa* are defined in the context of *Mārga Tālas*, but the term *Jāti* is not employed. However, *Tāladīpikā* explicitly uses the term *Jāti* while maintaining a similar classification.

Graha,Laya and Yati

The term *graha*, denotes ”grahanam” meaning to grasp or catch. This term is not found in Bharata's *Nāṭyaśāstra*, The term *pāṇi* was used instead [RN]. In most of texts, the term *graha* denotes the alignment, or lack thereof, between the start of the *tāla* and the music. When both the *tāla* and music begin simultaneously, this alignment is referred to as ”*sama*”. If the music starts before or after the *tāla*, the *graha* is classified as either ”*anāgata*” or ”*atīta*”, depending on which begins first. *Taladīpika* follows *Saṅgītaratnākara* and describes the *Grahās Sama*, *Atīta*, and *Anāgata* with respect to the lag between the commencement of *Tāla* and commencement of *Gīta/Vādya/Nṛtya*.

Laya is defined as the period of rest following a *kriyā*. In terms of its amplitude, it is categorized into three types: *Druta*, *Madhya*, and *Vilambita*. This terminology has remained consistent in all previous texts, and its meaning has remained unchanged in contemporary usage.

The manner of change in *laya* is termed as *yati*. This term has been in use since the time of the *Gāndharva* system. The change in *laya* can be observed in two ways: one within a *tāla* and the other between two renderings of a *tāla*. *Tāladīpikā* follows *Saṅgīta Ratnākara* and classifies *yati*-s into: *Sama*, *Srōtōgata*, and *Gōpuccha*.

Understanding the various meanings and the emergence of *Deśa Prāṇas* provides deeper insight into their underlying principles. Time is both confined and defined, establishing a fixed reference. The segmentation of time into measurable units (*aṅgas*) forms the basis for structuring rhythmic patterns (*jātis*). Specific actions (*kriyās*) are employed to measure time, while the intervals between these actions determine *laya*. Variations in *laya* manifest as different *yatis*. The combination of time and actions yields measurable unit (*kalā*). Manipulating these temporal divisions (kāla)gives rise to different *mārgas*, while diverse methods of initiating and developing musical phrases define various *grāhas*.This systematic structuring is essential for developing an intuitive understanding of time.Organizing these characteristic (*prāṇas*) into a cohesive framework under single board has significantly simplified the comprehension, classification, and practical application of *tālas*.It is a fair possibility that *Tippa Gopendra Bhūpāla* was among the first to formalize and integrate these elements into a unified format.His contribution has been pivotal in shaping many present-day rhythmic curations.

2.4.3 Mārga Tālas

Post the descriptions of some the fundamental components , *Tippa* proceeds to elaborate on the *Mārga Tālas*, detailing their structural characteristics and principles of amplification.[TD.6-13]. The initial divisions of *Caturaśra* and *Tiśra* are introduced, followed by a subsequent classification into the following subtypes: *Caccapuṭa*,*Cācāpuṭa*, *Ṣaṭpitāputra*, *Saṁpakveṣṭaka*, and *Udghaṭṭa*, ,where the first belongs to the Caturasra category, while the rest are classified under Tiśra. This classification is not an innovation but rather a continuation of an established tradition. Nearly all post-*Bharata* authors have adopted and reinforced this scheme in their treatises.

It is to be noted that, up to this point in *Tāladīpikā*, no formalized symbolic encoding system had been introduced. This section marks the initial use of distinct rhythmic symbols—ट, द,। or letters to denote specific elements.Bharata, in his *Nāṭyaśāstra*, does not employ any coded notation for the explication of Tālas, relying instead on descriptive prose. In contrast, *Saṅgītaratnākara* systematically incorporates a structured symbolic framework to represent *Tāla aṅgās*. Given these methodological differences, the most effective approach to deciphering the notation used in *Tāladīpikā* is by comparative analysis. *Ratnākara* has been thoroughly studied and revived, and the *aṅgas* of the tālas are easily accessible. Table 2.4.3(1) presents the *Mārga tālas* and their corresponding *aṅgas* documented in *Tāladīpikā* and *Saṅgīta Ratnākara*. Except for tālas *Caccapuṭa* and *Cācāpuṭa*, the rest exhibit similar structures. (s = ट, । = ।,ś = द), a .These codes are representatives for guru(G),laghu(L) and pluta(P) respectively.

Mārga Tālas	***Tāladīpikā***	***Ratnākara***
Caccapuṭa	टटटट।द्	s s।ś
Cācāpuṭa	टट।।ट	s।। s
Ṣaṭpitāputra	द्।टट।द्	ś।ss।ś
Saṁpakveṣṭaka	द्टटटद्	ś sss ś
Udghaṭṭa	टटट	sss

Table2.4.3(1):Comparison of Mārga Tālas in *Tāladīpikā* and *Saṅgītaratnākara*

*Ratnākara*and *Tāladīpikā* provide explanations for these tālas but explicitly mention only certain *aṅgas*, leaving the rest encoded. yet they only specify certain aṅgas, The missing elements must be deciphered through a careful analysis of the encoded structure. Given that numerous texts align with the structural framework established in *Saṅgītaratnākara*, Tippa largely adheres to the same explanatory model as *Ratnākara*, albeit with an erroneous code. This discrepancy warrants rectification to ensure conformity with the *Saṅgītaratnākara* format. Moreover, in the treatment of amplification of these tālas, *Tippa* accurately employs the appropriate notation, further corroborating the presence of a typographical error in the tālas concerning *Caccapuṭa* and *Cācāpuṭa*.

Amplification:

The tālas are subjected to amplification of different orders, which includes *dvikāla* , *chatuśkāla*, and further divisions. Amplification, in this context, refers to the extension of time units. For instance, in the case of dvikāla, the total time (kāla) is doubled. In order to facilitate smoother or easier transitions, all aṅgas are unified into a single unit. Since the entire system is measured in terms of kalā, where the unit of kalā is equivalent to that of a guru [RN], each aṅga is converted into the corresponding number of gurus for the amplification process.The process of converting elements into the guru format is a common practice, found in texts such as *Ratnākara*, *Nāṭanānirṇaya*, *Saṅgītasūryodayā*, and others.

(Note , (L=2D , G=4D , P=6D, if D=1 ; L=2, G=4, P=6))

Dvikāla Caccapuṭa:

Ekakāla format:

टट।द or $GGLP$

Converting into gurus:

$$G + G + L + (L + G) = G + G + (L + L) + G$$

$$= G + G + G + G = (G\ G\ G\ G)$$

Doubling:

$$GG\ GG\ GG\ GG$$

Catuśkāla Caccapuṭa:

Ekakāla format in guru:

G G G G

Four times:

GGGG GGGG GGGG GGGG

Similarly ,

Marga tālas	**Kkakāla**	**Dvikāla**	**Chatuśkāla**
Caccapuṭa:	GGLP (GGGG)	GG GG GG GG	GGGG GGGG GGGG GGGG
Cācāpuṭa	GLLG (GGG)	GG GG GG	GGGG GGGG GGGG
Udghaṭṭa	GGG	“	“
Ṣaṭpitāputra	PLGGLP (GGGGGG)	GG GG GG GG GG GG	GGGG GGGG GGGG GGGG GGGG GGGG
Saṁpakveṣṭaka	PGGGP (GGGGGG)	“	“

As observed, the *Dvikāla* and *Chatuśkāla* patterns of certain *tālas* may appear identical. However, the distinguishing factor between these two is rooted in the execution . It is important to note that these *tālas* are performed with specific *kriyas* represented by the letters *G*, *L*, and *P*. To precisely define each *Anga* of a Tala and its corresponding mode of action, the author employs the *Pātakalā Vidhi*. As previously discussed, the terms *pāta* and *kalā* (in their earlier contexts) were used before the introduction of *kriyas*, referring to the *shaśabdha* (sound-producing) and *niśabdha* (silent) kriyas . This segment introduces an alternative method of encryption, similar to the system outlined in the *Ratnakara*. The methodology is elucidated through some examples.

In Ṣaṭpitāputra

ekakāla: संताशताशता

dvikāla: निप्र तास निता निश ताप्र निसम्

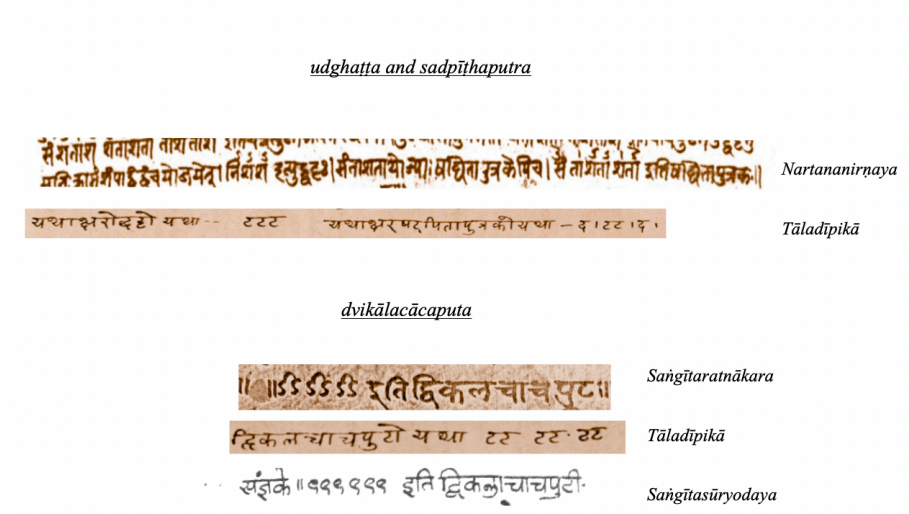
chatuśkāla: आनिविप्र आताविश आनिविता आनिविस आताविप्र आनिविसम्

These terms may initially seem arbitrary or devoid of meaning. However, upon closer examination, it becomes evident that specific letters are designated for *śāṣabdha* (sound-producing) actions and *niśabdha* (non-sound-producing) actions.In particular, the letters (स,श,त) are classified as *śāṣabdha*, whereas all remaining letters denote *niśabdha* actions.(*śāṣabdha kriyās*: superscript ^{s},*niṣabdha kriyās*: superscript ^{ns}). For instance, the *Ēkakāla* of this *Tāla* is represented by *PLGGLP*, comprising six *aṅgās*. Correspondingly, the

code consists of six letters: (सं ता श ता श ता), all of which are classified as *śāṣabdha*. This signifies that the execution of these *aṅgās* requires sound-producing actions.. Thus, the final notation $P^sL^sG^sG^sL^sP^s$. For dvikāla GG GG GG GG GG GG, the respective code given is (निप्र तास निता निश ताप्र निसम्).The final representation ,$G^nG^nG^sG^sG^nG^sG^nG^sG^sG^nG^nG^s$.For chatuśkāla, आनिविप्र आताविश आनिविता आनिविस आताविप्र आनिविसम् , the final representation is $G^nG^nG^nG^n$ $G^nG^sG^nG^s$ $G^nG^nG^nG^s$ $G^nG^nG^nG^s$ $G^nG^sG^nG^n$ $G^nG^nG^nG^s$

In **Saṁpakveṣṭaka** the dvikāla is represented by (निप्र निशा निता निश ताप्र निसम्), giving $G^nG^nG^nG^sG^nG^sG^nG^sG^sG^nG^nG^s$.This illustrates that although the aṅgas assigned to these tālas are identical, the kriyās (mode of action) serve as their distinguishing factor. Additionally, it is noteworthy that the number of *śābda kriyās* remains constant, matching that in *ekakāla*, while the amplification occurs through *niṣabda kriyās*.

One of the interesting observations when comparing the *mārga Tāla* notation schemes across various texts is the variety of letters used for the notation. Some systems adhere strictly to what *Rātnākara* introduced, while others introduce their own letter codes. The present-day *aṅga* notation precisely follows that given by *Rātnākara*. However, an intriguing question arises: how is it that texts like *Tāladīpikā* and *Saṅgītasūryodayā*,both of which cite *Rātnākara* and largely follow his principles have their own unique letter systems for the *aṅga* notation?It is important to note here that *Sārāṅgadeva*, hailing from the *Kashmir* region, is distantly separated geographically from the authors of these southern texts. Much of this traditional knowledge system was passed down orally, and yet, remarkably, the principles and concepts were preserved with accuracy across such vast geographical distances.One possible hypothesis for the variation in the notation schemes is that the availability of written sources of *Rātnākara* was limited, and thus the authors texts may not have had access to his written works. It is possible that the written *aṅga* notation was not known to them. Another plausible explanation is the linguistic diversity of the available manuscripts in the present time. For example, the *Tāladīpikā* exists in both *Telugu* and *Sanskrit* versions. Due to translation processes, it is possible that the notations were preserved in their original form even with language variations.



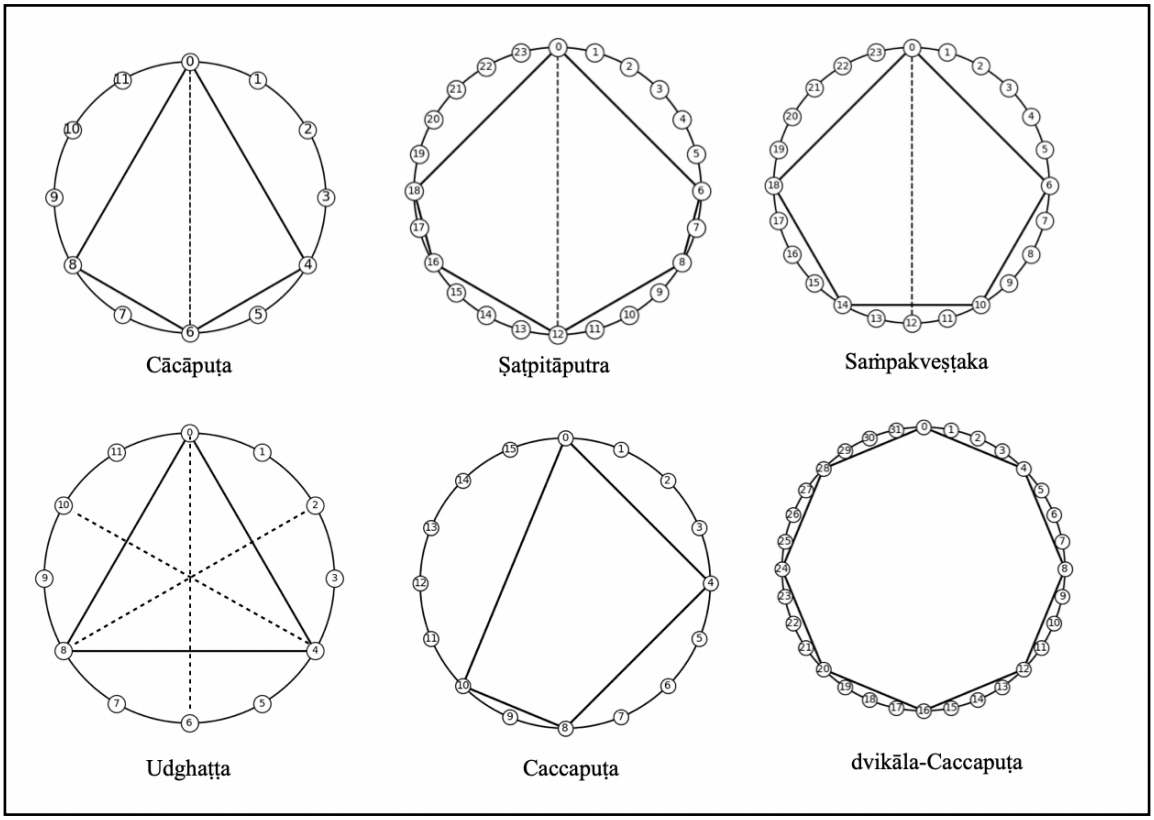
Mārga tālas from various sources

Comparing Marga tālas with present day tālas

With the exception of *Caccapuṭa*, all the other Mārga Tālas exhibit palindromic structures. This implies that these sequences possess mirror symmetry about a central point in time, such that they remain identical when read both forward and backward. In contemporary times , such structures are referred to as *palindromic rhythms* or *non-retrogradable rhythms*, as their mere reversal does not yield a new rhythmic sequence[geo]. Symmetric rhythms have historically been favored due to their inherent balance and structural stability. Their predictability makes them highly suitable for various artistic and performative applications, particularly in poetry, dance, and music compositions. According to Bharata's *Nāṭyaśāstra*, all forms of art were inherently interdependent, seamlessly blending into theatrical dance and musical performances. However, not all rhythmic structures are conducive to dance or aurally pleasing. As a result, most mārga tālas that exhibit symmetry were likely curated to ensure their practical applicability in performance contexts. In contrast, in the present scenario, not all art forms are necessarily performed together. While fundamental rhythmic structures remain predominant in dance and poetry, music and its instrumentals have evolved to incorporate a diverse range of rhythmic patterns, extending beyond the mārga framework.

