

Experiments on enhancement of inter-stage transfer efficiency in laser cooling of strontium atoms

विद्या वाचस्पति की
उपाधि की अपेक्षाओं की आंशिक पूर्ति में प्रस्तुत शोध प्रबंध

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degree of Doctor of Philosophy

द्वारा / By
कोरक बिश्वास/ Korak Biswas

पंजीकरण सं. / Registration No: २०१६३४८७/ 20163487

शोध प्रबंध पर्यवेक्षक / Thesis Supervisor:

उमाकांत रापोल/ Umakant Rapol



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by:

Korak Biswas

Registration ID 20163487



Department of Physics

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

PUNE - 411008, India

Certificate

Certified that the work incorporated in the thesis entitled “**Experiments on enhancement of inter-stage transfer efficiency in laser cooling of strontium atoms**” submitted by **Korak Biswas** was carried out by the candidate, under my supervision. The work presented here or any part of it has not been included in any other thesis submitted previously for the award of any degree or diploma from any other university or institution.

Date: 15-07-2024

A handwritten signature in blue ink, appearing to read "V. O. Bhat", enclosed within a blue oval.

(Supervisor)

Declaration

Name of student: Korak Biswas

Reg. No: 20163487

Thesis Supervisor: Umakant D. Rapol

Department: Physics

Date of joining program: 01-08-2016

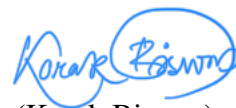
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I declare that this written submission represents my idea in my own words and where others' ideas have been included; I have adequately cited and referenced the original sources. I declare that I have acknowledged collaborative work and discussions wherever such work has been included. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

The work reported in this thesis is the original work done by me under the guidance of **Prof. Umakant D. Rapol**.

Date: 15-07-2024,


(Korak Biswas)

Abstract

Laser cooling is a matter of paramount significance, particularly when dealing with atoms from the alkaline earth metal group. These atoms offer ultra-narrow optical transitions, which can be utilized for quantum information processing, extremely precise optical atomic clocks and various other experiments including the detection of dark matter and gravitational waves. However, all of these experiments generically suffer from different sorts of noises arising from loss of atoms during different stages of experiments. The loss of atoms increases the various noise levels in different quantum experiments, including but not limited to atom number shot-noise in optical atomic clocks and atom interferometry, frequency instability in atomic clocks attributable to fluctuations in atom numbers etc. Additionally, the number of atoms is extremely important in atom-interferometry based gravitational wave (GW) detection. In a nutshell, the significance of the number of atoms becomes paramount in achieving a good signal-to-noise ratio for these experiments. However, the number of atoms is affected by various factors.

The conventional method of cooling alkaline earth metals involves a two-tier Doppler cooling approach to make the atoms adequately cold for various quantum experiments. For instance, for strontium atoms, the lowest achievable temperature (Doppler limited temperature) in the first stage is $760\text{ }\mu\text{K}$. This temperature, though relatively low, is unsuitable for experiments involving quantum phenomena. Consequently, a second stage of cooling is necessary, with its Doppler-limited temperature (proportional to the linewidth of the optical transition) reaching lower than the single photon recoil temperature (460 nK). Therefore, the two stages of cooling of alkaline earth atoms can be referred to as the initial magneto-optical trap (MOT) cooling phase and the narrow-line MOT cooling phase. In the aforementioned two-tier cooling method, there are two significant reasons for having a lower number of atoms:

Firstly, the suboptimal capture of atoms during the initial stage of cooling is often due to the poor optimization of experimental parameters. Manual tuning of multiple parameters can often fail to discover the optimal set of parameters that would result in the maximum number of atoms.

Secondly, the temperature and velocity distribution of atoms within a MOT are di-

rectly related to the linewidth of the corresponding laser cooling transition used in the MOT. The number of atoms interacting with light inside a MOT depends on both the linewidth of the transition and the laser detuning. A smaller linewidth leads to fewer interacting atoms within the MOT with a fixed light detuning. Therefore, during the transfer of alkaline atoms from the initial MOT with a broader linewidth to the narrow-line MOT, a significant number of atoms fail to interact with the laser, which is precisely locked to the narrow-line transition of the secondary MOT. This results in a substantial loss of atoms from the trap during the transfer process, despite artificially broadening the laser detuning through temporal modulation.

This thesis outlines the methodologies used to address these challenges in experiments with strontium 88 atoms. Specifically, it discusses the methods to enhance the number of atoms by:

1. Implementing machine learning techniques to optimize the parameters of the initial stage of cooling.
2. Employing a novel three-level intermediate cooling method, which reduces the temperature of the first stage cooling by approximately 75%, thereby narrowing the velocity distribution and enhancing the transfer efficiency to around 85%, as opposed to the conventional method that offers a transfer efficiency of around 40%.

Notably, these methods can be applied across all the elements belonging to the alkaline earth metal group as they share similar level structures. By applying this approach, we can effectively increase the number of atoms in the experiments, consequently elevating the SNR for various applications.

To my **Grandma**

Publications from the thesis work

- Machine-learning-based automated loading of strontium isotopes into magneto-optical trap; **Korak Biswas**, Kushal Patel, S. Sagar Maurya, Pranab Dutta, Umakant D. Rapol; AIP Advances 1 July 2023; 13 (7): 075313
- Electromagnetically Induced Transparency (EIT) aided cooling of strontium atoms; **Korak Biswas**, Kushal Patel, S. Sagar Maurya, Pranab Dutta, Umakant D. Rapol, Yeshpal Singh, arXiv: 2311.08318 (2023) (Under review)
- Spectroscopy of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition of strontium using laser cooled atoms; Kushal Patel, Palki Gakkhar, **Korak Biswas**, et al 2024 J. Phys. B: At. Mol. Opt. Phys. 57 105501

List of other publications

- Measurement of Sr-N₂ collision cross section in Magneto-Optical-Trap; Chetan Vishwakarma, Kushal Patel, Jay Mangaonkar, Jamie L. MacLennan, **Korak Biswas** and Umakant D. Rapol, arXiv: 1905.03202 (2019)
- Comparative Analysis of Phase Noise for different configurations of Bragg lattice for an Atomic Gravimeter with Bose-Einstein Condensate; Pranab Dutta, S. Sagar Maurya, **Korak Biswas**, Kushal Patel, Umakant D. Rapol; 2312.00628 (2023)
- A Decade of Advancement of Quantum Sensing and Metrology in India Using Cold Atoms and Ions. Dutta, P., Maurya, S.S., Patel, K., Biswas, K. et al., J Indian Inst Sci, 103, 609-632 (2023)
- Interplay between quantum diffusion and localization in the atom-optics kicked rotor; S. Sagar Maurya, J. Bharathi Kannan, Kushal Patel, Pranab Dutta, **Korak Biswas**, Jay Mangaonkar, M. S. Santhanam, and Umakant D. Rapol; Phys. Rev. E 106, 034207 (2022)

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Embarking on a career in physics has been quite the journey for me. Ever since my school days, I have been fascinated by this subject, relishing both its theoretical concepts and hands-on experiments. However, as I pursued my master's degree, I made the decision to specialize in theoretical high-energy physics. But upon completing my M.Sc and joining IISER Pune for my Ph.D., I discovered there were no openings in that particular area. Instead, I found myself in a group focused on computational statistical physics. While I made progress in my research, I could not shake the feeling that something was missing - that elusive "x factor" that truly ignites passion. After thoughtful discussions with my former supervisor, Prof. Apratim Chatterji, I eventually decided to leave the group after a year and a half of pursuing my Ph.D.

I found myself at a crossroads, unsure of my next steps. It was then that I stumbled upon the experimental group "Atomic Physics and Quantum Optics," led by Prof. Umakant Rapol. Initially I was reluctant to venture into the experimental physics. My lack of hands-on experience in experiments left me apprehensive about pursuing a Ph.D in experimental physics. Yet, Prof. Rapol's words resonated deeply within me. "Have you ever repaired a bicycle on your own? If you have, then you're already an experimentalist.", he quipped. Indeed, I had tackled bicycles with determination and grit. With renewed confidence, I resolved to join Prof. Rapol's group, eager to embark on this new chapter of my academic journey. I am immensely thankful to my supervisor, Prof. Rapol, whose mentorship has significantly advanced my career in physics. Equally, I owe a debt of gratitude to my former supervisor, Prof. Apratim Chatterji, for his invaluable guidance during a particularly uncertain period in my career. Their unwavering support has been crucial in shaping my path forward.

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Korak Biswas

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Chapter 1

Introduction

1.1 Laser cooling and trapping of neutral atoms

Laser cooling and trapping of neutral atoms represent transformative techniques that have reshaped the landscape of atomic physics, providing an unparalleled degree of control and precision over atomic motion [9, 10]. This cutting-edge technology has become indispensable across various scientific and technological domains [11]. By leveraging the principles of Doppler cooling and radiation pressure, laser cooling allows scientists to cool atoms to temperatures approaching absolute zero, effectively slowing their motion to a virtual standstill [12]. This meticulous control over atomic motion has not only facilitated the generation of Bose-Einstein condensates [13, 14], extraordinary quantum states with distinctive properties but has also found applications in quantum computing [15], quantum information processing [16], and fundamental investigations into quantum mechanics. The manipulation and confinement of neutral atoms with laser beams have ushered in a new era of exploration into the fundamental behaviours of matter at the quantum level, fostering advancements with the potential for significant scientific discoveries and technological innovations [17].

The historical narrative of laser cooling and trapping of neutral atoms traces back to the early 1970s when visionaries like Arthur Ashkin, Steven Chu, and Claude Cohen-Tannoudji independently conceived and developed the foundational principles that would later underpin laser cooling. Arthur Ashkin's introduction of optical trapping in 1978,

demonstrating the capability of laser beams to apply pressure and immobilize small particles [18], earned him the Nobel Prize in Physics in 2018. Concurrently, Steven Chu successfully pioneered laser cooling in 1985 by cooling a cloud of sodium atoms using six laser beams [19], achieving temperatures conducive to observe effects of quantum degeneracy.

Simultaneously, Claude Cohen-Tannoudji, along with collaborators Alain Aspect and Jean Dalibard [20, 21], made significant progress in laser cooling and trapping, focusing their efforts on cesium atoms. Cohen-Tannoudji's contributions were recognized with a share of the 1997 Nobel Prize in Physics, underscoring the profound insights gained into fundamental quantum phenomena through laser cooling.

The initial phase of laser cooling and trapping primarily involved alkali metals. Following Chu's success with sodium atoms, researchers expanded their focus to other alkali metals, including potassium, rubidium, and cesium. Subsequently, the collaborative efforts of Carl Wieman, Eric Cornell, and Wolfgang Ketterle culminated in the groundbreaking achievement of creating the first Bose-Einstein condensate in 1995 using alkali atoms [14, 22].

The pursuit of cooling and trapping alkaline earth atoms, a category encompassing elements such as Strontium, Ytterbium and Calcium, presented distinct challenges due to their electronic configurations. Despite these obstacles, researchers persevered in developing innovative approaches. In the early 2000s, Jun Ye's group at the National Institute of Standards and Technology (NIST) achieved a breakthrough by successfully laser-cooling and trapping strontium atoms, marking a significant stride toward extending laser cooling techniques beyond alkali metals [23, 24].

The cooling and trapping of both alkali and alkaline earth atoms have since played a pivotal role in the realm of ultracold physics, facilitating exploration into diverse quantum phenomena [25, 26]. These advancements have not only propelled applications in precision measurement, quantum information processing, and the study of novel quantum states but also continue to push the boundaries of what can be achieved with ultracold atoms, unlocking new frontiers in both fundamental physics and practical applications.

Cold alkali and alkaline earth atoms offer diverse applications. A noteworthy use of

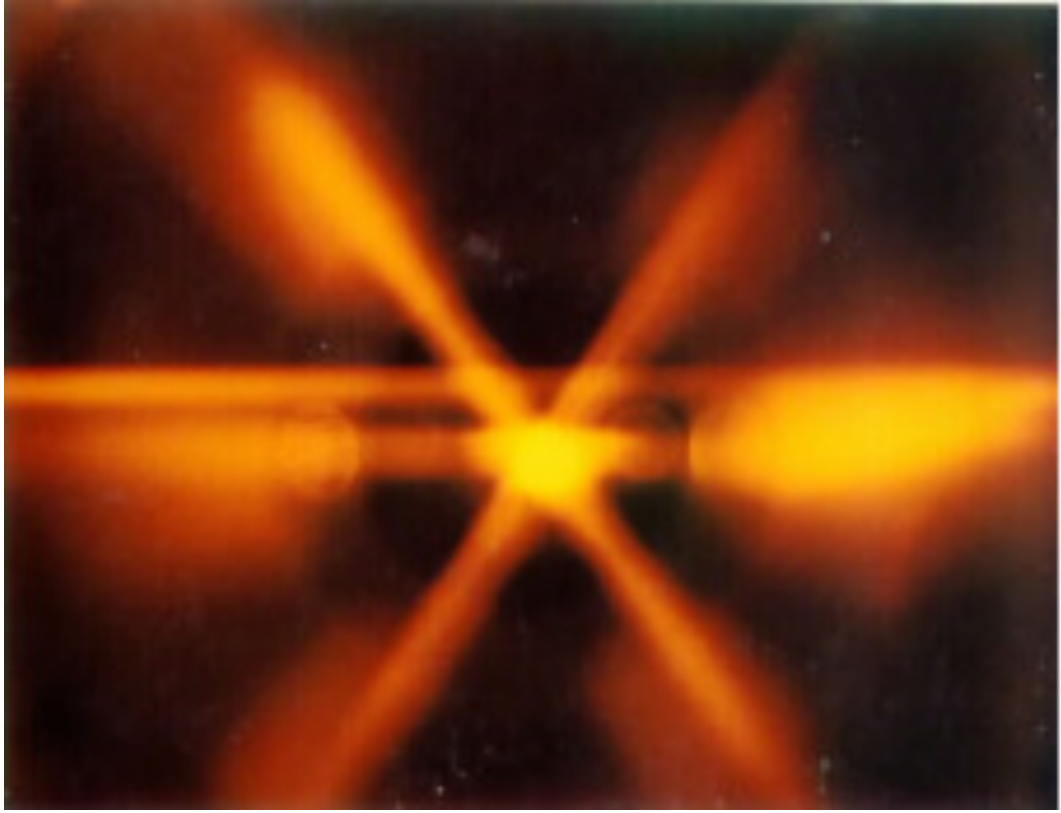


Figure 1.1: Optical molasses: One of the first images of cold sodium in an optical molasses at the intersection of six laser beams. This image has been taken from [1]

cold alkali atoms involves generating and studying Bose-Einstein condensates (BECs), which are unique quantum states of matter exhibiting remarkable coherence and behavior near absolute zero temperatures [14, 22]. These BECs, formed with alkali atoms, have been crucial for exploring fundamental aspects of quantum mechanics, shedding light on phenomena such as superfluidity [27, 28], quantum interference [29], and wave-particle duality [30]. On the other hand, laser-cooled alkaline earth atoms, with their distinctive atomic structure, provide a dependable platform for conducting quantum information processing [31] with extended coherence times. They also enable precision measurements through highly accurate optical atomic clocks [32]. In this way, cold alkaline earth atoms contribute not only to technological advancements but also offer insights into fundamental physics, such as the study of gravitational time delay at the quantum length scale.

As we approach the era of quantum technologies, it is anticipated that both cold alkali and alkaline earth atoms will play a pivotal role in both technological advancements and fundamental scientific exploration.

1.1.1 Applications of laser cooling

Laser cooling plays an important role in elucidating fundamental quantum mechanical phenomena. Quantum systems tend to exhibit classical behavior at higher temperatures even without encountering thermal decoherence. Consider a one-dimensional quantum harmonic oscillator in thermal equilibrium with a heat bath. The partition function of this system is given by [33]:

$$Z = \text{Tr}\{\exp(-\beta\hat{H})\} \quad (1.1)$$

Here, $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$ represents the Hamiltonian of the oscillator, and $\beta = \frac{1}{k_B T}$ where T denotes the temperature of the heat bath. The mass and angular frequency of the oscillation are represented by m and ω , respectively. Equation (1.1) can be equivalently expressed as:

$$Z = \sum_{n=0}^{\infty} \langle n | \exp(-\beta\hat{H}) | n \rangle \quad (1.2)$$

Here, $|n\rangle$ signifies the n -th eigenstate of $\hat{H}$. For a quantum harmonic oscillator, the energy eigenvalue equation is given by:

$$\hat{H} |n\rangle = \left(n + \frac{1}{2}\right) \hbar\omega |n\rangle \quad (1.3)$$

Hence, the partition function Z can be expressed as:

$$\begin{aligned} Z &= \sum_{n=0}^{\infty} \exp \left\{ -\beta \left(n + \frac{1}{2} \right) \hbar\omega \right\} \\ &= \frac{1}{\exp(\beta\hbar\omega/2) - \exp(-\beta\hbar\omega/2)} \end{aligned} \quad (1.4)$$

The average energy of the system is given by:

$$\begin{aligned}
 \langle E \rangle &= \text{Tr} \left\{ \exp(-\beta \hat{H}) \hat{H} \right\} \\
 &= - \frac{\partial}{\partial \beta} (\ln(Z)) \\
 &= \frac{\hbar\omega}{2} \frac{1 + \exp(-\beta\hbar\omega)}{1 - \exp(-\beta\hbar\omega)}
 \end{aligned} \tag{1.5}$$

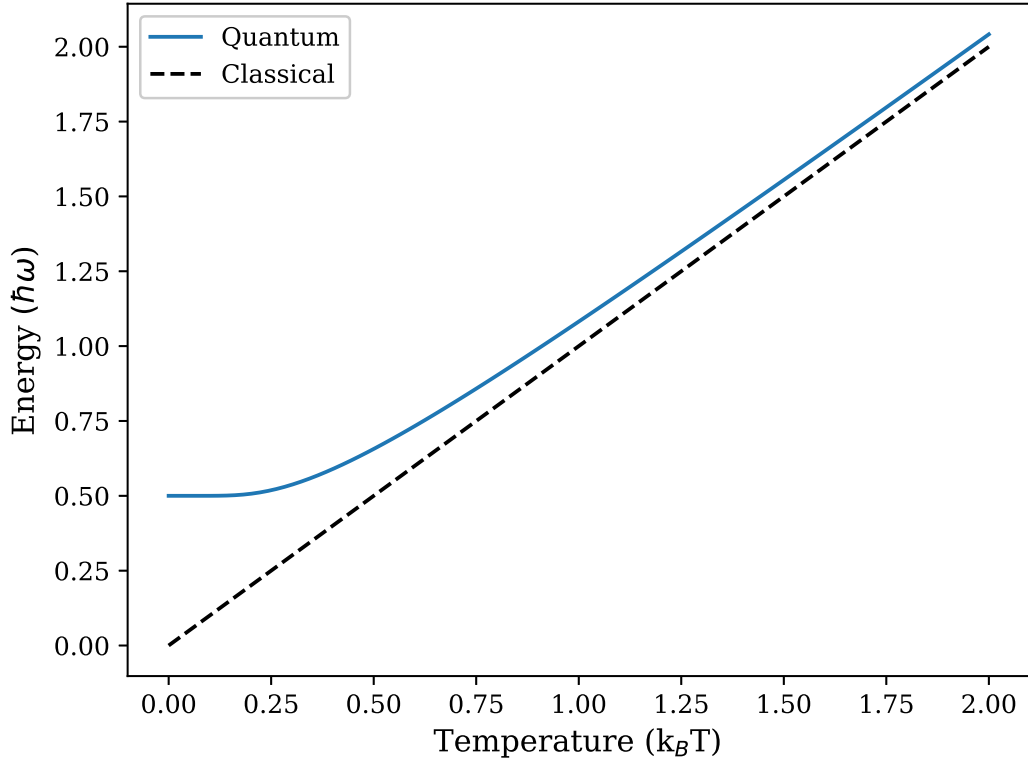


Figure 1.2: Harmonic oscillator at thermal equilibrium : Energy of the oscillator approaching the classical limit asymptotically at higher temperature.

From Equation (1.5), it is evident that at higher temperatures, $\langle E \rangle$ approaches the classical limit of $\langle E \rangle = \frac{1}{2}k_B T$ as derived from the equipartition theorem [33]. However, at lower temperatures, $\langle E \rangle$ saturates at a finite value ($\frac{1}{2}\hbar\omega$), consistent with the Heisenberg uncertainty principle. This indicates the emergence of quantum phenomena at lower temperatures, as illustrated in Figure(1.2).

Therefore, laser cooling serves as a gateway to unveil quantum phenomena that are masked by thermal effects. In the subsequent part of this section, we delve into the significance of laser cooling in various practical scenarios.

The Atom interferometry: Unraveling the wave nature of atoms

The laser cooling of neutral atoms is an excellent method for realizing the wave nature of atoms. One common approach to demonstrating this phenomenon is by implementing interferometry with laser-cooled atoms [34]. In an atom interferometry setup, an initial atomic wavepacket described by the state $|x, p\rangle$ (where x and p denote position and momentum, respectively) undergoes coherent splitting into two wave packets: $|x_1, p_1\rangle$ and $|x_2, p_2\rangle$. These wavepackets then evolve to a final state $|x_f, p_f\rangle$ after a time t . The final wavefunction is given as [35]:

$$|\psi_f\rangle = (\exp(i\theta_1) + \exp(i\theta_2)) |x_f, p_f\rangle \quad (1.6)$$

The phases θ_1 and θ_2 represent the accumulated phases of the two wavepackets during their paths, which can be determined using the path integral approach [36]. Since the contribution of the phase is significant for the classical trajectory, the phases θ_1 and θ_2 are approximated as:

$$\exp(i\theta_1) \approx \exp(iS_{cl}(x_1)/\hbar) \quad (1.7)$$

$$\exp(i\theta_2) \approx \exp(iS_{cl}(x_2)/\hbar) \quad (1.8)$$

Here, $S_{cl}(x)$ represents the action along the classical trajectory x . By substituting these expressions for θ_1 and θ_2 into the observable quantity $|\langle\psi_f|\psi_f\rangle|^2$, we find the intensity of the final wavefunction oscillates as a function of the phase difference, akin to the interference of light waves. This phenomenon reveals the wave-like nature of atoms. The phase difference corresponds to the difference in the actions along classical trajectories of the two wavepackets. For a coherent interference pattern, this phase difference must be preserved. However, at higher atom temperatures, the random thermal kicks disrupt this phase difference [37], leading to the disappearance of interference patterns. Laser cooling plays a crucial role in reducing the temperature of the atoms significantly,

thereby minimizing the effect of thermal disturbances and revealing the wave nature of atoms.

Atom interferometry (AI) has a wide range of applications across various fields of science and technology. For instance, Atom interferometry plays a pivotal role in the realm of precision navigation and inertial sensing, offering unparalleled accuracy in determining position, velocity, and orientation. Atom interferometers create ultra-sensitive gyroscopes and accelerometers that are vital in aerospace, geophysics, and autonomous vehicle navigation [38, 39, 40]. These devices provide precise measurements of angular rotation, acceleration, and gravitational forces, enabling aircraft, spacecraft, and submarines to navigate with exceptional accuracy. Inertial measurement units (IMUs) based on atom interferometry offer improved reliability and stability [41], crucial for applications where traditional sensors may be susceptible to drift or environmental interference. The advancements in atom interferometry for navigation not only enhance the safety and efficiency of transportation systems but also pave the way for future developments in geodetic surveying [42], autonomous vehicles and robotics.

AI serves as a powerful tool for testing fundamental physics principles and theories, providing a controlled environment to probe the fundamental laws governing the universe [43, 44]. One significant application is in testing the equivalence principle of general relativity [45, 46], which posits that gravitational and inertial mass are equivalent. Atom interferometers can precisely measure gravitational acceleration and detect even the smallest deviations from this principle, offering insights into the nature of gravity and the fabric of spacetime [47]. Additionally, these devices are used to search for deviations from known laws of physics and investigate new theories beyond the standard model, such as the presence of extra dimensions or the existence of new fundamental particles [48]. The high precision and sensitivity of atom interferometry make it an invaluable tool for exploring the frontiers of fundamental physics, pushing the boundaries of our understanding of the universe.

