Aspects of cosmological observables

विद्या वाचस् की

उपाधि की अपेक्षाओं की आंशिक पूर्ति में प्रस् शोध प्रबंध

A thesis submitted in partial fulfillment of the requirements of the degree of **Doctor of Philosophy**

द्वारा / By

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भारतीय विज्ञान शिक्षा एवं अनुसंधान संस् पुणे

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH PUNE

2025

Dedicated to my family
La familia es todo.

Certificate

Certified that the work incorporated in the thesis entitled "**Aspects of cosmological observables**" submitted by **Farman Ullah** was carried out by the candidate, under my supervision. The work presented here or any part of it has not been included in any other thesis submitted previously for the award of any degree or diploma from any other University or institution.

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Title of Thesis: Aspects of cosmological observables

I declare that this written submission represents my idea in my own words and where others' ideas have been included; I have adequately cited and referenced the original sources. I declare that I have acknowledged collaborative work and discussions wherever such work has been included. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

The work reported in this thesis is the original work done by me under the guidance of Prof. Diptimoy Ghosh.

Farman Ullah

Date: 15/05/2025

Acknowledgements

I want to start by thanking my PhD supervisor, Prof. Diptimoy Ghosh. He has been an excellent mentor and a source of constant support. I am grateful to him for spending countless hours discussing physics with me and explaining and clarifying many concepts. I also want to thank him for facilitating my visits to international conferences and, therefore, connecting me to the international community in my field. I would also want to express my sincere gratitude to the members of my RAC committee, Prof. Paolo Creminelli and Prof. Aleksandr Azatov. Prof. Creminelli, being a pioneer in the field, has always been a source of inspiration. He has always been very generous yet straightforward in his mentorship. I am extremely grateful to him for guiding me from time to time and also for clarifying a lot of my doubts regarding cosmology and inflation in particular. I also want to thank him for helping me with my postdoctoral applications. I want to thank Prof. Azatov for always highlighting the importance of experiments/observations in theoretical physics research. I also want to extend my deep gratitude to Prof. Enrico Pajer for being a wonderful collaborator and a mentor. In particular, for being extremely generous with his time and helping me with my postdoctoral applications. I also want to thank all of my wonderful collaborators, Prof. Arun Thalapillil, Amartya Harsh Singh, Kushan Panchal, Rajat Sharma, Supritha Bhowmick and Prof Enrico Pajer. I would like to thank all the faculty in the physics department. In particular, Prof. Sachin Jain, Prof. Sunil Mukhi and Prof. Suneeta Vardarajan for exciting discussions during the physics journal club. I am very thankful to all high-energy physics students for making the second floor an exciting place. I am grateful to Arhum, Ali, Saurabh, Nipun, Lalit, Sakil, Dhruva and Deep for stimulating discussions during our weekly discussion club.

I am very grateful to my friends at IISER for making these years exciting and enjoyable. In particular, I want to thank Ali, Arhum, Lalit, Nipun, Shakeel and Saurabh for making this journey worthwhile. I will miss our Saturday outings and stimulating discussions about religion, politics and life in general. I am thankful to Saurabh for making the office a lively place. I will miss our absurd yet hilarious conversations. I am also grateful to my Kashmiri friends, including Manzoor, Saleem, Yunus, Zahid, Ishtiyaq, Hilal, Saqlain, Saim and Usama for making me feel at home.

I am extremely grateful to my whole family for being a supportive force throughout this journey. In particular, I am thankful to my mother and father for always being there for me, especially during tough times. I am grateful to my wife for encouraging me and boosting my self-confidence from time to time, and hence helping me improve as a physicist. I am also thankful to my sisters for guiding me throughout my academic life and sparking my love for science.

Contents

Abstract

One of the goals of early universe cosmology is to understand the physics of inflation. Since we cannot visually access this epoch, we rely on indirect observations, e.g., measuring temperature correlations on the CMB. By analysing various features of these correlation functions, we can infer details of the inflationary epoch. However, the observational data is not yet powerful enough to give a detailed sketch of this period. This is because these correlations are very feeble and hence difficult to measure. So far, we have only measured the two-point function and its dependence on the wavelength. Measuring three- and higher-point functions is very challenging; however, one can still make theoretical progress. This can be achieved by understanding various mathematical structures present in the correlation functions and tracing their precise physical origin. This approach is very well developed in the case of flat space scattering amplitudes. For instance, it is well known that locality is related to the poles structure of the amplitude and unitarity is encoded in the Optical theorem and the Cutkosky cutting rules. In the case of cosmology, these ideas have been developed only very recently.

In this thesis, we develop these ideas further by first analysing the tree-level analytic structure of scalar correlation functions in momentum space. In particular, we try to establish a connection between the pole structure and the initial state of inflation. We find that while only excited states can produce folded poles, the absence of such poles does not imply a non-excited initial state, i.e., the vacuum. We then proceed to discuss well-known inflationary soft theorems. Although the proof of soft theorems is very general, it is always good to have explicit checks. By taking specific operators in the Effective Field Theory of Inflation (EFToI), we perform explicit checks of soft theorems for mixed correlation functions (scalar+tensor). Such an explicit check was missing in the case of mixed soft theorems (to the best of our knowledge). We then discuss the boostless bootstrap and apply it to certain higher-dimensional EFToI operators.

There has been very little progress in computing cosmological loops because of their mathematical complexity. Most of the literature has been devoted to computing the inflationary power spectrum at one-loop. We take the next step and compute the three-point function at one-loop in the EFToI. We explicitly compute the log running and find the following two structures: 1) $\log H/\mu$ & 2) $\log k_i/k_T$. We verify our results using both cut-off and dimensional regularisation. In the case of dim-reg, we notice that the unphysical logarithms of the form $\log k/\mu$, which are ratios of co-moving scale and a physical scale, cancel only after properly renormalising the results. In the case of cut-off regularisation, these unphysical logs never show up.

Finally, we discuss the implications of unitarity for cosmological wavefunction coefficients. In particular, we construct Cutkosky-like cutting rules for wavefunction coefficients at any order in perturbation theory, assuming a Bogoliubov initial state. Such a state is an excited state over the Bunch-Davies vacuum. We derive implications of these rules for n-point contact and four-point exchange diagrams. We also give an efficient prescription for getting the Bogoliubov wavefunction coefficients directly from the Bunch-Davies ones without performing complicated time integrals. Finally, we extend these cutting rules to fields of any mass and spin.

List of Publications

1. D. Ghosh, A. Thalapillil, F. Ullah, Astrophysical hints for magnetic black holes, *Phys. Rev. D 103 (2) (2021) 023006. arXiv:2009.03363.*

2. ⋆ D. Ghosh, A. H. Singh, F. Ullah, Probing the initial state of inflation: analytical structure of cosmological correlators, *JCAP 04 (2023) 007. arXiv:2207.06430.*

3. D. Ghosh, R. Sharma, F. Ullah, Amplitude's positivity vs. subluminality: causality and unitarity constraints on dimension 6 & 8 gluonic operators in the SMEFT, *JHEP 02 (2023) 199. arXiv:2211.01322.*

4. ⋆ D. Ghosh, K. Panchal, F. Ullah, Mixed graviton and scalar bispectra in the EFT of inflation: Soft limits and Boostless Bootstrap, *JHEP 07 (2023) 233. arXiv:2303.16929.*

5. ⋆ S. Bhowmick, D. Ghosh, F. Ullah, Bispectrum at 1-loop in the Effective Field Theory of Inflation, *JHEP 10 (2024) 057. arXiv:2405.10374.*

6. ⋆ D. Ghosh, E. Pajer, F. Ullah, Cosmological cutting rules for Bogoliubov initial states, *SciPost Phys. 18 (1) (2025) 005. arXiv:2407.06258.*

7. ⋆ D. Ghosh, F. Ullah, Cosmological cutting rules for Bogoliubov initial states: any mass and spin, *(2 2025). arXiv:2502.05630.*

8. S. Bhowmick, M. H. G. Lee, D. Ghosh, F. Ullah, Singularities in Cosmological Loop Correlators, *(3 2025). arXiv:2503.21880.*

Only entries marked with an asterisk are included in this thesis.

Chapter 1

Introduction

1.1 Background

It is believed that the structure in the universe, i.e., stars, galaxies, and clusters of galaxies, has a simple quantum mechanical origin. Quantum fluctuations during an early phase called inflation [1, 2, 3, 4], where the universe expands and accelerates exponentially fast, stretch out due to the expansion. Once the wavelength of the fluctuations crosses the Hubble scale, their amplitude freezes, which makes them potential seeds for structure formation [5, 6]. To be precise, these small fluctuations start clumping due to gravity and result in the formation of stars, galaxies and eventually us. Unfortunately, we do not have direct visual access to this early inflationary phase; rather, the earliest epoch visible through photons is roughly 380,000 years after the beginning, and this radiation is known as the Cosmic Microwave Background (CMB) radiation [7]. This radiation comes from an epoch when photons no longer had enough energy to ionize hydrogen and, therefore, they decoupled and travelled freely towards us. The quantum mechanical fluctuations during inflation lead to energy density fluctuations in this radiation, which eventually lead to temperature fluctuations in the CMB (see Fig. 1.1). It turns out that the statistics of these temperature fluctuations, encoded by their correlation functions, contain very useful information which can help us reconstruct the early phase of the universe, which is not visually accessible. In particular, these correlation functions contain information about the fields, particles and interactions present in the inflationary phase. These correlation functions encode spatial correlation at equal time since the CMB surface we see is an equal time surface present roughly at a redshift of $z = 1100$. Since inflation is believed to be the origin of these, the inflationary correlation functions we will be interested in are also evaluated at equal time.

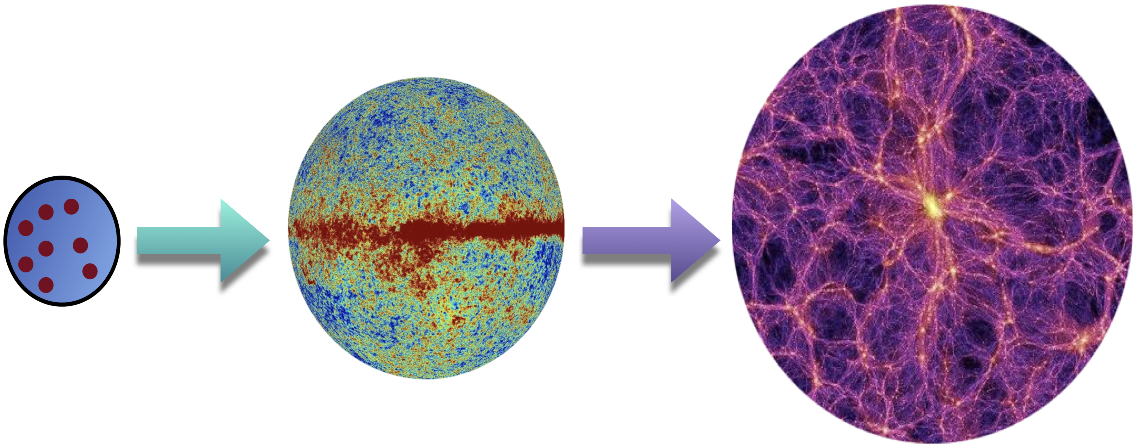


FIGURE 1.1: Primordial inflationary fluctuations (extreme left) give rise to temperature fluctuations in the CMB (middle), which through gravitational clumping give rise to stars, galaxies, clusters (extreme right)

So far, we have measured the two-point function, i.e., $\langle \delta T \delta T \rangle$ and its dependence on the wavelength (or momentum k). This is known as the spectral tilt, denoted by $n_s - 1$, defined by

$$n_s - 1 = \frac{d \log k^3 P(k)}{d \log k}\,, \tag{1.1}$$

where $P(k)$ is the two-point function in Fourier space. One often defines a modified quantity, $\mathcal{P}(k)$,

$$\mathcal{P}(k) = \frac{k^3 P(k)}{2\pi^2} = \mathcal{A}_s \times k^{n_s - 1} \tag{1.2}$$

Current observations [8, 9] tell us,

$$n_s - 1 = 0.9649 \pm 0.0042\,, \tag{1.3}$$

$$\mathcal{A}_s = (2.099 \pm 0.029) \times 10^{-9} \tag{1.4}$$

Notice that the amplitude of the two-point function is already quite small; higher point functions are even more suppressed. Although many future observational programs are planned to measure these, such as CMB S-4, CMB Bharat, LiteBIRD, and LISA, in the meantime, one would still want to make theoretical progress in understanding the higher point functions. This is very important since higher point functions carry information about the initial state, field content and interactions present during inflation. For instance, the three-point function (also known as the bispectrum) is a very good probe of field content and interactions [10, 11]. Due to translational invariance, it forms a triangle in Fourier space. By looking at where the bispectrum peaks, one can get clues about the fields and the interactions, e.g., for single-field inflation the bispectrum is small for squeezed configurations (see Fig. 1.2: right panel), but for multi-field models the bispectrum peaks at these configurations. In the case of the initial state of inflation, the analytic structure of the bispectrum is a very good probe, e.g., poles in the folded configuration (see Fig. 1.2: left panel) cannot be produced by a vacuum initial state [12, 13, 14, 15, 16, 17, 18, 19]. Also, it is important to understand the constraints various physical principles like locality, unitarity, etc, enforce on these correlation functions. In other words, how can one check whether a correlation function came from a unitary or a local theory? There has been a lot of progress in this direction [20, 21, 22, 23, 24]; in the case of unitarity, correlation functions follow a set of Cutkosky-like cutting rule relations, and to check locality, one uses the so-called Manifest Locality Test (MLT).

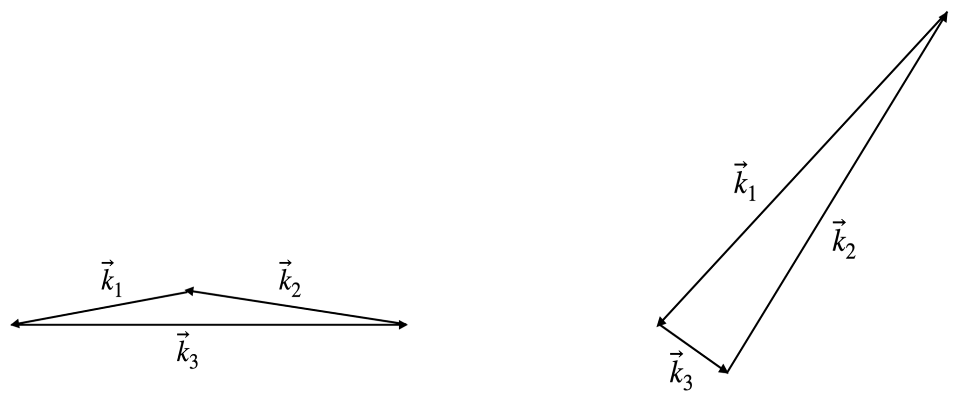


FIGURE 1.2: Left panel: Folded momentum configuration with $k_1 + k_2 \sim k_3$. Right panel: Squeezed momentum configuration with $k_3 \ll k_1, k_2$.

It is also important to understand the structure of observables at the loop level. Inflationary loops are extremely suppressed compared to tree-level, and the tree-level results themselves are very small. Therefore, loop effects do not seem to be observationally relevant. However, as Steven Weinberg emphasised [25, 26], it is important to explore and understand the mathematical aspects of our theories. This could shed light on their possible generalisations. Also, this theoretical data might be useful to make powerful statements about observables, e.g., as in the positivity bound program in flat space [27, 28, 29, 30, 31, 32]. Unlike flat space amplitudes, the structure of cosmological observables at the loop level is not very well known. Since inflation is a time-dependent background, there is no time-translational invariance, which leads to complicated nested time integrals in the computation of correlation functions. Also, the mode functions are not simple exponentials, i.e., e^{ikt}. These features make it difficult to compute the observables, sometimes even at tree-level. Since having a loop will include momentum integrals on top of these time integrals, naturally, the computations very quickly become intractable. It is only in a very few simple cases that one can compute explicit expressions.

Until this point, we have been discussing temperature fluctuations in the CMB, which result from scalar modes during inflation. However, inflation also produces tensor modes, i.e., gravitational waves. So far, no signal has been detected, although there is an experimental bound on their amplitude. This bound is often expressed in terms of the ratio (denoted by r) of the tensor two-point function amplitude and the scalar two-point function amplitude. The current bound [8, 9] is the following

$$r < 0.036\,. \tag{1.5}$$

1.2 Inflation

As mentioned above, inflation is a stage of rapid, near-exponential expansion of the universe where the scale factor accelerates, i.e., $\ddot{a} > 0$. In this section, we will discuss the concept of inflation through its simplest realisation, i.e., single-field slow-roll inflation.

1.2.1 Classical evolution

In the simplest model, inflation is achieved by a single homogeneous scalar field $\phi(t)$ called the inflaton with the following equation of motion,

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0\,, \tag{1.6}$$

where a prime (') represents a derivative with respect to ϕ. This field slowly rolls down a very flat potential (see Fig. 1.3). In other words, the field kinetic energy is smaller than the potential energy. To illustrate how this works, consider the second Friedmann equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right)\,, \tag{1.7}$$

where ρ, p are the energy density and pressure of the fluid present. In the case of a scalar field,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V\,, \tag{1.8}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V\,. \tag{1.9}$$

This implies, $\ddot{a} \propto \left(\dot{\phi}^2 - V\right)$ and therefore, when $\dot{\phi}^2 < V$, inflation will happen. This is the minimum requirement to have an inflationary phase. An efficient and relatively long-lasting phase of inflation can be achieved by the so-called slow-roll conditions given as,

$$\frac{1}{2}\dot{\phi}^2 \ll V\,, \tag{1.10}$$

$$\ddot{\phi} \ll 3H\dot{\phi}\,. \tag{1.11}$$

The first condition ensures inflation starts, and the second condition ensures that it lasts sufficiently long. These notions can be made more precise by defining a set of parameters,

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2M_p^2 H^2}\,, \tag{1.12}$$

$$\eta = 2\epsilon - \frac{\dot{\epsilon}}{2\epsilon H}\,. \tag{1.13}$$

The slow-roll conditions mentioned above imply $\epsilon, \eta \ll 1$. In the slow-roll limit, these parameters can be written in terms of the potential and its derivatives as follows,

$$\epsilon = \frac{1}{2}M_p^2\left(\frac{V'}{V}\right)^2\,, \tag{1.14}$$

$$\eta = M_{pl}^2\frac{V''}{V} \tag{1.15}$$

Since the energy density of the inflaton changes extremely slowly, it can be treated as a constant in the first approximation. Then, using the first Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_p^2}\,, \tag{1.16}$$

one obtains an exponentially expanding scale factor i.e., $a \simeq e^{Ht}$. The actual Friedmann equation also has a $-\frac{K}{a^2}$ term on the right. This term captures the spatial curvature, but since the accelerated expansion during inflation drives it to zero very quickly, we can safely ignore it. Notice that this implies the inflationary phase can be well approximated by de Sitter space in flat slicing, whose metric is given by

$$ds^2 = dt^2 + e^{Ht}d\vec{x}^2 \tag{1.17}$$

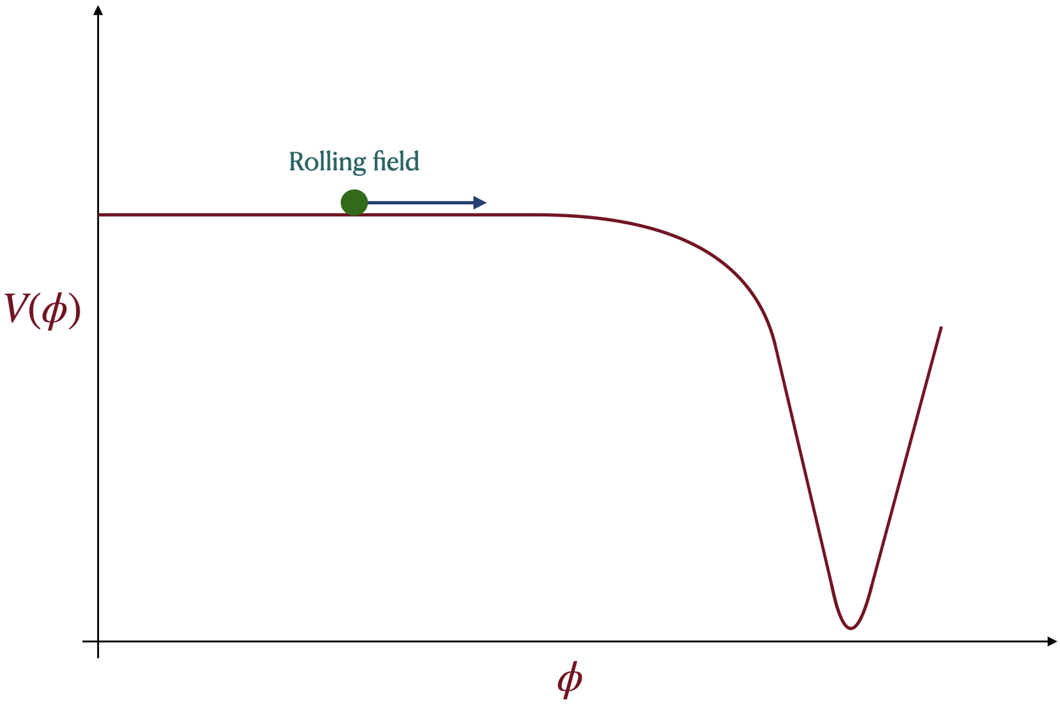


FIGURE 1.3

1.2.1.1 Properties of de Sitter spacetime

We digress slightly to discuss properties of de Sitter spacetime (we mainly follow T. Hartman's lecture notes on de Sitter space and [33]). These properties can be very useful for understanding inflationary observables (in first approximation) since the metric can be approximated by de Sitter space to a very good accuracy.

De Sitter space is a solution of Einstein's field equations with a positive cosmological constant. It is a maximally symmetric spacetime with $\frac{d(d+1)}{2}$ isometries. Isometries are transformations which leave the functional form of the metric invariant, i.e., under an isometry transformation $x^\mu \to \tilde{x}^\mu = x^\mu + \epsilon^\mu$,

$$g_{\mu\nu}(x) \to \tilde{g}_{\mu\nu}(\tilde{x}) = g(\tilde{x})\,. \tag{1.18}$$

A d-dimensional de Sitter spacetime can be thought of as a hyperbolic embedding in a $d+1$-dimensional Minkowski spacetime (see Fig. 1.4). Therefore, its metric can be written as,

$$ds^2 = -(dX^0)^2 + \Sigma_{i=1}^d dX^{i^2}\,, \tag{1.19}$$

with the constraint $-(X^0)^2 + \Sigma_{i=1}^d (X^i)^2 = l^2$. This is the equation for a Hyperbola. This construction makes the isometries of de Sitter space very obvious. We need to find symmetry transformations of a $d+1$ dimensional Minkowski space subject to the above-mentioned constraint. Notice that the constraint equation is just a Lorentz contraction $X_\mu X^\mu = l^2$. Therefore, the isometry group for a d-dimensional de Sitter spacetime is $SO(d,1)$. The symmetry group is not $ISO(d,1)$ since translations do not keep the constraint invariant. The group $SO(d,1)$ contains $\frac{d(d-1)}{2}$ rotations and d boost transformations adding up to $\frac{d(d+1)}{2}$ (expected for a maximally symmetric space). The generators of the symmetry transformations form the usual Lorentz algebra,

$$[M_{AB}, M_{CD}] = \eta_{BC} M_{AD} - \eta_{AC} M_{AD} + \eta_{AD} M_{BC} - \eta_{BD} M_{AC}\,, \tag{1.20}$$

where $\eta_{AB} = \mathrm{diag}(-1,1,1,1)$. One can instead work with the following linear combinations,

$$M_{ij}, \qquad M_{0,d} = D, \qquad M_{d,i} - M_{0,i} = K_i, \qquad P_i = M_{d,i} + L_{0,i} \quad , \tag{1.21}$$

where M_{ij} are rotations, D are called dilations, K_i & P_i are boosts and translations. Here $i,j = 1,2,\ldots,d-1$. These generators obey the following commutation relations

$$[D,P_i] = P_i,\ [D,K_i] = -K_i,\ [K_i,P_j] = 2\delta_{ij}D - 2M_{ij},\ [M_{ij},P_k] = \delta_{jk}P_i - \delta_{ik}P_j, \tag{1.22}$$

$$[M_{ij},K_k] = \delta_{jk}K_i - \delta_{ik}K_j,\ [M_{ij},M_{kl}] = \delta_{jk}M_{il} - \delta_{ik}M_{jl} + \delta_{il}M_{jk} - \delta_{jl}M_{ik}\,.$$

One can put different kinds of coordinates on the hyperbola. We discuss two of these here:

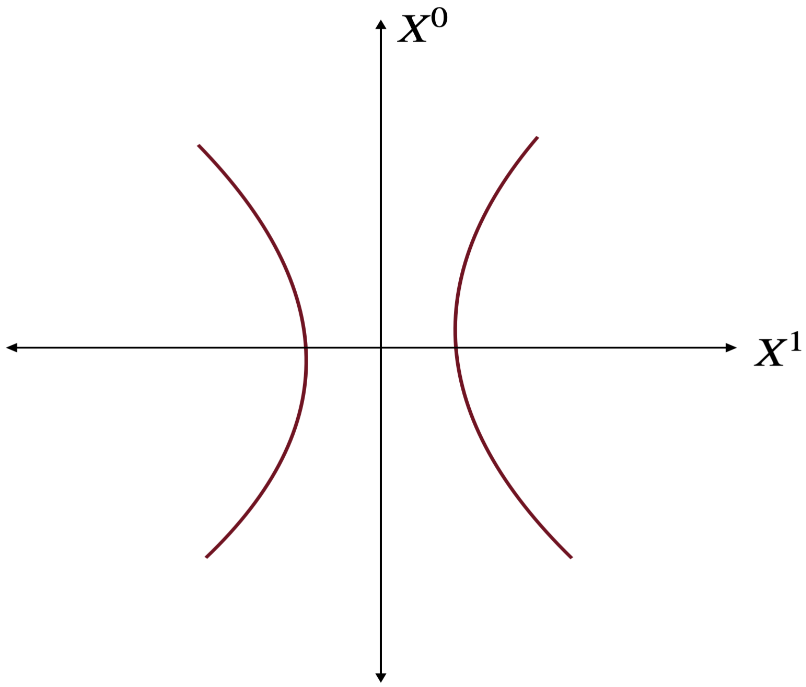


FIGURE 1.4: Here we show the two-dimensional version. However, the actual d dimensional de Sitter space is a higher dimensional hyperboloid

1.2.1.2 Global slicing

These coordinates cover the full hyperbola, hence the name global coordinates. These are defined by

$$X^0 = l \sinh(t/l) \quad \& \quad X^i = d^i l \cosh(t/l)\,, \tag{1.23}$$

Clearly, this parameterisation solves the above-mentioned constraint with $\vec{d}^2 = 1$. One can think of this choice of coordinates as slicing the hyperbola by horizontally oriented planes, as shown in Fig. 1.5. We plug this into Eq. (1.19) and get,

$$ds^2 = -dt^2 + l^2 \cosh^2(t/l) d\Omega_{d-1}^2\,, \tag{1.24}$$

where $d\Omega^2_{d-1}$ is the metric on a $d-1$-dimensional unit sphere which can be further foliated by $d-2$-spheres. Finally, we get,

$$ds^2 = -dt^2 + l^2 \cosh^2\left(t/l\right)\left(d\theta^2 + \sin^2\theta d\Omega^2_{d-2}\right) \; . \tag{1.25}$$

The above metric is a positively curved FLRW-type metric with $a(t) = l \cosh\left(t/l\right)$.

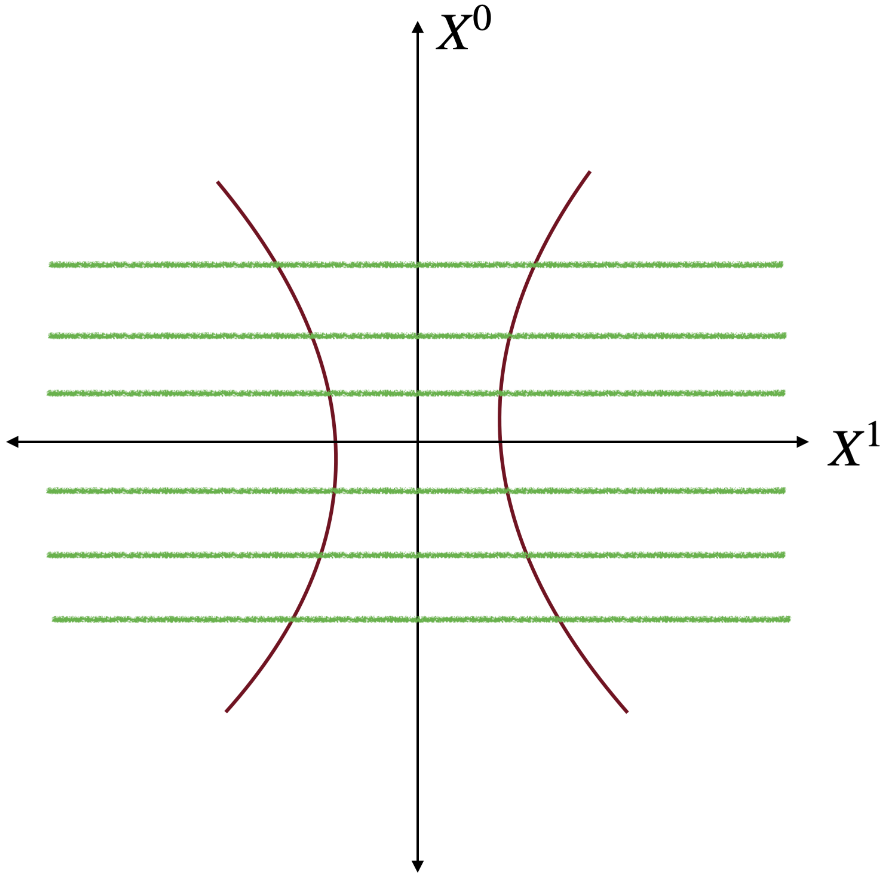


FIGURE 1.5: Here we show the two-dimensional version. However, the actual d dimensional de Sitter space is a higher dimensional hyperboloid

Penrose diagram Before going to the other slicing, we slightly digress to discuss the Penrose diagram of de Sitter space. As is well known, a Penrose diagram helps expose the causal structure of a spacetime. It is clear from Eq. (1.25) that the light rays in the $t-\theta$ plane do not travel at 45°. We will perform a transformation of coordinates so that light rays travel at 45° in the $t-\theta$ plane so that it is easy to expose regions that are causally connected (disconnected). Consider the following transformation of coordinates,

$$\frac{dt}{l\cosh(t/l)} = d\epsilon\,. \tag{1.26}$$

Under such a transformation, the metric takes the following form,

$$ds^2 = \Omega^2(\epsilon)\left(-d\epsilon^2 + d\theta^2 + \sin^2\theta d\Omega^2_{d-2}\right)\,. \tag{1.27}$$

It is clear that light rays in the $\epsilon - \theta$ plane now travel at 45°. To find the range of ϵ, we integrate Eq. (1.26) and get,

$$\tan\epsilon = \sinh(t/l)\,. \tag{1.28}$$

Since $t \in (-\infty, \infty)$, therefore, $\epsilon \in (-\pi/2, \pi/2)$. The polar angle θ has the range $[0, \pi]$ and therefore, the Penrose diagram (shown in Fig. 1.6) is a square of length π. Note the following features of the diagram:

- I^+ and I^- denote future and past conformal boundaries.
- The left and right boundaries are just points. The left one ($\theta = \pi$) we call the south pole, and the right ($\theta = 0$) as north pole.
- Each point in the interior of the diagram is a $d-2$-sphere.
- Even light rays do not travel an infinite distance in infinite time. For instance, note that a null ray emerging from the north pole (denoted by the yellow line) in the far past (I^-) can only travel up to the north pole in infinite time.
- The peculiar feature of de Sitter space is that no observer is causally connected to the entire spacetime. Consider an observer at the north pole, the region $B+C$ will never be able to send signals to the observer. Similarly, the observer can never send signals to the region $A+B$.

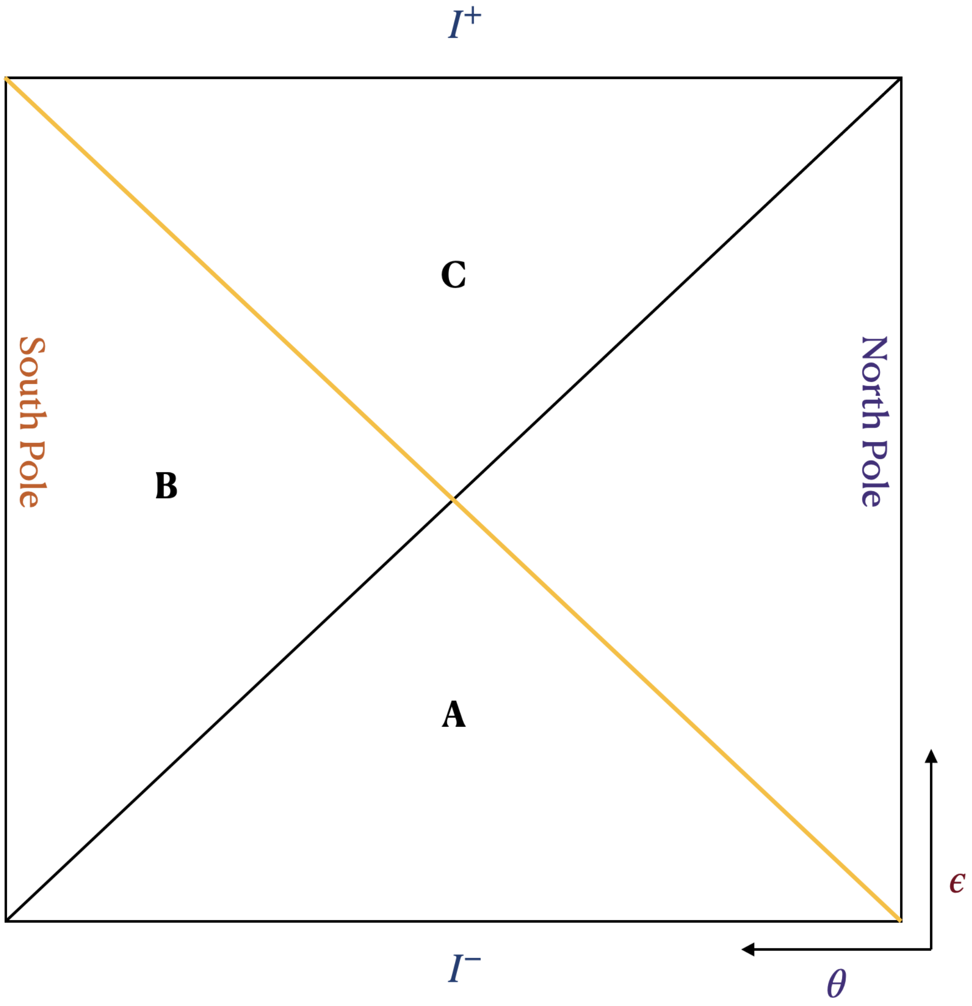


FIGURE 1.6

1.2.1.3 Flat slicing

Another set of coordinates one can choose is defined as,

$$X^0 = l \sinh{(t/l)} + \frac{r^2}{2l} e^{t/l} \quad \& \quad X^1 = l \cosh{(t/l)} - \frac{r^2}{2l} e^{t/l} \quad \& \quad X^i = e^{t/l} y^i \,, \qquad (1.29)$$

where $i = 2 \dots d$ & $r^2 = \vec{y}^2$. Plugging this into Eq. (1.19), one gets,

$$ds^2 = -dt^2 + e^{2t/l} d\vec{y}^2 \,, \qquad (1.30)$$

where $d\vec{y}^2$ is the euclidean $d-1$ dimensional metric. Now it is clear why this is known as the flat slicing: the spatial curvature in these coordinates is zero. These coordinates slice up the hyperbola diagonally (see Fig. 2.1). It is easy to check that

$$X^0 + X^1 > 0\,. \tag{1.31}$$

Therefore, these coordinates only cover half of the hyperbola. This metric is a flat FLRW type metric with $a(t) = e^{t/l}$. It is important to note that de Sitter space in flat slicing is relevant for inflation. This is because inflation very quickly (exponentially fast) drives the spatial curvature to zero, and therefore, even if one starts with appreciable spatial curvature, it goes to zero very fast. Therefore, it is most accurately described by the flat slicing of de Sitter space. To be precise, inflation does not get rid of the spatial curvature but makes the spatial curvature negligible compared to the total energy density. This is captured by just setting $K = 0$ directly in the equations, but one should remember that this is just a simplification and not take it literally.

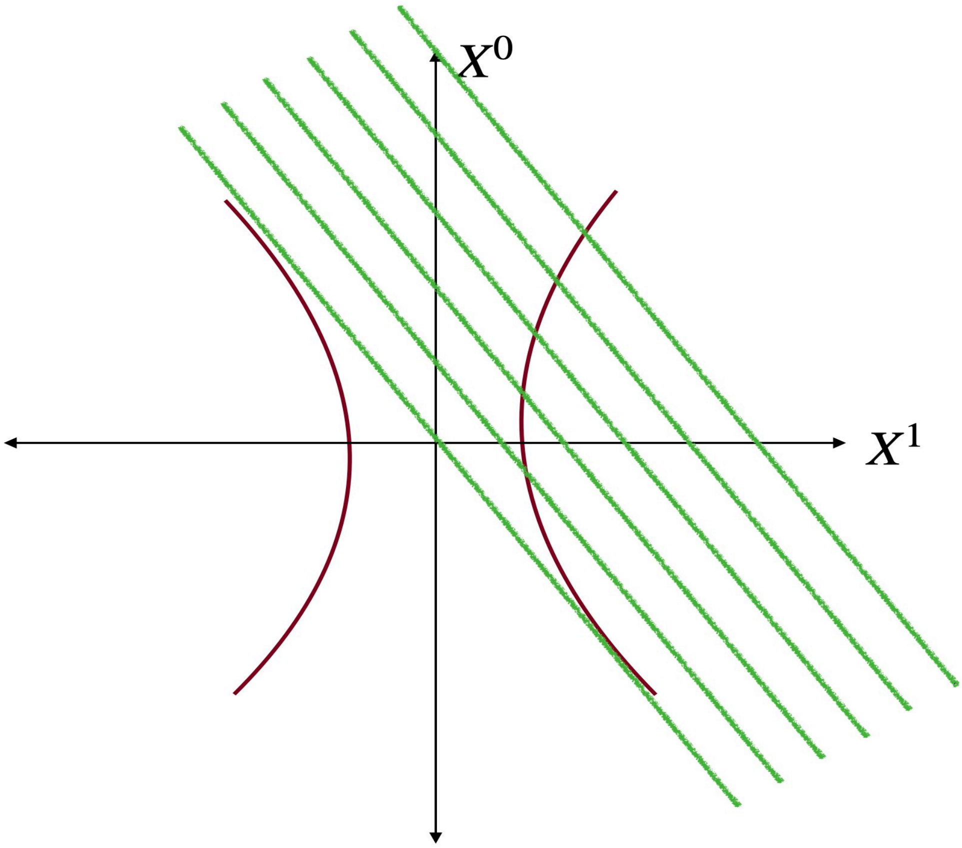


FIGURE 1.7: Here we show the two-dimensional version. However, the actual d dimensional de Sitter space is a higher dimensional hyperboloid

1.2.2 Quantum fluctuations

Until now, we have discussed the classical evolution of a homogeneous scalar field, which naturally generates a homogeneous and isotropic universe. However, the universe today looks homogeneous only on scales larger than $\mathcal{O}(100)$ Mpc and one needs a mechanism to generate these inhomogeneities. It turns out that quantum mechanical fluctuations during inflation are sufficient to seed the structure in the universe we see today. More importantly, the pattern of inhomogeneities (correlation functions) we observe is exactly what inflation predicts. To discuss these quantum predictions, we first illustrate how perturbation theory is defined in a cosmological setup. In cosmological perturbation theory (See [34] for a nice review), one always decomposes any relevant quantity (e.g., $\phi(x)$) in the following manner

$$\phi(\vec{x},t) = \bar{\phi}(t) + \delta\phi(\vec{x},t) \tag{1.32}$$

where $\bar{\phi}(t)$ is the solution of the following background EOM for the scalar field (FLRW coordinates)

$$\ddot{\bar{\phi}}(t) + 3H\dot{\bar{\phi}}(t) + V' = 0 \tag{1.33}$$

No matter what coordinates one works with, the split between background and perturbation will always have the same time-dependent (no spatial dependence) background function. The reason for such a split is the following: as mentioned above, our universe on very large scales looks very homogeneous (no spatial dependence) and isotropic to co-moving observers. This has been verified by the Large-scale structure data and the CMB temperature profile [9]. Of course, this is only an approximate statement, and there are small fluctuations present on top of this background. Since we are interested in studying these fluctuations, we always split any relevant quantity in any coordinates in terms of an FLRW background solution (homogeneous and isotropic) and perturbations about it. If one thinks in terms of a background spacetime, then this means that there is a preferred choice of the time coordinate (slicing) in which the background metric has no spatial dependence. For the inflationary background, this slicing corresponds to slices of constant inflation, which act as a physical clock (with values of this field keeping time). This is the reason why sometimes single-field inflation is also referred to as single-clock inflation.

We will assume (as discussed above) that the fluctuations are generated quantum mechanically and therefore will be quantised using the usual field theory prescription. Since the fluctuations are defined over a time-dependent background, they are not coordinate invariant. The field fluctuation for instance, under a transformation $\tilde{x}^\mu = x^\mu + \epsilon^\mu$ transforms as

$$\delta\tilde{\phi}(\tilde{x}) = \delta\phi(x) - \epsilon^0\dot{\bar{\phi}}(t) \tag{1.34}$$

This means one can remove the field fluctuations by making the appropriate coordinate transformation, which makes the meaning of having a perturbed universe ambiguous. However, the metric also has quantum mechanical fluctuations and the field perturbation, if removed, goes into the metric as a scalar mode. Therefore, one needs to keep track of both metric plus field perturbations. This can be achieved by defining convenient gauge-invariant variables. We will mostly work with the so-called co-moving curvature perturbation, $\zeta(x)$, defined as

$$\zeta(x) = \psi(x) + H\frac{\delta\phi(x)}{\dot{\phi}}\,, \tag{1.35}$$

where $\psi(x)$ is defined through the spatial part of the metric, $g_{ij} = e^{\psi}(e^{\gamma})_{ij}$ with the following transformation property for $\psi(x)$

$$\tilde{\psi}(\tilde{x}) = \psi(x) + H\epsilon^0\,. \tag{1.36}$$

This property, combined with Eq. (1.34) makes $\zeta(x)$ defined in Eq. (1.35) gauge-invariant. Having defined the perturbations, one needs a quantisation prescription. Since the background spacetime is FLRW and not Minkowski, therefore to proceed one needs the tools of QFT in curved spacetime.

1.2.2.1 QFT on a time-dependent background: a short detour

To quantise the perturbations produced during inflation, one needs to take into account the fact that the background spacetime is not Minkowski space. As mentioned earlier, inflationary spacetime can be very well approximated by flat slicing of de Sitter space with the metric given in Eq. (1.17). Therefore, one only needs to take into account the time dependence of the background spacetime. To illustrate the quantisation procedure (we primarily follow [35] & [36]), let us consider a massive scalar free field in a general flat FLRW background with the following metric,

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2\,, \tag{1.37}$$

where $a(t)$ is the scale factor, which for the de Sitter case is equal to e^{Ht}. The action for such a field is given by,

$$S = \frac{1}{2}\int d^3x dt \sqrt{-g}\left[(\partial\phi)^2 - m^2\phi^2\right] , \tag{1.38}$$

$$= \frac{1}{2}\int d^3x dt \left[a^3\dot{\phi}^2 - a(\partial_i\phi)^2 - m^2a^3\phi^2\right] , \tag{1.39}$$

where in the second line we used $\sqrt{-g} = a^3$. We now change the time variable and define the so-called conformal time, τ, as,

$$d\tau = \frac{dt}{a(t)} . \tag{1.40}$$

With this change, the FLRW metric becomes conformally equivalent to Minkowski space, i.e., $ds^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$. In this time coordinate, the scale factor for de Sitter becomes $a = -1/(H\tau)$ with $\tau \in (-\infty, 0)$. To simplify the action, we define $\chi = a\phi$, giving the following action (expressed in conformal time),

$$S = \frac{1}{2}\int d\tau d^3x \left[\chi'^2 - (\partial_i\chi)^2 - \left(m^2a^2 - \frac{a''}{a}\right)\chi^2\right] , \tag{1.41}$$

where we have dropped a boundary term. A prime (') denotes derivative wrt conformal time. Notice, the problem has been reduced to a field theory in Minkowski space with a time-dependent mass term. Clearly, the Hamiltonian of the theory will have explicit time dependence and therefore its spectrum will change with time. We proceed with the quantisation procedure, as usual, we impose,

$$[\chi(\vec{x}), \pi(\vec{x}')] = i\delta^3(\vec{x} - \vec{x}') , \tag{1.42}$$

where $\pi = \chi'$ is the usual canonical momentum. On then expands the field operator in terms of creation and annihilation operators as follows,

$$\chi = \int d^3k \left[\left(\chi_k^{\rm cl}\right)^* a_{\vec{k}} + \left(\chi_k^{\rm cl}\right) a^\dagger_{-\vec{k}}\right] e^{i\vec{k}.\vec{x}} , \tag{1.43}$$

where the function $\chi_k^{\rm cl}$, called the mode function, solves the classical equation of motion in Fourier space. In other words, it is the solution to the equation

$$\left(\chi_k{}^{\rm cl}\right)'' + k^2\chi_k^{\rm cl} + \left(m^2a^2 - \frac{a''}{a}\right)\chi_k^{\rm cl} = 0 \tag{1.44}$$

The state annihilated by $a_{\vec{k}}$ is usually called the "vacuum" state. In Minkowski space QFT, the mode function for the analogous problem is given by $\chi_k^{\rm cl} \propto e^{i\omega\tau}$ with $\omega = \sqrt{k^2 + m^2}$. This choice of mode functions minimises the energy of the state annihilated by $a_{\vec{k}}$. In other words, a state $|0\rangle$ defined by the condition $a_{\vec{k}}\,|0\rangle = 0$ is the minimum energy state in the spectrum. Once such a state is chosen, it is easy to construct the Fock space of multi-particle states by repeated application of $a^\dagger_{\vec{k}}$ on this state. One additional feature in usual Minkowski QFT is that all inertial observers agree on what the vacuum state is. This is easy to see; the general solution of the classical equation of motion in Fourier space for a free massive scalar in Minkowski space is

$$\chi_k^{\rm cl} = \alpha_k e^{i\omega\tau} + \beta_k e^{-i\omega\tau}\,, \tag{1.45}$$

where α_k and β_k are functions of k alone. If one demands that the vacuum state be invariant under isometries of the Minkowski spacetime, i.e, we demand that Fock space is invariant under a Poincaré transformation (equivalence of inertial frames), then $\beta_k = 0$. In other words, there is a unique state invariant under the isometries. This state is also the minimum energy state. Therefore, in Minkowski space, the notion of vacuum is unambiguous. Coming back to the FLRW case, first of all, since the Hamiltonian has an explicit time dependence, the question of a minimum energy state is not well defined, as the ground state keeps changing. The state, which was a ground state at a given time, may not generally remain a ground state at a later time (it may not even remain an eigenstate of the Hamiltonian). Also, as we will see in the specific example of de Sitter space, the state invariant under de Sitter isometries is not unique. This means that if different freely falling detectors are calibrated such that the "no particle" state is set to be a state which is de Sitter invariant, they will not, in general, agree on the count of particles since there are multiple (actually an infinite) number of invariant states. Therefore, the definition of vacuum as a zero particle state for a freely falling detector is ambiguous for de Sitter space. In other words, there are multiple invariant Fock spaces. To make these notions precise, we specialise to de Sitter space in Eq. (1.44) and get (we suppress the "cl" superscript),

$$\chi_k'' + \left[k^2 + \frac{1}{\tau^2}\left(\frac{m^2}{H^2} - 2\right)\right]\chi_k = 0\,. \tag{1.46}$$

The general solution to this equation is

$$\chi_k'' = \sqrt{-\tau}\left[\alpha_k H_\nu^{(2)}(-k\tau) + \beta_k H_\nu^{(1)}(-k\tau)\right]\,, \tag{1.47}$$

where $\nu = \sqrt{9/4 - m^2/H^2}$ and $H_\nu^{(1)}$ & $H_\nu^{(2)}$ are Hankel functions of first and second kind. To proceed with the quantisation procedure, one needs to specify the functions α_k & β_k. Recall that the reason inflation solves the horizon problem is that the modes one observes in the CMB were well inside the Hubble scale (H^{-1}) at the onset of inflation. Therefore, the initial condition for each Fourier mode is set when it is deep inside the Hubble scale. In this regime, the first term in the parenthesis in Eq. (1.46) dominates, and we get

$$\chi_k'' + k^2\chi_k = 0\,. \tag{1.48}$$

The problem has now been completely reduced to a Minkowski QFT. Therefore, we quantise the system by the usual choice of mode functions, i.e., $\frac{1}{\sqrt{2k}}e^{ik\tau}$. This sets β_k to zero since

$$H_\nu^{(1)}(x \gg 1) \sim e^{ix}, \qquad H_\nu^{(2)}(x \gg 1) \sim e^{-ix}\,. \tag{1.49}$$

This choice of mode functions corresponds to a choice of the vacuum state (state annihilated by $a_{\vec{k}}$). This is known as the Bunch-Davies vacuum. Such a state reduces to the usual Minkowski vacuum at short distances (in the UV). As mentioned above, since the background is time-dependent, there cannot be any state which is the ground state at all times. However, one can think of instantaneous ground states. It turns out that the Bunch-Davies vacuum is the minimum energy state at early times. Also, such a state is de Sitter invariant. Therefore, the Bunch-Davies vacuum is considered to be the most natural choice for the initial state of inflation. However, if one approaches from a purely symmetry perspective, then there is an infinite number of de Sitter invariant states to choose from. These states correspond to mode functions χ_k where α_k & β_k are independent of k. In the literature, they are known as α-vacua. There has been some discussion in the literature about whether these states are physically realisable [37, 38, 16]. The main reason behind the scepticism is that these states produce an infinite backreaction, hence spoiling the background de Sitter geometry. We will not go into further details here.

1.2.3 Back to inflation

Using the machinery of QFT in curved space discussed above, one can proceed and quantise cosmological perturbations and calculate various correlation functions. In particular, we quantise the co-moving curvature perturbation ζ introduced above. This is an appropriate variable to work with because it is conserved on very large scales during inflation and continues to be conserved even after inflation has ended [39]! This is particularly useful because we do not know much about the physics of reheating (inflaton decaying to usual standard model particles/radiation). The quadratic action for ζ is given by

$$S_2 = \int d^3x dt \epsilon \left[a^3 \dot{\zeta}^2 - a(\partial_i \zeta)^2 \right] , \tag{1.50}$$

where ϵ is the slow-roll parameter. We proceed by going to conformal time and defining a new variable $v = \sqrt{2\epsilon} a \zeta$, the action becomes,

$$S_2 = \int d^3x d\tau \left[v'^2 - (\partial_i v)^2 + \frac{z''}{z} v^2 \right] , \tag{1.51}$$

With the following equation of motion in Fourier space

$$v_k'' + \left(k^2 - \frac{z''}{z} \right) v_k = 0 \, . \tag{1.52}$$

This equation reduces to its Minkowski analogue for very large k, and following the arguments discussed above, we choose the Bunch-Davies vacuum as the initial state. This gives the following mode function,

$$u_k \sim H_\nu^{(2)}(-k\tau) \, , \tag{1.53}$$

with $\nu = \frac{3}{2} + \epsilon - \eta$. This solution is obtained by restricting to the leading order in slow-roll. One can now proceed to quantise ζ and compute the first non-trivial observable, i.e., the two-point function (in Fourier space). We skip the calculational details and directly give the final result,

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H_k^2}{4\epsilon_k M_{pl}^2 k^3} \,. \tag{1.54}$$

The subscript k indicates that the quantities are evaluated at the time of horizon crossing, i.e., when the wavelength of the mode becomes equal to H^{-1}. The $1/k^3$ factor is a signature of de Sitter vacuum fluctuations. The dependence on the scale k apart from this $1/k^3$ dependence is called the spectral tilt denoted by $n_s - 1$, which for the above correlation function is,

$$n_s - 1 = 2\eta - 6\epsilon \,. \tag{1.55}$$

Inflation also produces gravitational waves. Their spectrum can be computed via the action,

$$\frac{1}{8} \int d^3x dt \left[a^3 \dot{\gamma}_{ij}^2 - a (\partial_l \gamma_{ij})^2 \right] \,. \tag{1.56}$$

The two-point correlation function obtained from the above action is given as,

$$\sum_{s=+,\times} \langle \gamma^s_{\vec{k}_1} \gamma^s_{\vec{k}_2} \rangle = 4(2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H_k^2}{M_{pl}^2} \,, \tag{1.57}$$

with a spectral tilt of $n_t = -2\epsilon$. As mentioned at the end of Sec. 1.1, there is an observational upper bound on this quantity. Since this correlation only depends on the Hubble scale, therefore, from observations $H_k < 10^{14}\,\mathrm{GeV}$. Note that the tensor-to-scalar ratio defined in Sec. 1.1 can now be computed in this single-field model,

$$r = \frac{4H_k^2 / M_{pl}^2 k^3}{H_k^2 / 4\epsilon_k M_{pl}^2 k^3} = 16\epsilon \,. \tag{1.58}$$

And we arrive at a very important consistency relation for single-field slow-roll inflation,

$$r = -8n_t \,. \tag{1.59}$$

If we see a violation of this relation in the data, then it would most likely point towards a multi-field scenario.

1.3 Computational techniques

The computation of inflationary correlation functions requires two pieces of information. One needs to know the state of the system and the interaction Hamiltonian. With this in mind, one can then compute them using either the wavefunctional approach or the so-called in-in formalism. In this section, we will review both of these approaches.

1.3.1 Wavefunction formalism

The wavefunction approach [40, 41, 42, 43, 44, 45, 46, 47] is similar to the one familiar in quantum mechanics, where once the state of the system is known, the expectation value of any operator $\mathcal{O}$ is given by

$$\langle \mathcal{O} \rangle = \int D\phi \Psi^*[\phi] \mathcal{O} \Psi[\phi] \,, \tag{1.60}$$

where $\Psi[\phi]$ denotes the state of the system. Since we are dealing with a field theory, the state is represented by a functional instead of an ordinary function. In terms of bra-ket notation, the wavefunctional is given by

$$\langle \phi(\vec{x}) | \, \Omega \rangle \,, \tag{1.61}$$

where $|\phi(\vec{x})\rangle = |\Phi(\vec{x}, \tau_0)\rangle$. The interacting vacuum denoted by $|\Omega\rangle$ is the state at the onset of inflation, and $|\Phi(\vec{x}, \tau_0)\rangle$ are field eigenstates at time τ_0. The above expression is written in the Heisenberg picture defined at the beginning of inflation.