Fig. 2.4.3(2) illustrates the geometric representation of the Mārga and present-day Tālas. This diagrammatic representation is influenced by the work of *Godfried T. Toussaint*, who developed similar representations for Western classical music.The Aṅga time measures are assigned as follows: D = 1, L = 2, G = 4, and P = 6 units, respectively for mārga tālas. An important thing to note is that the values of Laghu in time units are fixed in the case of Mārga Tālas, whereas present-day Tālas have variable values (3, 4, 5, 7, 9).Originally, Laghu, Guru, Pluta, and smaller units like Druta represented single time units created by one Kriyā, followed by rests. However, over time, Laghu and Druta have come to represent groups of time units instead of individual ones. Today, Druta consists of two time units with two Kriyās, while Laghu generally consists of four time units and four Kriyās. This shift has led to a divergence from their original meanings.[RN] Most of the Tālas represented here have the value of Laghu set to four. On comparing Tripuṭa Tāla with a Laghu value of 4 and Laghu value of 7, it is seen that the structure remains similar, just shifted in time. Except for all the Jātis of *Matya* and *Caturasra Tripuṭa*, no other palindromic Tālas exist in the 35 tāla system.The Dvikāla and subsequent states of these Tālas are amplified by increasing each Akṣara, leading to an increased number of cycles of the same Tāla to return to the origin. This means the geometric shape remains unchanged. In contrast, in Mārga Tālas, the increase is with respect to Guru , and thus the geometric shape varies. While certain similarities between the present-day Tāla system and the Mārga system can be observed, various modifications and developments have led to very a distinct system.

a)



b)

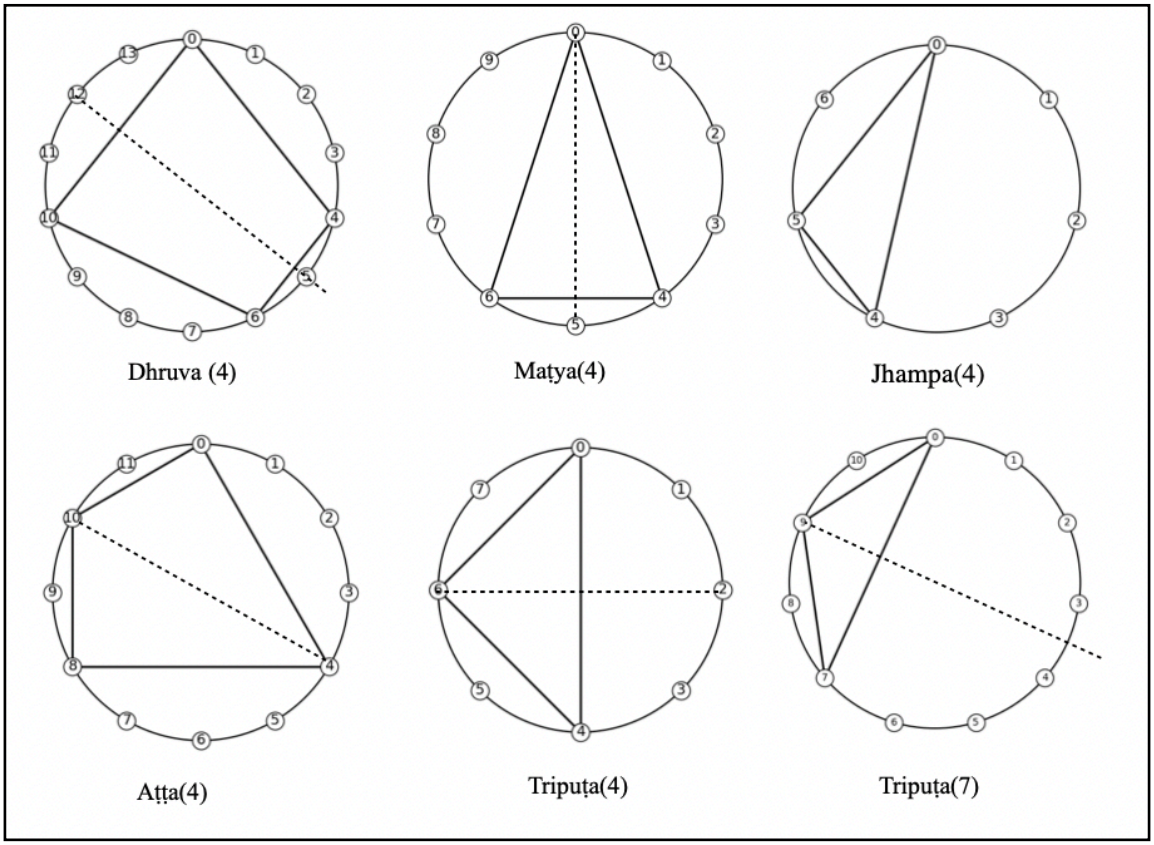


Fig:2.4.3(2). Comparing mārga tālas with present day tālas

2.5 Deśitālavivekaha

This section constitutes the most substantial portion of Tippa's *Tālapīpīkā*, wherein various forms of regional (*Deśī*) tālas are systematically enumerated, followed by an in-depth analysis of their structural characteristics. Tippa employs three distinct methods for codifying the tāla aṅgas. The first method utilizes symbolic notation, where each aṅga is represented by specific syllabic markers: druta (*ṭha* - ठ), laghu (।), guru (ṭa - ट), pluta (ḍa - द), kākapāda (+), and virāma (— or a bindu on a letter, e.g., ठँ). The second method adopts an alphabetic abbreviation system, using the initial letters of the respective aṅgas: druta (**द**), laghu (**ल**), guru (**ग**), and pluta (**प**). The third method uses words to codify the angas. In most instances, two methods are employed concurrently, ensuring clarity and precision in the representation of tāla structures. The concise notation facilitates quick identification of tāla structures, while the elaborate coding methods serves as a verification mechanism for accuracy. This dual-layered system is particularly useful in rectifying errors—if one representation contains inconsistencies, the other serves as a cross-check, reinforcing reliability.
Here are a few simple examples to illustrate the system:

1. गजेलघुचतुष्कं स्याद्रङ्गेगांताचतुर्द्रती ।।।। इति गजतालः ठठठठट इति रङ्गतालः।।
gajelaghucatuṣkaṁ syādraṅgegāṁtācaturdratī |||| *iti gajatālaḥ. ṭhaṭhaṭhaṭhaṭa iti raṅgatālaḥ.*

The direct and concise approach is used here. The first part of the verse directly describes the aṅgas, while the second part provides the concise notation.
Breakdown:

- *gajelaghucatuṣkaṁ* - Indicates four laghus in *gaja tāla*: ।।।।(LLLL)
- *syādraṅgegāṁtācaturdratī* - Specifies four drutas followed by **ga** (guru) in *raṅga tāla*: ठठठठट(DDDDG

2.पगलालागपालोगः प्लुतोलः प्लुतलद्रुतः इति चोलमण्डलपालकतालः

pagalālāgapālōgaḥ plutōlaḥ plutaladrutaḥ iti cōlamaṇḍalapālakatālaḥ

The tāla structure is suggested using the second method, employing the first letters of the respective aṅgas: **pa** = pluta, **la** = laghu, **ga** = guru, **da** = druta.
Breaking it down:

pagalālāgapālōgaḥ = pa + ga + la + la + ga + pa + la + ga

plutōlaḥ = pa + la

plutaladrutaḥ = pa + la + da

Joining them together:

PGLLGPLGPLPLD

Or, in Tippa's convention:

cōlamaṇḍalapālakatālaḥ: दट।।टद।टद।दलठ

3.ततस्सावरणराजादौ पौत्रताले पगौचतुः दटदटदटदट।

tatas sāvaraṇarājādau pautratāle pagaucatuḥ daṭa daṭa daṭa daṭa

This example demonstrates the complementary usage of two distinct coding techniques. The first part of the verse employs an alphabetic notation system to represent the tāla structure, whereas the second part provides a compact symbolic representation. **Analysis:**

- प (pa) - pluta
- गौ (ga) - guru
- चतुः (catuḥ) - four

The first code specifies that the tāla consists of four plutas and four gurus. However, it does not explicitly indicate the precise sequence in which these elements are arranged. Multiple configurations are theoretically possible, such as GGGGPPPP, GPGGPGPP, or other variations. This inherent ambiguity necessitates the inclusion of a secondary coding system for disambiguation. The second notation, "दटदटदटदट", systematically encodes the sequence by explicitly interspersing the plutas द and gurus ट. This structured representation ensures an unambiguous reconstruction of the tāla pattern, thereby enhancing the precision of the system.

4. **निश्शङ्केतु शरश्चापो विद्युच्चापद्वयं शरः। इति निश्शङ्कः**

niśśaṅketu śaraścāpo vidyuccāpadvayaṁ śaraḥ | iti niśśaṅkaḥ

The third system is in action here. The tāla is encoded using specific syllabic representations:

- शरः (śaraḥ) - laghu
- चापः (cāpaḥ) - guru
- विद्युत् (vidyut) - pluta

From this, the tāla structure can be decoded as follows:

- शरश्चापः (śaraścāpaḥ) = laghu + guru = ।ट (LG)
- विद्युच्चापद्वयं (vidyuccāpadvayaṁ) = pluta + guru + guru = दटट (PGG)
- शरः (śaraḥ) = laghu = । (L)

Combining them, the final the representation of Niśśaṅka tāla is LGPGGL,।टदटट।.

5. **पुर्वेप्लुतो बानचापौ बाणौ बिन्दूगपौ लपौ। लोबिन्दुगत्रयं लध्वोर्मध्ये काकपदं परम्।। दाट।।ठठटदादाठठ।टटट।+**

pūrve pluto bānacāpau bāṇau bindūgapau lapau | lobindugatrayaṁ ladhvormadhye kākapadaṁ param ||

The tāla structure for pūrvatala is encoded using all the above coding schemes. **Breakdown:**

- पुर्वेप्लुतः (pūrve plutaḥ) – The first aṅga is a pluta (P).
- बानचापौ (bānacāpau) – bāṇa (arrow) (L) and cāpa (bow) (G).
- बाणौ बिन्दूगपौ (bāṇau bindūgapau) – bāṇau (2L), bindū (2D), ga (G), pau (P).
- लपौ (lapau) – laghu (L) and pluta (P).
- लोबिन्दुगत्रयं लध्वोर्मध्ये काकपदं परम् (lo binduga-trayaṁ ladhvormadhye kākapadaṁ param) – ‘lo’ (L), bindū (2D), gatrayaṁ (3G), both interspersed between the laghus, and finally ending with a kākapāda (+).

Combining them: PLGLLDDGPLPLDDLGGGL+ (दाट।।ठठटदादाठठ।टटट।+

The interpretation of tāla structures in *Tāladīpikā* presents several complexities. Unlike texts such as *Saṅgītaratnākara Nartananirṇaya* or *Sangītasuryodaya*, where tāla definitions are distinctly separated for clarity, Tippa's treatise groups multiple tālas together.Though indented to find similarities, this makes it difficult to discern which specific tāla a given characteristic pertains to. When multiple tālas are discussed simultaneously, it stands unclear whether they are synonymous, share similar structural elements, or represent a single tāla with an extended name. For example:

अनन्दबाद्धक्रीडाविपञ्चीष्वन्तविरत्यदः ठठ। इत्यानन्दबद्धनिस्सारुक्रीडा तालो। विपञ्ची च। लघुशेखरकेत्वेको लघुरेव विरामवान्। इति लघुशेखरः।।[TD.25]

In this verse multiple components are involved, making the identification of individual *tāla* names and their structures particularly complex. Furthermore, Tippa employs three distinct modes of codification in his descriptions. Since different musical texts use varied encoding schemes to represent *tālas*, familiarity with only one system is insufficient. A comprehensive understanding of pre-existing *tāla* nomenclature is necessary for decoding newly introduced tālas in *Tāladīpikā*.

Tippa enumerates 223 tālas—103 more than Śārṅgadeva. He incorporates nearly all of Śārṅgadeva's 120 tālas, with the exception of a few such as *Caṇḍatālaḥ*, *Darpaṇaḥ*, *Ghaṭṭā*, *Śārṅgadevaḥ*, and *Yatilāgnāḥ*. The presence of common tālas has been instrumental in identifying and extracting the structural patterns within *Tāladīpikā*. Girija Easwaran's *Tālalakṣaṇa Samuccayam* has been of immense value in understanding tāla structures across different texts[GE]. By drawing from multiple sources, it provides an organic insight into the intended purpose of these classifications, thereby establishing a methodological framework necessary for analyzing *Tāladīpikā*.Through meticulous analysis and cross-referencing, it becomes possible to reconstruct at least some of the tāla structures encoded in Tippa's work.

Additional challenges included the legibility of the verses in the manuscript, as in some instances, only the first half of the verses remained visible. Furthermore, a missing page (page 32) has resulted in the loss of crucial details regarding the characteristics of certain tālas.The next few pages gives a tentative list of 215 tālas out of the 223 addressed in Taladipika.Between [TD.25] and [TD.26], the tālas listed on page 25 are enumerated, and so on.