AI has emerged as a promising technology for the detection of gravitational waves, the ripples in spacetime predicted by Einstein's theory of general relativity. Atom interferometers offer a unique approach to gravitational wave detection [49], complementing existing techniques such as laser interferometry. By precisely measuring the effects of

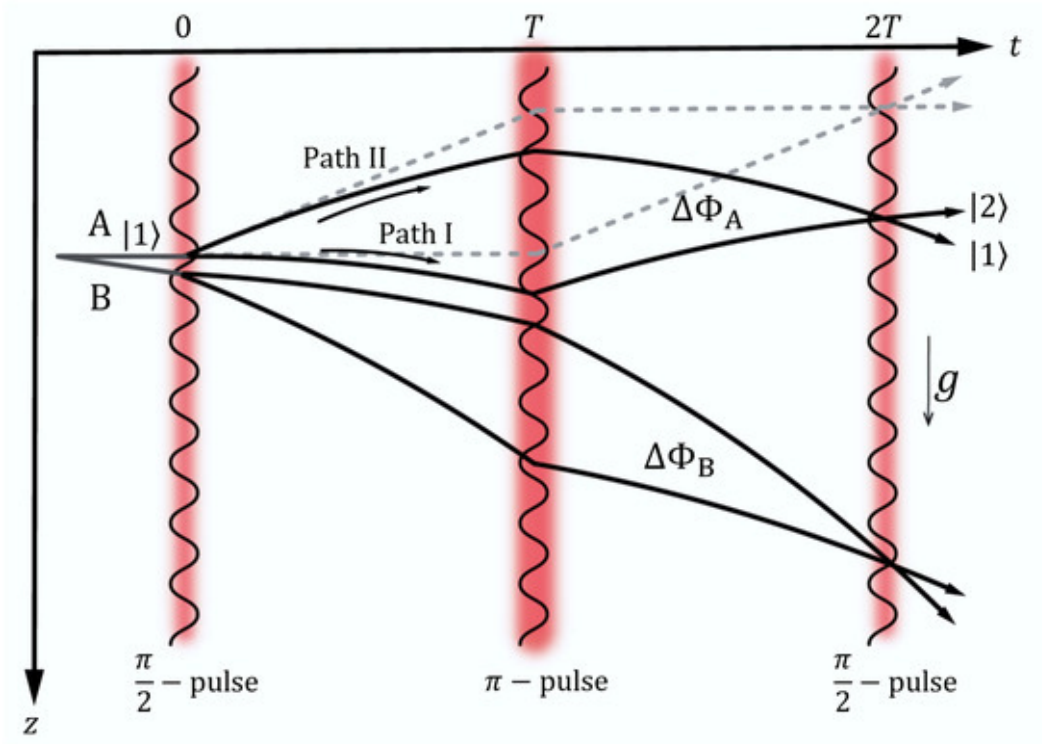


Figure 1.3: Atom interferometry: A schematic of standard Raman interferometry for measuring acceleration due to gravity. This image has been taken from [2]

gravitational waves on the quantum states of atoms, these devices can provide a new avenue for observing and studying these elusive cosmic phenomena [44]. Atom interferometers have the potential to detect gravitational waves from a variety of sources, including merging black holes, neutron star collisions, and other cataclysmic events in the universe. Their sensitivity to minute changes in spacetime curvature makes them invaluable tools for gravitational wave astronomy, offering a new perspective on the dynamics of the cosmos and the nature of gravity itself. Ongoing research and development in atom interferometry promise to enhance our ability to detect and study gravitational waves, opening up new frontiers in astrophysics and fundamental physics.

Atom interferometry stands at the forefront of the search for dark matter [50] and the exploration of new physics beyond the standard model. Dark matter, an enigmatic form of matter that makes up a significant portion of the universe's mass, has so far eluded direct detection. Atom interferometers offer a sensitive tool to detect hypothetical dark matter particles [51], such as axions [52], by observing their gravitational interactions

with ordinary matter. Additionally, these devices can be used to search for new particles or interactions predicted by theories beyond the standard model of particle physics [53]. The precision and sensitivity of atom interferometry enable scientists to probe for deviations from known physical laws and explore the existence of hidden dimensions or exotic particles. As research continues to push the boundaries of atom interferometry, it holds the promise of shedding light on the mysterious nature of dark matter and uncovering new facets of the fundamental forces governing the universe.

Apart from the instances mentioned above, AI finds numerous other applications, extensively documented in the existing literature. In the next section, we will explore the impact of laser cooling on the creation of Bose-Einstein condensate (BEC), an exotic quantum state of matter.

The Bose-Einstein condensate

The grand canonical partition function of a Bosonic gas is expressed as [33]

$$Z = \prod_i \frac{1}{1 - z \exp(-\beta E_i)} \quad (1.9)$$

Here, $z = \exp(\beta\mu)$ is known as the fugacity, where μ represents the chemical potential, and E_i denotes the energy of the i -th level. The case where $i = 0$ corresponds to the ground state. The total number of particles N and the number of particles in the i -th state n_i are given by:

$$\begin{aligned} N &= -z \frac{\partial}{\partial z} \log(Z) \\ &= \sum_i \frac{z \exp(-\beta E_i)}{1 - z \exp(-\beta E_i)} \end{aligned} \quad (1.10)$$

$$\begin{aligned} n_i &= -\frac{1}{\beta} \frac{\partial}{\partial E_i} \log(Z) \\ n_i &= \frac{z \exp(-\beta E_i)}{1 - z \exp(-\beta E_i)} \end{aligned} \quad (1.11)$$

The relationship between N and n_i is established by the equation $N = \sum_i n_i$. For non-interacting particles with $E_i = p_i^2/2m$ (p_i being the momentum) in a large volume

($V \rightarrow \infty$), the summation over i in 3D can be replaced by $\frac{4\pi V}{h^3} \int_{p_i} dp_i p_i^2$, resulting in

$$\begin{aligned}\rho &\equiv N/V \\ &= \frac{1}{\lambda^3} g_{3/2}(z) + \frac{n_0}{V}\end{aligned}\quad (1.12)$$

Here, $\lambda = \sqrt{2\pi\hbar^2\beta/m}$, $g_{3/2}(z) = \sum_j^\infty \frac{z^j}{j^{3/2}}$, and n_0 is the number of particles in the ground state. It can be further demonstrated that

$$\frac{n_0}{N} = \begin{cases} 0 & \text{if } T > T_c \\ 1 - \left(\frac{T}{T_c}\right)^{3/2} & \text{otherwise} \end{cases} \quad (1.13)$$

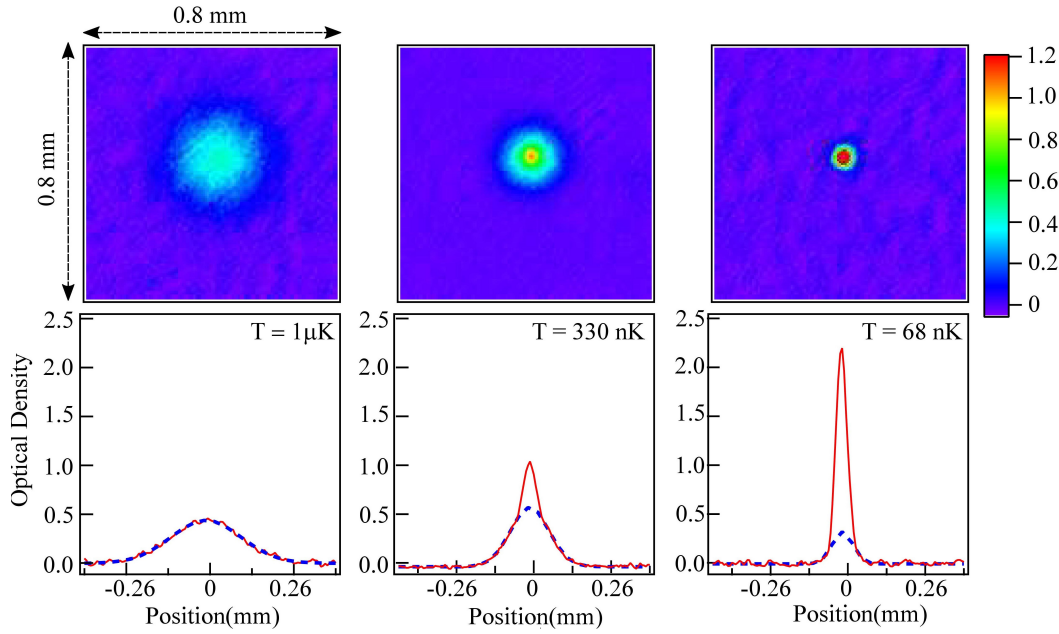


Figure 1.4: Bose-Einstein Condensate: Emergence of a BEC from cold atoms as the temperature reduces. This image is taken from [3].

In Eq.(1.13), the critical temperature T_c is defined as

$$T_c = \frac{2\pi\hbar^2\rho^{2/3}}{m \{g_{3/2}(1)\}^{2/3}} \quad (1.14)$$

If the temperature of the particles falls below the critical temperature, a substantial number of particles will occupy the ground state, leading to the manifestation of various exotic quantum phenomena. In this context, laser cooling serves a pivotal role in lowering the temperature of the Bosonic gas below its critical temperature.

BEC play a significant role in exploring fundamental physics and practical applications. For instance, superfluidity [27, 28], a remarkable phenomenon exhibited by BEC, has profound implications and applications in physics and engineering. Under certain conditions BECs transform into a superfluid state where it flows without any viscosity or resistance. This property enables extraordinary applications, such as the development of highly sensitive sensors for detecting tiny changes in rotation or acceleration [54]. Superfluid BECs can be employed in gyroscopes [55], gravitational wave detectors for probing the fabric of spacetime [56], and even in modeling astrophysical phenomena like neutron stars [57]. The study of superfluidity in BECs also sheds light on the behavior of quantum fluids, offering insights into the fundamental nature of matter and providing a platform for investigating new states of quantum matter.

The study of superfluidity is closely related to the vortex dynamics [58]. Vortex dynamics within BEC present a fascinating area of study with diverse applications. In BECs, vortices are quantized whirlpools of superfluid flow, offering insights into the behavior of these exotic quantum fluids. Understanding vortex dynamics in BECs is crucial for unraveling the mysteries of superfluidity. By manipulating and studying vortices within BECs, researchers gain a deeper understanding of quantum phenomena and pave the way for innovative technologies in sensing and measurement.

BEC serves as invaluable tools for simulating condensed matter systems, offering profound insights into the behavior of materials at a fundamental level. By manipulating the interactions and properties of the atoms within a BEC, scientists can create controlled environments that replicate the complex phenomena observed in condensed matter physics [59]. This capability enables the investigation of superfluidity [27, 28], supersolidity [60], quantum phase transitions, and other quantum mechanical phenomena in a controlled and reproducible manner. BECs provide a bridge between the microscopic world of individual atoms and the macroscopic properties of materials, offering a unique window into the quantum realm of condensed matter physics.

BECs are powerful tools for simulating and studying quantum many-body systems, offering a unique platform to investigate complex quantum phenomena in a controlled environment. By manipulating the interactions and confinement of atoms within a BEC, researchers can create analogs of systems found in condensed matter physics [61], quantum field theory [62], and even high-energy physics [63]. These simulations allow for the exploration of phenomena such as quantum magnetism [64], exotic phases of matter [65], and the dynamics of strongly correlated systems. The insights gained from BEC simulations help deepen our understanding of fundamental physics, offering new perspectives on the behavior of materials and paving the way for the development of novel technologies based on quantum principles.

BECs offer a remarkable avenue for simulating and studying cosmic phenomena in controlled laboratory settings [66]. Researchers can use BECs to recreate extreme conditions found in the cosmos, such as those near black holes or within neutron stars [67]. By manipulating the properties of the BEC, they can simulate gravitational fields, space-time distortions, and the effects of strong gravitational forces. These simulations provide valuable insights into the behavior of matter and light in extreme gravitational environments [68], aiding our understanding of phenomena like Hawking radiation [69], black hole dynamics [70], and the behavior of ultra-dense matter [71]. The ability to study cosmic phenomena in BECs not only contributes to fundamental physics research but also opens doors to new technologies and approaches for observing and understanding the universe on a larger scale. This includes potential applications in gravitational wave detection, astrophysical modeling, and the exploration of fundamental principles underlying our cosmos.

The optical atomic clock

The optical atomic clock stands as a fascinating application of laser cooling. A clock, by definition, relies on counting the periodic oscillations of a stable reference oscillator. In the case of an optical atomic clock, these reference oscillators are actual atoms. A significant error source in atomic oscillators arises from the Doppler shift [72] caused by the motion of the atoms, leading to a broadening of the natural linewidth of electronic transitions. To mitigate this error, it becomes necessary to decouple the motional de-

degrees of freedom from the internal degrees of freedom of the atoms. This is achieved by confining the atoms within an optical lattice and employing the technique of 'resolved sideband cooling' [32].

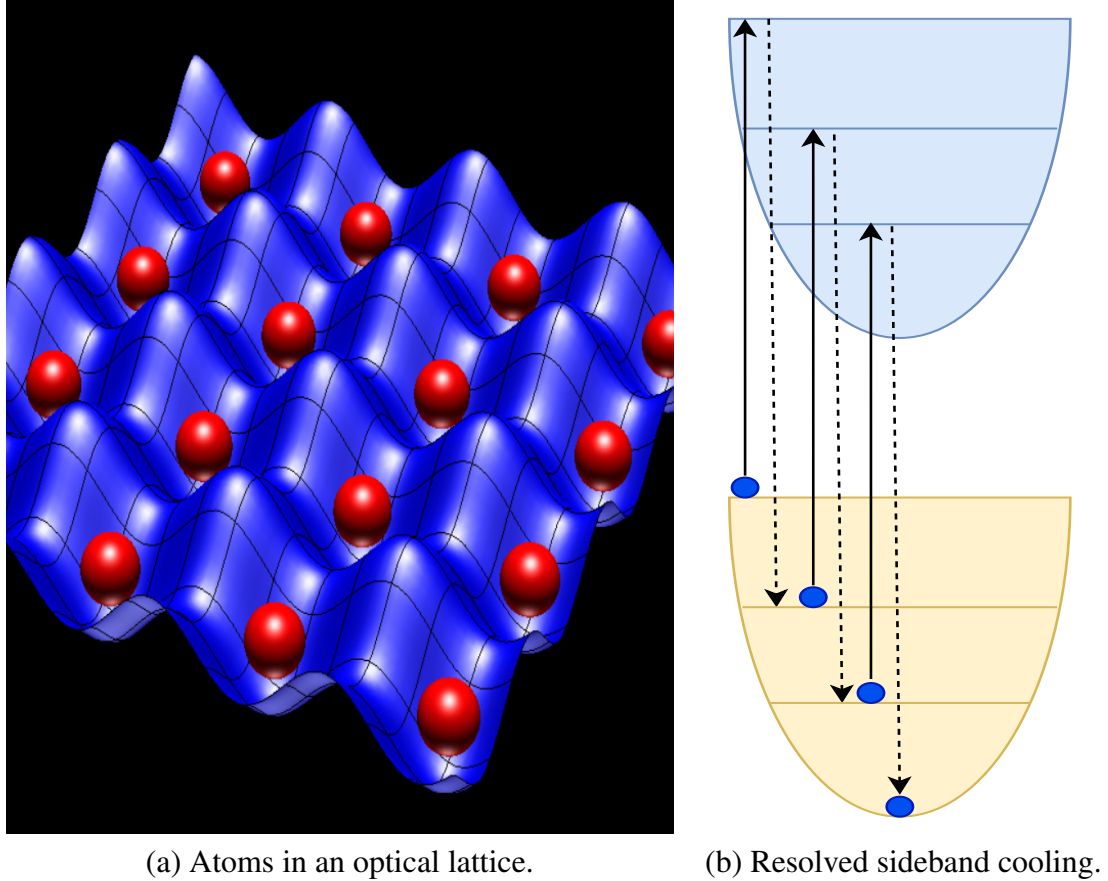


Figure 1.5: Resolved sideband cooling in optical lattice: (a) shows an representative image of atoms in an optical lattice [4]. (b) depicts the schematic of resolved sideband cooling. Straight arrows illustrate driven transitions, while dashed arrows denote spontaneous transitions. Each driven transition leads the atom to an excited state with one fewer motional quanta, guiding it to the motional ground state.

The optical lattice [73] creates a periodic trapping potential, which in the vicinity of a local minimum, can be approximated as a simple harmonic trap to the leading order. Within a one-dimensional system, the energy eigenstates of the trapped atoms are given by $E_n = (n + \frac{1}{2}) \hbar \omega_{\text{trap}}$, where ω_{trap} denotes the trap frequency of the optical lattice. Consequently, the energy difference between consecutive energy levels becomes $\Delta E = \frac{1}{2} \hbar \omega_{\text{trap}}$. If this energy difference surpasses the 'energy spread' of the transition

line due to the natural linewidth ($\hbar\Gamma$), the Doppler broadening of the natural linewidth is removed, and the motional states manifest as sidebands of the optical transition. Nevertheless, a shift in the resonance peaks occurs due to the recoil of photons ($\omega_{\text{rec}} = \frac{\hbar k^2}{2m}$). This shift is minimized in the Lamb-Dicke regime, where $\omega_{\text{rec}} \ll \omega_{\text{trap}}$.

Following the loading of atoms into the optical lattice, the process of resolved sideband cooling is initiated. This technique ensures that the atoms are primarily in their motional ground states, thereby effectively eliminating errors stemming from the Doppler shift. A schematic illustrating the concept of resolved sideband cooling is presented in Figure(105).

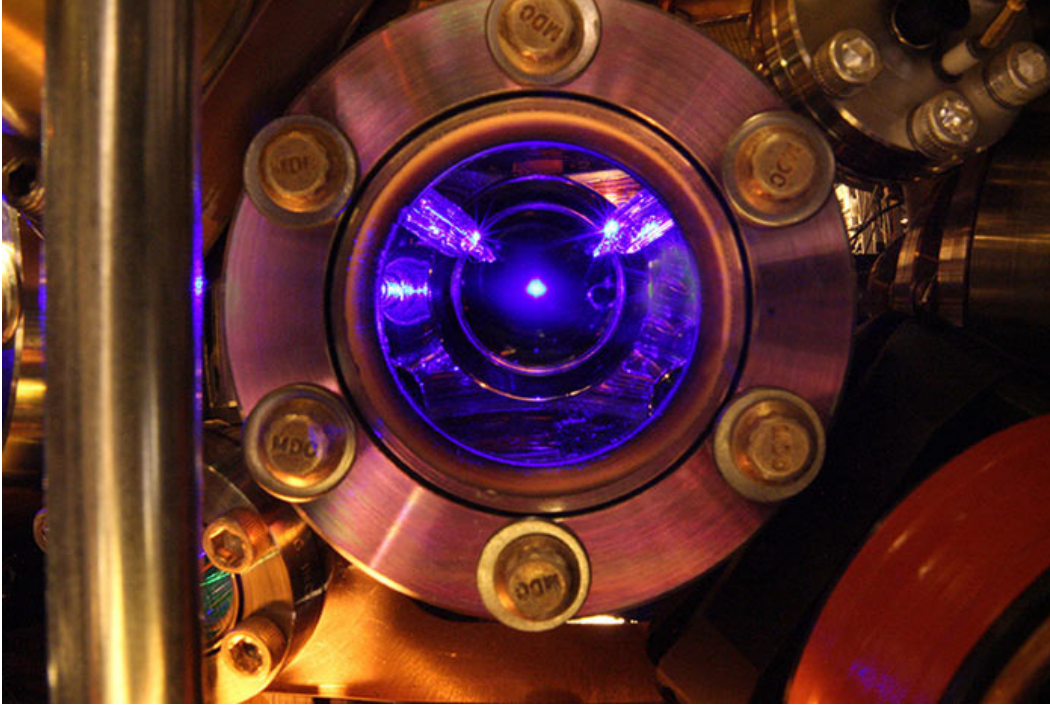


Figure 1.6: Optical atomic clock: The Strontium Optical Lattice Clock at NIST: The most accurate clock ever made. This image has been taken from [5].

In terms of temperature, the depth of the trapping potential within an optical lattice typically ranges around $\sim 50 \mu\text{K}$. When the atoms possess a temperature higher than the trap depth, the efficiency of loading atoms into the optical lattice drastically diminishes, resulting in an exceedingly low signal-to-noise ratio (SNR) that is hardly detectable. Consequently, laser cooling of the atoms becomes imperative prior to initiating the process of resolved sideband cooling.

Optical atomic clocks play a pivotal role in the functioning and enhancement of global navigation systems such as GPS (Global Positioning System) [74]. These highly precise clocks provide the critical timekeeping needed for satellite-based navigation [75], enabling accurate positioning, timing, and synchronization of signals. By using atomic clocks on board satellites, GPS systems can calculate the precise distances between satellites and receivers on Earth, allowing for accurate determination of a receiver's position. Optical atomic clocks improve the reliability and precision of GPS signals, reducing errors in navigation and enhancing the efficiency of location-based services in industries such as aviation, maritime navigation, transportation, surveying, and emergency response. The stability and accuracy of optical atomic clocks ensure that GPS users receive dependable and consistent positioning information, making them indispensable tools for global navigation systems used in everyday life and critical applications.

Optical atomic clocks serve as foundational components in secure communication systems and cryptographic protocols [76], providing precise timekeeping essential for encryption and decryption processes. In the realm of quantum key distribution (QKD) [77], optical atomic clocks play a crucial role in ensuring secure and tamper-proof communication channels [78]. These clocks enable precise synchronization of quantum states and time stamps, allowing for the creation and exchange of cryptographic keys with unmatched security. By leveraging the stability and accuracy of optical atomic clocks, QKD systems can generate encryption keys based on the principles of quantum mechanics, making them immune to eavesdropping attempts. This technology promises unhackable communication channels, safeguarding sensitive data in fields such as finance, government communications, healthcare, and defense. The applications of optical atomic clocks in secure communication and cryptography pave the way for a new era of ultra-secure data transmission, protecting vital information in an increasingly interconnected and data-driven world.

Optical atomic clocks are instrumental in experiments testing the constancy of physical constants [79], such as the fine-structure constant [80], providing insights into the stability of the universe over time. They also contribute to investigations into violations of Lorentz symmetry [81], the equivalence principle [82], and other fundamental princi-

ples of relativity [83] and quantum mechanics. By pushing the boundaries of precision measurement, optical atomic clocks aid in the quest to understand the fundamental nature of matter, energy, space, and time. They serve as indispensable tools in laboratories worldwide, advancing our knowledge of the universe and the underlying laws governing it.

The applications of laser cooling extend to numerous areas in fundamental physics and applications. However, this thesis does not delve into the exhaustive details of all these applications. Therefore, we conclude the discussion here and proceed to the next section of the thesis.

1.2 Signal-to-noise ratio in laser cooling experiments

Signal-to-noise ratio (SNR) [84] stands as the most important metric in the realm of experimental investigations. SNR is a fundamental concept in signal processing [85], representing the ratio of the power of a desired signal to the power of background noise. In essence, SNR quantifies the quality of a signal by comparing the strength of the signal to the level of unwanted noise present in the system. A higher SNR indicates a stronger, clearer signal relative to the noise, resulting in better performance and reliability of the communication or detection system. The SNR concept is essential for understanding the robustness and effectiveness of signal processing systems, guiding the design and optimization of systems for optimal performance in various practical scenarios.

In the realm of laser cooling and trapping of atoms, various factors introduce noise, stemming from a blend of classical and quantum sources. Classical noise sources can arise from fluctuations in laser intensity, frequency, and beam pointing stability. These variations can impact the precision and stability of the trapping process, leading to uncertainties in the final atomic states. Additionally, mechanical vibrations from the environment, such as from nearby equipment or the laboratory infrastructure, can introduce classical noise that affects the trapping efficiency. On the other hand, quantum noise sources, such as atom shot noise, photon shot noise, quantum projection noise arise from the quantum nature of atoms and photons and it can also limit the SNR of the experiments. Understanding and mitigating both classical and quantum noise sources is essential for optimizing the performance of laser cooling and trapping systems, enabling

advancements in precision atomic physics and quantum technologies.

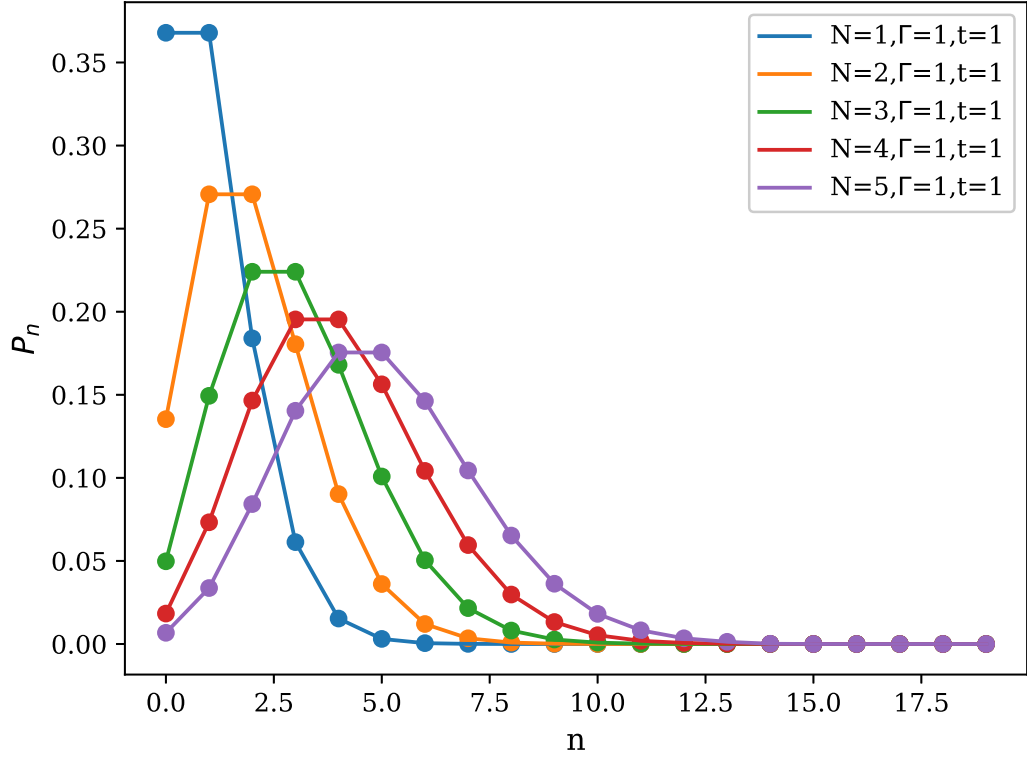


Figure 1.7: Atom shot noise: Poisson distributions of the atom shot noise for different number of total atoms.