The analogous expression for the wavefunction, $\psi(x)$ in quantum mechanics, is given as

$$\psi(x) = \langle \vec{x}, \tau_0 | \, \Omega \rangle \tag{1.62}$$

Since the wavefunctional above is defined as a transition from the vacuum to some field eigenstate at late times, this can naturally be obtained from a path integral,

$$\Psi[\phi] = \int_{\Phi(\vec{x},\tau\to-\infty^-)=0}^{\Phi(\vec{x},\tau_0)=\phi(\vec{x})} D\Phi e^{iS[\Phi]} \,, \tag{1.63}$$

where the early time limit is taken with an $i\epsilon$ to project on the interacting vacuum. Till now, everything seems very formal and abstract; to illustrate how computations in this formalism are performed, we discuss the simple example of tree-level correlators. In this case, it is sufficient to evaluate the action on the classical saddle

$$\Psi[\phi] \approx e^{iS[\Phi_{cl}]} \,, \tag{1.64}$$

where $\Phi_{\rm cl}$ is the solution of the classical equation of motion with the boundary conditions mentioned above. Consider a scalar field with the following action

$$S[\Phi] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}m^2\Phi^2 + \mathcal{L}_{\rm int}\right] . \tag{1.65}$$

The classical equation of motion is given by

$$(\Box - m^2)\Phi_{\rm cl} = \frac{\delta S_{\rm int}}{\delta\Phi} \,. \tag{1.66}$$

And the solution can be written as

$$\Phi_{\rm cl}(x) = \int \phi(\vec{x'})K(\vec{x},\vec{x}',\tau)d^3x' + i\int \frac{\delta S_{\rm int}}{\delta\Phi}G(x,x')d^4x \,, \tag{1.67}$$

where $K(\tau,\vec{x},\vec{x}')$ is the solution of homogeneous equation of motion

$$(\Box_x - m^2)K(\vec{x},\vec{x}',\tau) = 0 \,, \tag{1.68}$$

and the Greens function $G(x, x')$ is defined via,

$$(\Box_x - m^2)G(x, x') = \frac{i}{\sqrt{-g}}\delta^4(x - x')\,. \tag{1.69}$$

Since at late times $(\tau = \tau_0)$, $\Phi_{\text{cl}} = \phi(\vec{x})$, therefore $G(\tau, \tau_0) = G(\tau_0, \tau) = 0$ and $K(\tau_0) = \frac{1}{\sqrt{\gamma}}\delta^3(\vec{x} - \vec{x}')$. We usually work in Fourier space since spatial translational invariance simplifies calculations. In Fourier space Eq. (1.67) gives,

$$\Phi_{\text{cl}}(\vec{k}, \tau) = \phi(\vec{k})K(k, \tau) + \int d\tau' \frac{\delta S_{\text{int}}}{\delta\phi(\vec{k}, \tau')}G(k, \tau, \tau')\,. \tag{1.70}$$

The boundary conditions in Fourier space for K & G are given as follows,

$$(\Box_k - m^2)K(k, \tau) = 0, \qquad K(k, \tau_0) = 1, \qquad K(k, -\infty^-) = 0\,, \tag{1.71}$$

$$(\Box_k - m^2)G(k, \tau, \tau') = \frac{i}{\sqrt{-g}}\delta(\tau - \tau'), \quad \lim_{\tau.\tau' \to \tau_o} G(k, \tau, \tau') = 0, \quad \lim_{\tau.\tau' \to -\infty^-} G(k, \tau, \tau') = 0\,, \tag{1.72}$$

where the early time limit is taken with an $i\epsilon$ to project on the vacuum, i.e., $-\infty^- = -\infty(1 - i\epsilon)$. Computing the full wavefunction is a daunting task; therefore, it is very useful to parameterise the wavefunction using the following expansion,

$$\Psi[\phi, \tau_0] = \exp\left(+ \sum_{n=2}^{\infty} \int \frac{d^3k_1 \ldots d^3k_n}{n!(2\pi)^{3n}} \psi_n \phi_{\vec{k}_1} \ldots \phi_{\vec{k}_n} \right)\,, \tag{1.73}$$

where $\psi_n = \psi_n(\{\vec{k}\}, \tau_0)$ are the wavefunction coefficients evaluated at time τ_0. With this expression in mind, we iteratively plug Eq. (1.67) into Eq. (1.64) to obtain an expression of the form Eq. (1.73). One can then read off various tree-level wavefunction coefficients by matching the two expressions. Once these are obtained, using Eq. (1.60), we can compute tree-level correlation functions from them. We give a few formulae as examples,

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \rangle' = \frac{1}{2\mathrm{Re}\psi_2} \,, \tag{1.74}$$

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle' = -2 \prod_{a=1}^{3} \frac{1}{2\mathrm{Re}\psi_2(k_a)} \mathrm{Re}\psi_3 \,, \tag{1.75}$$

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle' = -2 \prod_{a=1}^{4} \frac{1}{2\mathrm{Re}\psi_2(k_a)} \left[\mathrm{Re}\psi_4 - \frac{\mathrm{Re}\psi_3(\vec{k}_1, \vec{k}_2, -s)\mathrm{Re}\psi_3(\vec{k}_3, \vec{k}_4, s)}{\mathrm{Re}\psi_2} - u - t \right] , \tag{1.76}$$

where primes denote that we have stripped off the momentum-conserving delta function.

Rather than following this cumbersome procedure every time, one can instead use Feynman rules for the computation of wavefunction coefficients. These are given as follows,

- Draw the late time boundary ($\tau = \tau_0$) where one wishes to compute the wavefunction.
- For an n-point function, draw all diagrams with n lines attached to the boundary.
- Assign a factor of iV to each interaction vertex insertion, where V is obtained by functional differentiation of the action.
- Assign a bulk-to-bulk propagator, G, to each internal line.
- Assign a bulk-to-boundary propagator, K, to each external line attached to the boundary.
- Integrate over all times from $-\infty^-$ to 0.
- In the case of loops, integrate over the loop momenta.

Another convention (introduced in [23]) is to define bulk-bulk propagator with an i, i.e., $G \to iG$ and therefore, each vertex gives a factor of V, and there is an overall factor of i^{1-L}, where L is the number of loops. We follow this convention in Chapter 5.

1.3.2 In-in formalism

In this section, we review the in-in formalism [42, 25, 48]. In Cosmology, we are interested in the late time ($\tau \to 0$) correlation functions provided we start in the interacting vacuum, $|\Omega(\tau_i)\rangle$, at some early time, τ_i, at the beginning of Inflation. Since the inflationary background is time-dependent, the state at late times is no longer the vacuum but a very high occupation number, semi-classical state, $|\psi(0)\rangle$. Therefore, the object of interest is,

$$ {}_I\langle\psi(0)|\, Q_I(0)\, |\psi(0)\rangle_I \,, \tag{1.77}$$

where $Q_I(0)$ is the operator we wish to compute the expectation value for at late times. The above equation is written in the interaction picture. We do not know what $|\psi(0)\rangle$ is, but we do know the initial state; therefore, we evolve backwards to the onset of inflation and get,

$$ {}_I\langle\Omega(\tau_i)|\, U_I^\dagger(0,\tau_i) Q_I(0) U_I(0,\tau_i)\, |\Omega(\tau_i)\rangle_I \,. \tag{1.78}$$

We identify all pictures at time τ_i, which means $|\Omega(\tau_i)\rangle = |\Omega\rangle$ (which is the Heisenberg picture ket) and get,

$$ \langle\Omega|\, U_I^\dagger(0,\tau_i) Q_I(0) U_I(0,\tau_i)\, |\Omega\rangle \,. \tag{1.79}$$

Now, to proceed further, we need to express the interacting vacuum in terms of something which we know how to calculate. In flat space QFT, we use the $i\epsilon$ prescription to project the free vacuum on the interacting vacuum in the following way: consider the object

$$ e^{-iHT}|0\rangle = e^{-iE_0 T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_n T}|n\rangle\langle n|0\rangle \,, \tag{1.80}$$

which is obtained by expanding the free vacuum in terms of eigenstates of the interacting Hamiltonian. Now take $T \to \infty(1-i\epsilon)$, this projects $|0\rangle$ on $|\Omega\rangle$ since

$$ \lim_{T\to\infty(1-i\epsilon)} \sum_{n\neq 0} e^{-iE_n T}\langle n|0\rangle \longrightarrow 0 \,. \tag{1.81}$$

And we have,

$$\lim_{T\to\infty(1-i\epsilon)} e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle\ . \tag{1.82}$$

Notice that the above equation seems to be telling us that the interacting vacuum at $t=0$ can be obtained from the free vacuum in the far past with a slight deformation of the contour into the complex plane. In other words, it looks as if the $i\epsilon$ has turned off interactions in the far past and turned the interacting vacuum into the free vacuum. This intuition is not correct because one could have projected any other state (Any state in the free or any generic state in the interacting theory) instead of $|0\rangle$, provided that it has a non-zero overlap with the interacting vacuum. For any general state $|\psi\rangle$ (with non-zero overlap with $|\Omega\rangle$) we have,

$$\lim_{T\to\infty(1-i\epsilon)} e^{-iHT}|\psi\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|\psi\rangle\ . \tag{1.83}$$

Therefore, it is best to only think of $i\epsilon$ as a projector for the interacting vacuum, mathematically,

$$\lim_{T\to\infty(1-i\epsilon)} e^{-iHT} = e^{-iE_0T}|\Omega\rangle\langle\Omega|\ , \tag{1.84}$$

provided that the projected state has a non-zero overlap with $|\Omega\rangle$. Now, let us get back to the cosmological case, Eq. (1.79). First, we consider the ket side (the bra side is just the adjoint), we have,

$$U_I(0,\tau_i)|\Omega\rangle\ . \tag{1.85}$$

We assume that inflation starts at a time (τ_i) early enough such that the relevant modes are deep inside the horizon and, because of the equivalence principle, are essentially in flat space, i.e. one can neglect time dependence due to the expansion. Therefore, for these early times, the interaction picture operator has the usual flat space form,

$$U_I(\tau,\tau_0) = e^{iH_0(\tau-\tau_i)}e^{-iH(\tau-\tau_0)}e^{-iH_0(\tau_0-\tau_i)}\ , \tag{1.86}$$

where the interaction picture is defined at time τ_i. Now, we use this object to project the free vacuum onto $|\Omega\rangle$ in the following way,

$$\begin{aligned}\lim_{T\to\infty(1-i\epsilon)} U_I(\tau_i,-T)|0\rangle &= \lim_{T\to\infty(1-i\epsilon)} e^{-iH(\tau_i+T)}e^{-iH_0(-T-\tau_i)}|0\rangle \\ &= \lim_{T\to\infty(1-i\epsilon)} e^{-iH(\tau_i+T)}|0\rangle \\ &= \lim_{T\to\infty(1-i\epsilon)} e^{-iE_0(T+\tau_i)}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_n(T+\tau_i)}|n\rangle\langle n|0\rangle\ ,\end{aligned} \tag{1.87}$$

where, we used the fact that $H_0|0\rangle = 0$. Again, like in the previous case, the summation goes to zero, and we get,

$$\lim_{T\to\infty(1-i\epsilon)} U_I(\tau_i,-T)|0\rangle = e^{-iE_0(T+\tau_i)}|\Omega\rangle\langle\Omega|0\rangle\ . \tag{1.88}$$

The bra side is the adjoint of this.

$$\lim_{T\to\infty(1+i\epsilon)} \langle 0|U_I^\dagger(\tau_i,-T) = e^{iE_0(T+\tau_i)}\langle\Omega|\langle 0|\Omega\rangle\ . \tag{1.89}$$

Put Eq. (1.88) and Eq. (1.89) in Eq. (1.79),

$$\langle 0|U_I^\dagger(\tau_i,-\infty^+)U_I^\dagger(0,\tau_i)Q_I(0)U_I(0,\tau_i)U_I(\tau_i,-\infty^-)|0\rangle\ , \tag{1.90}$$

which gives the final relation,

$$\boxed{\langle 0|U_I^\dagger(0,-\infty^+)Q(0)U_I(0,-\infty^-)|0\rangle}, \tag{1.91}$$

where, $\infty^\pm = \infty(1\pm i\epsilon)$. Again, one can use a set of in-in Feynman rules to evaluate correlation functions like the wavefunction formalism. These rules are as follows,

- Draw the late time boundary ($\tau = \tau_0$) where one wishes to compute the correlation function.
- Draw all relevant diagrams with vertices above and below the boundary line.

- Lines that cross or end on the time surface represent a "Wightman propagator",
$$W_k(\tau,\tau') = f_k^*(\tau) f_k(\tau')\,,$$
where the left argument is given by the time of the uppermost vertex, and the right argument is given by the lowermost vertex. Lines which lie entirely below the fixed time surface represent Feynman propagators,
$$G_F(k,\tau,\tau') = W_k(\tau,\tau')\theta(\tau-\tau') + W_k(\tau',\tau)\theta(\tau'-\tau)\,,$$
while lines which lie entirely above the surface are replaced with time-reversed Feynman propagators with $\tau \longleftrightarrow \tau'$.

- Vertices below the time surface get vertex factors $-iV$, where V is obtained by functional differentiation of the interaction Hamiltonian. Vertices above the time surface get factors of $iV^\dagger$. In Fourier space, momentum is conserved at each vertex, and there is an overall momentum-conserving delta function.

- Integrate over all times from $-\infty^-$ to 0.

- In the case of loops, integrate over the loop momenta.

- If there are symmetry factors in diagrams, divide by them. In the second step above, we can mod out by reflections across the final time surface, but then we have to take 2Re of the answer.

Note that sometimes the vertices below and above are denoted by L (left) and R (right), respectively. In this case, both vertices are drawn below the late time boundary. This nomenclature simply arises because the in-in formula Eq. (1.90) has two evolution operators (one on the left and one on the right). The vertices that arise from the right are labelled R, and similarly, the vertices arising from the left are labelled L. This convention is used in Chapter 3.

1.4 Effective Field Theory of Inflation (EFToI)

The Effective field theory of Inflation is an attempt to unify all models of inflation by constructing the most general low-energy action consistent with symmetries. In this approach, the action is directly written in terms of the fluctuations.
As mentioned above, we are finally interested in a theory of fluctuations about a homogeneous (time-dependent) background and therefore time diffeomorphism, while still being a symmetry, is now non-linearly realised on the fluctuations, i.e they are spontaneously broken. One can then choose a gauge to write the EFT in the so-called unitary gauge, which has no inflaton fluctuations $\delta\phi(\vec{x},t) = 0$. In this gauge, all fluctuations go into the metric. The transformation rule for $\delta\phi(\vec{x},t)$ under $x^\mu \to x^\mu + \epsilon^\mu$,

$$\delta\tilde{\phi}(\tilde{x}) = \delta\phi(x) - \epsilon^0 \dot{\bar{\phi}}(t)\,, \tag{1.92}$$

indicates that unitary gauge just fixes the time diffs and therefore, the EFT will only involve terms which are invariant under spatial diffeomorphisms. We are allowed to write full diff invariant terms, e.g.,${}^{(4)}R^{(4)}_{\mu\nu}R^{\mu\nu}$, terms with free upper time indices, e.g., δg^{00} (terms with lower free time indices are not invariant under spatial diffs) and terms describing the slicing. Hence, we have:

$$S_{EFT} = \int d^4x\sqrt{-g}\mathcal{L}({}^{(4)}R_{\mu\nu\sigma\rho}, {}^{(3)}R_{ijkl}, \nabla_\mu, \delta K_{ij}, g^{00}, \partial_t, ...)\,, \tag{1.93}$$

where K_{ij} is the extrinsic curvature. So far, we have identified the correct degrees of freedom for the action. Now, like for any sensible EFT, one must identify the correct expansion parameter(s) to organise the terms. The EFToI is an expansion in the number of metric perturbations and the number of derivatives on the metric perturbations, which (before any canonical normalisation) have dimension 0. We start by splitting the action into three parts,

$$S_{EFT} = S_0 + S_2 + S_{\geq 3}\,, \tag{1.94}$$

where,

$$S_2 = \int d^4x \, \sqrt{-g} M_{pl}^2 \left(m_1 \left(\delta K^i_j \delta K^j_i \right) + m_2 (\delta K)^2 + m_3 \left({}^{(3)}R \delta g^{00} \right) + M_2^2 (\delta g^{00})^2 \right. \tag{1.95}$$

$$\left. + M_3 \left(\delta g^{00} \right) \delta K + \frac{1}{M_4} {}^{(3)}R_{ij} \delta K^{ij} + \frac{1}{M_5} {}^{(3)}R \delta K + \frac{1}{M_6^2} {}^{(3)}R^2 + \frac{1}{M_7^2} {}^{(3)}R_{ij}^2 + \frac{1}{M_8^2} {}^{(3)}R_{ijkl}^2 \right)$$

$$+ \dots , \tag{1.96}$$

where, S_n has terms which start from n^{th} order in perturbations. Here, S_0 is Einstein-Hilbert action coupled with a scalar field with $\lambda(t) = -3H^2 + \dot{H}$ and $c(t) = -\dot{H}$ so that the background/zeroth-order equations of motion are satisfied. The dots in the above equations indicate the same terms but with more derivatives. $\{m_i\}$ have mass dimension zero whereas $\{M_i\}$ have mass dimension one. We have not made any assumption about how the EFT operators are generated from the UV, therefore, every operator comes with a different Mass scale, M_i. It is important to keep in mind that finally, we want a theory in terms of the scalar fluctuations ζ and the tensor fluctuation γ_{ij}. The EFT derivative power counting is clear only in terms of these variables, and to illustrate this, consider the operator $\partial_i \delta g^{00} \partial^i \delta g^{00}$. This naively looks like a leading two-derivative term and therefore is not suppressed or enhanced by any mass scale. However, in terms of ζ, depending on the constraint solution, it can generate higher derivative terms. Since we are ignorant about the short distance physics, i.e. $k \gg H$ and are interested in the modes which have exited the horizon and have left an imprint on the CMB, the typical length/energy scale is H. Hence, an expansion in H/M is obtained for our EFToI. Assuming for concreteness that all dimensional coefficients are order unity and $H \ll M_{4,5,6,7}$, one can easily arrange the action as a perturbative series. For instance, the quadratic action, S_2, is

$$S_2 \sim \left(m_1 + m_2 + m_3 + \frac{M_2^2}{H^2} + \frac{M_3}{H} + \frac{H}{M_4} + \frac{H^2}{M_5^2} + \frac{H^2}{M_6^2} + \frac{H^2}{M_7^2} \right) + ... , \tag{1.97}$$

For some explicit calculations, we take the action S_0+S_2 and write only the operators which have at most 2 derivatives on the metric :

$$S_0 + S_2 = \int d^4x \sqrt{-g} M_{pl}^2 \left(\frac{^{(4)}R}{2} + m_1 \delta K^i_j \delta K^j_i + m_2 (\delta K)^2 + m_3 {}^{(3)}R \delta g^{00} + D\delta K \right.$$
$$\left. - M_{pl}^2 \lambda(t) - c(t) g^{00} + M_1 g^{ij} \partial_i \delta g^{00} \partial_j \delta g^{00} + M_2^2 (\delta g^{00})^2 + M_3 \delta g^{00} \delta K \right) . \tag{1.98}$$

The leading (two derivatives) quadratic (scalar and tensor) actions are given by [49] :

$$S_{\zeta\zeta} = \frac{1}{2} \int d^4x \, \epsilon \left(a^3 \dot{\zeta}^2 - a (\partial\zeta)^2 \right) , \tag{1.99}$$

$$S_{\gamma\gamma} = \frac{1}{8} \int d^4x \, \left(a^3 \dot{\gamma_{ij}}^2 - a (\partial\gamma_{ij})^2 \right) , \tag{1.100}$$

i.e the action derived for a canonical scalar field minimally coupled to gravity. This is an element of the class of actions defined by equation Eq. (1.98) where the action only contains the first three terms of S_0. In passing we point out that from Eq. (1.99) and Eq. (1.100) one can easily see that the existence of ζ is tied to the fact that inflation is quasi de Sitter ($\epsilon \neq 0$), and such a variable does not exist in pure de Sitter ($\epsilon = 0$). On the other hand, the tensor perturbation γ_{ij} is also well defined in de Sitter. Coming back to Eq. (1.98), there is a large number of unknown parameters, but as usual, the EFT parameters have to be determined/constrained experimentally. Unfortunately, in cosmology, the number of observables is very small. This is because, unlike in flat space EFTs, we do not have experimental control. Therefore, one can constrain or fix only a handful of parameters.

1.5 Notation and Conventions

Here we introduce the notation and conventions used throughout this thesis. For completeness, we repeat some of the things already mentioned in previous sections. The background metric during inflation is approximated by flat slicing of de Sitter space given by,

$$ds^2 = \frac{-d\tau^2 + d\vec{x}^2}{(H\tau)^2}\,, \tag{1.101}$$

where τ is the conformal time with the range $\tau \in (-\infty, 0)$ and H is the Hubble parameter. Derivatives with respect to conformal time are denoted by a prime, as for example in $\partial_\tau \phi = \phi'$.
For the perturbed spacetime (background+fluctuations), the form of the metric (in unitary gauge) is given by the standard ADM formulation [50]:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,, \tag{1.102}$$

where we see that:

$$g_{00} = -N^2 + g_{ij} N^i N^j\,. \tag{1.103}$$

And we define:

$$g^{0i} = N^i\,, \qquad g_{ij} = a^2 e^{2\zeta} \left(e^{\gamma}\right)_{ij}\,, \qquad \partial_i \gamma_{ij} = 0\,, \qquad \gamma_{ii} = 0\,, \tag{1.104}$$

where a is the scale factor and $\frac{\dot{a}}{a} = H$ is the Hubble constant. γ_{ij} is the gauge-invariant tensor perturbation and $\zeta = \psi + H\frac{\delta\phi}{\dot{\phi}}$ is the gauge-invariant scalar perturbation. It is important to note that the quantity that actually appears in g_{ij} is ψ, but since in unitary gauge $\delta\phi = 0$, we have $\psi|_{\delta\phi=0} = \zeta$.
The field-theoretic wavefunction is parameterised as

$$\Psi[\phi, \tau_0] = \exp\left(+ \sum_{n=2}^{\infty} \int \frac{d^3k_1 \ldots d^3k_n}{n!(2\pi)^{3n}} \psi_n \phi_{\vec{k}_1} \ldots \phi_{\vec{k}_n} \right)\,, \tag{1.105}$$

where $\psi_n = \psi_n(\{\vec{k}\}, \tau_0)$ are the wavefunction coefficients evaluated at time τ_0. The n external momenta are denoted by $\{\vec{k}_a\}$ for $a = 1, 2, \ldots, n$ and the I internal energies by $\{p_m\}$ for $m = 1, 2, \ldots, I$. We call "energy" the norm of the momenta, $k = |\vec{k}|$. We denote by $\mathcal{K}$ and $\mathcal{G}$ the bulk-boundary and bulk-bulk propagators for a Bunch-Davies initial state, while we use K and G for the case of a Bogoliubov initial state. For useful references on the field-theoretic wavefunction in the cosmological context see [40, 41, 42, 43, 44, 45, 46, 47].

The divergences in the loop integrals are regulated by dimensional regularisation. The mode functions in d dimensions for massive fields are Hankel functions, $H_\nu^{(1)}(-k\tau)$, where $\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$. Even for the massless case, the mode functions are complicated Hankel functions $H_{d/2}^{(1)}(-k\tau)$ in d dimensions (unlike $d = 3$, where the mode functions are simpler to work with), which makes the loop computations difficult. Instead, we implement a slightly modified form of dimensional regularization by using the trick [23, 51] of analytically continuing the mass to d dimensions, i.e. $m^2 \to \frac{H^2}{4}(d^2 - 9)$. In this case, we get $\nu = \frac{3}{2}$ and $m \to 0$ when $d \to 3$. This gives back simple massless mode functions of the form,

$$f_k(\tau) = (-H\tau)^{\delta/2} \frac{H}{\sqrt{-\dot{H}M_{pl}^2}} \frac{1}{\sqrt{2k^3}} (1 - ik\tau)\, e^{ik\tau} \,. \tag{1.106}$$

1.6 Organisation of the Thesis

The rest of this thesis is organised as follows. In Chapter 2, we analyse the analytic structure (pole structure) of three-point tree-level inflationary correlation functions. The link between the initial state and the pole structure is made precise. Moreover, we illustrate that while folded poles cannot be generated by a vacuum (Bunch-Davies) initial state, however, their absence does not uniquely select the vacuum as the initial state.

In Chapter 3, we explicitly verify soft theorems for mixed (scalar+tensor) tree-level three-point inflationary correlators. More precisely, we consider higher-dimensional operators in the EFToI which correct both the three-point and the two-point functions. Since both sides of the soft theorem get corrected, this makes our checks non-trivial. We also comment on the possibility of bootstrapping the mixed correlators coming from higher-dimensional operators using the so-called "boostless bootstrap".

In Chapter 4, we go beyond the tree-level and discuss loop effects during inflation. In particular, we compute the three-point function at one-loop in the EFToI. Although we mainly implement dim-reg throughout this chapter, we also perform certain cross-checks via cut-off regularisation. We explicitly compute the logarithmic running after properly renormalising our results and find two structures: 1) $\log(H/\mu)$, 2) $\log(k_i/k_T)$.

In Chapter 5, we discuss implications of unitarity for the cosmological wavefunction. In particular, we derive Cutkosky-like cutting rules for the wavefunction coefficients at any order in perturbation theory for fields of any mass and (integer) spin. These relations are derived assuming a Bogoliubov initial state, which is an excited state over the Bunch-Davies vacuum. We also derive implications of these relations for n-point contact and four-point exchange diagrams. Finally, we give a prescription for obtaining Bogoliubov wavefunction coefficients from Bunch-Davies ones, saving a lot of computational work.

Chapter 2

Tree-level analytic structure as a probe of initial state of inflation

The material presented in this chapter is based on the author's publication [52].

As discussed in Chapter 1, in most models of inflation, the perturbations start their life in a special kind of vacuum state known as the Bunch-Davies vacuum, but this assumption is not free of contention([12, 13, 14, 15, 16, 17, 18, 19]). The primary motivation for this is that we know nothing about the (possibly Transplanckian) physics that preceded inflation, and assuming any boundary condition in the far past is an extrapolation of intuition to scales we don't understand. It turns out that the most general vacuum state possible is the general Bogoliubov transformed Bunch-Davies state ([16, 15]), and the most general vacuum state invariant under de Sitter isometries is the so-called α vacuum[53, 54]. The power spectrum alone cannot distinguish between these different choices of vacua and initial states. Therefore, we turn to higher point functions, e.g., bispectrum[55, 49, 56, 57, 58] and their analytic structures to look for distinguishing features. It is believed that perturbations that had a classical origin can give rise to present observations at the level of the power spectrum, without appealing to any quantum origin ([59, 60]); see [61] for interesting recent developments. The definition of what pole structure one calls classical, however, is unclear, and it is among the purposes of this chapter to clarify this point. The crucial insight is to study the analytic structure of the correlation functions as a function of the initial conditions, and we find that a simple definition of classical - such as a state with a large number of particles - is not enough to indicate the presence of a different pole structure than that of a true quantum vacuum. Coherent states, which are clearly excited - under certain assumptions - behave exactly like a Bunch-Davies vacuum at the level of correlators. Moreover, any other choice of initial state aside from the Bunch-Davies vacuum and its coherent states displays a recurring presence of physical poles. Therefore, in a sense, the analytic structure arising from a Bunch-Davies vacuum or coherent state is very unique, with everything else showing a common additional structure in the form of *physical poles*. It is the purpose of this chapter to trace precisely the origin of this difference, and study in detail the new features that in-in correlators start displaying as one changes the nature of the initial state.

The different pole structures can ultimately be tied down to the presence or absence of mixing of positive and negative frequency modes-in curved space, these can naturally arise depending on the initial condition. When we think of inflation as being driven by some Effective field theory, we need to understand what features of the physics before inflation are important to understand. New physics can manifest itself as either irrelevant operators (consistent with slow-roll conditions) or as initial state modifications (backreaction effects [37] must be consistently included/dropped - this is a subtle issue that we don't explore here. See [16, 38] for a discussion of this issue at the EFT level. See [62] for details about perturbation theory in the alpha vacuum. At the level of the power spectrum, given a cutoff Λ; initial state effects can add corrections of the order $\frac{H}{\Lambda}$, much larger than the $\frac{H^2}{\Lambda^2}$ effects coming from irrelevant operators [37], and therefore initial state effects cannot be disregarded, despite their somewhat more obscure origin (see [63] for a recent review of the EFT of cosmology). The observed power spectrum then allows us to infer bounds on the scale of new physics, Λ. More generally, they constrain the number density of particles in the initial state by constraining the Bogoliubov parameters, which, in this case, also modify the normalisation of the power spectrum. However, higher point correlation functions are more sensitive probes ([16]) for the simple reason that they are more sensitive to interactions of particles in the initial state, and have always been an object of interest either from a bootstrapping perspective or as a tool to systematically trace signatures of new physics in these correlators [64, 21, 65, 66, 67, 68, 69, 70, 71]; see [46] for a recent view. For this matter, we turn our attention to the bispectrum and the 4-point function [72, 73, 74, 75, 76] with various initial conditions, and observe how their structure changes. We don't make any pretence about the kind of physics before inflation that leads to the choice of the initial state. In particular, we emphasise that the universe has no obligation to exist in the vacuum state of the perturbations.

Correlation functions of the inflaton (more precisely, the scalar mode of the ADM metric after removing gauge freedom) are an obvious tool to study the physics of inflation, encoding both the nature of the interactions in the EFT of inflation, as well as the initial conditions for inflation itself. These correlators, on the other hand, must not be sensitive to whatever fundamental physics drove inflation. We have to be careful about the nature of singularities that are generated by different interactions, as well as the need for an IR regulator. For instance, the difference between a ζ^3 and a ζ'^3 interaction in an in-in calculation is reflected in the singularities they generate in a correlation function. The former generates logarithmic singularities, whereas the latter generates simple poles. The log divergences are contact terms in Fourier space, and their origin can be traced to the fact that conformal symmetry constrains position space correlators such that they are singular at coincident points. This relationship deserves a detailed analysis of it's own ([77, 78]), and we don't dwell on this subject here. We'll instead work with a derivative interaction where the poles are simple poles in momenta and no regulator is required, and we see how the correlation functions with this specific interaction respond to changes in initial conditions. Three-point functions, in particular, are especially interesting because they are a measure of the 'non-Gaussianity', which is a direct measure of the existence of interactions in the inflationary action. The important thing to notice is the word 'interaction', and this is where the nature of de Sitter geometry and corresponding vacua becomes important. In flat space, one typically has the convenience to turn off all interactions in the far past and future, so that the vacuum state at the asymptotic boundaries is free of excitations. In curved backgrounds, in contrast, one doesn't have this luxury, and in fact, the notion of particles and excitations is not well defined. This has consequences for the nature of correlation functions evaluated with different initial conditions. For example, if the initial state is excited relative to, say, the Bunch-Davies vacuum, then it is possible in some cases to find an observer which sees no particles. These cases are precisely those which are related via a Bogoliubov transform. The interpretation of correlators as

Feynman diagrams is therefore very different - the diagrams for a Bunch-Davies observer contain disconnected pieces because there are more particles in the initial state than those that can interact at a vertex-these will then have support only at localised points in phase space. The diagrams for an alpha-observer however, are fully connected as only the field insertions interact at the vertex.

In either case, we see the presence of physical poles where the singularities can be reached in the physical region. It is tempting to relate this to an interpretation in terms of particle scattering and decays in the initial state, but as we remarked, this picture is observer dependent. Clearly, the situation is more complicated than a relative particle density between the vacuum state of the modes and the initial state for inflation. An initial state, which is an alpha vacuum at the start of inflation, produces a similar structure to the one mentioned above, but here it is clear that there is no particle self-interaction interpretation that can be ascribed. If one insists on working with an observer who sees particles, then it is true that an analogy can be drawn with flat space scattering processes, but one must then deal with the fact that the phase space where these poles have support is restricted. Therefore, it isn't straightforward to trace various pole structures to the absence/presence of particles in the initial state. One has the same problem in flat space- LSZ only gives us information about fully connected diagrams that have the right pole structure, but the S-matrix contains contributions from both connected and disconnected diagrams in a highly excited state.

For example, if we had a cubic vertex ϕ^3 and we computed the leading order contribution to

$$\int d^4x \langle n_{out}|\phi^3(x)|n_{in}\rangle \supset \int d^4x \langle n_{k_1}+1, n_{k_2}+1, n_{k_3}-1, ..|n_{in}\rangle \sim \delta(k_1-k_2-k_3)A_{1\to 2} \tag{2.1}$$

where $n \equiv \otimes_k (a_k^\dagger)^n|0\rangle/\sqrt{n!}$. Here, $A_{1,2}$ is now a disconnected amplitude with k_1, k_2, k_3 interacting at a vertex, while the rest of the n_k's spectate. LSZ kills this diagram, and therefore it doesn't contribute to the fully connected part of the S-matrix; it exists nonetheless, and since with inflation we don't have the luxury of an S-matrix formalism, we must make our peace with such processes.

We are typically interested only in the fully connected part of the S-matrix, but such a requirement cannot be imposed when the initial states are highly occupied. Going to higher orders in perturbation theory partially remedies this situation. For instance, consider a $3 \to 2$ process in flat space with a cubic vertex. The Feynman diagrams for the first two orders in perturbation theory are shown in Figure 2.1. We see that going to higher orders in perturbation theory is one way to generate connected diagrams. However, this problem only worsens as we include more and more particles in the external states, because then the fully connected diagrams get harder and harder to measure. In curved space in-in correlators, as we shall see, the recipe to circumvent these is to switch to a frame where there are no particles and the state is the vacuum with respect to the new modes. We will then be left with only fully connected diagrams, whose pole structure can be analysed. Generic excited states display physical poles, but their signatures are clouded by the restricted points in phase space where they have support. We will see that these physical poles occur when particles in the initial state interact at a vertex along with some of the field insertions, while the remaining field insertion is a spectator and therefore has support only when its momentum coincides with one of the momenta of the spectator particles in the initial state. If we have all possible momenta in such a state (what one would expect in a 'classical' state), then at least one of these physical poles will be observable, but at an isolated point in phase space for a given momentum. Scanning all possible momenta for the field insertions will then generate a measurable physical pole. In the case where the initial state is a Bogoliubov transform of the Bunch-Davies vacuum, the physical poles are seen transparently.

Interestingly, there is a class of excited states over the Bunch Davies vacuum, so so-called coherent state $|C\rangle$ [79]

$$a_{\vec{k}}|C\rangle = C(\vec{k})|C\rangle \tag{2.2}$$

which, under certain assumptions, reproduces the pole structure for the vacuum state, i.e. it displays no physical poles despite being an excited state. This is in direct contradiction with the supposed association of physical poles with excited states. If instead we had a coherent state that was built over not the Bunch-Davies vacuum, but that of its Bogoliubov transform, then physical poles are restored.

FIGURE 2.1: (Left) Disconnected diagram at order λ. (Right) Fully connected at order λ^3

Let us elaborate on the role of disconnected pieces. Once the order in perturbation theory is fixed, we must allow the possibility of other particles to spectate the interaction and contribute as disconnected pieces. LSZ will have nothing to say about this, and therefore, the flat space intuition for physical poles in inflation is premature. This chapter is devoted to studying in detail the origin of pole structures in in-in correlators and the information it carries about the initial state of inflation, and probing the relationship between a highly excited quantum state versus a 'classical' state.

The broad outline of this chapter is as follows. In Sec. 2.2.1, we review the features of LSZ in flat space, and the corresponding intuition to be drawn for in-in correlators. In Sec. 2.3, we will analyse in-in 3-point correlators for a variety of initial states, including the anomalous case of coherent states. In addition, subsection 2.3.7 is devoted to checking explicitly, by computing the 4-point classical and quantum in-in correlation function, that going to higher orders in perturbation theory doesn't change the basic inferences we draw from the bispectrum about the singularity structure, extending the result of [59]. Next, in Sec. 2.4 we study the role of derivative interactions that require no IR regulator in generating singularities that are simple poles. In Sec. 2.5 we discuss the squeezed limit of the bispectrum. Finally, in Sec. 2.6 clarifies the link between physical poles in classical correlators versus those in quantum in-in correlators.

2.1 From flat space to curved space

2.1.1 Background

Provided with the field content, background geometry, and symmetries of the system, one can write an action functional that describes the theory. The equations of motion following from this action are solved in terms of *mode functions* $u_k(t,\vec{x})$ that multiply some coefficients a_k, and their hermitian conjugates. In time-independent, flat backgrounds, we have plane wave solutions for the mode functions of free fields; $u_k(t,\vec{x}) = \frac{1}{\sqrt{2\omega_k}} e^{i\vec{k}x-\omega t}$. We promote $a_k, a_k^\dagger$ to operators that satisfy canonical commutation rules that then get imparted to the fields themselves, and we say we have canonically quantised the theory. Studying this theory then amounts to studying the corresponding Hilbert space, and this discussion begins by first identifying a *vacuum state* from which other states are realised, defined as $a_k|0\rangle = 0$. In flat space, it is uniquely fixed by demanding invariance under the underlying Poincaré group. This means that the mode functions and the quantisation prescription of the theory are unique. This is traced to the fact that $u(k)$ is Lorentz invariant, and this is where the situation drastically changes in more general backgrounds.

If we allow for diffeomorphism invariance, a much larger group, then the x and t dependence of the mode functions can change since $(\vec{k}\cdot\vec{x}-\omega t)$ need not be invariant anymore. In fact, plane waves simply needn't be solutions to equations of motion in general backgrounds. Moreover, in time-dependent backgrounds is the case with inflation-ω has a time dependence, which reflects as an additional time dependence in the mode functions. Therefore, mode functions are, in general, not unique. Being solutions to second-order equations of motion, they require two additional inputs to be fixed. Consequently, different mode expansions are distinguishable in the sense that they differ, for example, in their measurement of particle density in a given state. Note that this difference lies in their different basis of modes (and consequently creation/annihilation operators), but the definition of a field itself, of course, is invariant under these transformations. Again, one has to keep in mind that this phenomenon occurs simply by allowing accelerated coordinate frames, even in flat space, and is therefore tied to the definition of an inertial observer. In special relativity, these are restricted to those obtained from Poincaré transformations on a

Minkowski background; in general relativity, we allow for the more general case of whatever isometry preserves the background geometry.

Unlike flat space, where demanding the vacuum to be invariant under the isometries of the background metric uniquely fixes the vacuum state, in de Sitter, the invariance under isometries does not completely fix the vacuum state [54]. Therefore, there is a class of vacua known as alpha vacua, which are invariant under de Sitter isometries and parametrised by α [54, 53]. The mode function corresonding to α vacua is given by

$$u_k(\tau) = \frac{\Delta_\zeta}{\sqrt{k^3}}\left(\alpha(1-ik\tau)e^{ik\tau} + \beta(1+ik\tau)e^{-ik\tau}\right) \tag{2.3}$$

where, $\Delta_\zeta = \left(\frac{H}{\sqrt{4\epsilon}M_{pl}}\right)$ and

$$|\alpha|^2 - |\beta|^2 = 1 \tag{2.4}$$

Eq. (2.3) is the solution of the EOM of a massless field in de Sitter,

$$\zeta_k'' - \frac{2}{\tau}\zeta_k' + k^2\zeta_k = 0 \tag{2.5}$$

If we call the α modes v_k and the Bunch-Davies modes by $u_k = (1-ik\tau)e^{ik\tau}$, then we can compare the associated initial conditions through

$$\dot{v}_k(\tau_0) = -i\omega_k' v_k(\tau), \dot{u}_k(\tau_0) = -i\omega_k u_k(\tau) \implies \frac{\alpha_k}{\beta_k} = \frac{\omega_k' - \omega_k}{\omega_k' + \omega_k} \tag{2.6}$$

The Bunch Davies vacuum ($\alpha = 1$, $\beta = 0$) corresponds to one particular choice of α vacua. This choice of vacuum is usually considered the most natural one since the modes that we are interested in were far inside the horizon at the beginning of inflation ($\tau \to -\infty$ limit), therefore these modes do not "feel" any de Sitter curvature and can be treated as Minkowski modes. But as mentioned above, this is not a settled issue. It is important to note that no co-moving/free-falling particle detector will register any particles in a particular α vacuum since all these observers are related by de Sitter isometries and these vacua are invariant under them.

2.1.2 What we learn

The purpose and findings of this chapter can be summarised as follows-

- Is there a unique pole structure for all in-in correlators? What distinguishes the physics leading to different singularities? *No, the pole's functional form depends on the choice of interaction. The physical origin we aim to see lies not in the functional form of the singularity but in the momentum configuration for which it occurs.* See Sec. 2.4 for a discussion.

- Are total energy poles a characteristic of only the Bunch Davies vacuum initial state with no excitations? Do physical poles arise only if the initial state was an excited state? *What is true is that most excited states relative to a Bunch Davies vacuum display physical poles. However, a class of coherent states displays the same pole structure as a state with no Bunch-Davies particles, and this is an important anomaly that we wish to highlight.* This result is derived in Sec. 2.3.4.

- Do initial states with a fixed number of Bunch Davies particles display observable physical poles? Is it necessary for each mode to have a high occupation number in the initial state to replicate the structure of a classical correlator? *With a finite number of particles, the physical poles are isolated in phase space. One needs to excite all possible momenta to guarantee an observable physical pole. The important ingredient is not to miss out on regions in phase space, and to excite every mode at least once. This state, however, has a large number of particles anyway, as one would expect for a classical state.* See Sec. 2.3.5 to understand the support for these new poles.

- If the initial state is a vacuum state, then do we not see physical poles? *The notion of vacuum state is not unique, and for any other vacuum other than Bunch Davies, one sees physical poles despite there being an observer who sees no particles in that state.* See Sec. 2.3.3 for discussion.

- What is the underlying reason for the similar mathematical structures of classical correlators and those from excited quantum states? *The classical Green's function has both signs of modes. If an excited quantum state is the vacuum* *for an observer related via a Bogoliubov transform to the Bunch-Davies vacuum, then in the picture where we work in the frame of this observer who sees a vacuum, the associated Wightman functions have both signs of modes too. If one instead continues to use the Bunch Davies picture, then the physical poles arise because of initial state particles interacting at a vertex with the field insertions produce modes with a relative sign between momenta.* See Sec. 2.6 for details.

2.2 The origin of singularities

We must clarify what we aim to understand from the calculations we wish to undertake. We wish to probe the interplay between the nature of the initial state and the structure of in-in correlators, through the momentum dependence of the singularity structures. What we mean by this is identifying the *configuration of momenta for which these correlation functions become singular.* The functional form of the singularity isn't fixed, and as we will see in a later section, it depends crucially on the nature of interactions. We have assumed the initial conditions are set at $\tau = -\infty$, and in Sec. 2.7 we see how things change at a finite early time cutoff.

The authors of [60] studied this in the context of understanding whether the initial conditions of inflation were classical perturbations in an ensemble or quantum zero-point fluctuations of the BD vacuum. The pole structure was linked to physical processes-such as particle decay in the initial (classical) state, producing poles at physical momenta. However, the results of [53] exhibit the physical poles despite considering α-mode insertions in an initial α-vacuum. This displays how the notion of calling a state 'classical' based on the particle content is subtle, as the α vacuum-while excited for a Bunch Davies observer, is devoid of α-particles. We will also see how the signatures of initial state effects reflect in the modification to Maldacena's consistency criteria (which is already modified in the presence of higher derivative operators [80])-in particular, non-Bunch Davies initial states produce severely enhanced deviations from the consistency criteria.

However, one needs to be careful about the physical implications of these singularity structures. To begin with, and as we'll elaborate on later, we must accept the inevitability of disconnected diagrams in correlation functions. Consider a flat space correlator with 3 field insertions and at cubic order. If the external state is a single-particle state of fixed momentum, then its natural contribution to the S-matrix element is schematically

$$\langle p|T\phi(k_1)\phi(k_2)\phi(k_3)|p\rangle \to^{LSZ} \langle p, k_1, k_2|p, k_3\rangle \tag{2.7}$$

where it is understood that crossed elements can be generated too; we have just written down a possible configuration. Now, at first order, it is easy to see that this amplitude will contain disconnected subgraphs where some of the particles are mere spectators- only 3 out of 5 particles can interact at a cubic vertex, and the remaining two will act as spectators, which don't participate in the interaction. These diagrams will produce factors of δ(subset of p, k_i), and will therefore only have support at isolated points in phase space. Generating fully connected contributions will require us to go to higher order in perturbation theory, where there are enough vertices to contract with every external leg-but these are subleading effects. It then becomes very tricky to find measurable imprints of these pole structures. In fact, the whole discussion is subtle because it is unclear how to translate from the flat space intuition of 'particles' and their Feynman diagrams, because the notion of particles is itself subtle in curved spaces. For example, it is entirely possible to switch to a different picture of particles, and the resulting correlators will have to be interpreted differently. As an example, which will be discussed in detail later, if we consider an initial state which is an excited Bunch Davies state such that it happens to be the vacuum for an α-observer, then the correlator can be computed in two ways. We can write the state in terms of Bunch-Davies excitations and compute the correlator that will now display disconnected pieces because of the initial state particles, see Sec. 2.3.3. Alternatively, we can go to a frame where there are no excitations-this frame corresponds to choosing the α-vacuum basis for mode expansion of our fields. Then, the diagrammatic interpretation of the involved terms will be entirely different, and we will see no disconnected pieces simply because now the interactions at a vertex don't involve initial state particles. Schematically, the two ways to compute the correlator amount to

$$_{\alpha}\langle 0|\zeta^{\alpha}\zeta^{\alpha}\zeta^{\alpha}|0\rangle_{\alpha} \quad \text{or} \quad _{BD}\langle \big(0|e^{\int d^3p a_p a_{-p}}\big)|\zeta^{BD}\zeta^{BD}\zeta^{BD}|\big(e^{\int d^3p a_p^{\dagger} a_{-p}^{\dagger}}|0\big)\rangle_{BD} \tag{2.8}$$

Where we have indicated what mode functions, i.e. what particle interpretation is being used. a_p are Bunch-Davies operators. Note that the second way of calculating requires including contractions representing interactions of in-state particles at a vertex, whereas the first doesn't. One expects that eventually some kind of resummation renders the two approaches equivalent, but it is not obvious.

The take-home message is this: once we begin considering excited initial states, then *because only finitely many of them can interact at a finite order in perturbation theory* (determined by the nature of the coupling and the order at which we are interested), correlation functions will be plagued by disconnected diagrams. Many pole structures will be obscured in observation because of the fact that their kinematic support lies on isolated points in phase space.

The disconnectedness of these diagrams clouds the interesting pole structure that comes from genuine interaction. If, then, we were in a situation where there are no real particles in the initial state, then one would expect to be free from these disconnected diagrams. This is, in fact, what happens whenever the initial state is a vacuum state of the perturbation modes, and when this vacuum state is 'excited' relative to a Bunch-Davies vacuum in a manner that all momenta are excited, one sees the presence of genuine, physically measurable poles. We would like to understand, then, to what extent pole structures encode information about the initial state. Let us first understand their origin.

2.2.1 Singularities and interacting vertices: flat space vs curved space

Let us elaborate on the remarks of the previous section, drawing an analogy from flat space results to see explicitly how the pole structure for in-in correlators is different from in-out and its interpretation in terms of physical processes. We will also see the precise origin of pole structures and the role of mode mixing. Our object of interest will be the structure of kinematics at a vertex in perturbation theory, since it is integrals over these vertices that produce analytic structures.

In flat space, we typically calculate time-ordered correlators via Wick's theorem. Let us first see how kinematics are constrained at a vertex in a simple $\lambda\phi^3/3!$ interaction in flat space. If we calculate the time ordered correlator, then the leading contribution to the 3-point correlator in perturbation theory is of the type

$$\langle T\phi(x_1)\phi(x_2)\phi(x_3)\rangle_{int} \sim \lambda \int d^4x D_F(x_1 - x)D_F(x_2 - x)D_F(x_3 - x) \tag{2.9}$$

where $D_F(x) = \int \frac{d^4p}{(2\pi)^4} e^{ipx}/(p^2 + i\epsilon)$ is the Feynman propagator, the use of which is tied to the presence of time ordering on the left. Let us focus on the integral over the vertex, where the momenta in the propagator's expression combine into a delta function, which tells us that four-momentum is conserved at every vertex. Since the p_i are being integrated over, and noting that $D_F(x) = D_F(-x)$, it is clear that it really doesn't matter if we choose to call a momentum p or $-p$. In fact, being a vertex or a correlation function, it doesn't immediately correspond to any physical process. Therefore, *we need to specify additional information to specify the correct kinematic constraints for any physical process*-this is precisely the LSZ prescription. This expression depends on x_i-the points of insertion. While the LHS is agnostic to their relative order because of time ordering within which everything commutes, the RHS needs greater care. Let us see how LSZ systematically teaches us what to do. Let us say that the field insertions at x_i correspond to one-particle states of momentum k_i in the asymptotic region, where the theory is assumed to be free(this is one of the assumptions that break down in curved space). For concreteness, let us say that x_1, x_2 correspond to insertions in the far past(in-state), and x_3 to an insertion in the far future(out-state)-this produces a $\delta^{(4)}(p_1 + p_2 - p_3)$ from the vertex integral. To see explicitly the interplay between presence of in/out states and time ordering, note that in a typical flat space in out correlator, everything including initial and final states is in time order; LSZ then converts it to an expression for $\langle f|i\rangle$, where a relative sign between momenta is induced because $i\int d^4x e^{ipx}(\partial_\mu\partial^\mu + m^2)\phi = \sqrt{2\omega_p}(a_\infty - a_{-\infty})$. This expression is, formally

$$|f\rangle = \sqrt{2k_1 \cdot 2k_2..}a^\dagger_{k_1}(\infty)a^\dagger_{k_2}(\infty)..|\Omega\rangle, |i\rangle = \sqrt{2p_1 \cdot 2p_2..}a^\dagger_{p_1}(-\infty)a^\dagger_{p_2}(-\infty)..|\Omega\rangle \tag{2.10}$$

$$\langle f|i\rangle = \sqrt{2k_1 \cdot 2k_2 \cdots 2p_1 2p_2..}\langle\Omega|a_{k_1}(\infty)a_{k_2}(\infty).....a^\dagger_{p_1}(-\infty)..|\Omega\rangle \tag{2.11}$$

And everything is naturally time-ordered, and we use Feynman propagators to evaluate this expression via Wick's theorem. However, in an in-in correlator, we only have $|i\rangle$ state available, and therefore a correlation function is of the form

$$\langle i|\mathcal{O}(t)|i\rangle \sim \langle 0|a_{p_1}(-\infty)a_{p_2}(-\infty)..\mathcal{O}(t)a^\dagger_{p_1}(-\infty)a^\dagger_{p_2}(-\infty)...|0\rangle$$

We must therefore resort to using Wightman functions, and this is precisely where destructive interference between modes can occur. For comparison, consider the difference between a sample-in-out and an in-in correlator. First,

$$\begin{aligned} &- i\lambda\delta(\sum \vec{k}_i)\int dt\langle k, out|T\phi_{k_1}(t)\phi_{k_2}(t)\phi_{k_3}(t)|p, p', in\rangle \\ &\qquad\qquad \supset \langle a_k(\infty)\phi_{k_1}\rangle\langle\phi_{k_2}a^\dagger_p(-\infty)\rangle\langle\phi_{k_3}a^\dagger_{p'}(-\infty)\rangle \\ &= -i\lambda\delta(\sum \vec{k}_i)\int dt\delta(\vec{k}+\vec{k}_1)\delta(\vec{k}_2-\vec{p})\delta(\vec{k}_3-\vec{p}')e^{i(k-p-p')t} = -i\lambda\delta^{(4)}(k-p-p') \end{aligned} \tag{2.12}$$

Each contraction here is a Feynman propagator. The $1/2k$ normalisations cancel in the propagator and the definition of the external states.

To see what happens without an out state, let us compute the in-in correlator at equal-time insertions for this example. The first difference in the vertex integrals is the replacement

$$\int_{\infty}^{\infty} dt' \to 2Im \int_{-\infty}^{t} dt' \tag{2.13}$$

where t is the time of insertion. With this replacement, the delta function for energies becomes a pole, with the expression(here p is p^0)

$$\int_{\infty}^{\infty} dt' e^{i(p_1+p_2+p_3)t'} = \delta(p_1+p_2+p_3) \to 2Im \int_{-\infty}^{t} dt' e^{i(p_1+p_2+p_3)t'} = \frac{2}{p_1+p_2+p_3} \tag{2.14}$$

The question now is, in flat space, can we have physical poles like $(k_1+k_2-k_3)^{-1}$ in equal time in-in correlators? Clearly, these poles would correspond to an energy-conserving delta function for a physical process $k_1, k_2 \to k_3$. But these, as we saw, arise *after* carrying out LSZ and placing some insertions in the far past and some in the far future-this cannot be done in an equal time in-in correlator. This was also observed in [60] where it was shown that LSZ killed all poles which didn't correspond to physical processes in flat space.

So then, how do physical poles arise in inflation? First, note that in a general background the modes are complicated functions $u_k(t)e^{\pm(\omega t - \vec{k}\cdot\vec{x})}$. Secondly, in the in-in correlators, one replaces the Feynman propagator $D_F(t_1, t_2)$ with the *Wightman function*, G_W. Recall that the flat space Wightman function and Feynman propagator are related as

$$D(t) = \langle 0|\phi_k(t)\phi_{k'}(0)|0\rangle = (2\pi)^3\delta(\vec{k}+\vec{k'})\frac{1}{2k}e^{-ikt} = D(-t)^*, \tag{2.15}$$

$$D_F(t) = \theta(t)D(t) + \theta(-t)D(t)^* \tag{2.16}$$

We see that while the Feynman propagator contains both mode functions formally, it only ever has support on one of them-whichever is picked by time ordering. In contrast, the classical Green's function and the Wightman function for a Bogoliubov-transformed mode are

$$G_c(k.t) = \frac{-i}{2k}(e^{ikt} + e^{-ikt}) \tag{2.17}$$

$$G_W^{BG}(0,\tau) = (2\pi)^3\delta(\vec{k}+\vec{k}')(Au_k(0) + Bu_k(0))(A^*u_k^*e^{ik\tau} + B^*u_ke^{-ik\tau}) \tag{2.18}$$

where $u_k = \frac{H}{\sqrt{2k^3}}(1+ik\tau)$. Both of these contain both frequencies of modes, and it is natural to expect that they will generate a similar pole structure in correlation functions. Note a number of things. Time ordering automatically ensures all these Wightman functions are just Feynman propagators, and the relative sign of the mode functions in the integral boils down to the existence of both in and out states.

To see this explicitly, suppose we instead calculated the (physically nonsensical) time-ordered quantity $\langle 0|\phi_1\phi_2\phi_3|k,p,p',in\rangle$ where there's no out state. Then, the first (time ordered) contraction above is replaced by $\langle a_k(\infty)\phi_{k_1}(t)\rangle \sim e^{ikt} \to \langle \phi_{k_1}(t)a_k^\dagger(-\infty)\rangle \sim e^{-ikt}$, where the physical meaning changes from creation at the vertex to annihilation. This new correlator evaluates to $-i\lambda\delta^{(4)}(k+p+p')$, which ofcourse cannot be physically reached, but has the same energy-momentum dependence as the Bunch Davies in-in correlator where we would see a $1/(k+p+p')$ pole for mode insertions with momenta k,p,p'. We see clearly the role of the presence of an out-state in reaching physical singularities, and the presence of unphysical total energy singularities in the presence of only in-states. The relationship between time ordering and the presence of out states can also be seen by noting that $\langle a_k(\infty)\phi(t)\rangle$ doesn't vanish because of how time ordering places the a so that it acts on the bra. If we had an in-state on the left, this would be replaced by $\langle a_k(-\infty)\phi(t)\rangle$, which would be killed by time ordering as it would take a to the right and annihilate the vacuum. Thankfully, such a correlator wouldn't be time ordered anyway, and therefore we'd be forced to use Wightman functions instead of the Feynman propagator, and they don't vanish.