1. Ekatāla [TD.25]
2. Druta Śekhara
3. Aditāla
4. Rasa Tāla
5. Ānanda Baḍḍha
6. Nissara Tāla
7. Krīḍā Tāla
8. Vipañci Tāla
9. Laghu Śekhara Tāla
10. Kāndarpa
11. Lagna Tāla
12. Viravikrama
13. AntaraKrīḍā Tāla [TD.26]
14. Tṛtīya Trivibhāga
15. Parathrīvita Tāla
16. Turanga Līlā Tāla
17. Pratyaya
18. Tunga Līlā Tāla
19. Vijayasthatta Tāla
20. Karunā Tāla
21. Cāṅcalya
22. Haṃsalīlā Tāla
23. Bhīṣaka
24. Samaraṅka Tāla
25. Dombalī Tāla
26. Rudradvīpaka
27. Haṃsa Tāla
28. Kamlasparśī Tāla
29. Simhavikrīḍa
30. Mathyā Tāla
31. Jaya
32. Jhampa Tāla
33. Vanamālī
34. Baḍḍhapāṇa Tāla[TD.27]
35. Syatu
36. Parāprati Tāla
37. Vaikuṇḍa Tāla
38. Kudukka Tāla
39. Parikrama
40. Ṣaṭ Tāla
41. Jhampatā Tāla
42. Nisśanka-ātta Tāla
43. Samā Tāla
44. Tāta Pratimatta
45. Jaya Maṇgala
46. Maṭṭikā Tāla
47. Jagat Jhampa Tāla
48. Viṣama
49. Kunda Sundara Tāla
50. Varṇamitaka [TD.28]
51. Simhalīlā Tāla
52. Chattā Tāla
53. Rāja Mṛgāṅka Tāla
54. Catus Tāla
55.RājaMartandaTāla[TD.29]
56. Kusumacaturtāla
57. Para Jagat Jhampa Tāla
58. Traśnā Varṇa Tāla
59. Vāñcitāpta Nissaru Tāla
60. Makaranda Tāla
61. Śrīkirti
62. Malla Kāmoda Tāla
63. Maṭallī Kāmoda Tāla
64. Gaja Tāla
65. Rāṅga Tāla
66. Parvatilocana
67. Cārabaḍḍha Tāla
68. Lalitā Tāla
69. Rati
70. Varṇa Bhinnaka Tāla
71. Maṅgala Tāla [TD.30]
72. Laḍhata Tāla
73. Lalitapriya
74. Gajalīlā Tāla
75. Panchama
76. VijayaPriyāṅgaMathyā
77. Utsava Tāla
78. Abhaṅga Tāla
79. Viṣama Tāla
80. Pratāpa Śekhara Tāla
81. Catuṣka
82. Siṃha Tāla
83. Sārasa Tāla
84. Rāga Vardhana Tāla
85. Pañca Ghaṭaka Tāla
86. Mathyā Gaurī Tāla
87. Kumuda Tāla
88. Mukunda Tāla
89. VarṇaMaṭṭikā[TD.31]
90. Tripurāntaka Tāla

91. Sarasatva
92. Abhinava Tāla
93. Para Vīra Vikrama Tāla
94. Parikrama Tāla
95. Nāndana Tāla
96. Candra Tāla
97. Vardhana Tāla
98. Candrākala —
99. Vallaba Tāla
100. Kuruvinda
101. Aṅga Mathyā
102. Sundarokthra Mathyā
103. Viṣama Kaṅkala Tāla
104. Pūrṇa Kaṅkala Tāla
105. RājaVidyādhara [TD.33]
106. Kāndarpa Tāla
107. Caturasra Varṇa Tāla
108. Śrī Kīrti Tāla
109. Tribhaṅgirarati Līlā Tāla
110. Tribinna Tāla
111. Kokilapriya Tāla
112. Kaṅkuta Tāla
113. Rayavankala Tāla
114. Parānandana Tāla
115. Lakṣmīśa Tāla
116. Rājapakṣī Tāla [TD.34]
117. Saluva Tāla
118. Sarasvatī Kaṇṭhābharaṇa
119. Dīpaka Tāla
120. Mohana Tāla
121. Mohana Mūrāri Tāla
122. Nārāyaṇa Tāla
123. Nāndī Tāla
124. Caturasra Varṇa Tāla
125. Lalitāpriya Tāla
126. Kuvinda Tāla
127. Caturmukha Tāla
128. ParaMakaranda [TD.35]
129. Śrī Nāndana Tāla
130. Bābara Bahuka Tāla
131. Vijaya Ānanda Tāla
132. Rāṅga Pradīpa Tāla
133. Pratyāṅga Tāla
134. Śrī Rāṅga Tāla
135. Kāla Dhvani Tāla
136. Rati Līlā Tāla
137. Siṃhanāda Tāla
138. Jaya Maṅgala Tāla
139. Rāja Cūḍāmaṇi Tāla
140. Para Nāndī Tāla [TD.36]
141. Chandas Tāla
142. Jayashrī Tāla
143. Haṃsanāda Tāla
144. Varṇayati Tāla
145. Aṅga Tāla
146. Saika Nissabdha Matta
147. Kathārī Saluva Tāla
148. Vasantā Tāla
149. Rāṅgābharaṇa Tāla
150. Vijaya Tāla [TD.37]
151. Jaya Tāla
152. Mukhya Matta Tāla
153. Saubhata Matta Tāla
154. Raṅgodyota Tāla
155. Pararaṅga Pradīpa Tāla
156. Kīrti Tāla
157. Rājātāla
160. Dharani Varaha Tāla
161. Ubbārāditya [TD.38]
162. Nisshanka Tāla
163. Nishanka Līlā Tāla
164. Śārṅgavega Tāla
165. TyāgaSimhāsanādheṣa
166. Para Rāja Tāla
167. Dvandva Tāla
168. Miśrāghanda Tāla
169. Kīrti Tāla
170. Para Matta Tāla
171. RaṅgaRājasthāpaka
172. Parvatilocona [TD.39]
173. Sāgara Durga Vipātaka
174. Janaka Tāla
175. DakṣiṇāSurātrāṇaVipāta
176. Anya Parvatilocona Tāla
177. Cāṇūra Malla Tāla
178. Candra Kāla Tāla
179. Siṃha Vikrama [TD.40]
180. Anya Janaka Tāla
181. DvinissabdhaMathya
182. Charchari Tāla

183. KanyanapuraNāyaka Tāla
184. Siṃha Vikṛḍita Tāla
185. Alākṣita Tyāga Śaunda Tāla
186. Chalikaya Nārayana Tāla
187. Laya Tāla
188. Miśra Varṇa Tāla [TD.41]
189. Bharatārṇava Pāriṇa Tāla
190. Sāvarṇa Rāja Pautra Tāla
191. Bopa Rāja Kumāra Tāla
192. Rāma Setu Stambha Sthāpaka Tāla
193. Cāṅkana Purādhipa Tāla
194. Parasiṃha Vikṛḍita
195. Candraśekhara Kiṅkara Tāla
196. Timmā Bhūpāla Sodara Tāla [TD.42]
197. Chola Maṇḍala Pālakā Tāla
198. Champu Raya Sthāpaka Tāla
199. Alampaka Kṣetravīra Tāla
200. Tribhuvanapurva Rāyāra Gaṇḍa Tāla
201. Utplutikaraṇa Samuddhāraka Tāla
202.ŚuddhaKāraṇa Samuddhāraka [TD.43]
203.Candra Vaṁśa Tilaka Tāla
204. Tippa Rāja Tāla
205. Sāvarṇa Rāja Tāla
206. Śrīraṅga Dhrohi Saptāṅga Harā Tāla
207. Athāraṇa Dhānī Śālaka Dhrāraka Tāla
208.ŚarṅgaPāṇiMaṇḍapa Kāraṇa[TD.44]
209. Anavārata Tyāgi Tāla
210. Timmā Bhūpāla Tāla
211. Kampa Rāja Sthāpaka Tāla
212. Vamanālankṛti Vyākhyākāra Tāla —
213. Pañcaganta Nāda Tāla
214. Pūrva Tāla [TD 45]
215. Siṃha Nāda Tāla

Ratnakara vs Tāladipika

Tippa's convention: *Druta* (ṭha - ठ),*Laghu* (।),*Guru* (ṭa - ट),*Pluta*(da - द), *Kākapāda* (+),*Virāma* ---, 's,' or a bindu on the letter ठ॰.

Sāraṅgadeva's convention: *Druta* (०) *Laghu* (।) *Guru* (s) *Pluta* (s') *Virāma* (bindu on the letter: ०', ।', or connecting letters) ४ for *niḥśabda kriyās*.

(The term kākapāda is not used by Saṅgītaratnākara; instead, it addresses it simply as a niḥśabdha kriyā.)

Given below are verses from Saṅgītaratnākara and Tāladīpikā that describe the Jaya tāla:

जो लघुर्द्वौ द्रुतौ पश्च इति जयः। [SR.5]

Jo laghur dvau drutau paśca iti jayaḥ.

जयेलाद्यन्तकाद्वक्राल्लघुर्बिन्दुयुगं प्लुतः। [TD.37]

Jayelādyantakād vakrāl laghur binduyugaṁ plutaḥ.

Tāla angās : ।ट।।ठठद or ।s।।००ś = LGLLDDP

Although both texts define Jaya tāla identically in terms of its rhythmic structure, the variation in linguistic expression highlights differences in technical nomenclature and descriptive approach. *Saṅgītaratnākara* provides a more direct enumeration of its components, whereas *Tāladipika* is more precise. The table (3.5.1) compares select *tālas* from *Tāladīpikā* with those in *Ratnākara*. At most, no significant variations in the structure of these *tālas* are observed. However, there are instances in *Tāladīpikā* where the dual coding contradicts itself. In such cases, the more elaborate coding is considered accurate, while *Ratnākara* is taken as the standard reference.

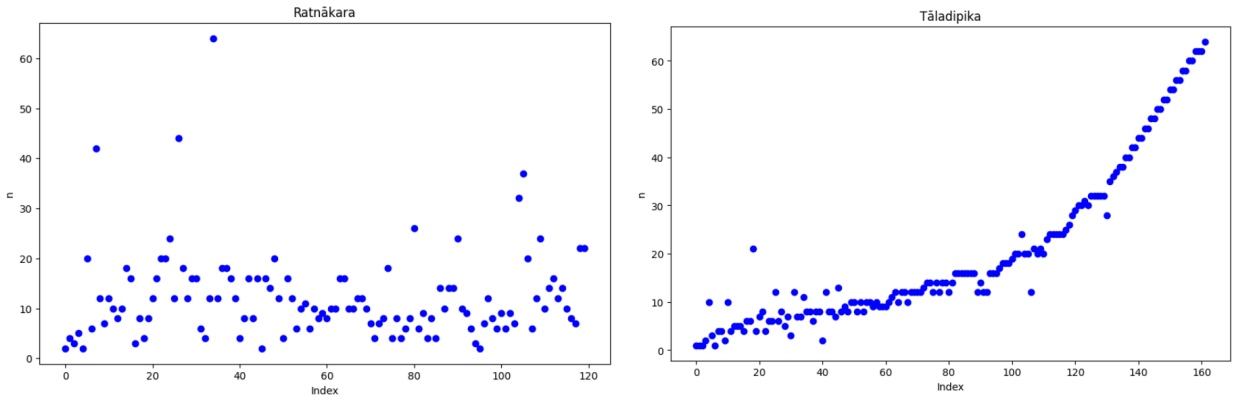
Apart from the differences in the basic coding scheme, there are a few key aspects that differentiate Tippa's approach to the tala system.

इत्युक्ता देशितालानां द्वे शते विंशतिस्त्रयः। इहत्वमस्माभिरुन्नीता: प्रच्येस्मो भिन्नलक्षणाः।
अस्मात्संबन्धिविरुदंनामाङ्कदेश्यतालकाः। एकचत्वारिंशदिति विशङ्कुर्वन्तु पण्डिताः।
एवमुद्देशमापन्नेष्वेषु तालेषु सूरिभिः। श्रद्धे यस्तालभेदस्तु वैषम्यान्नाममात्रयो:
एकस्मिन्नेव विन्यासे ताल भेदानुकीर्तनम् यदेतदेशदे पार्थक्येन गणा नार्हन्तीत्यत्रनिर्णयः।
विशेषतोत्र निर्देशे विरामस्तत्तदाश्रितः। अपिशेषेण निर्देशे मत्वाधैरुपर्वसंश्रिताः।
इत्युद्देशः[TD.25]

Upon completing the enumeration, the author offers a few insightful remarks. The first statement ascertains the systematic compilation of 223 *tālas*. The author further addresses some of the most trivial points and issues with this classification.The author acknowledges that there may be inconsistencies in naming and classification, urging scholars to analyze them carefully. He identifies the number of recursive tālas to be 41, and questions whether they truly deserve separate classification or they are just different by name.For instance , tālas *Gajah* and *Gauri* are both explained to have four laghus (।।।।), meaning they follow the same time structure but are named differently. As discussed earlier, Tippa resolves this by grouping structurally similar *tālas* together. While this approach may initially complicate comprehension, it represents a novel method of classification. One example of this grouping is a set of *tālas* that share the same total number of time units but differ in their internal arrangement. These are essentially different ordered permutations of *D*, *L*, *G*, and *P*, constrained to a sum of 12 units (*D* = 1, *L* = 2, *G* = 3, *P* = 6). They correspond to different *Prastāras* of the 12-DPs (see Section 2.6).

Kaṇṭuka Tāla	**।।।।ट**	**LLLLG**	**12**
Rāyavaṅkalā Tāla	**ट।ट ठठ**	**GLGDD**	**12**
Parānandana Tāla	**।।ठठद**	**LLDDP**	**12**
Lakṣmīśa Tāla	**ठठ।।द**	**DDLLP**	**12**

Note that these tālas are discussed collectively in [TD.33], whereas Ratnākara treats them separately and addresses each individually. Tippa adheres to systematic manner of arrangement, starting with tālas that have shorter time units and progressively moving towards those with longer time units. The figure below presents a comparative analysis of the order in which the tālas are arranged, correlating the total sum of angas (n) with their corresponding index numbers.The existence of this pattern allows for the easy identification of potential misinterpretations of tala structures or ambiguities within the code itself. Outliers naturally surface and can be addressed accordingly.



Fig(3.2.1)Comparison of Rātnakara and Tāladipika in Terms of Index

There exist quite a few instances where the dual codes are antagonistic. Here is an example:

चच्चर्या मष्टकृत्वोन्तविरामौ लपरौ द्रुतौ।
ठठठठठठठठठठठठठठठठ इति चर्चुरीलालः।।[TD.40]

The first half of the verse explicitly mentions the inclusion of laghu, whereas the second half does not account for it. This lack of coherence raises ambiguity in the intended structure of the tāla. Conversely, there are instances where the reverse occurs as well. In such cases, rigorous analysis with earlier texts is required to resolve these inconsistencies. However, for tāla structures that are entirely new and independently introduced, even these measures become ineffective, necessitating alternative methods for validation.