In this section, we will focus on specific noise factors related to the number of atoms in laser cooling experiments. The laser cooling and trapping of neutral atoms involve complex processes with multiple sophisticated stages. Consequently, experiments involving laser cooling are susceptible to atom loss at various stages and during inter-stage transfers. For example, while cooling strontium atoms, a notable number of atoms is often lost during the transition from the initial cooling stage to the subsequent stage of cooling there by limiting the SNR. This emphasizes the significance of understanding and addressing noise factors arising from a reduced number of atoms. Our discussion will now center on these specific noise sources related to fewer atoms. Subsequently, in the later chapters of this thesis, we will discuss strategies aimed at mitigating these noises within the framework of cooling of strontium atoms.

1.2.1 Atom shot noise

Atom shot noise [86] is a fundamental quantum noise source prevalent in the domain of laser cooling of atoms. This noise becomes particularly relevant during the detection of atoms. In typical experimental setups, atoms are commonly detected through their fluorescence or absorption characteristics. When probing these atoms with laser light, the detection process involves the interaction of photons with individual atoms. As a consequence of the quantized nature of light and atom, the detection of each photon can be subject to inherent statistical fluctuations. This noise source becomes integral to consider when performing high precision experiments, as it sets a fundamental limit on the measurement accuracy. Understanding and mitigating atom shot noise is crucial for enhancing the sensitivity of spectroscopic measurements and atomic clocks. Consider an ensemble comprising N atoms undergoing spontaneous photon emission starting from $t = 0$. Within a sufficiently small time interval δt , the following relationships hold true:

$$P_n(t + \delta t) = P_{n-1}(t)P_1(\delta t) + P_n(t)(1 - P_1(\delta t)) \quad (1.15)$$

$$P_0(t + \delta t) = P_0(t)(1 - P_1(\delta t)) \quad (1.16)$$

Here, $P_n(t)$ denotes the probability of detecting n atoms at time t . The dynamics of these probabilities can be derived, resulting in the equations:

$$\frac{dP_n}{dt} = -N\Gamma(P_n - P_{n-1}) \quad (1.17)$$

$$\frac{dP_0}{dt} = -N\Gamma P_0 \quad (1.18)$$

Here, Γ represents the linewidth of the emission. With the initial condition $P_n(0) = 0$, the solution is given by:

$$P_n(t) = \frac{(N\Gamma t)^n}{n!} \exp(-N\Gamma t) \quad (1.19)$$

Equation (1.19) reveals that the probability distribution follows a Poissonian distribution. The standard deviation is calculated as:

$$\sigma = \sqrt{\sum_n n^2 P_n - \left(\sum_n n P_n\right)^2} \propto \sqrt{N} \quad (1.20)$$

This inherent noise, known as atom shot noise, contributes to the fluctuations in measurements. Initially, one might assume that increasing the number of atoms, denoted as N , would degrade the signal quality due to the proportional relationship between the variance and N . However, upon careful examination of the SNR, it becomes evident that $\text{SNR} \propto N/\sqrt{N} = \sqrt{N}$.

The SNR due to atom shot noise actually improves as the number of atoms increases. Consequently, the number of atoms present in these experiments helps in minimizing atom shot noise, thereby enhancing the quality of signals across various experimental scenarios.

1.2.2 Quantum projection noise

Quantum projection noise [87] arises from the population fluctuations of a specific atomic state within a fixed number of atoms, which have significant implications as they contribute to the errors in atomic frequency standards.

Consider a system of N non-interacting two-level atoms with energy levels $|g\rangle$ and $|e\rangle$. The number operator for the $|e\rangle$ state is given by:

$$\hat{N}_e = \sum_{i=1}^N |e_i\rangle \langle e_i| \quad (1.21)$$

Assuming the steady-state wavefunction of the i -th atom as $|\psi_i\rangle = c_g |g_i\rangle + c_e |e_i\rangle$, the wavefunction of the entire atomic system becomes:

$$|\psi\rangle = \bigotimes_{i=1}^N |\psi_i\rangle \quad (1.22)$$

The standard deviation of $\hat{N}_e$ can then be calculated as

$$\Delta N_e = \sqrt{\langle \psi | \hat{N}_e^2 | \psi \rangle - \left(\langle \psi | \hat{N}_e | \psi \rangle \right)^2} \quad (1.23)$$

Upon evaluation, it is found that $\langle \psi | \hat{N}_e^2 | \psi \rangle = (N^2 - N)|c_e|^2 + N|c_e|$ and $\langle \psi | \hat{N}_e | \psi \rangle = N|c_e|$. Consequently, the standard deviation ΔN_e is given by:

$$\Delta N_e = \sqrt{N|c_e|^2(1 - |c_e|^2)} \quad (1.24)$$

This leads to the Signal-to-Noise Ratio (SNR) being proportional to $\sqrt{N}$, denoted as $\text{SNR} \propto \sqrt{N}$.

Quantum projection noise plays a critical role in the functioning of atomic clocks as it introduces intrinsic statistical fluctuations directly affecting the clock transition signal. The stability of an atomic clock is quantified by the Allan Deviation (σ_y), which is inversely proportional to the SNR, given by $\sigma_y \propto 1/\text{SNR} \propto 1/\sqrt{N}$. This clearly illustrates that the clock's stability improves with an increased number of atoms, highlighting the significance of quantum projection noise in atomic clock operations.

1.2.3 Quantum phase fluctuation

Quantum phase fluctuation is a significant factor that can significantly impact the precision and stability of various quantum systems, particularly atom interferometers where phase holds utmost importance [88]. The uncertainty in phase directly affects the quality of the interferometry signal. Let's delve into the inherent phase noise of a two-level atomic system.

A two-level atom essentially behaves like a pseudo-spin system, allowing us to define spin operators within the context of the system:

$$\hat{\sigma}_x = \frac{|e\rangle \langle g| + |g\rangle \langle e|}{2} \quad (1.25)$$

$$\hat{\sigma}_y = \frac{|g\rangle \langle e| - |e\rangle \langle g|}{2i} \quad (1.26)$$

$$\hat{\sigma}_z = |e\rangle \langle e| - |g\rangle \langle g| \quad (1.27)$$

The determination of phase fluctuation ideally involves defining a 'phase operator,' which remains a longstanding challenge in quantum mechanics. Instead, we can define phase fluctuation from classical analog. For Ramsey interferometry, the phase fluctuation is given by:

$$\Delta\phi = \sqrt{\frac{\Delta\sigma_y}{\langle\sigma_x\rangle^2}} \quad (1.28)$$

For a system comprising N atoms, it can be demonstrated that $\Delta\phi \propto 1/\sqrt{N}$, often referred to as the Standard Quantum Limit (SQL). The SQL implies that phase fluctuation can be reduced by increasing the number of atoms. However, methods exist to surpass the SQL, such as employing squeezed light, where the phase fluctuation is given by $\Delta\phi \propto 1/\sqrt{N}$, indicating a faster reduction in phase fluctuation compared to the SQL case.

In essence, several quantum noises are present in laser cooling experiments, in which the number of atoms playing a significant role. Increasing the number of atoms in laser cooling and trapping experiments can notably enhance the SNR.

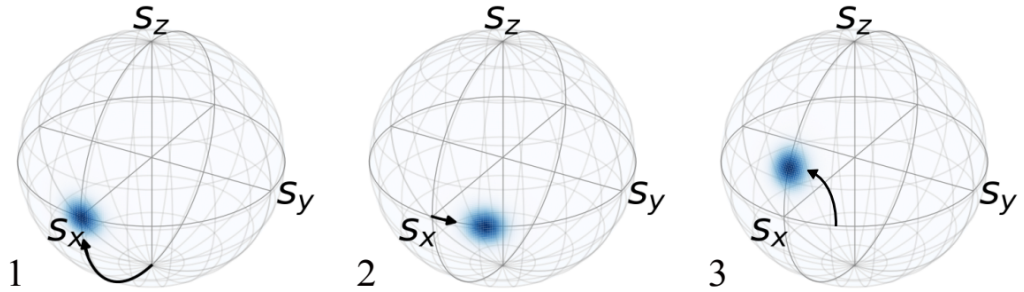


Figure 1.8: Quantum phase fluctuation: Depiction of quantum phase uncertainty on the Bloch sphere as the atomic state evolves. This image has been taken from [6].

1.3 Objective of the thesis

The enhancement of SNR in laser cooling experiments through the capture of a substantial number of atoms is an aspect that remains relatively underexplored compared to other topics. However, it holds an importance in the realm of laser cooling experiments. This thesis aims to shed light on the significance of this aspect, particularly focusing on the cooling process of strontium atoms. The central objective of this thesis is to amplify

the SNR by maximizing the number of atoms at various stages of the cooling process.

The process of cooling strontium atoms involves two sequential stages of Doppler cooling, commonly referred to as the blue Magneto Optical Trap (MOT) cooling and the red MOT cooling. The terms 'blue' and 'red' signify the wavelengths of 461 nm and 689 nm, falling within the visible electromagnetic spectra. The number of atoms captured in the blue MOT relies not only on the specific parameters of the blue MOT itself but also on the preceding stages such as Zeeman slowing and transverse cooling.

Optimizing the number of atoms in the blue MOT requires tuning various parameters to their optimal values. However, manual optimization process is time-consuming and does not guarantee finding the global maximum of atom numbers. This thesis introduces a methodology to streamline the optimization process by implementing the machine learning techniques, ensuring the discovery of the global maximum of atom numbers while saving time. Additionally, our proposed method offers the potential for automated loading of different isotopes into the MOT, expanding its versatility and efficiency in experimental setups.

After the completion of the blue MOT stage, the process of transferring strontium atoms to the red MOT poses a significant challenge. This challenge stems from the substantial difference in linewidth between the transitions of the blue MOT and the red MOT. Specifically, the issue arises from the difficulty of capturing atoms from the blue MOT using the red light, which is tuned to a much narrower linewidth, due to the Doppler shift effect.

Conventionally, this issue is tackled by artificially broadening the red light. However, even with the broadened red light, the efficiency of transferring atoms from the blue MOT to the red MOT remains around 40%, leading to a degradation in the SNR of the experiments.

To overcome this limitation, this thesis presents a novel three-level cooling scheme. This scheme significantly enhances the transfer efficiency of atoms from the blue MOT to the red MOT, achieving an efficiency of approximately 85%. This substantial improvement in transfer efficiency results in a considerable enhancement of the SNR in the experiments.

In a nutshell, the objective of this thesis is to enhance the SNR of laser cooling and

trapping experiments with strontium atoms by

- maximizing the number of atoms in the blue Magneto Optical Trap (MOT) by optimizing its parameters using a machine learning technique.
- improving the transfer efficiency of strontium atoms from the blue MOT to the red MOT through the implementation of a novel three-level cooling technique.

1.4 Organization of the thesis

Chapter 2: Chapter 2 begins with a discussion on the interaction between light and atoms, followed by an explanation of the semi-classical theory of Doppler cooling and trapping of neutral atoms. The latter part of this chapter focuses on the specific case of Doppler cooling applied to strontium atoms. Emphasis is placed on narrowline cooling techniques, crucial for achieving temperatures approaching the single-photon recoil limit.

Chapter 3: In this chapter, the construction of the experimental setup for the cooling and trapping of strontium atoms is detailed. It covers the essential components of the vacuum assembly, including the strontium oven, transverse cooling chamber, Zeeman slower, Magneto Optical Trap (MOT) chamber, ion pumps, and titanium sublimation filament. Furthermore, the chapter provides insights into the laser system and optical components required for the cooling and trapping process.

Chapter 4: This chapter delves into the blue MOT cooling process applied to strontium atoms. It provides a detailed discussion on the optimization of parameters using machine learning techniques, aiming to maximize the number of atoms in the blue MOT while saving valuable time. Additionally, this chapter covers the characterization of the blue MOT, offering insights into its performance and effectiveness.

Chapter 5: This chapter explores the intricacies and challenges associated with transferring atoms from the blue MOT to the red MOT. It discusses the conventional approach to deal with this process, highlighting its limitation. Additionally, it introduces a novel three-level cooling technique aimed at enhancing the transfer efficiency from the

blue MOT to the red MOT. The chapter also provides a comparative analysis between the traditional transfer methods and the new three-level cooling technique.

Chapter 6: Chapter 6 discusses the experimental methodologies for the red MOT and its characterization.

Chapter 7: Chapter 6 provides a comprehensive summary of the thesis, highlighting the significance of the experiments discussed. It emphasizes the practical implications and potential applications of the findings presented in this thesis. Furthermore, the chapter explores potential avenues for future experiments, building upon the groundwork laid out in the preceding chapters.

Chapter 2

Laser cooling and trapping of neutral atoms: A theoretical review

Laser cooling and trapping of neutral atoms is a revolutionary process that relies on the interaction between light and atoms. By harnessing the core principles of this interaction, it has been possible to effectively diminish the kinetic energy of atoms, leading to a substantial decrease in temperature. This groundbreaking technique, pioneered in the latter part of the 20th century, has now become a staple in laboratories worldwide, serving as a fundamental tool for a diverse arena of experimental investigations.

2.1 The light atom interaction

The Hamiltonian of a light interacting with a static atom is given by [89]

$$\mathcal{H} = \frac{1}{2} \int_V d^3x \left(\epsilon_0 \mathbf{E}^2 + \frac{\mathbf{B}^2}{\mu_0} \right) + \sum_i \frac{1}{2m_e} (\mathbf{p}_i + e\mathbf{A}(\mathbf{r}_i, t))^2 - e\phi(\mathbf{r}_i, t) + V_C(|\mathbf{r}_i|) \quad (2.1)$$

The index i indicates the i -th electron of the atom. The term $V_C(\mathbf{r}_i)$ includes the interaction between the nucleus and the electrons, as well as among the electrons within the atom. The term containing $\mathbf{A}(\mathbf{r}_i, t)^2$ resulting from the expansion of $(\mathbf{p}_i + e\mathbf{A}(\mathbf{r}_i, t))^2$ in the Hamiltonian can be neglected as it is considered 'sufficiently small'. Considering the atom's size is significantly smaller than the wavelength of the light allows for the removal of the dependence of the vector and scalar potential on $\mathbf{r}_i$. Therefore, $\mathbf{r}_i$ is re-

placed with $\mathbf{r}$, which is independent of the former quantity and represents the location of the atom in the light field. It should be noted that $\phi(\mathbf{r}, t) = 0$ as there is no static charge in the system.

Next, a Gauge transformation is performed to obtain the Hamiltonian in a more convenient form. The transformation is as follows [90]:

$$\phi'(\mathbf{r}_i, \mathbf{r}, t) = \phi(\mathbf{r}, t) - \frac{\partial}{\partial t}(\mathbf{A}(\mathbf{r}, t) \cdot \mathbf{r}_i) \quad (2.2)$$

$$\mathbf{A}'(\mathbf{r}_i, \mathbf{r}, t) = \mathbf{A}(\mathbf{r}, t) - \nabla_i(\mathbf{A}(\mathbf{r}, t) \cdot \mathbf{r}_i) \quad (2.3)$$

This provides us with $\phi'(\mathbf{r}, \mathbf{r}_i, t) = \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{r}_i$ and $\mathbf{A}'(\mathbf{r}, \mathbf{r}_i, t) = 0$. Finally, we have the following form of the Hamiltonian.

$$\mathcal{H} = \frac{1}{2} \int_V d^3x \left(\epsilon_0 \mathbf{E}^2 + \frac{\mathbf{B}^2}{\mu_0} \right) + \sum_i \left(\frac{\mathbf{p}_i^2}{2m_e} + V_C(|\mathbf{r}_i|) \right) - \mathbf{d} \cdot \mathbf{E}(\mathbf{r}, t) \quad (2.4)$$

Here, the expression $\mathbf{d} = e \sum_i \mathbf{r}_i$ represents the dipole moment of the atom. This allows us to express the Hamiltonian as follows: $\mathcal{H} = \mathcal{H}_{light} + \mathcal{H}_{atom} + \mathcal{H}_{int}$ with $\mathcal{H}_{light} = \frac{1}{2} \int_V d^3x \left(\epsilon_0 \mathbf{E}^2 + \frac{\mathbf{B}^2}{\mu_0} \right)$, $\mathcal{H}_{atom} = \sum_i \left(\frac{\mathbf{p}_i^2}{2m_e} + V_C(|\mathbf{r}_i|) \right)$ and $\mathcal{H}_{int} = -\mathbf{d} \cdot \mathbf{E}(\mathbf{r}, t)$.

For convenience, if we treat the light as classical and only the atom as a quantum object, we can readily omit $\mathcal{H}_{light}$, resulting in the following expression for the Hamiltonian:

$$\hat{\mathcal{H}} = \sum_i \left(\frac{\hat{\mathbf{p}}_i^2}{2m_e} + \hat{V}_C(|\mathbf{r}_i|) \right) - \hat{\mathbf{d}} \cdot \mathbf{E}(\mathbf{r}, t) \quad (2.5)$$

2.1.1 The two level approximation

Dealing with the Hamiltonian of a multi-electron atom is complex and computationally intensive. To simplify, we can consider an atom with two energy levels [17], denoted as $|g\rangle$ and $|e\rangle$, where $\hat{\mathcal{H}}_{atom} |g\rangle = \hbar\omega_g |g\rangle$ and $\hat{\mathcal{H}}_{atom} |e\rangle = \hbar\omega_e |e\rangle$. This implies that $\hat{\mathcal{H}}_{atom} = E_g |g\rangle \langle g| + E_e |e\rangle \langle e|$. Since the expectation value of the dipole moment operator, being an odd parity operator, is always zero, we can express $\hat{\mathbf{d}} = \mathbf{D} |g\rangle \langle e| + \mathbf{D}^\dagger |e\rangle \langle g|$. Consequently, the interaction Hamiltonian between the atom and

a light beam with a field vector $\mathbf{E}(\mathbf{r}, t) = \frac{1}{2}(\mathbf{E}_0(\mathbf{r}) \exp(i\omega_L t) \exp(i\phi(\mathbf{r})) + H.C)$ can be represented as:

$$\hat{\mathcal{H}}_{int} = \left(\frac{\hbar\Omega(\mathbf{r})}{2} \exp(i\omega_L t) \exp(i\phi(\mathbf{r})) |g\rangle \langle e| + \frac{\hbar\Omega^*(\mathbf{r})}{2} \exp(i\omega_L t) \exp(i\phi(\mathbf{r})) |g\rangle \langle e| \right) + H.C \quad (2.6)$$

Here, H.C represents the Hermitian conjugate of the counterpart. The quantity $\Omega = -(\mathbf{D} \cdot \mathbf{E}_0(\mathbf{r}))/\hbar$ is commonly referred to as the *Rabi frequency* [91].

2.1.2 The rotating wave approximation (RWA)

Although, the Hamiltonian is quite simplified, it has an explicit time dependence through $\hat{\mathcal{H}}_{int}$. It will be convenient to find a unitary transformation $\hat{U}(t)$ such that the new Hamiltonian $\hat{\mathcal{H}}_{new} = i\hbar \frac{\partial \hat{U}(t)}{\partial t} \hat{U}^\dagger(t) + \hat{U}(t) \hat{\mathcal{H}} \hat{U}^\dagger(t)$ is consistent with the Schroedinger equation and becomes independent of time. By means of a series of unitary transformations and adopting the RWA [92], we can remove the time dependence of the Hamiltonian. By transforming the Hamiltonian using the unitary operators $\hat{U}_1(t) = \exp(-iH_1 t)$, $\hat{U}_2(t) = \exp(-iH_2 t)$ and $\hat{U}_3(t) = \exp(-iH_3 t)$ with $H_1 = -\omega_g(|g\rangle \langle g| + |e\rangle \langle e|)$, $H_2 = \omega_{eg} |e\rangle \langle e|$ and $H_3 = \Delta |e\rangle \langle e|$ the time independent Hamiltonian can be derived. Here, $\omega_{eg} = \omega_e - \omega_g$. After the second unitary transformation, the Hamiltonian looks like

$$\hat{\mathcal{H}}' = \left(\frac{\hbar\Omega(\mathbf{r})}{2} \exp(i\Delta t) \exp(i\phi(\mathbf{r})) |g\rangle \langle e| + H.C \right) + \mathcal{O}\{\exp((\omega_L + \omega_{ge})t)\} \quad (2.7)$$

Here, $\Delta = \omega_L - \omega_{ge}$. Now, $\mathcal{O}\{\exp((\omega_L + \omega_{ge})t)\}$ oscillates much faster than $\exp(i\Delta t)$ and therefore can be ignored safely. This is known as the RWA. After the final transformation, the Hamiltonian has the following form.

$$\hat{\mathcal{H}}_{new} = -\hbar\Delta |e\rangle \langle e| + \left(\frac{\hbar\Omega(\mathbf{r})}{2} |g\rangle \langle e| + H.C \right) \quad (2.8)$$

From now onwards, the suffix of the new Hamiltonian will be dropped and represented as $\hat{\mathcal{H}}$.

2.2 The dynamics of an atom

To know the dynamics of an atom we must know the complete Hamiltonian of an atom in motion. This is given by

$$\hat{\mathcal{H}}_d = \frac{\mathbf{P}^2}{2m} + \hat{\mathcal{H}} \quad (2.9)$$

Using this Hamiltonian one can calculate the force on an atom. If we define the force operator $\hat{\mathbf{F}} = \frac{d\hat{\mathbf{P}}}{dt}$, then we can write $\hat{\mathbf{F}} = -\frac{i}{\hbar}[\hat{\mathbf{P}}, \hat{\mathcal{H}}_d] = -\frac{i}{\hbar}[\hat{\mathbf{P}}, \hat{\mathcal{H}}]$, where $\hat{\mathbf{P}}$ is the momentum operator of the atom. The expectation value of the force will be given by $\langle \hat{\mathbf{F}} \rangle = \text{Tr}(\hat{\rho} \hat{\mathbf{F}})$, where $\hat{\rho}$ is the density operator of the atomic states.

2.2.1 Evolution of the density operator

The evolution of the density operator is given by the Schrödinger equation which is as follows.

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}, \hat{\rho}] \quad (2.10)$$

However, there are always losses from the excited state of the atom due to the spontaneous emission. This can not be described by the Hamiltonian formalism and the Schrödinger equation should be modified in the following form ??.

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}, \hat{\rho}] - \frac{1}{2}\{\hat{\Gamma}_M, \hat{\rho}\} \quad (2.11)$$

Here, $\hat{\Gamma}_M$ represents the ‘decay operator’ that describes the losses due to the spontaneous emission. $\langle e | \hat{\Gamma}_M | e \rangle = \Gamma$ and $\langle g | \hat{\Gamma}_M | g \rangle = \langle g | \hat{\Gamma}_M | e \rangle = \langle e | \hat{\Gamma}_M | g \rangle = 0$. $\Gamma/2\pi$ is the decay rate of the atomic excited state population due to the spontaneous emission. Solution of Eq.(2.11) reveals the familiar Rabi oscillation observed in two-level systems (see Figure 2.1). It is worthy of mentioning here the evolution of the density operator described in Eq.(2.11) is a non-unitary one and that is why it does not preserve the trace of the same. One should compensate the loss from the excited state to preserve the norm to ensure the conservation of the probability.

By solving Eq.(2.11) with $\frac{\partial \hat{\rho}}{\partial t} = 0$ one can get the steady state solution of the density operator and find out the force on an atom exerted by the light.

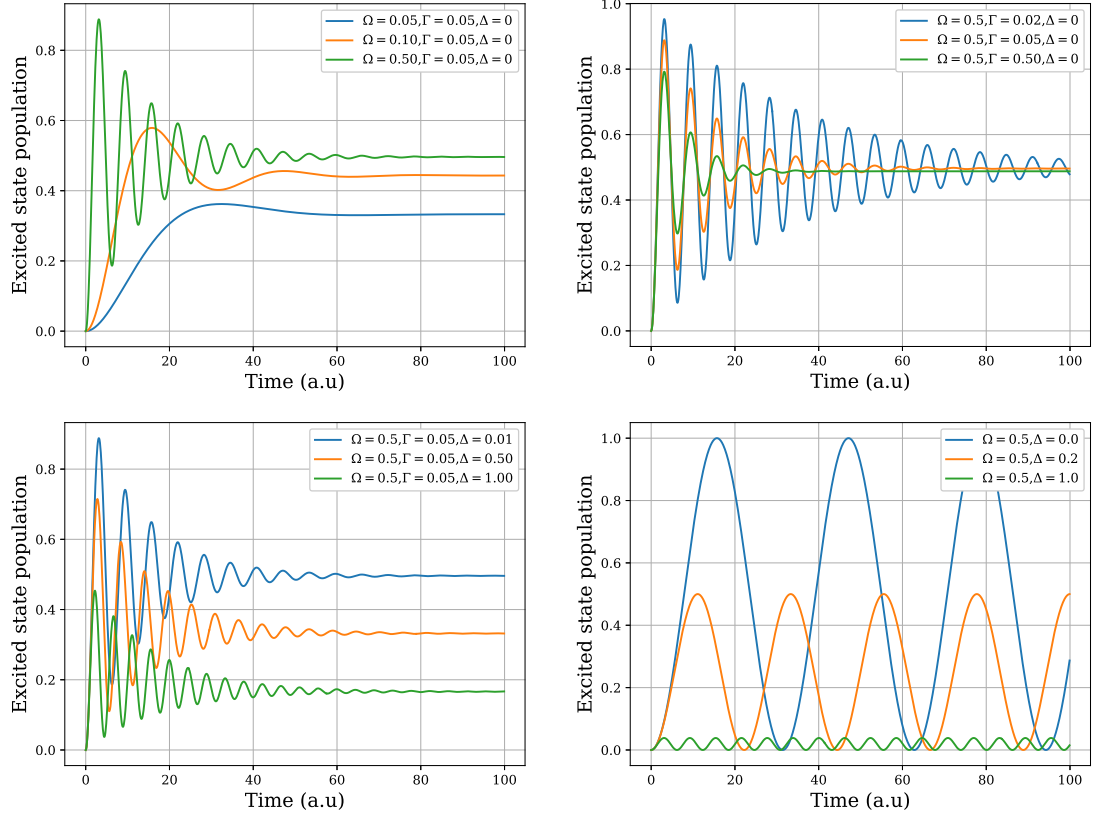


Figure 2.1: Rabi oscillation: Rabi oscillation in a two level system. Proceeding clockwise from the top left corner, these plots depict Rabi oscillations for various Rabi frequencies, different linewidths, zero linewidths with varying detunings, and finite linewidths with different detunings..