Now, consider a correlation function with a highly occupied state N, of the form $\int d^4x\langle N|T\phi\phi\phi|N\rangle$. By construction, this correlator is a physical process with external states put on shell. Therefore, integrating over the vertex will produce physical poles depending on which mode of the initial state we use to contract with the operators inside the correlator. We can pick out 2 $k_i's$ from the in state and one from the out state and get a delta function that conserves the in-out momentum, which in the in-in case would produce a physical pole. In the language of flat space Feynman rules, these physical delta functions arise because of the fact that contractions with in and out states induce exponentials with different signs of momenta-indeed this is due to the implicit LSZ that has been carried out already where we know that the exponentials for in/out states are hermitian conjugates of each other.

This is the state of affairs. In the presence of particles in the external states, the possibility of a physical pole arises because they may either annihilate or be created at a vertex. In an in-out correlator, the relative sign between the time of insertions for in and out states (which is, in practice, consistently accounted for via LSZ) between these processes reflects in physical singularities. In an in-in correlator, operationally, this reflects in the fact that $\langle a\phi_p(t)\rangle \sim e^{ikt}$ whereas $\langle \phi_p(t)a^\dagger\rangle \sim e^{-ikt}$. We will see that physical poles will come from contractions schematically of the type

$$\langle ...p..|\zeta_1\zeta_2\zeta_3|...p..\rangle \supset \langle p\zeta_1\rangle\langle\zeta_2\zeta_3 p\rangle \tag{2.19}$$

where we indicate the terms which contract together at a vertex(the second term), and the term that spectates(the first term). In the absence of particles in the initial state, or the inability to perform LSZ; correlation functions will only depend on the Wightman functions between field insertions, and in time dependent backgrounds it is possible-in fact necessary- to have Wightman functions that include both modes(as opposed to the Feynman propagator which picks only one based on time ordering) and therefore produce physical singularities through their interference terms, despite the absence of features like time ordering and LSZ in in-in correlators. In curved space, we will use the trick of performing calculations in a frame where there are no particles, but the Wightman functions include both modes and consequently help achieve the same physical singularities as one would expect in the presence of particles.

The occurrence of physical poles in cases where this can't be done, and therefore initial state particles interacting at a vertex cannot be avoided, is discussed in Sec. 2.6. With particles in the initial state, an in-in correlator receives contributions from terms where one of the initial state particles is contracted with the (cubic) vertex insertion along with 2 of the field insertions, while the third spectates. This contraction generates a physical pole at the right momentum support for the spectator.

In Sec. 2.3.5, the occurrence of this pole in the presence of a single particle in the initial state was studied in detail.

2.3 In-In Correlation functions and associated analytic structures

In this section, we will report, case by case, the analytic structure for correlators corresponding to various choices of initial states and mode functions (i.e., the choice of vacuum). We use the in-in formalism to compute quantum correlators. While our primary interest will be the 3-point function, we also write down the power spectrum for completeness. We will also look at the structure obtained with a coherent initial state and contrast it with a general excited initial state. Finally, we will look at the 4-point function calculated using both the in-in formalism and a classical perturbation theory formalism, and check if our intuition after studying 3-point functions holds true there too.

The results in this section will be organised in the following manner. We will first fix the mode functions, i.e. the vacuum state of the perturbations to be either BD or alpha-modes, and then compute the correlation functions with the following choices of initial states-

- The initial state coincides with the vacuum state
- The initial state is a one-particle state of definite momentum
- The initial state is an excited state with all modes occupied
- The initial state is a highly excited state which is a *Bogoliubov transform of Bunch Davies.*
- The initial state is a coherent state for either a Bunch-Davies or a Bogoliubov transformed observer

The results, to be detailed in the following sections, are summarised in the following table-

Initial state	Pole structure
BD vacuum	k_t pole
Excited over BD with finite modes present	disconnected physical poles, connected k_t pole
Excited over BD with all modes present	connected k_t pole, observable physical pole
Excited over BD but a BG	both physical and k_t poles
Excited over BD but coherent	k_t pole
Excited over BD but coherent for some BG	both k_t and physical poles
Classical	both k_t and physical poles

2.3.1 Normalisation of the expectation value

We must recognise that computing the expectation value for any state requires that we also divide by the norm of that state, which needn't always be one. For example, with a $|p\rangle$ state the norm is proportional to $\delta^{(3)}(\vec{p}-\vec{p})$ which is formally divergent. However, a correlation function of the form $\langle p|\zeta_1\zeta_2\zeta_3|p\rangle$ will also contain factors of $\langle p|p\rangle$ when we contract the two initial states together (nothing prevents us from doing it at the level of correlation functions, this is just a disconnected diagram). Therefore, after dividing by the normalisation, we will see that this disconnected piece's contribution vanishes, and we're left with a finite term whose pole structure can be read off. All other terms will be suppressed by this (infinite) normalisation factor, except when they have localised support in phase space through other delta function singularities, in which case they can peak at *those* points in phase space, but those points only. It is important to note that $\delta^{(3)}(0)$, wherever it occurs, will be regulated by a volume V, which we will see ultimately drops out of the correlator. Also, the factor of $2E_p$ that relativistically normalises a $|p\rangle$ drops out. In the below correlators (evaluated for a $\frac{\lambda}{3!}\dot{\zeta}^3$ interaction), therefore, such a normalisation is always implicit, even if not explicitly written.

2.3.2 BD vacuum initial state

We review the results for the most common choice of initial state.

2.3.2.1 Two point function

At $O(\lambda^0)$ the two-point function is given by

$$\langle 0|\, \zeta_{k_1}\zeta_{k_2}\,|0\rangle = \frac{\Delta_\zeta^2}{k_1^3}\delta^3(\vec{k_1}+\vec{k_2}) \tag{2.20}$$

2.3.2.2 Three point function

To calculate

$$\langle 0|\, \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\,|0\rangle \tag{2.21}$$

At $O(\lambda^0)$ the three point function vanishes so we compute at $O(\lambda)$[59]

$$\langle 0|\, \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\,|0\rangle = \frac{4H^{-1}\Delta_\zeta^6\lambda}{(k_1+k_2+k_3)^3 k_1k_2k_3}\delta^3(\sum \vec{k_i}) \tag{2.22}$$

2.3.3 Excited initial state from Bogoliubov transform: α-vacuum

In this section, we consider initial states which are excited relative to a Bunch-Davies observer, but can *also* be reached by a Bogoliubov transform of the Bunch-Davies vacuum. That is, we consider excitations that a different observer would see when observing a Bunch-Davies vacuum. What this means is that there exists an observer for which such a state is actually the vacuum, while it has particles for a Bunch-Davies observer. From a calculation standpoint, this is a massive simplification because vacuum states are simpler to deal with than excited states. For concreteness, let us study an initial state that would correspond to an α vacuum, which is also de Sitter invariant and can be reached from the Bunch-Davies initial state by a simple Bogoliubov transform.

Given the Bunch Davies mode functions $u_k(\tau)$ and it's set of creation and annihilation operators $a_k, a_k^\dagger$, the Bogoliubov creation annihilation operators are $b_k = \beta^* a_k - \alpha^* a_{-k}^\dagger$

where the α, β are parametrized by a single parameter (it's part of the 2 parameter family of dS invariant vacua, and we set one of the parameters to zero) $\tilde{\alpha}$; $\beta, \alpha = \cosh(\tilde{\alpha}), -i\sinh(\tilde{\alpha})$. The α vacuum can be expressed in terms of the Bunch-Davies expression through ([53])

$$|\alpha\rangle = \frac{1}{N}\exp\left(\frac{\alpha^*}{2\beta^*}\int \frac{d^3k}{(2\pi)^3} a_k^\dagger a_{-k}^\dagger\right)|0\rangle \tag{2.23}$$

Where N is an overall normalisation constant. We see straightaway that it contains Bunch Davies excitations, but they come in pairs of equal and opposite momenta, and therefore the state contains no net momentum. One would therefore hope that any disconnected pieces in the three-point function will only have support at zero momentum for some of the external momenta (since there is no nonzero spectator particle momentum that requires to be conserved along with them)-an uninteresting case. Let us explicitly see how the calculation of the three-point function goes through. We are interested in the object

$$\langle\alpha|\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}|\alpha\rangle = \frac{1}{N^2}\Bigg(\langle 0|\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}|0\rangle + \int \frac{d^3p}{(2\pi)^3}\frac{\alpha^*}{2\beta^*}\langle 0|\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}a_p^\dagger a_{-p}^\dagger|0\rangle$$

$$+\int \frac{d^3p}{(2\pi)^3}\frac{\alpha}{2\beta}\langle 0|a_{-p}a_p\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}|0\rangle + \frac{|\alpha|^2}{4|\beta|^2}\int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\langle 0|a_{-p}a_p\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}a_q^\dagger a_{-q}^\dagger|0\rangle + ...\Bigg) \tag{2.24}$$

Let us name these terms I, II, III, IV so that the correlator on the left is their sum. The terms can be arranged in powers of $\frac{\alpha}{\beta}$, and we therefore consider representative examples from each power. To start with, we see that at zeroth power, i.e. the term I, the bispectrum is the same as that for Bunch-Davies vacuum, and we have a total k pole only with no disconnected subgraphs. Let us move on to terms linear in α/β; consider III at first order in perturbation theory-

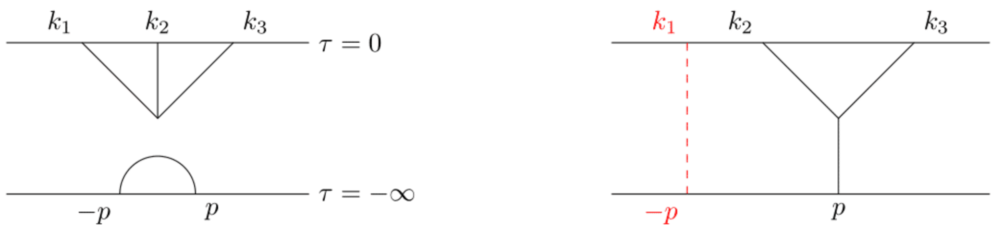


FIGURE 2.2: (Left) The initial state particles are spectators. (Right) The initial state particles interact at a vertex. There is a sum over all possible p values.

$$\int \frac{d^3P}{(2\pi)^3}\langle 0|a_{-P}a_P\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}|0\rangle = 2\mathrm{Im}\prod_{i=1}^{3}\int \frac{d^3p_i}{(2\pi)^3}e^{i\vec{p}_i\cdot\vec{x}}\int\frac{d\tau}{\tau}\int\frac{d^3P}{(2\pi)^3}$$

$$\times\langle 0|a_{-P}a_P\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta'_{p_1}(\tau)\zeta'_{p_2}(\tau)\zeta'_{p_3}(\tau)|0\rangle \quad (2.25)$$

$$= 2\mathrm{Im}\prod_{i=1}^{3}\int\frac{d^3p_i}{(2\pi)^3}e^{i\vec{p}_i\cdot\vec{x}}\int\frac{d\tau}{\tau}\int\frac{d^3P}{(2\pi)^3}\langle a_P\zeta_{k_1}\rangle\langle a_{-P}\zeta'_{p_1}\rangle\langle\zeta_{k_2}\zeta'_{p_2}\rangle\langle\zeta_{k_3}\zeta'_{p_3}\rangle + \text{perm.}$$

$$= \int \delta(\vec{P}+\vec{k_1})\delta(\vec{P}-\vec{p_1})\delta(\vec{k_2}+\vec{p_2})\delta(\vec{k_3}+\vec{p_3})(u^*_{k_1}(0)u'^*_{p_1}(\tau))(u_{k_2}(0)u'^*_{p_2}(\tau))(u_{k_3}(0)u^*_{p_3}(\tau))$$

$$= \delta(\vec{k}_t)2\mathrm{Im}\int\frac{d\tau}{\tau}\prod_{i=1}^{3}\left(u_{k_i}(0)u'^*_{k_i}(\tau)\right) \quad (2.26)$$

Thus we find, upto permutations(we have used that $u_{k_1}(0)$ is purely real in the first paranthesis)

$$III = \frac{H^6}{8k_1k_2k_3}2\mathrm{Im}\int_{-\infty}^{0} d\tau\tau^2 e^{i(k_1+k_2+k_3)\tau} = \frac{H^6}{2k_1k_2k_3}\frac{1}{(k_1+k_2+k_3)^3} \quad (2.27)$$

We see that we only have a total energy pole. Figure 2.2 explains diagrammatically the situation.

Let's check if this changes at the next order in (α/β). Take IV-

$$\int \frac{d^3P}{(2\pi)^3}\frac{d^3Q}{(2\pi)^3}\langle 0|a_{-P}a_P\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}a^\dagger_Q a^\dagger_{-Q}|0\rangle = 2\mathrm{Im}\prod_{i=1}^{3}\int\frac{d^3p_i}{(2\pi)^3}e^{i\vec{p}_i\cdot\vec{x}}\int\frac{d\tau}{\tau}\int\frac{d^3P}{(2\pi)^3}$$

$$\times\langle 0|a_{-P}a_P\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta'_{p_1}(\tau)\zeta'_{p_2}(\tau)\zeta'_{p_3}(\tau)a^\dagger_Q a^\dagger_{-Q}|0\rangle \quad (2.28)$$

This will obviously include disconnected pieces because there are more terms to contract than there are external points. For example, consider the contraction

$$IV \supset \int \langle a_P\zeta_{k_1}\rangle\langle a_{-P}\zeta_{k_2}\rangle\langle \zeta'_{p_1}a^\dagger_Q\rangle\langle\zeta'_{p_2}a^\dagger_{-Q}\rangle\langle\zeta_{k_3}\zeta'_{p_3}\rangle \propto \delta(\vec{k}_3)\delta(\vec{k}_2+\vec{k}_1) \quad (2.29)$$

which is disconnected and has support only when one of the momenta is zero.

Consider instead the contraction

$$\langle a_P\zeta_{k_1}\rangle\langle\zeta_{k_2}\zeta'_{p_2}\rangle\langle\zeta_{k_3}\zeta'_{p_3}\rangle\langle\zeta'_{p_1}a^\dagger_{-Q}\rangle\langle a_{-P}a^\dagger_Q\rangle$$

This yields a piece proportional to the integral of $\delta(\vec{k}_t)e^{i(-k_1+k_2+k_3)\tau}$, which is fully connected but has a physical pole at $k_1 = k_2 + k_3$. On the other hand, consider the contraction

$$\langle\zeta_{k_1}a^\dagger_{-Q}\rangle\langle\zeta_{k_2}\zeta'_{p_2}\rangle\langle\zeta_{k_3}\zeta'_{p_3}\rangle\langle a_P\zeta'_{p_1}\rangle\langle a_{-P}a^\dagger_Q\rangle$$

This piece yields again a connected result (proportional to $\delta(\vec{k}_t)$), but this time with a total energy pole at $(k_1 + k_2 + k_3)$.

Thus, with this way of calculating things, we see that we obtain both total and physical energy poles, but their physical meaning is obscured because of the fact that some of these might be disconnected. This diagrammatic interpretation in terms of connected/disconnected diagrams is obviously a handicap because of the unclear particle interpretation in curved spaces-in particular, we could use a different basis to expand the ζs and the resulting calculations would be a mess. One would hope that all such calculations are ultimately identical, but it is far from obvious at this stage.

But it is precisely this freedom of choosing a particle interpretation that rescues us. If the initial state is chosen to be among a family of Bogoliubov transforms of the Bunch-Davies states, then there is a natural choice of the mode functions where the action of the fields on the state is simple. For example, if the initial state is an α vacuum, it would be natural to use the mode expansion with α modes. This approach should be free of any artificial disconnected diagrams, because the initial state is now a vacuum state for an observer using this basis. Note that the field itself is fully invariant-observers are distinguished by their choice of mode functions and accompanying creation operators. Using the freedom of being able to choose any frame, we use the frame where *the initial state is a vacuum for the choice of observer*, and then we are left with fully connected pieces. With this approach, we can expand our field using the α mode functions $v_k = \alpha u_k^* e^{ik\tau} + \beta u_k e^{-ik\tau}$ with corresponding modifications to the creation operators via the Bogoliubov transform, and the calculation is extremely simple because the action of these on the external states is simple.

With this choice, we find the new mode functions, which we now call u_k

$$u_k(\tau) = \alpha(1 - ik\tau)e^{ik\tau} + \beta(1 + ik\tau)e^{-ik\tau} \tag{2.30}$$

where, α, β are complex constants. These vacua, parametrised by α and β, are invariant under de Sitter isometries[54]. In other words, all co-moving/free-falling observers will agree on the choice of α, β.

The free two point ($O(\lambda^0)$) function is given by

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle = A \frac{\Delta_\zeta^2}{k_1^3} \delta^3(\vec{k_1} + \vec{k_2}) \tag{2.31}$$

where, $A = |\alpha + \beta|^2$

We have at $O(\lambda)$ (all non-zero contractions are equal)

$$\begin{aligned} = Re\Bigg(&-2i\delta^3(\vec{k_1} + \vec{k_2} + \vec{k_3})(\alpha + \beta)^3 \frac{\Delta_\zeta^6 H^{-1}}{k_1 k_2 k_3} \int_{-\infty}^0 d\tau' \tau'^2 \left(\alpha^* e^{-ik_1\tau'} + \beta^* e^{ik_1\tau'}\right) \\ &\times \left(\alpha^* e^{-ik_2\tau'} + \beta^* e^{ik_2\tau'}\right) \left(\alpha^* e^{-ik_3\tau'} + \beta^* e^{ik_3\tau'}\right) \end{aligned} \tag{2.32}$$

$$
\begin{aligned}
=&4\lambda\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3})(\alpha+\beta)^3\frac{\Delta_\zeta^6 H^{-1}}{2k_1k_2k_3}\left[\left(\beta^{*2}\alpha^*-\alpha^{*2}\beta^*\right)\left(\frac{1}{(k_1+k_2-k_3)^3}\right.\right.\\
&\left.\left.+\frac{1}{(k_1-k_2-k_3)^3}+\frac{1}{(-k_1+k_2-k_3)^3}\right)+\left(\beta^{*3}-\alpha^{*3}\right)\frac{1}{(k_1+k_2+k_3)^3}\right]+c.c
\end{aligned}
\tag{2.33}
$$

In the limit where $\beta=0$, i.e we are back to Bunch-Davies vacuum, the physical poles vanish; therefore, everything is consistent. Notice that there are, by definition, no α particles in α vacuum, but still the three-point function exhibits physical poles. Therefore, the particle interpretation for physical poles [59] does not hold in this case. It seems that the particle interpretation is only clear when the observer is using a Bunch-Davies basis.

2.3.4 Coherent States

There is yet another special class of excited states-the so called coherent states, which are eigenstates for annihilation operators. Since this is an observer-dependent statement in time-dependent backgrounds, it is interesting to ask what poles these will generate, and will the answer change if they are defined with respect to different observers? In fact, as coherent states in quantum mechanics are known to show classical behaviour, their pole structure will be extremely instructive to sharpen our understanding of classical vs quantum initial states. It turns out that coherent states for a Bunch-Davies observer with certain assumptions can have the absence of physical poles [79] (in contrast with [59]). We do the same calculation with a $\frac{\lambda}{3!}\dot{\zeta}^3$. This is an outlier case which goes against the currently accepted correspondence between the initial state and pole structure.
The coherent states are simply eigenstates of the lowering operator

$$a_{\vec{k}}\,|C\rangle = C(\vec{k})\,|C\rangle \tag{2.34}$$

The calculation of the Bispectrum is done again using a $\frac{\lambda}{3!}\dot{\zeta}^3$ with one additional assumption i.e

$$C(\vec{k})+C^*(-\vec{k})=0 \tag{2.35}$$

Which is equivalent to setting the one-point function to zero at $\tau = 0$, that is $\langle C|\zeta(0)|C\rangle = 0$. We need to compute at $O(\lambda)$

$$\langle C|\, \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\, |C\rangle \tag{2.36}$$

We can compute it in the following way. By Wick's theorem

$$\zeta\zeta\zeta....\zeta =: \zeta\zeta\zeta....\zeta : + : \textit{all possible contractions} : \tag{2.37}$$

Since now we are not computing in the vacuum state, therefore all terms on the RHS of Eq. (5.81) contribute, but the condition Eq. (2.35) makes everything vanish except the term where all fields are contracted, and we get the usual BD answer. A simple proof is as follows. In every term that is not fully contracted, there exists, say, an uncontracted $a^\dagger$. There must also be a term which is different from this only by the replacement $a^\dagger \to a$. Adding these terms gives a result proportional to $C(\vec{k}) + C^*(-\vec{k})$, which vanishes by assumption. To illustrate it further, consider a term with just two contractions

$$\sim \lambda\, \langle C| : \wick{\c1 \zeta_{k_1} \c2 \zeta_{k_2} \zeta_{k_3} \c1 \zeta'_{q_1}(\tau) \c2 \zeta'_{q_2}(\tau)} \zeta'_{q_3}(\tau) : |C\rangle \tag{2.38}$$

where we have omitted the overall integral and the overall delta function. This simplifies to

$$\begin{aligned}\sim \lambda \wick{\c \zeta_{k_1} \c \zeta'_{q_1}}(\tau) \wick{\c \zeta_{k_2} \c \zeta'_{q_2}}(\tau)\, \langle C|\, a^\dagger_{k3} a^\dagger_{q3} u_{k3} u'_{q3}(\tau) + a^\dagger_{k3} a_{-q3} u_{k3} u'^*_{q3}(\tau) + a^\dagger_{q3} a_{-k3} u^*_{k3} u'_{q3}(\tau) \\ + a_{-k3} a_{-q3} u^*_{k3} u'^*_{q3}(\tau)\, |C\rangle\end{aligned} \tag{2.39}$$

where, u_k's are the de Sitter mode functions at $\tau = 0$. Using the fact that $a_k\,|C\rangle = C(k)\,|C\rangle$, we get

$$\begin{aligned}\sim \lambda \wick{\c \zeta_{k_1} \c \zeta'_{q_1}}(\tau) \wick{\c \zeta_{k_2} \c \zeta'_{q_2}}(\tau)\, \langle C|\, u'_{q_3} \Big(C^*(\vec{k_3}) + C(-\vec{k_3})\Big)\, C^*(\vec{q_3}) + u^*_{q_3} \Big(C^*(\vec{k_3}) \\ + C(-\vec{k_3})\Big)\, C(-\vec{q_3})\, |C\rangle\end{aligned} \tag{2.40}$$

Finally, using $C^*(-\vec{k}) + C(\vec{k}) = 0$ one can easily check that the expression vanishes. In this way, all partially contracted terms can be shown to vanish. Schematically, representing uncontracted operators in red, terms can be pairwise combined and cancelled as follows

$$\langle C|a^\dagger a^\dagger..a_K^\dagger..aaa|C\rangle + \langle C|a^\dagger a^\dagger..a_K..aaa|C\rangle \propto (C(K) + C^*(-K)) = 0 \tag{2.41}$$

thereby leaving behind only the fully contracted piece,

$$\begin{aligned} &\mathrm{Re} - 2i\int\left(\prod_i d^3\vec{q}_i\right)\delta^3(\vec{k_1}+\vec{q_1})\delta^3(\vec{k_2}+\vec{q_2})\delta^3(\vec{k_3}+\vec{q_3})\delta^3(\vec{q_1}+\vec{q_2}+\vec{q_3}) \\ &\qquad\qquad \frac{q_1^2q_2^2q_3^2\Delta_\zeta^6 H^{-1}\lambda}{\sqrt{(k_1k_2k_3q_1q_2q_3)^3}}\int_{-\infty}^0 d\tau'\tau'^2 e^{i(q_1+q_2+q_3)\tau'} \end{aligned} \tag{2.42}$$

$$= \mathrm{Re} - 2i\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3})\frac{\Delta_\zeta^6 H^{-1}\lambda}{k_1k_2k_3}\int_{-\infty}^0 d\tau'\tau'^2 e^{i(k_1+k_2+k_3)\tau'} \tag{2.43}$$

which then yields a result identical to the Bunch Davies case up to the normalisation $\langle C|\rangle$, which drops out of the overall expectation value. Thus,

$$\langle C|\,\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\,|C\rangle = \frac{4H^{-1}\Delta_\zeta^6\lambda}{(k_1+k_2+k_3)^3k_1k_2k_3}\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3}) \tag{2.44}$$

These results seem to contradict the conclusion of [59] that a total energy pole will uniquely single out BD vacuum as the initial state. The above class of coherent states, as seen above, also shares the same feature.

Since the coherent state's definition is observer dependent- more precisely, dependent on the basis of annihilation/ creation operators used; one can ask what happens if the initial state is coherent not for a Bunch Davies observer but for an observer that is related by a Bogoliubov transform, say the α-vacuum. We therefore now consider

$$b_{\vec{k}}|C\rangle_\alpha = C(\vec{k})|C\rangle_\alpha \tag{2.45}$$

This is not coherent for a Bunch-Davies observer because $b \sim a - a^\dagger$ and the second piece renders it not to be an eigenstate of both b and a. Therefore, it is a generic excited state for a Bunch-Davies observer, and one would expect to see physical poles. Indeed, by expanding the field insertions with an α observer's mode functions, one finds

$$
\begin{aligned}
{}_\alpha\langle C|\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}|C\rangle_\alpha = 4\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3})(\alpha+\beta)^3\frac{\Delta_\zeta^6 H^{-1}\lambda}{2k_1k_2k_3}\Bigg[(\beta^{*2}\alpha^{*}-\alpha^{*2}\beta^{*})& \\
\left(\frac{1}{(k_1+k_2-k_3)^3}+\frac{1}{(k_1-k_2-k_3)^3}\right.& \\
\left.+\frac{1}{(-k_1+k_2-k_3)^3}\right)+(\beta^{*3}-\alpha^{*3})\frac{1}{(k_1+k_2+k_3)^3}\Bigg]+c.c& \quad (2.46)
\end{aligned}
$$

The normal ordered product of the b's vanishes for the same reason as before. We therefore see plainly physical poles, and this is in tune with our intuition that a state that is coherent for an α observer is just some generic excited state for a BD observer and therefore must display physical poles.

2.3.5 BD modes, one particle state

The purpose of this section is to see the precise origin of physical poles starting from a single excitation, and their support in phase space

2.3.5.1 Two point function

Performing the contractions one gets at $O(\lambda^0)$

$$
=\frac{\Delta_\zeta^2}{k^3}\delta^3(\vec{k_1}+\vec{k_2})\left(1+\frac{1}{V}\left[\delta^3(\vec{k_1}+\vec{p})+\delta^3(\vec{k_1}-\vec{p})\right]\right) \qquad (2.47)
$$

The k dependence does not change.

2.3.5.2 Three point function

At $O(\lambda^0)$ the correlator vanishes. Therefore, we look at $O(\lambda)$ contribution, where a typical contraction gives terms like

$$-2i\int\left(\prod_i d^3\vec{q_i}\right)\delta^3(\vec{k_1}-\vec{p})\delta^3(\vec{q_3}-\vec{p})\delta^3(\vec{k_2}+\vec{q_1})\delta^3(\vec{k_3}+\vec{q_2})\delta^3(\vec{q_1}+\vec{q_2}+\vec{q_3})$$
$$\frac{q_1^2q_2^2q_3^2\Delta_\zeta^6H^{-1}\lambda}{\sqrt{(k_1k_2k_3q_1q_2q_3)^3}}\int_{-\infty}^0 d\tau'\tau'^2e^{-i(q_1+q_2+q_3)\tau'} \tag{2.48}$$

$$=-2i\delta^3(\vec{k_1}-\vec{p})\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3})\frac{\Delta_\zeta^6H^{-1}\lambda}{k_1k_2k_3}\int_{-\infty}^0 d\tau'\tau'^2e^{-i(k_1+k_2+k_3)\tau'} \tag{2.49}$$

Adding contributions from all contractions, we get

$$=\frac{-4\lambda\Delta_\zeta^6H^{-1}}{k_1k_2k_3}\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3})\left[\left(\frac{1}{k_1+k_2+k_3}\right)\right.$$
$$\left.+\frac{1}{V}\left(\frac{1}{(k_1+k_2+k_3)}\delta^3(\vec{k_1}-\vec{p})+\frac{1}{(-k_1+k_2+k_3)}\delta^3(\vec{k_1}+\vec{p})+k_1\leftrightarrow k_2+k_1\leftrightarrow k_3\right)\right] \tag{2.50}$$

Since this is real, the real part is the above expression itself. Apart from a total momentum conserving delta function in Eq. (2.47) and Eq. (2.50) we also have additional delta functions involving $\vec{p}$. Therefore, these are disconnected diagrams which contribute only at isolated points in phase space, where there is an additional V from the numerator that removes the delta function singularity in the denominator. Notice, in Eq. (2.50), the total energy pole survives, and reflects the fact that the vacuum fluctuations can always annihilate at a vertex among themselves without any reference to initial state particles.

Let us carefully analyse this expression in light of the discussion in literature [59, 60] where the presence of physical poles was ascribed to physical scatterings and decays in the initial- possibly 'classical'-state. Note that there are indeed physical poles that correspond to scattering-like processes in the initial state (wherein the support of the disconnected pieces depends on the initial state momentum, as required by momentum conservation in any process), as well as a total energy pole that corresponds to vacuum fluctuations. Note that now if we divide by the norm of the state, the only unsuppressed contribution is the last term, which has the same structure as a Bunch-Davies vacuum 3-point function. It is true that formally there are physical poles that when extrapolated to flat space would indicate interactions between the initial state particle and the field insertions-what this means is that there exist diagrams with interaction vertices which can connect the external p state with two of the k_i modes. The remaining p state is then contracted with the remaining field insertion, and momentum conservation produces the offending delta function that makes the support localised. Dividing by the norm then forces this term to always vanish, except when the support is reached, i.e. the unique point in phase space where one of the field insertions has the same momentum as the initial state. It is natural to ask what happens when the initial state contains all possible momenta instead of a single particle with a fixed momentum p. The physical pole should now be more likely to be observable because of the larger support. In fact, it was shown in [81] that an initial state with a finite number of quanta is already at odds with data (which is largely compatible with Bunch Davies initial conditions), and therefore it is natural to ask how this changes when there is a very large number of quanta in the initial state-atleast one for every momentum. In this case, the isolated support for the physical poles is expected to smear out, because there will always be a momentum in the initial state that has support at the same momentum as one of the k_i. Then, while the physical poles will still formally have support at isolated momenta, the fact that the phase space of support is continuous will ensure that at least one of the physical poles is always reached. Let us see how this happens explicitly. Consider an initial state where multiple momenta are occupied; $\sqrt{2p_1 \cdot 2p_2 \cdot 2p_3 \cdots}|p_1, p_2, p_3, ...\rangle$. This state will be normalised with respect to $\prod \langle p_i|p_i\rangle$; all other contractions are zero by orthogonality. Then, the 3-point function will be schematically-

$$\frac{\langle p_1, p_2, p_3, ...|\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}|p_1, p_2, p_3, ..\rangle}{\prod \langle p_i|p_i\rangle} \propto \delta(\vec{k}_t)\Bigg[\frac{1}{(k_1+k_2+k_3)^3}$$

$$+\frac{1}{(k_1+k_2+k_3)^3}\sum_i \frac{\delta(\vec{k}_1-\vec{p}_i)}{\langle p_i|p_i\rangle} + \frac{1}{(k_2+k_3-k_1)^3}\sum_i \frac{\delta(\vec{k}_1+\vec{p}_i)}{\langle p_i|p_i\rangle} + \text{perm.}\Bigg] \quad (2.51)$$

All other contractions vanish because they involve spectator terms like $\delta(\vec{p}_i - \vec{p}_j)$, which for unequal momenta vanish. Note now that in comparison to the 1 particle state, the physical poles have more support - at each of the p_is now, and the observational imprint of these states. If the momenta were continuously occupied-that is to say that p_is were a continuous variable, then we would expect a continuous distribution. Therefore, taking the continuum limit and making the replacement

$$\sum_i \frac{\delta(\vec{k}_1 \pm \vec{p}_i)}{\langle p_i|p_i\rangle} \to \sum_{\vec{p}\in\mathbb{R}^3} \frac{\delta(\vec{k}_1 \pm \vec{p})}{\langle p|p\rangle} = 1 \tag{2.52}$$

because the sum now has all possible momenta, one of which must be k_1 and therefore this term always has a support. The numerator and denominator are then both $\delta^{(3)}(0)$ which can be regulated in the usual way [82] to be taken to V, and therefore the limit $V \to \infty$ of this expression is just 1.This is similar to how the infinities coming from phase space volumes drop out in the final formulae for cross sections. One can think of this infinite sum schematically as an integral $\int d^3k\delta(\vec{k}_1 \pm \vec{p})$=1, where it is understood that in the denominator the limit $d^3p \to 0, V \to \infty$ is taken such that the $d^3pV \to$ finite.

We therefore see that in the case where *all* momenta states are occupied, the physical pole is always reached. While it is true that it comes from a disconnected diagram, the point now is that *now there always exists a disconnected diagram that has the required support to be observable.* We can therefore conclude that

$$\text{initial state} = \tag{2.53}$$

$$\prod_{k\in\mathbb{R}^3} |\vec{k}\rangle \implies \text{sum of disconnected pieces always contains observable physical pole} \tag{2.54}$$

It turns out that if one wants a connected contribution from $|p\rangle$ state correlation function then one needs to go to higher order in perturbation theory. e.g., for our case, we need to compute contribution at $O(\lambda^3)$. The in-in formula at $O(\lambda^3)$ gives

$$-i\int_{-\infty}^{0} dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_2} dt_3 [H(t_3), [H(t_2), [H(t_1), Q(0)]]] \tag{2.55}$$

Clearly, there will be many terms but it serves our purpose to just pick one of these terms and one of the contractions. We pick

$$\begin{aligned} &-2Re\int_{-\infty}^{0} \frac{d\tau_1}{H\tau_1} \int_{-\infty}^{\tau_1} \frac{d\tau_2}{H\tau_2} \int_{-\infty}^{\tau_2} \frac{d\tau_3}{H\tau_3} i\langle Q(0)H(t_1)H(t_2)H(t_3)\rangle \\ = &-2Re\int_{-\infty}^{0} dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_2} dt_3 i \prod_{i=1}^{9}\int d^3\vec{q_i}\delta^3(\vec{q_1}+\vec{q_2}+\vec{q_3})\delta^3(\vec{q_4}+\vec{q_5}+\vec{q_6}) \\ &\times\delta^3(\vec{q_7}+\vec{q_8}+\vec{q_9})\,\langle p|\,\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta'_{q_1}(\tau_1)\zeta'_{q_2}(\tau_1)\zeta'_{q_3}(\tau_1)\zeta'_{q_4}(\tau_2) \\ &\times\zeta'_{q_5}(\tau_2)\zeta'_{q_6}(\tau_2)\zeta'_{q_7}(\tau_3)\zeta'_{q_8}(\tau_3)\zeta'_{q_9}(\tau_3)\,|p\rangle \end{aligned} \tag{2.56}$$

It can be easily checked that if one performs the following contraction

$$\langle a_p\zeta_{q_1}(\tau_1)\rangle\langle\zeta_{q_2}(\tau_1)\zeta_{q_4}(\tau_2)\rangle\langle\zeta_{q_5}(\tau_2)\zeta_{q_7}(\tau_3)\rangle\langle\zeta_{k_1}\zeta_{q_3}(\tau_1)\rangle\langle\zeta_{k_2}\zeta_{q_6}(\tau_2)\rangle\langle\zeta_{k_3}\zeta_{q_9}(\tau_3)\rangle\langle\zeta_{q_8}(\tau_3)a_p^\dagger\rangle \tag{2.57}$$

then after performing all momenta integrals one is left with only one delta function i.e $\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3})$ which means we have a connected diagram. In the next section, we will see in detail the nature of contractions for a highly occupied initial state, by which we mean not only that all momenta are excited, but also each is excited by a large number.

2.3.6 BD modes, large occupation number initial state

Following [60] and the preceding discussion, we compute the correlation functions for excited high occupation number states above the Bunch-Davies vacuum. All momentum modes for this $|n\rangle$ state are highly occupied. To be more precise we have

$$|n\rangle \equiv \bigotimes_{\vec{k_i}} |n_{\vec{k}}\rangle \tag{2.58}$$

where,

$$|n_{\vec{k}}\rangle = \frac{1}{\sqrt{n!}}\left(a_{\vec{k}^\dagger}\right)^n|0\rangle \tag{2.59}$$

where n is very large.

2.3.6.1 Two-Point Function

The two point function at $O(\lambda^0)$ is given by

$$\langle n|\, \zeta_{k_1}\zeta_{k_2}\, |n\rangle = (2\pi)^3\delta^3(\vec{k_1}+\vec{k_2})\left((n_{k_1}+1)u^*_{k_2}u_{k_1} + n_{k_1}u_{k_2}u^*_{k_1}\right) \tag{2.60}$$

$$= (2pi)^3\delta^3(\vec{k_1}+\vec{k_2})\frac{\Delta^2_\zeta}{k_1^3}(2n_k+1) \tag{2.61}$$

where, the action of field ζ_k on the state is given as follows

$$\zeta_k\,|n\rangle = \sqrt{n_{-k}+1}u_{k_1}\,|n_{-k_1}+1\rangle\left|\hat{n};-\vec{k_1}\right\rangle + \sqrt{n_k}u^*_k\,|n_k-1\rangle\left|\hat{n};\vec{k}\right\rangle \tag{2.62}$$

2.3.6.2 Three-Point Function

At $O(\lambda)$ the three point function is the following

$$Re\,\langle 0|\prod_i (a_{p_i})^n\left(-2i\zeta^I_{k_1}\zeta^I_{k_2}\zeta^I_{k_3}\int\left(\prod_i d^3\vec{q_i}\right)\delta^3(\sum\vec{q_i})\int_{-\infty}^0\frac{d\tau'}{H\tau'}\left(\zeta'^I_{q_1}\zeta'^I_{q_2}\zeta'^I_{q_3}\right)\right)\prod_i (a^\dagger_{p_i})^n\,|0\rangle \tag{2.63}$$

It is clear from the above expression and previous computations that there are contraction where ζ'_qs on the right can be contracted with lowering operators on the left and the ζ_ks on the left with raising operators on the right. These types of contractions will give rise to physical poles. One such contraction is shown below

$$\prod_i (a_{p_i})^n a_{p_j}(-2i\zeta^I_{k_1}\zeta^I_{k_2}\zeta^I_{k_3}\int\left(\prod_i d^3\vec{q_i}\right)\delta^3(\sum\vec{q_i})\int_{-\infty}^0\frac{d\tau'}{H\tau'}\left(\zeta'^I_{q_1}\zeta'^I_{q_2}\zeta'^I_{q_3}\right))\,a^\dagger_{p_j}\prod_i (a^\dagger_{p_i})^n \tag{2.64}$$

Such a contraction will produce an integral of the form

$$\sim\int_{-\infty}^0 \tau'^2 e^{-i(k_2+k_3-k_1)\tau}d\tau' \tag{2.65}$$

giving rise to physical poles but again there will be disconnected diagrams.

2.3.7 Four point function

All these calculations assume that there is no non-Gaussianity in the initial state. Then the non-Gaussianity results only from the interaction Hamiltonian time evolving the initial state to the present state. We aim to study the analytic structures allowed, and in this section we extend the results of [59] to four-point functions [83, 84, 73, 85]. We calculate both classical and quantum four-point functions and comment about their pole structures.

2.3.7.1 Classical correlator

Consider a massless field in de Sitter space with the following action

$$S = \int d\tau d^3x \left(\frac{1}{2}\left(a^2\zeta'^2 - a^2\partial^2\zeta\right) + a\frac{\lambda}{6}\zeta'^3\right) \tag{2.66}$$

where a is the scale factor and prime denotes a derivative w.r.t to conformal time τ. The equation of motion is given by

$$\frac{1}{a^2}(\zeta'' + 2\frac{a'}{a}\zeta' - \partial^2\zeta) = -\frac{\lambda}{a^4}(a\zeta'\zeta'' + \frac{a'\zeta'^2}{2}) \tag{2.67}$$

The Greens function corresponding to the operator on the left acting on ζ and its derivative are given by [59, 86]

$$G_{\vec{k}}(\tau,\tau') = \theta(\tau-\tau')\frac{2\Delta_\zeta^2}{k^3}\left((1+k^2\tau\tau')\sin k(\tau-\tau') - k(\tau-\tau')\cos k(\tau-\tau')\right) \tag{2.68}$$

$$\partial_\tau G_{\vec{k}}(\tau,\tau') = 2\Delta_\zeta^2[k^{-1}\tau'\sin k(\tau-\tau') - \tau\tau'\cos k(\tau-\tau')] \tag{2.69}$$

We can solve Eq. (2.67) perturbatively by expanding ζ as follows

$$\zeta = \zeta^{(1)} + \lambda\zeta^{(2)} + \lambda^2\zeta^{(3)} + O(\lambda^3) \tag{2.70}$$

Plugging this into Eq. (2.67) we get the following equations

$$\frac{1}{a^2}(\zeta''^{(1)} + 2\frac{a'}{a}\zeta'^{(1)} - \partial^2\zeta^{(1)}) = 0 \quad (2.71)$$

$$\frac{1}{a^2}(\zeta''^{(2)} + 2\frac{a'}{a}\zeta'^{(2)} - \partial^2\zeta^{(2)}) + \frac{1}{a^4}\left(a\zeta'^{(1)}\zeta''^{(1)} + \frac{a'\zeta'^{(1)2}}{2}\right) = 0 \quad (2.72)$$

$$\frac{1}{a^2}(\zeta''^{(3)} + 2\frac{a'}{a}\zeta'^{(3)} - \partial^2\zeta^{(3)}) + \frac{1}{a^4}\left(a\zeta'^{(1)}\zeta''^{(2)} + a\zeta'^{(2)}\zeta''^{(1)} + a'\zeta'^{(1)}\zeta'^{(2)}\right) = 0 \quad (2.73)$$

Therefore the solution to Eq. (2.72) and Eq. (2.73) can be written in terms of Greens function given above.

$$\zeta^{(2)}(\tau, \vec{x}) = \int d^3x' \int d\tau' \frac{a}{2}\partial_{\tau'}G(x, x')(\zeta'^{(1)2}) \quad (2.74)$$

$$\zeta^{(3)}(\tau, \vec{x}) = \int d^3x \int d\tau' a\left(\partial_{\tau'}G(x, x')\zeta'^{(1)}\zeta'^{(2)}\right) \quad (2.75)$$

where we have neglected a total derivative since $G(\tau, \tau + 0^+)$ is zero (retarded GF) and $G(\tau, -\infty)$ is also set to zero with an appropriate $i\epsilon$ prescription. These equations in fourier space read

$$\zeta^{(2)}_{\vec{k}}(\tau) = \int d\tau' \frac{d^3p}{2(2\pi)^3} a\left(\partial_{\tau'}G_{\vec{k}}(\tau, \tau')\zeta'^{(1)}_{\vec{p}}\zeta'^{(1)}_{\vec{k}-\vec{p}}\right) \quad (2.76)$$

$$\zeta^{(3)}_{\vec{k}}(\tau) = \int d\tau' \frac{d^3p}{(2\pi)^3} a\left(\partial_{\tau'}G_{\vec{k}}(\tau, \tau')\zeta'^{(2)}_{\vec{p}}\zeta'^{(1)}_{\vec{k}-\vec{p}}\right) \quad (2.77)$$

The free field solution is given by

$$\zeta^{(1)}_{\vec{k}(\tau)} = \frac{\Delta_\zeta}{\sqrt{k^3}}\left(a^\dagger_{\vec{k}}u_k(\tau) + a_{-\vec{k}}u^*_k(\tau)\right) \quad (2.78)$$

defined with the following statistics

$$< a_{\vec{k}}a^\dagger_{\vec{k'}} >=< a^\dagger_{\vec{k}}a_{\vec{k'}} >= \frac{1}{2}\delta^3(\vec{k} - \vec{k'}) \quad (2.79)$$

where, $u_{\vec{k}}(\tau) = (1 - ik\tau)e^{ik\tau}$
This produces the correct two point function which is the only observable we have measured so far. Note that this statistical nature is due to our lack of knowledge and not due to some inherent uncertainty as is the case for quantum mechanical operators.

The two point function is given by

$$< \zeta^{(1)}_{\vec{k}}(\tau)\zeta^{(1)}_{\vec{k'}}(\tau') >= \frac{\Delta_\zeta^2}{k^3}\delta^3(\vec{k}+\vec{k'})Re[u_k(\tau)u_k^*(\tau')] \tag{2.80}$$

Let us now calculate the classical 4 point function upto $O(\lambda^2)$

$$\begin{aligned} < \zeta\zeta\zeta\zeta >=< (\zeta^{(1)}_{k_1} + \lambda\zeta^{(2)}_{k_1} + \lambda^2\zeta^{(3)}_{k_1})(\zeta^{(1)}_{k_2} + \lambda\zeta^{(2)}_{k_2} + \lambda^2\zeta^{(3)}_{k_2})(\zeta^{(1)}_{k_3} + \lambda\zeta^{(2)}_{k_3} + \lambda^2\zeta^{(3)}_{k_3})\times \\ (\zeta^{(1)}_{k_4} + \lambda\zeta^{(2)}_{k_4} + \lambda^2\zeta^{(3)}_{k_4}) > \end{aligned} \tag{2.81}$$

Terms of $O(\lambda)$ vanish (odd number of terms) and the non-trivial contribution to the correlator comes from two types of terms:

I: $< \zeta^{(1)}_{k_1}\zeta^{(1)}_{k_2}\zeta^{(2)}_{k_3}\zeta^{(2)}_{k_4} >$

II: $< \zeta^{(1)}_{k_1}\zeta^{(1)}_{k_2}\zeta^{(1)}_{k_3}\zeta^{(3)}_{k_4} >$

Rest of the terms are just permutations of these two terms.

Let us first evaluate **I**:

$$\begin{aligned} \mathbf{I} = \int \frac{d\tau' d\tau'' d^3p d^3q}{(2\pi)^6}(-H\tau')^{-1}(-H\tau'')^{-1}\partial_{\tau'}G_{k_3}(\tau,\tau')\partial_{\tau'}G_{k_3}(\tau,\tau'')\times \\ < \zeta^{(1)}_{k_1}\zeta^{(1)}_{k_2}\zeta'^{(1)}_{\vec{k_3}-\vec{p}}(\tau')\zeta'^{(1)}_{p}(\tau')\zeta'^{(1)}_{\vec{k_4}-\vec{q}}(\tau'')\zeta'^{(1)}_{q}(\tau'') > \end{aligned} \tag{2.82}$$

Using wick's theorem for these gaussian random fields, we get

$$\begin{aligned} = \frac{12\Delta_\zeta^{10}H^{-2}k_{13}}{6k_1k_2k_3k_4}\left[\int d\tau'\tau'^2\cos k_1\tau'\cos k_{13}\tau'\sin k_3\tau' \int d\tau'\tau'^2\cos k_2\tau''\cos k_{13}\tau''\sin k_4\tau''\right] \\ +k_1 \leftrightarrow k_2 \end{aligned} \tag{2.83}$$

where, $k_{13} = |\vec{k_1} + \vec{k_3}|$. After some more algebra this simplifies to

$$\frac{4\Delta_\zeta^{10} H^{-2}}{k_1 k_2 k_3 k_4} \left(k_{13} \times \alpha \times \beta + k_1 \leftrightarrow k_2\right) \tag{2.84}$$

where,

$$\alpha = \frac{1}{(k_1+k_3+k_{13})^3} + \frac{1}{(k_1+k_3-k_{13})^3} + \frac{1}{(-k_1+k_3+k_{13})^3} - \frac{1}{(k_1-k_3+k_{13})^3} \tag{2.85}$$

$$\beta = \frac{1}{(k_2+k_4+k_{13})^3} + \frac{1}{(k_2+k_4-k_{13})^3} + \frac{1}{(-k_2+k_4+k_{13})^3} - \frac{1}{(k_2-k_4+k_{13})^3}. \tag{2.86}$$

Now, term **II** gives

$$\begin{aligned} &= \int \frac{d\tau' d^3p}{(2\pi)^3} (-H\tau')^{-1} \partial_{\tau'} G_{k_4}(\tau,\tau') < \zeta^{(1)}_{k_1} \zeta^{(1)}_{k_2} \zeta^{(1)}_{k_3} \zeta'^{(1)}_{\vec{k_4}-\vec{p}}(\tau') \times \\ &\partial_{\tau'} \left(\int \frac{d\tau'' d^3q}{(2\pi)^3} (-H\tau'')^{-1} \partial_{\tau''} G_p(\tau',\tau'') \zeta'^{(1)}_{q}(\tau'') \zeta'_{\vec{p}-\vec{q}}(\tau'') \right) > \end{aligned} \tag{2.87}$$

Using Leibniz rule to simply this further

$$\begin{aligned} &\int \frac{d\tau' d^3p d^3q d\tau''}{(2\pi)^6} (-H\tau')^{-1} (-H\tau'')^{-1} \partial_{\tau'} G_{k_4}(\tau,\tau') \partial_{\tau'} \partial_{\tau''} G_p(\tau',\tau'') \\ &\qquad < \zeta^{(1)}_{k_1} \zeta^{(1)}_{k_2} \zeta^{(1)}_{k_3} \zeta'^{(1)}_{\vec{k_4}-\vec{p}}(\tau') \zeta'^{(1)}_{q}(\tau'') \zeta'^{(1)}_{\vec{p}-\vec{q}}(\tau'') > \end{aligned} \tag{2.88}$$

We proceed as before and after some lengthy algebra one gets

$$\begin{aligned} &\frac{4H^{-2}\Delta_\zeta^{10} k_{14}}{16 k_1 k_2 k_3 k_4} \Bigg[\frac{4}{u_{x1}^3}\left(\frac{1}{x^3} + \frac{6}{x u_{x1}^2}\right) + \frac{4}{u_{x2}^3}\left(\frac{1}{x^3} + \frac{6}{x u_{x2}^2}\right) + \frac{4}{u_{x3}^3}\left(\frac{1}{x^3} + \frac{6}{x u_{x3}^2}\right) \\ &+ \frac{4}{u_{x4}^3}\left(\frac{1}{x^3} + \frac{6}{x u_{x4}^2}\right) + \frac{12}{x^2}\left(-\frac{1}{u_{x1}^4} + \frac{1}{u_{x2}^4} - \frac{1}{u_{x3}^4} + \frac{1}{u_{x4}^4}\right) + (u_y, y) + (u_z, z) + (u_w, w) \\ &\qquad + k_1 \leftrightarrow k_2(\text{of previous term}) + k_2 \leftrightarrow k_3(\text{of previous term}) \Bigg] \end{aligned} \tag{2.89}$$

where,

$$(u_m, m) = \frac{4}{u_{m1}^3}\left(\frac{1}{m^3} + \frac{6}{mu_{m1}^2}\right) + \frac{4}{u_{m2}^3}\left(\frac{1}{m^3} + \frac{6}{mu_{m2}^2}\right) + \frac{4}{u_{m3}^3}\left(\frac{1}{m^3} + \frac{6}{mu_{m3}^2}\right) \tag{2.90}$$

$$+\frac{4}{u_{m4}^3}\left(\frac{1}{m^3} + \frac{6}{mu_{m4}^2}\right) + \frac{12}{m^2}\left(-\frac{1}{u_{m1}^4} + \frac{1}{u_{m2}^4} - \frac{1}{u_{m3}^4} + \frac{1}{u_{m4}^4}\right) \tag{2.91}$$

$$x = k_{14} + k_2 + k_3 \tag{2.92}$$

$$y = k_{14} + k_2 - k_3 \tag{2.93}$$

$$z = k_{14} - k_2 + k_3 \tag{2.94}$$

$$w = k_{14} - k_2 - k_3 \tag{2.95}$$

$$u_{x1} = k_1 + k_4 + k_{14} - x \tag{2.96}$$

$$u_{x2} = k_1 + k_4 - k_{14} - x \tag{2.97}$$

$$u_{x3} = -k_1 + k_4 - k_{14} - x \tag{2.98}$$

$$u_{x4} = -k_1 + k_4 - k_{14} - x \tag{2.99}$$

For u_y and u_z terms we just replace x in u_x terms by y and z. Again, the poles are located at physical momenta consistent with the interpretation that if initial state has physical particles then processes like decays can give rise to physical poles. The final expression for the classical four-point function is the sum of Eq. (2.84) and Eq. (2.89) + permutations.

2.3.7.2 Quantum correlator

We recall the mode expansion

$$\zeta_{\vec{k}}(\tau) = \frac{\Delta_\zeta}{\sqrt{2k^3}}(u_k(\tau)a_{\vec{k}} + u_k^*(\tau)a^\dagger_{-\vec{k}}), u_k(\tau) = (1+ik\tau)e^{-ik\tau} \tag{2.100}$$

From which we find

$$\zeta'_k(\tau) = \frac{\Delta_\zeta k^2\tau}{\sqrt{2k^3}}(a_{\vec{k}}e^{-ik\tau} + a^\dagger_{-\vec{k}}e^{ik\tau}) \tag{2.101}$$

We will need the following 2 point functions-

$$\langle\zeta'_p(\tau_1)\zeta'_q(\tau_2)\rangle = (2\pi)^3\delta(\vec{p}+\vec{q})\frac{p\Delta_\zeta^2\tau_1\tau_2}{2}e^{ip(\tau_2-\tau_1)} \equiv (2\pi)^3\delta(\vec{p}+\vec{q})G'_p(\tau_1,\tau_2) \tag{2.102}$$

$$(2\pi)^3\delta(\vec{p}+\vec{k})G_k(\tau) \equiv \langle\zeta'_p(\tau)\zeta_k(0)\rangle = (2\pi)^3\delta(\vec{p}+\vec{k})\frac{\Delta_\zeta^2\tau}{2k}e^{-ik\tau} = (2\pi)^3\delta(\vec{p}+\vec{k})\langle\zeta_k(0)\zeta'_p(\tau)\rangle^* \tag{2.103}$$

At order λ^2, we have-

$$\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle = -2Re\int_{-\infty}^0 d\tau_2\int_{-\infty}^{\tau_2}d\tau_1\Big(\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}H(\tau_2)H(\tau_1)\rangle \tag{2.104}$$

$$-\langle H(\tau_1)\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}H(\tau_2)\rangle\Big) \tag{2.105}$$

We define

$$\vec{k}_{12} = \vec{k}_1+\vec{k}_2 = -\vec{k}_{34} \quad \vec{k}_t = \sum_{i=1}^4\vec{k}_i \tag{2.106}$$

This becomes, suppressing momentum conserving delta functions and permutations for brevity,

$$= -2Re\int_{-\infty}^{0}\frac{d\tau_2}{H\tau_2}\int_{-\infty}^{\tau_2}\frac{d\tau_1}{H\tau_1}\Big(G^*_{k_1}(\tau_2)G^*_{k_2}(\tau_2)G^*_{k_3}(\tau_1)G^*_{k_4}(\tau_1)G'_{k_{12}}(\tau_2,\tau_1)$$
$$- G_{k_1}(\tau_1)G_{k_2}(\tau_1)G^*_{k_3}(\tau_2)G^*_{k_4}(\tau_2)G'_{k_{12}}(\tau_1,\tau_2)\Big) + \text{perm.} \quad (2.107)$$

$$= -2Re\frac{\Delta_\zeta^{10}k_{12}}{32k_1k_2k_3k_4}\int_{-\infty}^{0}\frac{d\tau_2}{H\tau_2}\int_{-\infty}^{\tau_2}\frac{d\tau_1}{H\tau_1}\tau_1^3\tau_2^3\Big(e^{i(k_1+k_2)\tau_2+(k_3+k_4)\tau_1+k_{12}(\tau_1-\tau_2)}$$
$$- e^{-i(k_1+k_2)\tau_1+(k_3+k_4)\tau_2+k_{12}(\tau_2-\tau_1)}\Big) \quad (2.108)$$

And therefore the full 4 point function becomes, after including all possible contractions

$$\frac{4\lambda^2\Delta_\zeta^{10}H^{-2}k_{12}}{k_1k_2k_3k_4}\left(\frac{2}{k_t^3(k_3+k_4+k_{12})^3}+\frac{6}{k_t^4(k_3+k_4+k_{12})^2}+\frac{12}{k_t^5(k_3+k_4+k_{12})}\right.$$
$$\left.+\frac{1}{(k_1+k_2+k_{12})^3(k_3+k_4+k_{12})^3}\right) + permutations \quad (2.109)$$

We see the presence of additional poles besides the one for total energy, but this is no physical pole, consistent with our expectations from a Bunch Davies vacuum.