Tippa Gopendra has composed a remarkable set of original and unique tālas. His compositions are marked not only by their unique rhythmic configurations but also an interesting approach to nomenclature, where many tālas reference themes of kingship, power, and war. The tālas he has introduced, such as Kalyānapura Nāyaka Tāla, Alakṣita Tyāga Śaunda Tāla, Tippa Rāja Tāla, and Śāraṅga Drohī Saptāṅga Haraṇa Tāla, showcase a creative fusion of two of his key interests.In addition to composing entirely new tālas, Tippa also introduces variations of existing ones. He adds a para or anya (alternative) to the names of these modified tālas which makes it easier to identify them. Some examples include Anya Pārvatī Lochana Tāla, Anya Janaka Tāla, and Para Raṅgapradīpaka. This process of transformation and reinterpretation is a trademark of the creative evolution in Indian classical music, where traditional forms are valued yet reimagined to explore new possibilities.

The similar trend is seen across many musical works. Robert Brown, in his Perspectives on Tāla,explores an existing format in which the talas are structured. He notes that about twenty to thirty tālas exhibit variant aṅga patterns compared to those in the Saṅgīta Ratnākara, while the remaining tālas—approximately two-thirds of the total—align with it. These original compositions are usually indicative of the authorship. For instance, in Ratnākara, the last twenty tālas are likely inventions of Śārṅgadeva. These tālas do not appear in any other lists unless cited directly in full. Just as there exists Śārṅgadeva Tāla, there exists Tippa Rāja Tāla and many others. The important question to address is whether these tālas were mere theoretical constructs or were they part of active performance traditions?Table (3.5.2) presents some original tālas and unique compositions of Tippa Gopendra.

Table 3.3.1: Comparing *Ratnākara* and *Tāladipika*

	Deśi tālas		Ratnākara	Tāladipika	[TD]pg .no.
1	अभङ्गः	*Abhaṅgaḥ*	।ऽ	।द	30
2	आदितालः/रासः	*Āditālaḥ/Rāsaḥ*	।	।	25
3	अनङ्गः	*Anangaḥ*	।ऽ।।s	।द।।ट	36
4	चच्छरी	*Caccarī*	००’००’००’००’००’००’०० ।	ठठं ठठं ठठं ठठं ठठं ठठं ठठं ठठं ।	40
5	चन्द्रकला	*Candrakalā*	sssऽऽऽ।	टटट ददद ।	39
6	चतुरश्रवर्णः	*Caturāśravarṇaḥ*	s ।०० s	ट।ठठ ट	34
7	चतुर्थकः	*Caturthakaha*	।।०	।।ठ	27
8	दीपकः	*Dīpakaḥ*	०० । ।s s	ठठ।।टट	34
9	द्वन्द्वः	*Dvandvaḥ*	। । s ss। ऽ	।।टटट।द	38
10	एकताली	*Ekatāli*	o	—ठ—	25
11	गजः,धारा	*Gajaḥ, Dhārā*	।।।।	।।।।	29
12	गजझम्पातालः	*Gajajjhampātālaḥ*	s००’०	ट ठ ठ ठ	29
13	गारुगी	*Gārugī*	०००‘०	ठठठठ—/ ठठ-ठठ-	27
14	गौरी	Gaurī	। । । ।	।।।।	30
15	हंसनादः	*Haṁsanādaḥ*	।ऽ००ऽ	।टठ ठद	36
16	जनकः	*Janakaḥ*	। ।।। s s।।s s	।।।।टट।।टट	39
17	जयः	*Jayāḥ*	।s । ।००ऽ	।ट।। ठ ठद	37
18	जयमङ्गलः	*Jayamaṅgalaḥ*	।। s ।। s	।।ट।। ट	35
19	झम्पातालः	*Jhampātālaḥ*	०‘०	—ठठ—	26
20	कन्दर्पः	*Kaṇḍarpaḥ*	०० । ss	ठ ठ।टट	33
21	कङ्कालाः	*Kaṅkālāḥ*	। s s	।टट	31
22	करणः	*Karaṇaḥ*	s	ट	26
23	कोकिला प्रियः	*Kokilāpriyāḥ*	s।ऽ	ट।द	35
24	कुविन्दकः	*Kuvindakaḥ*	। ०० s ऽ	।ठठटद	34
25	लक्ष्मीशः	*Lakṣmīśaḥ*	०‘०। ऽ	ठठ।।द	35
26	ललितप्रियः	*Lālitapriyāḥ*	। । s । s	।।ट।ट	34
27	लयः	*Layaḥ*	s।ऽऽऽ s ऽ०००	ट।द द द ट द ठ ठ ठ	40
28	मकरन्दः	*Makarandaḥ*	०० ।।।	ठ ठ।।।	29
29	मल्लिकामोदः	*Mallikamodaḥ*	।।००००	।। ठ ठ ठ ठ	29
30	मण्ठिका	*Maṇṭikā*	। ।’००	—।।ठठ	28
31	मुकुन्दः	*Mukundaḥ*	। ०००० s	। ठ ठ ठ ठ ट	31
32	नन्दनः	*Nandanaḥ*	। ००ऽ	। ठ ठ द	31
33	नि शङ्कलीलकः	*Ni Śaṅkalīlākaḥ*	ऽऽ s s	द द ट ट	38
34	निःशङ्कः	*Niḥśaṅkaḥ*	। s ऽ s s ।	।ट द ट ट।	38
35	पार्वतिलोचनः	*Pārvatilocanaḥ*	s s s।ऽ s s ००	टटट।दटट ठठ	39
36	प्रतापशेखरः	*Pratāpaśekharaḥ*	ऽ०‘०	द ठठ	30
37	प्रतिमण्ठकः, कोल्लकः	*Pratimaṇṭhakaḥ, Kollakaḥ*	।।s or s।।	ट।।	30
38	प्रत्यङ्गः	*Pratyāṅgaḥ*	s s s।।	ट ट ट ।।	35
39	राजचूड़ामणिः	*Rājacūḍamaṇiḥ*	०० ।।।०० । s	ठठ ।।। ठठ ।ट	35
40	राजमार्तण्ड :	*Rājamārtaṇḍaḥ*	s । ०	ट।ठ	29
41	राजतालः	*Rājatālaḥ*	sऽ ००s । ऽ	टद ठठ ट।द	38
42	राजविद्याधरः	*Rājavidyādharaḥ*	। s ००	।टटठठ	33
43	रङ्गाभरणः	*Raṅgabharaṇaḥ*	ss ।। ऽ	टट।।द	36
44	रङ्गः	*Raṅgaḥ*	०००० s	ठठठठ ट	29
45	रङ्गप्रदीपकः	*Raṅgaprādīpakaḥ*	s s । sऽ	टट। टद	33
46	रङ्गोद्योतः	*Raṅgodyotaḥ*	s s s। ऽ	टटट।द	37
47	समः	*Samāḥ*	। । ‘०‘०	।।ठठ	28
48	सारसः	*Sāraḥ*	। ०००। ।	।ठठठ।।	30
49	सरस्वतीकण्ठाभरणः	*Sarasvatīkaṇṭhābharaṇaḥ*	s s । । ००	टट।।ठठ	34
50	षट्तालः	*Ṣaṭtālaḥ*	००००००	ठठठठठठ	27
51	सिंहः	*Simhaḥ*	। ०। । ।	।ठ।।।	30
52	सिंहलीलः	*Simhalīlāḥ*	। ०००	।ठठठ	29
53	सिंहनादः	*Simhanādaḥ*	। s s ।s	। ट ट । ट	35
54	सिंहनन्दनः	*Simhanandanaḥ*	ss।ऽ।s ०० s s ।ऽ।ऽऽ।-४x	टट I द ।टठठ टट।द।दट।+	45
55	सिंहविक्रमः	*Simhavikramaḥ*	s s s।ऽ।s ऽ	टटटादाटद	40
56	श्रीकीर्तिः	*Śrīkīrthīḥ*	।।s s	।।।टट	33

Table 3.3.2: Some unique *tālas* in *Tāladipika*

	Deśi Tālas [TD]		In Tippa's convention	In D,L,G,P
1	रङ्गराजस्थापकतालः	*Raṅgarājasthāpakatālaḥ*	ठठटठदठठद	GDDGDPDDP
2	दक्षिणसुरत्राणपूर्वविपाटकतालः	*Dakṣiṇasuratrāṇapūrvavipāṭakatālaḥ*	दटठ।।ठटदठ	PGDLLLDGPD
3	अन्यपार्वतीलोचनतालः१	*Anyapārvatīlocanatālaḥ 1*	टटट।दटटठठ	GGGLPGGDD
4	अन्यपार्वतीलोचनतालः२	*Anyapārvatīlocanatālaḥ 2*	।ठठ।ठठटट।।।।ट।।	LDDLDDGGLLLLGLL
5	चाणूरमल्लतालः	*Cāṇūramallatālaḥ*	ददठ।ठ।ठदद	PPDLDLDPPP
6	द्विनिःशब्दमध्यतालः	*Dviniḥśabdha Madhya Tālaḥ*	ट।।टट++	GLLGG++
7	कन्यनपुरनायकतालः	*Kanyānapura Nāyaka Tālaḥ*	टट ठठठठ टठट ठठठठ टट	GGDDDDGDGDDDDGG
8	सिंहविक्रीडिततालः	*Siṁha Vikrīḍita Tālaḥ*	।।दट द।द	LLPGPLP
9	अलाक्षितत्यागशौण्डतालः	*Alākṣita Tyāga Śauṇḍa Tālaḥ*	।। ठ।।।ठठठठ ।।ठ।।ठठठठ ।।ठठ ।।	LLDLLLDDDDLLDLLDDDDLLDDLL
10	चालिकायनारायणतालः	*Chālikāya Nārāyaṇa Tālaḥ*	।टद।टद।टद	LGPLGPLGP
11	मिश्रवर्णतालः	*Miśra Varṇa Tālaḥ*	ठठठठठठठठठठठठठदटटठठटट।	DDD*DDDD*DDDD*D
12	भारतार्णवपारिणतालः	*Bhāratārṇava Pāriṇa Tālaḥ*	ठठ।।ठठ।।ठठ।।ठठ।।ठठ	DDLLDDLLDDLLDDLLDD
13	सावर्णराजपौत्रतालः	*Sāvarṇa Rāja Pautra Tālaḥ*	दटदटदटद	PGPGPGPG
14	गोपराजकुमारतालः	*Gopa Rāja Kumāra Tālaḥ*	ठदट।ठठठ।टद।टदठ	DPGLDDDLGPGPD
15	रामसेतुस्थम्भस्थापकतालः	*Rāma Setu Stambha Sthāpaka Tālaḥ*	ठदा।दा द।द।दठ	DPLLPLPLPLPD
16	चङ्कणपुरधिपतालः	*Caṅkaṇa Purādhipa Tālaḥ*	।द।।द।।द।।द।।द।।	LPLLPLLPLLPLLPLL
17	परसिंहविक्रीडित	*Parasiṁha Vikrīḍita*	।दटा।दटा।दटाद	LPGLLPGLLPGP
18	चन्द्रशेखरकिङ्करतालः	*Candraśekhara Kiṅkara Tālaḥ*	दा ठदटठ।दटठ टठ	PLDPGDLPGDGD
19	तिम्मभूपालसौदरतालः	*Timma Bhūpāla Saudara Tālaḥ*	।दट।दटा दटा दट	LPGLPGLPGLPG
20	चोलमण्डलपालकतालः	*Chola Maṇḍala Pālaka Tālaḥ*	दट।।दटा दटा द।ठ	PGLLPGLPGLPLD
21	अलम्पकक्षेत्रवीरतालः	*Alampaka Kṣetravīra Tālaḥ*	टद।टद।टद।टद।ठ	GPLGPLGPLGPLD
22	त्रिभुवनपूर्वरायरगण्डतालः	*Tribhuvanapūrva Rāyara Gaṇḍa Tālaḥ*	ठ।।ट।।ट।।ट।।ट।।ट।।ट।।ठ	DLLGLLGLLGLLGLLGLLGLLD
23	उत्प्लुतिकरणसमुद्धारकतालः	*Utplutikaraṇa Samuddhāraka Tālaḥ**	।ट।ट।ट।ट।ट।ट।ट।ठ	LGLGLGLGLGLGLGLD
24	शुद्धकरणसमुद्धारकतालः	*Śuddha Karaṇa Samuddhāraka Tālaḥ*	ठ ठ ट।।टट।।टट।।टट।।टठठ	DDGLLGGLLGGLLGGLLGDD
25	चन्द्रवंशतिलकतालः	*Candra Vaṁśa Tilaka Tālaḥ**	टट।। ठटट।। ठटट।। ठटट।। ठठ	GGLLDGGLLDGGLLDGGLLDD
26	तिप्पराजतालः	*Tippa Rāja Tālaḥ*	ठठठ।s ।ठठठ।s ।ठठठ ।s।ठठठ। s।ठठठ।s।	DDDLs LDDDLs LDDDLsLDDDLs
27	सावर्णराजतालः	*Sāvarṇa Rāja Tālaḥ*	टददठठटददठठटददठठठ	GPPDDGPPDDGPPDDD
28	श्रीरङ्गध्रोहिसप्ताङ्गहरातालः	*Śrīraṅga Dhrohi Sapthāṅga Harā Tālaḥ*	ददटदद।।दटददट	PPGPPLLPGPPG
29	अथहरणधानीशालकधप्तकतालः	*Athaharaṇa DhānīśālakadaptakaTālaḥ*	टटद टटद टटद टटद	GGPGGPGGPGGP
30	शार्ङ्गपाणिमण्डपकारकतालः	*Śārṅga Pāṇi Maṇḍapa Kāaraka Tālaḥ*	।द।द।द।द।द।++	LPLPLPLPLPL ++
31	अनवरत्यागितालः	*Anavarata Tyāgi Tālaḥ*	दद।।। टट।।। टट।।।ददठ	PPLLLGGLLLGGLLLPPD
32	तिम्मभूपालतालः	*Timma Bhūpāla Tālaḥ*	टददट टददट टददट टददट	GPPGGPPGGPPGGPPG
33	कम्पराजस्थापकतालः	*Kampa Rāja Sthāpaka Tālaḥ*	ददटददटदठ	PPDPPDPPD
34	वामनालङ्कृतिव्याख्याकारतालः	*Vāmanālaṅkṛti Vyākhyākāra Tālaḥ*	।ठ।ठटठदठदठटठ।ठ।ठटठदठटठ।ठठ	LDLDGDPDPDGDLDLDGDPDGDLDD
35	पञ्चगण्टानादतालः	*Pañcagaṇṭā Nāda Tālaḥ*	टददट।ठठ।दद++++++	GPPGLDDLDD++++++
36	पूर्वतालः	*Pūrva Tālaḥ*	द।ट।।ठठटद।द।ठठ।टटट।+	PLGLLDDGPLPLDDLGGGL