2.2.2 Force on an atom

Inserting the steady state solution of Eq.(2.11) into the expression of the expectation value of the force operator and taking time average over one period of oscillation of the light field, it can be shown that the force on an atom due to the light can be given by

$$\langle \mathbf{F} \rangle = -\frac{s(\mathbf{r})}{1 + s(\mathbf{r})} \left(\hbar \delta \frac{\nabla \Omega(\mathbf{r})}{\Omega(\mathbf{r})} + \frac{\hbar \Gamma}{2} \nabla \phi(\mathbf{r}) \right) \quad (2.12)$$

Here, $s(\mathbf{r}) = \frac{\Omega^2(\mathbf{r})/2}{\delta^2 + \Gamma^2/4}$ and δ is the detuning of the light. For a static atom without any other source of energy level shift, δ is identical to Δ . For a moving atom in an magnetic field δ includes the Doppler shift and the Zeeman shift. We can write $2\Omega^2(\mathbf{r})/\Gamma^2 =$

$I(\mathbf{r})/I_s$ where $I(\mathbf{r})$ is the intensity of the light and I_s is the *saturation intensity* of the corresponding atomic transition.

The first term of $\langle \mathbf{F} \rangle$ indicates the force on an atom due the intensity gradient of the light beam where as the second term comes from the gradient of the phase. If the light is not tightly focused, the first term can be ignored and the dependence of $s(\mathbf{r})$ on $\mathbf{r}$ can be neglected and as a result the expression of the force due to the light with plane wavefront ($\phi(\mathbf{r}) = -\mathbf{k} \cdot \mathbf{r}$) is given by

$$\begin{aligned} \langle \mathbf{F} \rangle &= \hbar \mathbf{k} \frac{\Gamma}{2} \frac{s}{1+s} \\ &= \hbar \mathbf{k} \frac{\Gamma}{2} \frac{s_0}{1+s_0+4\delta^2/\Gamma^2} \end{aligned} \quad (2.13)$$

Here, $s_0 = I/I_s$. Eq.(2.13) is also written as $\langle \mathbf{F} \rangle = \hbar \mathbf{k} \gamma_{sc}$, where $\gamma_{sc} = \frac{\Gamma}{2} \frac{s_0}{1+s_0+4\delta^2/\Gamma^2}$ is called the scattering rate.

2.2.3 Force due to the spontaneous emission

Eq. (2.12) and consequently Eq. (2.13) does not include the random force experienced by an atom due to the spontaneous emission. The force exerted by the spontaneous emission on an atom ($\boldsymbol{\eta}(t)$) is completely random in nature and as a result the time average of this force $\langle \boldsymbol{\eta}(t) \rangle_t = 0$. It is to be mentioned here that although, $\langle \boldsymbol{\eta}(t) \rangle_t = 0$ the autocorrelation function of the same is given by $\langle \boldsymbol{\eta}(t) \boldsymbol{\eta}(t + \tau) \rangle_t = 2D\delta(\tau)$. It can be that an atom moving in an n dimensional space, $D = n\Gamma\hbar^2\mathbf{k}^2s_0$. The quantity $\langle \boldsymbol{\eta}(t) \boldsymbol{\eta}(t + \tau) \rangle_t$ plays a very crucial role in the cooling of neutral atoms using light and restricts us from achieving the absolute zero temperature which will be discussed in the latter part of this chapter.

2.3 Cooling of neutral atoms

The force described in the Eq. (2.13) can be utilized to reduce the kinetic energy of atoms and thereby reducing the temperature. In practice, for cooling atoms, an atomic beam is generated by heating the atomic source at high tempearture. Then the atoms are cooled and trapped using light in a few successive stages.

2.3.1 The Zeeman slower

The atoms ejected from the thermal effusive oven are quite fast (~ 500 m/s). The most obvious way of slowing down these atoms is to imparting force on the atomic beam from the opposite direction of the beam propagation. This is done by making the wavevector $\mathbf{k}$ of the light opposite to the atomic beam propagation. The light is kept red detuned from the atomic transition that is used for exerting force on the atoms. Otherwise in the reference frame of atoms, due to the Doppler shift ($\Delta_D = -\mathbf{k} \cdot \mathbf{v}$) the light won't be in resonance with the atomic transition. As a result, it is evident from Eq. (2.13) as δ increases the magnitude of the force decreases. But, making the light red detuned is not enough. Because, in a slowing process the velocity of atom is constantly changing. As a result a fixed detuning will not work. Our goal is to maximize the force on atom by making $\delta = 0$ always during the slowing process. This problem can be addressed using an external magnetic field. Without any external perturbation, the total detuning is given by $\delta = \Delta - \mathbf{k} \cdot \mathbf{v}$. But in the presence of an external magnetic field, it can be written that $\delta = \Delta - \mathbf{k} \cdot \mathbf{v} + \Delta_Z$. Here Δ_Z is the Zeeman shift of the atomic energy level and is given by $\Delta_Z = -\mu_B g m_f B_{ext} / \hbar$. By spatially modulating the external magnetic field it is possible to nullify the effect of gradually changing Doppler shift in a slowing process and thus $\delta = 0$ is maintained throughout the process and the atoms get slowed.

If $\delta = 0$ is maintained throughout the process, the force exerted on an atom is constant and given by

$$\langle \mathbf{F} \rangle_{res} = \hbar \mathbf{k} \frac{\Gamma}{2} \frac{s_0}{1 + s_0} \quad (2.14)$$

While considering the slowing process, we can assume the kinematics of the atoms is one dimensional as the transverse components of the velocity of atoms ejected from the atomic oven are very small compared to the longitudinal components. So we can drop the vector notation and adopt scalar equation. Now, an atom with an initial velocity u after moving a distance L will have the final velocity

$$v = \sqrt{u^2 + \frac{2}{m} \langle F \rangle_{res} L} \quad (2.15)$$

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Now, $\delta = 0$ implies

$$B_{ext}(L) = \frac{\hbar}{\mu_B g m_f} \left(\Delta - k \sqrt{u^2 + \frac{2}{m} \langle F \rangle_{res} L} \right) \quad (2.16)$$

Thus, by spatially modulating the external magnetic field, it is possible to slow down the atoms efficiently.

2.3.2 The optical molasses

The optical molasses is an optical setup designed to reduce the kinetic energy of atoms moving at sufficiently low velocities. This arrangement consists of three pairs of counter-propagating laser beams, each pair coming from mutually independent directions. The force acting on an atom with velocity v in 1D is described by the equation:

$$F = \hbar k \frac{\Gamma}{2} \frac{s_0}{1 + s_0 + 4\delta_+^2/\Gamma^2} - \hbar k \frac{\Gamma}{2} \frac{s_0}{1 + s_0 + 4\delta_-^2/\Gamma^2} \quad (2.17)$$

Here, $\delta_{\pm} = \delta_0 + \Delta_d^{\pm}$, where δ_0 represents the light detuning and $\Delta_d^{\pm} = \pm kv$ denotes the detuning arising from the Doppler shift due to the motion of the atoms. When the atom's velocity is sufficiently low ($v \ll \frac{1+s_0\delta_0^2}{2k\delta_0}$), Eq.(2.17) simplifies, yielding the acceleration equation:

$$\frac{dv}{dt} = \alpha v \quad (2.18)$$

Here, α is defined as:

$$\alpha = 2 \frac{\hbar k^2}{m} \frac{s_0}{1 + s_0} \frac{\delta_0 \Gamma}{\delta_0^2 + \Gamma^2/4} \quad (2.19)$$

According to Eq.(2.19), the force (α) exhibits damping characteristics ($\alpha < 0$) only when the light (δ_0) is red-detuned from the atomic transition. Henceforth, we consider the light to be red-detuned, and the force acting on the atom due to the optical molasses is denoted as $F = -\alpha v$. Notably, to achieve this, we replace δ_0 with $|\delta_0|$ in Eq.(2.19).

Considering the damping force alongside the force due to spontaneous emission, the complete force equation becomes:

$$\frac{dv}{dt} = -\alpha v + \eta(t) \quad (2.20)$$

Here, the force due to spontaneous emission is scaled by a factor of $1/m$. The general solution to Eq.(2.20) is [93]:

$$v(t) = v_0 \exp(-\alpha t) + \int_0^t dt' \eta(t') \exp(-\alpha(t-t')) \quad (2.21)$$

This solution suggests that the average velocity $\left(\langle v \rangle = \frac{1}{t} \int_0^t v(t) dt\right)$ of an atom in the optical molasses tends towards zero over time. However, fluctuations in velocity persist due to kicks from spontaneous emission. The variance of velocity is then given by:

$$\Delta v^2 = \frac{D}{\alpha m^2} (1 - \exp(-2\alpha t)) \quad (2.22)$$

Here, D represents the diffusion coefficient mentioned in Section(2.2.3). As $t \rightarrow \infty$, Δv^2 does not vanish and converges to $\frac{D}{\alpha m^2}$. By applying the equipartition theorem, we derive the temperature of the cooling process:

$$k_B T = \frac{D}{\alpha m} \quad (2.23)$$

Upon substituting the values of α and D into Eq.(2.23), we obtain:

$$k_B T = \frac{\hbar \Gamma}{8|\delta_0|} (1 + s_0) (4\delta_0^2 + \Gamma^2) \quad (2.24)$$

It can be shown that at low intensity ($s_0 \rightarrow 0$), the temperature reaches a minimum at $|\delta_0| = \Gamma/2$. The corresponding value is:

$$T_{\text{lim}} = \frac{\hbar \Gamma}{2k_B} \quad (2.25)$$

This process of cooling is known as ‘Doppler cooling’, and T_{lim} is termed the ‘Doppler limited temperature’. Notably, the Doppler limited temperature depends on the linewidth of the cooling transition, resulting in different species of atoms being cooled to varying temperatures based on the linewidths of their cooling transitions.

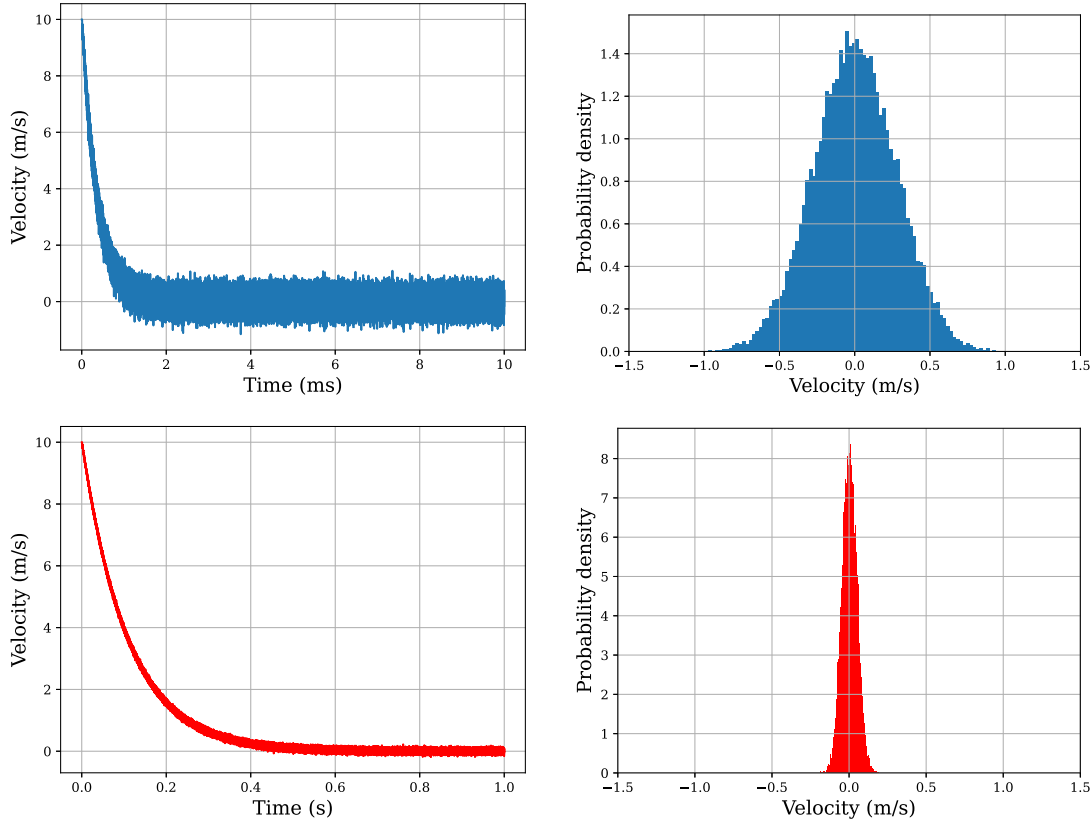


Figure 2.2: Optical molasses effect: Left column : Simulation of the deceleration of a strontium atom (initial velocity = 10 m/s) in blue optical molasses ($5s^2\ ^1S_0 \rightarrow 5s5p\ ^1P_1$) and red optical molasses ($5s^2\ ^1S_0 \rightarrow 5s5p\ ^3P_1$) with a saturation parameter of 0.01 in both cases. The fluctuation around zero velocity reflects the temperature of the molasses. Right column: Histogram plots of velocity distribution at steady state are shown, where the width of the distributions corresponds to the molasses temperature. The red molasses exhibits more efficient cooling due to the narrower linewidth.

2.3.3 The Magneto Optical Trap

Even after the atoms are cooled using the optical molasses setup, they must be spatially confined to prevent their escape. This is where magnetic field is used for achieving the spatial confinement. Typically, in a cooling transition, the lower energy states of atoms are insensitive to magnetic fields, while the excited states experience splittings due to the Zeeman effect.

To create this spatial confinement, a linear anti-symmetric magnetic field is employed with respect to the center of the optical molasses. This arrangement ensures

that the Zeeman splittings become spatially anti-symmetric as well. The outcome is the creation of potential minima right at the center of the molasses, effectively trapping the atoms in place. This combined setup of the optical molasses with the anti-symmetric magnetic field is aptly known as the Magneto Optical Trap (MOT).

When under the influence of such a linear anti-symmetric magnetic field, the first-order Zeeman splitting of a state is described by the following formula (in terms of angular frequency):

$$\Delta_Z = \frac{\mu_B}{\hbar} g m_j \frac{dB}{dx} x \quad (2.26)$$

In this scenario, g represents the Landè g-factor specific to the state, m_j denotes the angular momentum component along the magnetic field, $\frac{dB}{dx}$ signifies the magnetic field gradient, and x indicates the atom's position. When the angular momentum of the excited state is $j = 1$, the permitted values for m_j are $m_j = 0, \pm 1$. Considering an optical transition using only σ^+ and σ^- lights, the transition from the ground state to the excited state with $m_j = 0$ is forbidden due to angular momentum conservation. Hence, within the context of σ^+ and σ^- polarizations, we can express the Zeeman splitting of a state with $j = 1$ as follows.

$$\Delta_Z^\pm = \pm \frac{\mu_B}{\hbar} g \frac{dB}{dx} x \quad (2.27)$$

This leads to the total detuning (δ) in Eq.(2.17) now including the Zeeman effect:

$$\delta = \delta_0 \pm kv \pm \frac{\mu_B}{\hbar} g \frac{dB}{dx} x \quad (2.28)$$

Now, with four pairs of available combinations based on wavevectors and polarizations of light beams: $(+k, \sigma^+)$, $(+k, \sigma^-)$, $(-k, \sigma^+)$, and $(-k, \sigma^-)$, a strategic choice ensures that atoms moving with positive velocity interact more strongly (less detuning) with the $-k$ wavevector than with $+k$, and vice versa for negative velocity atoms. Consequently, atoms attempting to escape the trap will invariably be pushed towards its center.

By inserting the Doppler and Zeeman detunings due to the chosen wavevector and polarization combinations into Eq.(2.17), the equation simplifies to (with the effects of

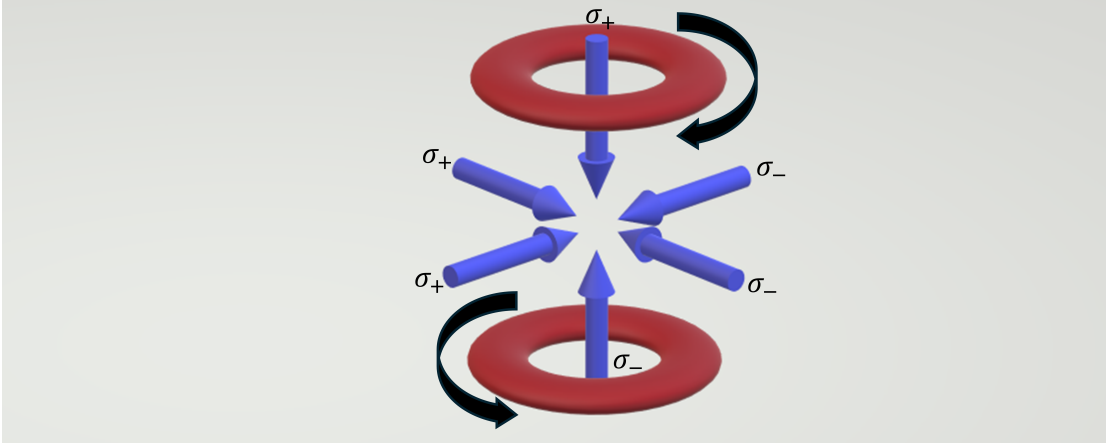


Figure 2.3: MOT Configuration: Brown tori depict magnetic quad coils arranged in an anti-Helmholtz configuration, ensuring anti-symmetry in the magnetic field vectors along all three directions. The direction of currents in the coils is indicated by black arrows. Counter-propagating light beams are represented by blue arrows, with their polarizations noted next to them.

spontaneous emissions included):

$$\frac{dv}{dt} = -\alpha v - \beta x + \eta(t) \quad (2.29)$$

Here, $\beta = \frac{g}{|k|} \left| \frac{dB}{dx} \right| \alpha$. The term $-\beta x$ provides the atoms with the spatial confinement. The resulting trapping potential $V(x)$ is given by $V(x) = \frac{1}{2}\beta x^2$. As the magnetic field gradient (β) increases, the trap tightens, leading to a compression of the trapped atoms.

2.4 Cooling of strontium atoms

In this section, we explore the process of cooling strontium atoms, an element with an atomic number of 38, represented by the symbol 'Sr', and classified within the second group of the periodic table known as the alkaline earth elements or group-II elements.

As a member of the group-II elements, strontium features two valence electrons in its outermost shell. Similar to atomic helium, its energy levels can be categorized into two distinct classes:

- The singlet state, characterized by an anti-parallel arrangement of electronic spins resulting in a total spin of $S = 0$

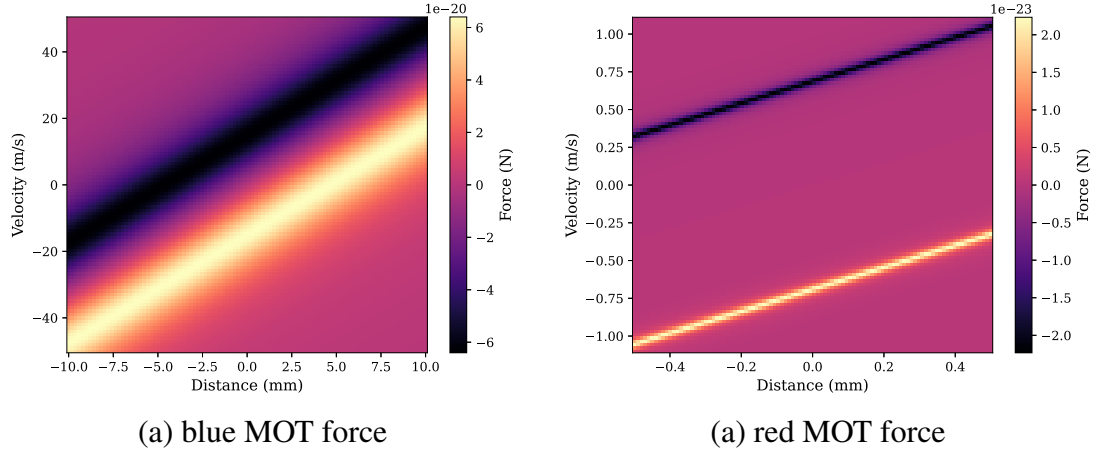


Figure 2.4: MOT Force: MOT force distribution over classical phase space on strontium atoms. (a) shows blue MOT force with $s_0 = 1$ and $\Delta = -2\pi \times 32$ MHz. (a) shows blue MOT force with $s_0 = 1$ and $\Delta = -2\pi \times 32$ MHz. (b) represents red MOT force with $s_0 = 100$ and $\Delta = -2\pi \times 1$ MHz

- The triplet state, where electronic spins are aligned parallelly, yielding a total electronic spin of $S = 1$

Within strontium, there are prominent transitions occurring between the ground state and the singlet state, following the selection rules of dipole transitions. Conversely, transitions between the ground state and the triplet states, also referred to as inter-combination lines, are forbidden by the dipole selection rules. This characteristic results in narrower transition linewidths for these inter-combination lines. Consequently, these transitions can be effectively utilized to cool strontium atoms to temperatures corresponding to the single-photon recoil limit using Doppler cooling techniques. Furthermore, these transitions hold potential for applications in quantum information processing and the development of highly precise optical atomic clocks.

For instance, in the case of ^{88}Sr (as depicted in Figure 2.5), the transition linewidth from $(5s^2)^1S_0$ to $(5s5p)^3P_1$ measures 7.5 kHz, leading to a Doppler-limited temperature of 180 nK. This temperature range enables the observation of quantum phenomena while mitigating the effects of thermal decoherence. Additionally, the transition from $(5s^2)^1S_0$ to $(5s5p)^3P_0$ exhibits a linewidth of approximately 1 mHz, making it suitable for applications in quantum information processing and the construction of highly accurate optical atomic clocks due to the extended lifetime of these transitions.

38 Chapter 2. Laser cooling and trapping of neutral atoms: A theoretical review

There exist four naturally occurring stable isotopes of the strontium atom. Table 2.1 provides a list of these strontium isotopes, presenting their respective nuclear spins, relative atomic masses, and relative abundances:

Isotope	Nuclear spin	Relative atomic mass	Relative abundance (%)
^{84}Sr	0	83.913 425(4)	0.56(1)
^{86}Sr	0	85.909 2624(24)	9.86(1)
^{87}Sr	9/2	86.908 8793 (24)	7.00(1)
^{88}Sr	0	87.905 6143(24)	82.58(1)

Table 2.1: Isotopes of Strontium (Sr): Naturally stable isotopes of the strontium atom along with their respective data from NIST.

In the following discussion, our focus will be specifically on the cooling process applied to ^{88}Sr atoms. As indicated in Table 2.1, ^{88}Sr stands out as the most abundant isotope of strontium which offers a high SNR during the experiments. Moreover, owing to its zero nuclear spin, the energy levels of ^{88}Sr are relatively simple. Consequently, the cooling and trapping of ^{88}Sr atoms are straightforward.

As previously discussed, the transition from $(5s^2)^1S_0$ to $(5s5p)^3P_1$ of the ^{88}Sr atom offers the potential to cool the atom to the desired temperature for observing quantum coherence phenomena. However, direct utilization of this transition is not the most efficient approach. To understand this, we need to delve into the concept of the 'capture velocity' of the optical molasses.

When an atom with an initial velocity v needs to be captured by the optical molasses, its velocity should decrease to approximately zero within the length scale of the optical molasses beams (R_m). Mathematically, this condition can be expressed as:

$$v^2 - 2\frac{F}{m}R_m = 0 \quad (2.30)$$

This equation sets a limit on the velocity that can be trapped by the optical molasses configuration. The condition for capture velocity is stated as:

$$v_{\text{lim}} \leq \sqrt{2\frac{F}{m}R_m} \quad (2.31)$$

This velocity is commonly known as the 'capture velocity'. Upon substituting the expression of F from Eq.(2.13), it is evident that $v_{\text{lim}} \propto \Gamma$. This implies that a higher transition linewidth corresponds to a higher capture velocity and, consequently, a higher capture probability. Hence, the transition from $(5s^2)^1S_0$ to $(5s5p)^1P_1$ (refer to Figure 2.5), known as the 'blue MOT' transition, is utilized prior to the narrow line transition $(5s^2)^1S_0 \rightarrow (5s5p)^3P_1$, referred to as the 'red MOT'.