2.4 Choice of interaction and their imprint on singularities

The role of singularities of the S-matrix in flat space has a clear physical meaning and origin. The role of poles and branch cuts are easy to interpret from their physical meaning, which lies in the Kallen-Lehmann decomposition of correlators. The nature of singularities is determined by the nature of interactions considered and the kind of processes they involve (for instance, tree level exchanges generates simple poles whereas loops are generally accompanied by branch cuts). Correlation functions are expected to be constrained by Lorentz invariance, and in flat space for example we have delta function singularities indicating Poincare invariance that conserves momenta. In deSitter, the correlation functions are similarly constrained by the de Sitter isometries which are conveniently those of a Euclidean CFT [87, 77, 78]. We will, for completeness, briefly discuss the nature of singularities with different interactions in de Sitter. The point is to emphasize that *while the functional form of singularities can change depending on the choice of interaction, the object we are trying to probe is the momentum structure encoded in the singularities, which carries a clear physical signature of the choice of quantization.* Let us begin by noting that far outside the horizon, the behaviour of a scalar field ϕ that has a very small coupling to the inflaton is of the form

$$\phi \sim \tau^{\Delta} \qquad \Delta \equiv \frac{3}{2}\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}}\right) \tag{2.110}$$

In this section we will focus on the case of Bunch Davies modes in a Bunch Davies initial state. Next, note that the correlators must be functions of the de Sitter invariant distance-

$$d(x_i, x_j) = \frac{|\vec{x}_i - \vec{x}_j|^2}{\tau_i \tau_j} - \left(\frac{\tau_i}{\tau_j} + \frac{\tau_j}{\tau_i}\right) \to \frac{|\vec{x}_i - \vec{x}_j|^2}{\tau_i \tau_j}|_{\tau \to 0} \tag{2.111}$$

Note that, in particular, this means that the spatial dependence of the correlation function is specified automatically once the time dependence of the field(which is related to its mass) is specified at late times [87]. The Fourier transform of this correlator contains the momentum dependence we are interested in, and this tells us that once the late time behaviour of the field is specified, the momentum dependence is specified too. Let us see how this works with the simple example of an interaction $\sqrt{-g}\lambda\phi^3/6$. For a general Δ, the time dependence of the 3 point function will be fixed as $\sim \tau_1^\Delta \tau_2^\Delta \tau_3^\Delta$, and therefore a generic position space function can only take the form

$$\langle \phi_1 \phi_2 \phi_3 \rangle \sim \prod \frac{\tau_i^\Delta}{|\vec{x}_i - \vec{x}_j|^\Delta} \tag{2.112}$$

Let us see what happens for different values of Δ. For a field that has mass $m = \sqrt{2}H$ we have a conformally coupled scalar with $\Delta = 1$. The Fourier transform of $\prod 1/|\vec{x}_i - \vec{x}_j|$ goes as $1/(k_1 k_2 k_3)$, and indeed in momentum space we find with a full in-in calculation, upto a 3-momentum conserving delta function

$$\langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle = \frac{\pi \lambda H^2}{8} \tau_*^3 \frac{1}{k_1 k_2 k_3} \tag{2.113}$$

where $\tau_i = \tau_*$ is imposed for an equal time correlator. Note that the result is dependent on τ_*. To study the limit $\Delta \to 0$, which is more subtle because now the late time behaviour of fields isn't a simple power law, one can use the fact that $\lim_{\Delta \to 0} \tau^\Delta \sim 1 + \Delta \log \tau$ and anticipate a logarithmic dependence on the late time, and therefore, on momenta. This is to be expected, since the field has no mass of it's own and therefore the only kinematic quantity available to make the logarithm dimensionless is a linear combination of momenta. The classical scale invariance of the massless theory is broken when one imposes a regulator(in the form of a finite cutoff τ_*), and this breaking is captured in the logarithmic piece. Maldacena's consistency requirement directly relates this nongaussianity to a breaking of scale invariance, as we shall later see. It is important to note that the massless case is subtle, because the limits $\tau \to 0, \Delta \to 0$ don't commute, and a naively τ-independent field outside the horizon acquires a logarithmic time dependence due to the cubic interaction. This logarithmic dependence on time then fixes the momentum dependence because of the nature of dS invariant distances. The in-in calculation in momentum space yields, as expected

$$\langle\phi_{k_1}\phi_{k_2}\phi_{k_3}\rangle = \frac{H^2\lambda}{12(k_1k_2k_3)^3}\left(k_1k_2k_3 - \sum k_ik_j^2 + \sum k_i^3(-1+\gamma+\log(k_t\tau_*))\right) \quad (2.114)$$

This is where the role of derivative interactions becomes important. For a massless field, the apparent singularity at $\tau_* \to 0$ can be avoided by considering only derivative interactions. The mathematical origin is simple-the interaction hamiltonian in conformal time is of the form $\int \frac{\lambda}{6\tau^4 H^4}\phi^3$ which is ill defined at $\tau = 0$. In the above 3 point correlator, the origin of the logarithm is from integrals of the form $\int e^{ik_t\tau}/\tau$ which are singular at the point $\tau = 0$. If we had sufficient powers of τ in the numerator, then this problem can be cured. With a purely cubic interaction, the Wightman functions entering the in-in formula are $\int \frac{d\tau}{\tau^4}\prod_i\langle\phi_i(0)\phi(\tau)\rangle \sim \int\prod_i(\frac{1}{2k_i^3}(1-ik_i\tau)e^{ik_i\tau})$ where τ is at the vertex being integrated over in the interaction hamiltonian. If, however, we consider the Wightman function for $\langle\zeta\zeta'\rangle \sim \sqrt{k}\tau e^{ik\tau}$, then we see that the numerator's factors of τ can compensate for the ill defined $\tau \to 0$ behaviour of the Hamiltonian. With an interaction like ζ'^3, the interaction hamiltonian is $\int \frac{\lambda}{6H\tau}\zeta'^3$, and the late time divergence is softer. Then, the in in calculation will contain integrals of the type $\int \frac{d\tau}{\tau}\prod_i\langle\phi_i(0)\phi(\tau)\rangle \sim \int d\tau\tau^2\prod_i e^{ik_i\tau}$, which has no log divergences. The singularities are in fact simple poles of the form $1/k_t$. In fact, it suffices to have an interaction of the type $\zeta\zeta'^2$-this is indeed a leading cubic interaction in Hamiltonian, where the $1/\tau^2$ in the denominator of the interaction hamiltonian can get compensated by two powers of τ coming from $\zeta\zeta'$ contractions; the remaining $\zeta\zeta$ contraction is free of any $1/\tau$ divergences.

Thus, we see that considering derivative interactions softens the behaviour of the $\tau \to 0$ limit of the correlators, and one needn't conduct any explicit regularization. We see that one needs *a minimum of two derivatives* in the interaction to get rid of this regulator requirement, and for simplicity we consider a ζ'^3 interaction.

Now, the crucial point we wish to emphasize. While the nature of singularities might be different in their functional forms (logarithmic, simple poles, or a combination); the momentum dependence of these singularities for the Bunch Davies case is always $(k_1+k_2+k_3)$. What we aim to understand is-having fixed an interaction and therefore the functional form of the singularity- does the nature of this momentum dependence change once we allow different initial states and/or mode functions? The answer, as we see, is quite generically yes.

2.5 Comments on the Maldacena consistency criteria

The Maldacena consistency criteria is a consequence of frequency-dependent freezing times of various modes. Consider the three point function in the squeezed limit $k_1 << k_2, k_3$. In this case, the mode ζ_{k_1} freezes in (becomes a classical variable) before the modes ζ_{k_2,k_3} have crossed the horizon. It then serves as a background over which the latter modes propagate. In effect, the 3 point function reduces to

$$\langle \zeta_2 \zeta_3 \rangle_{\zeta_1} \approx \langle \zeta_1 \zeta_2 \rangle_0 + \zeta_1 \frac{\partial}{\partial_{\zeta_1}} \langle \zeta_1 \zeta_2 \rangle_0 \tag{2.115}$$

The fact that ζ_1 has frozen in can be accounted for by a rescaling of coordinates-in particular-

$$ds^2 = dt^2 - a^2 e^{2\zeta_1}(dx^2 + dy^2 + dz^2) \implies x \to (1 + \zeta_1)x \tag{2.116}$$

Thus, the squeezed limit can be see to be accounted for by spatial reparametrizations. Then, in position space one can account for invariance under spatial reparametrizations as

$$\langle \zeta(x)\zeta(0) \rangle_{\zeta_1} = \langle \zeta(x + \zeta_1 x)\zeta(0) \rangle_0 \tag{2.117}$$

We can now Taylor expand the RHS too, to get a term of the form $\zeta_1 x \frac{d}{dx} \langle \zeta\zeta \rangle$ and then multiply both sides with ζ_1 before averaging. Converting to momentum space, this yields

$$\lim_{k_1 << k_2, k_3} \langle \zeta_1 \zeta_2 \zeta_3 \rangle \approx \langle \zeta_{k_1} \zeta_{-k_1} \rangle k \frac{d}{dk} \langle \zeta_{k_2} \zeta_{k_3} \rangle \tag{2.118}$$

Now, for a *nearly scale invariant power spectrum*, $\langle\zeta\zeta\rangle \sim k^{-3+n_s}$, and therefore we derive a consistency equation

$$\lim_{k_1<<k_2,k_3}\langle\zeta_1\zeta_2\zeta_3\rangle = -n_s\langle\zeta_{k_1}\zeta_{-k_1}\rangle\langle\zeta_{k_2}\zeta_{k_3}\rangle \tag{2.119}$$

Note the crucial step where we absorbed the background field into the metric, therefore finding ourselves a coordinate system where the power spectrum is insensitive to the background wave[88]. This step was valid because at leading order, ζ interactions come from the nonlinearities of the metric itself, without any reference to the features of the potential. However, the interaction we have considered comes from a potential with derivative interactions $\dot{\phi}^4$. It is known [80] that nongaussianities from such interactions are suppressed by $\frac{k_L^2}{k_S^2}$ in the squeezed limit, where S, L are the large and small momenta respectively(the subscripts are conventionally chosen to denote short and large distances). The basic idea is that in the squeezed limit, the relevant interactions are those where one of the insertions has no derivatives, and this is the insertion that's chosen to be the short wavelength mode. Explicitly, the relevant action at cubic order is [89]

$$S = \int d^4x\epsilon a^3\Big((1+3\bar{\zeta})\zeta'^2 - (1+\bar{\zeta})(\partial_i\zeta)^2\Big) \tag{2.120}$$

where the insertion that's taken to be the squeezed wavelength has been made explicit. Any corrections to this will therefore be from derivative corrections, and it was argued in [89] that the corrections are atleast quadratic in (k_L/k_S). There are many ways to see this, starting from nothing that the power spectrum at late time recieves genuine corrections only from second derivatives, because $\langle\zeta_1\zeta_2\rangle = P(k_1)(1+k^2\tau^2)$ and therefore if one was to organise this expression as a power series in $k\tau$, the leading correction at $\tau \to 0$ would come from ∂_τ^2, not before. Yet another way to see this is to note that schematically, the Wightman functions $\langle\zeta\zeta\rangle$ and $\langle\zeta\zeta'\rangle$ differ by a factor of $k^2\tau$ and therefore in the in-in formalism, we replace the former with the latter(as is what happens if we switch from inflation's leading interaction to our), we get additional powers of $(k_L/k_S)^2$. On more general grounds, focusing on the momentum dependence and ignoring the prefactors, the bispectrum is always of the form

$$\langle\zeta_1\zeta_2\zeta_3\rangle \sim \frac{1}{(k_1k_2k_3)^3}f(k) \tag{2.121}$$

And in the squeezed limit where we denote $k_1 \to k_L, k_{2,3} \to k_S$, we find that in the case where ζ is only a (scalar) mode of the metric, $f(k) \to k_L^3$ therefore restoring the consistency criteria. However, with an interaction like ours, $f(k)$ includes term of the nature $k_L^2 k_S$ because now the short wavelength can be put onto any of the three insertions in the interaction hamiltonian without being ignored unless we wish to kill the entire contribution. This produces the required $(k_L/k_S)^2$ suppression. As has been emphasized in [89], $f(k) \to 0$ as k_L^2, not k_L. On the other hand, without higher derivative features in the potential, $f(k)$ is a constant in k_L. For an explicit example, consider our eq(2.22), where the bispectrum is

$$\Delta\langle\zeta_1\zeta_2\zeta_3\rangle \sim \frac{1}{(k_1k_2k_3)(k_1+k_2+k_3)^3} \to P(k_s)P(k_L)\left(\frac{k_L}{k_S}\right)^2 \tag{2.122}$$

And therefore we see that the consistency condition is violated by an amount, as has been argued on general grounds, $\left(1+\left(\frac{k_L}{k_S}\right)^2\right)$.

The behaviour of $f(k)$ is more interesting in the case we have physical poles. The general conclusion is that this time, the behaviour is modified by a reciprocal quantity- k_S/k_L which is severely enhanced in the squeezed limit. In the squeezed limit, for all cases other than choosing purely BD initial conditions, the consistency criteria fails. The reason is the presence of physical poles, which generate terms proportional to k_1/k_3 which swamp the $\vec{k}_3 \to 0$ limit and spoil the consistency. The spoiling depends on the angle between the momenta, because we have a constraint $\sum \vec{k}_i = 0$ from which we can study the poles in terms of the magnitudes k_i as well as the angles. It is clear that such a structure wouldn't occur if we had only physical poles, because then all 3 momenta are on an equal footing inside the correlator and setting any one to zero can only generate $k_1 + k_2$ poles, not a $k_1 - k_2$ pole that can potentially compete with the $k_3 \to 0$ pole and therefore require to be kept intact before taking the limit. Since the consistency condititon is linked to spatial reparametrization invariance, it is tempting to think that OPE, gauge choice and the BD initial conditions are all intimately linked. In fact, it was argued in [53] how the reparametrization invariance is a feature of Einstein's equations, and with a non-BD vacuum, we must account for the effect of it's contributions to the stress energy tensor's vacuum expectation value. Neglecting this contribution means that we have not fully accounted for the modification to the ADM action-the piece which actually solves Einstein's equations. Therefore, we are working with an action that doesn't yield the correct equations of motion, and therefore we have lost the freedom to carry out spatial reparametrizations, and consequently the consistency condition.

We therefore see that while the squeezed limit doesn't directly contain information about the analytic structure of the bispectrum because the modes have been in a sense fixed; it does carry information about the initial states and the vacuum of the modes in the form of departures from Maldacena consistency. In particular, a $(k_L/k_S)^2$ departure is a signature of a Bunch Davies initial state with higher order interactions, whereas everything else produces a large deviation in the form of k_S/k_L poles.

2.6 Classical vs Quantum

The authors of [59, 60] argued that physical poles are a reflection of a classical initial state. At the level of calculation, it amounted to using the classical Green's function,

$$G_c(0,t) = \frac{-i}{2k}(e^{ik\tau} - e^{-ik\tau}) \tag{2.123}$$

which contains both positive and negative frequency modes, and therefore the product of these Green's functions generates poles with varying momentum dependence depending on the interference terms between the modes. However, the Wightman function for purely Bunch Davies modes contains only one of these modes, whereas the one for Bogoliubov transformed states(for example the alpha vacuum) contains both-

$$G_W^{BD}(0,\tau) \sim u_k(0)u_k^*(\tau)e^{ik\tau}, G_W^{\alpha}(0,\tau) \sim v_k(0)v_k^*(\tau) = v_k(0)(A^*u_k^*(\tau)e^{ik\tau} + B^*u_k(\tau)e^{-ik\tau}) \tag{2.124}$$

Therefore, we clearly see the link between the appearance of mode mixing in the case of classical as well as Bogoliubov transformed states, and the subsequent appearance of physical poles. What about the case of excited states? The presence of particles in the initial state means that there are additional possible interactions at a vertex, where some of the field insertions annihilate not just each other but also these initial state particles at a vertex. This is drastically different from the case of a vacuum initial state where only the field insertions annihilate(or create) among themselves at a vertex, and the only kinematic constraint then is conservation of their spatial momenta. The next crucial difference is the interplay of energies(stacked in the exponentials in the mode functions) at the vertex. In the cases where the initial state particles don't interact at a vertex but merely spectate, the vertex represents a *creation* of particles from the vacuum, as in the figure (cite figure). Since the vertex factor for creation at a vertex is accompanied by the mode $e^{ik\tau}$, the creation of three particles comes with a $\tau-$integral over $e^{i(k_1+k_2+k_3)\tau}$, and produces a total k pole as we have seen. However, the vertex factor that accompanies an annihilation is quite different, and has the opposite energy as the mode function for creation. If a particle in the initial state with momentum P annihilated at a vertex to produce 2 particles of momenta k_1, k_2; then we would find the prefactor at the vertex to be $e^{i(k_1+k_2-P)\tau}$, along with a momentum conserving delta function for the remaining particles which is just an exchange of momentum, and not a genuine interaction. Together, they produce a physical pole, and it's origin is clear-it's the relative sign of momenta in the mode functions corresponding to annihilation vs creation. However, because of these same momentum conserving delta functions, these physical poles have a very restricted support and will therefore not be observable except at isolated points in phase space.

An obvious way to get rid of the isolated nature of these singularities is to average over all possible momenta of initial state particles-this smears out the phase space where the poles have support. Crucially, *this cannot be accomplished with a state where a only fixed number of momentum states are occupied, no matter how highly.* That is to say, states of the type $\otimes_k (a_k^\dagger)^n|0\rangle$ cannot display observable physical poles unless we consider a sum over all possible momenta at each order in n. Schematically, a correlator like $\langle P|\zeta_1\zeta_2\zeta_3|P\rangle$ won't generate observable physical poles, but a correlator like $\int d^3P\langle P|\zeta_1\zeta_2\zeta_3|P\rangle$ will. Indeed, this is what we saw for states excited with respect to the Bunch Davies vacuum. As we introduce more and more particles, we must accept the possibility of more and more possible disconnected pieces and consequently isolated poles at a given order in perturbation theory, and it seems unlikely that we will ever be rid of this problem. However, for *some* excited states (relative to a Bunch Davies observer), it is possible to go to a frame where this state is a vacuum, and there are no particles in the initial state to interact at a vertex. These states are precisely the Bogoliubov transforms of the Bunch Davies vacuum, which are excited for a Bunch Davies observer but have zero net momentum. If we compute these correlators in a frame where the particles actually exist, we see in Sec. 2.3.3 indeed the presence of physical poles corresponding to physical interactions of initial state particles at a vertex, but it is difficult to find a clear interpretation due to additional disconnected pieces that must exist for reasons mentioned above. Instead, calculating them using the accelerated observer's basis ensures that these particles are not seen, but their physics is now captured in the modified mode functions-specifically, we must now also include negative frequency modes that otherwise appear due to annihilation of particles at a vertex. The physics of annihilation at a vertex is now absorbed into the physics of the new mode functions, and the mathematical correspondence is clear. Figure 2.3 demonstrates this discussion. (Here, by 'create/annihilate' we simply mean the way in which they contract-either with an a_p or with $a_p^\dagger$).

To contrast the situation with flat space time ordered in-out correlators, recall that the frequency of the mode functions there is fixed via LSZ, and time ordering picks the right frequencies in the Feynman propagators. In contrast, for in-in correlators, both the classical Green's function and the quantum Wightman functions for Bogoliubov states contain both signs of frequencies. Even in flat space, the Wightman function can have either frequency depending on the time order of the field insertions $\langle\phi_k(t)\phi_{k'}(0)\rangle \sim \delta(\vec{k}+\vec{k'})e^{-ipt}/2k = \langle\phi_k(0)\phi_{k'}(t)\rangle^*$; where the complex conjugation is equivalent to a negative frequency. It is precisely time ordering that eventually tells us which frequency to choose. The lack of this luxury with in-in correlators is what is ultimately tied to physical poles.

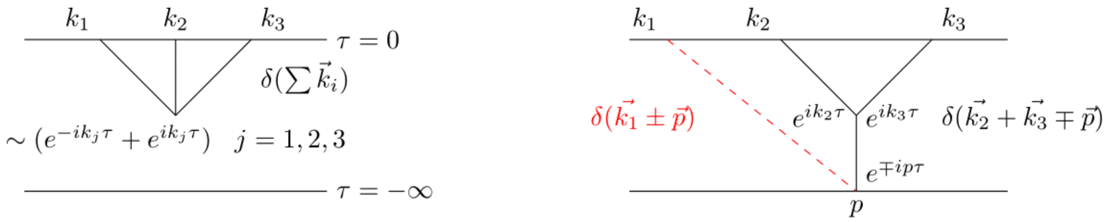


FIGURE 2.3: (Left) the case of classical or accelerated observer's mixed mode functions, (Right) the case of genuine interaction of initial state particle. The $\pm$ sign is determined by whether the particle is created/annihilated at the vertex. The latter generates physical poles. The red dashed line represents a spectator.

2.7 Choosing a cutoff

While carrying out the integrals in the in-in correlators, the lower limit $\tau \to -\infty$ must be taken with care [90, 91]. Generally, the exponential factor at the lower limit is taken to vanish because of it's oscillatory nature [15]. The general idea is that [16] the initial conditions need to be set when the EFT of inflation can still be trusted, and the Bunch Davies prescription is singled out by setting them at $\tau \to -\infty$. However, assuming that there is some cutoff scale Λ upto which our EFT can be trusted, then the validity of this EFT is reached at some $k \sim a(\tau_0)\Lambda$ where k is a physical momentum scale today-note that inflation redshifts energies. Scales that were once in the UV become part of the IR due to the universe's expansion. Setting a cutoff at a finite τ_0 automatically takes us away from the Bunch Davies prescription, and we need to introduce more general initial conditions, which yields modes which are Bogoliubov transformations of Bunch Davies. We would like to understand if taking the lower limit $\tau \to -\infty$ in our calculations hides any information about the physical poles seen by an accelerating observer-in particular, if the singular behaviour at physical energies is still seen. If we instead keep a cutoff intact, then we see that the integrals containing physical poles are modified to($k_j' \equiv k_t - 2k_j$)

$$\text{Im}\int_{\tau_0}^{0} d\tau\tau^2 e^{ik'_j\tau} = \text{Im}\frac{i(2+e^{ik'_j\tau_0}(k'^2_j\tau_0^2+2ik'_j\tau_0-2))}{k'^3_j} \quad \text{(with } \zeta'^3 \text{ interaction)}$$

$$= \frac{2}{k'^3_j}(1-\cos\big(k'_j\tau_0\big)) + \frac{\tau_0^2\cos\big(k'_j\tau_0\big)}{k'_j} - 2\frac{\tau_0\sin\big(k'_j\tau_0\big)}{k'^2_j} \quad (2.125)$$

We need to check the leading singular behaviour as $k'_j \to 0$. Expanding the numerators for small k'_j(but finite τ_0) we find

$$\text{Im}\int_{\tau_0}^{0} d\tau\tau^2 e^{ik'_j\tau}|_{k'_j\to 0} = \frac{2\tau_0^2}{k'_j} - \frac{2\tau_0^2}{k'_j} + \text{terms regular in } (k'_j\tau_0) \quad (2.126)$$

From this expression, it is clear that the limits $k'_j \to 0$ and $\tau_0 \to -\infty$ don't commute, and in fact taking $\tau_0 \to -\infty$ before going to the folded configuration $k'_j \to 0$ displays a divergence(from the terms regular in τ_0 which blow up in this limit), whereas the opposite order of limits is finite. This transparently demonstrates that the divergence in the folded limit is an artifact of the initial conditions being set at $\tau_0 = -\infty$. The absence of a divergence can directly be seen by plotting Eq. (2.125) as a function of k'_j. Therefore, clearly there is no singularity in the folded limit. This can be seen directly from Fig 2.4 where we have plotted Eq. (2.125) as a function of k'_j. It has been noted earlier too, for example in [90], that the divergence in the folded limit can be cured by introducing an early time cutoff. In [16], a similar phenomenon was shown for the interaction of inflation, where the presence of a cutoff rendered the folded limit finite but regular in the cutoff, and would therefore be subject to the same conclusion as ours. For an initial state that is a Bogoliubov transform of the Bunch Davies vacuum, setting the initial conditions away from $-\infty$ is perhaps reasonable because one can argue that a Bogoliubov transformed operator basis at τ_0 can be thought to be time evolved from a Bunch Davies basis which was imposed at $\tau \to -\infty$. However, there is no first-principles argument for this, and there are other excited states where this reasoning wouldn't be straightforward.

In the literature, one will find plentiful examples of both cases - where initial conditions are either set at $-\infty$ or at a finite cutoff. Since the primary motive of our

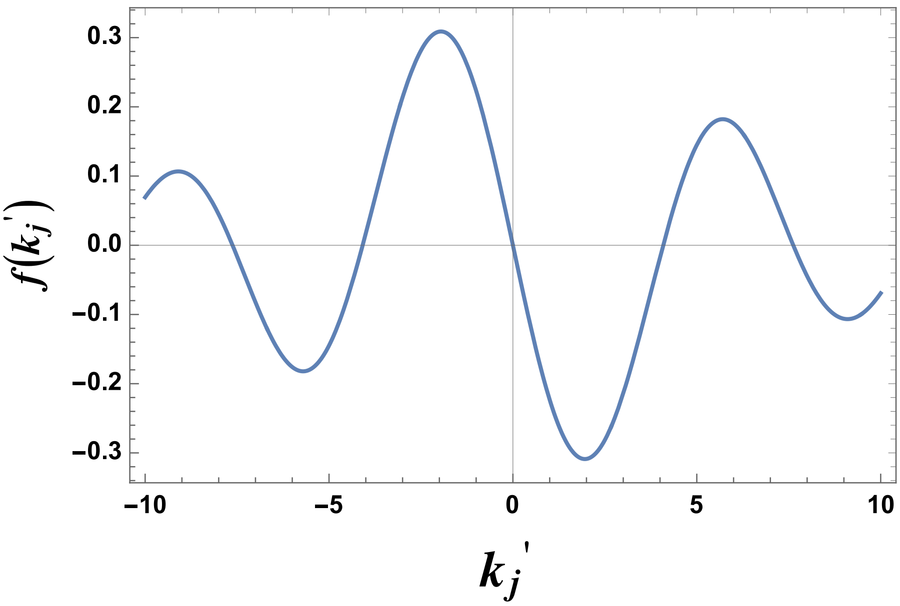


FIGURE 2.4: Plot of $f(k_j')$(see (2.125)) as a function of k_j'. This shows that there is no singularity in the folded limit. In addition to this there is an oscillatory feature.

analysis is to study and compare the pole structure of correlators, and trace the mathematical origin of physical poles in quantum and classical correlators, we will not dwell on this issue further, and treat our initial conditions set at $-\infty$.

Chapter 3

Soft theorems and boostless bootstrap

The material presented in this chapter is based on the author's publication [92].

There is a wide variety of models for inflation, and one can compute observables for each of them. However, one can adopt a more general (model-independent) approach by constructing an Effective Field Theory of Inflation (EFToI) [93, 94, 95] (as discussed in Chapter 1) consistent with symmetries. This allows us to go beyond the minimally coupled canonical single field slow-roll inflation [49]. In this chapter, we consider higher derivative operators which contribute to the mixed three-point correlation functions (i.e. $\langle\zeta\zeta\gamma\rangle$, $\langle\gamma\gamma\zeta\rangle$, which we compute using the in-in formalism). It is important that the EFT respects soft limits [49, 96, 97, 98] provided that we do not violate any of the assumptions implicit in their derivations. We clearly state under what assumptions these theorems are expected to hold and explicitly check (for mixed correlators) that they are satisfied for operators that modify both scalar as well as tensor power spectra. This provides important consistency checks for our calculations.

It is also interesting to bootstrap these correlators from a pure boundary perspective without referring to the bulk evolution. This approach has a lot of advantages, for instance, it was shown in [99, 100] that using certain field redefinitions (that vanish at the boundary) some operators can be removed from the EFT without affecting the late-time correlators. This redundancy, by construction, is not present once we have a purely boundary perspective and therefore can potentially lead to a lot of simplifications. There is a large amount of literature on both the conformal/pure de-sitter bootstrap [101, 102, 103, 104] as well as the "boostless bootstrap" approach, the latter being more recent [105, 106, 83, 107]. The boostless bootstrap program focuses on properties such as the analytical structure of the correlators, soft limits etc. The analytical structure of correlators on its own gives a lot of information such as the initial state [108, 109, 110] and the flat space amplitude [97, 103] corresponding to the interaction. This technique has had considerable success for pure graviton correlators [111, 112, 106] (sometimes in conjunction with tools like spinor helicity formalism and results related to parity and the cosmological optical theorem). In this chapter, we follow the rules entailed in [105] to bootstrap three-point mixed correlators arising from operators in our EFToI that do not modify power spectra, and check to what extent the boostless bootstrap works. In doing so, we separately consider local and non-local interactions.

Finally, we give a prescription for obtaining the correlators for states that are Bogoliubov tranformations (BT) of Bunch-Davies (BD) vacuum. As an example, we calculate the correlators for α- vacua (which is the most general family of vacuum states consistent with de Sitter isometries [36]) from the answers for BD vacuum. This offers significant computational benefit since one does not have to repeat the cumbersome in-in calculation for BT states.

3.1 EFToI: Explicit check of soft theorems

In this section, we will explicitly check the validity of soft theorems for some of our EFT operators but first, we quickly review the key assumptions implicit in their derivations. It was shown in [49, 88, 113, 96] that every single field inflation model must satisfy consistency conditions relating three-point functions (in the squeezed limit) to two-point functions.

$$\lim_{\vec{k}_1\to 0}\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\boldsymbol{k_3}}\rangle = -(n_s-1)\langle\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle\langle\zeta_{\vec{k}_2}\zeta_{-\vec{k}_2}\rangle\left(1+\mathcal{O}(k_1^2/k_2^2)\right) \tag{3.1}$$

$$\lim_{\vec{k}_1\to 0}\langle\zeta_{\vec{k}_1}\gamma^h_{\vec{k}_2}\gamma^{h'}_{\vec{k}_3}\rangle = -n_t\langle\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}\rangle\langle\gamma^h_{\vec{k}_2}\gamma^{h'}_{-\vec{k}_2}\rangle\left(1+\mathcal{O}(k_1^2/k_2^2)\right) \tag{3.2}$$

$$\lim_{\vec{k}_1\to 0}\langle\gamma^h_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = -\langle\gamma^h_{\vec{k}_1}\gamma^h_{-\vec{k}_1}\rangle\epsilon^h_{ij}k_2^ik_2^j\frac{\partial}{\partial k_2^2}\langle\zeta_{\vec{k}_2}\zeta_{-\vec{k}_2}\rangle\left(1+\mathcal{O}(k_1^2/k_2^2)\right) \tag{3.3}$$

For the leading order (two derivatives) minimally coupled canonical action, we have $n_s-1 = 2(\eta-3\epsilon)$ and $n_t = -2\epsilon$ [49]. Besides requiring only a single degree of freedom during inflation and the condition that the curvature perturbation does not grow outside the horizon (not true for non-attractor models [114, 115, 116, 117, 118], see [119, 120] for modified soft theorems in shift symmetric non-attractor models), these relations also rely on the fact that there is a negligible contribution to the three-point function when all the modes are within the horizon. In fact, these theorems assume that the contribution to the correlators only starts becoming sizable when the long mode has left the horizon and has already frozen acting as a classical background. This is always the case when the initial state is the BD vacuum. The BD three-point correlators only have a total energy pole which comes from integrals of the form

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \sim \int_{-\infty}^{0}\tau^m e^{iK\tau}d\tau \tag{3.4}$$

where, $K = k_1+k_2+k_3$. To compute such an integral, a regularisation scheme is required. This integral can be regularized by taking $\tau\to\tau(1-i\epsilon)$, and therefore, damping the contribution in the far past. It only starts giving appreciable contribution once $K\tau\sim -1$ which in the equilateral case ($k_1=k_2=k_3$) corresponds to the epoch of horizon crossing for all the modes and in the squeezed case ($k_1\ll k_2\approx k_3$, relevant for soft theorems) corresponds to epoch of horizon crossing for the short modes since $K\sim k_s$. Therefore, in the squeezed limit the in-in contribution remains negligible till the epoch of the horizon crossing for the short modes. By this time the long mode has already frozen and therefore, the soft theorems must hold. There exist cases where the soft theorems are violated even in single field models [118, 121, 122, 110], for instance, if one starts in an excited initial state, for e.g., α-vacuum [36] then apart from total energy pole, one also has a $(k_1+k_2-k_3)$ kind of pole structure arising from mode-mixing [123]. Therefore the integral contains terms,

$$\langle \zeta_{\vec{k_1}} \zeta_{\vec{k_2}} \zeta_{\vec{k_3}} \rangle \sim \int_{-\infty}^{0} \tau^m e^{i(k_1+k_2-k_3)\tau} d\tau \tag{3.5}$$

which, in the squeezed limit ($k_1 \ll k_2 \approx k_3$) starts giving appreciable contribution when $k_1\tau \sim -1$ i.e when the long mode is near horizon exit. Therefore, the soft theorem proof does not hold in this case.

3.1.1 Explicit Checks

In this section, we explicitly calculate and verify the soft limits mentioned above. We start with the scalar three-point function calculated for the action in Eq. (1.98) with only $M_2 \neq 0$ as a check for calculations with $c_s \neq 1$ while for the mixed correlators, we take the canonical minimally coupled quadratic action for simplicity of calculation. Although the scalar soft theorems have been explicitly checked [113] the mixed correlator ones have not been explicitly checked for higher derivative EFToI operators and therefore, according to our knowledge, this is the first such an attempt.

3.1.1.1 Soft limit for $\langle\zeta\zeta\zeta\rangle$

The terms which contribute to the soft 3-point correlator $\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{\vec{k}_1\to 0}$ are given by:

$$S_0 + S_2 = \int M_{pl}^2 \left[\left(\epsilon^2 \zeta_c \dot{\zeta_c}^2 a^3 + \epsilon^2 \zeta_c (\partial\zeta_c)^2 a \right) + 4\,(3\epsilon - 2\eta) \left(\frac{M_2}{H}\right)^2 \zeta_c \dot{\zeta_c}^2 a^3 \right] \tag{3.6}$$

where the first two terms are from the Maldacena cubic action. It is important to note that the final Maldacena cubic action is derived after performing a field redefinition [49]

$$\zeta = \zeta_c + \frac{1}{2}(2\epsilon - \eta)\zeta_c^2 + \frac{\dot{\zeta}_c \zeta_c}{H} + ...$$

which removes terms proportional to the equation of motion. Since M_2 also gives quadratic corrections to ζ action, the above field redefinition generates additional cubic terms. Therefore, the last term in Eq. (3.6) is a combination of the cubic part generated by M_2 and the terms generated by the redefinition. The soft correlator is given by,

$$\begin{aligned}\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{\vec{k}_1\to 0} =& \left(\frac{H^2}{4\epsilon c_s M_{pl}^2}\right)^2\left(\frac{11}{2}\epsilon-2\eta+\frac{\epsilon}{2}c_s^2+4\frac{M_2^2}{H^2}c_s^2\left(\frac{3}{2}-\frac{\eta}{\epsilon}\right)\right)\frac{1}{k_1^3}\frac{1}{k_2^3}\\ =&-(n_s-1)\langle\zeta(\vec{k}_1)\zeta(-\vec{k}_1)\rangle\langle\zeta(\vec{k}_2)\zeta(-\vec{k}_2)\rangle\end{aligned} \tag{3.7}$$

3.1.1.2 Soft limits for mixed correlators

We classify the operators in the EFT into two classes, cubic and purely cubic operators. Cubic operators are those which contribute to $\langle\zeta\zeta\rangle$ or $\langle\gamma\gamma\rangle$, as well as the mixed bispectra, while purely cubic operators contribute only to the latter. We check soft theorems for cubic operators since they give a non-trivial contribution to both sides of the theorem. We take two cubic operators from Eq. (1.95) as examples:

(1) $\boldsymbol{\int d^4x\sqrt{-g}\frac{M_{pl}^2}{M_5}{}^{(3)}R\delta K}$ **:**

Since δK contains both δN and N^i, i.e. the ADM constraint variables, they get modified and are now given by:

$$\delta N =\frac{\dot{\zeta}}{H}+\frac{2\partial^2\zeta a^{-2}}{M_5 H} \tag{3.8}$$

$$N^i =\partial_i\left(-\frac{\zeta}{H}a^{-2}+\epsilon\partial^{-2}\dot{\zeta}+\frac{2\epsilon\zeta a^{-2}}{M_5}\right) \tag{3.9}$$

Note that the two equations above are valid only when $\frac{H}{M_5}\ll 1$ i.e. they're 1st order expressions in $1/M_5$. Using this, the corrections to the quadratic and cubic actions for $\gamma\gamma\zeta$ are given up to first order in $1/M_5$ by:

$$\mathcal{O}_{\zeta\zeta} = \int 4\frac{M_{pl}^2}{M_5} a\partial^2\zeta\left(\epsilon\dot{\zeta} - \frac{\partial^2\zeta}{H}a^{-2}\right) \tag{3.10}$$

$$\mathcal{O}_{\gamma\gamma\zeta} = \int \frac{M_{pl}^2}{4M_5}\left(a\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{H}\partial^2\zeta - 2a^{-1}\frac{\partial_l\gamma_{ij}\partial_l\gamma_{ij}\partial^2\zeta}{H} + \epsilon a\partial_l\gamma_{ij}\partial_l\gamma_{ij}\dot{\zeta} - 2\epsilon\gamma_{ij}\partial_l\dot{\gamma}_{ij}\partial_l\zeta\right) \tag{3.11}$$

The correction to the scalar power spectrum is given by,

$$\delta P_\zeta = -\frac{M_{pl}^2}{M_5}\frac{H^3}{4\epsilon^2 M_{pl}^4 k^3}(5 - 3\epsilon) \tag{3.12}$$

For perturbation theory to work here, the quadratic correction δP_ζ must be small compared to $P_\zeta(k) = \frac{H^2}{4\epsilon M_{pl}^2 k^3}$. We henceforth assume that $\frac{H}{M_5\epsilon} \ll 1$ so that the enhancement to the power spectrum remains small. This also ensures that we can expand the action (as well as the ADM constraints) in powers of M and the analysis above holds. In the soft limit, $\mathcal{O}_{\gamma\gamma\zeta_{\vec{k}\to 0}} = 0$ i.e its contribution to the mixed correlator in the soft limit vanishes at leading order. However, the following "exchange diagram" diagram gives a non-zero contribution:

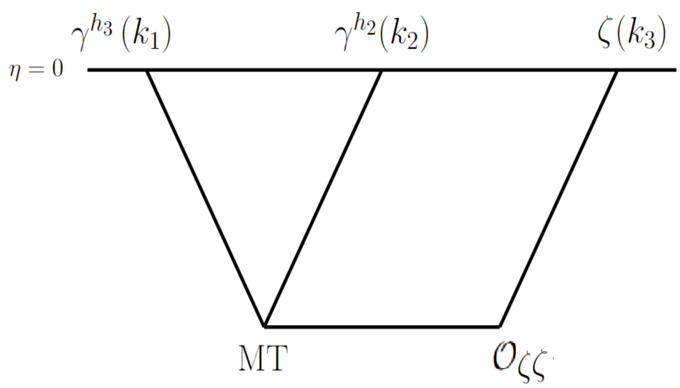


FIGURE 3.1: The MT (Maldacena term) vertex is the usual cubic $\gamma\gamma\zeta$ vertex of the Maldacena action

where the Maldacena term refers to the mixed cubic vertex ($\zeta\gamma\gamma$) computed in [49]. This 3-point correlator is a sum of two terms,

$$\langle\gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{\vec{k}_3\to 0} = 2\left(\langle\gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{LR,\vec{k}_3\to 0} - \langle\gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{RR,\vec{k}_3\to 0}\right) \tag{3.13}$$

where R and L indicate the time and anti-time orderings of the interaction Hamiltonian respectively. For instance, RR means both vertices are time ordered (See [56] for more details on these notations and conventions). The RR contribution is given by [1]

$$\begin{aligned}\langle\gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{RR,\vec{k}_3\to 0} &= \left(\frac{H^6}{M_{pl}^6 k_1^3 k_2^3 k_3^3}\right)\left(\frac{M_{pl}^2}{8\epsilon H M_5}\right) e_{ij}^{h_1}(k_1) e_{ij}^{h_2}(k_2)\left[k_1^2 k_2^2\left[\frac{5-3\epsilon}{4k_T} + \frac{(3-\epsilon)k_3}{k_T^2} - \right.\right.\\ &\left.\frac{\epsilon k_3^2}{k_T^3} - \frac{5k_3^3}{k_T^4} - \frac{8k_3^4}{k_T^5}\right] - (\vec{k}_1\cdot\vec{k}_2)\left[\frac{5-3\epsilon}{4}k_T + \epsilon k_3 I - \frac{5-3\epsilon}{2}k_3 + (3-\epsilon)\frac{k_3^2}{k_T} - k_1 k_2\left(\frac{5-3\epsilon}{4k_T} + \frac{5-3\epsilon}{2k_T^2} + \right.\right.\\ &\left.\frac{k_3^2}{k_T^3} - 2\frac{k_3^3}{k_T^4}\right) + (k_2 k_3 + k_1 k_3)\left(\frac{5-3\epsilon}{4k_T} + \frac{(1+\epsilon)k_3}{2k_T^2} + \frac{k_3^2}{k_T^3} + \frac{2k_3^3}{k_T^4}\right) + k_1 k_2 k_3\left(\frac{5-3\epsilon}{4k_T^2} + \frac{(1+\epsilon)k_3}{k_T^3} + \frac{3k_3^2}{k_T^4}\right.\\ &\left.\left. + \frac{8k_3^3}{k_T^5}\right)\right] - \left[k_2^2(\vec{k}_1\cdot\vec{k}_3)\left(\frac{5-3\epsilon}{4k_T} + \frac{5-3\epsilon}{4k_T^2}(k_1+2k_3) + \frac{k_1k_2(5-3\epsilon)+k_3^2}{k_T^3} + \frac{3k_1k_3^2 - 2k_3^3}{k_T^4} - \frac{8k_3^3k_1}{k_T^5}\right)\right.\\ &-(k_1\leftrightarrow k_2)]]\end{aligned} \tag{3.14}$$

where $I = \gamma_E\ +\ \log(-k_T\eta_0)$, $\eta_0 \to 0$ [2] and $k_T = \sum_{a=1}^{3} k_a$. It is easy to show that the LR contribution is simply given by,

$$\langle\gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{LR} = \frac{B_{MT}\delta P_\zeta(k_3)}{4P_\zeta(k_3)} \tag{3.15}$$

Therefore, the final 3-point function is given by:

$$\langle\gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{RR} = \frac{B_{MT}\delta P_\zeta(k_3)}{2P_\zeta(k_3)} - 2\langle\gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{RR} \tag{3.16}$$

---

[1] The results given from here on do not include the divergent terms of the type: $\lim_{\eta\to 0}\frac{cos(k\eta)}{\eta}$. These are ignored while calculating contact diagrams since they contribute only to the imaginary part of the L and R vertices and so, they get cancelled. Here, however, these terms come from both the vertices of the "exchange" diagram and get multiplied, because of which they contribute to the real part. However, one can easily check that the contributions from RR, RL, LR and LL add up to 0.

[2] here the logarithmically divergent term comes from an integral of the type $\int_{\infty}^{\eta_0}\frac{e^{ik\eta}}{\eta}d\eta$

where B_{MT} is the mixed bispectrum of the Maldacena cubic action which is given by [49]:

$$B_{MT}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{H^4}{M_p^4 l} \frac{1}{\prod_a 2k_a^3} e_{ij}^{h_1}(\vec{k}_1) e_{ij}^{h_2}(\vec{k}_2) \left(-\frac{1}{4}k_3^3 + \frac{1}{2}k_3(k_1^2 + k_2^2) + 4\frac{k_2^2 k_3^2}{k_T} \right) \tag{3.17}$$

One can easily check that:

$$\langle \gamma(\vec{k}_1)\gamma(\vec{k}_2)\zeta(\vec{k}_3)\rangle_{\vec{k}_3 \to 0} = -\frac{n_t P_\gamma(k_1) P_\zeta(k_3)}{2P_\zeta(k_3)} - \frac{1}{2} n_t P_\gamma(k_1) \delta P_\zeta(k_3) = -n_t P_\gamma(k_1) \delta P_\zeta(k_3) \tag{3.18}$$

i.e the soft limit Eq. (3.2) is satisfied.

(2) $\boldsymbol{\int d^4x \sqrt{-g} \frac{M_{pl}^2}{M_7^2} {}^{(3)}R_{ij} {}^{(3)}R_{ij}}$:

This operator gives the corrections:

$$\mathcal{O}_{\gamma\gamma\zeta} = \int \frac{M_{pl}^2}{M_7^2} a^{-1} \left[-\frac{1}{4} \zeta \partial^2 \gamma_{ij} \partial^2 \gamma_{ij} + \frac{\dot{\zeta}}{4H} \partial^2 \gamma_{ij} \partial^2 \gamma_{ij} \right] \tag{3.19}$$

$$O_{\zeta\zeta} = \int 6 \frac{M_{pl}^2}{M_7^2} a^{-1} (\partial^2 \zeta)^2 \quad O_{\gamma\gamma} = \int \frac{M_{pl}^2}{4M_7^2} a^{-1} \partial^2 \gamma_{ij} \partial^2 \gamma_{ij} \tag{3.20}$$

The corresponding corrections to the power spectra are,

$$\delta P_\zeta = 15 \frac{M_{pl}^2}{M_7^2} \frac{H^4}{8M_{pl}^4 \epsilon^2} \quad \delta P_\gamma = 5 \frac{M_{pl}^2}{M_7^2} \frac{H^4}{2M_{pl}^4 k^3} \delta_{h_1 h_2} \tag{3.21}$$

Again, as also noted in [97] one should be worried about the $\frac{\Delta\langle\zeta\zeta\rangle}{\langle\zeta\zeta\rangle_0} \sim \frac{M_{pl}^2}{M_7^2} H^2/\epsilon M_{pl}^2$ enhancement of the power spectrum and thus we can remove the correction by taking an extra $({}^{(3)}R)^2$ term or assume that this ratio is small, which places a lower bound on M_7. As pointed out in [97], after the field redefinition $\gamma_{ij} \to \gamma_{ij} - \dot{\gamma}_{ij}\zeta/H + ...$

which removes the terms proportional to the equation of motion in the Maldacena mixed cubic action, and after integration by parts, we get:

$$\mathcal{O} = \int \sqrt{-g}\frac{M_{pl}^2}{M_7^2}{}^{(3)}R_{ij}{}^{(3)}R_{ij} + \frac{1}{2}\frac{M_{pl}^2}{M_7^2}\zeta\partial^2\frac{\dot{\gamma}_{ij}}{H}\partial^2\gamma_{ij}a^{-1} = -\int \frac{1}{4}\epsilon\zeta\frac{M_{pl}^2}{M_7^2}\partial^2\gamma_{ij}\partial^2\gamma_{ij}a^{-1} + ... \tag{3.22}$$

After taking the soft limit $\zeta_{\vec{k}_3\to 0}$, the terms represented by dots go to 0 at leading order as mentioned in [97]. The term proportional to ϵ however, does contribute and we have:

$$\Delta_A\langle\zeta(\vec{k}_3)\gamma^{h_1}(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\rangle_{\vec{k}_3\to 0} = -\frac{5}{8}\frac{M_{pl}^2}{M_7^2}\left(\frac{H}{M_{pl}}\right)^6\frac{1}{k_1^3}\frac{1}{k_3^3}\delta_{h_1h_2} \tag{3.23}$$

This term in Eq. (3.22) is not mentioned in [97] and there, the 3-point function in the soft limit is shown to vanish at $\mathcal{O}((k_1/k_3)^0)$. The difference lies in the order of calculations, i.e. whether you compute and simplify the operator first and then do the in-in computation or just do the in-in computation for all the terms and add the answers as done in [97]. The difference in the answers arises because H is not a constant when we simplify the operators but is taken to be time-independent while doing the in-in integrals. The reason for this is that the in-in integral pick up most of the contribution near horizon crossing and therefore, it is a good approximation to take $H(t) = H_*$ i.e Hubble at horizon crossing.[3] To get the correct soft limit, we should thus always try to eliminate H dependence in the action and get everything ordered in powers of ϵ, η, as in Eq. (3.22), before calculating the correlator. This prescription is also followed in standard Maldacena action calculations [49]. Proceeding with the calculations, we again have the "exchange diagrams" as before as shown below. The left one gives a contribution:

[3] for instance if the operator is $\mathcal{O} = \int a^3 M_{pl}(M_2/H)^2\dot{\zeta}^3$ and we want to compute $\langle\zeta\zeta\zeta\rangle$ we'll compute it as follows

$$\langle\zeta\zeta\zeta\rangle \sim \left(\frac{M_2}{H_*}\right)^2\int_{-\infty}^0 d\eta k_1^2k_2^2k_3^2\eta^4 e^{ik_T\eta}$$

i.e. we keep H_* outside the integral. There are also multiple factors of H_* (and M_{pl}), which come from $a = -1/H\eta$ and the mode functions that are also taken outside.

$$\Delta_B\langle\zeta(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle_{\vec{k}_1\to 0} = 2\left(\frac{B_{MT}|_{k_1\to 0}\delta P_\gamma(k_1)}{2P_\gamma(k_1)}\right) - 2\langle\zeta(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle_{RR,\vec{k}_1\to 0} \tag{3.24}$$

$$= 2\left(\frac{5}{8}\frac{M_{pl}^2}{M_7^2}\frac{H^6}{M_{pl}^6}\frac{1}{k_1^3k_3^3}\delta_{h_1h_2}\right) + \frac{15}{8}\frac{M_{pl}^2}{M_7^2}\frac{H^6}{M_{pl}^6}\frac{1}{k_1^3k_3^3}\delta_{h_1h_2} \tag{3.25}$$

where the first term in the last line just follows from the Maldacena soft limit and the second term is calculated in Appendix B. Calculating the modified spectral tilt of $\langle\gamma\gamma\rangle$ yields:

$$\widetilde{n}_t = \frac{1}{H}\partial_t \log\langle\gamma\gamma\rangle = -2\epsilon - 5\epsilon\frac{M_{pl}^2}{M_7^2}\frac{H^2}{M_{pl}^2} \tag{3.26}$$

Adding both the contributions Δ_A and Δ_B gives:

$$\langle\zeta(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle_{A+B,\ \vec{k}_1\to 0} = -\widetilde{n}_t(P_\gamma(k_1)+\delta P_\gamma(k_1))P_\zeta(k_3) \tag{3.27}$$

The second diagram is just like the one we considered for ${}^{(3)}R\delta K$, and hence adding it to the previous answer gives:

$$\langle\zeta(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle_{\vec{k}_1\to 0} = -\widetilde{n}_t(P_\gamma(k_1)+\delta P_\gamma(k_1))(P_\zeta(k_3)+\delta P_\zeta(k_3)) \tag{3.28}$$

where the equality holds at $\mathcal{O}((H/M_{pl})^6)$. These calculations for the soft limits of $\langle\gamma\gamma\zeta\rangle$ also hold for $\langle\gamma\zeta\zeta\rangle$ and one can verify it for ${}^{(3)}R\delta K$ (see Appendix C). Since the operator above changes P_γ, one might also be interested in obtaining the soft limit results for pure graviton correlators, which can be found in [98].

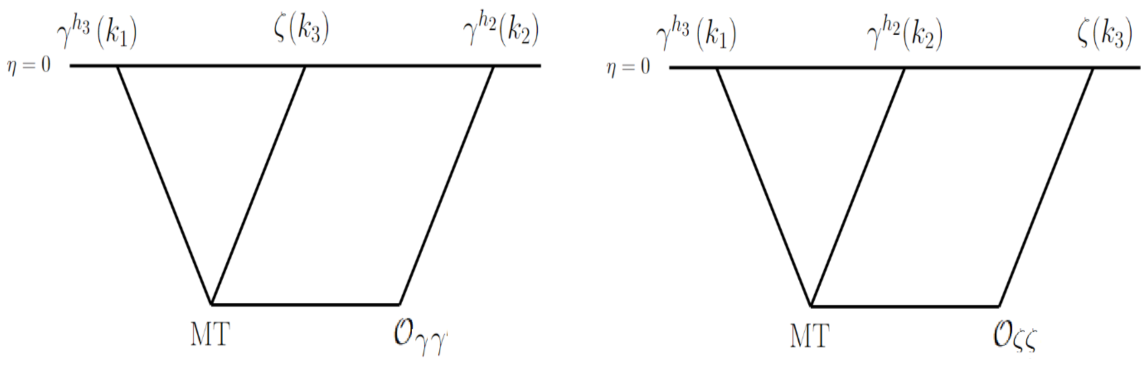


FIGURE 3.2: The MT (Maldacena term) vertex is the usual cubic vertex of the Maldacena action and $\mathcal{O}_{\gamma\gamma}, \mathcal{O}_{\zeta\zeta}$ the respective quadratic correction operators from ${}^{(3)}R^{(3)}_{ij}R_{ij}$

In summary, we have shown explicitly how the soft limits are obeyed for various higher derivative operators. These provide a consistency check for models beyond the Maldacena action and as we'll see next, these have an important role in the boostless bootstrap of correlators.

3.2 Boostless bootstrap

Cosmological bootstrap, like any other bootstrap program, tries to fix the mathematical form of physical observables without doing explicit computations and by using very general principles such as symmetries, analytical structure, etc. As mentioned before, as of yet we have measured only the two-point scalar correlation function and its tilt [8, 9]. The two-point function can be bootstrapped just by using rotational, translational and scale invariance. Note that the two-point function is not exactly scale-invariant but since the deviation, i.e the spectral tilt, is very small, one can use scale invariance to fix the leading contribution (i.e. the result which survives in the pure dS limit). Since the momentum dependence is completely fixed without using dS boosts, from the bootstrap perspective, it is not clear whether boosts are good symmetries of cosmological correlators or not. In single-field slow-roll models of inflation, both boost and scale invariance are broken by a slowly-evolving background (governed by $\dot{\phi}(t)$) parametrised by the slow-roll parameter ϵ. This allows one to bootstrap these correlations up to slow-roll corrections, i.e up to ϵ corrections using the full de Sitter isometries [124]. In terms of ζ, it is not very clear that this should give the correct answer since it is a gauge mode in pure de Sitter [49] but one can instead bootstrap $\delta\phi$ correlators and use $\zeta = \frac{\delta\phi}{\sqrt{2\epsilon}M_{pl}} - \frac{\eta\delta\phi^2}{8\epsilon M_{pl}^2}$ to get the corresponding correlators for the curvature perturbations. In a general theory of inflation, boost breaking can be very large while still preserving approximate scale invariance. E.g., as already mentioned in the first example of the EFToI section, M_2 can potentially give a speed of sound $c_s \ll 1$, thus breaking boosts by a large amount. Therefore, one can develop either a boost-preserving, i.e. a conformal bootstrap [103, 125, 126] or a boost-breaking or boostless bootstrap [105, 106, 127].