Māgadeśiprastaranirūpanam

हेमाद्रिरित दानपारकगकरो हेमाद्रिचापोप
मोध्यस्सालववंशवारिधिशशी धर्मेकरक्षा-
मणिः। विद्याराजसुता स्वयंवरपतिर्विद्वेषिशेला-
शनिः युत्पत्तेः परमावधिः जयति
यो विश्वभ्भरामण्डले ।। कुर्वन्गां
महिषींकुलक्रमगतां तस्यां वृषं तादृशं
निष्पाद्यैनमजामुरागसर सान्नित्यं समुल्लासयत्।
तिम्मश्मापतिरीदृगद्भुतनिधिस्तिम्यद्यशोभूषण-
स्यागरव्यातिनिरस्तनन्दनतरुर्जगर्ति
यत्पूर्वजाभा कामान् साधयितुं सतां
निपुणधीः सोयं सुरेन्द्रोपमः श्रीमान्
गोपयतिप्पभूपरिवृढस्सोमान्वयग्रामणीः।
सीमानन्नयति प्रसिद्धिगणनां
पातालमेरु- स्थितिश्यामाकामुकबर्हबन्ध
शिर सोवामाङ्गजानेस्तुतिः।।
त्रिपञ्चाशत्प्रत्ययानां मध्ये बुध्यैक लक्षिताः।
पातार्हा अविपाताः स्युस्ते मीदलगपा रह।
आलीयन्त इति प्रोक्तः पातालोड उणादिजे।
अथवास्त्रनलनं पातो द्रुतादीनां तदेष हि।
आलानातीति पातालो डेस्त्यौणादिके परे।
इत्थं मता हनुमतोस्तारादीनां निरुक्तयः।
अथत्रयाणामेतेषां आकारं व्याहरामहे।
ऊर्ध्वप्रस्तार कूटस्थादुरोरल्पमयोलिखेत्।
शेषं यथोपरिप्रायोन्यूनेवामगतानु रुन्।
संभवे संभवेत्वल्पांस्तारप्रत्येततोलिखेत्।
स्फुटं तावत्प्रयोक्तव्यः पुनःपुनरयं
विधिः। सर्वद्रुतावधिर्यावत् प्रस्तारः
परिनिष्ठितः। लघोर्गुरौ प्लुतौ तेव्यक्ते
समस्ते च समत्ययौ। प्रस्तारभङ्गिरुदिता
नतुव्यक्तेवृतेस्तिहि। इति प्रस्तारः॥
एकद्याङ्कौ विलिख्यादौ आनुपूर्व्यादधस्तनैः।
द्वितीयतुर्यषष्ठाङ्कैर्घटयेदङ्कमन्तिमम्।

स्तरो मेरुश्च पातालस्तालध्यौव्येक हेतवः।
शिष्टास्तु केवलं सिद्धाश्शिल्प मात्र प्रयोजनाः।
तम्मार्गदेशितालेषु प्रत्येकं परिनिष्ठिताः
। प्रस्तारमेरुपातालप्रक्रियोपक्रमेण साः।
संख्यास्तदङ्कवक्क्त्रार्थमय मद्य ममोद्यमः।
स्तृणाति धातुस्सन्तानच्छादनार्थप्रवाचकम्।
तस्तात्प्रपूर्वाद्धन्तिप्रस्तार शब्दः पथां गतः।
कूटस्थमात्रातभसन्तनोति सर्व द्रुतावधिः।
आत्मीयकारणाकारञ्चात्मनिच्छादयत्ययं।
माशेब्देनोच्यते मानमीशब्देन विचित्रता।
रुशब्देन वसंशब्दस्तत्प्रस्तारहि दा मितेः।
सद्रुताद्यद्रुतादित्ववैचित्र्येणात्रशब्दनम्। अस्तीति
प्रत्ययस्सोयम्मेरुशब्देन कथ्यते।
तुरीयष्ठालाभेतु कुर्यात्प्रतिनिधी तयोः।
तृतीयपञ्चमाङ्कौहि क्रमादग्रे मरन्दृशम्। योगं
लिखे दक्षिण स्थात्वेव मङ्का वलिर्भवेत्।
सेयं वाञ्छिततताकास्थं द्रुतसंख्याङ्क काविधिः।
द्रुतो लघुस्सद्रुतो लघु गुरुस्स द्रुतो गुरुः।
प्लुतः प्लुतः सद्रुतश्चेदेकैक द्रुत वर्धितैः।
संख्यायन्ते तालभेदाः क्रमादङ्कै रिहस्थितैः।।
एषां हि योगादङ्कानां सिद्धोन्त्योङ्कोल्लतस्तुतैः।
संख्यायन्ते क्रमाद्भेदा यतोदलग पान्तकाः।
अतस्संख्येयमुक्ता सावपिमेरुः कपेः मतात्।
इति संख्यापक्षान्तरसिद्धो मेरुश्च।।
लिखेत्पङ्क्तिन्तिरश्ची नामिष्टतालद्रुतोचितैः।
कोष्ठेर्युक्तां ततश्चैकां परां कोष्ठनवर्जिताम्।

ततश्च स्वस्वपूर्वाभ्यो दिदिकोष्ठानिताः पराः। संख्यायन्ते यत्र भेदास्सासंरव्या मेकरूपसौ। अथाङ्क योगस्सर्वासां पङ्क्ती नामाद्य कोष्ठयोः। इति द्विधाहतु, मेरु लक्ष्मावतर कंबलौ। द्वयोर्द्वयोस्तु विलिखेदेकाङ्कामविशेषतः। अथो उभयोः पक्षयो सुल्यं फलं भेदविवेचनम्। विधिस्तृतीयादौ कोष्ठे तुविषमे लिखेत्। संख्या इत्यत्र हनुमङ्गन्ये मेरुरुद्दिश्य पक्षयोः। ङ्कवत्प्रतिनियता मन्त्यादिचतुर्र्युतमं। समे कोष्ट अन्तिम पक्षमाश्रित्य मेरुरत्रोच्यते मया। उपान्तादि त्रियोगमथ वीथिषु। परासूक्तं चतुर्योगं प्रत्येकम्मार्ग तानेषु देशितालेषुच क्रमात्। शेषको ष्ठेषु निर्वहेत्। नास्थेयोत्र प्रतिनिधिरङ्क प्रस्तार मेरुपातालनिर्णय श्चेन्ननीयते। तदागोष्ठयां योस्तुर्यषष्ठयोः। निसर्गसिद्धास्तासूर्ध्वाः विदुष्मत्यां ताम्येदेव हि तालवित्। अमुं पङ्क्तयः प्रस्फुरन्ति साः।[49] तन्मध्ये निर्णयमा रख्यात मनुतालं मुनीश्वराः। शैलादि समकोष्ठेन्त्यस्थानेन्त्याधस्तजोन्तकः। विधेयो कपिशाण्डिल्यशार्ङ्ग द्या अपि शास्त्रिणः। थ तु तैरङ्कैः भेदसंख्या प्रसिद्ध्यति॥ अङ्के कमनीषा कम्पचानाम्मादृशामिह का कथा। स्थितैरूर्ध्वपङ्क्तौ विषमायामधः क्रमात्। तथापि जानकी जाने स्तादृशानुग्रहादहम्। एकद्रुतादिविषम संख्या भूतभिदायुताः। अधिकृत्यानुपूर्व्येण तालानुक्रमणीमिह। सर्व द्रुतान्ता गम्यन्तेङ्कैस्सभायां पुनस्थितैः। कूटस्थादिस्तारमेरु पातालां कालिसाधिकाम्। अद्रुताद्याद्विद्रुतादि समसंख्यद्रुताभिदाः। खिश्वाधिकस्तुतिं वक्ष्ये पदेः कठपयात्मभिः। सर्वद्रुतान्ता गम्यन्ते रचनाप्राच्यमेरुवत्। अतो भेदश्होद्दिष्टस्तपरे यावतिथो भवत्। एकैकस्मिन्नाद्य कोष्ठे तासामेकाङ्कमालिखेत्। तत्पूर्वापर संरन्ख्यानुवर्गसंख्येव कथ्यते।

अधः पङ्के रन्यतुर्यषष्ठ योगं प्रयोजयेत्। एकमात्रात्मके ताले तथाध्यर्धमात्रके । शेष कोष्ठेष्वथान्या सामुपान्त्याधस्तनाधिकम्। पदैः संख्याङ्कसंसिद्धिं पञ्चभिः प्रतिपादयेत्। एनन्नियोगं शेषेषु कोष्ठेषु विनियोजयेत्। हानादाने पदं प्रस्तारमात्राद्यं द्वितीयद्रुतमेरुकम्। प्रतिनिधे त्रापि द्रुतमेरुवत्। ऊर्ध्वपङ्क्तिस्थकोष्ठाकै तृतीयंलघुमेरुं दुतपातालंतु रायकम्। पञ्चमं शरभ्य लघुवर्जितम्। एकाघेकोत्तरोद्दृध्दलाभिदा लघु पातालं विदाङ्कुर्वन्तु वित्त गाः। ताले सर्वकान्तकः। विज्ञेयास्थाधिपंख्याच सर्वाङ्कानां द्विमात्रे गप्यस्मिन् सार्धेङ्के सत्पभिः पदैः। समानयात्। इति लघुमेरुः।। गुरु मेरावधः त्रीणि प्राग्वदिहाप्यादौ पदं तुर्याङ्ग मेरुकम्। पङ्के रूनाकोष्ठैस्थितिःपरा। पराश्चतुरचतुः पञ्चमं द्रुतपातालं लघ्वातं तु षष्ठकम्। कोष्ठो नास्वस्वप्राच्यपङ्किकः। सर्वपङ्याद्याद्य सप्तमं गुरु पातालं विजानीयुर्विपश्चितः। अतः कोष्ठा से कैकाङ्केन मुद्रिताः। कोष्ठो परं तु ये तालास्त्रिमात्राद्यास्त्रितिं गताः। द्वितीयोधः पङ्केः यङ्कवाञ्चेषां कोष्ठकाः तेषां स्तारादि संख्याकान् भावयेन्नवभिः [50] अत्राह भगवान्नेरुविषये त्वञ्ननासुतः। पदैः। पदमाद्यमिहस्तारन्द्रुतमेरुद्वितीयकम्। स्यान्मेरुसद्रुतायद्रुताद्यद्रुतादिस्त्रिति भिदामितिः। लघुमेकतृतीयं च गुरुमेरु चतुर्थकम्। प्लुतमेरं यद्वाविन्द्धन्तपुङ्खान्तधनुरन्तप्लुतान्तकाः। पञ्च दपातालं तु षष्ठकं।

लपाताल सप्तमंच गपातालं तथाष्टमम्। प्लुतपातालकमपि नवमं भावयेत्बुधः। विसर्जनीयश्लोकान्ते तदर्धे वास्तियत्र हि। नपृथक्पतातस्य किन्तूर्ध्वपदशेषता। स्तारादिविनियुक्तेभ्यः पदेभ्यो यस्तुशिष्यते। श्लोकांशस्तत्रतालाख्या मात्रात् समभये। श्लोकांशो यद्यपर्यात्पश्लोकेभ्यश्चात्र बध्यते। तालमात्रास्तुवर्ण्यन्ते तादृश्या भूतसंरव्यया। तालनाम्नःकचित् स्फूर्तिः पद भङ्गस्य मुद्रया। चतुरश्तं त्र्यश्तमिश्त भेदानामसतामपि। अभ्युपेत्यैव दृश्येषु तत्तदायक्षराङ्कनम्। स्तानाद्युक्त वधौ तद्व्यवहित क्वचित्। उक्त भेदत्रयाङ्काक्षराङ्को यत्र न दृश्यते। ते खण्डताला इत्येव प्रत्येतव्याः परीक्षकैः। तालस्यैकस्य यः प्रस्तारादिप्रवादुकैः। सएव लमात्रेष्ववसेयोतिदेशतः। तालानां तुल्यमात्राणां अतिदेशे समागते। उद्देशक्रमभङ्गोत्र नदोषयिति मन्यताम्। नवस्युर्नव पञ्चाष्टौ क्रमात्कटपयादनः। वर्णद्वित्वे त्वधोवर्णं ग्रात्यं शून्यौन्जनौ मतौ। यादयस्तु ककारेण युक्ता नवमितान्तरा। शून्य स्थानाभिधायित्वं यद्यपि ज्योतिषे त्वचाम्। तथाप्येषां पदाद्यानां अत्रालङ्कारमत्रता। स्यदुवर्णमुवर्णं च हलपृक्तं शून्यवाचकम्। हालोवर्णानुर्बन्धस्य पारतन्त्र्यादमर्थता। जेनेन्द्रदृष्टां वक्ष्येथ संख्याङ्कस्थान पद्धतिम्।

2.6 Māgadeśiprastaranirūpanam

Indian classical music is mathematics in disguise.The soothing melodies are stratagic orchastrations with fine flavours of artistic exploration within the logical framework. Its originality and profoundness can be attributed to the intense mathematical rigor in exploring the melody and rhythm dynamics ; *"The Raga and Tala system"*.This is a very distinctive system, featured exclusively in the Indian aesthetics domain. Both *rāgas* and *tālas* have been explored significantly in the eye of mathematics and symphonies. The Earliest treatises that address the matter of prosody by means of combinatorial tools called *pratyayas* are *Pingala's Chandas-shastra*(300BC) and *Bharata's Natya shastra*(500BC). The ingenuity of these theorists is reflected in their ability to:

>Express complex principles concisely while maintaining clarity and precision.

>Develop a compact tabulation methods for systematic organization.

>Simplify intricate formulations into basic arithmetic operations for ease of computation.

>Illustrate the practical relevance of these theoretical constructs in order to make them a lot easier.[AK]

Chandaḥ Śāstra was a part of the Vedic learning and played a crucial role in preserving various kinds of metric alliterations. This śāstra defines a system based on short and long syllables (laghu and guru) to enumerate all possible rhythmic arrangements of words over time.This is essentially a form of binary notation, akin to representing values as true or false, or 0 and 1. Ambikā Kulkarṇi notes that the Indian discovery of the binary number system largely went unnoticed by Western scholars, likely because *Chandaḥ Śāstra* was primarily regarded as a treatise on poetic meter rather than a mathematical text. [CK]

Piṅgala introduces a concept called *pratyayas* (six in number), in which the metrical elements are explored. He provides sūtras that mathematically formulate *prastāra*, *nāṣṭa*, *uddiṣṭa*, *sāṅkhyā*, *adhvayoga*, and *eka-dvi* (laghu-guru) kriyā-s. Cross-disciplinary exchange of ideas is not that uncommon in the Indian intellectual tradition, as literary studies, aesthetics, and mathematics were not essentially treated as distinct individual domains. Therefore, there exists a certain possibility that the *pratyaya* treatment in music was borrowed from grammatical constructs [AK.3].

Tāla-prastāra is one such concept that has been extensively studied by ancient musicologists. It explores the expansion of a prototype metrical structure into every possible variation according to principles of permutation. Though the concept and the idea remain almost the same, different authors have dealt with and explained them in varied ways. Due to the underlying arithmetic rigor and construction of tables in *prastāras*, mere verses can lead to misunderstandings. A minor misplacement or an error in the directional arrangement of the table can lead to significantly different results.Sārṅgadeva's *Saṅgīta Ratnākara* is one such gem that has documented verses along with figures pertaining to the *prastāra* concept. Upon initial examination, Tippa's verses in this section may seem somewhat perplexing.However, with the help of the *Ratnākara*,one can conclude that Tippa has effectively summarized most of the content presented in it. The figures of the *Ratnākara* and a few recent studies by Akella Mallikarjuna Sharma in the tāla-prastāra methodology have been immensely helpful in understanding Tippa's verses.