2.4.1 The blue MOT cooling

The blue MOT cooling employs the strong dipole-allowed cyclic transition $5s^2\ ^1S_0 \rightarrow 5s\ 5p\ ^1P_1$, which necessitates a laser with a wavelength of 460.8 nm. This transition is characterized by a short lifetime of approximately 5 ns, corresponding to a linewidth of 32 MHz. The relatively broad linewidth of this transition results in a higher Doppler limited temperature of around 770 μK . Additionally, the transition is not entirely closed, as atoms excited to the 1P_1 state can decay to the 1D_2 state with a branching ratio of $1 : 2 \times 10^{-5}$. These decayed atoms can further transition to the triplet states $5s\ 5p\ ^3P_2$ and $5s\ 5p\ ^3P_1$ with a branching ratio of 1 : 2 respectively.

2.4.2 The red MOT cooling

The red MOT cooling method utilizes the dipole-forbidden transition $5s^2\ ^1S_0 \rightarrow 5s\ 5p\ ^3P_1$, requiring a laser with a wavelength of 689 nm. This transition is notable for its ultra-narrow linewidth of 7.5 kHz. Due to the exceptionally narrow linewidth, the Doppler limited temperature associated with this transition is approximately 180 nK, which closely aligns with the single photon recoil temperature (460 nK). As a result, this transition proves to be effective in cooling strontium atoms to temperatures where thermal decoherence effects are significantly reduced.

However, the behavior of the red MOT process is intricate due to the involvement of a transition with an ultra-narrow linewidth. As previously discussed, the interaction between light and atoms is described by Eq.(2.11), which highlights the significance of Γ , Δ , and Ω in the dynamics. When either Ω or Γ is much larger than Δ , the dynamics can be approximated using semiclassical methods. This occurs because the impact of Doppler shifts resulting from photon scattering is predominantly influenced by Ω or Γ .

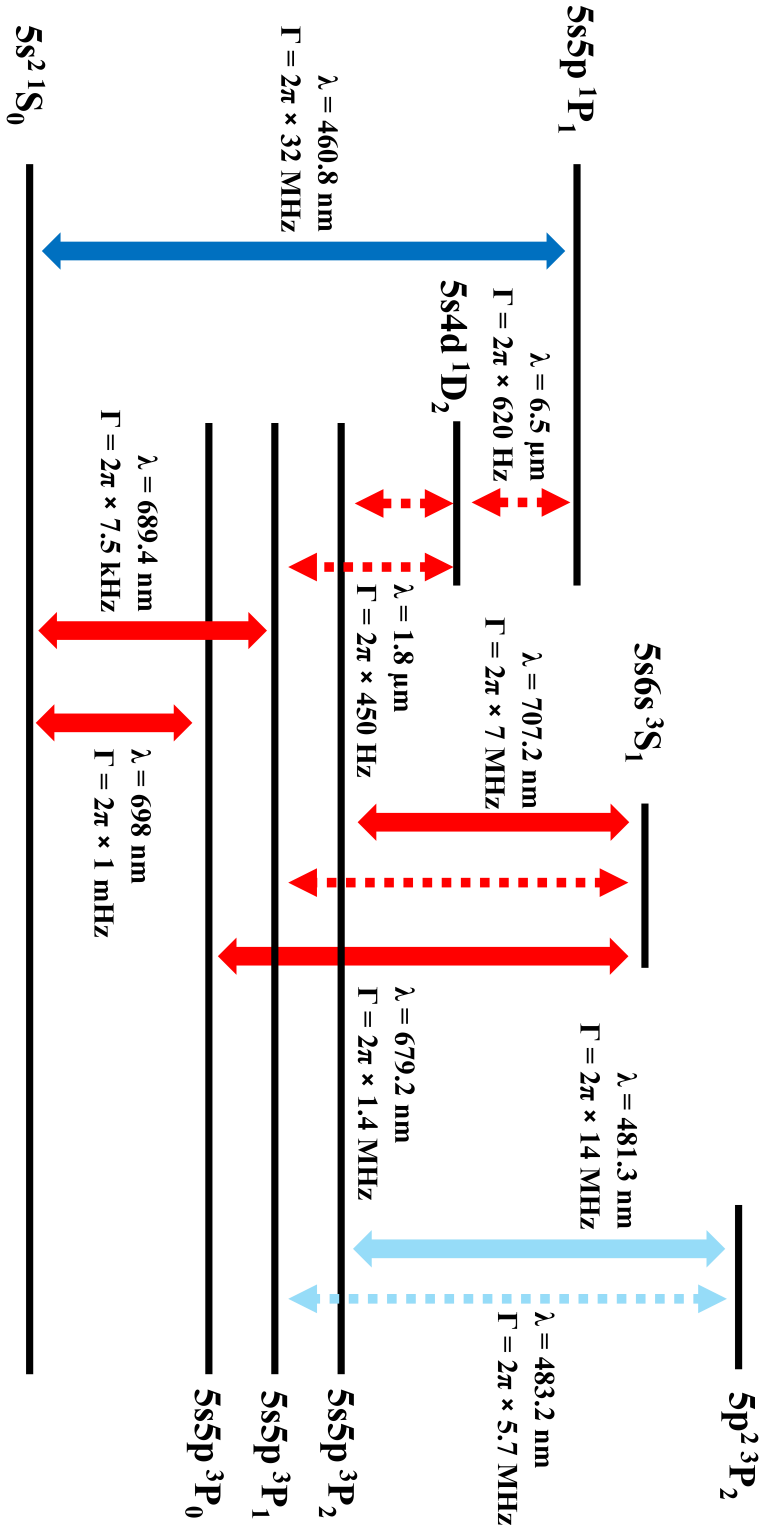


Figure 2.5: Low lying energy level diagram of ^{88}Sr . The energy levels have been labeled by Russel-Saunders notation ($^{2S+1}L_J$).

Consequently, when the intensity of the light is high, such as when $\Omega \gg \Delta$, semiclassical treatment is applicable even if Γ remains narrow. However, if all three parameters are comparable, a full quantum treatment becomes necessary. Because in this parameter regime, a single photon recoil can drive the system out of resonance, particularly due to the significantly small value of Γ , leading to a divergent result from the semiclassical approximation.

Transferring atoms from the blue MOT stage to the red MOT stage presents its own set of challenges. Following the cooling in the blue MOT, direct capture of atoms into the red MOT is not feasible. This is because the 689 nm light, tuned to the narrow $5s^2 \ ^1S_0 \longrightarrow 5s \ 5p \ ^3P_1$ transition, cannot effectively interact with atoms in the blue MOT due to their broadened velocity distribution caused by thermal motion. To overcome this hurdle, the red MOT light is initially artificially broadened by modulating its frequency. This process, known as the broadband red MOT, captures a significant number of atoms and cools them down to a temperature where the red MOT light can interact without being affected by the broadened velocity distribution. Subsequently, the red MOT operates in a single frequency mode, referred to as the single frequency red MOT.

In this chapter, a concise overview of the theoretical principles behind laser cooling and trapping of neutral atoms has been presented, with a specific focus on the cooling procedures for ^{88}Sr atoms. Drawing from these theoretical foundations, the subsequent chapter will delve into the description of the experimental setup employed for the laser cooling and trapping of ^{88}Sr atoms.

Chapter 3

Experimental apparatus and methods

In this chapter, we will demonstrate the design and construction of the experimental setup employed for laser cooling of strontium atoms.

The experimental setup described in this chapter is meticulously engineered to meet the stringent requirements for laser cooling of strontium atoms. We begin by outlining the key components of the setup, including the laser systems, optical elements, and vacuum apparatus. Each component plays a crucial role in the overall functionality of the experiment, from generating precisely tuned laser beams to creating ultra-high vacuum conditions necessary for atomic manipulation. Additionally, we explore the challenges associated with mitigating systematic effects and external perturbations such as stray magnetic fields and laser frequency fluctuations that may impact the performance of the system.

Overall, this chapter provides a comprehensive overview of the experimental apparatus employed for laser cooling of strontium atoms, highlighting the design and implementation of each component. Through careful optimization and rigorous experimental techniques, we aim to maximize the numbers of atoms in MOT, thereby increasing the SNR of the physics experiments in the subsequent stages.

3.1 The vacuum system

Laser cooling operates on the principle of transferring momentum from directed light to atoms. This is accomplished by conducting the laser cooling and trapping process

within an ultra-high vacuum (UHV) chamber. Within this UHV environment, the lifetime of atoms confined in a trap is extended due to the scarcity of background collisions. The UHV regime is defined by pressures below 10^{-9} torr. Achieving these operational conditions entails designing a vacuum chamber equipped with a suitable pumping mechanism to establish and sustain the desired pressure levels. This aspect stands as a pivotal requirement for experiments involving cold and ultracold atoms or molecules, necessitating meticulous design considerations.

In the experiments detailed in this thesis, the vacuum chamber has been designed using a 3D CAD software. The assembly incorporates standard vacuum components. Most of these components have been sourced from SMP Enterprises and Fourvac Technologies, located in Pune, India.

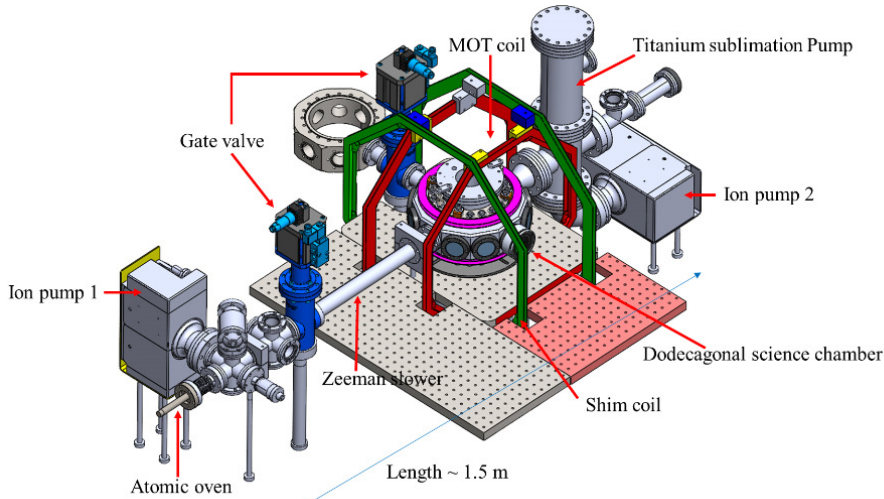


Figure 3.1: Vacuum chamber: 3D isometric view of the vacuum chamber used for the production of cold strontium atoms.

The schematic diagram of the vacuum chamber is shown in Figure (3.1). The crucial components of this setup are as follows:

1. The strontium oven
2. The transverse cooling chamber.
3. The Zeeman slower
4. The MOT chamber

5. The magnetic shim coils
6. The ion pumps
7. The titanium sublimation pump.

The strontium oven serves as the primary source of atomic strontium in the setup, utilizing thermal effusion. The transverse cooling chamber is used for the enhancement of the collimation of the atomic beam. Following this, the Zeeman slower is employed to decelerate atoms emitted from the thermal oven, thereby maximizing the loading efficiency of the Magneto-Optical Trap (MOT). Within the MOT chamber, laser cooling and trapping of atoms occurs. To cancel external magnetic field, magnetic shim coils are employed. Additionally, the maintenance of ultra high vacuum quality is ensured through the use of ion pumps and titanium sublimation pumps.

3.1.1 The strontium oven

A critical requirement for laser cooling experiments is the availability of a high-flow atomic source capable of producing a collimated stream of atoms. To meet this demand, an effusive thermal oven is constructed. Effusion, in this context, describes the process through which a gas escapes from a container via a hole significantly smaller than the mean free path of its molecules. The mean free path (μ) at a given pressure (P) and temperature (T) is determined by $\mu = \frac{k_B T}{\pi \sigma^2 P}$ [94]. Here, σ represents the atomic diameter. For strontium within the temperature range of 400°C to 600°C, the vapor pressure (in Pascals) is modeled by [95]:

$$\log_{10}(P) = A - \frac{B}{T} \quad (3.1)$$

where $A = 9.584 \pm 0.132$ and $B = 7566 \pm 101$. Strontium is heated to approximately 500°C. At this temperature, the mean free path of strontium atoms is approximately 3 mm. These atoms are directed through holes with diameters much smaller than 500 μm . The effusive flux is given by [94]:

$$\phi = \frac{\Delta P d^2}{2m v_{\text{av}}} \quad (3.2)$$

Here, ΔP represents the pressure difference across the hole, d is the hole's diameter, m stands for the atom's mass, and v_{av} denotes the average velocity according to the Maxwell-Boltzmann distribution, given by $\sqrt{\frac{8k_B T}{\pi m}}$. The temperature dependence of the flux is illustrated in Figure 3.2.

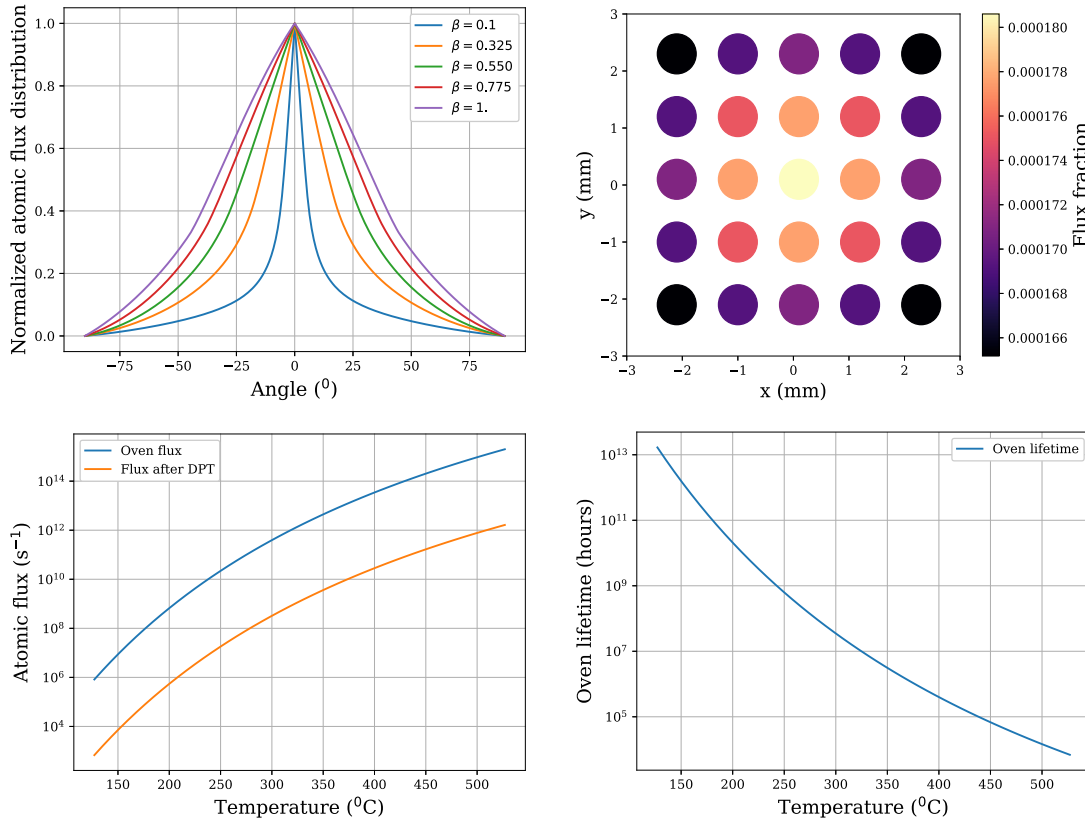


Figure 3.2: Oven characteristics: Starting from the top left corner and proceeding clockwise the figures respectively demonstrate the theoretical angular distribution of the atomic beam of effusive capillaries, the fraction of flux contributed by different effusive holes arranged in a rectangular array, the life time of the oven with ~ 5 gm strontium and the flux of the oven before and after the differential pumping tube (DPT). The corresponding parameters of these plots are discussed in detail later in this section.

From Eq.(3.2), it is evident that the flux will increase if the diameter of the hole is increased. However, increasing the diameter will decrease the quality of collimation of the atomic beam. Within the limit of molecular flow, the angular distribution of the beam divergence (ρ) is given by [96]:

$$\begin{aligned}\rho(\theta) &= \alpha \cos(\theta) + \frac{2}{\pi} \cos(\theta) \left[(1 - \alpha)R(q) + \frac{2}{3q}(1 - 2\alpha)(1 - (1 - q^2)^{3/2}) \right] \quad \text{if } q \leq 1 \\ &= \alpha \cos(\theta) + \frac{4}{3\pi q}(1 - 2\alpha) \quad \text{if } q > 1\end{aligned}\quad (3.3)$$

Here, θ represents angle, $R(q) = \cos^{-1}(q) - q\sqrt{1 - q^2}$, $\alpha = \frac{1}{2} - \frac{3}{\beta^2} \left\{ \frac{1 - 2\beta^3 + (2\beta^2 - 1)\sqrt{1 + \beta^2}}{\sqrt{1 + \beta^2} - \beta^2 \sinh^{-1}(1/\beta^2)} \right\}$, and $\beta = d/l$ with l being the length of the hole. The angular distribution of the atomic beam is illustrated in Figure 3.2 for different values of β . Equations (3.2) and (3.3) together suggest the need for an optimized value of d to achieve optimal performance. In our setup, we have $d \approx 500 \mu\text{m}$ and $l = 2 \text{ cm}$, resulting in $\beta \approx 0.025$. The 3D CAD design of the oven is given in Figure 3.3.

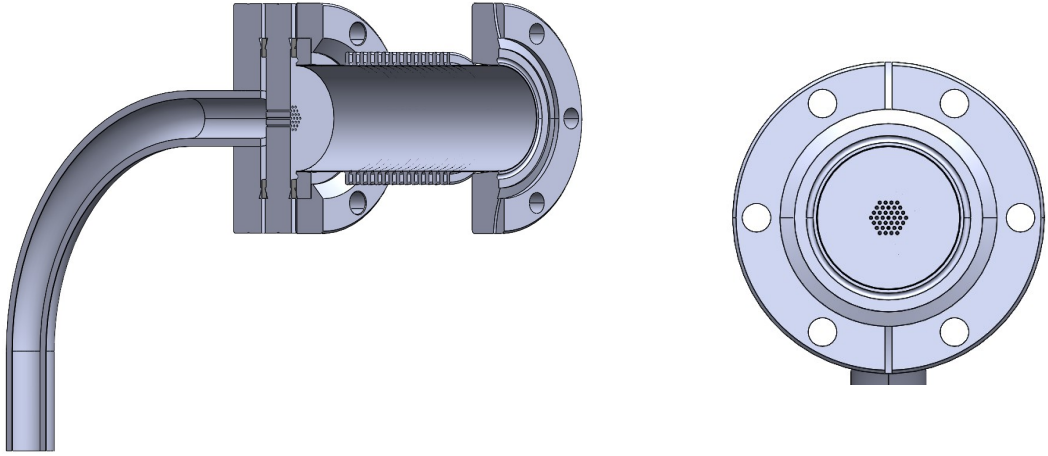


Figure 3.3: Oven assembly: On the left side of the illustration, the cross-section of the thermal oven assembly is depicted. The curved cylindrical region represents the strontium reservoir, which is connected to a CF35 flange. On the right side, micro-drills are visible on the face of the flange, serving as the effusive holes of the thermal oven.

To further narrow the width of the angular distribution, a collimator, also acting as a differential pumping tube (DPT) is employed. A DPT is a long, narrow tube that allows only atoms with low angular divergence to pass through. Assuming the wall of the DPT to be perfectly ‘dark’, the flux contribution of a particular oven hole is given by [96]:

$$f = \frac{1}{\pi W} \int_{\phi} d\phi \int_{\theta} \rho(\theta) \sin(\theta) d\theta \quad (3.4)$$

Here, the limits over θ and ϕ depend on the relative position of the oven hole and the collimator, as well as the geometry of the DPT. In Equation (3.4), W is a normalization constant and is given by $W = \frac{1}{\pi} \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \rho(\theta) \sin(\theta) d\theta$. The collimator also plays a crucial role in maintaining the pressure difference between the oven side and the MOT chamber of the assembly. This pressure difference is essential for efficient cooling and trapping of atoms. The pressure difference between the two ends of the DPT scales as $\propto 1/D^4$ (D - diameter of the DPT) [97], indicating that a smaller diameter will maintain a higher pressure difference. Mathematically, it can also be shown that the smaller the diameter, the smaller the divergence of the atomic beam. However, a smaller diameter of the DPT affects the atomic flux. Therefore, careful optimization of the DPT is necessary for optimal performance.

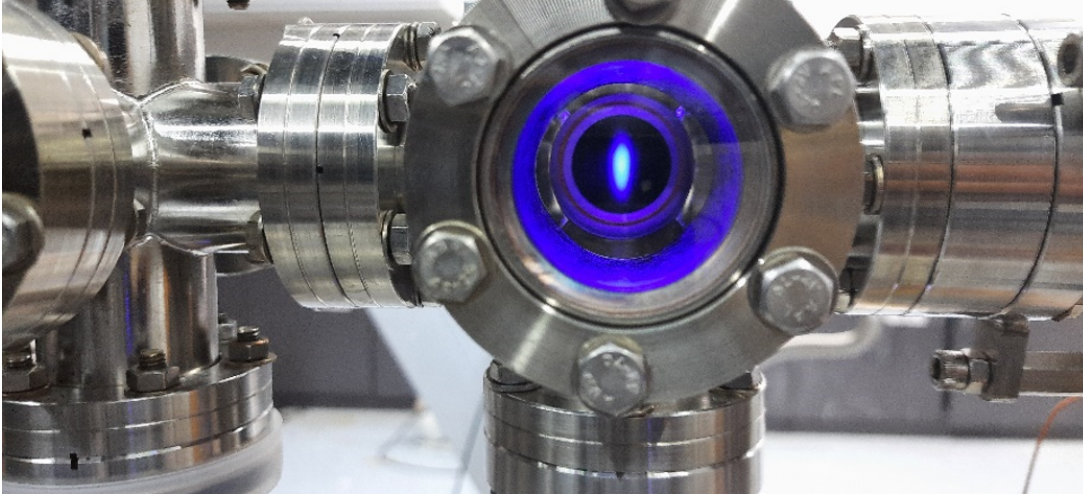


Figure 3.4: The Oven: Image of the fluorescence of the atoms ejecting from the oven.

In our setup, the DPT is located approximately 5 cm away from the oven. The radius and length of the DPT are approximately 5 mm and 10 cm, respectively. The calculated total flux contribution after the DPT is illustrated in Figure (3.2). The DPT in our setup effectively maintains the pressure in the MOT chamber at 10^{-11} Torr, while the pressure on the oven side reaches as high as 10^{-8} Torr. The detailed experimental results for the oven are presented in Mr. Kushal Patel's PhD thesis.

3.1.2 The transverse cooling chamber

Even after passing through the DPT, the atomic beam retains some finite divergence. To mitigate this, we employ the transverse cooling method. Transverse cooling involves directing resonant light from two orthogonal directions relative to the atomic beam's propagation, spatially “squeezing” the beam to improve collimation. This enhancement in collimation boosts the capture efficiency of the Magneto-Optical Trap (MOT). The chamber of transverse cooling is shown in Figure (3.5).

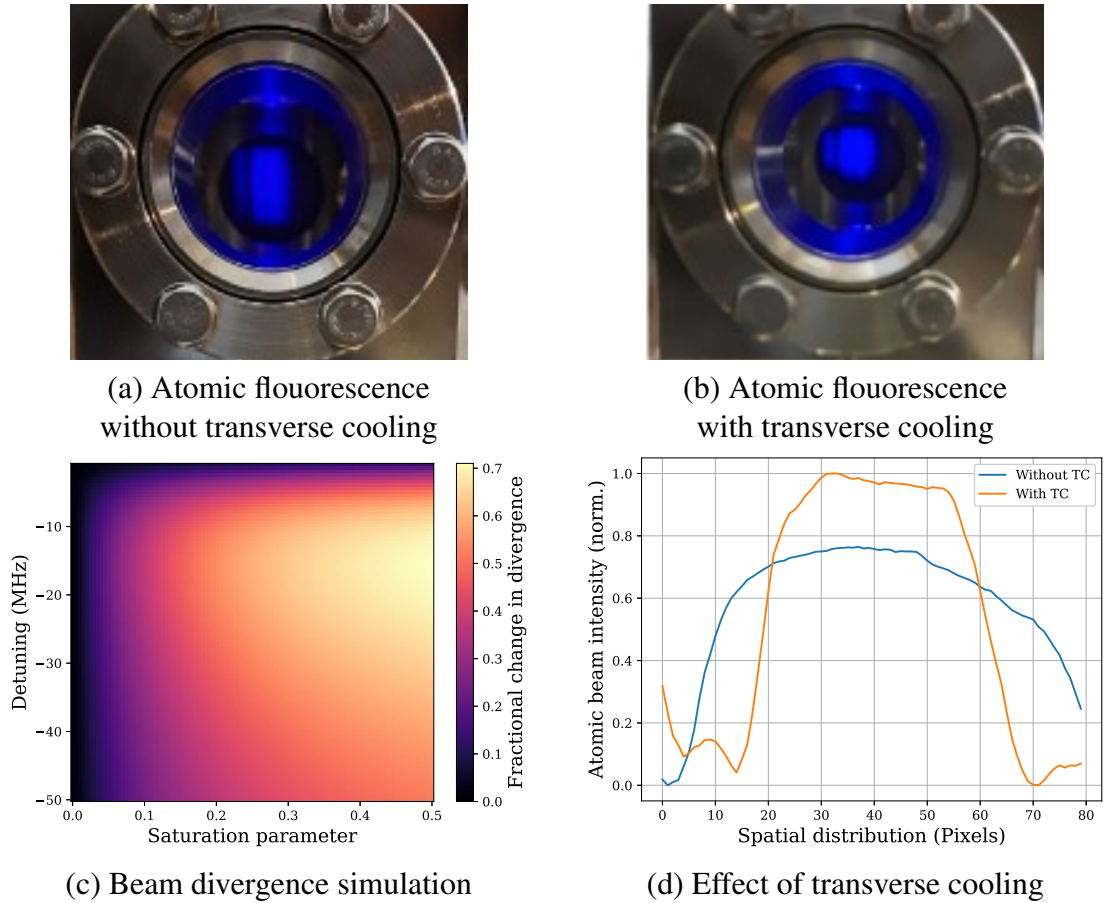


Figure 3.5: Transverse cooling: Snapshot comparison of the atomic fluorescence with (a) and without (b) transverse cooling. (c) presents the theoretical values of beam divergence with saturation parameter and light detuning considering beam diameter to be 1.5 cm and the longitudinal velocity to be 500 cm. (d) provides the quantitative representation of (a) and (b).