The conformal bootstrap basically uses Ward identities of boost symmetries, along with results related to the Operator Product Expansion(OPE) [126] and soft limits to solve for the relevant correlators. The boostless bootstrap (BB), however, focuses more on the analytical structure of the correlators along with results motivated by locality, soft limits, etc. A strong motivation for considering boostless theories is provided by the theorem proved in [108], which states that all non-gaussianities in ζ approximately vanish if we consider only dS invariant theories. In the upcoming sections, we explore the BB and use the rules mentioned in [105] to construct mixed correlators from higher derivative (more than two) operators in the EFToI.

3.2.1 Relevant terms for $\langle\gamma\gamma\zeta\rangle$

Term	No. of Derivatives	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\Lambda)$ or $\mathcal{O}(M_{pl})$
${}^{(3)}R$	2	1	M_{pl}^2
${}^{(3)}R\delta g^{00}$	2	1	Λ^2
$\delta K_{ij}\delta K_{ij}$	3	$1, \epsilon$	M_{pl}^2
$\delta K_{ij}\delta K_{ij}\delta g^{00}$	3	1	Λ^2

$\delta K_{ij}\delta K_{jk}\delta K_{ki}$	3	$1, \epsilon$	Λ
$\delta K_{ij}\delta K_{ji}\delta K$	3	$1, \epsilon$	Λ
${}^{(3)}R_{ij}\delta K_{ij}$	3	$1, \epsilon$	Λ
${}^{(3)}R\delta K$	3	$1, \epsilon$	Λ
${}^{(3)}R_{ij}\delta K_{ij}\delta g^{00}$	3	1	Λ
${}^{(3)}R_{ij}^{(3)}R_{ij}$	4	1	1
${}^{(3)}R_{ij}^{(3)}R_{ij}\delta g^{00}$	4	1	1
${}^{(3)}R_{ij}\delta K_{jk}\delta K_{ki}$	4	$1, \epsilon$	1
${}^{(3)}R_{ij}^{(3)}R_{jk}\delta K_{ki}$	5	$1, \epsilon$	$1/\Lambda$
${}^{(3)}R_{ij}^{(3)}R_{ij}\delta K$	5	$1, \epsilon$	$1/\Lambda$
${}^{(3)}R_{ij}^{(3)}R\delta K_{ij}$	5	$1, \epsilon$	$1/\Lambda$
${}^{(3)}R_{ij}^{(3)}R_{jk}^{(3)}R_{ki}$	6	1	$1/\Lambda^2$
${}^{(3)}R_{ij}^{(3)}R_{ij}^{(3)}R$	6	1	$1/\Lambda^2$

TABLE 3.1: Quadratic and cubic operators and their contributions to $\langle\gamma\gamma\zeta\rangle$. The 3rd column shows what powers of ϵ can be present in the terms generated by the operators, while the last column shows the coefficients of the operators in terms of some energy scale Λ. The term in red is redundant as it can be removed using $\int A(t)^{(3)}R_{ij}K_{ij} = \int A(t)^{(3)}RK + \dot{A(t)}^{(3)}R/2N +$ total derivative, while the term in blue is present in the Maldacena action. Here $\delta g^{00} = g^{00} + 1$.

As mentioned before, While writing the operators in Table 3.1, we use covariant objects (i.e. which are covariant, at least w.r.t. the 3d metric). We also take the operators defined in [100] :

$$V = \frac{\delta\dot{N} - N^i\partial_i N}{N} \tag{3.29}$$

$$A_\mu = \frac{h^\nu_\mu \nabla_\nu N}{N} \tag{3.30}$$

where $h_{\mu\nu}$ is the 3d spatial metric. Note that using A_0 and V we can obtain $\delta\dot{N}$ and $N^i\partial_i N$, so we'll use them instead. Also, using N_i and $\partial_i N$ we can obtain A_i's contribution to various operators. and so, we will not use A_i explicitly. For $\langle\gamma\gamma\zeta\rangle$, only $\delta\dot{N}$ is relevant, which is just a higher derivative operator derived from δN and hence it has not been included in Table 3.1. All the higher derivative operators will of course be suppressed by an energy or mass scale Λ. Some of the operators give non-local terms which we shall discuss below.

3.2.2 Purely cubic local terms

Let us take a local term (which is not present in the Maldacena action) as an example of bootstrapping. We shall primarily use 4 rules of bootstrapping, developed in [105, 107]. They are:

- Symmetry between identical Bosons. For instance, $\langle\zeta(\vec{k}_1)\gamma(\vec{k}_2)\gamma(\vec{k}_3)\rangle$ should be symmetric under $k_2 \leftrightarrow k_3$.
- Amplitude rule [83, 111, 128]:

$$\lim_{k_T\to 0}\langle\zeta_1\zeta_2\zeta_3..\zeta_n\rangle = \frac{(-1)^n H^{p+n-1}(p-1)!}{2^{n-1}}\frac{\mathrm{Re}(i^{n+p+1}A_n)}{k_T^p\prod_{a=1}^n k_a^2} \tag{3.31}$$

  Here p, for $n = 3$, is found by the total number of space and time derivatives in the interaction Hamiltonian.

- Manifest Locality Test (MLT)[107]

$$\left.\frac{\partial \mathcal{B}}{\partial k_1}\right|_{k_1=0} = \left.\frac{\partial \mathcal{B}}{\partial k_2}\right|_{k_2=0} = \left.\frac{\partial \mathcal{B}}{\partial k_3}\right|_{k_3=0} = 0 \tag{3.32}$$

where $\mathcal{B}(k_1, k_2, k_3) \sim k_1^3 k_2^3 k_3^3 \langle \zeta_1 \zeta_2 \zeta_3 \rangle_{tr}$, "tr" signifying that the tensor contractions have been trimmed. This equation is valid only for local operators [4].

- Soft limits for purely cubic operators. We shall not bootstrap correlators arising from cubic operators since the procedure would require knowing what diagrams we need, and in this case, an explicit in-in computation might be much more simpler and economical.

We take the purely cubic vertex

$$R^{(3)}_{ij} \delta K_{ij} \delta g^{00} = \int a \frac{\Lambda}{H} \partial^2 \gamma_{ij} \dot{\gamma}_{ij} \dot{\zeta} \tag{3.33}$$

Direct in-in calculation gives:

$$\langle \zeta(\vec{k}_1) \gamma^{h_2}(\vec{k}_2) \gamma^{h_3}(\vec{k}_3) \rangle = \frac{\Lambda}{H} e^{h_2}_{ij}(\vec{k}_2) e^{h_3}_{ij}(\vec{k}_3) k_1^2 k_2^2 k_3^2 \frac{H^6}{\epsilon M_{pl}^6 k_1^3 k_2^3 k_3^3} \left[\frac{2}{(k_1+k_2+k_3)^3} + 3 \frac{k_2+k_3}{(k_1+k_2+k_3)^4} \right] \tag{3.34}$$

We have $p = 4$ for this vertex. The corresponding flat space amplitude is given by,

$$A[1^0, 2^{h_2}, 3^{h_3}] \sim e^{h_2}_{ij}(\vec{k}_2) e^{h_3}_{ij}(\vec{k}_3) k_1 k_2 k_3 (k_2 + k_3) \tag{3.35}$$

[4]This can be seen easily by taking the most general local operator $\mathcal{O} \sim \zeta \cdot \zeta^{(m)} \cdot \zeta^{(n)}$ where n, m denote the no. of derivatives on ζ. Suppose $\boldsymbol{q}$ is the soft mode. Note that if all three ζ's had derivatives then the MLT would be trivially satisfied due to powers of q coming from the derivatives. Hence the non-trivial contribution comes when we have $\boldsymbol{q}$ associated with the ζ with no derivatives. In this case, the main term to focus on is:

$$\langle \zeta_{\boldsymbol{q}\to 0} \zeta \zeta \rangle_{tr} \sim \frac{\partial}{\partial q} \int_{-\infty}^{0} d\eta (1 - iq\eta) e^{ik_T \eta} |_{\boldsymbol{q}\to 0} + \frac{\partial}{\partial q} \int_{-\infty}^{0} d\eta (1 + iq\eta) e^{-ik_T \eta} |_{\boldsymbol{q}\to 0} = 0$$

where we have omitted the mode functions involving the other 2 momenta. The analysis is easily extended to correlators with γ.

where we have kept the normalisation arbitrary (which contains information about things like Λ and ϵ dependence). From this we write an ansatz using the first rule (where $k_T = k_1 + k_2 + k_3$, $e_1 = k_2 + k_3$, $e_2 = k_2 k_3$ and $e_3 = k_1 k_2 k_3$):

$$\langle\zeta(\vec{k}_1)\gamma(\vec{k}_2)\gamma(\vec{k}_3)\rangle \sim \frac{e_{ij}^{h_2}(\vec{k}_2)e_{ij}^{h_3}(\vec{k}_3)}{k_T^4 e_3^3}\left[e_3 k_1 k_2 k_3(k_2+k_3) + k_T(A_1 e_2^3 + A_2 e_2^2 e_1^2 + A_3 e_2 e_1^4 + A_4 e_1^6)\right.$$
$$+k_T^2(A_5 e_2^2 e_1 + A_6 e_2 e_1^3 + A_7 e_1^5) + k_T^3(A_8 e_2^2 + A_9 e_2 e_1^2 + A_{10} e_1^4) + k_T^4(A_{11} e_2 e_1 + A_{12} e_1^3)$$
$$\left.+k_T^5(A_{13} e_2 + A_{14} e_1^2) + A_{15} k_T^6 e_1 + A_{16} k_T^7\right] \tag{3.36}$$

We get the following set of equations after applying MLT and soft limits for various momenta:

$$\text{Soft limit for } k_2, k_3: \quad A_4 = A_7 = A_{10} = A_{12} = A_{14} = A_{15} = A_{16} = 0 \tag{3.37}$$
$$\text{MLT for } k_2, k_3: \quad A_3 = A_6 = A_9 = A_{11} = A_{13} = 0 \tag{3.38}$$
$$\text{Soft limit for } k_1: \quad A_2 + A_5 + A_8 = 0 \tag{3.39}$$
$$\text{MLT for } k_1: \quad A_1 = 0 \quad 3A_2 + 2A_5 + A_8 = 0 \tag{3.40}$$

which fixes our correlator to be:

$$\langle\zeta(\vec{k}_1)\gamma(\vec{k}_2)\gamma(\vec{k}_3)\rangle \sim \frac{e_{ij}^{h_2}(\vec{k}_2)e_{ij}^{h_3}(\vec{k}_3)}{k_T^4 e_3^3} e_3^2\left[A_2 k_T + e_1\right] \tag{3.41}$$

which means we're able to fix the bispectra up to an overall factor and another arbitrary constant. This expression agrees with the explicit calculation in Eq. (3.34) with $A_2 = 2/3$.

3.2.3 Non-local terms

We have the following non-local terms (see Appendix D) at various orders in ϵ and energy scales:

Operator	$\mathcal{O}(\epsilon)$	$\mathcal{O}(H/M_{pl})$ or $\mathcal{O}(H/\Lambda)$
$\delta K_{ij}\delta K_{ij}$	ϵ	H/M_{pl}
$\delta K_{ij}\delta K_{jk}\delta K_{ki}$	ϵ	$(H/M_{pl})^2(\Lambda/M_{pl})$
$\delta K_{ij}\delta K_{jk}{}^{(3)}R_{ki}$	ϵ	$(H/M_{pl})^3$
$\delta K_{ij}{}^{(3)}R_{jk}{}^{(3)}R_{ki}$	ϵ	$(H/M_{pl})^3(H/\Lambda)$

(For the sake of brevity, we have not included the parity-odd terms, but their contributions are of the same order as the last 3 terms.) The second term in the table gives:

$$\int\sqrt{-g}\ \Lambda\delta K_{ij}\delta K_{jk}\delta K_{ki} = \int -\frac{3}{4}a^3\Lambda\epsilon\dot{\gamma_{ij}}\dot{\gamma_{jk}}\partial_i\partial_k\partial^{-2}\dot{\zeta} \tag{3.42}$$

for which the explicit in-in calculation yields:

$$\langle\zeta(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle = -\frac{3}{2}\frac{H^6}{M_{pl}^6}\left(\frac{\Lambda}{H}\right)e_{ij}^{h_2}(\vec{k}_2)e_{jm}^{h_3}(\vec{k}_3)k_{1i}k_{1m}\frac{k_2^2k_3^2}{k_T^3e_3^3} \tag{3.43}$$

The non-local term in Eq. (3.42) naively doesn't seem to have a proper flat space counterpart, but as pointed out in [105], it can be considered to come from a toy model:

$$S_{\text{flat}} = \int d^4x\ \ \epsilon(\partial_\mu\zeta)^2 - \frac{1}{2}(\partial_i X)^2 + \frac{M_{pl}^2}{8}(\partial_\mu\gamma_{ij})^2 - \frac{3\Lambda\epsilon\dot{\zeta}_0}{8}X + \frac{3\Lambda\epsilon}{8\dot{\zeta}_0}X\dot{\zeta}^2 + \Lambda\dot{\gamma_{ij}}\dot{\gamma_{jk}}\partial_i\partial_k X \tag{3.44}$$

Here $\zeta_0(t) = -(\sqrt{2\epsilon})^{-1}\bar{\phi}(t)$ is the background value of the scalar field. Integrating out the field X above gives us an EFT with the desired non-local term, which gives an amplitude (with no of derivatives, $p = 3$):

$$A[1^0, 2^{h_2}, 3^{h_3}] = e_{ij}^{h_2}(\vec{k}_2)e_{jm}^{h_3}(\vec{k}_3)k_{1i}k_{1m}\frac{k_2k_3}{k_1} \tag{3.45}$$

Taking the ansatz as before and using soft limits gives:

$$\langle\zeta(\vec{k}_1)\gamma(\vec{k}_2)\gamma(\vec{k}_3)\rangle \sim \frac{e_{ij}^{h_2}(\vec{k}_2)e_{jm}^{h_3}(\vec{k}_3)k_{1i}k_{1m}}{k_T^3e_3^3}\left[\frac{k_2k_3}{k_1}e_3 + Ak_Te_1e_2 + Bk_T^2e_2\right] \tag{3.46}$$

Applying MLT with respect to k_2 and k_3 fixes $A = B = 0$ and hence, the correlator up to an overall factor, matching with Eq. (3.43). Note that as the value of p increases, we'll get more and more unknown parameters. Specifically for the non-local terms, we cannot use the MLT w.r.t the non-local momenta, which removes one condition and increases the no. of arbitrary constants. We want to point out that the source of these non-localities is rooted in the fact that not all metric components are dynamical variables. Since we have constraint equations for these non-dynamical variables, one plugs in their solutions in the action, which can potentially involve inverse differential operators since the constraint equations are differential equations.

3.2.4 Extending results to $\langle\gamma\zeta\zeta\rangle$

A similar bootstrap analysis can be carried out for non-local and local terms separately. One again finds that the non-local terms start appearing at $\mathcal{O}(\epsilon)$. The operators are summarized in Table 3.2 below

Term	No. of Derivatives	$\mathcal{O}(\epsilon)$	$\mathcal{O}(\Lambda)$ or $\mathcal{O}(M_{pl})$
$c(t)\delta g^{00}$	0	ϵ	M_{pl}^2
$\delta K_{ij}\delta K_{ij}$	2	$1, \epsilon$	M_{pl}^2
$\delta K_{ij}\widetilde{N_i}\partial_j\delta g^{00}$	2	$1, \epsilon$	Λ^2
${}^{(3)}R_{ij}\delta K_{ij}$	3	$1, \epsilon$	Λ
$\delta K_{ij}\delta K_{jk}\delta K_{ki}$	3	$1, \epsilon, \epsilon^2$	Λ
$\delta K_{ij}\delta K_{ji}\delta K$	3	$1, \epsilon, \epsilon^2$	Λ

${}^{(3)}R_{ij}\delta K_{ij}\delta g^{00}$	3	$1, \epsilon$	Λ
$\delta K_{ij}\partial_i\delta g^{00}\partial_j\delta g^{00}$	3	1	Λ
${}^{(3)}R_{ij}\widetilde{N}_i\partial_j\delta g^{00}$	3	$1, \epsilon$	Λ
${}^{(3)}R_{ij}\delta K_{ij}\delta K$	4	$1, \epsilon, \epsilon^2$	1
${}^{(3)}R_{ij}\delta K_{jk}\delta K_{ki}$	4	$1, \epsilon, \epsilon^2$	1
${}^{(3)}R_{ij}\partial_i\delta g^{00}\partial_j g^{00}$	4	1	1
${}^{(3)}R_{ij}^{(3)}R_{ij}\delta g^{00}$	4	1	$1/\Lambda$
${}^{(3)}R_{ij}^{(3)}R_{jk}\delta K_{ki}$	5	$1, \epsilon$	$1/\Lambda$
${}^{(3)}R_{ij}^{(3)}R_{ij}\delta K$	5	$1, \epsilon$	$1/\Lambda$
${}^{(3)}R_{ij}^{(3)}R_{jk}^{(3)}R_{ki}$	6	1	$1/\Lambda^2$
${}^{(3)}R_{ij}^{(3)}R_{ij}^{(3)}R$	6	1	$1/\Lambda^2$

TABLE 3.2: Quadratic and cubic operators and their contributions to $\langle\gamma\zeta\zeta\rangle$. Again, the blue terms are Maldacena terms, and the red term is removable by the identity mentioned below Table 3.1.

We take the term (which is the operator $\mathcal{O}_3$ in Appendix D) :

$$\delta K_{ij}\partial_i\delta g^{00}\partial_j\delta g^{00} \sim \int a\frac{1}{\Lambda}\dot{\gamma}_{ij}\partial_i\dot{\zeta}\partial_j\dot{\zeta} \tag{3.47}$$

The explicit in-in calculation gives:

$$\langle\gamma^h(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = 6\frac{H^7}{\epsilon^2 M_{pl}^6\Lambda}\frac{e^h_{ij}(\vec{k}_1)k_{2i}k_{3j}}{e_3 k_T^5} \tag{3.48}$$

We have the corresponding flat space amplitude:

$$A[1^h, 2^0, 3^0] \sim e_3 e^h_{ij}(\vec{k}_1)k_{2i}k_{3j} \tag{3.49}$$

which gives us the ansatz for the correlator to be:

$$\begin{aligned}\langle\gamma^h(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \sim \frac{e^h_{ij}(\vec{k}_1)k_{2i}k_{3j}}{k_T^5 e_3^3}&\left[e_3^2 + k_T(A_1e_2^2e_1 + A_2e_2e_1^3 + A_3e_1^5)\right.\\ &+k_T^2(A_4e_2^2 + A_5e_2e_1^2 + A_6e_1^4)k_T^3(A_7e_2e_1 + A_8e_1^3)\\ &\left.+k_T^4(A_9e_2 + A_{10}e_1^2) + A_{11}k_T^5e_1 + A_{12}k_T^6\right]\end{aligned} \tag{3.50}$$

Soft limits and MLTs give the following equations:

$$\text{Soft limit } \vec{k}_1 \to 0 \begin{cases} A_1 + A_4 = 0 \\ A_2 + A_5 + A_7 + A_9 = 0 \\ A_3 + A_6 + A_8 + A_{10} + A_{11} + A_{12} = 0 \end{cases} \tag{3.51}$$

$$\text{MLT for } k_1 \begin{cases} 4A_1 + 3A_4 = 0 \\ 4A_2 + 3A_5 + 2A_7 + A_9 = 0 \\ 4A_3 + 3A_6 + 2A_8 + A_{10} - A_{12} = 0 \end{cases} \tag{3.52}$$

$$\text{Soft limit } \vec{k}_2 \to 0 \quad \text{Already satisfied due to the tensor structure} \tag{3.53}$$

$$\text{MLT for } k_2 \begin{cases} 5A_3 + A_2 + 2A_6 = 0 \\ A_3 = 0 \\ A_5 + 4A_6 + 3A_8 = 0 \\ A_7 + 3A_8 + 4A_{10} = 0 \\ A_9 + 2A_{10} + 5A_{11} = 0 \\ A_{11} + 6A_{12} = 0 \end{cases} \tag{3.54}$$

which leads to the following correlator with just one arbitrary parameter:

$$\begin{aligned} \langle \gamma^h(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \sim \frac{e^h_{ij}(\vec{k}_1)k_{2i}k_{3j}}{k_T^5 e_3^3} &\left[e_3^2 + A_{10}\left(-2k_T e_2 e_1^3 + k_T^2(2e_2e_1^2 + e_1^4)\right.\right. \\ &\left.\left. + k_T^3(e_2e_1 - 2e_1^3) + k_T^4(-2e_2 + e_1^2)\right)\right] \end{aligned} \tag{3.55}$$

which matches the explicit result in Eq. (3.48) for $A_{10} = 0$.

3.3 Going to Bogoliubov states

In our calculations, we are not compelled to fix the initial condition (and the subsequent evolution) by taking the Bunch-Davies (BD) vacuum. A simple family of excited states one might be motivated to take is the family of states that are Bogoliubov transformations of the BD vacuum, since they just involve taking a linear combination of the creation and annhilation operators of BD. In particular, within this family, one can take the well-known family of α vacua [36, 125] since they respect the symmetries of the quasi dS background. Bootstrapping α-vacua answers directly using the BB is difficult since the soft theorems are not satisfied. This is because of the pole structures mentioned in Sec. 3.1. However, once we have bootstrapped

$\mathcal{B}(k_1, k_2, k_3)$(as defined above) for BD, we can extend the result to α vacua easily. For a (k-independent) Bogoliubov transformation (BT), just by noting the form of the mode functions, which are given by :

$$\begin{gathered} u_k(\eta) = \alpha(1 - ik\eta)e^{ik\eta} + \beta(1 + ik\eta)e^{-ik\eta} \\ |\alpha|^2 - |\beta|^2 = 1 \end{gathered} \tag{3.56}$$

we can give an ansatz for the BT bispectra as follows:

$$\begin{aligned} \mathcal{B}_{BT}(k_1, k_2, k_3, \{\vec{k}\}) = \text{Re}\Bigg[(\alpha + \beta)^3 \Bigg(&\psi_3'(k_1, k_2, k_3, \{\vec{k}\})\alpha^{*3} + \sum_{\text{cyclic}} \psi_3'(-k_1, k_2, k_3, \{\vec{k}\})\alpha^{*2}\beta^* \\ &+ \sum_{\text{cyclic}} \psi_3'(-k_1, -k_2, k_3, \{\vec{k}\})\alpha^*\beta^{*2} + \psi_3'(-k_1, -k_2, -k_3, \{\vec{k}\})\beta^{*3}\Bigg)\Bigg] \end{aligned} \tag{3.57}$$

where ψ_3' is the trimmed cubic wavefunction coefficient in BD vacuum [83, 106]. From the cosmological optical theorem, if we have odd parity interactions i.e. odd number of momenta contracted with the polarization tensors, the correlator for BD is 0 and we need the wavefunction coefficients to get the final answer for BT states. However, for even parity interactions, we have $\mathcal{B}_{BD} = \psi_3'$ and in this case we can get the answers for BT states directly from the BD answers. Putting $\alpha = \cosh\alpha$ and $\beta = i\ \sinh\alpha$, we get the α vacua result. Using this equation to bootstrap $\langle\gamma\gamma\gamma\rangle$ in α vacua for the Maldacena action, we take the well-known result for BD which was bootstrapped in [105] (also explicitly calculated in [49]), and get :

$$\langle\gamma^{h_1}(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle_\alpha = -\frac{2H^4}{M_{pl}^4}\frac{1}{(\prod_{a=1}^3 k_a^3)}e^{h_1}_{ii'}e^{h_2}_{jj'}e^{h_3}_{kk'}t_{ijk}t_{i'j'k'}\left[\left(-k_T+\frac{\sum k_ik_j}{k_T}+\frac{k_1k_2k_3}{k_T^2}\right)+\right.$$
$$\left.\sinh^2 2\alpha\left(-(-k_1+k_2+k_3)+\frac{k_2k_3-k_1k_2-k_1k_3}{(-k_1+k_2+k_3)}-\frac{k_1k_2k_3}{(-k_1+k_2+k_3)^2}\right)+\text{cyclic}\right]$$

$$t_{ijk} = k_{2i}\delta_{jk}+k_{3j}\delta_{ki}+k_{1k}\delta_{ij} \tag{3.58}$$

which agrees with the explicit in-in result in [111, 129]. We also get the following expression for the pure scalar correlator:

$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle_\alpha = \frac{H^4}{32M_{pl}^4\epsilon^2}\frac{1}{(\prod_{a=1}^3 k_a^3)}\left[2(\epsilon-\eta)\sum_a k_a^3+\epsilon\left(\sum_a k_a^3+\sum_{a\neq b}k_a^2k_b+8\sum_{a>b}\frac{k_a^2k_b^2}{k_T}\right)+\right.$$
$$\left.\sinh^2 2\alpha\left(2(\epsilon-\eta)\sum_a k_a^3+\epsilon\left(\sum_a k_a^3+\sum_{a\neq b}k_a^2k_b+\sum_{a>b}8\frac{k_a^2k_b^2}{k_2+k_3-k_1}+8\frac{k_a^2k_b^2}{k_3+k_1-k_2}+8\frac{k_a^2k_b^2}{k_1+k_2-k_3}\right)\right)\right] \tag{3.59}$$

which agrees with the calculation done in [125]. This demonstrates that the precription given above indeed works.

Similarly, using the BD results, we get the following for mixed correlators for Maldacena action in α vacua:

$$\langle\gamma^h(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle_\alpha = \frac{H^4}{4M_{pl}^4\epsilon}\frac{1}{(\prod_{a=1}^3 k_a^3)}e^h_{ij}(k_1)k_{2i}k_{3j}\left[\left(-k_T+\frac{\sum k_ik_j}{k_T}+\frac{k_1k_2k_3}{k_T^2}\right)+\right.$$
$$\left.\sinh^2 2\alpha\left(-(-k_1+k_2+k_3)+\frac{k_2k_3-k_1k_2-k_1k_3}{(-k_1+k_2+k_3)}-\frac{k_1k_2k_3}{(-k_1+k_2+k_3)^2}\right)+\text{cyclic}\right] \tag{3.60}$$

$$\langle\zeta(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle_\alpha = \frac{H^4}{8M_{pl}^4}\frac{1}{(\prod_{a=1}^3 k_a^3)}e^{h_2}_{ij}(k_2)e^{h_3}_{ij}(k_3)\left[\left(-\frac{1}{4}k_1^3+\frac{1}{2}k_1(k_2^2+k_3^2)+4\frac{k_2^2k_3^2}{k_T}\right)\right.$$
$$\left.+\sinh^2 2\alpha\left(-\frac{1}{4}k_1^3+\frac{1}{2}k_1(k_2^2+k_3^2)+4\frac{k_2^2k_3^2}{(k_2+k_3-k_1)}+4\frac{k_2^2k_3^2}{(k_3+k_1-k_2)}+4\frac{k_2^2k_3^2}{(k_1+k_2-k_3)}\right)\right] \tag{3.61}$$

To take an example of a parity-odd interaction, we can take the Weyl action and calculate the un-trimmed wavefunction coefficient ψ_3 (up to some numerical factors):

$$S_3 = \int d\eta d^3x a^{-5} \left(\partial_\eta \Pi^+_{ij} \partial_\eta \Pi^+_{jk} \partial_\eta \Pi^+_{ki} - \partial_\eta \Pi^-_{ij} \partial_\eta \Pi^-_{jk} \partial_\eta \Pi^-_{ki}\right) \tag{3.62}$$

$$\text{where} \quad \partial_\eta \Pi^\pm_{ij} = \frac{1}{2}\left(\partial_\eta(a\partial_\eta \gamma_{ij}) \mp i\epsilon_{jab}\partial_b \partial_\eta \gamma_{ia}\right) \tag{3.63}$$

$$\psi_3 \sim \frac{k_1^2 k_2^2 k_3^2}{k_T^6}\left(\epsilon_{iab}\epsilon_{jcd}\epsilon_{kfg} e^{h_1}_{jb} e^{h_2}_{kd} e^{h_3}_{ig} k_{1a} k_{2c} k_{3f} - \sum_{cyclic} k_2 k_3 \epsilon_{jab} k_{1b} e^{h_1}_{ia} e^{h_2}_{ki} e^{h_3}_{kj}\right) \tag{3.64}$$

We can simplify the last equation using the relation: $\epsilon_{iab} k_b e^h_{ja} = -ikhe_{ij}$. However, we must keep in mind that while using the ansatz Eq. (3.57) we have to take the trimmed part as the one before we use the relation above to simplify Eq. (3.64), i.e we consider the unsimiplified levi-cevita contractions to be the tensor contractions. Hence we don't flip the signs of k's generated from this contraction while using the ansatz. We also note that Eq. (3.64) has multiple tensor contractions and for each contraction, the trimmed part obeys Eq. (3.57), so we can just add up the answers. This finally gives the following correlators for arbitrary helicities $h_i = \pm 1$:

$$\langle \gamma^{h_1}(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\gamma^{h_3}(\vec{k}_3)\rangle_\alpha \sim \sinh 4\alpha e^{h_1}_{ij} e^{h_2}_{jk} e^{h_3}_{ki}\left[\frac{3}{k_T^6}(h_1 + h_2 + h_3 + h_1 h_2 h_3) - \sum_{cyclic} \frac{1}{(-k_1 + k_2 + k_3)^6}(h_1 - h_2 - h_3 + h_1 h_2 h_3)\right] \tag{3.65}$$

$$\implies \langle \gamma^-(\vec{k}_1)\gamma^+(\vec{k}_2)\gamma^+(\vec{k}_3)\rangle_\alpha \sim \sinh 4\alpha e^{h_1}_{ij} e^{h_2}_{jk} e^{h_3}_{ki} \frac{1}{(k_2 + k_3 - k_1)^6} = - \langle \gamma^+(\vec{k}_1)\gamma^-(\vec{k}_2)\gamma^-(\vec{k}_3)\rangle_\alpha \tag{3.66}$$

Again, these results match with the explicit calculations done in [130]. While all the calculations above were done for cases where the Bogoliubov coefficients α, β

are k-independent, one can easily generalise Eq. (3.57) to cases where they are k-dependent. In that case, one would have to flip the signs of k's for α, β like we did for ψ in Eq. (3.57).

We see that the prescription saves us the effort of doing the cumbersome in-in calculation which involves simplifying a lot of integrals. Note that, our prescription is for *the inflationary correlations functions* and works for general Bogoliubov states (thus, goes beyond the pure de-Sitter/CFT results in α vacua [131]).

Chapter 4

Loops in inflation

The material presented in this chapter is based on the author's publication [132].

4.1 Introduction

Loop corrections to cosmological correlation functions were first studied by Weinberg [25], where he computed the 1-loop correction to the 2-point correlation function. However, it was pointed out in [133] that there were mistakes in the computation, since the loop correction computed in [25] and subsequent related studies [134, 135, 136, 137, 138] have a $\log(k/\mu)$ dependence. Such a term is unphysical because it is the ratio of the co-moving momentum k and the physical renormalization scale μ. In [133] it was shown that once the mode functions are properly continued to d dimensions in dimensional regularization, the unphysical logarithms cancel and the log dependence instead is $\log(H/\mu)$. Loop effects were studied in further detail in [139, 140].

It is crucial to go beyond the 1-loop 2-point function to capture features of interactions in the shape of the correlation function. Moreover, the analytic structure of higher point cosmological correlators at the loop level is mostly unexplored. It therefore demands a thorough theoretical investigation. In this chapter, we study

1-loop corrections to the scalar 3-point function in the decoupling limit of the Effective Field Theory of Inflation (EFToI). Since our main objective is to explore the analytic structure of the correlation functions, we perform our calculations for a tuned scenario (see Sec. 4.2) reducing the number of possible Feynmann diagrams. We find a few interesting features in our final results:

- The correlation function is not a rational function unlike the parity odd correlators computed in [51].
- Unlike [133], the unphysical logs do not cancel despite using correct d dimensional mode functions. They are removed only after renormalizing the results.
- Our final renormalized result for the correlation function contains logarithmic structures of the form $\log \frac{k_i}{k_T}$ and $\log \frac{H}{\mu}$, where $k_T = k_1 + k_2 + k_3$.

The rest of the chapter is organised as follows. In Sec. 4.2 we briefly revisit the decoupling limit of EFToI and explicitly write down the tuned action we use for computations. In Sec. 4.3 we give details of the loop computations. In Sec. 4.4 we renormalise our answers and show that unphysical logarithms cancel. In Sec. 4.5 we perform a cross check where we compute one of our diagrams in cutoff regularisation.

4.2 Decoupling limit of EFToI

The EFToI [141, 142, 143] lagrangian is constructed in the unitary gauge by fixing time diffeomorphisms, and therefore no longer invariant under time diffs. One can restore this invariance by introducing an additional field $\pi(x)$ in the lagrangian using the so-called Stueckelberg trick. The action for $\pi(x)$ is given by,

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Big[&\frac{1}{2}M_{pl}^2 R - M_{pl}^2\left(3H^2(t+\pi) + \dot{H}(t+\pi)\right) + \\
&M_{pl}^2\dot{H}(t+\pi)\left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi + 1\right) + \\
&\frac{M_2(t+\pi)^4}{2!}\left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi + 1\right)^2 + \\
&\frac{M_3(t+\pi)^4}{3!}\left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi + 1\right)^3 + \\
&\frac{M_4(t+\pi)^4}{4!}\left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi + 1\right)^4 + ...\Big]\,.
\end{aligned}
\tag{4.1}
$$

This action is invariant under the transformation, $t \to t + \epsilon(x)$ & $\pi \to \pi(x) - \epsilon(x)$. One can define a limit of the above action where the energy scale associated with the coupling between metric and scalar fluctuations, $E_{\rm mix}$ is much smaller than the energy scale of the processes under consideration, i.e. $E \gg E_{\rm mix}$. This is known as the decoupling limit of EFToI. If the observables we wish to compute are not dominated by mixing with gravity, the action in the decoupling limit will give the correct result. This is very useful because the above action in the decoupling limit for the scalar sector simplifies to,

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Big[&\frac{1}{2}M_{pl}^2 R - M_{pl}^2\left(3H^2(t+\pi) + \dot{H}(t+\pi)\right) + \\
&M_{pl}^2\dot{H}(t+\pi)\left(-(1+\dot{\pi})^2 + \frac{1}{a^2}(\partial_i\pi)^2 + 1\right) + \\
&\frac{1}{2!}M_2(t+\pi)^4\left(-(1+\dot{\pi})^2 + \frac{1}{a^2}(\partial_i\pi)^2 + 1\right)^2 + \\
&\frac{1}{3!}M_3(t+\pi)^4\left(-(1+\dot{\pi})^2 + \frac{1}{a^2}(\partial_i\pi)^2 + 1\right)^3 + \\
&\frac{1}{4!}M_4(t+\pi)^4\left(-(1+\dot{\pi})^2 + \frac{1}{a^2}(\partial_i\pi)^2 + 1\right)^4 + ...\Big]\,.
\end{aligned}
\tag{4.2}
$$

We assume that the time dependence of EFT coefficients is slow-roll suppressed and therefore can be neglected in the first approximation. Since the goal of this chapter is to explore the general analytic structure for the loop-level bispectrum, we consider a scenario where the coefficients of the EFT are tuned, such that only one cubic and two quartic interactions are important. Then we have the following action for π,

$$S = \int d^4x\sqrt{-g}\Big[- M_{pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial\pi)^2}{a^2}\right) + g_3\dot{\pi}^3 + g_4\dot{\pi}^4 - \frac{3g_3}{2}\dot{\pi}^2\left(\frac{\partial\pi}{a}\right)^2\Big] , \quad (4.3)$$

where we have set $M_2 = 0$ and

$$g_3 = -\frac{4}{3}M_3^4 , \quad (4.4)$$

$$g_4 = -2M_3^4 + \frac{2}{3}M_4^4 . \quad (4.5)$$

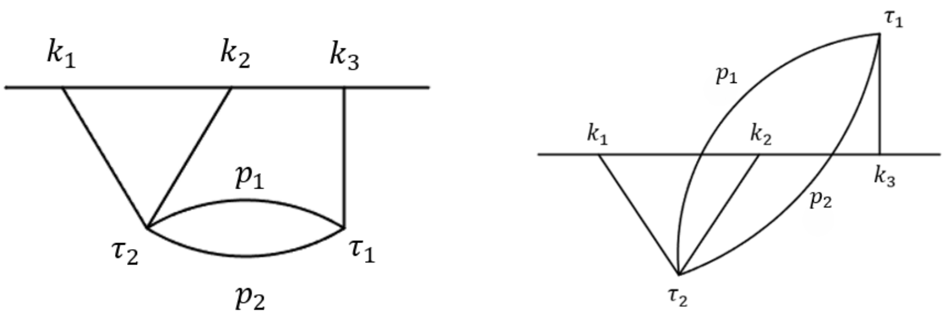
4.3 Explicit computation of Bispectrum at 1-loop

In this section, we compute the 1-loop contribution to the 3-point correlator coming from the leading cubic and quartic operators in the EFToI, Eq. (4.3). The computations are done using the in-in formalism (reviewed in Sec. 1.3.2),

$$\langle \pi^3(0)\rangle = \frac{\langle 0|\bar{T}e^{i\int_{-\infty(1+i\epsilon)}^{0} H_I d\tau}\ \pi^3(0)\ Te^{-i\int_{-\infty(1-i\epsilon)}^{0} H_I d\tau}|0\rangle}{\langle 0|\bar{T}e^{i\int_{-\infty(1+i\epsilon)}^{0} H_I d\tau}\ Te^{-i\int_{-\infty(1-i\epsilon)}^{0} H_I d\tau}|0\rangle} , \quad (4.6)$$

where the correlation functions are computed at the future boundary of de Sitter, i.e. $\tau \to 0$. Here $(\bar{T})$ T denotes (anti-)time ordering. The time integrals are performed using the usual continuation $\tau \to \tau(1 \pm i\epsilon)$, which rotates the time contour and projects the free vacuum on the interacting vacuum.

The 1-loop corrections can arise from one insertion of a cubic and a quartic operator each, given by Fig. 4.1, which we refer to as Diagram I. In Sec. 4.3.2, we provide details of their computation. All other contributions from a cubic and a quartic interaction are different time orderings of Fig. 4.1, obtained by complex conjugation of the same. At the same order in the EFT expansion, one can also have contributions from diagrams with three insertions of the cubic operator. Some of these diagrams are given in Fig. 4.2, which we refer to as Diagram II. We calculate Fig. 4.2a in Sec. 4.3.3 and show that the analytic structure is similar to Diagram I, leaving an explicit computation of all other diagrams for the future. Finally, there are contributions from diagrams where the loop closes on itself, including tadpole



(A) (B)

FIGURE 4.1: Diagram I: Loop contribution to the bispectrum from one insertion of the cubic and a quartic operator. k_1, k_2, and k_3 denote external momenta. p_1 and p_2 denote loop momenta. τ_1 and τ_2 are points in the bulk. Time-ordered diagram is shown in the left panel and the diagram with no specific time ordering is drawn in the right panel.

diagrams, resulting in a scaleless integral. Such diagrams are given in Fig. 4.3. These contributions vanish in dim-reg, as discussed in Sec. 4.3.4.

4.3.1 Calculational scheme

Since we will compute many diagrams using the same method, it is useful to first give the general calculational outline and then proceed to compute explicitly. To compute any diagram $D(k_1, k_2, k_3)$, we use the well-known Feynman rules [144]. When working in $d = 3 + \delta$ dimensions, the mode function and the measure will have δ dependence, given by

$$f_k(\tau) = (-H\tau)^{\delta/2} \frac{H}{\sqrt{-\dot{H}M_{pl}^2}} \frac{1}{\sqrt{2k^3}} (1 - ik\tau)\, e^{ik\tau} = \left(1 + \frac{\delta}{2}\log(-H\tau)\right) f_k^{3d}(\tau) + \mathcal{O}(\delta^2)\,,$$
$$a(\tau)^\delta = 1 - \delta \log(-H\tau) + \mathcal{O}(\delta^2)\,. \tag{4.7}$$

In our computations, the external mode functions (at $\tau = 0$) and the external momenta conserving delta function ($\delta^d(\sum_i \vec{k}_i)$) are taken to be in 3 dimensions, since the counter terms cancel the $O(\delta)$ contribution from these terms. Our integrals have

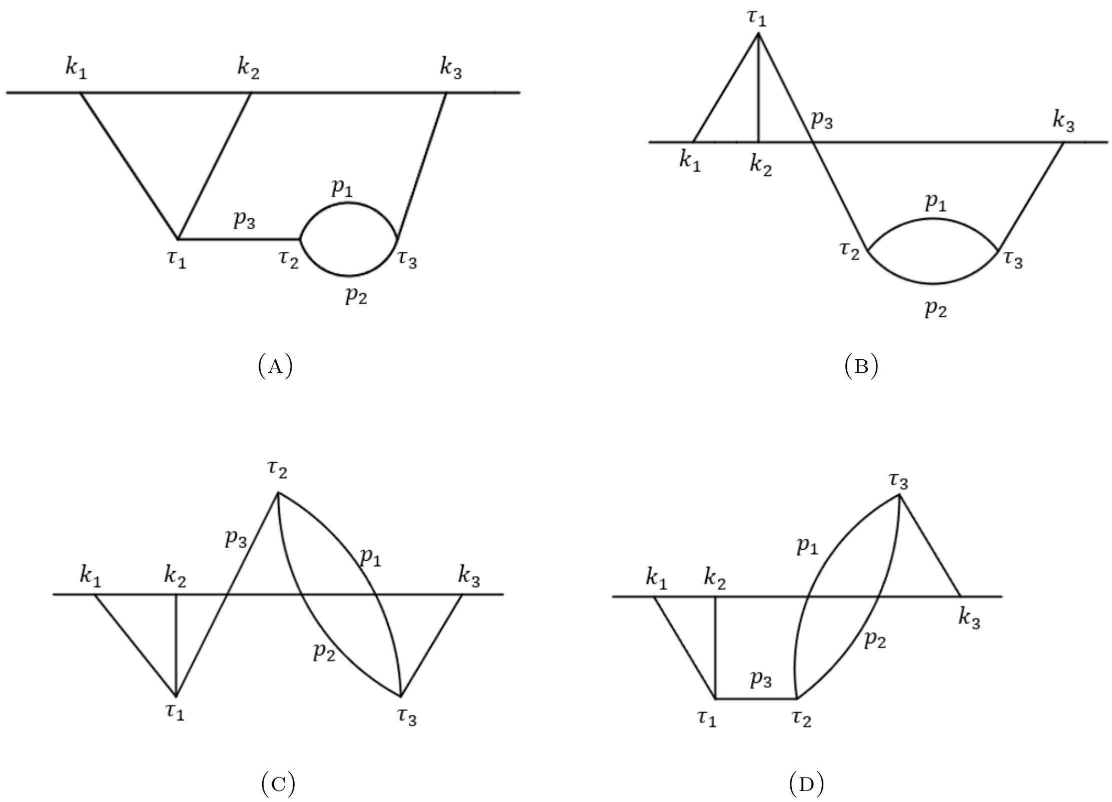


FIGURE 4.2: Diagram II : Loop contribution to the bispectrum from three insertions of the cubic operator. k_1 , k_2 and k_3 denote external momenta. p_1 , p_2 are loop momenta and $p_3 = k_3$ due to momentum conservation. τ_1 , τ_2 and τ_3 are points in the bulk.

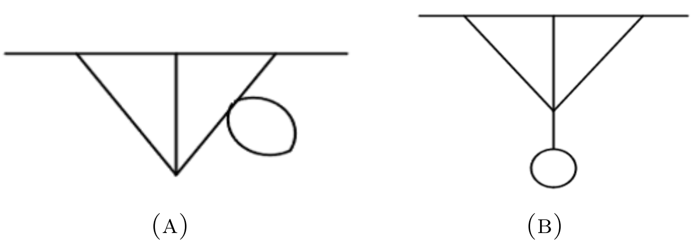


FIGURE 4.3: Diagrams with scaleless loops (left panel) and tadpoles (right panel). Contributions from scaleless loops vanish in dimensional regularization. Since all the interactions have derivatives, the tadpole diagram integrand either vanishes, by virtue of the tadpole leg $k = 0$, or is finite and its contribution vanishes from scaleless loop arguments (see Sec. 4.3.4)

the following form,

$$D(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \int d^d\vec{p}_1 d^d\vec{p}_2\ P(p_1,p_2,\{k_i\},\{\tau_i\})\delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3) \times \prod_i \left[(-H\tau_i)^{(n_i)\delta} d\tau_i\right],$$
$$= \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \int d^d\vec{p}_1 d^d\vec{p}_2 P(p_1,p_2,\{k_i\},\{\tau_i\})\delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3) \times \left[1+\sum_i n_i\delta\log(-H\tau_i)\right]\prod_i d\tau_i\,, \quad (4.8)$$

where μ is the renormalisation scale. In passing we note that from Eq. (4.7) we get $n_i = -1$ from each scale factor, $a(\tau_i)^\delta$, and $n_i = \frac{1}{2}$ from each mode function. Now, we can write $D = D_1 + D_2$, with,

$$D_1(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \int d^d\vec{p}_1 d^d\vec{p}_2 P(p_1,p_2,\{k_i\},\{\tau_i\})\delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3)\prod_i d\tau_i\,,$$
$$D_2(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \int d^d\vec{p}_1 d^d\vec{p}_2 P(p_1,p_2,\{k_i\},\{\tau_i\})\sum_i n_i\delta\log(-H\tau_i) \times \delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3)\prod_i d\tau_i\,, \quad (4.9)$$

where D_1 is the computation of the correlator with 3-dimensional mode functions and scale factors, and D_2 includes the $\mathcal{O}(\delta)$ contribution from them. For D_1, computation of the time integrals gives,

$$D_1(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \int d^d\vec{p}_1 d^d\vec{p}_2 Q(p_1,p_2,\{k_i\})\delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3)\,. \quad (4.10)$$

To compute momentum integrals we use the identity mentioned in [23] (See Appendix F) and get,

$$D_1(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \frac{S_{d-2}}{2}\int_{k_3}^{\infty} dp_+ \int_{-k_3}^{k_3} dp_- \frac{p_1^{d-2}p_2}{k_3} Q(p_1,p_2,\{k_i\})\,. \quad (4.11)$$

To simplify the computations further, we turn the integral dimensionless by factoring out powers of one of the momenta (say k_m), and work with ratios of momenta

$$D_1(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \frac{S_{d-2}\ k_m^{\delta+N}}{2} \int_1^\infty dq_+ \int_{-1}^1 dq_-\ {q_1}^{d-2}\ q_2\ Q(q_1,q_2,\{x_i\}), \tag{4.12}$$

where $q_i = \frac{p_i}{k_3}$ and $x_i = \frac{k_i}{k_m}$. Computation of the integrals gives the following,

$$D_1(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \mu^{-\frac{3\delta}{2}} \frac{S_{d-2}\ k_m^{\delta+N}}{2} R(d,\{x_i\})\,. \tag{4.13}$$

Finally, we expand everything close to $d = 3$ by substituting $d = 3 + \delta$ and series expanding near $\delta = 0$. The function $R(3+\delta,\{x_i\})$ may or may not have a divergence in the limit $\delta \to 0$. If there is no divergence then we can simply take the limit $\delta \to 0$ everywhere and get,

$$D_1(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \frac{S_{d-2}\ k_m^{N}}{2} R(3,\{x_i\})\,. \tag{4.14}$$

In the case of a divergence, the function $R(d,\{x_i\})$ will have the form

$$R(3+\delta,\{x_i\}) = \frac{F_0(\{x_i\})}{\delta} + F_1(\{x_i\}) + \mathcal{O}(\delta)\,,$$

where F_0 is the coefficient of the divergent term, and F_1 is a function which can have logarithmic dependence on x_i. Therefore, we have

$$D_1(k_1,k_2,k_3) = \delta^3\big(\sum_i \vec{k}_i\big)\ \frac{S_{d-2}\ k_m^{N}}{2} \left\{ \frac{F_0(\{x_i\})}{\delta} + F_0(\{x_i\}) \left(\log k_m - \frac{3}{2}\log\mu\right) + F_1(\{x_i\}) \right\}\,. \tag{4.15}$$

For the computation of D_2, the resulting contribution is equivalent to the logs being pulled outside the integrals as $\log(H/k_T)$, plus some rational functions of momenta

(See Appendix H). Therefore, D_2 becomes,

$$\begin{aligned} D_2(k_1,k_2,k_3) &= \delta\left[\int d^d\vec{p}_1 d^d\vec{p}_2 P(p_1,p_2,\{k_i\},\{\tau_i\})\delta^{(d)}(\vec{p}_1+\vec{p}_2-\vec{k}_3)\prod_i d\tau_i\right] \\ &\qquad \times \sum_i n_i \log(H/k_T)\delta^3\big(\sum_i \vec{k}_i\big) \;, \\ &= \delta\; D_1 \sum_i n_i \log(H/k_T)\,. \end{aligned} \tag{4.16}$$

Combining D_1 and D_2 gives,

$$D = D_1\left(1+\delta\sum_i n_i \log(H/k_T)\right)\,. \tag{4.17}$$

Therefore, the final expression is given by,

$$\boxed{\begin{aligned} D = \frac{S_{d-2}\, k_m^N}{2}&\left[\frac{F_0(\{x_i\})}{\delta} + F_0(\{x_i\})\left(\log k_m - \frac{3}{2}\log\mu + \sum_i n_i \log(H/k_T)\right)\right. \\ &\left. + F_1(\{x_i\})\right]\; \delta^3\big(\sum_i \vec{k}_i\big)\,. \end{aligned}} \tag{4.18}$$

If D_1 does not have a divergence then $D = D_1$.

For the loop correction to the power spectrum in [133], $\sum_i n_i = 1$. It is clear from the equation above, that unless the number of internal mode functions (i.e. $f_k(\tau)$ at $\tau \neq 0$) and scale factors is such that $\sum_i n_i = 1$, the unphysical logs in D_1 will not cancel with contributions from D_2.

4.3.2 Loop contribution from Diagram I

Here, we follow the procedure outlined in Sec. 4.3.1 to compute the loop contributions coming from diagrams with one quartic and one cubic vertex (Fig. 4.1). We consider the diagram where k_3 contracts with the cubic vertex τ_1. There will be 2 more contributions coming from contractions of k_1 and k_2 with the cubic vertex, obtained by substituting $k_3 \longleftrightarrow k_1$ and $k_3 \longleftrightarrow k_2$ respectively in the final expressions.

We first consider the time-ordered diagram Fig. 4.1a, with the quartic interaction having only time derivatives, i.e. the $\dot{\pi}^4$ vertex. Using Feynman rules,

$$D = -72\ g_3 g_4\ \delta^3\big(\sum_i \vec{k}_i\big) f^*_{k_1}(0) f^*_{k_2}(0) f^*_{k_3}(0) \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} f'_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'_{k_3}(\tau_1)$$
$$a^{d-2}(\tau_1) a^{d-3}(\tau_2) \int d^d\vec{p}_1 d^d\vec{p}_2 \{ f'_{p_1}(\tau_2) f'^*_{p_1}(\tau_1) f'_{p_2}(\tau_2) f'^*_{p_2}(\tau_1) \Theta(\tau_1 - \tau_2)$$
$$+ f'_{p_1}(\tau_1) f'^*_{p_1}(\tau_2) f'_{p_2}(\tau_1) f'^*_{p_2}(\tau_2) \Theta(\tau_2 - \tau_1)\}\ \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,, \tag{4.19}$$

As outlined in Sec. 4.3.1, we work with 3d mode functions and ignore δ dependence from scale factors to compute $D_1(k_1, k_2, k_3)$. We obtain,

$$D_1 = 72\ g_3 g_4\ \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{k_3(k_1+k_2)(k_3 - 2k_1 - 2k_2)}{k_1 k_2 k_T^7} \big(\frac{1}{\delta} + \frac{1}{2}\log\left(k_T k_3\right)$$
$$- \frac{3}{2}\log\,\mu\big) +\ \mathrm{NLf}\,, \tag{4.20}$$

where NLf (Non-logarithmic functions) are simple rational functions of external momenta. Next, we compute D_2. As mentioned in Section. 4.3.1 (explicit check performed in Appendix G), D_2 can be evaluated by bringing the log term out of the time integrals as $\log(H/k_T)$, along with some non-logarithmic functions (NLf) of k_i. We note that there are 3 internal mode functions and 1 scale factor at τ_1, 4 internal mode functions and 1 scale factor at τ_2. Hence $D_2(k_1, k_2, k_3)$ is given by

$$D_2 = \delta \left\{ \left(\frac{3}{2} - 1\right) + \left(\frac{4}{2} - 1\right) \right\} \log\left(\frac{H}{k_T}\right) D_1 +\ \mathrm{NLf}\,.$$

Adding this $\mathcal{O}(\delta)$ correction to D_1, alongwith contribution from diagrams where k_1 and k_2 contract with the cubic vertex, gives :

$$\boxed{\begin{aligned} D = 72\ g_3 g_4\ \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{k_3(k_1+k_2)(k_3 - 2k_1 - 2k_2)}{k_1 k_2 k_T^7} \{ \frac{1}{\delta} + \frac{1}{2}\log\frac{k_3}{k_T} \\ - \frac{1}{2}\log\,k_T + \frac{3}{2}\log\frac{H}{\mu} \} + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} +\ \mathrm{NLf}\,. \qquad (4.21) \end{aligned}}$$

Next, we compute Fig. 4.1b for quartic interactions with temporal derivatives only. Using Feynman rules we get,

$$D = -72\ g_3 g_4\ \delta^3\Big(\sum_i \vec{k}_i\Big) f^*_{k_1}(0) f^*_{k_2}(0) f_{k_3}(0) \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} f'_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'^*_{k_3}(\tau_1)$$
$$a^{d-2}(\tau_1) a^{d-3}(\tau_2) \int d^d\vec{p}_1 d^d\vec{p}_2\ f'_{p_1}(\tau_2) f'^*_{p_1}(\tau_1) f'_{p_2}(\tau_2) f'^*_{p_2}(\tau_1)\ \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,. \quad (4.22)$$

The evaluated answer does not produce any divergences or unphysical logarithms and is given by,

$$\boxed{D = D_1 = 72\ g_3 g_4\ \delta^3\Big(\sum_i \vec{k}_i\Big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{k_3(k_1+k_2)\,(k_T+k_1+k_2)}{2k_1k_2\,(k_3-k_1-k_2)^{\,7}} \log\left(\frac{k_T}{2k_3}\right) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{ NLf}\,. \quad (4.23)}$$

The apparent folded pole in the above expression actually goes away once we series expand close to the folded limit i.e. $k_3 \to k_1 + k_2$ with NLf given by,

$$\text{NLf} = 72\ g_3 g_4\ \delta^3\Big(\sum_i \vec{k}_i\Big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{1}{8k_1k_2k_3} \frac{1}{120(k_3-k_{12})^7 k_T^4}\Big(- 9k_3^8 + 388k_{12}k_3^7$$
$$+ 1380k_{12}^2k_3^6 + 2180k_{12}^3k_3^5 - 190k_{12}^4k_3^4 - 2196k_{12}^5k_3^3 - 1388k_{12}^6k_3^2 - 180k_{12}^7k_3 + 15k_{12}^8$$
$$- 480k_{12}k_3^7\log(2) - 2880k_{12}^2k_3^6\log(2) - 6720k_{12}^3k_3^5\log(2) - 7680k_{12}^4k_3^4\log(2)$$
$$- 4320k_{12}^5k_3^3\log(2) - 960k_{12}^6k_3^2\log(2)\Big) + \ldots \quad (4.24)$$

where the dots represent finite terms in NLf that do not have folded singularity.