Before stepping into the vast sea of ideas and terminologies, here is a gist of essential concepts that will aid in further understanding.Consider a permutation of four elements (*aṅgas*) (in unit time/mātra)

- D (Druta) = 1 unit
- L (Laghu) = 2 units
- G (Guru) = 4 units
- P (Pluta) = 6 units

Given a total mātras n, the objective is to determine all possible ordered arrangements (permutations) of these elements such that their cumulative sum remains constant n. Mathematically, this can be formulated as:

$$x_1 D + x_2 L + x_3 G + x_4 P = n, \tag{2.1}$$

where

$$x_i \in \mathbb{Z}_{\geq 0}, \quad \sum_{i=1}^{4} x_i \geq 1, \quad i \in \{1, 2, 3, 4\}. \tag{2.2}$$

Here, x_i represents the count of each respective *aṅga* in a given sequence. The enumeration of all valid sequences satisfying this equation is known as *Prastāra*.Writing out all the possible combinations as a pyramid is the *meru*. The total number of possible combinations is denoted by *sankhya*. The total sum of occurrences of D, L, G, P across all combinations in the prastāra, (total rhythmic weight of the system) is the *Pātala*. As the above process considers four elements (*aṅgas*), it is termed as the *Chaturaṅga Prastāra*.

The unrestricted enumeration of all possible permutations of the given *aṅgas* while maintaining the sum condition is known as *Akhaṇḍa Prastāra*. A restricted form of *prastāra* where certain *aṅgas* are either omitted entirely or their occurrences are constrained is called *Khaṇḍa Prastāra*. This type of *prastāra* is categorized into two forms:

- *Hīna* – Permutations are generated without a specific *aṅga*.
- *Yukta* – A particular *aṅga* must appear a specified number of times [AK.3]

The process of permutation of all the above divisions is further classified into two based on the directionality of arranging the patterns:

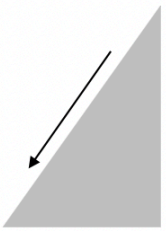
- *Anuloma Prastāra* – The permutations are generated in an obverse (forward) order.
- *Viloma Prastāra* – The permutations are generated in a reverse order.[AK.1]

Beyond the four-elements, expanding the set of rhythmic units to five, six,seven leads to more complex combinatorial structures -*Pañcāṅga Prastāra*, *Ṣaḍaṅga Prastāra* and *Miśritāṅga / Samyutāṅga Prastāras*. Figure (3.7) provides a pictorial representation of the various types of *prastāras*. considering units, expressed in *Druta Permutation* (DP). The directional flow of enumeration is indicated by the grey triangles. c) shows 2 *Druta-Containing Permutation* restriction.One thing to note, both the process and the outcome are referred to as *prastāras*, just as in mathematics, where ”combinations” and ”permutations” denote both the method of arrangement and the resulting set of patterns.The directionality of construction, whether forward (*anuloma*) or backward (*viloma*), may initially seem ambiguous, as it depends on the reference framework used for enumeration, this problem will be taken up in the following sections.

a)

Akhaṇḍa-Anulōma-prastara 6-DP

1						P	*plutānta*
2					L	G	
3				D	D	G	*guruvānta*
4					G	L	
5				L	L	L	
6			D	D	L	L	
7			D	L	D	L	
8			L	D	D	L	
9		D	D	D	D	L	*laghvānta*
10				D	G	D	
11			D	L	L	D	
12			L	D	L	D	
13		D	D	D	L	D	
14				G	D	D	
15			L	L	D	D	
16		D	D	L	D	D	
17		D	L	D	D	D	
18		L	D	D	D	D	
19	D	D	D	D	D	D	*Drutānta*



c)

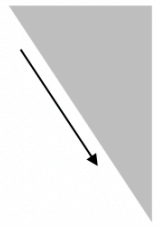
Khaṇḍa:Yukta prastāra (2DCPs)

1				D	D	G	
2			D	D	L	L	
3			D	L	D	L	
4			L	D	D	L	
5				D	G	D	
6			D	L	L	D	
7			L	D	L	D	
8				G	D	D	
9				D	G	D	

b)

Akhaṇḍa-vilōma-prastara 6-DP

1	D	D	D	D	D	D
2		L	D	D	D	D
3		D	L	D	D	D
4		D	D	L	D	D
5			L	L	D	D
6				G	D	D
7		D	D	D	L	D
8			L	D	L	D
9			D	L	L	D
10				D	G	D
11		D	D	D	D	L
12			L	D	D	L
13			D	L	D	L
14			D	D	L	L
15				L	L	L
16					G	L
17				D	D	G
18					L	G
19						P



d)

Khaṇḍa:Hīna prastāra (DDP)

1						P
2					L	G
3					G	L
4				L	L	L

Fig(3.7): Types of *prastāras*, n=6DP

2.6.1 Prastāra, Meru, and Pātāla

Tippa states that the *Prastāra* methodology belongs to both the Mārga and Deśī-Tāla systems (classical and regional rhythmic frameworks)[TD.47]. However, towards the end of the chapter, he names it "Mārgatālaprastārādi Nirūpaṇam", which contradicts his previous statement. Additionally, The Journal of Madras Music Academy (1993) refers to the chapter as Mārgadeśītālaprastārādi Nirūpaṇam. Given that Ratnākara presents his Prastāras within the section on Deśī Tālas, this discrepancy adds to the uncertainty regarding the correct title. Nevertheless, in contemporary practice, Prastāras are predominantly discussed within the Deśī Tāla framework.

Tippa begins his exposition by stating that there exist 53 pratyayās, with the most important among them being *prastāra*, *meru*, *pātāla* and *sankhya* .He also mentions that sages consider the remaining (*śiṣṭāḥ*) *pratyayas* as merely ornamental or artistic elements (*śilpamātra-prayojanāḥ*).This deviates from the concept of 19 *pratyāyas* that *Ratnākara* begins with.

प्रस्तारसङ्ख्ये नष्टं चोद्दिष्टं पातालकस्ततः।
द्रुतमेरुर्लघोर्मेर्गुरुमेरुः प्लुतस्य च॥
मेरुः संयोगमेरुश्च खण्डप्रस्तारकस्ततः।
प्राचां चतुर्णां मेरुणां नष्टोद्दिष्टे पृथक्पृथक्।
एकोनविशतिरिति प्रत्ययास्तान् ब्रुवेऽधुना।। [SR.5(313-315)]

Among these, *Saṅkhyā* and the different kinds of *Meru* are discussed in the later segments of *Tālādīpikā*, but not all concepts are addressed. *Saṅgītaratnākara* directly goes into the methodology of constructing the *Prastāra*, whereas *Tippa* provides the etymology of *Prastāra*, *Meru*, and *Pātāla*.

Tippa's prespective,

The term *prastāra* is derived from the root (dhātu) स्तृ (stṛ), which signifies "to spread" (from *sṛṇāti iti stṛ*). This root conveys the notion of extending or expanding, which in the context of *prastāra*, refers to the systematic enumeration and organization of all possible combinations or patterns.The prefix *pra-* (प्र, *upalakṣaṇa-pratyayaḥ*) adds the sense of sequential arrangement, indicating that *prastāra* involves the structured unfolding of rhythmic combinations in a methodical manner.

Prastāra is not merely an act of expansion but also encompasses the concept of *chādanam* (covering). When an object is placed on a surface, it is understood to cover or conceal the preexisting space. Thus, as the *prastāra* expands, it metaphorically covers or envelops, symbolizing a dual process of unfolding and encompassing.

The boundaries of *prastāra* are delineated by specific limits. The enumeration commences at *kuṣṭa* (the fixed unit at the top), which represents a foundational starting point, and progresses to the *sarva drutā* (all *drutā* combinations), such as "DDDD" for $n = 4$. In this framework, "D" of unit value 1 is considered the basic unit, which does not simplify into smaller elements. At the first glance, both *sarva drutā* and *kuṣṭa* appear to be fixed elements, raising a potential contradiction: how can fixed units, which seem immutable, be part of *prastāra*, a process that by definition implies spreading and transformation? To resolve this apparent contradiction, the concept of *chādanam* within *prastāra* must be invoked. *Chādanam* suggests that even fixed elements, such as *kuṣṭa* and *sarva drutā*, can be elemental participants of the *prastāra*.
It is critical to recognize that the prāstara/patterns originate from a single foundational unit. This unit, in turn, serves not only as the cause but also as the source of multiple combinations. It functions simultaneously as the initiator and the product of the *prastāra* process (*ātmanikāraṇakārya*).

The term *meru* (मरु= म,इ ,रु)is defined using its encompassing letters as its characteristics.The *mā* refers to measurement (*mānam*), while *ī* implies diversity (*vicitratā*). The term *ru* denotes stability or the property of remaining in place(*vasaṃśabdaḥ*).Within the *Prastāra*, some combinations consist of *Drutas*, while others do not (*Adruta*). Consequently, the overall structure comprises distinct different patterns, contributing to its *variety* (*Vicitratā*). Since one of the defining characteristics of *Meru* is *Vicitratā*, it follows that *Meru* is also a form of *Prastāra*.
Pātāla, (पातार्ह अविपाताः स्युः). The first interpretation of Pātāla is derived by analyzing its components: पाता (Patā) + ल (la). The root पत् (Pat) connotes the meanings "to fall" or "to descend" (पतति), while the prefix वि (Vi) suggests the idea of scattering.Consequently, the term विपात (Vipāta) conveys the notion of *dispersion* or *scattering*.The negation अविपातः (Avipātaḥ), therefore, denotes the quality of being impervious to scattering or dispersion, implying that Pātāla remains fixed and does not undergo dispersion, in contrast to the dynamic expansiveness of Meru. In an alternative interpretation, (पत् (Patā) + आलः (Ālaḥ)) is proposed.

In this context, आलः (Ālaḥ) is understood as (आलीयन्ते इति आलः), which translates to "that which merges or combines." This interpretation suggests that Pātāla is characterized by the integration or merging of multiple elements. The implication is that Pātāla signifies a quantity obtained by combining various parts into a unified whole. *Drutādīnām Ālaḥ*, meaning a combination of Druta (rapid or fixed elements) and similar entities. With the clarity provided on the underlying elements, the author proceeds to further explore each of them in detail.

2.6.2 Construction of Prastāra

Tippa largely follows Sarangadeva's methodology in constructing the prastāra elements. Rather than altering the system, he adds connectors and simple explanations to enhance clarity, ensuring a more structured understanding. Before analyzing their methods , here is a simple illustration of the construction of a prastāra for n=4 / 4DP (Druta Permutations) (Fig 3.7.1). The *prastāra* follows a systematic approach, progressing from higher-order units to lower-order units. The directionality is such that larger units are positioned towards the right, while successively smaller units are arranged towards the left. The stepwise decomposition is

a Begin by selecting the largest available unit that can fully represent the target sum. - Since the sum n= 4, the *prastāra* is initialized with G, (referred to as the base unit.

b Place the next lower-order unit directly beneath the base unit. - Here, $L = 2$ is positioned below G. - To maintain the total sum of 4, an additional L is introduced to the left to compensate (LL).

c Analyze and identify the first non druta element from left to right. - This chosen element is replaced by the next lower-order unit. - Here, $D = 1$ is positioned below L. - To maintain the sum, another D is placed accordingly, yielding the configuration DDL.

d The same process is recursively applied to the remaining non-druta elements. - L at the rightmost is the non druta element.D is placed right below. To ensure the sum , 3 units must be placed. The higher order unit amoung them is placed closer to right;and the lower to the left . DLD.

e Repeat the process until all elements are decomposed into the fundamental unit D. The final configuration consists exclusively of Druta (D), $DDDD$ .This marks the completion of the prastāra for $n = 4$.

>Ratnakara's veres

न्यायाल्पमादान्महतोऽधस्ताच्छेषं यथोऽपि। प्रागूने वामसंस्थांस्तु संभवे महतो लिखेत्।।
अल्पानसंभवे तालपूर्यै भोयोऽयं विधिः। सर्वद्राविधः कार्यः प्रस्तारोऽयं लघौ गुरौ ।।
प्लुते व्यस्ते समस्ते च न तु व्यस्ते द्रुतेऽस्ति सः।[SR.5(316-318)]

Tippa largely adheres to the same phrasing as *Ratnākara*, with minimal deviations. The key addition he introduces is the concept of *Ūrdhva-prastāra*, which accounts for directionality within the structure. However, this terminology introduces potential confusion, as the term ūrdhva (meaning "upward") intuitively suggests that the *prastāra* extends upward. In reality, the arrangement begins at the top and expands downward. One positive addition Tippa makes is the clarification of the placement of lesser elements. While their alignment on the left remains relative, his more elaborate wording suggests that these subordinate components should consistently be positioned to the left of the main *kuṣṭa* side. One key aspect to remember is that both texts follow the convention that *kuṣṭa*, the smaller element must be placed at the top. However, "smaller" here does not refer to numerical value but rather to quantity. The element that can singularly represent the entire value is the one that is indicated.