Transverse cooling can be likened to a 2D optical molasses. By manipulating the light-atom interaction, the transverse components of atomic velocity are exponentially

damped. The force equation in any of the transverse directions can be mathematically expressed as [17]:

$$\frac{dv_T}{dt} = -\frac{\beta}{m}v_T \quad (3.5)$$

Here, m represents the atom's mass, and β denotes the damping factor in an optical molasses configuration, as discussed previously. Equation (3.5) indicates that for an atom with an initial transverse velocity v_T ($\ll v_L$, where v_L is the longitudinal velocity), the final transverse component becomes $v_T \exp(-\frac{\beta R}{mV_L})$, where R is the extent of the laser beam along the longitudinal velocity component.

When $v_T \ll v_L$, the angle of divergence can be approximated as $\theta \approx v_T/v_L$. Therefore, the change in the divergence angle after transverse cooling is given by $\delta\theta = \left(\exp(-\frac{\beta R}{mV_L}) - 1\right)\theta$, where θ_i represents the divergence before transverse cooling. In a linear approximation, the fractional change in the divergence angle is given by:

$$\frac{\delta\theta}{\theta} \approx -\frac{\beta R}{mV_L} \quad (3.6)$$

The negative sign in Equation (3.6) indicates that the final divergence will be smaller than the initial divergence. It also implies that the divergence will decrease if we increase R while keeping β constant. This can be achieved by making the light beam elliptical, with the semi-major axis aligned with the longitudinal velocity component. The elliptical beam assists in increasing R while maintaining the intensity of the light almost constant, thereby keeping β intact.

In our experimental setup, we maintained the saturation intensity of the transverse cooling at approximately 0.1 and the detuning at around 20 MHz. This led to a reduction in the beam divergence by a factor of 1.5, corresponding to a fractional change of 0.33. This effect is illustrated in Figure 3.5.

3.1.3 The Zeeman slower

The Zeeman slower plays a critical role in the laser cooling process of strontium atoms, serving as the initial cooling stage in the setup. Following the transverse cooling stage, the atoms' temperature remains at 600° C, with velocities too high for capture by an

optical molasses configuration. Calculating the capture velocity of the optical molasses, denoted as v_{lim} , using Eq.(2.31), it becomes apparent that with typical parameters, $v_{\text{lim}} \sim 20$ m/s. The velocity distribution of the atoms at 600°C is significantly broad and the fraction of atoms below 20 m/s is nearly negligible. Hence, without slowing down the atoms, the loading of atoms into the MOT would be considerably low.

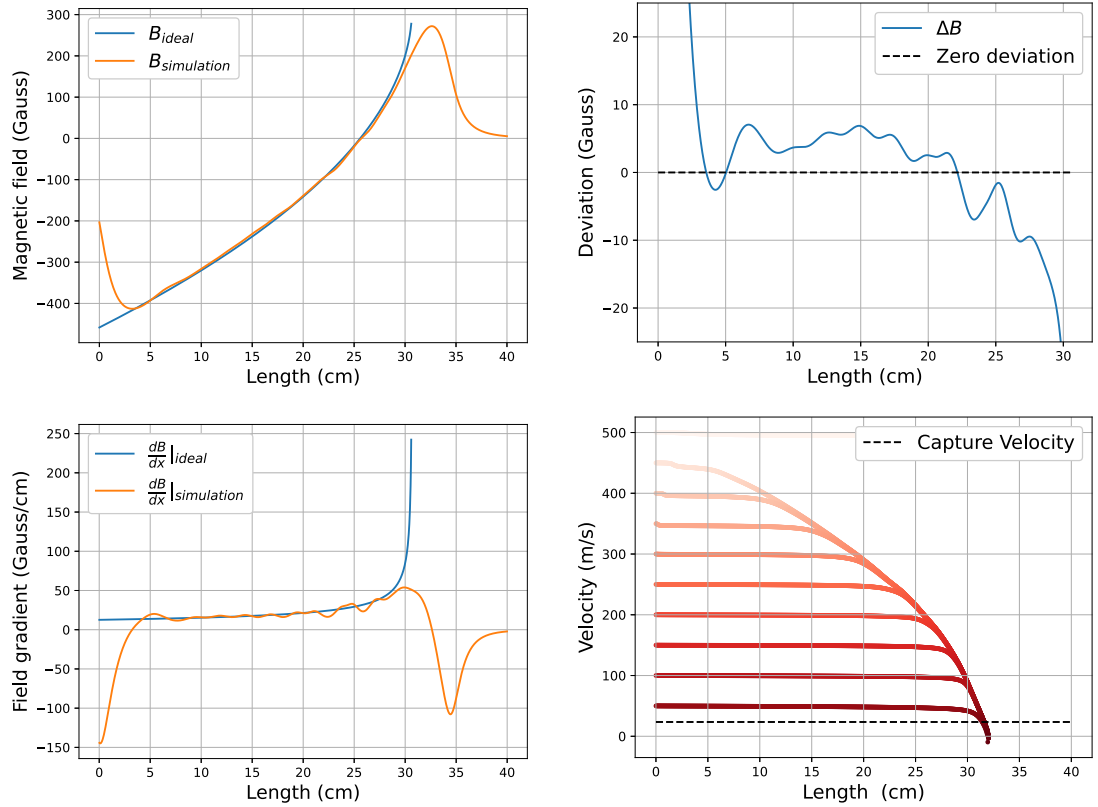


Figure 3.6: Zeeman slower: Clockwise from the top-left corner: The magnetic field profile of the custom-designed Zeeman slower, deviation from the ideal magnetic field, velocity distribution of atoms within the Zeeman slower, and the resulting field gradient. The velocity distribution reveals that atoms with different velocities up to 450 m/s achieve final velocities below the capture threshold.

The theoretical concept of the Zeeman slower has been detailed in Section 2.3.1. Although Eq.(2.14) should be modified and written as

$$\langle \mathbf{F} \rangle_{\text{res}} = \epsilon \hbar \mathbf{k} \frac{\Gamma}{2} \frac{s_0}{1 + s_0} \quad (3.7)$$

The factor ϵ is introduced to counter the non-uniform (Gaussian) distribution of laser intensity that naturally arises as the solution of Maxwell's equations. The $5s^2\ ^1S_0 \rightarrow 5s5p\ ^1P_1$ transition line allows for a strong force to be exerted on the atoms by the shining light, owing to its high transition linewidth Γ .

Based on the theoretical calculations presented in Section 2.3.1, we have computed the ideal magnetic profile required to slow down atoms with an initial velocity of approximately 500 m/s, which aligns closely with the RMS velocity of strontium atoms at 600°C. Using these magnetic field calculations, we have designed the Zeeman slower for our experimental setup, optimized specifically for ^{88}Sr atoms.

Layer	Primary coil	Secondary coil
1	195	60
2	185	55
3	165	45
4	150	38
5	130	35
6	110	33
7	90	25
8	70	20
9	40	18
10	35	16
11	20	14
12	-	12
13	-	10
14	-	8

Table 3.1: Configuration of the coils in the Zeeman slower.

For our Zeeman slower, we utilized a 35 cm long DN16 tube. The length of the slower is determined by the maximum target velocity and the force provided by Eq.(3.7). Our Zeeman slower is specifically designed as a spin-flipping Zeeman slower, consisting of two magnetic coils crafted using class H enameled copper wire. The primary coil operates at a current of 4.5 A, while the secondary coil runs at 3.3 A, resulting in a total

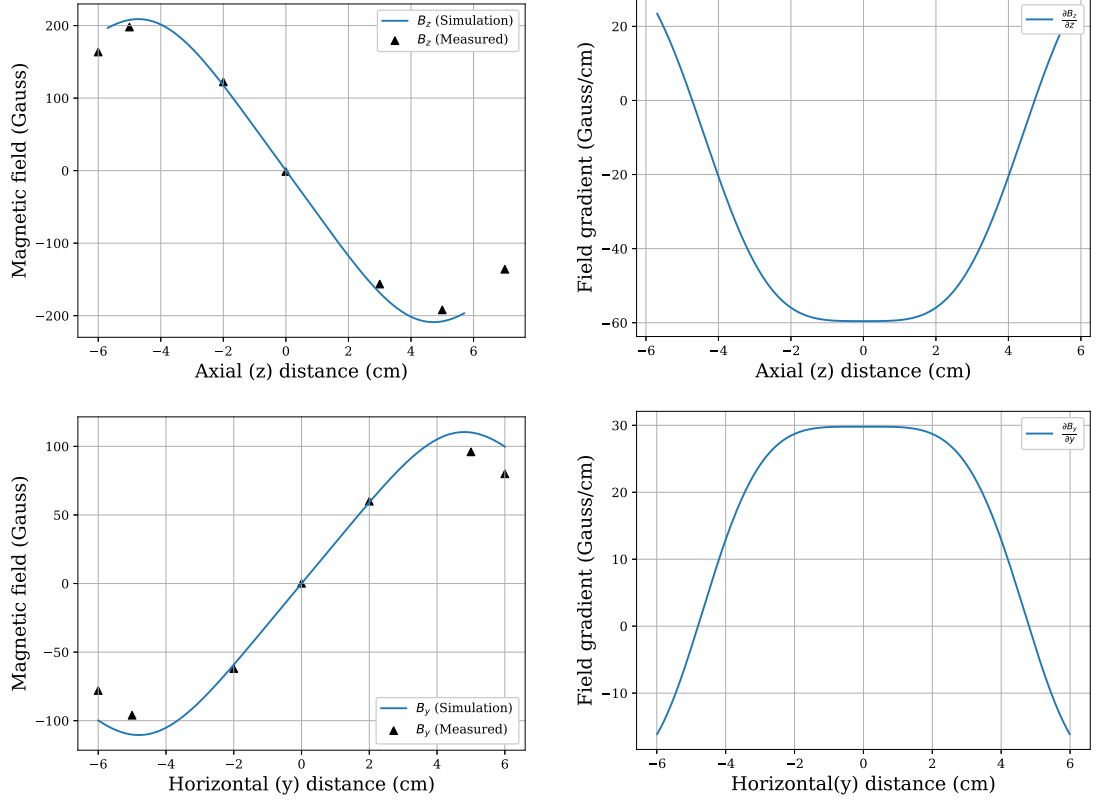


Figure 3.7: Magnetic field of MOT coils: Upper row: Axial field and axial field gradient. Lower row: y -component of the field and its gradient along the y -axis. The uniformity of the field gradients near the center indicates the well-maintained linearity of the field up to approximately 3 cm distance around the center.

power consumption of 35 W. These coils are wound in mutually opposite directions. The direction of the currents is determined based on the polarization of the light used, as indicated by Eq.(2.16), where m_f plays a crucial role. To provide the necessary force, the intensity of the laser beam is set to approximately 50 mW/cm^2 , while the detuning is maintained at 440 MHz. This detuning value is chosen to counteract the significant Doppler shift arising from the atoms' high thermal velocities. The results of our simulations are illustrated in Figure 3.6. Refer to Appendix A for more details.

3.1.4 The MOT chamber

In our experimental setup, the MOT chamber is a dodecagonal structure featuring 14 CF35 flanges, with view ports attached to 11 of these flanges. Two of the remaining

flanges are directly connected to the vacuum assembly, while the remaining one is left free and linked to a gate valve for attaching a second chamber in future experiments. The dimensions of the MOT chamber are uniform, with each side measuring 10 cm and a height of 6.5 cm. Notably, three pairs of view ports, oriented orthogonally to each other, are designated for the optical molasses configuration, serving simultaneously for both the blue and red molasses.

A critical aspect of the MOT chamber design is the integration of a magnetic coil to generate an anti-symmetric magnetic field at its center, as discussed in Section 2.3.3. This anti-symmetric magnetic field is achieved through the operation of a pair of circular coils in an anti-Helmholtz configuration. For a pair of coaxial coils, aligned along the z -axis, with identical properties and negligible spatial extent separated by a distance a and having radii R , the field near the center can be described as [98]:

$$\mathbf{B} = -\frac{B_0}{2} (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) + B_0 z\hat{\mathbf{k}} \quad (3.8)$$

Here, the expression for the field strength is given by $B_0 = \frac{3\mu_0 I}{2} \frac{aR^2}{(a^2 + R^2)^{5/2}}$, indicating a linear anti-symmetry in the magnetic field near the center of the trap. The resulting magnetic trap exhibits a stiffer confinement along the z -axis compared to the other two axes. The gradient of the magnetic field can be adjusted by varying the parameters R and a , as well as the current. However, practical coils have spatial extent, leading to slight deviations from the theoretical equation (3.8).

Coil	Innermost layer dia	Dist. from the center	Layers	Turns per layer
Coil 1	90 mm	34.4 mm	8	16
Coil 2	90 mm	-34.4 mm	8	16

Table 3.2: MOT coil specifications

With a 15 A current, the designed pair of coils generates a 60 Gauss/cm field gradient, as required for the operation of the blue MOT. At this current, the power consumption per coil is approximately ~ 195 W. Consequently, water cooling is essential to prevent thermal damage to the coils. Our magnetic coil is housed within a delrin holder, which is a moderate conductor of heat. The coil housing, equipped with inlet

and outlet connections, allows for water cooling. The water temperature is maintained at approximately $\sim 10^\circ\text{C}$. Further details of the magnetic field are provided in Figure 3.7.

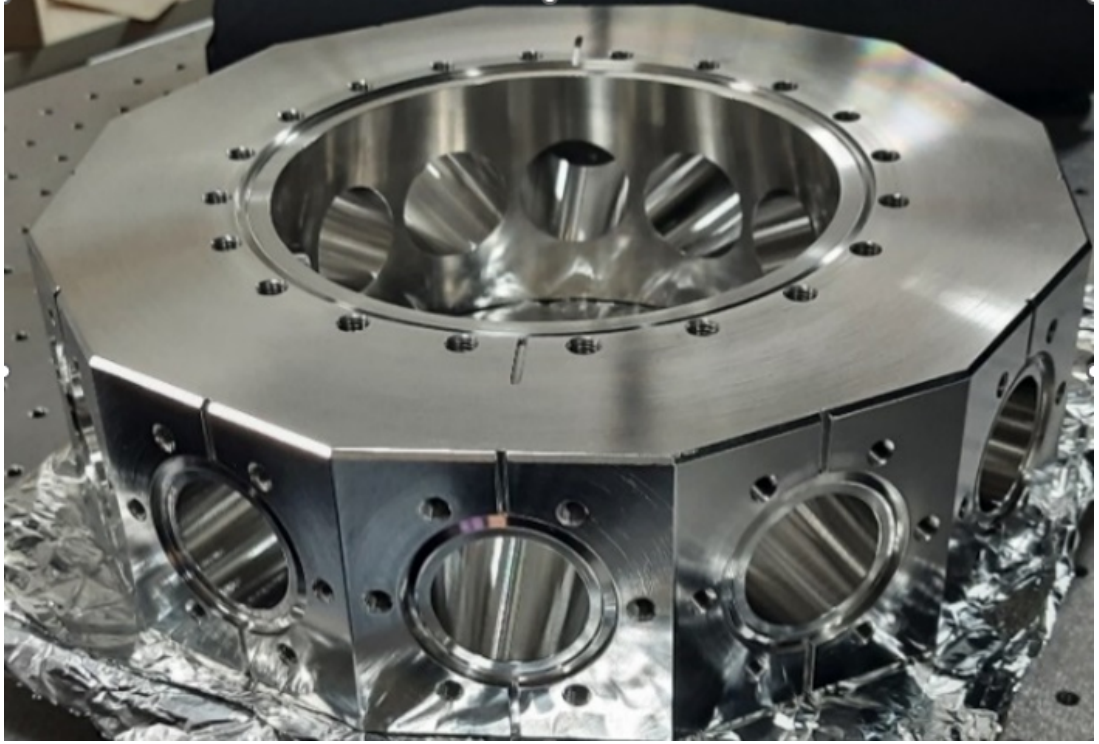


Figure 3.8: MOT chamber: Image of the dodecagonal MOT chamber.

3.1.5 The magnetic shim coils

Magnetic shim coils are essential components in cold atom experiments, offering precise control over the magnetic field environment crucial for manipulating and trapping atoms effectively. They are particularly vital in the context of strontium atom cooling due to the involvement of the narrow line cooling (7.5 kHz), making the system sensitive to even minute magnetic field variations. The earth's magnetic field ($\sim 50 \mu\text{T}$), for instance, can induce a significant Zeeman shift ($\sim 1 \text{ MHz}$), which can disrupt the cooling process. To counteract such effects, shim coils are employed.

Operating in a Helmholtz configuration, the shim coils ensure a homogeneous magnetic field near the center of the experimental setup. The distance between the coils in this configuration is critical for achieving field homogeneity and is determined by the

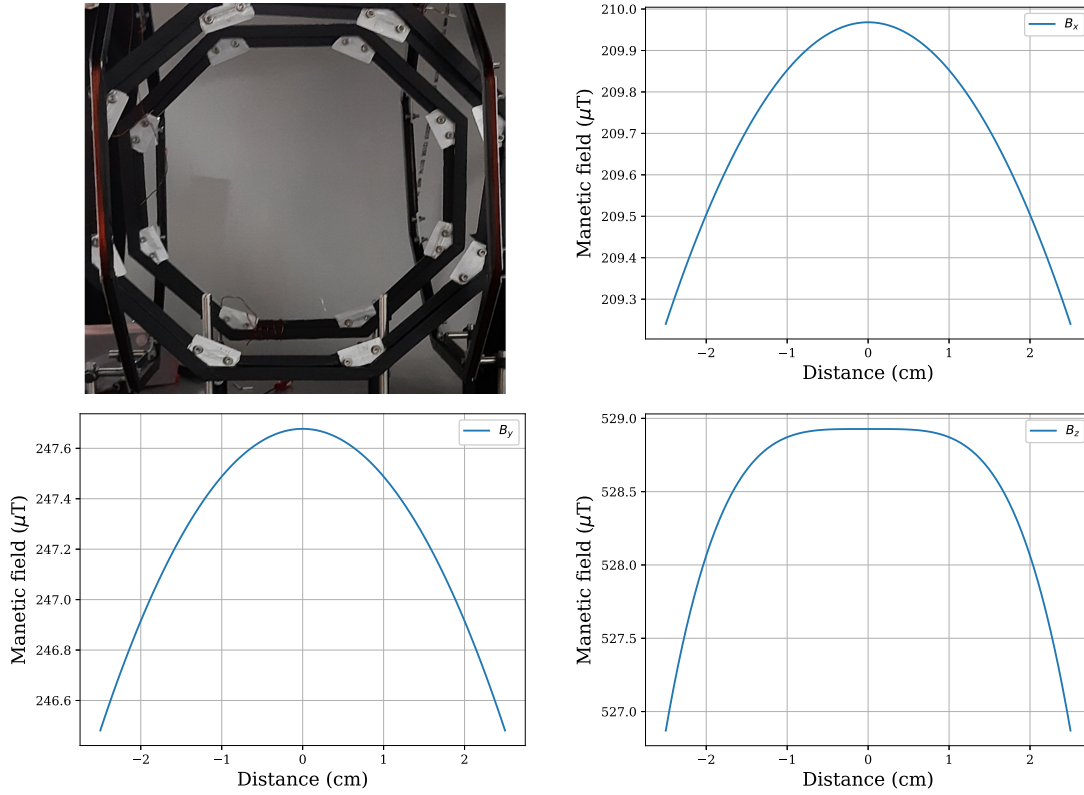


Figure 3.9: Shim coils and their magnetic fields: Within a 5 cm radius of the center, the magnetic field variation from the x and y coils at 1 A is under $0.5 \mu\text{T}$, while the z coils produce a variation of approximately $1 \mu\text{T}$.

coils' geometry and dimensions. Larger coil dimensions result in better field homogeneity.

Our setup includes three pairs of shim coils designed to nullify stray magnetic fields from three independent directions. Two pairs of coils, aligned along the x and y directions, have an octagonal shape, while the third pair, along z direction, is circular. Further details of the coil configurations are presented in Table 3.3.

Coil	Geometry	Side length	Dist. between pairs	Layers	Turns per layer
Pair 1	Octagon	236 mm	224 mm	10	6
Pair 2	Octagon	200 mm	190 mm	10	6
Pair 3	Circle	102 mm (rad)	102 mm	10	6

Table 3.3: Shim coil specifications

3.1.6 The ion pumps

Ion pumps are vital components in cold atom experiments, serving as a critical tool for establishing and preserving the ultra-high vacuum (UHV) conditions required for such precise investigations. The presence of even minute traces of residual gas molecules or contaminants within the vacuum chamber can disrupt these delicate experiments, leading to unwanted atom collisions, increased heating, or loss of atoms. Ion pumps effectively address these challenges by continuously pumping residual gas molecules from the vacuum chamber.

Operating on the principle of generating high electric fields, ion pumps ionize gas molecules within the chamber, transforming them into ions. These ions, being guided by magnetic fields, are then propelled towards electrodes along long spiral tracks, leading to a high ion yield and effectively “pumping” them out of the chamber at low gas densities. By maintaining UHV conditions, ion pumps ensure that cold atom experiments can proceed with minimal interference from external factors.

In our experimental setup, the vacuum assembly is initially baked and evacuated using a turbo molecular pump to establish UHV conditions. Once the desired pressure is achieved, the ion pumps, specifically two Agilent ion pumps with pumping capacities of 55 L/s each, are activated. One ion pump is strategically positioned adjacent to the thermal oven to uphold vacuum levels in the oven area. The second ion pump is located near the MOT chamber to maintain a UHV environment within the chamber.

Pressure readings from the ion pumps indicate that the pressure within the oven area remains around $\sim 10^{-8}$ torr, while within the MOT chamber, the pressure is maintained at approximately $\sim 10^{-11}$ torr. These ion pumps serve as indispensable elements in our setup, ensuring the stability and reliability of our cold atom experiments.

3.1.7 The titanium sublimation pump

The titanium sublimation pump is a critical component in cold atom experiments, playing a pivotal role in achieving and maintaining UHV. It operates by heating titanium metal within the pump, causing it to sublime directly from a solid to a gas. The titanium vapor then reacts with residual gases, forming non-volatile titanium compounds that are permanently trapped on the chamber walls. This continuous process effectively

“pumps” out unwanted gases, creating and maintaining ultra-high vacuum levels. The reliability and efficiency of the titanium sublimation pump make it an indispensable tool for cold atom experiments. We have installed a titanium sublimation pump in our setup, providing the capability to restore vacuum levels as needed.

3.2 The Laser system and optical components

Laser cooling and trapping of neutral atoms rely heavily on the precise control and manipulation of lasers and various other optical components. The process involves using carefully tuned laser beams to cool the atoms to extremely low temperatures, approaching absolute zero. This requires lasers with specific wavelengths and intensities to interact with the neutral atoms, allowing for the absorption and emission of photons that result in the cooling process. Optical components are also crucial for directing and focusing the laser beams onto the atoms, creating the necessary trapping potentials. The importance of adjusting and stabilizing these optical systems is paramount for achieving the desired trapping and cooling conditions, enabling the study of quantum phenomena and precision measurements in atomic physics.

3.2.1 Blue MOT Laser

As mentioned earlier, the blue Magneto-Optical Trap (MOT) operates on a 461 nm laser. In our setup, we generate this light with sufficient power using a master-slave laser locking configuration. Master-slave locking is a technique used in laser systems to synchronize the frequency and phase of two lasers: the master laser, which serves as the reference, and the slave laser, which is the laser being controlled or locked to the master laser. In this technique, the master laser provides the reference frequency and phase. A portion of the master laser’s output power is tapped and fed into the slave laser. This feedback adjusts the operating conditions of the slave laser to match the frequency and phase of the master laser. Master-slave locking is particularly essential in situations where a system requires a higher laser power than what is available from a single source with a narrow linewidth. In such cases, a second laser with higher power is precisely locked to the reference laser. This synchronization not only boosts the overall power of the system but also ensures stability and coherence between the lasers, enabling optimal

performance in applications demanding increased power output.