Next, we take up quartic interactions with spatial derivatives, i.e. $\dot{\pi}^2(\partial\pi)^2$, and compute the time-ordered loop contribution (Fig. 4.1a). When both spatial derivatives act on k_i modes (i.e. f_{k_i}), the loop contribution is given by

$$D = 18\ g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) f^*_{k_1}(0) f^*_{k_2}(0) f^*_{k_3}(0) \{\vec{k}_1 \cdot \vec{k}_2\} \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2)$$
$$f_{k_1}(\tau_2) f_{k_2}(\tau_2) f'_{k_3}(\tau_1) \int d^d\vec{p}_1 d^d\vec{p}_2\ \{f'_{p_1}(\tau_2) f'^*_{p_1}(\tau_1) f'_{p_2}(\tau_2) f'^*_{p_2}(\tau_1) \Theta(\tau_1 - \tau_2)$$
$$+ f'_{p_1}(\tau_1) f'^*_{p_1}(\tau_2) f'_{p_2}(\tau_1) f'^*_{p_2}(\tau_2) \Theta(\tau_2 - \tau_1)\} \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,, \tag{4.25}$$

where $\vec{k}_1 \cdot \vec{k}_2 = -\vec{k}_1 \cdot \left(\vec{k}_1 + \vec{k}_3\right) = \frac{k_3^2 - k_1^2 - k_2^2}{2}$.

The spatial derivative can act on a loop mode (i.e. f_{p_j}) and on a k_i mode. The contribution will be given by,

$$D = 72\ g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) f^*_{k_1}(0) f^*_{k_2}(0) f^*_{k_3}(0) \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2)$$
$$\Big\{ f_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'_{k_3}(\tau_1) \vec{k}_1 + f_{k_2}(\tau_2) f'_{k_1}(\tau_2) f'_{k_3}(\tau_1) \vec{k}_2 \Big\} \cdot \int d^d\ \vec{p}_1 d^d\vec{p}_2\ \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)$$
$$\{\ f'^*_{p_1}(\tau_1) f'^*_{p_2}(\tau_1) \Theta(\tau_1 - \tau_2) \left(f'_{p_1}(\tau_2) f_{p_2}(\tau_2)\ \vec{p}_2 + f_{p_1}(\tau_2) f'_{p_2}(\tau_2)\ \vec{p}_1\right)$$
$$+ f'_{p_1}(\tau_1) f'_{p_2}(\tau_1) \Theta(\tau_2 - \tau_1) \left(f'^*_{p_1}(\tau_2) f^*_{p_2}(\tau_2)\ \vec{p}_2 + f^*_{p_1}(\tau_2) f'^*_{p_2}(\tau_2)\ \vec{p}_1\right)\ \}\,, \tag{4.26}$$

with

$$\vec{k}_1 \cdot \vec{p}_1 = k_1 p_1 \left[\cos\theta_1 \cos\theta_k + \sin\theta_1 \sin\theta_k \cos\theta_2\right]\,, \tag{4.27}$$

where $\cos\theta_k$ is angle between k_1 and k_3, given by $\cos\theta_k = \frac{k_2^2 - k_1^2 - k_3^2}{2k_1 k_3}$. θ_1, θ_2 are polar angles of p_1, with $\cos\theta_1 = \frac{k_3^2 + p_1^2 - p_2^2}{2k_3 p_1}$. However we only need to keep the first term in Eq. (4.27) when evaluating the momentum integrals using the identity in Appendix F, since the integrand is odd over $\theta_2 \in (0, \pi)$ due to the $\cos\theta_2$ term. Other dot products can be converted to $\vec{k}_1 \cdot \vec{p}_1$ using the momentum conserving delta functions.

Finally, the contribution when both spatial derivatives act on loop mode functions is given by,

$$D = 18\ g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2)$$
$$f'_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'_{k_3}(\tau_1) \int d^d\ \vec{p}_1 d^d \vec{p}_2\ \{f_{p_1}(\tau_2) f'^*_{p_1}(\tau_1) f_{p_2}(\tau_2) f'^*_{p_2}(\tau_1) \Theta(\tau_1 - \tau_2)$$
$$+ f'_{p_1}(\tau_1) f^*_{p_1}(\tau_2) f'_{p_2}(\tau_1) f^*_{p_2}(\tau_2) \Theta(\tau_2 - \tau_1)\}\ \{\vec{p}_1 \cdot \vec{p}_2\}\ \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,, \quad (4.28)$$

with $\vec{p}_1 \cdot \vec{p}_2 = \vec{p}_1 \cdot \left(\vec{k}_3 - \vec{p}_1\right) = p_1 k_3 \cos\theta_1 - p_1^2$.

The $\mathcal{O}(\delta)$ contribution from the mode functions to this computation is given by $D_2 = \frac{3\delta}{2} \log\left(\frac{H}{k_T}\right) D_1$, obtained from the 3 internal mode functions and 1 scale factor at τ_1, 4 mode functions and 1 scale factor at τ_2 (Appendix H). Computing the integrals above and adding them with D_2 gives the following contribution to the 1-loop correction with quartic interactions having spatial derivatives,

$$D = -\frac{3}{40} g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\big(-\dot{H}M_{pl}^2\big)^5} \frac{k_3}{k_1^3 k_2^3} \frac{\vec{k}_1\cdot\vec{k}_2}{k_T^7} \big(40k_1^4 + 280k_2k_1^3 + 55k_3k_1^3 + 480k_2^2k_1^2$$
$$+ 21k_3^2k_1^2 + 105k_2k_3k_1^2 + 280k_2^3k_1 + 7k_3^3k_1 + 42k_2k_3^2k_1 + 105k_2^2k_3k_1 + 40k_2^4$$
$$+ k_3^4 + 7k_2k_3^3 + 21k_2^2k_3^2 + 55k_2^3k_3\big) \left(\frac{1}{\delta} + \frac{1}{2}\log\left(\frac{k_3}{k_T}\right) - \frac{1}{2}\log k_T + \frac{3}{2}\log\left(\frac{H}{\mu}\right)\right)$$
$$- \frac{3}{40} g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\big(-\dot{H}M_{pl}^2\big)^5} \frac{1}{k_1^3k_2^3k_3} \frac{1}{k_T^7} \big(20k_1^8 + 140k_2k_1^7 + 140k_3k_1^7 + 200k_2^2k_1^6$$
$$- 152k_3^2k_1^6 + 840k_2k_3k_1^6 - 140k_2^3k_1^5 - 264k_3^3k_1^5 - 1904k_2k_3^2k_1^5 + 560k_2^2k_3k_1^5$$
$$- 440k_2^4k_1^4 + 260k_3^4k_1^4 + 56k_2k_3^3k_1^4 - 248k_2^2k_3^2k_1^4 - 1540k_2^3k_3k_1^4 - 140k_2^5k_1^3$$
$$+ 124k_3^5k_1^3 + 1764k_2k_3^4k_1^3 - 2592k_2^2k_3^3k_1^3 + 3008k_2^3k_3^2k_1^3 - 1540k_2^4k_3k_1^3 + 200k_2^6k_1^2$$
$$- 128k_3^6k_1^2 - 896k_2k_3^5k_1^2 + 3008k_2^2k_3^4k_1^2 - 2592k_2^3k_3^3k_1^2 - 248k_2^4k_3^2k_1^2 + 560k_2^5k_3k_1^2$$
$$+ 140k_2^7k_1 - 896k_2^2k_3^5k_1 + 1764k_2^3k_3^4k_1 + 56k_2^4k_3^3k_1 - 1904k_2^5k_3^2k_1 + 840k_2^6k_3k_1 + 20k_2^8$$
$$- 128k_2^2k_3^6 + 124k_2^3k_3^5 + 260k_2^4k_3^4 - 264k_2^5k_3^3 - 152k_2^6k_3^2 + 140k_2^7k_3\big)$$
$$\left(\frac{1}{\delta} + \frac{1}{2}\log\left(\frac{k_3}{k_T}\right) - \frac{1}{2}\log k_T + \frac{3}{2}\log\left(\frac{H}{\mu}\right)\right) + \frac{9}{4} g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\big(-\dot{H}M_{pl}^2\big)^5}$$
$$\frac{1}{k_1k_2k_3k_T^7} \big(k_1^4 + 4k_2k_1^3 + 7k_3k_1^3 + 6k_2^2k_1^2 + 7k_3^2k_1^2 + 21k_2k_3k_1^2 + 4k_2^3k_1 - 23k_3^3k_1$$
$$+ 14k_2k_3^2k_1 + 21k_2^2k_3k_1 + k_2^4 - 48k_3^4 - 23k_2k_3^3 + 7k_2^2k_3^2 + 7k_2^3k_3\big)$$
$$\left(\frac{1}{\delta} + \frac{1}{2}\log\left(\frac{k_3}{k_T}\right) - \frac{1}{2}\log k_T + \frac{3}{2}\log\left(\frac{H}{\mu}\right)\right) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \ \text{NLf}\,. \tag{4.29}$$

Lastly, we compute the unordered diagram Fig 4.1b with quartic interaction having spatial derivatives. The correlator when both spatial derivatives act on k mode functions, is given by,

$$D = 18\ g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}(0) \{\vec{k}_1\cdot\vec{k}_2\} \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2)$$
$$f_{k_1}(\tau_2) f_{k_2}(\tau_2) f'^*_{k_3}(\tau_1) \int d^d\vec{p}_1 d^d\vec{p}_2\ \ f'_{p_1}(\tau_2) f'^*_{p_1}(\tau_1) f'_{p_2}(\tau_2) f'^*_{p_2}(\tau_1) \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,. \tag{4.30}$$

The correlator when the two spatial derivatives act on a k mode and a loop mode function, is given by

$$D = 72\ g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) f^*_{k_1}(0) f^*_{k_2}(0) f_{k_3}(0) \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2)$$
$$\Big\{ f_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'^*_{k_3}(\tau_1) \vec{k}_1 + f_{k_2}(\tau_2) f'_{k_1}(\tau_2) f'^*_{k_3}(\tau_1) \vec{k}_2 \Big\} \cdot \int d^d\ \vec{p}_1 d^d \vec{p}_2$$
$$f'^*_{p_1}(\tau_1) f'^*_{p_2}(\tau_1) \left(f'_{p_1}(\tau_2) f_{p_2}(\tau_2)\ \vec{p}_2 + f_{p_1}(\tau_2) f'_{p_2}(\tau_2)\ \vec{p}_1 \right)\ \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,. \tag{4.31}$$

When both spatial derivatives act on loop mode functions, we have

$$D = 18\ g_3^2\ \delta^3\big(\sum_i \vec{k}_i\big) f^*_{k_1}(0) f^*_{k_2}(0) f_{k_3}(0) \int_{-\infty}^{0} d\tau_1 d\tau_2\ \ \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2) f'_{k_1}(\tau_2) f'_{k_2}(\tau_2)$$
$$f'^*_{k_3}(\tau_1) \int d^d\ \vec{p}_1 d^d \vec{p}_2\ f_{p_1}(\tau_2) f'^*_{p_1}(\tau_1) f_{p_2}(\tau_2) f'^*_{p_2}(\tau_1)\ \ \{\vec{p}_1 \cdot \vec{p}_2\}\ \ \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,. \tag{4.32}$$

Once again the contribution from this unordered diagram does not produce any divergences or unphysical logarithms, and is given by,

$$
\begin{aligned}
D = D_1 = &\frac{3}{20} g_3^2\, \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{\{\vec{k}_1 \cdot \vec{k}_2\}}{k_1^3 k_2^3} \frac{k_3}{(k_1+k_2-k_3)^7} \log\left(\frac{2k_3}{k_T}\right) (40k_1^4 \\
&+ 280k_2k_1^3 - 55k_3k_1^3 + 480k_2^2k_1^2 + 21k_3^2k_1^2 - 105k_2k_3k_1^2 + 280k_2^3k_1 - 7k_3^3k_1 + 42k_2k_3^2k_1 \\
&- 105k_2^2k_3k_1 + 40k_2^4 + k_3^4 - 7k_2k_3^3 + 21k_2^2k_3^2 - 55k_2^3k_3) \\
&+ \frac{3}{80} g_3^2\, \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{1}{k_1^3 k_2^3 k_3} \frac{1}{(k_1+k_2-k_3)^7} \log\left(\frac{2k_3}{k_T}\right) (20k_1^8 + 140k_2k_1^7 \\
&+ 200k_2^2k_1^6 - 152k_3^2k_1^6 - 840k_2k_3k_1^6 - 140k_2^3k_1^5 + 264k_3^3k_1^5 - 1904k_2k_3^2k_1^5 - 560k_2^2k_3k_1^5 \\
&- 440k_2^4k_1^4 + 260k_3^4k_1^4 - 56k_2k_3^3k_1^4 - 248k_2^2k_3^2k_1^4 + 1540k_2^3k_3k_1^4 - 140k_2^5k_1^3 - 124k_3^5k_1^3 \\
&+ 1764k_2k_3^4k_1^3 + 2592k_2^2k_3^3k_1^3 + 3008k_2^3k_3^2k_1^3 + 1540k_2^4k_3k_1^3 + 200k_2^6k_1^2 - 128k_3^6k_1^2 \\
&+ 896k_2k_3^5k_1^2 + 3008k_2^2k_3^4k_1^2 + 2592k_2^3k_3^3k_1^2 - 248k_2^4k_3^2k_1^2 - 560k_2^5k_3k_1^2 + 140k_2^7k_1 \\
&+ 896k_2^2k_3^5k_1 + 1764k_2^3k_3^4k_1 - 56k_2^4k_3^3k_1 - 1904k_2^5k_3^2k_1 - 840k_2^6k_3k_1 + 20k_2^8 - 128k_2^2k_3^6 \\
&- 124k_2^3k_3^5 + 260k_2^4k_3^4 + 264k_2^5k_3^3 - 152k_2^6k_3^2 - 140k_2^7k_3 - 140k_3k_1^7) \\
&+ \frac{9}{8} g_3^2\, \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{1}{k_1 k_2 k_3} \frac{1}{(k_1+k_2-k_3)^7} \log\left(\frac{k_3}{k_T}\right) (-k_1^4 - 4k_2k_1^3 + 7k_3k_1^3 \\
&- 7k_3^2k_1^2 + 21k_2k_3k_1^2 - 4k_2^3k_1 - 23k_3^3k_1 - 14k_2k_3^2k_1 + 21k_2^2k_3k_1 - k_2^4 + 48k_3^4 - 23k_2k_3^3 \\
&- 6k_2^2k_1^2 - 7k_2^2k_3^2 + 7k_2^3k_3) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{ NLf}\,. \qquad (4.33)
\end{aligned}
$$

The apparent folded poles are cancelled by contribution from non-log (NLf) terms.

Eq. (4.21) and Eq. (4.29) depend on the logarithm of a dimensionful quantity, i.e., $\log k_T$. This is due to the fact that we have inserted a factor of $\mu^{-\frac{3\delta}{2}}$ to keep mass dimensions of couplings unchanged in d-dimensions, and in doing so the correlator's mass dimension changes and depends on δ. Factors of μ^δ are scheme dependent (unphysical), only affecting the constant finite part of the answer, they can therefore be inserted at will (provided one does similar manipulation in the counter-term as well). Hence, if one instead inserts a factor of $\mu^{-\delta}$, such that the mass dimension of the correlator is continued to d-dimensions, then the unrenormalized answer will not feature such dimensionful logarithms and instead become $\log \frac{k_T}{\mu}$. This is precisely the unphysical logarithm of ratio of co-moving momenta and the renormalisation scale.

As mentioned above, these unphysical logarithms, i.e. $-\frac{1}{2}\log k_T$ in Eq. (4.21) and

Eq. (4.29) are present even after adding the $\mathcal{O}(\delta)$ contributions from mode functions. These will cancel once contribution from counter terms is taken into account in Sec 4.4, and the final renormalised result features logarithms of dimensionless quantities.

4.3.3 Loop contribution from Diagram II

As previously stated, we will only give results for the explicit computation of Fig. 4.2a. There will be 2 more contributions coming from contractions of k_1 and k_2 with the cubic vertex at τ_3, obtained by substituting $k_3 \longleftrightarrow k_1$ and $k_3 \longleftrightarrow k_2$ respectively in the final expressions. As we will show in Sec. 4.4, the unphysical logarithms cancel diagram-by-diagram in the 2 vertex case, as well as for the 3-vertex diagram we are about to compute. Hence, we expect this cancellation to also occur for each diagram in the 3-cubic vertex case.

The time ordered diagram (Fig. 4.2a) is given by

$$
\begin{aligned}
D = -648i\ g_3^3\ \delta^3\big(\sum_i \vec{k}_i\big) f^*_{k_1}(0) f^*_{k_2}(0) f^*_{k_3}(0) \iiint_{-\infty}^{0} d\tau_1 d\tau_2 d\tau_3\ \mu^{-\frac{3\delta}{2}} f'_{k_1}(\tau_1) f'_{k_2}(\tau_1) f'_{k_3}(\tau_3)& \\
\{a(\tau_1)a(\tau_2)a(\tau_3)\}^{d-2} \int d^d\ \vec{p}_1 d^d\vec{p}_2\ \delta^d(\vec{p}_1+\vec{p}_2-\vec{k}_3)& \\
\{f'_{k_3}(\tau_1) f'^*_{k_3}(\tau_2) f'_{p_1}(\tau_2) f'^*_{p_1}(\tau_3) f'_{p_2}(\tau_2) f'^*_{p_2}(\tau_3)\Theta(\tau_3-\tau_2)\Theta(\tau_2-\tau_1)& \\
+ f'_{k_3}(\tau_1) f'^*_{k_3}(\tau_2) f'^*_{p_1}(\tau_2) f'_{p_1}(\tau_3) f'^*_{p_2}(\tau_2) f'_{p_2}(\tau_3)\Theta(\tau_2-\tau_1)\Theta(\tau_2-\tau_3)& \\
+ f'^*_{k_3}(\tau_1) f'_{k_3}(\tau_2) f'_{p_1}(\tau_2) f'^*_{p_1}(\tau_3) f'_{p_2}(\tau_2) f'^*_{p_2}(\tau_3)\Theta(\tau_1-\tau_2)\Theta(\tau_3-\tau_2)& \\
+ f'^*_{k_3}(\tau_1) f'_{k_3}(\tau_2) f'^*_{p_1}(\tau_2) f'_{p_1}(\tau_3) f'^*_{p_2}(\tau_2) f'_{p_2}(\tau_3)\Theta(\tau_1-\tau_2)\Theta(\tau_2-\tau_3)\}\,,&
\end{aligned}
\tag{4.34}
$$

To compute D_2, we note 3 internal mode functions and 1 scale factor at each of the three time vertices τ_1, τ_2 and τ_3. From arguments given in Sec. 4.3.1, we have $D_2 = \frac{3\delta}{2}\log\left(\frac{H}{k_T}\right) D_1$. Adding this $\mathcal{O}(\delta)$ contribution, this diagram is computed to be,

$$
\begin{aligned}
D = & -648\ g_3^3\ \delta^3\Big(\sum_i \vec{k}_i\Big)\frac{H^9}{\left(-\dot{H}M_{pl}^2\right)^6}\frac{1}{8k_1k_2k_3}\frac{1}{15k_T^8}\big(k_1^5+5k_2k_1^4+11k_3k_1^4+10k_2^2k_1^3 \\
& +58k_3^2k_1^3+44k_2k_3k_1^3+10k_2^3k_1^2+198k_3^3k_1^2+174k_2k_3^2k_1^2+66k_2^2k_3k_1^2+5k_2^4k_1-99k_3^4k_1 \\
& +396k_2k_3^3k_1+174k_2^2k_3^2k_1+44k_2^3k_3k_1+k_2^5+3k_3^5-99k_2k_3^4+198k_2^2k_3^3 \\
& +58k_2^3k_3^2+11k_2^4k_3\big)\left\{\frac{1}{\delta}+\log\left(\frac{k_3}{k_T}\right)-\frac{1}{2}\log k_T+\frac{3}{2}\log\left(\frac{H}{\mu}\right)\right\} \\
& +648\ g_3^3\ \delta^3\Big(\sum_i \vec{k}_i\Big)\frac{H^9}{\left(-\dot{H}M_{pl}^2\right)^6}\frac{1}{8k_1k_2}\frac{2k_3^3}{15k_T^8\,(k_3-k_1-k_2)^3}\log\left(\frac{k_T}{k_3}\right)\big(150k_1^4 \\
& +600k_2k_1^3-195k_3k_1^3+900k_2^2k_1^2+133k_3^2k_1^2-585k_2k_3k_1^2+600k_2^3k_1-25k_3^3k_1 \\
& +266k_2k_3^2k_1-585k_2^2k_3k_1+150k_2^4+k_3^4-25k_2k_3^3+133k_2^2k_3^2-195k_2^3k_3\big) \\
& +\{k_3\longleftrightarrow k_1\}+\{k_3\longleftrightarrow k_2\}+\ \mathrm{NLf}\,. \qquad (4.35)
\end{aligned}
$$

Once again, the apparent folded poles in the second term containing $\log(k_T/k_3)$ cancel from the following contribution from NLf,

$$
\begin{aligned}
\mathrm{NLf} = & \ 648\ g_3^3\ \delta^3\Big(\sum_i \vec{k}_i\Big)\frac{H^9}{\left(-\dot{H}M_{pl}^2\right)^6}\frac{1}{64k_1k_2k_3k_T^8}\frac{1}{(450(k_3-k_{12})^3)}\big(3k_3^8(80\log(2)-2593) \\
& +96k_{12}k_3^7(229-145\log(2))+24k_{12}^2k_3^6(2380\log(2)-4173)+8k_{12}^3k_3^5(29681-13380\log(2)) \\
& +10k_{12}^4k_3^4(5520\log(2)-19729)+16k_{12}^5k_3^3(2293-840\log(2))+8k_{12}^6k_3^2(943-840\log(2)) \\
& +8k_{12}^7k_3(173-240\log(2))+k_{12}^8(173-240\log(2))\big)+\dots\,, \qquad (4.36)
\end{aligned}
$$

with $k_{12}=k_1+k_2$ and the dots denote the finite terms in NLf having no folded poles.

Again, the unphysical logarithms $-\frac{1}{2}\log k_T$ in Eq. (4.35) is present even after adding the $\mathcal{O}(\delta)$ contributions from mode functions, and it cancels after renormalization in Sec 4.4.

4.3.4 Contribution from scaleless loops

It is well known that all scaleless integrals vanish in dimensional regularization. Here, we show that the contributions from diagrams similar to the ones in Fig. 4.3a vanish [51]. In all these diagrams the loop closes on itself, therefore there is no injection of external kinematics. The bulk-to-bulk propagator for the loop is given by,

$$G_p(\tau,\tau) = \frac{H^2}{2p^3}\left(1+p^2\tau^2\right) . \tag{4.37}$$

There are no exponentials with energy p, which implies that after time integration we only get scale-less integrals of p of the form,

$$\int p^\alpha d^3p \, , \tag{4.38}$$

with k dependent coefficients. Such integrals are zero in dim-reg. This can be shown very easily : substituting $p = \lambda p'$ in the above integral in d dimensions, we get,

$$\int \lambda^{d+\alpha} p'^\alpha d^d p' \, , \tag{4.39}$$

and we have,

$$\int p^\alpha d^d p = \lambda^{d+\alpha} \int p'^\alpha d^d p' \, . \tag{4.40}$$

The above equality follows because the limits are from $-\infty$ to ∞. Now, since $d+\alpha \neq 0$, the only possibility is that the integral is zero. Therefore, all 1-loop diagrams where the loop closes on itself are zero. However, one needs to be careful about diagrams which have a tadpole attached (e.g. Fig. 4.3b). Since by translational invariance the tadpole leg has zero momentum therefore in some cases the integrand for the diagram may diverge. This does not happen in our case since every field operator in the interaction Hamiltonian comes with a derivative. Therefore, either the intergand of such diagrams is zero or finite. The finite case evaluates to zero via the above-mentioned scaleless loop argument.

To see this, note that the propagator arises due to contraction of the tadpole leg coming from the cubic ($\dot{\pi}^3$) vertex with a mode function coming from either the cubic or a quartic ($\dot{\pi}^4$, $\dot{\pi}^2(\partial\pi)^2$) vertex. When mode functions with time derivatives contract to give the propagator, the integrand is proportional to the tadpole leg momentum, k and hence vanishes for $k = 0$. When modes with a time and a spatial derivative contract, the propagator is non-vanishing but finite for $k = 0$, hence the integrand is finite and the tadpole contribution vanishes by the scaleless arguments given in this section.

4.4 Renormalisation: cancelling the unphysical logarithms

It is well known that Effective Field theories can be renormalised like usual renormalisable theories provided we restrict to a particular order in the EFT expansion. In a renormalisable field theory ($\mathcal{L}^{(d\leq 4)}$), to renormalise any diagram one does not need to introduce any higher dimensional operators, all divergences can be absorbed in a finite set of parameters present in the lagrangian itself. On the other hand, for an EFT, one needs to include higher dimensional counter-term operators to renormalise the theory, but thankfully only a finite number of counter-terms are needed at a particular order. For instance, a loop diagram computed with two insertions of a dimension six operator is of order $\left(\frac{E}{\Lambda}\right)^4$ and therefore, it is necessary to have an eight-dimensional counter operator to renormalise it.

In this chapter, we computed the three-point function at 1-loop for the following EFT,

$$S = \int d^4x\sqrt{-g}\left[\left(\dot{\pi}_c{}^2 - \frac{(\partial\pi_c)^2}{a^2}\right) + \frac{\tilde{g}_3}{\Lambda^2}\dot{\pi}_c{}^3 + \frac{\tilde{g}_4}{\Lambda^4}\dot{\pi}_c{}^4 - \frac{3\tilde{g}_3}{2\Lambda^4}\dot{\pi}_c{}^2\left(\frac{\partial\pi_c}{a}\right)^2\right], \quad (4.41)$$

where we have now written the EFT lagrangian given in Eq. (4.2) in terms of inverse powers of some UV cutoff scale Λ. This makes the power counting manifest. We have assumed that all operators are generated by integrating out the same scale in the UV. In the expression above $\{\tilde{g}_i\}$ are all dimensionless and $\pi_c = \sqrt{-\dot{H}}M_{pl}\pi$ (canonically normalised). The EFToI Lagrangian was written in a similar form also in [65]. It is now clear that our loop results are of order $\left(\frac{H}{\Lambda}\right)^6$, therefore, we need ten-dimensional cubic counter term operators to renormalise it. We do not explicitly write down all possible operators. Instead, we show the following: given that the counter-terms, which give the same kinematical structures as that of terms multiplying the divergence, exist, then once added to the bare answers, they get rid of the unphysical logarithms. The most important takeaway from this section is that the unphysical structures of the form $\log \frac{k}{\mu}$ we got in the previous sections are corrected after renormalisation, and we finally get logarithms of ratios of comoving momenta. The expected structure of counter-term operators is the following,

$$\mathcal{O}^{\mathrm{CT}} \sim \frac{1}{\delta} \int \mu^{-\delta/2} a^{-3+\delta} \partial^7 \left(\pi^3\right) , \tag{4.42}$$

where ∂^7 are seven space/time derivatives. The three internal mode functions plus the scale factors bring the following δ dependence in the in-in integrals,

$$\frac{1}{\delta} \int d\tau \mu^{-\delta/2} (-H\tau)^{\delta/2} \ldots \ , \tag{4.43}$$

where the dots represent the remaining terms in the in-in integral. This expression can be simplified to yield,

$$\int d\tau \left(\frac{1}{\delta} - \frac{1}{2} \log \mu + \frac{1}{2} \log (-H\tau) \right) \ldots \ . \tag{4.44}$$

Once again we can pull $\log(-H\tau)$ out of the integral, as we have done before, as $\log\left(\frac{H}{k_T}\right)$. Hence, we factor out $\left(\frac{1}{\delta} + \frac{1}{2}\log\left(\frac{H}{\mu}\right) - \frac{1}{2}\log k_T\right)$ from all the counter terms. This implies that the coefficient of the unphysical logarithm and the divergent piece have a relative factor of $-1/2$, which is exactly the case for the unrenormalised answer of each diagram. Therefore if the counter-term is set appropriately to cancel the divergence, then it will automatically get rid of the unphysical logarithm. As mentioned above we do not explicitly renormalise the theory. Instead we assume such counterterms exist, i.e. we can get the kinematic structures that multiply the divergent terms in Sec. 4.3, ensuring that the divergence can be canceled by the counter terms. More explicitly, we saw that for diagrams computed in Sec. 4.3,

$$\begin{aligned} D(k_1, k_2, k_3) = \frac{S_{d-2} k_3^N}{2} F_0(\{x_i\}) \times \Bigg(& \frac{1}{\delta} + \frac{1}{2} \log\left(\frac{k_3}{k_T}\right) - \frac{1}{2} \log k_T + \frac{3}{2} \log\left(\frac{H}{\mu}\right) \\ & + F_1(\{x_i\}) \Bigg) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} \ . \end{aligned} \tag{4.45}$$

Now, we assume that a linear combination of counter-term operators with an appropriate choice of coefficients can yield an answer of the form,

$$D^{\text{CT}} = -\left[\frac{S_{d-2}k_3^N}{2}F_0(\{x_i\}) \times \left(\frac{1}{\delta} + \frac{1}{2}\log\left(\frac{H}{\mu}\right) - \frac{1}{2}\log k_T\right)\right.$$
$$\left. + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\}\right] . \tag{4.46}$$

Adding Eq. (4.45) and Eq. (4.46) we get,

$$\boxed{D^{\text{renorm}} = \delta^3\Big(\sum_i \vec{k}_i\Big)\frac{S_{d-2}k_3^N}{2}\left(F_0(\{x_i\})\log\left(\frac{k_3}{k_T}\right) + F_0(\{x_i\})\log\left(\frac{H}{\mu}\right) + F_1(\{x_i\})\right) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} .} \tag{4.47}$$

This gets rid of unphysical logarithms of co-moving momenta in all the diagrams computed.

It is important to note that such a feature of log cancellation from counter terms was missing from the two-point function calculation performed in [133]. The reason is pretty straightforward, in this case, there are two scale factors and six mode functions and this is just right to cancel the unphysical logs. Let us review this very quickly, the δ dependence of the loop integral is,

$$\mu^{-\delta/2}\int d\tau d^{3+\delta}p_1 d^{3+\delta}p_2(-H\tau)^{\delta}\ldots.. , \tag{4.48}$$

Again the logarithm can be factored out at scale k which is the only scale in the problem and the integral evaluates to the form given in Eq. (4.18) with $\sum_i n_i = 1$, and we get,

$$\begin{aligned} D &= \frac{\mu^{-\delta/2}S_{d-2}k^N}{2}\left(\frac{F_0}{\delta} + F_0\log k + F_0\log(H/k) + \text{finite}\right) \\ &= \frac{S_{d-2}k^N}{2}\left(\frac{F_0}{\delta} + F_0\log\frac{H}{\mu} + \text{finite}\right) . \end{aligned} \tag{4.49}$$

So in this case the unphysical logarithms cancel without renormalising. We believe that for general one-loop diagrams, the unphysical logarithms will be canceled only after renormalizing the results unless there is some cancellation as in the two-point case.

4.5 Cutoff Regularisation: a cross check

If the full mode functions are used in our computations instead of the Taylor expanded ones, evaluation of the time integrals results in complicated hypergeometric functions, and the problem becomes intractable. Thus we work with the modes expanded about $\delta = 0$. Now, the expansion in δ for $(-H\tau)^{\delta} = \exp\{\delta \log(-H\tau)\}$ converges for all values of δ. Also, we only had to evaluate the $\mathcal{O}(\delta)$ term, since the higher order terms did not produce higher order poles upon evaluation of the loop integrals, hence vanishing when $\delta \to 0$.

Note that in our computations, we had to commute the summation operator across the integrals. There is a subtlety involved here. For the loop integrals to converge, δ must be analytically continued to some point in the complex plane, and arguing the convergence of the sum of integrals is not straightforward at this value of δ. This makes the commutation of summation and integration operations tricky. However, to demonstrate that our method of implementing the dimensional regularisation still computes the logarithmic running reliably, we compute one of the diagrams in the cutoff-regularisation, as follows.

We focus on the time ordered contribution to Diagram I coming from the interactions with only time derivatives i.e., $\dot{\pi}^3$, $\dot{\pi}^4$. Note that there is another contribution to this diagram coming from the quartic interaction $\dot{\pi}^2(\partial\pi)^2$, but since it has a different coupling constant, this contribution cannot alter the log structure we compute below. It is interesting to note that in cutoff regularisation, the mode functions are 3-dimensional, hence the contribution from the counter terms, which are tree-level diagrams, will never produce $\log k$ terms. Thus we expect the unrenormalised answer not to feature any unphysical logarithms, which turns out to be true in the following.

To evaluate Eq. (4.19) in cutoff regularisation, we regulate the momentum integrals by placing a cutoff at $\Lambda a(\tau_f)$, where Λ is the physical cutoff and τ_f is the time being integrated first, e.g. for the $\Theta(\tau_1 - \tau_2)$ part of the integrand $\tau_f = \tau_2$. We integrate the loop integrals first, since they are now τ dependent, using the following identity :

$$\int d^3\vec{p}_1 d^3\vec{p}_2 Q(p_1,p_2,k_i)\delta^3(\vec{p}_1+\vec{p}_2+\vec{k}_3) = 2\pi\int_0^{\Lambda a(\tau_f)} dp_1 \int_{|\vec{p}_1-\vec{k}_3|}^{\vec{p}_1+\vec{k}_3} dp_2 \frac{p_1 p_2}{k_3} Q(p_1,p_2,k_i)\,, \tag{4.50}$$

Evaluating the $\tau_1 > \tau_2$ piece of the loop integrals gives the following :

$$\begin{aligned}
D = & -36\ g_3 g_4\ \delta^3\big(\sum_i \vec{k}_i\big)\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1\left(1+\frac{H}{\Lambda}\right)} d\tau_2\ \ \{\frac{1}{16k_3\,(\tau_1-\tau_2)^6}\tau_1^2\tau_2^4\big(e^{i(k_1+k_2+k_3)\tau_2} \\
& \big(-k_3^2\,(\tau_1-\tau_2)^2 + 5ik_3\,(\tau_1-\tau_2)+8\big) + e^{i(k_1+k_2+3k_3)\tau_2-2ik_3\tau_1}(8k_3^4\,(\tau_1-\tau_2)^4 \\
& -20ik_3^3\,(\tau_1-\tau_2)^3 + k_3^2\,(\tau_1-\tau_2)^2\,(-27+8e^{2ik_3(\tau_1-\tau_2)}) \\
& -ik_3\,(\tau_1-\tau_2)\,\big(-21+16e^{2ik_3(\tau_1-\tau_2)}\big) - 16e^{2ik_3(\tau_1-\tau_2)}+8)\big) + \ldots \\
& + \frac{1}{240k_3\,(\tau_1-\tau_2)^6}\tau_1^2\tau_2^4(-2ik_3^5\,(\tau_1-\tau_2)^5\,e^{2ik_3(\tau_1-\tau_2)} - 10k_3^4\,(\tau_1-\tau_2)^4\,\big(12+e^{2ik_3(\tau_1-\tau_2)}\big) \\
& + 20ik_3^3\,(\tau_1-\tau_2)^3\,\big(15+2e^{2ik_3(\tau_1-\tau_2)}\big) - 15k_3^2\,(\tau_1-\tau_2)^2\,\big(-27+e^{2ik_3(\tau_1-\tau_2)}\big) + 15ik_3 \\
& (\tau_1-\tau_2)\,\big(-21+5e^{2ik_3(\tau_1-\tau_2)}\big) + 120\,\big(-1+e^{2ik_3(\tau_1-\tau_2)}\big))e^{(i(k_1+k_2)\tau_2 - ik_3(2\tau_1-3\tau_2))}\}\,,
\end{aligned} \tag{4.51}$$

where we regulate the time integrals with the same cutoff Λ. The first 4 lines correspond to $q_1 > k_3$, the last 3 lines correspond to $q_1 < k_3$. Similarly, evaluating the $\tau_2 > \tau_1$ piece gives,

$$D = -36\ g_3 g_4\ \delta^3\big(\sum_i \vec{k}_i\big) \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{\tau_2\left(1+\frac{H}{\Lambda}\right)} d\tau_1\ \ \{\frac{1}{16k_3\,(\tau_1-\tau_2)^{\,6}}\tau_1^2\tau_2^4 e^{i(2k_3\tau_1+(k_1+k_2-3k_3)\tau_2)}$$
$$(8k_3^4\,(\tau_1-\tau_2)^{\,4}e^{2ik_3\tau_1} + 20ik_3^3\,(\tau_1-\tau_2)^{\,3}e^{2ik_3\tau_1} - k_3^2\,(\tau_1-\tau_2)^{\,2}\left(27e^{2ik_3\tau_1} - 7e^{2ik_3\tau_2}\right)$$
$$- ik_3\,(\tau_1-\tau_2)\left(21e^{2ik_3\tau_1} - 11e^{2ik_3\tau_2}\right) + 8\left(e^{2ik_3\tau_1} - e^{2ik_3\tau_2}\right)) + \ldots$$
$$+ \frac{1}{4k_3\,(\tau_1-\tau_2)^{\,3}}\tau_1^2\tau_2^4 e^{i(k_3(2\tau_1-\tau_2)+(k_1+k_2)\tau_2)}(\frac{1}{5}ik_3^5\,(\tau_1-\tau_2)^{\,2}e^{i(k_3(2\tau_1-\tau_2)+(k_1+k_2)\tau_2)}$$
$$+ \frac{1}{2}k_3^4\,(\tau_1-\tau_2)\,(1-ik_3\,(\tau_1-\tau_2))\,e^{i(k_3(2\tau_1-\tau_2)+(k_1+k_2)\tau_2)} + \frac{1}{3}ik_3^3(k_3^2\,(\tau_1-\tau_2)^{\,2}$$
$$+ 2ik_3\,(\tau_1-\tau_2) - 2) - \frac{1}{4\,(\tau_1-\tau_2)^{\,3}}e^{i(2k_3\tau_1+(k_1+k_2-k_3)\tau_2)}\left(k_3^2\,(\tau_1-\tau_2)^{\,2} + 5ik_3\,(\tau_1-\tau_2) - 8\right)$$
$$+ \frac{1}{4\,(\tau_1-\tau_2)^{\,3}}e^{i(4k_3\tau_1+(k_1+k_2-3k_3)\tau_2)}(-8k_3^4\,(\tau_1-\tau_2)^{\,4} - 20ik_3^3\,(\tau_1-\tau_2)^{\,3}$$
$$+ 27k_3^2\,(\tau_1-\tau_2)^{\,2} + 21ik_3\,(\tau_1-\tau_2) - 8))\}\,, \tag{4.52}$$

where the first 3 lines correspond to $q_1 > k_3$ and the last 5 lines correspond to $q_1 < k_3$. In Eq. (4.51) and Eq. (4.52) the dots represent contributions that are proportional to the term $(\mathrm{Exp}\{\frac{\Lambda}{H}\big(\frac{\tau_s}{\tau_f}-1\big)\})$, where the Θ function imposes $\tau_s > \tau_f$. These terms do not produce any logarithms (as was also noted in [133]).

Evaluating the time integrals and adding all contributions, we arrive at the following answer,

$$D = 72\ g_3 g_4\ \delta^3\big(\sum_i \vec{k}_i\big)\frac{H^9}{\left(-\dot{H}M_{pl}^2\right)^5}\frac{k_3(k_1+k_2)(k_3-2k_1-2k_2)}{k_1k_2k_T^7}\{\log\frac{H}{\Lambda}$$
$$+ \frac{1}{2}\log\frac{k_3}{k_T}\} + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \mathrm{L}\,. \tag{4.53}$$

where L denotes power divergences in Λ and other non-log finite terms.

After renormalisation $\log\frac{H}{\Lambda} \longrightarrow \log\frac{H}{\mu}$. Therefore the logarithm running, as well as the structure of the coefficients multiplying them, matches with the results of our previous dim reg computation after adding contribution from counter terms. This shows that the procedure of expanding in δ gives the correct log running.

Chapter 5

Cosmological cutting rules for Bogoliubov initial states

The material presented in this chapter is based on the author's publication [145].

When the full system is described using quantum mechanics, one expects unitarity to leave an imprint on the observables of the theory. In flat space, unitarity gives rise to the well-known optical theorem for scattering amplitudes and the associated Cutkosky cutting rules [146, 82]. One expects a similar set of relations to exist also in the context of cosmology and this has been confirmed by recent results. In [20, 147, 24], an infinite set of equations were derived that relate wavefunction coefficients to all orders in perturbation theory, which are now known as the cosmological optical theorem or cosmological cutting rules. A crucial assumption of these results was an initial Bunch-Davies initial state in the infinite past. A formal extension to more general states was presented in [148]. In this chapter *we derive explicit cosmological cutting rules assuming a general Bogoliubov initial state* [12, 13, 17, 18, 16, 19, 14, 15, 52]. It should be noted that, in contrast to flat space, the cosmological optical theorem has been formulated so far only perturbatively and research is still in progress to find non-perturbative consequences of unitarity in cosmological spacetimes (see e.g. [149, 150, 151, 152, 153, 154, 155]).

The choice of a Bogoliubov initial state is motivated by the desire to see how the cosmological optical theorem generalises to a broader class of initial states, which leads nonetheless to a tractable problem. Keeping this goal in mind, here we consider a class of excited states which are Bogoliubov transformations of the usual Bunch-Davies state. As discussed in the previous chapters, these states are characterised by two rotation-invariant functions of momenta, (α_k, β_k), which are known as Bogoliubov coefficients and are constrained by the normalization condition $|\alpha_k|^2 - |\beta_k|^2 = 1$. We assume throughout that these functions are chosen such that the gravitational back-reaction is negligible and the spacetime remains well approximated by de Sitter space. A modified initial state during inflation leaves an imprint on cosmological correlators and this possibility has been constrained by the data [17, 18, 16, 19, 14, 15, 8].

The first obstacle in deriving the cutting rules for general Bogoliubov initial states is that the bulk-boundary propagator no longer satisfies the so-called Hermitian analyticity condition, i.e. $K_k^*(\alpha_k, \beta_k) \neq K_{-k}(\alpha_{-k}, \beta_{-k})$, which was the crucial ingredient in the original derivation [20]. However, new sets of relations for the propagator emerge when leveraging the simple analytic dependence on the initial state parameters α_k and β_k. These new relations, which play the same role as Hermitian analyticity in the Bunch-Davies case, form a $\mathbf{Z}_2 \times \mathbf{Z}_2$ discrete symmetry group action on $K_k(\alpha_k, \beta_k)$. Two of these relations can be used to define an appropriate "discontinuity" operation, which in turn leads to two classes of cutting rules. From the new cutting rules we derive constraints for IR-finite n-point contact and four-point exchange diagrams. We confirm our results with explicit examples. First, we focus only on massless and conformally coupled scalar fields, and then we generalise to fields of any mass and spin.

In passing, we were able to derive relations expressing the Bogoliubov wavefunction coefficients in terms of the corresponding Bunch-Davies coefficients for both n-point contact and four-point exchange diagrams. In [92], such a relation was derived only for the three-point contact wavefunction coefficient. The relations we derive provide a welcome technical simplification since one does not need to explicitly calculate Bogoliubov wavefunction coefficients from scratch, which would involve performing complicated time integrals over both positive- and negative-frequency modes. Instead, one can just evaluate the Bunch-Davies wavefunction coefficients and use a simple formula to obtain the Bogoliubov equivalent.

The rest of this chapter is organized as follows. In Sec. 5.1 we present a brief review of the cosmological optical theorem and the associated cosmological cutting rules in the case of a Bunch Davies initial state, as derived in [20, 24, 147]. In Sec. 5.2 we derive a new infinite set of cosmological cutting rules for wavefunction coefficients in the case of a Bogoliubov initial state. In Sec. 5.3 we discuss the implications of our cutting rules for n-point contact and four-point exchange diagrams with IR-finite interactions. In Sec. 5.5 we give relations between n-point and four-point exchange Bogoliubov and the corresponding Bunch-Davies wavefunction coefficients. In Sec. 5.6 we generalise the Bogoliubov cutting rules to fields of any mass and spin.

5.1 Review of Bunch-Davies Cutting Rules

Here we illustrate the basic idea of the cosmological optical theorem [20] using only the four-point tree-level exchange and n-point contact diagrams computed for $\frac{\lambda}{3!}\phi^3$. In [147], these relations were extended to all orders in perturbation theory, including any number of loops and in [24] they were shown to be valid for fields of any mass and spin, for arbitrary interactions, as long as the theory is unitary and the initial state is the Bunch-Davies state. Other constraints from unitarity on the wavefunction were discussed in [156].

The four-point exchange wavefunction Feynman rules give

$$\psi_4(\{k\},p) = i\lambda^2 \int_{-\infty}^{0} d\tau_1 d\tau_2 \mathcal{K}_{k_1}(\tau_1)\mathcal{K}_{k_2}(\tau_1)\mathcal{G}_p(\tau_1,\tau_2)\mathcal{K}_{k_3}(\tau_2)\mathcal{K}_{k_4}(\tau_2)\,, \tag{5.1}$$

where $\{k\}$ collectively denotes the four external energies $k_a = |\mathbf{k}_a|$ for $a = 1, \ldots, 4$. Using the following property of the Bunch-Davies bulk-boundary propagator,

$$\mathcal{K}^*_{-k}(\tau) = \mathcal{K}_k(\tau)\,, \tag{5.2}$$

it follows that we can write[1]

$$\begin{aligned} \psi_4^*(\{-k\}, p) &= -i\lambda^2 \int_{-\infty}^0 d\tau_1 d\tau_2 \left[\mathcal{K}_{-k_1}(\tau_1)\mathcal{K}_{-k_2}(\tau_1)\mathcal{G}_p(\tau_1,\tau_2)\mathcal{K}_{-k_3}(\tau_2)\mathcal{K}_{-k_4}(\tau_2)\right]^*\,, \\ &= -i\lambda^2 \int_{-\infty}^0 d\tau_1 d\tau_2 \mathcal{K}_{k_1}(\tau_1)\mathcal{K}_{k_2}(\tau_1)\mathcal{G}_p{}^*(\tau_1,\tau_2)\mathcal{K}_{k_3}(\tau_2)\mathcal{K}_{k_4}(\tau_2)\,, \end{aligned} \tag{5.3}$$

Adding Eq. (5.1) and Eq. (5.3), we get,

$$\begin{aligned} \psi_4(\{k\}, p) + \psi_4^*(\{-k\}, p) = i\lambda \int_{-\infty}^0 & d\tau_1 d\tau_2 \mathcal{K}_{k_1}(\tau_1)\mathcal{K}_{k_2}(\tau_1) \\ 2P_p \mathrm{Im}[\mathcal{K}_p(\tau_1)]&\mathrm{Im}[\mathcal{K}_p(\tau_2)]\mathcal{K}_{k_3}(\tau_2)\mathcal{K}_{k_4}(\tau_2)\,, \end{aligned} \tag{5.4}$$

where we have used the following property,

$$\mathrm{Im}\mathcal{G}_p(\tau_1,\tau_2) = 2P_p \mathrm{Im}[\mathcal{K}_p(\tau_1)]\mathrm{Im}[\mathcal{K}_p(\tau_2)]\,. \tag{5.5}$$

The key simplification here is that taking the imaginary part of $\mathcal{G}$ removes the Heaviside theta function, which nested the two time integrals. Without the Heaviside theta function, the time integrals are independent and can be recognised to be

$$\begin{aligned} &\psi_4(k_1,k_2,k_3,k_4,p) + \psi_4^*(-k_1,-k_2,-k_3,-k_4,p) = \\ &- P_s\left[\psi_3(k_1,k_2,p) + \psi_3^*(-k_1,-k_2,p)\right]\left[\psi_3(k_3,k_4,s) + \psi_3^*(-k_3,-k_4,p)\right]\,. \end{aligned} \tag{5.6}$$

Therefore, the cutting rule relates higher point coefficients to lower point ones. Similarly, one can show that for any n-point contact diagram, the cutting rule reads,

$$\psi_n(\{k_i\}) + \psi_n^*(\{-k_i\}) = 0\,. \tag{5.7}$$

This equation has a simple interpretation i.e. there are no internal lines to cut.

[1]Equations in this section are valid for all interactions with an even number of spatial derivatives. In the presence of an odd number of spatial derivatives one needs to simultaneously transform $k_a \to -k_a$ *and* $\vec{k}_a \to -\vec{k}_a$, which ensures $\partial_{\vec{x}}$ remains invariant.

For parity-odd interactions, we need to be careful about the transformation of the three-momenta. The more precise version of the above formula is

$$\psi_4(\{k_i\}, p, \{\vec{k}_i\}) + \psi_4^*(\{-k_i\}, p, \{-\vec{k}_i\}) =$$
$$- P_s \left[\psi_3(\{k_i\}, p, \{\vec{k}_i\}) + \psi_3^*(\{-k_i\}, p, \{-\vec{k}_i\})\right] \left[\psi_3(\{k_i\}, p, \{\vec{k}_i\}) + \psi_3^*(\{-k_i\}, p, \{-\vec{k}_i\},)\right] , \tag{5.8}$$

and for contact one has,

$$\psi_n(\{k_i\}, \{\vec{k}_i\}) + \psi_n^*(\{-k_i\}, \{-\vec{k}_i\}) = 0\,. \tag{5.9}$$

5.2 Cutting Rules at any order: Bogoliubov states

In this section, we will derive cutting rules assuming that the initial state is a general Bogoliubov state rather than the Bunch Davies state. Our derivation mimics that presented in [147] and is hence valid to all orders in perturbation theory i.e. for a diagram with any number of vertices, internal lines and loops.

We first briefly review some salient aspects of our setup. The bulk-bulk and bulk-boundary propagators for the Bogoliubov states are denoted by $K_k(\tau)$, $G_s(\tau, \tau')$ and by $\mathcal{K}_k(\tau)$, $\mathcal{G}_s(\tau, \tau')$ for the Bunch-Davies case. The generic expression for the bulk-bulk propagator is given by

$$G_p(\tau_1, \tau_2) = i\left[\theta(\tau - \tau')\left(\phi_p^+(\tau')\phi_p^-(\tau) - \frac{\phi_p^-(\tau_0)}{\phi_p^+(\tau_0)}\right)\phi_p^+(\tau)\phi_p^+(\tau') + (\tau \leftrightarrow \tau')\right], \tag{5.10}$$

and the bulk-boundary propagator is given by

$$K_k(\tau) = \frac{\phi_k^+(\tau)}{\phi_k^+(\tau_0)}\,, \tag{5.11}$$

where $\phi_k^+(\tau)$ and $\phi_k^-(\tau)$ are linearly independent solutions of the equation of motion also known as mode functions. For $k \in \mathbb{R}$ we have ${\phi^*}_k^+(\tau) = \phi_k^-(\tau)$, which leads to a useful relation between the two propagators,

$$G_p(\tau_1, \tau_2) = iP_p \left[{K^*}_p(\tau_1)K_p(\tau_2)\theta(\tau_1 - \tau_2) + {K^*}_p(\tau_2)K_p(\tau_1)\theta(\tau_2 - \tau_1) - K_p(\tau_1)K_p(\tau_2)\right] , \tag{5.12}$$

where P_p is the power spectrum evaluated at the late time boundary, i.e

$$P_p = \langle\phi_{\vec{k}}(\tau_0)\phi_{-\vec{k}}(\tau_0)\rangle' = \left|\phi_k^+\right|^2 . \tag{5.13}$$

The prime here means that we have stripped off the momentum conserving delta function. Our conventions for the factors of i in the propagators and the Feynman rules are the same as in [147]. A particular choice for the mode function selects the vacuum state. In particular, given the following Fourier expansion of the field operator ϕ_k,

$$\phi_k = \phi_k^+ {a^\dagger}_{-\vec{k}} + \phi_k^- a_{\vec{k}} \,, \tag{5.14}$$

the state annihilated by the operator $a_{\vec{k}}$ is defined as the vacuum state. Therefore, the most general vacuum state (Bogoluibov state) is given by the following choice of the mode functions

$$\begin{aligned} \phi_k^+(\tau) &= \frac{H}{\sqrt{2k^3}}\left(\alpha_k(1 - ik\tau)e^{ik\tau} + \beta_k(1 + ik\tau)e^{-ik\tau}\right) && \text{(massless scalar)}\,, \\ \phi_k^+(\tau) &= \frac{H}{\sqrt{2k}}\left(\alpha_k(-i\tau)e^{ik\tau} + \beta_k(i\tau)e^{-ik\tau}\right) && \text{(conformally coupled scalar)}\,. \end{aligned} \tag{5.15}$$

The usual Bunch-Davies vacuum initial state corresponds to the choice of $\alpha_k = 1$, $\beta_k = 0$. For a Bogoliubov state (which is the subject of this chapter) α_k and β_k can be anything given that they satisfy the constraint equation $|\alpha_k|^2 - |\beta_k|^2 = 1$. This relation imposes the right commutation condition on the creation and annihilation operators i.e. $\left[a_{\vec{k}}, a^\dagger_{\vec{k}'}\right] = \delta^3(\vec{k} - \vec{k}')$. Although we do not work with any particular choice of the Bogoliubov coefficients we assume that they are chosen such that the backreaction is controlled and the background geometry is well approximated by de Sitter space. The bulk-boundary propagator for a massless scalar with the Bunch-Davies initial condition is then given by

$$\mathcal{K}_k(\tau) = (1 - ik\tau)e^{ik\tau} \,, \tag{5.16}$$

and for general Bogoliubov states is given by,

$$K(\alpha_k, \beta_k, k) = \frac{1}{(\alpha_k + \beta_k)} \left(\alpha_k(1 - ik\tau)e^{ik\tau} + \beta_k(1 + ik\tau)e^{-ik\tau}\right) . \tag{5.17}$$

For the conformally coupled scalar, the bulk-boundary propagator for a Bogoliubov initial state is given by

$$K(\alpha_k, \beta_k, k) = \frac{\tau \left(\alpha_k e^{ik\tau} - \beta_k e^{-ik\tau}\right)}{\tau_0 \left(\alpha_k - \beta_k\right)} , \tag{5.18}$$

where τ_0 is the time on the boundary. Again, the Bunch-Davies case corresponds to $\alpha_k = 1$, $\beta_k = 0$. See [157] for earlier discussions of Bogoliubov states in de Sitter. Notice that the Bogoliubov bulk-boundary propagator does not have a linear in k term when expanded around $k = 0$, just like in Bunch Davies case,

$$\frac{\partial}{\partial k} K(\alpha_k, \beta_k, k)|_{k=0} = 0 . \tag{5.19}$$

Here the derivative with respect to k is taken keeping α_k and β_k fixed. This simple property leads to the manifestly local test for wavefunction coefficients [22], which must be satisfied for all theories with manifestly local interactions and more generally for soft interactions, as discussed in [158].