>Comparison of the Prastāra Systems: Ratnākara, Tippa Gopendra vs Pingala, Pingala in his *Chandas Shastra* gives four sūtras for *prastāra*: Dvikau glau, Misrau ca, Pruthaglāmishrāh, and Vasavastrikāh [CK]. All these sūtras pertain only to two elements, namely *laghu* and *guru*. Amba Kulkarni represents these sūtras with matrices as follows:

1. **Dvikau glau:**

$$\begin{bmatrix} G \\ L \end{bmatrix}$$

2. **Misrau ca:**

$$\begin{bmatrix} G & L \\ L & L \end{bmatrix} \quad \begin{bmatrix} G & G \\ L & G \end{bmatrix} \quad \begin{bmatrix} G & G \\ L & G \\ G & L \\ L & L \end{bmatrix}$$

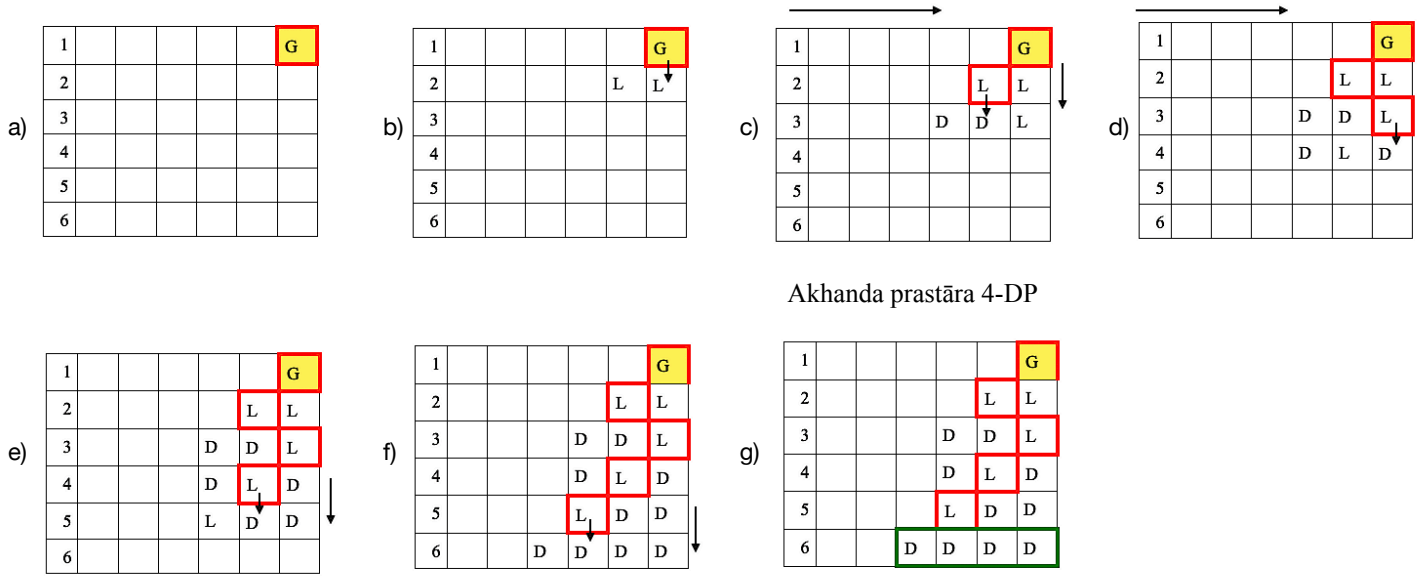
3. **Pruthaglāmishrāh:**

$$\left[\begin{array}{ccc} G & G & G \\ L & G & G \\ G & L & G \\ L & L & G \\ \hline G & G & L \\ L & G & L \\ G & L & L \\ L & L & L \end{array}\right]$$

In comparison to the elaborate verses found in *Saṅgītaratnākara* and *Tāladīpikā*, *Pingala's* work primarily consists of concise *sūtras*. His approach is highly algorithmic, focusing on step-by-step functions to systematically derive the *prastāra*. In contrast, musicological texts like *Ratnākara* and *Tāladīpikā* emphasize direct construction methods for generating the final *prastāra* table, rather than detailing with abstract functional steps. A notable difference is that *Pingala* deals with only two elements (laghu and guru), whereas in musical texts, laghu is not further converted into smaller units. However, the top-down hierarchy, where elements of higher value are placed at the top, followed sequentially by lower-valued elements, remains consistent across all texts.

One key observation is that *Pingala's* sūtras do not explicitly mention directionality. This raises an important question—should the higher-valued elements always be placed on the right? What is the significance of this positional value?This question must be investigated further, as each row in the prastāra corresponds to a tāla used in musical rendition. If the arrangement of elements carries structural significance, does it imply a functional difference in how these tālas are performed or interpreted?

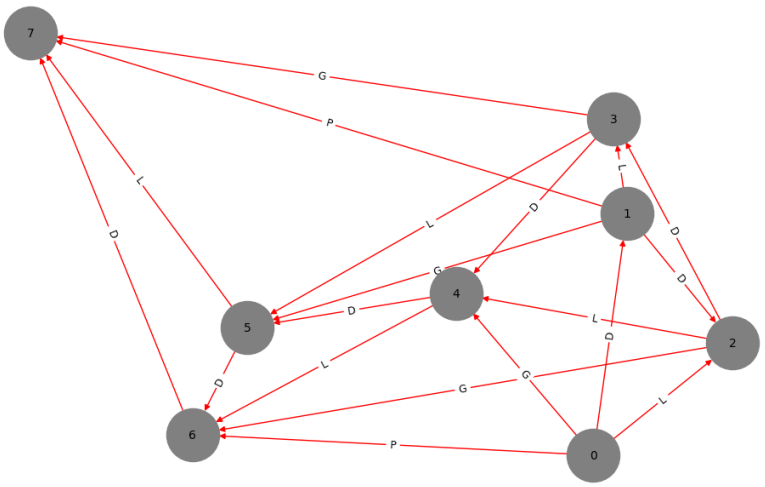
Fig(2.6.2):Illustrating construction of the Prastāra , 4DP



Akhanda prastāra 4-DP

Akhanda prastāra7-DP

1						D	P
2					D	L	G
3					L	D	G
4				D	D	D	G
5					D	G	L
6				D	L	L	L
7				L	D	L	L
8			D	D	D	L	L
9					G	D	L
10				L	L	D	L
11			D	D	L	D	L
12			D	L	D	D	L
13			L	D	D	D	L
14		D	D	D	D	D	L
15						P	D
16					L	G	D
17				D	D	G	D
18					G	L	D
19				L	L	L	D
20			D	D	L	L	D
21			D	L	D	L	D
22			L	D	D	L	D
23		D	D	D	D	L	D
24				D	G	D	D
25			D	L	L	D	D
26			L	D	L	D	D
27		D	D	D	L	D	D
28				G	D	D	D
29			L	L	D	D	D
30		D	D	L	D	D	D
31		D	L	D	D	D	D
32		L	D	D	D	D	D
33	D	D	D	D	D	D	D



2.6.3 Calculation of saṅkhyā

Saṅkhyā represents the total number of rhythmic patterns or possible combinations generated within a given prastāra. Most musical texts calculate the sankhya based on identifying repeated endings. These prastaras or combinations which end with certain respective elements are as follows:

- Druta-ending permutations (DEP) (*Drutānta*)
- Laghu-ending permutations (LEP) (*Lagvānta*)
- Guru-ending permutations (GEP) (*Gurvānta*)
- Pluta ending permutations (PEP) (*Plutānta*)

The construction of *prastāra* can be understood through a systematic approach. This method, originally described by *Akella Mallikarjuna Sharma* for *Panchāṅga Prastāra*, has been simplified into *Chaturāṅga Prastāra* to aid comprehension. In this system, permutations are classified based on their ending elements. The first proxy consists of all Druta-ending permutations (DEP), which include sequences that terminate in druta. Similarly, the second proxy, Laghu-ending permutations (LEP), consists of permutations that conclude with laghu. The third proxy, Guru-ending permutations (GEP), categorizes patterns that end with guru, while the fourth proxy, Pluta-ending permutations (PEP), includes those that conclude with pluta. These classifications serve as fundamental components in the structured analysis of *prastāra.* For any given n , the classification of tāla patterns is determined by systematically identifying occurrences of each ending type. This structured approach enables a clear understanding of how rhythmic variations develop and how different permutations contribute to the overall rhythmic framework. The summation of all proxy classifications provides the total count of unique rhythmic patterns for the given n . For example, when n=2, the possible permutations are: DD (one Druta-ending combination) and L (one Laghu-ending combination). Resulting in 3 unique permutations. Likewise , for n=6, 10 DEP, 6 LEP, 2 GEP and 1 PEP leading to the total number of patterns to be 19. The table (2.6.3) tabulates the same for values of n upto 12.

n.	1st proxy (DEP)	2nd proxy (LEP)	4th proxy (GEP)	6th proxy (PEP)	Total
1	1	-	-	-	**1**
2	1	1	-	-	**2**
3	2	1			**3**
4	3	2	1		**6**
5	6	3	1		**10**
6	10	6	2	1	**19**
7	19	10	3	1	**33**
8	33	19	6	2	**60**
9	61	33	10	3	**106**
10	108	61	19	6	**191**
11	196	108	33	10	**347**
12	350	196	61	19	**626**

n.	1	2	3	4	5	6	7	8	9	10	11	12
Sn	**1**	**2**	**3**	**6**	**10**	**19**	**33**	**60**	**106**	**191**	**347**	**626**

Table:2.6.3 : Tabulation of *sankhya(Sn)*

n.	1st proxy (DEP)		2nd proxy (LEP)		4th proxy (GEP)		6th proxy (PEP)		Total
	S	M	S	M	S	M	S	M	
1	1	-	-	-	-	-	-	-	**1**
2	1	1	1	-	-	-	-	-	**3**
3	2	3	1	1	-	-	-	-	**7**
4	3	7	2	3	1	-	-	-	**16**
5	6	16	3	7	1	1	-	-	**34**
6	10	34	6	16	2	3	1	-	**72**
7	19	72	10	34	3	7	1	1	**147**
8	33	147	19	72	6	16	2	3	**298**
9	61	299	33	147	10	34	3	7	**594**
10	108	596	61	299	19	72	6	16	**1177**
11	196	1182	108	596	33	147	10	34	**2306**
12	350	2316	196	1182	61	299	19	72	**4495**

n.	1	2	3	4	5	6	7	8	9	10	11	12
Mn	**1**	**3**	**7**	**16**	**34**	**72**	**147**	**298**	**595**	**1177**	**2306**	**4495**

Table:2.6.3 : Tabulation of *mahapātala(Mn)*

Both *Saṅgīta Ratnākara* and *Tāla Dīpikā* state that the total possible permutations in the n-druta prastāra is obtained by adding the possible combinations of the first previous, second previous, fourth previous, and sixth previous. This can be represented as :

$$S_n = S_{n-1} + S_{n-2} + S_{n-4} + S_{n-6}. \tag{2.3}$$

where S_n, represents the number of *tāla* forms in the *n-druta-prastāra*.In addition, the respective representatives (*pratinidhi*) of the fourth and sixth terms are assigned as the third and fifth terms, respectively.

>Establishing the Recurrence Formula

Given that compositions in this system use only the elements $\{1, 2, 4, 6\}$, each valid sequence that sums to S_n can be formed by appending one of these values to a smaller sequence. This results in the following recurrence cases:

- If the last element is 1, the remaining sum is S_{n-1}.
- If the last element is 2, the remaining sum is S_{n-2}.
- If the last element is 4, the remaining sum is S_{n-4}.
- If the last element is 6, the remaining sum is S_{n-6}.

By summing all possible extensions of previous valid compositions, we obtain the recurrence relation:

$$S_n = S_{n-1} + S_{n-2} + S_{n-4} + S_{n-6}.$$

>Base Case Verification

To validate this recurrence small values are considered,

$$S_0 = 1, S_1 = 1, S_2 = 2, S_3 = 3, S_4 = 5.$$

Using the recurrence relation:

$$\begin{aligned} S_6 &= S_5 + S_4 + S_2 + S_0 \\ &= 8 + 5 + 2 + 1 = 16. \end{aligned}$$

This confirms the correctness of the recurrence relation for small cases.Refer to [CK] for a rigorous proof.

2.6.4 Construction of different *merus*

The formulation of combinatorial tables, particularly *Meru* structures, serves a crucial role in the systematic derivation of rhythmic entities. A *druta meru*, for instance, facilitates the enumeration of all possible rhythmic combinations for a given n, while imposing constraints on the number of permissible *drutas*. The structured organization of these tables play a pivotal role in facilitating musical compositions and choreographic design, offering a mathematical approach to rhythmic construction.A simple approach to construction methodology :

- Fundamental Table Construction: The final table is assembled progressively using a sequence of intermediate tables.
- Base Layer: The initial *sankhyā* and *mahāpatāla* tables are formulated under the assumption of zero *drutas*, thereby eliminating the need for the 1st proxy.
- Incremental Constraints: Successive layers introduce progressive constraints:
  - The first upper layer incorporates a single *druta* constraint (*1DCP*).
  - The next layer enforces two *druta* constraints (*2DCP*), followed by subsequent iterations extending up to n constraints.
- Final Table Formulation: The rightmost column values of these intermediary tables systematically populate the rows of the final table. (Refer to Fig. 2.6.4 for detailed visualization.)

Let $D_{n,k}$ denote the number of tāla-forms with exactly k drutas in an n-unit prastāra. The recurrence relation is given by:

$$D_{n,k} = D_{n-1,k-1} + D_{n-2,k} + D_{n-4,k} + D_{n-6,k} \tag{2.4}$$

where:

- $D_{n-1,k-1}$ accounts for forms ending in a druta, decreasing the druta count by 1.
- $D_{n-2,k}$, $D_{n-4,k}$, and $D_{n-6,k}$ represent forms ending in a laghu, guru, or pluta respectively, preserving the count of drutas.

Verification for $D(6,0)$

Computing $D(6,0)$ explicitly using the recurrence relation:

$$D(6,0) = D(5,-1) + D(4,0) + D(2,0) + D(0,0)$$

Since $D(5,-1) = 0$ (as negative druta counts are invalid)

$$D(6,0) = D(4,0) + D(2,0) + D(0,0)$$

From base cases:

$$D(0,0) = 1, \quad D(2,0) = 1, \quad D(4,0) = D(3,-1) + D(2,0) + D(0,0) = 0 + 1 + 1 = 2$$

Thus,

$$D(6,0) = 2 + 1 + 1 = 4$$

This value is represented in the final table, sixth column and second row.

Taladipika and Ratnakara's method:

For constructing an *n-druta prastāra*, the process begins with an initial row containing *n* entries. Each subsequent row omits the first column, with two additional columns removed at each successive stage.

- The first two entries in every row are always set to 1.
- In the first row, values in odd-numbered positions are determined by summing the preceding, second-preceding, fourth-preceding, and sixth-preceding entries, following the pattern of *saṅkhyāṅkas*.
- Even-numbered positions are computed using the same sum, except that the immediate previous entry is excluded.
- If the fourth or sixth-preceding entry is absent, the third or fifth-preceding entry is used as a substitute.

For subsequent rows:

- Odd-positioned entries continue to be derived using the summation of the previous, second-preceding, fourth-preceding, and sixth-preceding values.

- Even-positioned entries follow the same summation rule, with the distinction that the immediate preceding value is taken from the row below.
- If the fourth or sixth-preceding value is unavailable, no replacement is applied.

Upon completion of the table, reading values from bottom to top in the n-th column reveals the number of possible *tāla* structures within the *n-druta prastāra*:

- When n is odd, the sequence represents the count of *tālas* with 1, 3, 5, etc., *drutas*.
- When n is even, the sequence corresponds to the count of *tālas* with 0, 2, 4, etc., *drutas*.

One important observation is that this method is defined for odd and even numbers of *drutas* separately. Although *Tippa Gopendra* does not explicitly outline the construction procedure for *mahāpatala*, it is implicit that knowledge of it is assumed. Similarly, the construction of other *merus*, such as *laghu*, *guru*, and *pluta merus*, follows the same foundational principles. *Tāladipika* does not introduce any new elements beyond what is described in *Saṅgīta Ratnākara*; rather, this segment of his work primarily consists of summarizing the existing descriptions.Even though various texts employ different coding schemes and mediums of conveyance, it is remarkable that they all converge toward similar structural and tabular formulations. Figure 3.7.5 illustrates the same, highlighting the underlying mathematical consistency across these treatises.