In our setup, the master laser is produced through a process known as second harmonic generation (SHG) from a 922 nm laser. Some power from the master laser is tapped and supplied to the slave laser to synchronize and lock the slave laser with the master laser.

The master laser

As mentioned in the last paragraph, we employ the SHG process to prime the laser for the blue magneto-optical trap (MOT). SHG is a nonlinear optical process in which two photons with the same frequency interact with a nonlinear medium, resulting in the emission of a single photon with twice the frequency (halved wavelength) of the initial photons. Mathematically, this is given by ($\omega_{\text{SHG}} = 2\omega_0$), where ω_{SHG} is the angular frequency of the generated light, and ω_0 is the angular frequency of the initial laser beam. The efficiency of SHG is highly dependent on the phase matching condition, which ensures that the two input beams are in phase when they interact with the nonlinear medium. The quality factor of mode matching is given by

$$M = \frac{\int E_{\text{SHG}}(\vec{r}) E_0^*(\vec{r}) d\vec{r}}{\sqrt{\int |E_{\text{SHG}}(\vec{r})|^2 d\vec{r} \int |E_0(\vec{r})|^2 d\vec{r}}} \quad (3.9)$$

Here, $E_{\text{SHG}}(\vec{r})$ and $E_0(\vec{r})$ represent the electric field distributions of the second harmonic and fundamental beams, respectively. The mode matching factor (M) should ideally be close to unity for efficient SHG. Achieving proper mode matching is crucial for optimizing the generation of the 461 nm laser for our blue MOT.

In our experimental setup, we employ a periodically poled Potassium Titanyl Phosphate (PPKTP) crystal as the nonlinear medium. Periodically poled crystals like PPKTP have a unique property where the domain orientation is reversed in a periodic manner along the crystal length. This periodic reversal allows us to achieve quasi phase matching conditions, which are crucial for efficient frequency doubling.

The efficiency of the PPKTP crystal is highly dependent on maintaining its temperature. Fluctuations in the crystal temperature can significantly impact the phase matching conditions, affecting the efficiency of the frequency doubling process. By carefully

controlling the temperature of the PPKTP crystal, we ensure optimal phase matching conditions for efficient frequency doubling. This enables us to generate the desired 461 nm light for the blue Magneto-Optical Trap (MOT), facilitating the trapping and cooling of strontium atoms.

We utilize a Toptica DL 100 922 nm 1W laser as the seed source within our system. This laser navigates through a bow tie cavity, a custom-made design housing the PPKTP crystal. Our setup offers the flexibility to adjust the cavity length via a piezo actuator. Employing the Pound Drever Hall technique, we lock the cavity to the seed laser, ensuring optimal intensity within the cavity. This maximizes the efficiency of Second Harmonic Generation (SHG), resulting in a produced 461 nm light with a peak power of around 250 mW. However, maintaining this power level destabilizes the crystal's temperature stability, prompting us to operate at a more stable power of 100 mW [99].

From the generated 461 nm light, several tasks are managed:

- A portion is allocated to the slave laser for master-slave locking.
- Another portion is dedicated to transverse cooling.
- A separate portion is utilized for frequency locking.
- The remaining power is directed towards dumping.

Despite the cavity being locked to the 922 nm laser, the laser-frequency drifts over time, leading to a shift in the output laser frequency. To counteract this, we continually measure the laser frequency using a HighFinesse WS7-30 wavemeter. This data is fed back to the seed laser, to precisely lock it at the desired frequency, ensuring stable and accurate operation of our system.

The slave laser

We utilize an in-house built laser housing for the slave laser, providing precise control over the laser diode's temperature and current. This custom-built housing integrates a controlling circuit from Toptica that controls current and temperature of the laser diode. To stabilize the laser diode's temperature, we employ a Peltier cooler from Thorlabs within the housing. Temperature readings are facilitated by the TH10K thermistor by Thorlabs, which provides feedback to the controlling circuit.

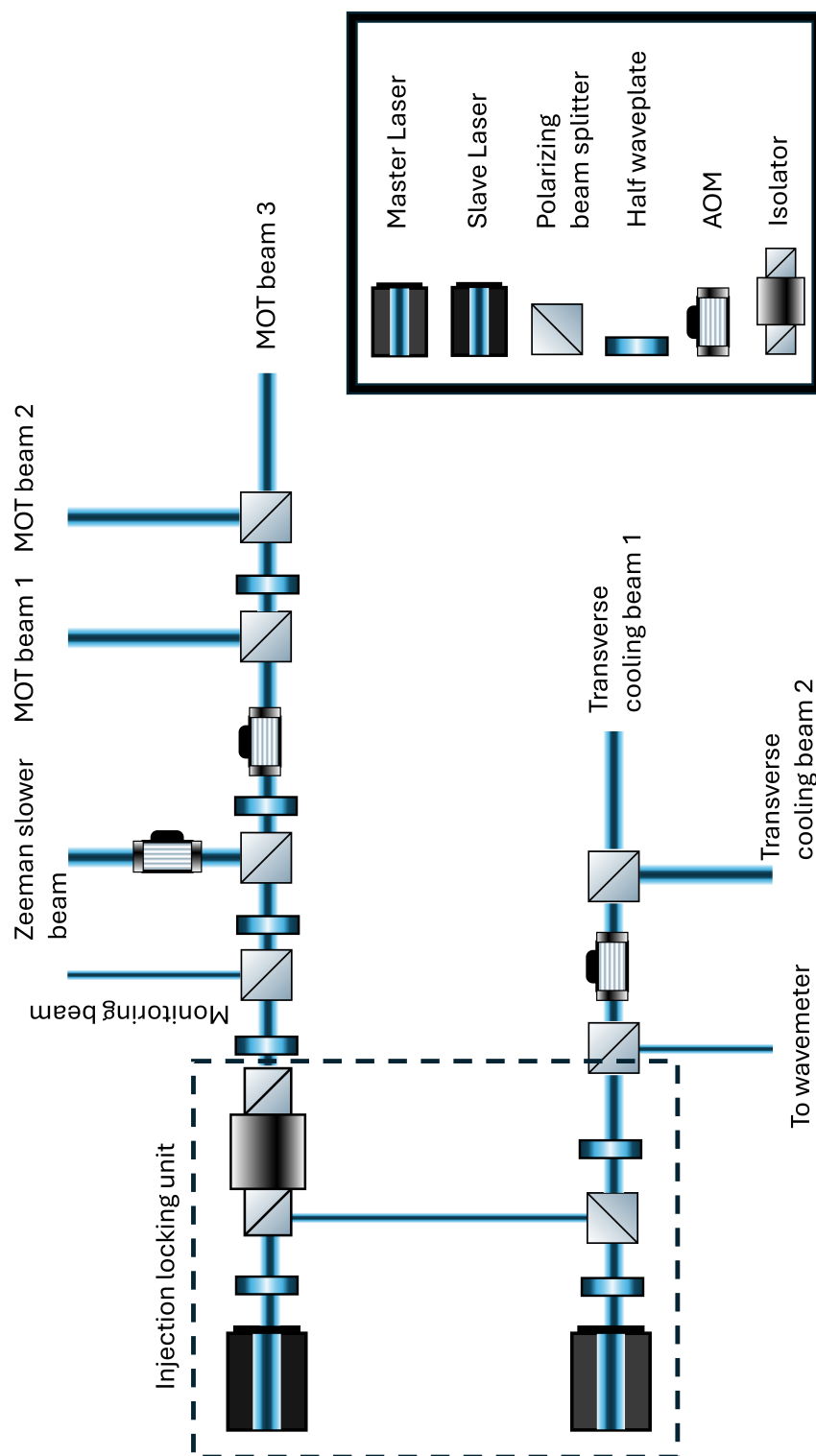


Figure 3.10: A layout of the optical system for preparing laser for the blue MOT. The SHG unit has not been shown here. For a more detailed view refer to

The light generation from the slave laser is powered by a Nichia 460 nm 500 mW diode (Part number). The laser operates at approximately 300 mW power output. This power is split into two channels: one dedicated to the Zeeman slower, and the other allocated for the blue magneto-optical trap (MOT).

To synchronize the slave laser with the master laser, 4 mW of power is tapped from the master laser. It is crucial to note that an optimal power level is necessary for this process. Insufficient power from the master laser may lead to instability in the locking mechanism. Conversely, excessive power from the master laser can potentially damage the slave laser diode. Once the light from the master laser is precisely aligned to maximize the feedback effect, the slave laser is locked to the master laser at the appropriate current and temperature settings for the laser diode. The optical layout of the preparation of 461 nm light is given in Figure 3.10.

3.2.2 Red MOT Laser

The red magneto-optical trap (MOT) operates using a 689 nm laser. This particular transition for the red MOT is a dipole forbidden transition with a linewidth of 7.5 kHz. Therefore, preparing 689 nm light for the red MOT poses a significant challenge due to the broad linewidth of commercially available lasers. Even after locking the laser through Doppler-free spectroscopy of strontium atoms at 689 nm, the laser's fluctuations remain around ~ 1 MHz, which is unsuitable for the operation of the red MOT. To overcome this limitation, a high finesse cavity is employed to filter the laser's frequency modes, producing light with high spectral purity for the red MOT.

The high-finesse cavity

The simplest optical cavity (Fabry-Perot configuration) is a fundamental optical resonator consisting of two parallel partially reflective mirrors. This cavity enhances the intensity of light by allowing multiple reflections between the mirrors, leading to constructive interference for certain frequencies. Simultaneously, the cavity improves the spectral purity of the laser modes.

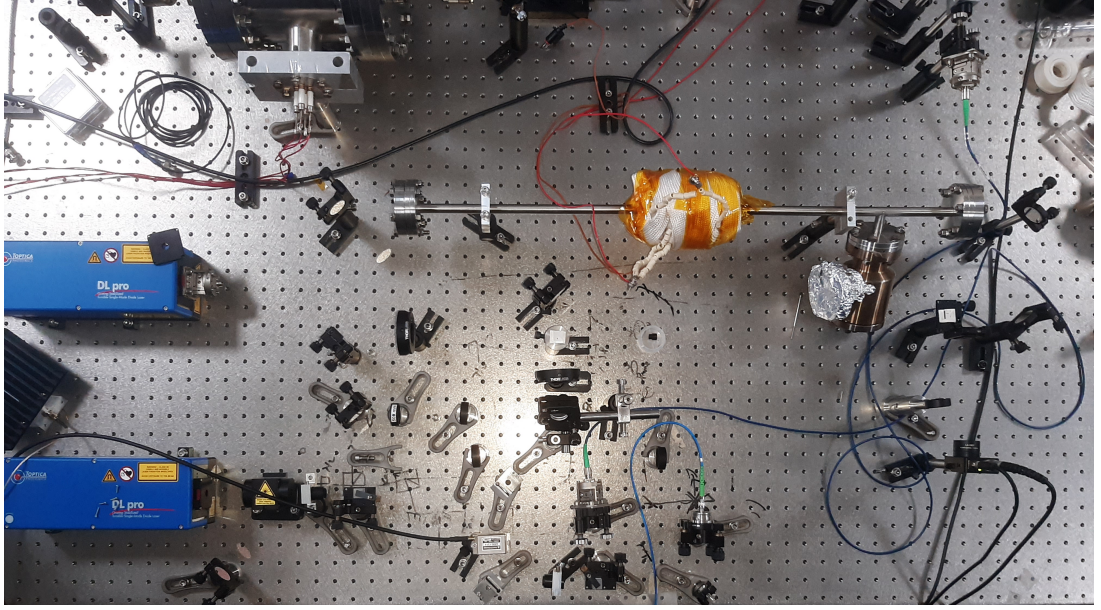


Figure 3.11: Heat pipe spectroscopy for the red MOT: Top view of the heat pipe spectroscopy setup.

The transmission of a Fabry-Perot cavity is given by [100]:

$$\mathcal{T} = \frac{1}{1 + \frac{4\mathcal{F}^2}{\pi^2} \sin^2(\phi/2)} \quad (3.10)$$

where the phase (ϕ) is given by:

$$\phi = 4\pi\mu l/\lambda \quad (3.11)$$

Here μ denotes the refractive index of the material of the cavity, l is the length of the cavity and λ is the wavelength of the light. The finesse ($\mathcal{F}$) of the cavity is defined as:

$$\mathcal{F} = \frac{\pi\sqrt{R_1 R_2}}{1 - \sqrt{R_1 R_2}} \quad (3.12)$$

where R_1 and R_2 are the reflectivities of the two mirrors.

Eq.(3.10) and Eq.(3.11) illustrate that the transmission ($\mathcal{T}$) is maximum when $l = m\lambda/2\mu$, where m is an arbitrary integer, denoting the resonance condition of the cavity. Furthermore, the Full Width at Half Maximum (FWHM) of the resonance peak, $\delta\phi$, is given by $\delta\phi = 2\pi/\mathcal{F}$. This indicates that a higher finesse of the cavity results in a higher

spectral purity of the cavity mode. In terms of frequency this translates to:

$$\delta\nu = \frac{c}{2\mu L\mathcal{F}} \quad (3.13)$$

The finesse of an optical cavity can be determined through the cavity ring down experiment, which involves measuring the reflectivity of the mirrors by tracking the intensity decay rate of the cavity. In a round-trip journey inside the cavity starting from an arbitrary point, a group of N photons will lose $\Delta N = N - NR_1R_2$ of them due to the partial transparency of the mirrors. Consequently, on average, the following equation can be written [91]:

$$\frac{dN}{dt} \approx \frac{\Delta N}{\Delta t} = \frac{c}{2\mu L}(1 - R_1R_2)N \quad (3.14)$$

This leads to the solution $N(t) = N_0 \exp(-t/\tau)$, where the characteristic time τ is given by:

$$\tau = \frac{2\mu L}{c(1 - R_1R_2)} \quad (3.15)$$

From Equation (3.15), it becomes apparent that by observing the intensity decay profile of the cavity, we can determine R_1R_2 , providing us with the finesse of the cavity.

In our experimental setup, we employ a high finesse Fabry-Perot cavity from Stable Laser Systems. High finesse cavities are particularly sensitive to external perturbations; therefore, the cavity is enclosed in a vacuum chamber to isolate it completely from the environment. Additionally, the cavity is temperature-stabilized to minimize length fluctuations due to temperature variations. The support structure of the cavity is designed to minimize deformations that could affect its performance.

Through the ring-down experiment, we have determined that the finesse of the cavity we are using is approximately 40000. The length of the cavity is 10 cm. From Equation (3.13), we have found that the linewidth of the resonance peak is approximately 37.5 kHz. Therefore, by implementing the Pound-Drever-Hall (PDH) locking technique, the laser can be locked with an uncertainty of approximately 4 kHz (a rule of thumb being 1/10th of the width of the peak). This uncertainty is lower than the transition linewidth of the 689 nm transition of strontium atoms, ensuring preparation of the suitable laser for the red Magneto-Optical Trap (MOT). The complete laser preparation process for

the cooling of strontium atoms is described in greater detail in Mr. Kushal Patel's PhD thesis.

The Doppler free spectroscopy

To ensure precise laser frequency stabilization near the linewidth of the atomic transition, the laser must first be locked to the atomic spectrum. However, when performing spectroscopy with strontium gas, the spectrum is significantly broadened due to the thermal velocities of the atoms. To overcome this challenge and obtain Doppler-free spectra, we employ saturation absorption spectroscopy.

Saturation absorption spectroscopy is a technique used to study the absorption properties of atoms or molecules with high sensitivity, based on the principle of optical saturation. In a typical setup, two counter-propagating laser beams, known as the pump and probe beams, are utilized. Initially, at low pump beam intensity, the atoms absorb photons from both the pump and probe beams. As the pump beam intensity increases, more atoms are excited to the upper energy level, leading to increased pump beam absorption.

At a saturation intensity, the pump beam absorption by the atoms reaches a maximum. This occurs because most atoms are already in the excited state, and further intensity increase does not result in additional absorption. Conversely, the absorption of the probe beam decreases as the pump beam intensity rises, due to the population redistribution between the ground and excited states induced by the pump beam. It is crucial to note that the effectiveness of saturation absorption spectroscopy is optimized when the atoms' velocities are close to zero. Otherwise, the Doppler shift, caused by the velocity component along the wavevector of the pump and probe beams, results in predominant interaction of the atoms with either the pump or the probe beam.

By monitoring the transmission of the probe beam through the atomic sample as a function of the pump beam intensity, a saturation curve is obtained. This curve typically exhibits a characteristic dip at the saturation intensity, providing valuable information about the near-Doppler-free atomic transition linewidth. This narrow dip can be effectively utilized for laser locking purposes, enabling precise laser frequency stabilization near the linewidth of the atomic transition.

Simultaneously, the laser must also be locked to the high finesse cavity frequency to

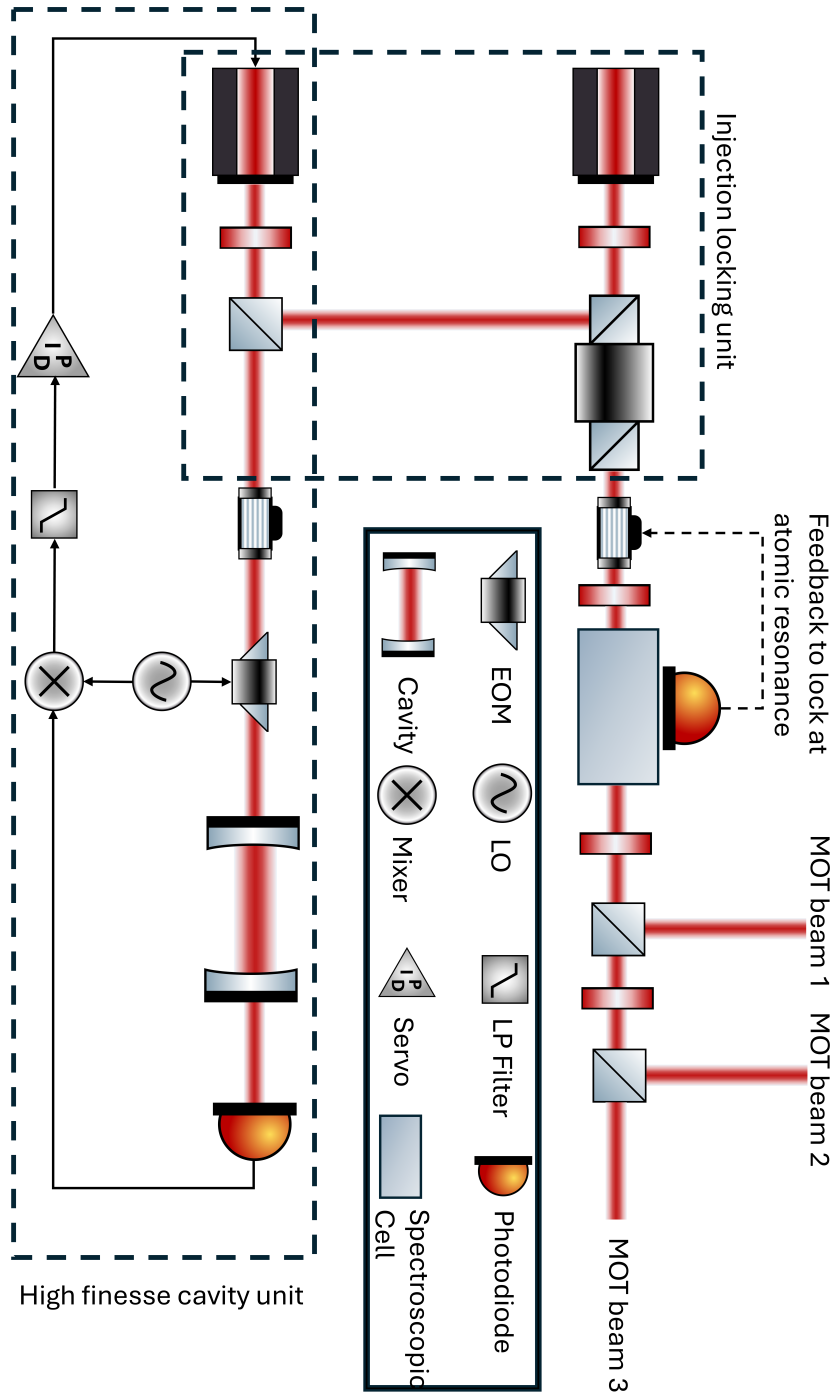


Figure 3.12: A layout of the optical system for preparing laser for the red MOT.

ensure a linewidth narrower than the transition linewidth. Equations (3.10) and (3.11) suggest that the cavity can resonate at multiple wavelengths. The separation between two resonant peaks is given by the frequency expression:

$$\Delta\nu = \frac{c}{2\mu L} \quad (3.16)$$

This quantity represents the free spectral range (FSR) of the cavity. For our specific cavity, the FSR is approximately 1.5 GHz. In the worst-case scenario, when the laser is locked to the cavity, the frequency will be approximately 750 MHz away from the atomic transition. To overcome this, we employ an acousto-optic modulator (AOM) that appropriately shifts the laser frequency, enabling simultaneous locking of the laser to both the high-finesse cavity and the atomic spectra. In our experimental setup, the minimum difference between the cavity spectra and the atomic spectra is ~ 220 MHz. This shift is accomplished by implementing a double-pass configuration of the light beam through a 110 MHz AOM.

We employ a master-slave locking configuration to prepare the laser for the red MOT. The master laser is a Toptica DL Pro 30 mW laser, while the slave laser is home-made (as described in the previous section), utilizing a Thorlabs laser diode (~ 100 mW). The detailed optical layout for preparing laser for the red MOT is provided in Figure 3.12.

3.2.3 The repumper laser

While operating the blue magneto-optical trap (MOT) for strontium atoms, a fraction of the atoms deviate from the desired cooling cycle ($5s^2\ ^1S_0 \rightarrow 5s5p\ ^1P_1$) and populate the $5s4d\ ^1D_2$ state. From this state, the atoms further decay to the $5s5p\ ^3P_2$ and $5s5p\ ^3P_1$ states with a branching ratio of 1:2. The atoms in the $5s5p\ ^3P_1$ state subsequently decay to the ground state by emitting photons at a wavelength of 689 nm, re-entering the blue MOT cycle. However, the $5s5p\ ^3P_2$ state has an exceptionally long lifetime, leading to permanent loss of these atoms from the blue MOT cycle, resulting in a decreased atom population in the MOT.

To address this issue, a 481 nm laser is employed to repump the atoms from the $5s5p\ ^3P_2$ state to the $5p^2\ ^3P_2$ state. From the $5p^2\ ^3P_2$ state, the atoms decay back to

the $5s5p\ ^3P_1$ and $5s5p\ ^3P_2$ states. The probability of decay from the $5p^2\ ^3P_2$ state to other levels is relatively low. The atoms in the $5p^2\ ^3P_1$ state return to the ground state as mentioned earlier, while those in the $5p^2\ ^3P_2$ state are repumped back to the same state, effectively increasing the number of atoms in the MOT.

For the repumping of strontium atoms, we utilize the Toptica DL Pro 60 mW 481 nm laser. Applying approximately 1 mW of power results in an increase in the atom population in the MOT by a factor of approximately 15.

In this chapter, we provided a concise overview of our experimental setup. We have detailed the essential components of the setup, including the vacuum assembly, laser system, and optical layout designed for the preparation of lasers. The setup has been used to cool and trap strontium atoms, enabling the preparation of cold atoms for a variety of physics experiments. The subsequent chapters of this thesis will provide a more detailed description of the cooling and trapping processes involved with strontium atoms, providing a comprehensive understanding of our experimental procedures and results.

Chapter 4

The blue MOT: Optimization and characterization

In this chapter, the focus will be on the blue MOT cooling technique applied to strontium atoms, with particular attention given to strategies for increasing the number of atoms within the blue MOT. This is a crucial aspect as the number of atoms significantly impacts the reduction of various noises present in experiments involving laser-cooled atoms. The significance of the number of atoms in such experiments has been previously discussed in the introductory chapter. Within this chapter, methods for optimizing parameters to capture the maximum number of atoms in the blue MOT will be explored, along with the characterization of the resulting optimized blue MOT.

The number of atoms (N) loaded into MOT can be maximized by tuning various parameters (detuning of lasers from the atomic transitions, currents in magnetic coils, etc.) of an experiment. For different isotopes of a particular atomic sample, the parameter values corresponding to the maximum number of atoms in a MOT (N_{max}) are different in general. In principle, one should be able to compute the parametric dependence of N . But obtaining the set of optimum values through manual optimization by monitoring the atomic fluorescence/absorption can be complicated and time-consuming as the number of parameters increases. It may lead to finding only one of the local maxima points. To enhance the time-efficiency of the optimization process, machine learning algorithms can be adopted instead of manual optimization, especially in versatile setups

where multiple experiments with different isotopes are performed.

In our experiment, we have maximized the number of ^{88}Sr , ^{86}Sr , and ^{87}Sr atoms in a ‘blue MOT’ that operates on the $(5s^2)^1S_0 \rightarrow (5s5p)^1P_1$ optical transition. We implemented the Gaussian method (GM) [101] and the neural network (NN) method [102] for the maximization. The results showed that the optimized parameters for each isotope belong to different exclusive regimes in the parametric space and are not suitable for the others. The optimization algorithms were also found to be more time-efficient than manual optimization and it allowed us to perform fast switching between MOTs of different isotopes.