Finally, we note that both Bunch-Davies and Bogoliubov bulk-bulk propagators satisfy the following factorization properties,

$$\mathrm{Im}\mathcal{G}(p, \tau_1, \tau_2) = 2P_p \mathrm{Im}[\mathcal{K}(p, \tau_1)]\mathrm{Im}[\mathcal{K}(p, \tau_2)] , \tag{5.20}$$

$$\mathrm{Im}G(p, \tau_1, \tau_2) = 2P_p \mathrm{Im}[K(p, \tau_1)]\mathrm{Im}[K(p, \tau_2)] . \tag{5.21}$$

5.2.1 Propagator identities

The key ingredient to derive the cosmological optical theorem is the Hermitian analyticity of the Bunch-Davies bulk-boundary propagator [20, 24, 147],

$$\mathcal{K}^*_{-k}(\tau) = \mathcal{K}_k(\tau) . \tag{5.22}$$

This is then combined with an infinite set of identities for products of the bulk-bulk propagator, the simplest of which is given in Eq. (5.5). The bulk-bulk propagator for the Bogoliubov case still has the same factorization properties but now the bulk-boundary propagator satisfies three relations,

$$K^*_{-k}(\alpha^*_k, \beta^*_k) = K_k(\alpha_k, \beta_k)\,, \tag{5.23}$$

$$K^*_{k}(\beta^*_k, \alpha^*_k) = K_k(\alpha_k, \beta_k)\,, \tag{5.24}$$

$$K_{-k}(\beta_k, \alpha_k) = K_k(\alpha_k, \beta_k)\,. \tag{5.25}$$

This set of transformations actually form a $\mathbf{Z_2}\times\mathbf{Z_2}$ group ($\{a,e\}\times\{b,e\} = \{a,b,ab,e\}$),

$$a\left(K_k\left(\alpha_k, \beta_k\right)\right) = K^*_{-k}\left(\alpha^*_k, \beta^*_k\right)\,, \tag{5.26}$$

$$b\left(K_k\left(\alpha_k, \beta_k\right)\right) = K^*_{k}\left(\beta^*_k, \alpha^*_k\right)\,, \tag{5.27}$$

$$ab\left(K_k\left(\alpha_k, \beta_k\right)\right) = K_{-k}\left(\beta_k, \alpha_k\right)\,, \tag{5.28}$$

where $a^2 = b^2 = e$. It is important to mention that the above relations are derived by keeping the k dependence of α_k, β_k separate from the rest of the k dependence in $K_k(\alpha_k, \beta_k)$. We will assume the same throughout this chapter. Note that the first of these relations is the natural extension of Eq. (5.22) to the Bogoliubov case but the other two are new and emerge only because we allow ourselves to vary the parameters α_k and β_k appearing in the initial state. This was not possible in the case of a Bunch-Davies state where these parameters are fixed to $\alpha_k = 1$ and $\beta_k = 0$. Now we proceed to prove the cutting rules for the Bogoliubov case to all orders in perturbation theory.

5.2.2 Proof of cutting rules for Bogoliubov initial states

We will follow [147] in proving cutting rules for a general diagram with any number of internal lines and any number of loops. In that work, one first proves a set of "propagator identities" at the level of integrands, which results in an infinite set of identities for the product of propagators. Second, one integrates these relations crucially assuming that coupling constants are real, hence arriving at the final relations among wavefunction coefficients ψ_n. However, in our case we do not need to go through this whole procedure. The reason is very simple, the cutting rules proved in [147] rely on the form of the bulk-bulk propagator as a function of the bulk-boundary propagator. This is unchanged in the case of Bogoliubov initial state, Eq. (5.12). The only difference is the introduction of two new kinds of "discontinuities" (Discs) because the Bogoliubov bulk-boundary propagator satisfies the new identities mentioned in the preceding section. These are given as

$$\underset{\{\alpha_{p_m},\beta_{p_m}\}}{\mathrm{Disc}(1)}\left[f(\{\alpha_{k_i},\beta_{k_i}\},\{\alpha_{p_m},\beta_{p_m}\},\{k_i\},\{\vec{k}_i\},\{p_m\})\right]=$$
$$f(\{\alpha_{k_i},\beta_{k_i}\},\{\alpha_{p_m},\beta_{p_m}\},\{k_i\},\{\vec{k}_i\},\{p_m\})-f^*(\{\beta^*_{k_i},\alpha^*_{k_i}\},\{\alpha_{p_m},\beta_{p_m}\},\{k_i\},\{-\vec{k}_i\},\{p_m\})\,, \tag{5.29}$$

$$\underset{\{\alpha_{p_m},\beta_{p_m}\}\&\{p_m\}}{\mathrm{Disc}(2)}\left[f(\{\alpha_{k_i},\beta_{k_i}\},\{\alpha_{p_m},\beta_{p_m}\},\{k_i\},\{\vec{k}_i\},\{p_m\})\right]=$$
$$f(\{\alpha_{k_i},\beta_{k_i}\},\{\alpha_{p_m},\beta_{p_m}\},\{k_i\},\{\vec{k}_i\},\{p_m\})-f^*(\{\alpha^*_{k_i},\beta^*_{k_i}\},\{\alpha_{p_m},\beta_{p_m}\},\{-k_i\},\{-\vec{k}_i\},\{p_m\})\,, \tag{5.30}$$

where $\{k_i\}$ & $\{p_m\}$ are external and internal energies. These two definitions follow from the propagator relations, Eq. (5.23) & Eq. (5.24). With all this in mind, we directly start from the propagator lemma mentioned in Eq. (4.7) of [147],

$$\sum_{\mathrm{cuts};\,C\subseteq I}^{2^I}\hat{D}_C=0\,, \tag{5.31}$$

where $\hat{D}_C$ ("C" stands for a "cut" and "I" for all internal lines) denotes the imaginary part of the product of internal and "cut" propagators (with appropriate factors of $(2i)$ as given below) present in the "cut" diagram D_C. "Cutting" a line with momentum say $\vec{p}$ is defined by replacing the corresponding bulk-bulk propagator $G_p(\tau_1,\tau_2)$ by $-2P_pK_p(\tau_1)K_p(\tau_2)$. The sum above runs over all kinds of cut diagrams $\{D_C\}$, which one can produce from the original diagram D. Now, If D_C is disconnected by cut(s) then it is given by the union of connected subdiagrams $D_C^{(n)}$ i.e $D_C=\cup_n D_C^{(n)}$. Mathematically,

$$\hat{D}_C=\prod_n \mathrm{Im}\left[(2i)^{L_n}\hat{D}_C^{(n)}\right]\,, \tag{5.32}$$

where L_n is the number of loops in the connceted subdiagram $D_C^{(n)}$. One can then write $\hat{D}_C$ in terms of the Discs defined above, Eq. (5.29) & Eq. (5.30). Let us illustrate this for the simplest case where none of the internal lines are cut, the corresponding term is denoted by $\hat{D}_{\{\}}$. Now, modulo the external lines, the integrand for the wavefunction coefficient for diagram D reads as follows,

$$\hat{\psi}^{(D)} = i^{1-L}\left[\prod_{i=1}^{I}\int d^3p_i G_{p_i}\right] . \tag{5.33}$$

Since the Disc operation does not change the α_{p_m}, β_{p_m} and p_m dependence of the internal lines, it simply takes the imaginary part of this expression, which in turn is related to $\hat{D}_{\{\}}$ by the above propagator identity,

$$-i\text{Disc}(m)\left[i\hat{\psi}^{(D)}\right] = \left[\prod_{i=1}^{I}\int d^3p_i\right](-2)^{1-L}\hat{D}_{\{\}}\,, \tag{5.34}$$

where m can be either 1 or 2. Similarly, for diagrams where some internal line(s) are cut, $\hat{\psi}^{D_C}$ is given by the same expression as above except the cut line(s) propagator(s), $G(\tau_1,\tau_2)$ are replaced by $-2PK(\tau_1)K(\tau_2)$. For all cut diagrams, the Disc of $\hat{\psi}^{D_C}$ can be related to $\hat{D}_C$ and we get the following expression,[2]

$$\begin{aligned}\int d^3p_1...d^3p_I(-2)^{1-L}\sum_{C\subseteq I}^{2^I}\hat{D}_C = \sum_{C\subseteq I}^{2^I}&\left[\prod_{a\in C}\int d^3q_a d^3q_a' P_{q_a q_a'}\right]\\ &\times\prod_n(-i)\underset{I_n\{q_a\}}{\text{Disc}(m)}\left[i\hat{\psi}^{(D_C^{(n)})}\right] ,\end{aligned} \tag{5.35}$$

where I_n ($I_n \subseteq I$) are internal lines contained within $D_C^{(n)}$. Now, the LHS of the above expression vanishes from the propagator lemma, Eq. (5.31). Therefore, we have

$$\sum_{C\subseteq I}^{2^I}\left[\prod_{a\in C}\int d^3q_a d^3q_a' P_{q_a q_a'}\right]\prod_n(-i)\underset{I_n\{q_a\}}{\text{Disc}(m)}\left[i\hat{\psi}^{(D_C^{(n)})}\right] = 0\,. \tag{5.36}$$

Finally, we need to multiply this expression by bulk-boundary propagators and integrate over all times. Using propagator identities (Eq. (5.23) & Eq. (5.24)) for the bulk-boundary propagator, we can move them inside the Disc which proves the

[2] We refer the reader to the original cutting rules paper for a detailed derivation.

cutting rules. Let us show this for Eq. (5.23).

$$\text{Disc}(2)\left[\prod_a^n K_{k_a}(\alpha_{k_a},\beta_{k_a})R\right] = \prod_a^n K_{k_a}(\alpha_{k_a},\beta_{k_a})R - \left(\prod_a^n K_{-k_a}(\alpha^*_{k_a},\beta^*_{k_a})R\right)^* , \tag{5.37}$$

using Eq. (5.23) for any R, this becomes,

$$\prod_a^n K_{k_a}(\alpha_{k_a},\beta_{k_a})\text{Disc}(2)\left[R\right] . \tag{5.38}$$

A similar argument holds for Disc(1). Therefore, the final equation of the cutting rule employing either only Disc(1) or Disc(2) is given by

$$\boxed{i\text{Disc}(m)\left[i\psi^{(D)}\right] = \sum_{C\subseteq I, C\neq\{\}}^{2^I-1}\left[\prod_{a\in C}\int d^3q_a d^3q'_a P_{q_a q'_a}\right]\prod_n (-i)\underset{I_n\{q_a\}}{\text{Disc}(m)}\left[i\psi^{(D_C^{(n)})}\right]}, \tag{5.39}$$

where the cut diagrams on the RHS do not include the "no cut" ($\{\}$) contribution since that has been shifted to the LHS. Also, ψ in the above expressions is the full wavefunction coefficient including bulk-boundary propagators and time integrals. Two comments are in order. First, while applying the Disc operations, the couplings are assumed to be real and since we are considering real fields, therefore, this implies the Hamiltonian is Hermitian. This is how unitarity is encoded in our cutting rules. Second, the Disc operations in the cutting rules take the complex conjugate of the Bogoliubov coefficients and sometimes interchanges α and β (depending on the type of Disc operation performed). Therefore, it relates wavefunction coefficients with different initial conditions. However, this is not so different from cutting rules for amplitudes, which relate scattering processes with different numbers and types of particles. This is more explicit in Sec. 5.3.

Mode-by-mode cutting rules Above we mainly focus on relations that arise from employing only either Disc(1) or Disc(2) for *all* external propagators. These relations are useful in deriving certain properties of wavefunction coefficients, as we will see in Sec. 5.3. However, we are not obliged to employ the *same* propagator identity for all external propagators. We could use any one of the two Disc operators for *each* external propagator. This gives us a total of 2^n cutting rules for an n-point diagram. The additional relations can be very useful if one tries to bootstrap the final answer starting from an ansatz (see [70] for a brief collection of results on the cosmological bootstrap). For example, for a n-point contact wavefunction coefficient we have the 2^n relations:

$$\psi_n(1,2,3,4,\dots) + \psi_n^*(\bar{1},\bar{2},\bar{3},\bar{4},\dots) = 0\,, \tag{5.40}$$

where a bar on the i-th external leg denotes either the a or the b operation in Eq. (5.26) on the kinematics of that external leg. So for example

$$\bar{1} = (\alpha^*_{k_1}, \beta^*_{k_1}, -k_1 - \vec{k}_1) \quad \text{or} \quad \bar{1} = (\beta^*_{k_1}, \alpha^*_{k_1}, k_1, -\vec{k}_1)\,, \tag{5.41}$$

and so on. We do not explore this any further.

5.3 Implications of the Cutting Rules for IR-finite interactions

In this section we derive some general implications of our cutting rules, focusing for concreteness on contact and exchange diagrams. Throughout the discussion, we assume IR-finite interactions, so that the scaling of ψ_n fully fixes the scaling with k (i.e. there are no η_0 regulators).

5.3.1 Contact

We have derived the following two relations for contact wavefunction coefficients,

$$\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) + \psi_n^*\left(\{\alpha^*_{k_i},\beta^*_{k_i}\},\{-k_i\},\{-\vec{k}_i\}\right) = 0\,, \tag{5.42}$$

$$\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) + \psi_n^*\left(\{\beta^*_{k_i},\alpha^*_{k_i}\},\{k_i\},\{-\vec{k}_i\}\right) = 0\,. \tag{5.43}$$

Substituting[3] $k_i \to -k_i$ and $\alpha_{k_i} \to \beta_{k_i}$ in the second relation and using the first, gives another relation,

$$\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right)=\psi_n\left(\{\beta_{k_i},\alpha_{k_i}\},\{-k_i\},\{\vec{k}_i\}\right)\,. \tag{5.44}$$

Using scale invariance we know,

$$\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right)=k_T^{3(1-n)+\sum\Delta}f_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i/k_T\},\{\vec{k}_i/k_T\}\right)\,, \tag{5.45}$$

where Δ is the scaling dimension for the fields in question. For scalars it is defined by,

$$\Delta=\frac{3}{2}+\sqrt{\frac{9}{4}-\frac{m^2}{H^2}}\,, \tag{5.46}$$

where m is the mass of the field. Clearly $\Delta=3$ for massless scalars and $\Delta=2$ for conformally coupled scalars, which have $m^2=2H^2$.
Combining Eq. (5.45) with Eq. (5.44), we get,

$$f_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i/k_T\},\{\vec{k}_i/k_T\}\right)-(-1)^{3(1-n)+\sum\Delta}f_n\left(\{\beta_{k_i},\alpha_{k_i}\},\{k_i/k_T\},\{\vec{k}_i/k_T\}\right)=0\,. \tag{5.47}$$

Two more properties are important. First, for an interaction with N_s spatial derivatives we must have the scaling $\psi_n(-\vec{k}_i)=(-1)^{N_s}\psi_n(\vec{k}_i)$.. Second, for massless fields, $3(1-n)+\sum\Delta=3$. Therefore, for massless scalars, combining these observations with Eq. (5.47) leads to

$$\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right)=-(-1)^{N_s}\psi_n\left(\{\beta_{k_i},\alpha_{k_i}\},\{k_i\},\{\vec{k}_i\}\right)\qquad(\text{massless scalar})\,, \tag{5.48}$$

From this, we conclude that *contact wavefunction coefficients for massless fields are anti-symmetric (symmetric) under exchange $\alpha_{k_i}\leftrightarrow\beta_{k_i}$ for parity even (odd) interactions.*

[3] As we mentioned early, when we take $k_i \to -k_i$ we leave the k_i dependence in α and β unchanged. In other words, we are thinking of ψ_n as a function of independent variables α, β and k_i.

For conformally coupled fields, $3(1-n)+\sum\Delta = 3-n$ and therefore,

$$\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) = -(-1)^{N_s-n}\psi_n\left(\{\beta_{k_i},\alpha_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) \ \text{(conformally coupled)}\,. \tag{5.49}$$

From this we conclude that *for an even number n of conformally coupled scalars and parity even (odd) interaction, ψ_n is anti-symmetric (symmetric) under the exchange $\alpha_{k_i} \leftrightarrow \beta_{k_i}$. For odd n and parity even (odd) interaction, ψ_n is symmetric (anti-symmetric).*

Scale invariance and Eq. (5.42) imply one more relation for the contact wavefunction coefficient,

$$\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) = \psi_n^*\left(\{\alpha^*_{k_i},\beta^*_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) \qquad \text{(massless scalar)}\,, \tag{5.50}$$

which is a Schwarz reflection property and,

$$\begin{aligned}\psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) &= \psi_n^*\left(\{\alpha^*_{k_i},\beta^*_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) \qquad (\text{c.c.},\ n=\text{even})\,,\\ \psi_n\left(\{\alpha_{k_i},\beta_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) &= -\psi_n^*\left(\{\alpha^*_{k_i},\beta^*_{k_i}\},\{k_i\},\{\vec{k}_i\}\right) \quad (\text{c.c.},\ n=\text{odd})\,.\end{aligned} \tag{5.51}$$

These relations we dub Schwarz reflection positive and Schwarz reflection negative. These properties hold for both parity even and parity odd interactions. The relation for massless fields in (5.50) is similar to the corresponding relation for Bunch-Davies initial states but crucially does *not* imply that ψ_n is real. In particular, any real function of α and β still satisfies (5.50), but ψ_n is in general complex for complex α and β. This means that the no-go theorem for the absence of parity-odd interactions at tree level[4] for tensors [159, 106] and scalars [160, 112, 161] do not apply. Indeed this was noticed in [162] where a new shape of parity-odd graviton non-Gaussianity was computed assuming an "α-vacuum" initial state, which is a special case of a Bogoliubov transformation. In the special case of real α and β, the no-go theorems still apply.

[4]Parity-odd loop contributions are non-vanishing [51].

5.3.2 Exchange

Here we derive properties for four-point exchange wavefunction coefficient for massless and conformally coupled scalars. The cutting rules for a 4-point exchange diagram read

$$\begin{aligned}
&\psi_4\left(\{\alpha_{k_i},\beta_{k_i}\},\alpha_p,\beta_p,\{k_i\},\{\vec{k}_i\},p\right)+\psi_4^*\left(\{\alpha^*_{k_i},\beta^*_{k_i}\},\alpha_p,\beta_p,\{-k_i\},\{-\vec{k}_i\},p\right) \quad (5.52)\\
&=-P_p\left[\psi_3\left(\{\alpha_{k_i},\beta_{k_i}\},\alpha_p,\beta_p,k_1,k_2,\{\vec{k}\},p\right)+\psi_3^*\left(\{\alpha^*_{k_i},\beta^*_{k_i}\},\alpha_p,\beta_p,-k_1,-k_2,\{-\vec{k}\},p\right)\right]\\
&\quad\times\left[\psi_3\left(\{\alpha_{k_i},\beta_{k_i}\},\alpha_p,\beta_p,k_3,k_4,\{\vec{k}\},p\right)+\psi_3^*\left(\{\alpha^*_{k_i},\beta^*_{k_i}\},\alpha_p,\beta_p,-k_3,-k_4,\{-\vec{k}\},p\right)\right]\,,
\end{aligned}$$

and,

$$\begin{aligned}
&\psi_4\left(\{\alpha_{k_i},\beta_{k_i}\},\alpha_p,\beta_p,\{k_i\},\{\vec{k}_i\},p\right)+\psi_4^*\left(\{\beta^*_{k_i},\alpha^*_{k_i}\},\alpha_p,\beta_p,\{k_i\},\{-\vec{k}_i\},p\right) \qquad (5.53)\\
&=-P_p\left[\psi_3\left(\{\alpha_{k_i},\beta_{k_i}\},\alpha_p,\beta_p,k_1,k_2,\{\vec{k}\},p\right)+\psi_3^*\left(\{\beta^*_{k_i},\alpha^*_{k_i}\},\alpha_p,\beta_p,k_1,k_2,\{-\vec{k}\},p\right)\right]\\
&\times\left[\psi_3\left(\{\alpha_{k_i},\beta_{k_i}\},\alpha_p,\beta_p,k_3,k_4,\{\vec{k}\},p\right)+\psi_3^*\left(\{\beta^*_{k_i},\alpha^*_{k_i}\},\alpha_p,\beta_p,k_3,k_4,\{-\vec{k}\},p\right)\right]\,.
\end{aligned}$$

The right-hand side of both equations is identical since one can use,

$$K^*(\beta^*_k,\alpha^*_k,k)=K^*(\alpha^*_k,\beta^*_k,-k)\,. \tag{5.54}$$

Therefore,

$$\begin{aligned}
\psi_4\left(\{\alpha_{k_i},\beta_{k_i}\},\alpha_p,\beta_p,\{k_i\},\{\vec{k}_i\},p\right)&=\psi_4\left(\{\beta_{k_i},\alpha_{k_i}\},\alpha_p,\beta_p,\{-k_i\},\{\vec{k}_i\},p\right)\\
&=(-1)^{N_s}\psi_4\left(\{\beta_{k_i},\alpha_{k_i}\},\alpha_p,\beta_p,\{-k_i\},\{-\vec{k}_i\},p\right)\\
&=-(-1)^{N_s}\psi_4\left(\{\beta_{k_i},\alpha_{k_i}\},\beta_p,\alpha_p,\{-k_i\},\{-\vec{k}_i\},-p\right)\\
&=-(-1)^{3(1-n)+\sum\Delta+N_s}\psi_4\left(\{\beta_{k_i},\alpha_{k_i}\},\beta_p,\alpha_p,\{k_i\},\{\vec{k}_i\},p\right)\,.
\end{aligned} \tag{5.55}$$

In the above expression while going from the second to the third line we have used the following property of the bulk-bulk propagator,

$$G_{-p}(\beta_p, \alpha_p, \tau_1, \tau_2) = -G_p(\alpha_p, \beta_p, \tau_1, \tau_2)\,. \tag{5.56}$$

This can be derived very easily by combining Eq. (5.12) & Eq. (5.25). If a time derivative is acting on the internal line then in such a case Feynman rules tell us that we ought to replace (suppressing Bogoliubov arguments) $G_p(\tau_1, \tau_2) \to \partial_{\tau_1}\partial_{\tau_2} G_p(\tau_1, \tau_2)$ and this produces an additional term shown as follows,

$$\begin{aligned}\partial_{\tau_1}\partial_{\tau_2} G_p(\tau_1, \tau_2) = iP_p \Big[&{K'^*}_p(\tau_1)K'_p(\tau_2)\theta(\tau_1-\tau_2) + {K'^*}_p(\tau_2)K'_p(\tau_1)\theta(\tau_2-\tau_1) - K'_p(\tau_1)K'_p(\tau_2) \\ &+ \underbrace{2i\mathrm{Im}\left[K^*_p(\tau_1)K'_p(\tau_2)\right]\delta(\tau_1-\tau_2)}_{\text{additional term}}\Big]\,.\end{aligned} \tag{5.57}$$

Clearly $\partial_{\tau_1}\partial_{\tau_2} G_{-p}(\beta_p, \alpha_p, \tau_1, \tau_2) = -\partial_{\tau_1}\partial_{\tau_2} G_p(\alpha_p, \beta_p, \tau_1, \tau_2)$. Therefore, Eq. (5.55) implies that the 4-point exchange wavefunction for a massless scalar (and even conformally coupled scalars) is symmetric (anti-symmetric) under exchange $\alpha_{k_i,p} \longleftrightarrow \beta_{k_i,p}$ for a parity even (odd) interaction and anti-symmetric (symmetric) for an odd number of conformally coupled fields. We can prove one more property for the exchange wavefunction as follows,

$$\begin{aligned}\psi_4\left(\{\alpha_{k_i}, \beta_{k_i}\}, \alpha_p, \beta_p, \{k_i\}, \{\vec{k}_i\}, p\right) &= -\psi^*_4\left(\{\alpha^*_{k_i}, \beta^*_{k_i}\}, \alpha^*_p, \beta^*_p, \{-k_i\}, \{-\vec{k}_i\}, -p\right) \\ &= -(-1)^{3(1-n)+\sum\Delta}\psi^*_4\left(\{\alpha^*_{k_i}, \beta^*_{k_i}\}, \alpha^*_p, \beta^*_p, \{k_i\}, \{\vec{k}_i\}, p\right)\,.\end{aligned} \tag{5.58}$$

Here, we have used the following property of the bulk-bulk propagator,

$$G^*_{-p}\left(\alpha^*_p, \beta^*_p, \tau_1, \tau_2\right) = G_p\left(\alpha_p, \beta_p, \tau_1, \tau_2\right) \tag{5.59}$$

This property also holds for Eq. (5.57) and so Eq. (5.58) also is valid in this case as well. Therefore, the (anti-)Schwarz reflection properties of contact wavefunctions derived above continue to hold at the four-point level. Again, the no-go theorems mentioned above do not apply except when all the Bogoliubov coefficients are real.

5.4 Explicit examples

In this section, we perform some explicit checks of the above-derived relations. For simplicity, we only take parity-even interactions.

5.4.1 On the convergence of time integrals

Before discussing concrete examples, we need to address the issue of convergence of time integrals for Bogoliubov initial states[5]. In the Bunch-Davies case, convergence in the far past is ensured by the prescription $-\infty \to -\infty(1-i\epsilon)$, which projects the Fock vacuum on the interacting vacuum. In the case of Bogoliubov states, there are a few distinct possibilities. Often one imposes the initial condition at a *finite* initial time $\tau_0 > -\infty$ (see e.g. [16]). This introduces an early time cutoff in the integral. This approach is usually justified by noting that the modes blueshift in the past and therefore the EFT should break down as $\frac{k}{a(\tau_0)} \geq M$, where M is the cutoff for the EFT and k is a mode of interest. This approach displays at least two characteristic properties. First one assumes a Gaussian state at a fixed moment in time for an interacting theory. Second, the final result depends on the cutoff. For example, we get oscillatory features for a sharp cutoff and non-oscillatory behaviour for a smooth cutoff. In this chapter, we instead propose a different physical picture and mathematical prescription to regulate the integral[6]. We have in mind a situation in which a free Bogoliubov initial state is prepared in the infinite past where interactions are switched off and the theory is free. Then, interactions are turned on adiabatically and the state evolves according to the Schrödinger equation in the interacting theory. This picture is implemented mathematically by explicitly multiplying the interaction Hamiltonian by a suitable regulator,

$$H(\tau) \to H_\epsilon = H(\tau)R(\epsilon\tau)\,. \tag{5.60}$$

[5]In this chapter, we only consider IR-finite interaction, so we only have to regulate the possible divergence from the far past ($\tau \to -\infty$).

[6]The same approach was implicitly also taken in [75]. Instead, in [15] a procedure of averaging over η_0 is mentioned but not developed in detail. In practice, in that reference, the contribution from the early boundary is dropped just like we do here invoking the adiabatic switching off of interactions in the infinite past. Finally, it's interesting to notice that the presence of an environment that induces dissipation regulates the integral in a physical way and no artificial cutoffs are necessary [163].

where $R(0) = 1$ and $R(-\infty) = 0$. One can then choose an appropriate regulator (e.g., $e^{\epsilon\tau}$ with $\epsilon > 0$) which turns off the interactions in the past while making the integrals converge (see e.g. [164], or more recent discussions in the cosmological context in [165, 154]). If the limit $\epsilon \to 0$ exists, we can define the results of the regulated calculation. One might ask whether the final answer depends on the choice of R, namely the details of how one turns on and off interactions in the past. In Appendix I we show that, for a suitable class of regulators, the final answer is independent of the choice of the regulator. This property puts this prescription on a firmer footing.

Folded singularities Although using a regulator does help with the convergence in the past, we still are faced with the issue of folded singularities. In particular, ψ_n diverges as $k_i + k_j \to k_l$ for $i \neq j \neq l$. We call these *folded* configurations. If one uses a cutoff M, irrespectively of whether it is sharp or smooth, the folded limit is proportional to $\frac{M}{H}$, which means that the prediction of the EFT is very sensitive to the choice of M for kinematics close to folded triangles. Our interpretation of this is that we can trust our EFT only for configurations that are sufficiently distant from folded divergences. In other words, the EFT prediction is trustworthy everywhere except for configurations very close to folded triangles. Let us make this more precise.

Very generally, if the EFT expansion has to remain valid, we must demand that the corrections given by various operators decrease as the operator dimension increases[7]. Therefore, the correction given by an operator $\mathcal{O}_\Delta$ of dimension Δ must be larger than the one given by an operator $\mathcal{O}_{\Delta'}$ of dimension $\Delta' > \Delta$. To be more quantitative, consider for simplicity correlators induced by a single contact interaction. On general grounds, we expect that the n-point correlation functions have the following

[7]Here we have in mind a simple one-scale power-counting scheme. Of course, more intricate power counting schemes can and do arise in concrete models. See for example [166] for a discussion in the context of inflationary cosmology.

dependence on momenta,

$$\begin{aligned} B_{\varnothing_\Delta} &\sim \left(\frac{H}{M}\right)^{\Delta-4} \frac{k^{-3n+\Delta}}{\tilde{k}_i^{\Delta-3}}\,, \\ B_{\varnothing_{\Delta'}} &\sim \left(\frac{H}{M}\right)^{\Delta'-4} \frac{k^{-3n+\Delta'}}{\tilde{k}_i^{\Delta'-3}}\,. \end{aligned} \tag{5.61}$$

where we introduced the folded pole $\tilde{k}_i = k_T - 2k_i$ and we assumed that momenta k_i are all of the same order denoted by k. The power of $\tilde{k}_i$, call it p, is fixed by the following formula given in [21],

$$p = 1 + \sum_\alpha (\Delta_\alpha - 4)\ , \tag{5.62}$$

where Δ_α is the mass dimension of operators used to compute the correlation function (only one for this contact case). For the EFT to remain valid we want to satisfy the inequality $B_{\varnothing_{\Delta'}} \ll B_{\varnothing_\Delta}$ and therefore we obtain the condition

$$\frac{k}{\tilde{k}_i} \ll \frac{M}{H}\,. \tag{5.63}$$

This roughly quantifies how close one can go to the folded limit ($\tilde{k}_i \to 0$) while remaining within the validity of the EFT.

5.4.2 Contact

Let us check some of above-derived relations explicitly with some simple contact examples. We first compute n-point contact wavefunction coefficient for a massless scalar field ϕ, using the following interaction Hamiltonian,

$$H_\epsilon^\phi = \frac{\lambda}{n!}\int a^{4-n}\phi'^n e^{\epsilon\tau} d^3x \quad \text{with } n \geq 3\,, \tag{5.64}$$

where a is the scale factor and λ is the coupling constant. The standard Feynman rules give

$$\psi_n = i\lambda(-H)^{n-4}\prod_{i=1}^n \frac{k_i^2}{\alpha_{k_i}+\beta_{k_i}}\int e^{\epsilon\tau}\tau^{2n-4}\prod_{j=1}^n \left(\alpha_{k_j}e^{ik_j\tau}+\beta_{k_j}e^{-ik_j\tau}\right)d\tau\,. \tag{5.65}$$

To integrate the above expression we note,

$$\lim_{\epsilon\to\ 0}\int_{-\infty}^{0} \tau^m e^{ik\tau} e^{\epsilon\tau} d\tau = \frac{(-1)^m m!}{(ik)^{m+1}}\,. \tag{5.66}$$

Now, Eq. (5.65) simplifies to,

$$\begin{aligned}\psi_n &= \lambda(-1)^n(-H)^{n-4}(2n-4)!\prod_{i=1}^{n}\left(\frac{k_i^2}{\alpha_{k_i}+\beta_{k_i}}\right)\\ &\times\sum_{m=0}^{n}\sum_{\sigma}\frac{\prod_{j=1}^{m}\alpha_{k_{\sigma_j}}\prod_{l=m+1}^{n}\beta_{k_{\sigma_l}}}{(\sum_{j=1}^{m}k_{\sigma_j}-\sum_{j=m+1}^{n}k_{\sigma_j})^{2(n-2)+1}}\,,\end{aligned} \tag{5.67}$$

where the sum over σ runs over all distinct partitions of $(1, 2, \ldots, n)$ into two subsets of m and $n-m$ elements, such that, after the sum over m, there are 2^n terms. Clearly, the above expression is antisymmetric under exchange $\alpha_{k_i} \leftrightarrow \beta_{k_i}$ and therefore, Eq. (5.48) is satisfied. This expression is also Schwarz reflection positive in agreement with Eq. (5.50).

Now, let us compute an n-point contact wavefunction coefficient for a conformally coupled scalar, σ, using the following interaction Hamiltonian,

$$H^{\sigma}_{\epsilon} = \frac{\lambda}{n!}\int a^4\sigma^n e^{\epsilon\tau} d^3x \quad \text{with } n \geq 4\,. \tag{5.68}$$

Notice that here we impose $n \geq 4$ because for $n = 3$ this interaction gives an IR-divergence. The wavefunction coefficient is given by

$$\psi_n = \frac{i\lambda}{H^4\tau_0^n\prod_{i=1}^{n}(\alpha_{k_i}-\beta_{k_i})}\int e^{\epsilon\tau}\tau^{n-4}\prod_{j=1}^{n}\left(\alpha_{k_i}e^{ik\tau}-\beta_{k_i}e^{-ik\tau}\right)d\tau\,, \tag{5.69}$$

where τ_0 is a late-time cutoff we introduce to be able to extract the leading term for $\tau_0 \to 0$. This expression integrates to,

$$\psi_n = \frac{i^{-n}\lambda(-1)^n(n-4)!}{H^4\tau_0^n\prod_{i=1}^{n}(\alpha_{k_i}-\beta_{k_i})}\left[\sum_{m=0}^{n}\sum_{\sigma}\frac{\prod_{j=1}^{m}\alpha_{k_{\sigma_j}}\prod_{j=m+1}^{n}\beta_{k_{\sigma_j}}-\prod_{j=1}^{m}\beta_{k_{\sigma_j}}\prod_{j=m+1}^{n}\alpha_{k_{\sigma_j}}}{2(\sum_{j=1}^{m}k_{\sigma_j}-\sum_{j=m+1}^{n}k_{\sigma_j})^{n-3}}\right]\,, \tag{5.70}$$

where sum over σ runs again over all distinct partitions of $(1, 2, 3, \ldots, n)$ into two sets of m and $n-m$ elements respectively. The expression in the square brackets is anti-symmetric in $\alpha_{k_i}, \beta_{k_i}$ for all n but the prefactor $\frac{1}{\prod_{i=1}^{n}(\alpha_{k_i}-\beta_{k_i})}$ is symmetric for even n and anti-symmetric for odd n, agreeing with Eq. (5.49). Also, this expression has an i^{-n} present making it Schwarz reflection positive (negative) for $n =$ even (odd) in agreement with Eq. (5.51). Let us now discuss what these relations imply for the contact correlation functions, B_n,

$$B_n = -2\left[\prod_{a=1}^{n} \frac{1}{2\text{Re }\psi_2}\right] \text{Re }\psi_n\,, \tag{5.71}$$

where, for a massless scalar

$$P(k) = \frac{1}{2\text{Re }\psi_2(k)} = \frac{H^2}{2k^3}|\alpha_k + \beta_k|^2\,. \tag{5.72}$$

The above relation implies that if the wavefunction coefficient is a symmetric (anti-symmetric) function of $\alpha_{k_i}, \beta_{k_i}$ then the correlation function is also symmetric (anti-symmetric). Therefore, B_n is anti-symmetric for a massless scalar or for an even number of conformally coupled scalars, while it is symmetric for an odd number of conformally coupled scalars. Finally, we note that for real $\alpha_{k_i}, \beta_{k_i}$, any correlation function involving an arbitrary number of massless scalars and an odd number of conformally coupled scalars must vanish as was already proved in [20].

5.4.3 Exchange

Here, we compute the four-point exchange wavefunction coefficient of a massless scalar for $\frac{\lambda}{3!}\dot{\phi}^3$ interaction.

$$\psi_4 = \frac{-\lambda^2(k_1k_2k_3k_4)^2 p}{2\prod_{i=1}^4(\alpha_{k_i}+\beta_{k_i})}\int d\tau_1 d\tau_2 e^{\epsilon(\tau_1+\tau_2)}\tau_1^2\tau_2^2\left[\prod_{i=1}^{2}\left(\alpha_{k_i}e^{ik_i\tau_1}+\beta_{k_i}e^{-ik_i\tau_1}\right)\right.$$
$$\times\prod_{i=3}^{4}\left(\alpha_{k_j}e^{ik_j\tau_2}+\beta_{k_j}e^{-ik_j\tau_2}\right)\left(\left(\alpha_p^*e^{-ip\tau_1}+\beta_p^*e^{is\tau_1}\right)\left(\alpha_p e^{ip\tau_2}+\beta_p e^{-ip\tau_2}\right)\theta(\tau_1-\tau_2)\right.$$
$$\left.\left.+(\tau_1\leftrightarrow\tau_2)-\frac{(\alpha_p+\beta_p)^*}{(\alpha_p+\beta_p)}\left(\alpha_p e^{ip\tau_1}+\beta_p e^{-ip\tau_1}\right)\left(\alpha_p e^{ip\tau_2}+\beta e^{-ip\tau_2}\right)+\frac{i}{p}|\alpha_p+\beta_p|^2\delta(\tau_1-\tau_2)\right)\right]\,, \tag{5.73}$$

which gives,

$$
\begin{aligned}
&\frac{-\lambda^2(k_1k_2k_3k_4)^2p}{2\prod_{i=1}^4(\alpha_{k_i}+\beta_{k_i})}\Bigg\{\Big[\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\alpha_{k_3}\alpha_{k_4}\alpha_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{k_3+k_4+p\})\\
&\qquad+\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\alpha_{k_3}\beta_{k_4}\alpha_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{k_3-k_4+p\})\\
&\qquad+\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\beta_{k_3}\alpha_{k_4}\alpha_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{-k_3+k_4+p\})\\
&\qquad+\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\beta_{k_3}\beta_{k_4}\alpha_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{-k_3-k_4+p\})\\
&\qquad+\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\alpha_{k_3}\alpha_{k_4}\beta_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{k_3+k_4-p\})\\
&\qquad+\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\alpha_{k_3}\beta_{k_4}\beta_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{k_3-k_4-p\})\\
&\qquad+\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\beta_{k_3}\alpha_{k_4}\beta_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{-k_3+k_4-p\})\\
&\qquad+\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\beta_{k_3}\beta_{k_4}\beta_p\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1+k_2-p,\{-k_3-k_4-p\})\\
&\qquad+\left(\alpha_{k_1}\beta_{k_2}\alpha_p^*\{...\}+\alpha_i\leftrightarrow\beta_i\right)f(k_1-k_2-p,\{...\})\leftarrow\text{Eight terms}\\
&\qquad+\left(\beta_{k_1}\alpha_{k_2}\alpha_p^*\{...\}+\alpha_i\leftrightarrow\beta_i\right)f(-k_1+k_2-p,\{...\})\leftarrow\text{Eight terms}\\
&\qquad+\left(\beta_{k_1}\beta_{k_2}\alpha_p^*\{...\}+\alpha_i\leftrightarrow\beta_i\right)f(-k_1-k_2-p,\{...\})\leftarrow\text{Eight terms}\Big]\\
&\qquad+[k_1,k_2\longleftrightarrow k_3,k_4]\\
&\quad-\Bigg(\frac{\alpha_{k_1}\alpha_{k_2}\alpha_p-\alpha_i\leftrightarrow\beta_i}{(k_1+k_2+p)^3}+\frac{\alpha_{k_1}\beta_{k_2}\alpha_p-\alpha_i\leftrightarrow\beta_i}{(k_1-k_2+p)^3}+\frac{\beta_{k_1}\alpha_{k_2}\alpha_p-\alpha_i\leftrightarrow\beta_i}{(-k_1+k_2+p)^3}\\
&\qquad\frac{-\beta_{k_1}\beta_{k_2}\alpha_s+\alpha_i\leftrightarrow\beta_i}{(k_1+k_2-p)^3}\Bigg)\times(k_1,k_2\longrightarrow k_3,k_4)\times\frac{(\alpha_p+\beta_p)^*}{(\alpha_p+\beta_p)}\\
&\qquad+\frac{24\left(|\alpha_p|^2-|\beta_p|^2\right)}{p}\Bigg(\frac{\alpha_{k_1}\alpha_{k_2}\alpha_{k_3}\alpha_{k_4}-\beta_{k_1}\beta_{k_2}\beta_{k_3}\beta_{k_4}}{(k_1+k_2+k_3+k_4)^5}\\
&\qquad+\frac{\alpha_{k_1}\alpha_{k_2}\alpha_{k_3}\beta_{k_4}-\beta_{k_1}\beta_{k_2}\beta_{k_3}\alpha_{k_4}}{(k_1+k_2+k_3-k_4)^5}+\frac{\alpha_{k_1}\alpha_{k_2}\beta_{k_3}\beta_{k_4}-\beta_{k_1}\beta_{k_2}\alpha_{k_3}\alpha_{k_4}}{(k_1+k_2-k_3-k_4)^5}\\
&\qquad+\text{perm}\Bigg)\Bigg\}.
\end{aligned}
\tag{5.74}
$$

The result turned out to be quite lengthy and therefore several shorthand expressions were used. Whenever we have $k_i \leftrightarrow k_j$ it also applies to subscripts i.e $\alpha_{k_i} \leftrightarrow \beta_{k_j}$. The function $f(a,b)$ is given by,

$$
f(a,b)=\frac{-4(a^2+5ab+10b^2)}{b^3(a+b)^5}\,. \tag{5.75}
$$

Finally, anything within {} is repeated in each set of "Eight terms" indicated above as the first eight terms. As expected Eq. (5.74) is in agreement with Eq. (5.55) and Eq. (5.58). To expose the pole structures, the above result can be simplified further by noting that terms appearing in $[k_1, k_2 \longleftrightarrow k_3, k_4]$ are the same as terms above it except with $f(g_1(k_1, k_2) \pm p, g_2(k_3, k_4) \pm p) \longleftrightarrow f(g_2(k_3, k_4) \pm p, g_1(k_1, k_2) \pm p)$, which in words means interchanging only external energy functions appearing in the argument of f without touching p. Therefore, one can pairwise sum these functions and simplify the expression further. This does not simplify terms with opposite sign of p in two arguments, e.g., terms like $f(k_1 + k_2 - p, k_3 + k_4 + p) \sim \frac{1}{(k_3+k_4+p)^3(k_1+k_2+k_3+k_4)^5}$ but it does simplify terms with the same sign of p in both arguments, in other words, terms that would otherwise give poles of the form $\frac{1}{k_1+k_2+k_3+k_4-2p}$. The pairwise sum of such terms makes these poles disappear and we get a product of three-point poles, e.g.,

$$\begin{aligned} &\left(\alpha_{k_1}\alpha_{k_2}\alpha_p^*\{\beta_{k_3}\beta_{k_4}\beta_p\} + \alpha_i \leftrightarrow \beta_i\right) \\ \times &\left[f(k_1 + k_2 - p, -k_3 - k_4 - p) + f(-k_3 - k_4 - p, k_1 + k_2 - p)\right] \\ &= \frac{4}{(k_1 + k_2 - p)^3 (k_3 + k_4 + p)^3} . \end{aligned} \tag{5.76}$$

5.5 From Bunch-Davies to Bogoliubov

Since the Bogoliubov mode functions are just linear combinations of positive and negative frequency solutions of Bunch-Davies ones, it is reasonable to expect that one could write the Bogoliubov answer in terms of linear combinations of the Bunch-Davies answer with various signs of energies flipped and weighted by the product of Bogoliubov coefficients. In this section, we derive such relations for interactions without derivatives. This can be generalised in a straightforward way for derivative interactions.

5.5.1 Contact Prescription

The contact wavefunction coefficient is given by,

$$\psi_n = \frac{i\lambda}{\prod_{i=1}^{n}(\alpha_{k_i}+\beta_{k_i})}\int\prod_{i=1}^{n}K_{k_i}(\alpha_{k_i},\beta_{k_i})d\tau\,, \tag{5.77}$$

$$= \frac{i\lambda}{\prod_{i=1}^{n}(\alpha_{k_i}+\beta_{k_i})}\int\prod_{i=1}^{n}\left(\alpha_{k_i}\mathcal{K}_{k_i}(\tau)+\beta_{k_i}\mathcal{K}^*_{k_i}(\tau)\right)d\tau\,, \tag{5.78}$$

where $\mathcal{K}_k(\tau)$ is the Bunch-Davies bulk-boundary propagator. Using $\mathcal{K}_{-k}(\tau)=\mathcal{K}^*_k(\tau)$, the above equation can be rewritten as,

$$\psi_n = \frac{i\lambda}{\prod_{i=1}^{n}(\alpha_{k_i}+\beta_{k_i})}\int\prod_{i=1}^{n}\left(\alpha_{k_i}\mathcal{K}_{k_i}(\tau)+\beta_{k_i}\mathcal{K}_{-k_i}(\tau)\right)d\tau\,, \tag{5.79}$$

which can be expanded as follows,

$$\psi_n = \frac{i\lambda}{\prod_{i=1}^{n}(\alpha_{k_i}+\beta_{k_i})}\sum_{m=0}^{n}\sum_{\sigma}\left(\prod_{j=1}^{m}\alpha_{k_{\sigma_j}}\prod_{j=m+1}^{n}\beta_{k_{\sigma_j}}\int d\tau\prod_{j=1}^{m}\mathcal{K}_{k_{\sigma_j}}\prod_{k=m+1}^{n}\mathcal{K}_{-k_{\sigma_j}}\right)\,, \tag{5.80}$$

where the sum over σ runs over all distinct bipartitions of $(1,2,\ldots,n)$ into two sets of size m and $n-m$, respectively. The above relation can be expressed in terms of the Bunch-Davies wavefunction coefficients,

$$\psi_n = \sum_{r=0}^{n}\sum_{\sigma}\left[\prod_{p=1}^{r}\alpha_{k_{\sigma_p}}\prod_{q=r+1}^{n}\beta_{k_{\sigma_q}}\psi_n^{BD}\left(\{k\}_r,-\{k\}_{n-r}\right)\right]\,, \tag{5.81}$$

where $\{k\}_r=\{k_{\sigma_1},....,k_{\sigma_r}\}$ and $\{k\}_{n-r}=\{k_{\sigma_{r+1}},....,k_{\sigma_n}\}$ and σ runs over all distinct partitions of $(1,2,3,\ldots,n)$ into two sets of r and $n-r$ elements respectively.

5.5.2 Exchange prescription

For the contact case, we only needed the expression of Bogoliubov bulk-boundary propagator in terms of Bunch-Davies one but for the four-point exchange case, one needs a similar expression for the bulk-bulk propagator. We find,

$$\begin{aligned} G_p(\alpha_p, \beta_p, \tau_1, \tau_2) &= |\alpha_p|^2 \mathcal{G}_p(\tau_1, \tau_2) - |\beta_p|^2 \mathcal{G}_{-p}(\tau_1, \tau_2) + A\mathcal{K}_p(\tau_1)\mathcal{K}_p(\tau_2) \\ +B\mathcal{K}_{-p}(\tau_1)\mathcal{K}_{-p}(\tau_2) &- \frac{(\alpha_p+\beta_p)^* \alpha_p\beta_p}{2p^3(\alpha_p+\beta_p)} \left[\mathcal{K}_p(\tau_1)\mathcal{K}_{-p}(\tau_2) + \mathcal{K}_{-p}(\tau_1)\mathcal{K}_p(\tau_2)\right] , \end{aligned} \tag{5.82}$$

where

$$A = \frac{1}{2p^3}\left(|\alpha_p|^2 + \alpha_p\beta_p^* - \frac{\alpha_p^2(\alpha_p+\beta_p)^*}{(\alpha_p+\beta_p)}\right) , \tag{5.83}$$

$$B = \frac{1}{2p^3}\left(|\beta_p|^2 + \alpha_p^*\beta_p - \frac{\beta_p^2(\alpha_p+\beta_p)^*}{(\alpha_p+\beta_p)}\right) , \tag{5.84}$$

and $\mathcal{G}_p(\tau_1, \tau_2)$ is the bulk-bulk propagator for the Bunch-Davies case. Let us now compute the four-point exchange diagram for a massless scalar field.

$$\begin{aligned} \psi_4 = \frac{-\lambda^2}{\prod_{i=1}^4(\alpha_{k_i}+\beta_{k_i})} \int d\tau_1 d\tau_2 \prod_{i=1}^{2}\left(\alpha_{k_i}\mathcal{K}_{k_i}(\tau_1) + \beta_{k_i}\mathcal{K}_{-k_i}(\tau_1)\right) \prod_{i=3}^{4}\left(\alpha_{k_i}\mathcal{K}_{k_i}(\tau_2) + \beta_{k_i}\mathcal{K}_{-k_i}(\tau_2)\right) \\ \times \big(|\alpha_p|^2\mathcal{G}_p(\tau_1,\tau_2) - |\beta_p|^2\mathcal{G}_{-p}(\tau_1,\tau_2) + A\mathcal{K}_p(\tau_1)\mathcal{K}_p(\tau_2) + B\mathcal{K}_{-p}(\tau_1)\mathcal{K}_{-p}(\tau_2) \\ -\frac{(\alpha_p+\beta_p)^*\alpha_p\beta_p}{2p^3(\alpha_p+\beta_p)}\left[\mathcal{K}_p(\tau_1)\mathcal{K}_{-p}(\tau_2) + \mathcal{K}_{-p}(\tau_1)\mathcal{K}_p(\tau_2)\right]\Big) , \end{aligned} \tag{5.85}$$

which simplifies to,

$$\begin{aligned} |\alpha_p|^2 \big[\alpha_{k_1}\alpha_{k_2}\{\alpha_{k_3}\alpha_{k_4}\}\psi_4^{BD}(k_1,k_2,\{k_3,k_4\}) + \alpha_{k_1}\alpha_{k_2}\{\alpha_{k_3}\beta_{k_4}\}\psi_4^{BD}(k_1,k_2,\{k_3,-k_4\}) \\ \alpha_{k_1}\alpha_{k_2}\{\beta_{k_3}\alpha_{k_4}\}\psi_4^{BD}(k_1,k_2,\{-k_3,k_4\}) + \alpha_{k_1}\alpha_{k_2}\{\beta_{k_3}\beta_{k_4}\}\psi_4^{BD}(k_1,k_2,\{-k_3,-k_{-4}\}) \\ + \underbrace{\alpha_{k_1}\beta_{k_2}\{...\}\psi_4^{BD}(k_1,-k_2,\{...\})}_{\text{Four terms}} + \underbrace{\alpha_{k_2}\beta_{k_1}\{...\}\psi_4^{BD}(-k_1,k_2,\{...\})}_{\text{Four terms}} \\ + \underbrace{\beta_{k_1}\beta_{k_2}\{...\}\psi_4^{BD}(-k_1,-k_2,\{...\})}_{\text{Four terms}}\Bigg] - |\beta_p|^2\left[p \longrightarrow -p\right] \\ +A\left[\psi_4^{BD}(k_i,k_j,k_m,k_n) \longrightarrow \psi_3^{BD}(k_i,k_j,p)\psi_3^{BD}(k_m,k_n,p)\right] \\ +B\left[p \longrightarrow -p, \psi_4^{BD}(k_i,k_j,k_m,k_n) \longrightarrow \psi_3^{BD}(k_i,k_j,p)\psi_3^{BD}(k_m,k_n,p)\right] \\ -\frac{(\alpha_+\beta_p)^*}{2p^3(\alpha_p+\beta_p)}\big[\psi_4^{BD}(k_i,k_j,k_m,k_n) \longrightarrow \psi_3^{BD}(k_i,k_j,p)\psi_3^{BD}(k_m,k_n,-p) \\ +\psi_4^{BD}(k_i,k_j,k_m,k_n) \longrightarrow \psi_3^{BD}(k_i,k_j,-p)\psi_3^{BD}(k_m,k_n,p)\big] . \end{aligned} \tag{5.86}$$

Anything within {} is repeated in each set of "Four terms" indicated above in the same order as the first four terms. As one can see the relation is already quite complicated for four-point exchange and is expected to get even worse for higher-point non-contact diagrams, therefore such a prescription may not be of much practical use for these higher-point non-contact diagrams.

5.6 Extension to any mass and (integer) spin

In this section, we *generalise Bogoliubov cutting rules to fields of any mass and spin*. It turns out that it is straightforward to generalise cutting rules to spinning fields once they are generalised to scalar fields of any mass. However, there are two apparent obstructions to extending the rules to massive scalars:

- **Propagator identities:** In the case of a Bunch-Davies initial state, the propagator identity, $\mathcal{K}^*_{-k} = \mathcal{K}_k$, is valid for scalar fields of any mass provided we approach the negative energies from below i.e., $-k = e^{-i\pi}k$[8]. However, in the case of a Bogoliubov initial state, there are two propagator identities,

$$K^*_{-k}(\alpha^*_k, \beta^*_k) = K_k(\alpha_k, \beta_k)\,, \tag{5.87}$$

$$K^*_k(\beta^*_k, \alpha^*_k) = K_k(\alpha_k, \beta_k)\,. \tag{5.88}$$

  and the identity, $K^*_{-k}(\alpha^*_k, \beta^*_k) = K_k(\alpha_k, \beta_k)$, does not extend to massive scalars. Since the mode function includes both types of Hankel functions (see (5.94), (5.95) & (5.96)), no matter how one approaches the negative energies (from above or below), the propagator identity mentioned above is never satisfied owing to the transformation properties of the two Hankel functions given in (5.102) & (5.103). Upon some inspection, it turns out that the massive scalar propagator instead satisfies a modified identity,

$$K^*_{-k}((\alpha_k + 2i\beta_k \cos\pi\nu)^*, \beta^*_k, \tau) = K_k(\alpha_k, \beta_k, \tau)\,, \tag{5.89}$$

[8]See [20, 147, 24] for more details.

where $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$. Thankfully, this identity reduces to $K^*_{-k}(\alpha^*_k, \beta^*_k) = K_k(\alpha_k, \beta_k)$ in the case of a massless ($\nu = 3/2$) or a conformally coupled scalar field ($\nu = 1/2$). Therefore, one obtains a generalised identity valid for scalar fields of any mass. The propagator identity, $K^*_k(\beta^*_k, \alpha^*_k) = K_k(\alpha_k, \beta_k)$, extends trivially to massive scalars.

- **Analytical continuation:** Since the negative energies are always approached from below, one analytically continues the wavefunction coefficients to the lower half complex plane. In the case of Bunch-Davies initial state, since the mode functions contain only positive frequencies, the analytic continuation to the lower half plane is well defined i.e., the time integrals in the past converge $\forall k_i \in \mathbb{C}^{n-}$. In the case of Bogoliubov initial states, the mode functions have both positive and negative frequency which for the propagator implies,

$$\lim_{\tau \to -\infty} K_k(\alpha_k, \beta_k, \tau) \sim e^{i\mathrm{Re}(k)\tau} e^{-\mathrm{Im}(k)\tau} + e^{-i\mathrm{Re}(k)\tau} e^{\mathrm{Im}(k)\tau} \,, \qquad \text{where } k \in \mathbb{C} \,. \tag{5.90}$$

  The time integrals will converge neither in the upper nor in the lower plane since one of the two terms will always diverge in the far past. However, since we always employ an adiabatic function to turn off interactions in the past, we will, with some care, be able to analytically continue the wavefunction to the complex plane.

5.6.1 Cutting rules for any mass and spin

In this section, we will extend the Bogoliubov cutting rules to fields of any mass and spin. Since the form of the bulk-bulk propagator as a function of the bulk-boundary propagator is unchanged, one only needs to define appropriate "Discontinuity" operations (Disc) to extend the cutting rules. The definition of the Disc operation will be determined by the propagator identities discussed below.