Comparing with Piṅgala and Kedārabhaṭṭa's Meru Constructions

The *meru* construction in the grammatical tradition consists of only two elements—*laghu* and *guru*—and the algorithm is known as the *ka-dvi-ādi-l-g-kriyā*. The sequence of numbers $1, 6, 15, 20, 15, 6, 1$ represents the number of possible combinations when selecting all *gurus*, one *laghu*, two *laghus*, three *laghus*, and so forth, up to the case where all are *laghus*. This expansion closely resembles Pascal's triangle, which provides the binomial coefficients $\binom{n}{r}$, representing the number of ways to choose r elements from n.[com] Though the structures appear different from those found in musical treatises, the underlying numerical entities and principles remain the same. This further strengthens the argument that these grammatical tools were adapted and extended to the domain of music.

Lower line

Sankhya

n.	2nd proxy	4th proxy	6th proxy	Total
1	-	-	-	0
2	1	-	-	1
3	0	-	-	0
4	1	1	-	2
5	0	0	-	0
6	2	1	1	4
7	0	0	0	0
8	4	2	1	7
9	0	0	0	0
10	8	4	2	14
11	0	0	0	0
12	15	8	4	27

Mahapātala

n.	2nd proxy		4th proxy		6 th proxy		Total
	S	M	S	M	S	M	
1	-	-	-	-	-	-	0
2	1	-	-	-	-	-	1
3	0	0	-	-	-	-	0
4	1	1	1	-	-	-	3
5	0	0	0	0	-	-	0
6	2	3	1	1	1	-	8
7	0	0	0	0	0	0	0
8	4	8	2	3	1	1	19
9	0	0	0	0	0	0	0
10	8	20	4	8	2	3	45
11	0	0	0	0	0	0	0
12	15	47	8	20	4	8	102

Upper line

Sankhya

n.	1st proxy of the lower line	2nd proxy	4th proxy	6th proxy	Total
1	1	-	-	-	1
2	0	-	-	-	0
3	1	1	-	-	2
4	0	0	-	-	0
5	2	2	1	-	5
6	0	0	0	-	0
7	4	5	2	1	12
8	0	0	0	0	0
9	8	12	5	2	27
10	0	0	0	0	0
11	15	28	12	5	60
12	0	0	0	0	0

Mahapātala

n.	1st proxy of the lower line		2nd proxy		4th proxy		6 th proxy		Total
	S	M	S	M	S	M	S	M	
1	1	-	-	-	-	-	-	-	1
2	0	0	-	-	-	-	-	-	0
3	1	1	1	1	-	-	-	-	4
4	0	0	0	0	-	-	-	-	0
5	2	3	2	4	1	1	-	-	13
6	0	0	0	0	0	0	-	-	0
7	4	8	5	13	2	4	1	1	38
8	0	0	0	0	0	0	0	0	0
9	8	20	12	38	5	13	2	4	102
10	0	0	0	0	0	0	0	0	0
11	15	47	28	104	12	38	5	13	262
12	0	0	0	0	0	0	0	0	0

	1	2	3	4	5	6	7	8	9	10	11	12	
	1	**0**	**2**	**0**	**5**	**0**	**12**	**0**	**27**	**0**	**60**	**0**	1-DCP
	1	0	4	0	13	0	38	0	102	0	262	0	
1	**0**	**1**	**0**	**2**	**0**	**4**	**0**	**7**	**0**	**14**	**0**	**27**	No DCP
	0	1	0	3	0	8	0	19	0	45	0	102	

						1							
					1		1						
				1		2		1					
			1		3		3		1				
		1		4		6		4		1			
	1		5		10		10		5		1		
1		6		15				15		6		1	
1		7		21					21		7		1

Pińgala , laghu meru

	1	1	1	1	1	**1**
1	2	3	4	5	**6**	
1	3	6	10	**15**		
1	4	10	**20**			
1	5	**15**				
1	**6**					
1						

Kedār Bhatt, laghu meru

Laghu Meru

	1	2	3	4	5	6	7	
		1	**2**	**3**	**4**	**7**	**12**	1-LCP
		1	4	9	16	29	54	
1	**1**	**1**	**1**	**2**	**3**	**5**	**7**	No-LCP
	1	2	3	5	9	16	27	

Guru Meru

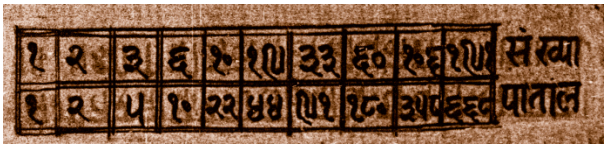
	1	2	3	4	5	6	7	
				1	**2**	**5**	**10**	1-GCP
				1	4	13	34	
1	**1**	**2**	**3**	**5**	**8**	**14**	**23**	No-GCP
	1	3	7	15	30	59	113	

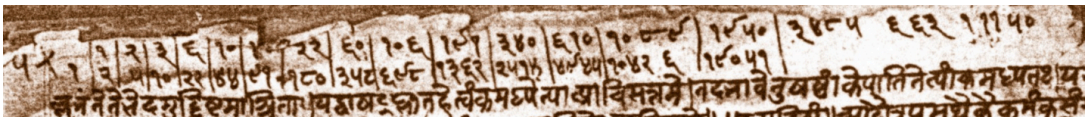
Pluta Meru

	1	2	3	4	5	6	7	
						1	**2**	1-PCP
						1	4	
1	**1**	**2**	**3**	**6**	**10**	**18**	**31**	No-PCP
	1	3	7	16	34	71	143	

Fig(3.7.5): Tracing Prastāra Across Generations and Traditions

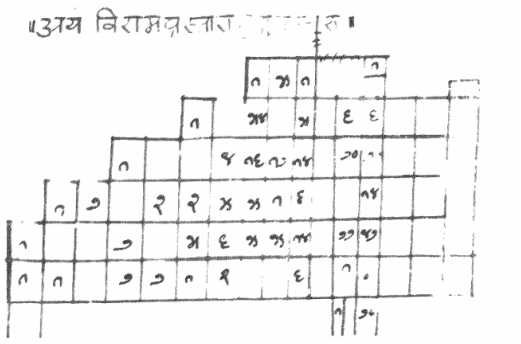
sangīta ratnakara



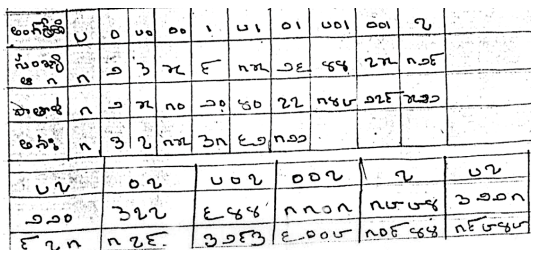


nartana nirnayam

deśha prana-prastāra (telugu)



sangīta suryōdaya



tāla prastaramu (telugu)

2.6.5 Order of usage and the Number system

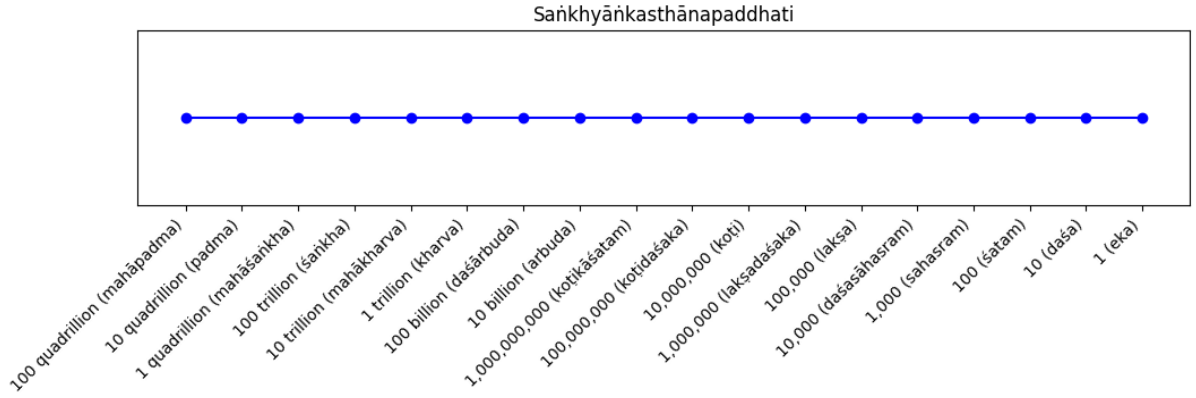
The final section of the *Mārgadeśīprastāranirūpaṇam* examines the application of previously discussed *prastāra* concepts and explains the *Saṅkhyāṅkapaddhati*—the numerical system. In addition to this, Tippa also provides insights into the *Kaṭapayādi* system, elaborating on its governing rules and its applications in phonetics.The author systematically presents the order of usage of *prastāra* constructions with respect to the number of *mātrās* in a *tāla*. *Mātrās* serve as fundamental units, normalized to *laghu*.[RN] Consequently, the values of *Druta* (D), *Laghu* (L), *Guru* (G), and *Pluta* (P) in terms of *mātrās* are as follows: *Druta* (D) $\frac{1}{2}$ *mātrā*, *Laghu* (L) 1 *mātrā* ,*Guru* (G) 2 *mātrās* and *Pluta* (P) 3 *mātrās*

Upon understanding the fundamental principles governing the generation of these structures, the appropriate constructions naturally follow. For instance, when dealing with a single *mātrā*, it is evident that the only possible compositions are either one *Laghu* (L) or two *Druta*-s (D). Given this constraint, it becomes redundant to systematically tabulate the *Druta*, *Guru*, or *Pluta Meru*, as their presence in this case is either singularly defined or nonexistent. Specifically, there exists only one valid combination that includes *Druta*, while *Guru* and *Pluta* do not feature in any permissible configurations. To eliminate any confusion, the author presents all possible calculations for *mātrā* values of 1, 1.5, 2, and greater than 3. For all *tālas* with *mātrās* exceeding three, all computations become feasible due to the inclusion of all structural elements up to *Pluta* The table(2.6) illustrates the same.

Mātrā Count	**No. of Methods**	**Methods**
1 Mātrā	5	Prastāra, Druta-Meru, Laghu-Meru, Druta-Pātāla, Laghu-Pātāla
1.5 Mātrās	5	Prastāra, Druta-Meru, Laghu-Meru, Druta-Pātāla, Laghu-Pātāla
2 Mātrās	6	Prastāra, Turyaṅga-Meru, Druta-Meru, Laghu-Meru, Druta-Pātāla, Laghu-Pātāla
3+ Mātrās	9	Stāra, Druta-Meru, Laghu-Meru, Guru-Meru, Pluta-Meru, Druta-Pātāla, Laghu-Pātāla, Guru-Pātāla, Pluta-Pātāla

Table 2.6.5:Combinatorial Methods for Different Tāla Mātrā Counts

Elucidating the number system or the *Kaṭapayādi* system within a musical treatise might seem unconventional.However, given the vast scale of *prastāras* and combinatorial possibilities—reaching exceedingly large numbers—it becomes a logical necessity for the author to address these systems. While his primary focus remains on *Caturaṅga Prastāra*, he also introduces an additional *aṅga* called *Ardhadruta*. This later solidifies into *Anudruta*, ultimately contributing to the evolution of *Pañcāṅga Prastāra* constructions, which exhibit even greater complexity.The number system he presents is presently recognized as the Indic numerical system.The same is depicted in Fig (2.6.5).



2.7 Conclusion

This study has examined the mathematical structures and encoded methodologies employed by *Tippa Gopendra Bhūpāla* in *Tāladīpikā*. The framework curated by Tippa, known as the *tāla deśa prāṇas*, has been explored in detail. Various forms of coded representations have been identified, revealing the systematic formulation of *mārga tālas* along with their symmetrical properties and applications.A total of 223 *deśī tālas* have been enlisted, decoded, and compared with *Saṅgīta Ratnākara*, demonstrating a more structured and refined classification in *Tāladīpikā*. Further, the study of the *prastāra* methodology confirms its fundamental alignment with *Saṅgīta Ratnākara*, while incorporating certain refinements and structural enhancements introduced by Tippa. The construction of various combinatorial rhythmic frameworks has also been analyzed, shedding light on the mathematical underpinnings of these rhythmic structures.

Both *Tāladīpikā* and *Tantraratna* emerge as invaluable texts, highlighting the scholarly contributions of *Tippa Gopendra Bhūpāla* and bringing his intellectual legacy to the forefront. Additionally, this study reinforces the richness of the Vijayanagara Empire's academic and artistic traditions. A deeper exploration of these works is anticipated to further unravel the intricacies of these remarkable creations.

2.8 Remarks

This study serves as a preliminary analysis and represents only an initial step in the broader exploration of this subject.It is a humble attempt to grasp a small fragment of the vast ocean of knowledge. While this study marks the conclusion of the preliminary analysis of the manuscript, the depth and complexity of *Tippa Gopendra's* work necessitate further inquiry. His extensive contributions continue to provide vast opportunities for deeper exploration and critical examination.

Tippa Gopendra's final remarks in Tāladipika:

> "Each tāla system—Mārga and Deśī—must be systematically analyzed within its respective framework. If the rules for Prastāra, Meru, and Pātāla are not well established, an expert in tāla science may struggle to apply them effectively. Therefore, only a learned tāla expert, possessing deep understanding, can truly comprehend and apply these principles. Great sages like Śailādi, Kapiśāṇḍilya, and Śārṅgadeva have deliberated upon these principles in depth. For ordinary scholars like us, understanding these great methodologies is a difficult endeavor. O revered sages, the ocean of tāla-prastāra is vast, and I am but a humble seeker attempting to document this profound knowledge. Even if my understanding is incomplete, I seek the guidance of tāla experts to refine my comprehension. May those who have expounded upon the tāla tradition forgive my shortcomings in this endeavor." [TD]

2.9 References

1.Lakshminarayana *Sangitasuryodaya of Lakshminarayana - Adyar Library*. MusicResearchLibrary, accessed.

2.Premalatha, V. (2009). Talakalabdhi of Acyutaraya: Discovery of a new work on Tāla. *Journal of the Madras Music Academy*, 80, 94-102.

3.Pandarika Vitthala *Nartananirnaya of Pandarika Vitthala 1 - OIB-11620*. MusicResearchLibrary, accessed.

4.Ramanathan, N. (1997). *Essays On Tala And Laya*. Percussive Arts Centre, Bangalore.

5.Ramanathan, N. (1999). Musical Forms in *Saṅgītaratnākara*. *Sampradaya*, Chennai.

6.Sarngadeva. *Sangitaratnakara of Sarngadeva -1 - Asiatic Society, Kalkatta: 9051*. MusicResearchLibrary, accessed.

7.Sharma, A. M. (2003). *Systematisation of Prastara Details of Deshi Talas*. Hyderabad: Sai Sannidhi Sangita Publications.

8.Srinivas, M. D. (2010). Combinatorial Methods in Indian Music: Pratyayas in *Saṅgītaratnākara* of Śārṅgadeva. Springer.

9.Tippabhupala. *Taladipika of Tippabhupala - GOML-R770*. MusicResearchLibrary, accessed.

10.Akella, M. S. (2001). *Indian Genius in Talaprastara*. Hyderabad: Sai Sannidhi Sangita Publications.

11.Toussaint, Godfried T. 2013. *The Geometry of Musical Rhythm: What Makes a "Good" Rhythm Good?* Boca Raton, FL: Chapman and Hall/CRC.