5.6.1.1 Propagator identities and corresponding "Discontinuities"

One of the key ingredients to deriving the cutting rules is propagator identities. In the case of a Bunch-Davies initial state, the identity reads,

$$\mathcal{K}^*_{-k} = \mathcal{K}_k \,. \tag{5.91}$$

This identity is crucial in identifying the appropriate Disc operation for the cutting rules. It was shown in [24] that this identity generalises to fields of any mass and spin. In [145], we discussed analogous propagator identities for a Bogoliubov initial state for massless and conformally coupled scalars,

$$K^*_{-k}(\alpha^*_k, \beta^*_k) = K_k(\alpha_k, \beta_k) \,, \tag{5.92}$$

$$K^*_k(\beta^*_k, \alpha^*_k) = K_k(\alpha_k, \beta_k) \,. \tag{5.93}$$

We stress that these relations are derived by keeping the k dependence of α_k, β_k separate from the rest of the k dependence in $K_k(\alpha_k, \beta_k)$.

Massive scalars

To see which of these identities extends to the massive case, we highlight some salient properties of the massive mode functions. It is well known that the mode functions in this case are Hankel functions,

$$\varphi^+_k(\tau) = ie^{-i\frac{\pi}{2}(\nu+\frac{1}{2})}\sqrt{\pi}\frac{H}{2}(-\tau)^{\frac{3}{2}}H^{(2)}_\nu(-k\tau) \,, \tag{5.94}$$

$$\varphi^-_k(\tau) = -ie^{i\frac{\pi}{2}(\nu+\frac{1}{2})}\sqrt{\pi}\frac{H}{2}(-\tau)^{\frac{3}{2}}H^{(1)}_\nu(-k\tau) \,, \tag{5.95}$$

where $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$. In the case of a Bunch-Davies initial state only $\varphi^+_k(\tau)$ survives. However, for a Bogoliubov initial state, the mode functions multiplying the raising operator, Φ^+_k, is a linear combination of the two solutions given above,

$$\Phi^+_k(\alpha_k, \beta_k, \tau) = \alpha_k \varphi^+_k(\tau) + \beta_k \varphi^-_k(\tau) \,. \tag{5.96}$$

The corresponding bulk-boundary propagator is given by,

$$K_k(\alpha_k, \beta_k, \tau) = \frac{\Phi_k^+(\alpha_k, \beta_k, \tau)}{\Phi_k^+(\alpha_k, \beta_k, \tau_0)} \,. \tag{5.97}$$

Let us list some useful properties of Hankel functions before we derive the propagator identities.

$$H_\nu^{(1)*}(z^*) = H_{\nu^*}^{(2)}(z)\,, \tag{5.98}$$
$$H_{-\nu}^{(1)}(z) = e^{i\pi\nu} H_\nu^{(1)}(z)\,, \tag{5.99}$$
$$H_{-\nu}^{(2)}(z) = e^{-i\pi\nu} H_\nu^{(2)}(z)\,. \tag{5.100}$$

Using above equations one can immediately see that $[\Phi_k^+(\beta_k^*, \alpha_k^*, \tau)]^* = \Phi_k^+(\alpha_k, \beta_k, \tau)$ and consequently the propagator identity (5.93) is true for massive scalars also. Therefore, we define the corresponding Disc operation as Disc(1) given in [145],

$$\begin{aligned}
&\underset{\{\alpha_{p_m},\beta_{p_m}\}}{\text{Disc}(1)} \left[f(\{\alpha_{k_i}, \beta_{k_i}\}, \{\alpha_{p_m}, \beta_{p_m}\}, \{k_i\}, \{\vec{k}_i\}, \{p_m\})\right] = \\
&f(\{\alpha_{k_i}, \beta_{k_i}\}, \{\alpha_{p_m}, \beta_{p_m}\}, \{k_i\}, \{\vec{k}_i\}, \{p_m\}) - f^*(\{\beta_{k_i}^*, \alpha_{k_i}^*\}, \{\alpha_{p_m}, \beta_{p_m}\}, \{k_i\}, \{-\vec{k}_i\}, \{p_m\})\,.
\end{aligned} \tag{5.101}$$

The cutting rules involving Disc(2) are non-trivial to extend because the analytical continuation of the bulk-boundary propagator to negative energies is not as straightforward as in the Bunch-Davies case. We explain this below.

Note that Hankel functions have a branch point at $z = 0$. Therefore, we will follow the standard convention of choosing the principal branch i.e., $-\pi < \arg(z) < \pi$ (branch cut on the negative x-axis). Approaching the branch cut from above, we get,

$$H_\nu^{(1)}(e^{i\pi} z) = -e^{-i\pi\nu} H_\nu^{(2)}(z)\,, \tag{5.102}$$
$$H_\nu^{(2)}(e^{i\pi} z) = e^{i\pi\nu} H_\nu^{(1)}(z) + (2\cos\pi\nu) H_\nu^{(2)}(z)\,. \tag{5.103}$$

Using the properties mentioned above, it was shown that for the Bunch-Davies case if one approaches the negative k-axis from below, the propagator identity (5.91) remains intact. The reason is very simple: the bulk-boundary propagator for the

Bunch-Davies case is given by,

$$\mathcal{K}_k(\tau) = \frac{\varphi_k^+(\tau)}{\varphi_k^+(\tau_0)}\,, \tag{5.104}$$

and if one defines $-k = e^{-i\pi}k$ then, using the properties above, it is straightforward to check that $[\varphi_{-k}^+(\tau)]^* = i\varphi_k^+(\tau)$ and therefore, $\mathcal{K}_{-k}^* = \mathcal{K}_k$. Let us now analyse the Bogoliubov case. It is immediately clear from (5.102) & (5.103) that $K_{-k}^*(\alpha_k^*, \beta_k^*) \neq K_k(\alpha_k, \beta_k)$ since, $[\Phi_{-k}^+(\alpha_k^*, \beta_k^*, \tau)]^* \neq c\Phi_k^+(\alpha_k, \beta_k, \tau)$, where $c \in \mathbb{C}$. More precisely,

$$[\Phi_{-k}^+(\alpha_k^*, \beta_k^*, \tau)]^* = i\Phi_k^+(\alpha_k - 2i\beta_k \cos \pi\nu^*, \beta_k, \tau)\,. \tag{5.105}$$

However, one is still able to define a modified propagator identity as follows,

$$K_{-k}^*((\alpha_k + 2i\beta_k \cos \pi\nu^*)^*, \beta_k^*, \tau) = K_k(\alpha_k, \beta_k, \tau)\,. \tag{5.106}$$

Notice that in the massless or conformally coupled limit the above expression reduces to $K_{-k}^*(\alpha_k^*, \beta_k^*, \tau) = K_k(\alpha_k^*, \beta_k^*, \tau)$ as expected. Motivated by (5.106), we can define a new disc operation (we call it Disc$(\tilde{2})$) as follows,

$$\begin{aligned}
&\underset{\{\alpha_{p_m},\beta_{p_m}\}}{\mathrm{Disc}(\tilde{2})}\,[f(\{\alpha_{k_i}, \beta_{k_i}\}, \{\alpha_{p_m}, \beta_{p_m}\}, \{k_i\}, \{\vec{k}_i\}, \{p_m\})] = \\
&f(\{\alpha_{k_i}, \beta_{k_i}\}, \{\alpha_{p_m}, \beta_{p_m}\}, \{k_i\}, \{\vec{k}_i\}, \{p_m\}) - \\
&f^*(\{\alpha_{k_i}^* - 2i\beta_{k_i}^* \cos \pi\nu, \beta_{k_i}^*\}, \{\alpha_{p_m}, \beta_{p_m}\}, \{-k_i\}, \{-\vec{k}_i\}, \{p_m\})\,.
\end{aligned} \tag{5.107}$$

This generalises the cutting rules involving Disc(2) to massive scalars. As expected for $\nu = 3/2$ & $1/2$ this Disc operation reduces to the Disc(2) operation for massless and conformally coupled scalars mentioned in [145]. Again, we stress that negative energies are approached from below i.e., $-k = e^{-i\pi}k$. To summarize, we give the two propagator identities for massive scalars,

$$K_k^*(\beta_k^*, \alpha_k^*) = K_k(\alpha_k, \beta_k)\,, \tag{5.108}$$

$$K_{-k}^*((\alpha_k + 2i\beta_k \cos \pi\nu^*)^*, \beta_k^*, \tau) = K_k(\alpha_k, \beta_k, \tau)\,. \tag{5.109}$$

Spinning fields

Now, we extend the cutting rules to spinning fields. We consider a very generic action for a spin s field $\sigma^{i_1 i_2 \ldots i_s}$ as given in [167],

$$S_2 = \frac{1}{2s!}\int a^3 d^3x dt \left[\left(\dot{\sigma}^{i_1 i_2 \ldots i_s}\right)^2 - c_s^2 a^{-2}\left(\partial_j \sigma^{i_1 i_2 \ldots i_s}\right)^2 - \delta c_s^2 a^{-2}\left(\partial_j \sigma^{j i_2 \ldots i_s}\right)^2 \right.$$
$$\left. - \left(m\sigma^{i_1 i_2 \ldots i_s}\right)^2\right], \quad (5.110)$$

where $\sigma^{i_1 i_2 \ldots i_s}$ is a totally symmetric traceless tensor. Such an action arises in inflationary setups where the spinning field couples to the inflaton. As shown in [24, 167], one can decompose the field in helicity basis to make the propagators diagonal i.e. different helicity components decouple (the term $\delta c_s^2 a^{-2}\left(\partial_j \sigma^{j i_2 \ldots i_s}\right)^2$ gives the non-diagonal part). It is simplest to illustrate this for the spin-1 case. In this case the non-diagonal term in the action in Fourier space is given by,

$$\sigma^i(\vec{k}) k_i k_j \sigma^j(-\vec{k})\,. \quad (5.111)$$

In matrix notation (we will suppress the momentum dependence of fields), this term reads,

$$\boldsymbol{\sigma}^T \boldsymbol{M} \boldsymbol{\sigma}\,, \quad (5.112)$$

where $M_{ij} = k_i k_j$. Now, we decompose the fields in terms of eigenvectors of $\boldsymbol{M}$ i.e.,

$$\sigma_i = \Sigma_{h=-1}^{+1} \sigma^h \epsilon_i^h\,, \quad (5.113)$$

where $\boldsymbol{\epsilon}^h$ are eigenvectors of $\boldsymbol{M}$ with eigenvalues λ^h. Using this fact and the condition $(\epsilon^h)^i (\epsilon^{h'})^i = \delta^{hh'}$, we get,

$$\boldsymbol{\sigma}^T \boldsymbol{M} \boldsymbol{\sigma} = \Sigma_{h,h'} \sigma_h \sigma_{h'} \lambda^h (\boldsymbol{\epsilon}^{h'})^T \boldsymbol{\epsilon}^h = (\sigma^h)^2 \lambda^h\,, \quad (5.114)$$

which is diagonal in helicities. Therefore, the equation of motion for each helicity component is given by,

$$(\sigma^h)'' - \frac{2}{\tau}(\sigma^h)' + \left((c_s^h k)^2 + m^2\right)\sigma^h = 0\,, \quad (5.115)$$

where each helicity component propagates with speed c_s^h which is only a function of c_s & δc_s [167]. The same logic follows for spin-s fields and therefore, one again gets the same equation of motion for each helicity component. The solutions to this equation are Hankel functions i.e., $H_\nu^{(1,2)}(-c_s^h k\tau)$, with $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$. These are the same mode functions as that of massive scalars, therefore, the cutting rules are naturally extended to fields of any spin.

5.6.1.2 Cutting rules

Given that we now have appropriate Disc operations for massive scalar fields we present the final cutting rule relation. First note that the relation between the bulk-bulk propagator and the bulk-boundary propagator is exactly as the massless and conformally coupled scalar case. The relation reads,

$$G_p(\tau_1,\tau_2) = iP_p\left[{K^*}_p(\tau_1)K_p(\tau_2)\theta(\tau_1-\tau_2) + {K^*}_p(\tau_2)K_p(\tau_1)\theta(\tau_2-\tau_1) - K_p(\tau_1)K_p(\tau_2)\right] . \tag{5.116}$$

Therefore, the imaginary part of a string of bulk-bulk propagators follows the same factorisation property as the one proved in [147] for the Bunch-Davies case. The only thing which changes is the definition of the cut i.e., the Disc operations. Therefore, the cutting rule reads,

$$\begin{aligned} &i\mathrm{Disc}(m)\left[i\psi^{(D)}\right] \\ &= \sum_{\text{cuts}}\left[\prod_{\text{cut momenta}}\int P\right]\prod_{\text{sub-diagram}}(-i)\underset{\text{internal \& cut lines}}{\mathrm{Disc}(m)}\left[i\psi^{(\text{sub-diagram})}\right] , \end{aligned} \tag{5.117}$$

where m can take values 1 or $\tilde{2}$. In (5.117) we have suppressed the spin labels. *This equation is valid for fields of any mass and spin.*

5.6.2 Convergence in the far past

Here we address the issue of convergence of the time integrals in the far past. In [145], we employed an adiabatic function of the form $R(\epsilon\tau) = e^{\epsilon\tau}$ [9] to ensure convergence as $\tau \to -\infty$. Such a regularisation scheme is required for a Bogoliubov initial state since the mode functions include both negative- and positive-frequencies. Now, we know

[9] In [145], it was shown that the final results are independent of the choice of the adiabatic function $R(\epsilon\tau)$.

that the cutting rules involve wavefunction coefficients where some of the energies are taken to negative values. In the case of massless and conformally coupled scalars, one can simply replace k by $-k$ since the mode functions have no branch points. However, for massive scalars due to the branch point, one needs to specify how one approaches the negative axis. In this chapter, we resolved this by performing an analytic continuation to the lower half complex plane. However, note that the asymptotic limit of bulk-boundary for the Bogoliubov case has both positive and negative exponentials, therefore, in the complex plane one has,

$$\lim_{\tau\to-\infty} K_k(\alpha_k,\beta_k,\tau) \sim e^{i\mathrm{Re}(k)\tau}e^{-\mathrm{Im}(k)\tau} + e^{-i\mathrm{Re}(k)\tau}e^{\mathrm{Im}(k)\tau}\,, \qquad \text{where } k\in\mathbb{C}\,. \tag{5.118}$$

Naively, one might think that the integrals therefore will converge neither in the upper nor in the lower half complex plane. However, as mentioned above we always use an adiabatic function, $R(\epsilon\tau)$ to turn off interactions in the far past. This improves the convergence of the integrals. Therefore, one can perform the integrals assuming that $\epsilon > |\mathrm{Im}(k)|$ and then take the limit $\epsilon\to 0$. This procedure defines a well-behaved analytic continuation in the lower-half complex plane.

Chapter 6

Conclusion and outlook

In this thesis, we discussed various aspects of cosmological observables, i.e., correlation functions and wavefunction coefficients. We developed a connection between the tree-level analytic structure of three-point correlators and the initial state of inflation. Moreover, we explicitly illustrated that not every excited state can produce folded poles ($k_1 + k_2 \sim k_3$). We then discussed soft theorems satisfied by these tree-level correlation functions and provided an explicit verification of mixed soft theorems (scalar+tensor) in the Effective Field Theory of Inflation (EFToI). We also commented on the possibility of bootstrapping such correlators using the so-called boostless bootstrap. We then moved on to discuss correlators and wavefunction coefficients beyond tree-level. We discussed loop effects during inflation and explicitly performed computations of correlation functions at one-loop in the EFToI. In particular, we derived explicit log running of the three-point function at one-loop. Finally, we derived a set of unitarity constraints on the cosmological wavefunction coefficients valid at any order in perturbation theory.

Some of the interesting future directions include:

- Deriving unitarity constraints for excited states which are not Bogoliubov transforms of the vacuum, e.g., coherent states.
- Understanding non-perturbative implications of unitarity for cosmological observables. If they exist, such relations can in principle be used to constrain de Sitter EFTs.

- Understanding implications of unitarity using the recently developed formalism of de Sitter S-matrix [168, 169, 154]. It might be simpler to derive a non-perturbative relation for the S-matrix directly via $S^{\dagger}S = 1$ than wavefunction or correlation functions.

- It is well known in the context of flat space QFT that a violation of unitarity is tantamount to missing on-shell states in the spectrum. In light of this, a violation of cosmological unitarity relations can, in principle, be used to probe additional degrees of freedom during inflation.

- In the context of loop cosmology, it would be interesting to develop a systematic understanding of one-loop correlation functions. In particular, classify the possible analytic structures for very generic field content and interactions. There has been some progress in this direction, e.g., a systematic study of two-site one-loop diagrams was done in [170].

- Computing three- and higher-site loop diagrams is very complicated and, as of now, seems intractable. However, since we are mostly interested in the non-analytic part, there might be a way of calculating this.

- In this thesis, we discussed loop effects generated by simple derivative operators present in the EFToI in the decoupling limit. It would be very interesting to compute, say three-point function at one-loop in the canonical case of single-field slow-roll inflation (Einstein gravity+minimally coupled scalar field with a potential $V(\phi)$). This computation is phenomenologically most relevant if we ever measure loop effects in the data.

Appendix A

Calculating the action in Unitary Gauge

From the definitions of the ADM metric variables, we have $\widetilde{N^i} = -g_{00}N^i = N^2N^i$. We further write $N = 1 + \delta N$ and then consider the following quadratic action:

$$\mathcal{L}_2 = \sqrt{-g}M_{pl}^2 \bigg(\frac{1}{2}R^{(4)} + m_1 \delta K^i_j \delta K^j_i + m_2(\delta K)^2 + D\delta K - M_{pl}^2 \lambda(t) - c(t)g^{00}$$
$$+ M_1 g^{ij}\partial_i(g^{00}+1)\partial_j(g^{00}+1) + M_2^2(g^{00}+1)^2 + M_3(g^{00}+1)\delta K + m_3^{(3)}R(g^{00}+1) \bigg) \tag{A.1}$$

where $c(t)$,$\lambda(t)$ are as defined before. Taking $\widetilde{N^i} = \partial_i \chi$, the equations of motion for N and $\widetilde{N^i}$ gives

$$\delta N\left((m_1+3m_2-1)H - M_3\right) + (m_1+m_2)\partial^2\chi = \dot{\zeta}(m_1+3m_2-1) \tag{A.2}$$

$$\partial^2\chi = \frac{-1}{(m_1+3m_2-1)H - M_3}\left(-\partial^2\zeta a^{-2} + c(t)\delta N + 4M_2^2\delta N - 4a^{-2}M_1\partial^2\delta N\right) - 3\dot{\zeta} \tag{A.3}$$
$$+ \frac{3\dot{\zeta}H(m_1+3m_2-1)}{(m_1+3m_2-1)H - M_3} - \frac{3\dot{\zeta}H(m_1+3m_2-1)M_3}{((m_1+3m_2-1)H - M_3)^2}$$

To solve these equations, one will have to take $m_1 + m_2 = 0$ so that the equations separate.

Appendix B

Calculating the RR vertex for ${}^{(3)}R_{ij}^{(3)}R_{ij}$

The expression for the diagram where both the vertices are time ordered, i.e. RR vertices [56] after taking the soft limit is given by:

$$
\begin{aligned}
&\langle \gamma^{h_1}(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\zeta(\vec{k}_3)|_{\vec{k}_3\to 0}\rangle|_{RR} = \frac{H^6}{M_{pl}^6 k_1^9 4\epsilon k_3^3}\left(\frac{M_{pl}^2\epsilon}{M_7^2(4\cdot 8)}2\cdot 2\cdot 2\cdot 2\; e_{ij}^{h_1}(\vec{k}_1)e_{ij}^{h_2}(\vec{k}_2)\right)\\
&\left[\int_{-\infty}^{0}\left(k_1^2k_2^2e^{2ik_1\eta} - (k_1\cdot k_2)\frac{(1-ik_1\eta)(1-ik_2\eta)}{\eta^2}e^{2ik_1\eta}\right)\left(\int_{\eta}^{0}(1+ik_2\eta')k_2^4(1-ik_2\eta')\right)d\eta d\eta' + \right.\\
&\left.\int_{-\infty}^{0}\left(k_1^2k_2^2 - (k_1\cdot k_2)\frac{(1-ik_1\eta)(1+ik_2\eta)}{\eta^2}\right)\left(\int_{-\infty}^{\eta}(1-ik_2\eta')^2k_2^4e^{2ik_2\eta}\right)d\eta d\eta'\right]
\end{aligned}
\tag{B.1}
$$

where we have taken $\vec{k}_3 \to 0$ but have not yet put $\vec{k}_1 = -\vec{k}_2$ for clarity. All the combinatorial and numerical/coupling factors are in the bracket in the first line. Note that for the Maldacena vertex, we have not taken the non-local term [49], as that term is 0 (or rather subleading) in the soft limit. After putting $\vec{k}_1 = -\vec{k}_2$ and simplifying we get:

$$
\langle \gamma^{h_1}(\vec{k}_1)\gamma^{h_2}(\vec{k}_2)\zeta(\vec{k}_3)|_{k_3\to 0}\rangle|_{RR} = -\frac{15c_1H^6}{16M_{pl}^6k_1^3k_3^3} \tag{B.2}
$$

Appendix C

Soft limit of $\langle\gamma\zeta\zeta\rangle$ for ${}^{(3)}R\delta K$

The "exchange diagram" is the same as the right diagram in Figure 2 and we have, to the 1st order in c_1:

$$O_{2\zeta} = \int 4a\frac{M_{pl}^2}{M_5}\partial^2\zeta\left(\epsilon\dot{\zeta} - \frac{\partial^2\zeta}{H}a^{-2}\right) \tag{C.1}$$

$$O_{\gamma\zeta\zeta} = \text{Maldacena terms} \quad + 4\frac{M_{pl}^2}{M_5}\left(2\frac{\gamma_{ij}a^{-1}}{H}\partial_i\partial_j\zeta\partial^2\zeta - a\gamma_{ij}\partial_i\partial_j\zeta\dot{\zeta}\right) \tag{C.2}$$

This gives a correction to the 3-point function, in the limit $\vec{k}_1 \to 0$

$$\langle\gamma^h(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = -B_{LL} - B_{RR} + B_{RL} + B_{LR} + B_{\text{contact}} \tag{C.3}$$

where the contact vertex is the one from $\mathcal{O}_{\gamma\zeta\zeta}$. One finds that $B_{RR} + B_{LL} = B_{\text{contact}}$ and we can easily see that

$$\begin{aligned} B_{LR}(k_1 \to 0) = B_{RL}(k_1 \to 0) &= \frac{1}{2}\frac{B_{MT}(k_1 \to 0, k_2 = k_3)\delta P_\zeta(k_3)}{P_\zeta(k_3)} \\ &= \frac{3}{4}e^h_{ij}(\vec{k}_1)\frac{1}{k_2^2}k_{2i}k_{2j}P_\gamma(k_1)\delta P_\zeta(k_2) \end{aligned} \tag{C.4}$$

and hence the soft limit (Equation 3.3) is satisfied.

Appendix D

Purely cubic operators

Here, we give explicit expressions for operators that contribute to the mixed correlators. Note that for these calculations, we have taken 1.99, 1.100 as the quadratic actions, i.e. the Maldacena quadratic action. Λ_i's are the UV cutoffs for each term while $c_i's$ are dimensionless quantities.

Purely Cubic Operators for $\langle\gamma\gamma\zeta\rangle$

$$\mathcal{O}_1 = \int \sqrt{-g}\Lambda_1^2 \delta K_{ij}\delta K_{ij}\delta g^{00} = \int \Lambda_1^2 a^3 \frac{1}{2H}\dot{\gamma_{ij}}\dot{\gamma_{ij}}\dot{\zeta} \tag{D.1}$$

$$\mathcal{O}_2 = \int \sqrt{-g}\Lambda_2 {}^{(3)}\!R_{ij}\delta K_{ij}\delta g^{00} = \int \Lambda_2 a \frac{1}{2H}\partial^2\gamma_{ij}\dot{\gamma_{ij}}\dot{\zeta} \tag{D.2}$$

$$\mathcal{O}_3 = \int \sqrt{-g}c_3 {}^{(3)}\!R_{ij} {}^{(3)}\!R_{ij}\delta g^{00} = \int c_3 a^{-1}\frac{1}{2H}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\dot{\zeta} \tag{D.3}$$

$$\mathcal{O}_4 = \int \sqrt{-g}c_4 {}^{(3)}\!R_{ij}\delta K_{jk}\delta K_{ki} = \int c_4 a\left(\frac{1}{2}\partial^2\gamma_{ij}\dot{\gamma_{ij}}\dot{\zeta} - \frac{1}{2H}a^{-2}\partial^2\gamma_{ij}\dot{\gamma_{jk}}\partial_i\partial_k\zeta\right. \tag{D.4}$$

$$\left. + \frac{1}{2}\epsilon\partial^2\gamma_{ij}\dot{\gamma_{jk}}\partial_i\partial_k\partial^{-2}\dot{\zeta} - \frac{1}{4}\dot{\gamma_{ij}}\dot{\gamma_{jk}}\partial_i\partial_k\zeta - \frac{1}{4}\dot{\gamma_{ij}}\dot{\gamma_{ij}}\partial^2\zeta\right) \tag{D.5}$$

$$\mathcal{O}_5 = \int \sqrt{-g}\frac{1}{\Lambda_5}{}^{(3)}R^{(3)}_{ij}R_{jk}\delta K_{ki} = \int \frac{1}{\Lambda_5}\left(\frac{1}{4H}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{jk}\partial_k\partial_i\zeta\right. \tag{D.6}$$

$$\left.-\frac{1}{4}\epsilon a^{-1}\partial^2\gamma_{ij}\partial^2\gamma_{jk}\partial_k\partial_i\partial^{-2}\dot{\zeta}\right) + \frac{1}{2}a^{-1}\partial^2\gamma_{ij}\dot{\gamma}_{ik}\partial_j\partial_k\zeta + \frac{1}{2}a^{-1}\partial^2\gamma_{ij}\dot{\gamma}_{ij}\partial^2\zeta \tag{D.7}$$

$$\mathcal{O}_6 = \int \sqrt{-g}\frac{1}{\Lambda_6}{}^{(3)}R^{(3)}_{ij}R_{ij}\delta K = \int \frac{1}{\Lambda_6}\left(\frac{1}{4H}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\partial^2\zeta - \frac{1}{4}\epsilon a^{-1}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\dot{\zeta}\right) \tag{D.8}$$

$$\mathcal{O}_7 = \int \sqrt{-g}\frac{1}{\Lambda_7}{}^{(3)}R_{ij}\delta K_{ij}{}^{(3)}R = \int \frac{1}{\Lambda_7}a^{-1}\partial^2\gamma_{ij}\dot{\gamma}_{ij}\partial^2\zeta \tag{D.9}$$

$$\mathcal{O}_8 = \int \sqrt{-g}\frac{1}{\Lambda_8^2}{}^{(3)}R_{ij}{}^{(3)}R_{jk}{}^{(3)}R_{ki} = \int \frac{1}{\Lambda_8^2}\left(-\frac{3}{4}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{jk}\partial_k\partial_i\zeta - \frac{3}{4}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\partial^2\zeta\right) \tag{D.10}$$

$$\mathcal{O}_9 = \int \sqrt{-g}\frac{1}{\Lambda_9^2}{}^{(3)}R_{ij}{}^{(3)}R_{ij}{}^{(3)}R = \int -\frac{1}{\Lambda_9^2}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\partial^2\zeta \tag{D.11}$$

$$\mathcal{O}_{10} = \int \sqrt{-g}\Lambda_{10}\delta K_{ij}\delta K_{jk}\delta K_{ki} = \int \frac{3}{4}a^3\Lambda_{10}\dot{\gamma}_{ij}\dot{\gamma}_{jk}\left(\partial_k\partial_i\frac{\zeta}{H}a^{-2} - \epsilon\partial_k\partial_i\partial^{-2}\dot{\zeta}\right) \tag{D.12}$$

$$\mathcal{O}_{11} = \int \sqrt{-g}\Lambda_{11}\delta K_{ij}\delta K_{ji}\delta K = \int \frac{3}{4}a^3\Lambda_{11}\dot{\gamma}_{ij}\dot{\gamma}_{ji}\left(\partial^2\frac{\zeta}{H}a^{-2} - \epsilon\dot{\zeta}\right) \tag{D.13}$$

Purely Cubic operators for $\langle\gamma\zeta\zeta\rangle$

$$\mathcal{O}_1 = \int \sqrt{-g}\Lambda_1^2 \delta K_{ij} \widetilde{N}_i \partial_j \delta g^{00} = \int \Lambda_1^2 \left(-a\frac{\dot{\gamma}_{ij}}{H^2}\partial_i \zeta \partial_j \dot{\zeta} + a^3 \frac{\epsilon}{H}\dot{\gamma}_{ij}\partial_i \dot{\zeta}\partial_j \partial^{-2}\dot{\zeta} \right) \tag{D.14}$$

$$\mathcal{O}_2 = \int \sqrt{-g}\Lambda_2 {}^{(3)}R_{ij} \delta K_{ij} \delta g^{00} = \int \Lambda_2 \left(-a^{-1}\frac{\partial^2 \gamma_{ij}}{H^2}\partial_i \partial_j \zeta \dot{\zeta} + \epsilon a \frac{\partial^2 \gamma_{ij}}{H^2}\partial_i \partial_j \dot{\zeta}\dot{\zeta} \right) \tag{D.15}$$

$$\mathcal{O}_3 = \int \sqrt{-g}\Lambda_3 \delta K_{ij} \partial_i \delta g^{00} \partial_j \delta g^{00} = \int 2\Lambda_3 a \dot{\gamma}_{ij}\partial_i \dot{\zeta}\partial_j \dot{\zeta} \tag{D.16}$$

$$\mathcal{O}_4 = \int \sqrt{-g}\Lambda_4 {}^{(3)}R_{ij} \widetilde{N}_i \partial_j \delta g^{00} = \int \Lambda_4 \left(a^{-1}\frac{\partial^2 \gamma_{ij}}{H^2}\partial_i \zeta \partial_j \dot{\zeta} - \epsilon a \frac{\partial^2 \gamma_{ij}}{H}\partial_i \dot{\zeta}\partial_j \partial^{-2}\dot{\zeta} \right) \tag{D.17}$$

$$\mathcal{O}_5 = \int \sqrt{-g} c_5 {}^{(3)}R_{ij} \delta K_{ij} \delta K = \int c_5 \left(-a^{-3}\frac{\partial^2 \gamma_{ij}}{2H^2}\partial_i \partial_j \zeta \partial^2 \zeta + \epsilon a^{-1}\frac{\partial^2 \gamma_{ij}}{2H}\partial^2 \zeta \partial_i \partial_j \partial^{-2}\dot{\zeta} \right. \tag{D.18}$$

$$\left. + \epsilon a^{-1}\frac{\partial^2 \gamma_{ij}}{2H}\dot{\zeta}\partial_i \partial_j \zeta - \epsilon^2 a \frac{\partial^2 \gamma_{ij}}{2}\dot{\zeta}\partial_i \partial_j \partial^{-2}\dot{\zeta} \right) \tag{D.19}$$

$$\begin{aligned} \mathcal{O}_6 = \int \sqrt{-g} c_6 {}^{(3)}R_{ij} \delta K_{jk} \delta K_{ki} = &\int -\frac{1}{2}c_6 a \partial^2 \gamma_{ij}\partial_j \partial_k \left(-\frac{\zeta}{H}a^{-2} + \epsilon \partial^{-2}\dot{\zeta} \right) \partial_i \partial_k \left(-\frac{\zeta}{H}a^{-2} + \epsilon \partial^{-2}\dot{\zeta} \right) \\ &- c_6 a \dot{\gamma}_{ij}\partial_j \partial_k \zeta \partial_k \partial_i \left(-a^{-2}\frac{\zeta}{H} + \epsilon \partial^{-2}\dot{\zeta} \right) - c_6 a \dot{\gamma}_{ij}\partial^2 \zeta \partial_j \partial_i \left(-a^{-2}\frac{\zeta}{H} + \epsilon \partial^{-2}\dot{\zeta} \right) \end{aligned} \tag{D.20}$$

$$\mathcal{O}_7 = \int \sqrt{-g} c_7 {}^{(3)}R_{ij} \partial_i \delta g^{00} \partial_j \delta g^{00} = \int -2c_7 a^{-1}\frac{\partial^2 \gamma_{ij}}{H^2}\partial_i \dot{\zeta}\partial_j \dot{\zeta} \tag{D.21}$$

$$\mathcal{O}_8 = \int \sqrt{-g} c_8 {}^{(3)}R_{ij} {}^{(3)}R_{ij} \delta g^{00} = \int 2c_8 a^{-1}\partial^2 \gamma_{ij}\partial_i \partial_j \zeta \frac{\dot{\zeta}}{H} \tag{D.22}$$

$$\mathcal{O}_9 = \int \sqrt{-g}\frac{1}{\Lambda_9}{}^{(3)}R_{ij}{}^{(3)}R_{jk}\delta K_{ki} = \int \frac{1}{\Lambda_9}\left(a^{-3}\frac{\partial^2\gamma_{ij}}{H}\partial_i\partial_k\zeta\partial_j\partial_k\zeta + a^{-3}\partial^2\frac{\gamma_{ij}}{H}\partial_i\partial_j\partial^2\zeta\right. \tag{D.23}$$

$$\left. +\frac{1}{2}a^{-1}\dot{\gamma_{ij}}\partial_i\partial_k\zeta\partial_j\partial_k\zeta + a^{-1}\dot{\gamma_{ij}}\partial_i\partial_j\zeta\partial^2\zeta - \epsilon a^{-1}\partial^2\gamma_{ij}\partial_j\partial_k\zeta\partial_k\partial_i\partial^{-2}\dot{\zeta} - \epsilon a^{-1}\partial^2\gamma_{ij}\partial^2\zeta\partial_i\partial_j\partial^{-2}\dot{\zeta}\right) \tag{D.24}$$

$$\mathcal{O}_{10} = \int \sqrt{-g}\frac{1}{\Lambda_{10}}{}^{(3)}R_{ij}{}^{(3)}R_{ij}\delta K = \int \frac{1}{\Lambda_{10}}\left(a^{-3}\frac{\partial^2\gamma_{ij}}{H}\partial_i\partial_j\zeta\partial^2\zeta - \epsilon a^{-1}\partial^2\gamma_{ij}\partial_i\partial_j\zeta\dot{\zeta}\right) \tag{D.25}$$

$$\mathcal{O}_{11} = \int \frac{1}{\Lambda_{11}^2}{}^{(3)}R_{ij}{}^{(3)}R_{jk}{}^{(3)}R_{ki} = \int -\frac{3}{2}\frac{1}{\Lambda_{11}^2}a^{-3}\partial^2\gamma_{ij}\partial_i\partial_k\zeta\partial_j\partial_k\zeta - 3\frac{c_{11}}{\Lambda^2}a^{-3}\partial^2\gamma_{ij}\partial_i\partial_j\zeta\partial^2\zeta \tag{D.26}$$

$$\mathcal{O}_{12} = \int \sqrt{-g}\frac{1}{\Lambda_{12}^2}{}^{(3)}R_{ij}{}^{(3)}R_{ij}{}^{(3)}R = \int -4\frac{1}{\Lambda_{12}^2}a^{-3}\partial^2\gamma_{ij}\partial_i\partial_j\zeta\partial^2\zeta \tag{D.27}$$

$$\mathcal{O}_{13} = \int \sqrt{-g}\Lambda_{13}\delta K_{ij}\delta K_{jk}\delta K_{ki} = \int \frac{3}{2}a^3\Lambda_{13}\dot{\gamma_{ij}}\partial_j\partial_k\left(\frac{\zeta}{H}a^{-2} - \epsilon\partial^{-2}\dot{\zeta}\right)\partial_k\partial_i\left(\frac{\zeta}{H}a^{-2} - \epsilon\partial^{-2}\dot{\zeta}\right) \tag{D.28}$$

$$\mathcal{O}_{14} = \int \sqrt{-g}\Lambda_{14}\delta K_{ij}\delta K_{ji}\delta K = \int a^3\Lambda_{14}\dot{\gamma_{ij}}\partial_j\partial_i\left(\frac{\zeta}{H}a^{-2} - \epsilon\partial^{-2}\dot{\zeta}\right)\left(\partial^2\frac{\zeta}{H}a^{-2} - \epsilon\dot{\zeta}\right) \tag{D.29}$$

Appendix E

Issue with Gauge

The EFT of inflation is written after fixing the time diffs, so naturally, the operators in the EFT do not respect time diffs. Therefore, they are NOT gauge-invariant. Consider the following operator

$$\sqrt{-g}\left(\delta g^{00}(t)\right)^2 {}^{(3)}R(t) \tag{E.1}$$

Naively if we just calculate the cubic ζ interactions coming from this operator, one can easily see that we get a non-zero answer in the unitary gauge, but zero in the flat gauge. Now, one can make this operator completely gauge-invariant by introducing the Stueckelberg field, π. The gauge invariant operator reads

$$\sqrt{-g}\left(\frac{\partial(t+\pi)}{\partial x^\mu}\frac{\partial(t+\pi)}{\partial x^\nu}g^{\mu\nu}+1\right)^2 {}^{(3)}R(t+\pi) = \sqrt{-g}\left[(1+\dot{\pi})^2 g^{00} + \partial_i\pi\partial_j\pi g^{ij}\right. \tag{E.2}$$

$$\left. +2g^{0i}\partial_i\pi(1+\dot{\pi})+1\right]^2 {}^{(3)}R(t+\pi) \tag{E.3}$$

Since we are interested in the cubic vertex, we need the expression for ${}^{(3)}R$ only up to first order.

$$\begin{aligned} {}^{(3)}R &= \partial_k\Gamma^k_{ii} - \partial_i\Gamma^k_{ik} &\text{(E.4)}\\ &= -a^{-2}\partial^2 g_{ii} &\text{(E.5)}\end{aligned}$$

The operator becomes

$$= -\left[(1+\dot{\pi})^2 g^{00} + \partial_i\pi\partial_j\pi g^{ij} + 2g^{0i}\partial_i\pi(1+\dot{\pi}) + 1\right]^2 e^{-2\rho}\partial^2 g_{ii}(t+\pi) \tag{E.6}$$

Let us evaluate this operator in the two gauges [49]
Flat gauge: $\delta\phi \neq 0$

$$= -6(\delta N|_{\psi=0} - \dot{\pi})^2 H\partial^2\pi \tag{E.7}$$

$$= -6\left(\frac{\dot{\zeta}}{H}\right)^2 \partial^2\zeta \tag{E.8}$$

Co-moving gauge: $\delta\phi = 0$

$$= -6\frac{\dot{\zeta}^2}{H^2}\partial^2\zeta \tag{E.9}$$

The vertex for ζ matches in the two gauges as expected. Since, this operator just contains derivatives, at leading order, this gives a vanishing contribution to the local bispectrum.

Appendix F

Loop Momentum Identity

Here, we compute momentum integrals given by

$$D_1(k_i) = \int d^d\vec{p}_1 d^d\vec{p}_2 \; Q(p_1, p_2, \{k_i\})\delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3)\,. \tag{F.1}$$

To evaluate this, we note that Eq. (F.1) can be rewritten, by carrying out the integral over $\vec{p}_2$ using the delta function, as follows

$$\begin{aligned}
&= \int d^d\vec{p}_1 \; Q(p_1, |\vec{k}_3 - \vec{p}_1|, \{k_i\}) = \int p_1^{d-1} Q(p_1, |\vec{k}_3 - \vec{p}_1|, \{k_i\}) \; dp_1 d\Omega_d\,, \\
&= \int p_1^{d-1} \; Q(p_1, |\vec{k}_3 - \vec{p}_1|, \{k_i\}) \; dp_1 \sin^{d-2}\theta_1 \sin^{d-3}\theta_2 ... \sin\theta_{d-2} \; d\theta_1 d\theta_2 d\theta_{d-2} d\phi\,,
\end{aligned} \tag{F.2}$$

where ϕ is the azimuthal angle and θ_i's denote polar angles. If $\hat{k}_3$ is chosen along axis with respect to which θ_1 is measured, then

$$\begin{aligned}
&|\vec{k}_3 - \vec{p}_1| = \sqrt{k_3^2 + p_1^2 - 2k_3 p_1 \cos\theta_1} = p_2 \\
&\text{or, } \frac{kp_1}{p_2} \sin\theta_1 d\theta_1 = dp_2
\end{aligned}$$

Thus, a variable change from θ_1 to p_2 in Eq. (F.2) gives us

$$\begin{aligned} &= \int \frac{p_1^{d-2} p_2}{k_3}\, Q(p_1, p_2, \{k_i\}) \sin^{d-3}\theta_1\; dp_1 dp_2 d\Omega_{d-1}\,, \\ &= S_{d-2} \int \frac{p_1^{d-2} p_2}{k_3}\, Q(p_1, p_2, \{k_i\}) \sin^{d-3}\theta_1\; dp_1 dp_2\,, \end{aligned} \tag{F.3}$$

where S_{d-2} is surface area of a $(d-2)$ - unit sphere, and $\sin\theta_1 = \sqrt{1 - \left(\frac{k_3^2 + p_1^2 - p_2^2}{2k_3 p_1}\right)^2}$. Defining $p_+ = p_1 + p_2$ and $p_- = p_1 - p_2$, we get

$$\begin{aligned} &\int d^d\vec{p}_1 d^d\vec{p}_2\; Q(p_1, p_2, \{k_i\}) \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \\ &= \frac{S_{d-2}}{2} \int_{k_3}^{\infty} dp_+ \int_{-k_3}^{k_3} dp_- \frac{p_1^{d-2} p_2}{k_3} \sin^{d-3}\theta_1\; Q(p_1, p_2, \{k_i\})\,. \end{aligned} \tag{F.4}$$

The factor of $\sin^{d-3}\theta_1$ was dropped in [23]. While it does not contribute to the $\mathcal{O}(\frac{1}{\delta})$ divergent terms in the correlator, it does give finite contributions and hence is important in determining the shape and pole structure of the correlator away from the equilateral limit.

Appendix G

Computation of D_2 for Diagram I

Here we explicitly compute the $\mathcal{O}(\delta)$ contribution from mode functions and scale factors for the diagram with time ordered vertices in Fig. 4.1. The integral is given by

$$D_2 = g_3 g_4 \; \delta^3\big(\sum_i \vec{k}_i\big) \frac{H^9}{\left(-\dot{H}M_{pl}^2\right)^5} \frac{1}{8k_1k_2k_3} \int d^d\vec{p}_1 d^d\vec{p}_2 \; \frac{p_1p_2}{4} \delta(I_1 + I_2)\,, \tag{G.1}$$

where I_1 and I_2 are the integrals with time orderings $\tau_1 > \tau_2$ and $\tau_2 > \tau_1$ respectively,

$$\begin{aligned}
I_1(k_i,p_j) &= \int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \; e^{i(k_{12}+p_+)\tau_2} e^{i(k_3-p_+)\tau_1} \left(\frac{1}{2}\log(-H\tau_1) + \log(-H\tau_2)\right) \\
&= I_1^{\tau_1}(k_i,p_j) + I_1^{\tau_2}(k_i,p_j)\,, \\
I_2(k_i,p_j) &= \int_{-\infty}^{0} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 \; e^{i(k_{12}-p_+)\tau_2} e^{i(k_3+p_+)\tau_1} \left(\frac{1}{2}\log(-H\tau_1) + \log(-H\tau_2)\right) \\
&= I_2^{\tau_1}(k_i,p_j) + I_2^{\tau_2}(k_i,p_j)\,,
\end{aligned} \tag{G.2}$$

where we refer to the terms having log dependence on the time integrated from $-\infty$ to 0 as $I_1^{\tau_1}(k_i,p_j)$ and $I_2^{\tau_2}(k_i,p_j)$, i.e., $\log(-H\tau_1)$ in I_1 and $\log(-H\tau_2)$ in I_2 respectively. Next, we consider terms having log dependence on the time being integrated from $-\infty$ to $\tau_{1(2)}$, and call them $I_1^{\tau_2}(k_i,p_j)$ and $I_2^{\tau_1}(k_i,p_j)$, i.e., $\log(-H\tau_2)$ in I_1 and $\log(-H\tau_1)$ in I_2 respectively.

- From $I_1^{\tau_1}$ and $I_2^{\tau_2}$, we get functions of p_+, k_{12} and k_3, plus terms dependent on $\log k_T$. The former gives a $\frac{1}{\delta}$ divergent term after the loop integrals, independent of logarithms, hence giving a finite contribution to D_2. The log term gives the following finite contribution to D_2 after performing loop integrals,

$$g_3 g_4 \ \delta^3\Big(\sum_i \vec{k}_i\Big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{k_3(k_1+k_2)(k_3-2k_1-2k_2)}{k_1 k_2 k_T^7} \frac{3}{2} \log\left(\frac{H}{k_T}\right) . \quad \text{(G.3)}$$

- From $I_1^{\tau_2}$ and $I_2^{\tau_1}$, we once again get functions of q_+, k_{12} and k_3, and a $\log k_T$ term. Additionally, one also gets terms dependent on $\log(p_+ + k_{12})$ and $\log(p_+ + k_3)$ for $I_1^{\tau_2}$ and $I_2^{\tau_1}$ respectively. However, upon p_- integration, the p_+ dependence of the result

$$\sim \frac{p_+^4 \left(p_+^2 - k_{12}^2\right)^{\delta/2} \log\left(p_+ + \frac{k_{12}}{k_3}\right)}{\left(p_+ - k_{12}\right)^3 \left(p_+ + \frac{k_{12}}{k_3}\right)^5} , \quad \text{(G.4)}$$

vanishes in the limit $p_+ \to \infty$. Thus the contributions from such terms to D_2 are of $\mathcal{O}(\delta)$, and vanish when $d \to 3$.

The remaining log independent terms give a finite, log-free contribution to D_2, and the contribution from $\log\, k_T$ terms after performing loop integrals is the same as Eq. (G.3)

Hence, summing the contributions, we have

$$D_2 = g_3 g_4 \ \delta^3\Big(\sum_i \vec{k}_i\Big) \frac{H^9}{\left(-\dot{H} M_{pl}^2\right)^5} \frac{k_3(k_1+k_2)(k_3-2k_1-2k_2)}{k_1 k_2 k_T^7} 3 \log\left(\frac{H}{k_T}\right) + \text{finite} . \quad \text{(G.5)}$$

Adding this to D_1 gives Eq. (4.21)

Appendix H

Computation of $\mathcal{O}(\delta)$ terms containing $\log(-H\tau)$

In Appendix G, we showed explicitly that D_2 can be computed by pulling the log term out of the integrals and substituting $\tau \to 1/k_T$, along with some finite terms. In this appendix, we show that this is also true for the D_2 contributions for all diagrams computed in Sec. 4.3

Note that D_2 is given by the same integral as D_1 with an additional factor of $\log \tau$. To compute this contribution, one would need to evaluate up to three time integrals, with logarithms dependent on any of the time vertices. The explicit computation of D_2 will be lengthy since each such integral can have various time orderings. Instead, let us take a look at the general structure of these integrals.

For diagrams with a cubic and a quartic interaction, the time integrals have the form,

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \ \tau_1^n \tau_2^m e^{ia\tau_1} e^{ib\tau_2} \log\left(-H\tau_{1(2)}\right) , \tag{H.1}$$

where $a + b = k_T$. In the integrals we compute, (m, n) take the values $(2, 2)$, $(2, 3)$ in Eqns. (4.26), (4.25), (4.28), and $(2, 4)$ in Eqns. (4.19), (4.26), (4.25), (4.28). The log acts on τ_1 or τ_2 . For three cubic diagrams, the time integrals are of the type

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \; \tau_1^m \tau_2^n \tau_3^l e^{ia\tau_1} e^{ib\tau_2} e^{ic\tau_3} \log\left(-H\tau_{1(2,3)}\right) \,, \tag{H.2}$$

where $a+b+c=k_T$ and $m,n,l=2$ for our computations. The log acts on τ_1,τ_2 or τ_3.

- **Integrals with** $\log(-\mathbf{H}\boldsymbol{\tau_1})$ **:** Performing the integral over τ_2 for Eq. (H.1) (and over τ_3 for Eq. (H.2)), we are left with

$$\int_{-\infty}^{0} d\tau_1 \; \tau_1^m e^{ik_T\tau_1} g(k_i, p_j, \tau_1) \log(-H\tau_1) \,, \tag{H.3}$$

  where g is a polynomial of τ_1, with coefficients depending on k_i and p_j. Such integrals evaluate to the following,

$$f_1(k_i, p_j) + \log\left(\frac{H}{k_T}\right) f_2(k_i, p_j) \,, \tag{H.4}$$

  where $f_1(k_i, p_j)$ is a finite term and $f_2(k_i, p_j)$ is the result of the time integrals evaluated without any logs.

- **Integrals with** $\log(-\mathbf{H}\boldsymbol{\tau_2})$ **:** Evaluation of τ_2 integral in Eq. (H.1) (and τ_3 integral in Eq. (H.2)) gives the following sum of exponential integral functions (Ei) and polynomials in τ_1,

$$\begin{aligned} &\int_{-\infty}^{0} d\tau_1 \; \tau_1^m e^{ia\;\tau_1} \{\mathrm{Ei}\left[i(b+c)\tau_1\right] g_1(k_i, p_j) + e^{i(b+c)\;\tau_1} g_2(k_i, p_j, \tau_1) \\ &+ e^{i(b+c)\;\tau_1} \log\tau_1 \; g_3(k_i, p_j, \tau_1)\} \,, \end{aligned} \tag{H.5}$$

  where g_2 and g_3 are polynomials in τ_1 and g_1 is a function of k_i and p_j. Upon evaluation of this integral, the first and second terms give some finite functions of k_i and p_j. The first term also gives a log term of the form $\log\left(\frac{k_T}{p_+ + k_{12}}\right)$. There are however no additional log contributions from this piece since it converges as $p_+ \to \infty$, which implies this contribution dies at $\delta \to 0$. This was the case for e.g. in Fig 4.1a, where the functional form of the extra log term in Eqn. G.4 was such that the loop integrals were convergent.

It turns out that the function g_3 is the result of τ_2,τ_3 time integrals performed without logarithms. Thus the third term in Eq. (H.5) has the same structure as Eq. (H.3), and hence the result will have the structure of Eq. (H.4).

- **Integrals with** $\log(-\mathbf{H}\tau_3)$ **:** Evaluation of τ_3 in Eq. (H.2) once again gives a sum of exponential integral functions and polynomials of τ_2,

$$\int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2\ \tau_1^m \tau_2^n \{\mathrm{Ei}\,[ic\tau_2]\, h_1(k_i,p_j) + e^{ic\tau_1} h_2(k_i,p_j,\tau_2) + e^{ic\tau_2} \log\tau_2\ h_3(k_i,p_j,\tau_2)\}\,, \tag{H.6}$$

where h_2 and h_3 are polynomials of τ_2 and h_1 is a function of k_i and p_j.

The second term gives some finite non-log contribution when the remaining time integrals are carried out. The τ_2 integral on the first term having exponential integral yields some finite answer along with more exponential integral(s), both of which give non-log finite contributions upon performing τ_1 integral. The latter however also gives a $\log\left(\frac{k_T}{p_+ + k_{12}}\right)$, which goes to zero as $\delta \to 0$ and does not contribute any additional logs, since this term is finite at $p_+ \to \infty$.

The third term in Eq. (H.6) has the same structure as Eq. (H.5), as the function h_3 is the result τ_3 integral performed without the log. Hence this evaluation will once again give the same structure as Eq. (H.4).

Hence we conclude that in all our computations of $\mathcal{O}(\delta)$ correction terms, D_2 is given by

$$\sim \delta \log\left(\frac{H}{k_T}\right)\ D_1 + \ \text{finite}\,. \tag{H.7}$$

Appendix I

Proof of regulator independence

To show regulator independence we follow the derivation given in a physics stack exchange post by Sangchul Lee[1]. Since throughout the paper we only consider IR-finite interactions, the integrals we encounter are of the form,

$$I_\epsilon = \int_{-\infty}^{0} \tau^n e^{ik\tau} R(\epsilon\tau) d\tau, \tag{I.1}$$

where $n \geq 0$ and R is a regulator controlled by a small parameter ϵ. We assume that the regulator satisfies the property, $R(0) = 1$. We are interested in the limit $\epsilon \to 0$ of the regulated expression, $I_0 = \lim_{\epsilon\to 0} I_\epsilon$. Let us substitute $\tau = -\tau'$, we get,

$$= (-1)^n \int_{0}^{\infty} \tau^n e^{-ik\tau} \tilde{R}(\epsilon\tau) d\tau, \tag{I.2}$$

where $\tilde{R}(\epsilon\tau) = R(-\epsilon\tau)$. Let us write $\tilde{R}(\epsilon\tau)$ in the following integral form,

$$\tilde{R}(x) = \int_{x}^{\infty} \rho(t) dt, \tag{I.3}$$

[1] https://math.stackexchange.com/questions/1968568/regulating-int-0-infty-sin-x-mathrmd-x

where $\rho(t)$ is integrable for $t \in [0, \infty]$. This simply means that the derivative of the regulator is integrable. Using these facts we write,

$$\begin{aligned} I_0 &= \lim_{\epsilon \to 0} \int_0^\infty \frac{1}{i^n} \frac{d^n}{dk^n} e^{-ik\tau} \tilde{R}(\epsilon\tau) d\tau, \\ &= \lim_{\epsilon \to 0} \frac{1}{i^n} \frac{d^n}{dk^n} \int_0^\infty e^{-ik\tau} \tilde{R}(\epsilon\tau) d\tau, \\ &= \frac{1}{i^n} \frac{d^n}{dk^n} \lim_{\epsilon \to 0} \lim_{a \to \infty} \int_0^a e^{-ik\tau} \tilde{R}(\epsilon\tau) d\tau \end{aligned} \tag{I.4}$$

Since $\left| e^{-ik\tau} \tilde{R}(\epsilon\tau) \right| = \tilde{R}(\epsilon\tau)\ \forall k \in \mathbb{R}$, which is integrable (we have $\epsilon > 0$), we can take the derivative outside the integral.

$$= \frac{1}{i^n} \frac{d^n}{dk^n} \lim_{\epsilon \to 0} \lim_{a \to \infty} \int_0^a e^{-ik\tau} \left(\int_{\epsilon\tau}^\infty \rho(x) dx \right) d\tau \tag{I.5}$$

Now using Fubini's theorem we interchange the order of integration keeping in mind that we originally have x integration from $\epsilon\tau$ to ∞ which means $\tau < \frac{x}{\epsilon}$ but notice also, $\tau < a$ (from (I.5)), therefore, $\tau < \min\{a, \frac{x}{\epsilon}\}$,

$$\begin{aligned} &= \frac{1}{i^n} \frac{d^n}{dk^n} \lim_{\epsilon \to 0} \lim_{a \to \infty} \int_0^\infty \left(\int_0^{\min\{a, \frac{x}{\epsilon}\}} e^{-ik\tau} d\tau \right) \rho(x) dx, \\ &= \frac{1}{i^n} \frac{d^n}{dk^n} \lim_{\epsilon \to 0} \lim_{a \to \infty} \int_0^\infty \left(\frac{e^{-ik\min\{a, \frac{x}{\epsilon}\}} - 1}{-ik} \right) \rho(x) dx \end{aligned} \tag{I.6}$$

Now, one can use the Dominated convergence theorem to commute the $\lim_{a \to \infty}$ inside the integral

$$\frac{1}{i^n} \frac{d^n}{dk^n} \lim_{\epsilon \to 0} \int_0^\infty \left(\frac{e^{-ik\frac{x}{\epsilon}} - 1}{-ik} \right) \rho(x) dx = \frac{1}{i^n} \frac{d^n}{dk^n} \int_0^\infty \frac{\rho(x)}{ik} dx = \frac{1}{i^n} \frac{d^n}{dk^n} \frac{1}{ik}, \tag{I.7}$$

where to derive the last line we have used the Riemann-Lebesgue lemma and the fact that

$$\int_0^\infty \rho(x) dx = \tilde{R}(0) = R(0) = 1\,. \tag{I.8}$$

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