4.1 Optimization of the blue MOT

4.1.1 The setup: A review

Before going into the details, let us summarize the key components and aspects of our system for clarity and convenience. In our setup, a beam of strontium atoms, prepared from an effusive oven [103], passes through ‘transverse cooling’ [104] region followed by a Zeeman slower [105] and subsequently enters the MOT region. The transverse cooling collimates the diverging atomic beam, and the Zeeman slower slows the atoms down to velocity within the “capture velocity” of the MOT using a spatially modulated magnetic field [106]. The MOT reduces the velocity further in all the three directions and confines the atoms spatially using an anti-Helmholtz magnetic field. Therefore, the loading of atoms into the MOT depends not only on the parameters of the MOT, but also on the parameters of all the preceding processes. The parameters involved in the loading of the MOT are as follows:

1. Detuning of transverse cooling beam (Δ_{TC})
2. Detuning of Zeeman slower beam (Δ_{ZS})
3. Detuning of MOT beams (Δ_{MOT})
4. Magnetic field of the Zeeman slower (I_{Z1} and I_{Z2})

5. Magnetic field of the MOT coil (I_{MOT}).

The detuning of the laser beams are tuned by acousto-optic modulators (AOM) and the magnetic fields are controlled by varying currents in the slower coils and (I_{Z1} and I_{Z2}) and in the MOT coil (I_{MOT}). The number of atoms (N) in the MOT depends on all of these parameters. Due to certain hardware limitations in the experimental setup, the detuning of the MOT beams was kept at a fixed value (red detuned by one line-width of the $(5s^2)^1S_0 \rightarrow (5s5p)^1P_1$ transition, $\Gamma = 2\pi \times 32$ MHz) [107]. Hence, we kept the MOT beam detuning fixed throughout the experiment and tune the other parameters to achieve $N = N_{max}$.

4.1.2 Optimization methods: A theoretical review

The problem can be stated as follows.

A surjective map $f : \mathbb{R}^5 \rightarrow \mathbb{R}$ is given by $f(\Delta_{TC}, \Delta_{ZS}, I_{Z1}, I_{Z2}, I_{MOT}) = N$, where N is the number of atoms in the MOT and can be obtained empirically. Using some physical constraints, the domain of the mapping can be restricted and can be taken as a closed subset ($\mathcal{S}$) of $\mathbb{R}^5$. Our goal is to find out at least one element $(\Delta_{TC}^{opt}, \Delta_{ZS}^{opt}, I_{Z1}^{opt}, I_{Z2}^{opt}, I_{MOT}^{opt}) \in \mathcal{S}$ for which $N = N_{max}$.

To incorporate optimization methods, we utilized the *Machine-Learning Online Optimization Package* (MLOOP) [108], which allowed us to implement the Bayesian technique to identify global minima of the *cost function*, and consequently, locate the global maxima of the number of atoms in the parametric space. As mentioned earlier, we implemented GM and NN methods in our experiment for optimization.

Gaussian Method: Let us denote the observations as follows: $\mathcal{O} = \{x_i, \mathcal{C}(x_i) : i \in \mathbb{N}\}$ where i denotes the i -th observation and $x_i = (\Delta_{TC}^i, \Delta_{ZS}^i, I_{Z1}^i, I_{Z2}^i, I_{MOT}^i)$ and $\mathcal{C}$ is called the ‘cost function’ [109]. According to the *Bayes theorem*- $P(\mathcal{C}|\mathcal{O}) \propto P(\mathcal{O}|\mathcal{C})P(\mathcal{C})$ where $P(X|Y)$ denotes the *conditional probability* of event X and event Y . In GM, the mapping ($\mathcal{C}$) is modeled as $\mathcal{C} \sim \mathcal{GP}(\mu(x), \sigma(x, x'))$. MLOOP assumes $\sigma(x, x') = \exp(-0.5||x - x'||^2)$ and evaluate $\mu(x)$ based on $\mathcal{O}$ to calculate $P(\mathcal{C}|\mathcal{O})$. Depending on $P(\mathcal{C}|\mathcal{O})$, MLOOP tries to predict the parameters for which $\mathcal{C}$ is minimized.

Neural Network: Neural network (NN) method is realized by building a network of

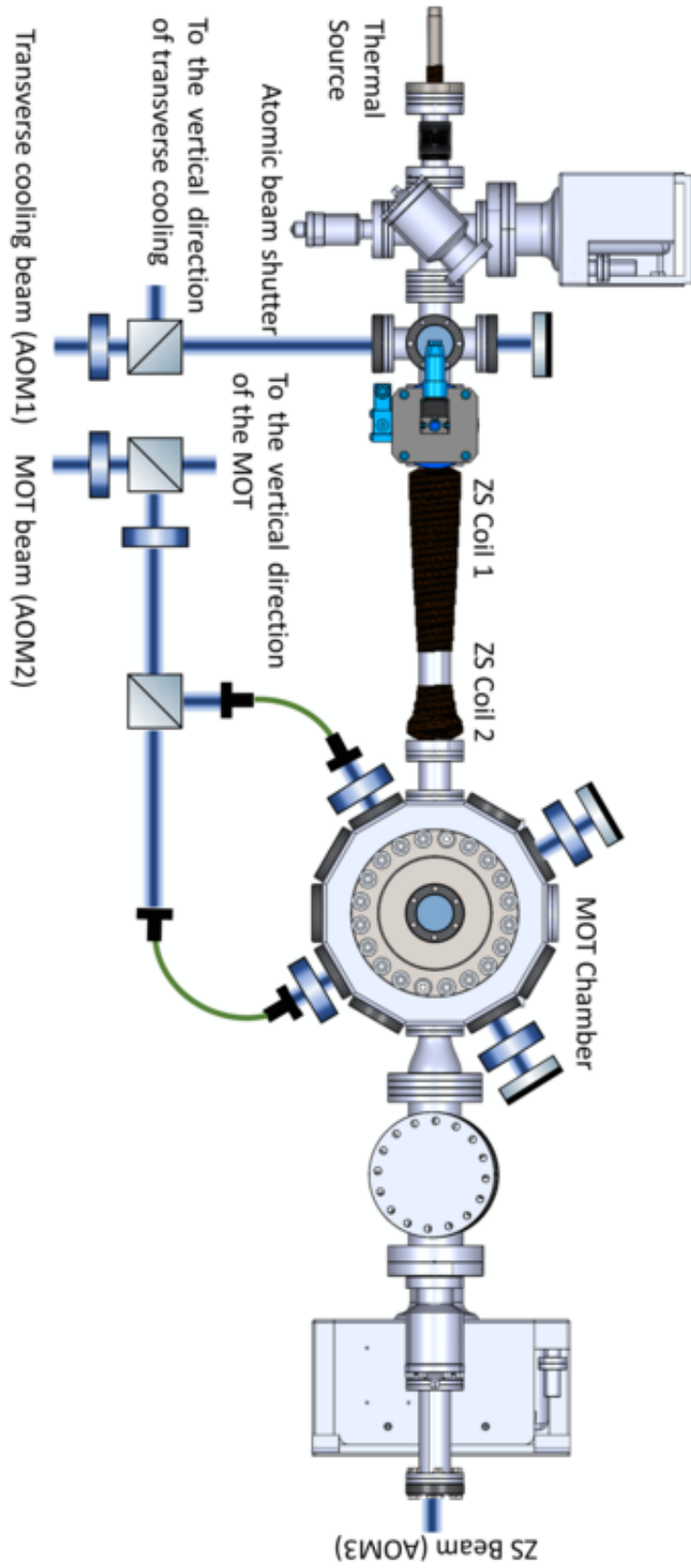


Figure 4.1: A simplified schematic (topview) of the experimental set up. The transverse cooling beam and the Zeeman slower beam passes through AOM 1 and AOM 3 respectively which are connected to an AOM controller. The Zeeman slower coils and the MOT coils (not shown in the figure) are attached to programmable power supplies. The current can be tuned by applying voltage in the programming ports of the power supplies. An FPGA based analog waveform generator, built in-house and accessible via serial communication, is used to program the power supplies.

interconnected nodes. NN has been used to establish a ‘black-box’ mapping between the physical parameters and the number of atoms in the MOT. The structure and the weights of network connections decide the nature of mapping. Refer to Appendix B.

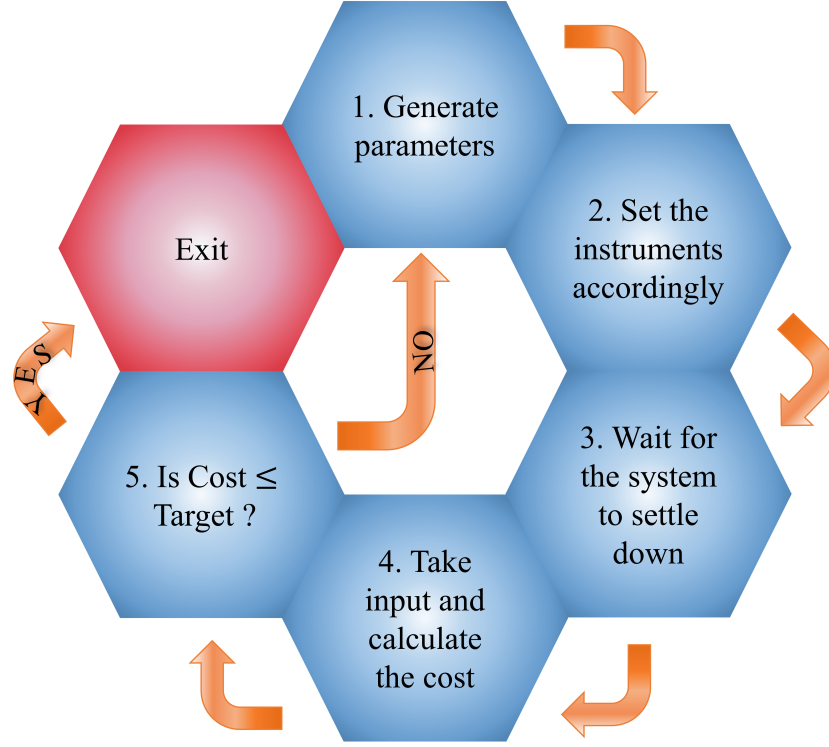


Figure 4.2: A simplified flowchart of the optimization program.

4.1.3 Experimental methods

To optimize a process, the program must be fed the costs of various parameters. The cost function is then modeled based on cost values, with lower cost values indicating better optimization. In our experiment, we defined the cost of optimization $\mathcal{C}$ as a function of the intensity of atomic fluorescence (I) in the MOT. To be specific, the cost function was given by $\mathcal{C} = I_0 - I$, where I_0 was a predefined constant. The optimization algorithms iteratively generated parameters to reduce the cost and as a result, the MOT intensity increased. MOT intensity I is directly proportional to the number of atoms in MOT ($I \propto N$) for a fixed detuning and intensity. Therefore, the less the cost, the more the number of atoms. The choice of I_0 can be arbitrary. One can start from a ‘meaningful’

arbitrary value of I_0 and increase it gradually after each optimization run until the target cost ($\mathcal{C}_T$) is just lower than the lowest cost ($\mathcal{C}_{min}$) returned by the optimization algorithm. The conditions, $\mathcal{C}_{min} \gtrsim \mathcal{C}_T$ and $\mathcal{C}_T = 0$, imply that I_0 is the upper limit of the MOT intensity that corresponds to N_{max} .

Params	Units	Exp. min.	Exp. max.
Δ_{TC}	MHz	-60	0
Δ_{ZS}	MHz	-380	-400
I_{Z1}	A	2.5	5.0
I_{Z2}	A	-5.0	-2.5
I_{MOT}	A	10	18

Table 4.1: Expected ranges of parametric values. The program is allowed to pick values from these bounded regions.

FIG. 4.1 shows the simplified schematic of our experimental setup. To perform the experiment, we locked the laser with a wavemeter and a digital servo controller at the frequency of $(5s^2)^1S_0 \rightarrow (5s5p)^1P_1$ transition of a particular isotope of strontium. All the physical parameters in our experiment were computer controlled. A multi-channel AOM controller, accessible via serial communication, precisely controlled the detunings of the transverse cooling and the Zeeman slower beam. An FPGA based home-built slow analog waveform generator communicated with the programmable power supplies that controlled the current in Zeeman slower and MOT coils. The waveform generator was also accessed via serial communication. We used a computer controlled analog voltage reader to read the output of a photo multiplier tube (PMT) that collected the fluorescence of the atoms in the MOT. The optimization program (algorithm given in FIG. 4.2) initially generated a set of parameters within the predefined ranges. Range of each parameter was decided by a theoretically calculated value. For example, we evaluated the value of Δ_{TC} by estimating the transverse velocity component of the atomic beam. Additionally, we simulated the values of Δ_{ZS} , I_{Z1} , and I_{Z2} from the geometric configuration of the Zeeman slower coils and the most probable velocity of the longitudinal velocity distribution of atoms. Sometimes technical limitations determined the range of a parameter. For example, the Joule heating restricted the upper limits of currents in

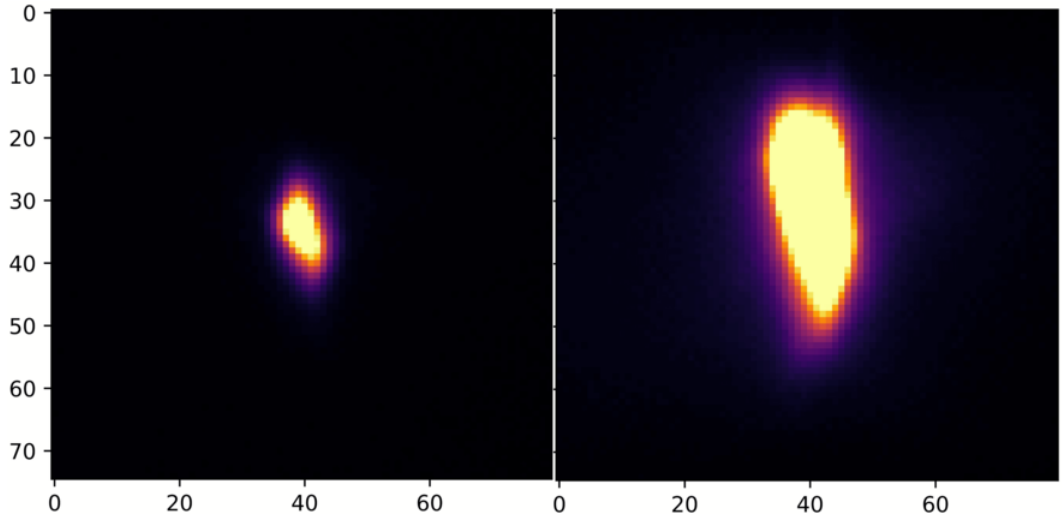


Figure 4.3: Snapshots of MOT fluorescence. The picture on the left-hand side shows the MOT fluorescence for some random initial parameters whereas the picture on the right hand side shows the same for the optimized parameters. The number of atoms in MOT after maximization is almost one order higher than the number of atoms due to some initial arbitrary set of parameters.

the coils. Thus, a close subset $\mathcal{S}$ was chosen from $\mathbf{R}^5$ that acted as the domain of the optimization parameters. The domain of parameters we set for our experiment is given in TABLE 4.1.

After generating the parameters, the program set the AOM controller and the power supplies accordingly. Then it allowed the system to equilibrate with the new set of parameters. Once the MOT intensity saturated to a particular value, the program collected the output of the PMT and took the average over 200 k samples (2 seconds of data acquisition at a sampling rate = 100 ksps) to calculate $\mathcal{C}$. Based on the computed value of $\mathcal{C}$, a new set of parameters was returned by the program and the process continued until the system was optimized. Once the optimization was completed for a particular isotope, we locked the laser near the transition of another isotope and followed the similar procedure to obtain the optimized parameters for the same.

4.1.4 Results and discussions

In our experiment, both GM and NN successfully maximized the number of atoms in MOT (FIG. 4.3) and eventually returned the mutually exclusive sets of optimized pa-

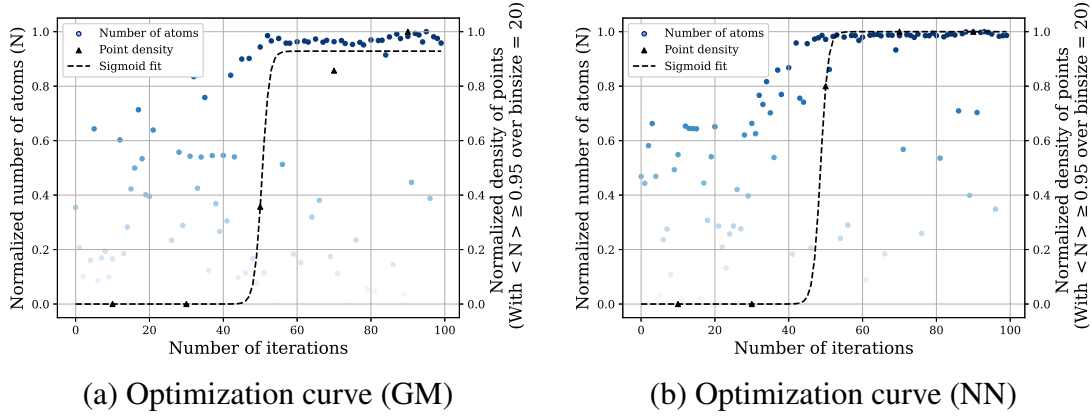


Figure 4.4: [a] GM optimization curve for ^{88}Sr . [b] NN optimization curve for ^{88}Sr . Circular dots represent the normalized number of atoms (N) where as the black triangles are the density of the circular dots with $<N> \geq 0.95 \times N_{max}$ calculated over 20 iterations.

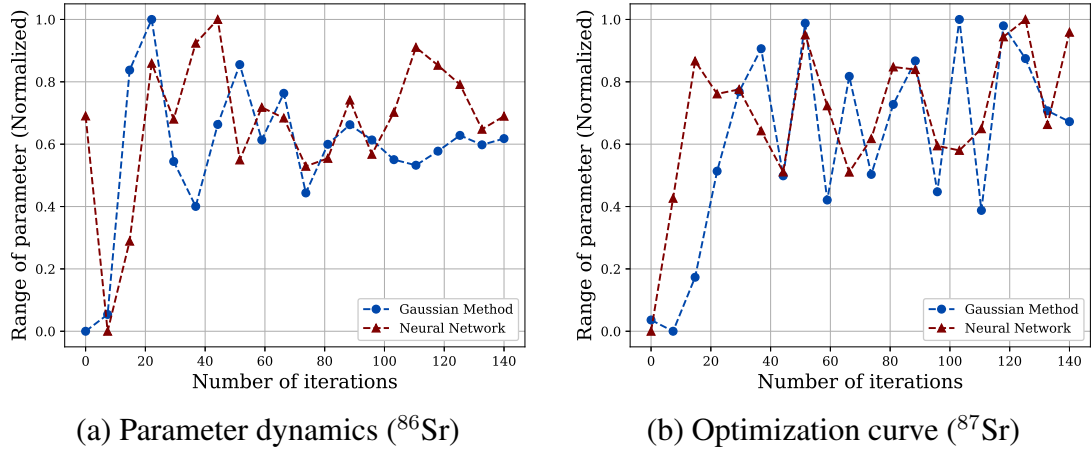


Figure 4.5: Saturation curves of one of the physical parameters for (a) ^{86}Sr and (b) ^{87}Sr . The points plotted here are the averaged value of the parameter over 7 iterations. $f(t) = a - b \exp(-ct)$ (not show here) can be fitted to the data points to estimate the time of optimization alternatively.

rameters for different isotopes. The implementation of the two types of algorithms was to do a comparative study of their ‘optimization speed’. In our system, we found that the numbers of iteration for optimization were almost similar for different isotopes in both of the methods. The completion of optimization can be defined by the time region where the number of atoms in the MOT due to the parameters returned by the program lie mostly in vicinity of the maximum number of atoms achieved by the program. One way to quantify this is to calculate the density of points in the vicinity of the maximum

number of atoms in the time vs atom number plot (FIG.4.4).

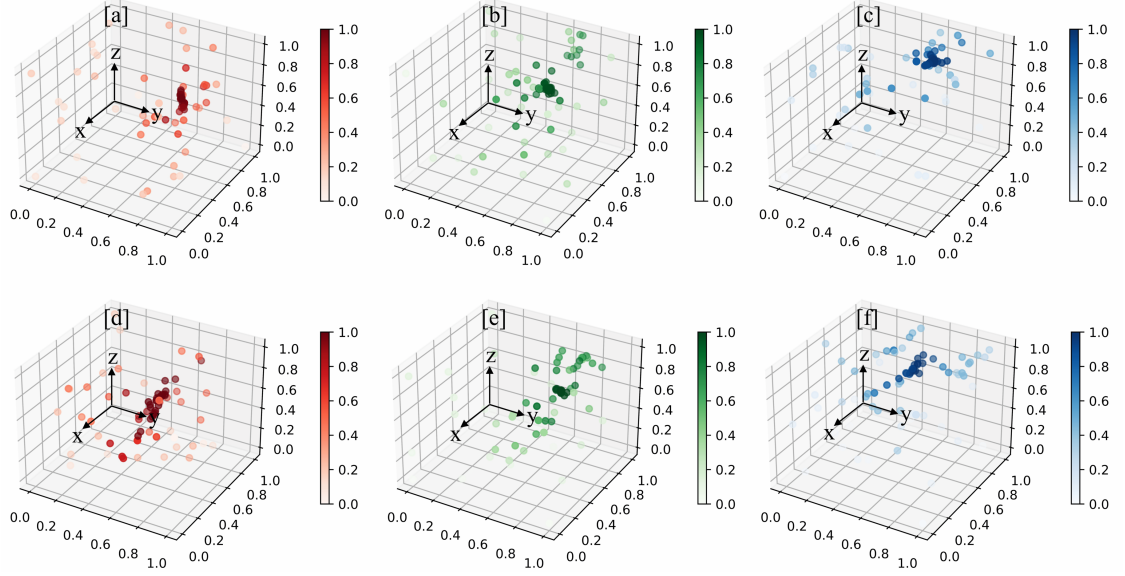


Figure 4.6: Number of atoms in a 3D parametric space. The colorbar represents the normalized number of atoms and the three axes represent three different parameters. The plots in the first row ([a],[b],[c]) are obtained from GM and the plots in the second row ([d],[e],[f]) are obtained from NN . The plots in the first, second and the third column are for ^{88}Sr , ^{86}Sr and ^{87}Sr respectively.

A sigmoid function can be fit to estimate the time of optimization. We can take the middle point of the sigmoidal transition of the point density as the characteristic optimization time. The values of the parameters also saturate gradually to the optimized values with time. One can also quantify the optimization time from the saturation curves of the parameters (FIG.4.5). It is worth mentioning that the number of ^{88}Sr atoms in the MOT was approximately 3×10^7 under optimized parameters without using any repumper, while the number of ^{86}Sr and ^{87}Sr was found to be approximately 3×10^6 after the optimization. After repumping ($(5s5p)^3P_2 \rightarrow (5p^2)^3P_2$, using 481.3 nm light) the number of atoms increases by a factor of 10.

We trained both GM and NN for 10 iterations and allowed them to optimize the system. From FIG. 4.4, it is evident that both of the algorithms reached the desired goal within ~ 50 runs. We can get a qualitative idea about how fast these optimizations were with respect to the manual optimization. Let us assume that the effect of a parameter on the number of atoms is almost invariable if we vary the parameter within a range of $\pm 10\%$. If we divide each of the parameter ranges into 10 distinct regions, a multi-

dimensional space spanned by 5 parameters will have 10^5 distinct regions. To find out one region corresponding to N_{max} randomly among 10^5 choices is undoubtedly a time consuming job. However, no experimentalists go through 10^5 possibilities. Usually the experimentalists change the parameters on the basis of their own experience which might lead to a local maxima. On the contrary, the optimization algorithms took only about ~ 50 iterations to find out $(\Delta_{TC}^{opt}, \Delta_{ZS}^{opt}, I_{Z1}^{opt}, I_{Z2}^{opt}, I_{MOT}^{opt})$ for which $N = N_{max}$.

Params	Units	^{88}Sr	^{87}Sr	^{86}Sr
Δ_{TC}	MHz	-20	-16	-20
Δ_{ZS}	MHz	-392	-391	-389
I_{Z1}	A	4.7	3.3	5.0
I_{Z2}	A	-4.0	-3.8	-4.2
I_{MOT}	A	13.3	18.0	11.4

Table 4.2: Optimized parametric values for different isotopes.

In FIG. 4.6, the normalized number of atoms has been plotted against three of the tunable parameters. It is evident from the figure that the best parametric regime for a particular isotope returned by the two different algorithms are almost same. But the best set of parameters for different isotopes are mutually exclusive. This is more comprehensible when the normalized atom number is plotted against one parameter (FIG. 4.7). In the two subplots of FIG. 4.7, normalized atom number has been plotted against two individual parameters. The peak-shifts indicate that the maximization of number of atoms for different isotopes occurs at different values of different physical parameters. For some of the parameters, the peak shift is large whereas for some of the parameters the peaks stand close to one another. One should also notice the existence of multiple local maxima in the plot. It is quite possible to get trapped in one of these local maxima while tuning the parameters manually. The occurrence of the mutually exclusive sets for different isotopes can be explained by the unique nuclear potential of each isotope, which give rise to distinct energy levels. Consequently, one can expect that each isotope will respond differently to various sets of experimental parameters. The optimized parameters for different isotopes obtained from our experimental setup have been provided in absolute units in Table 4.2.

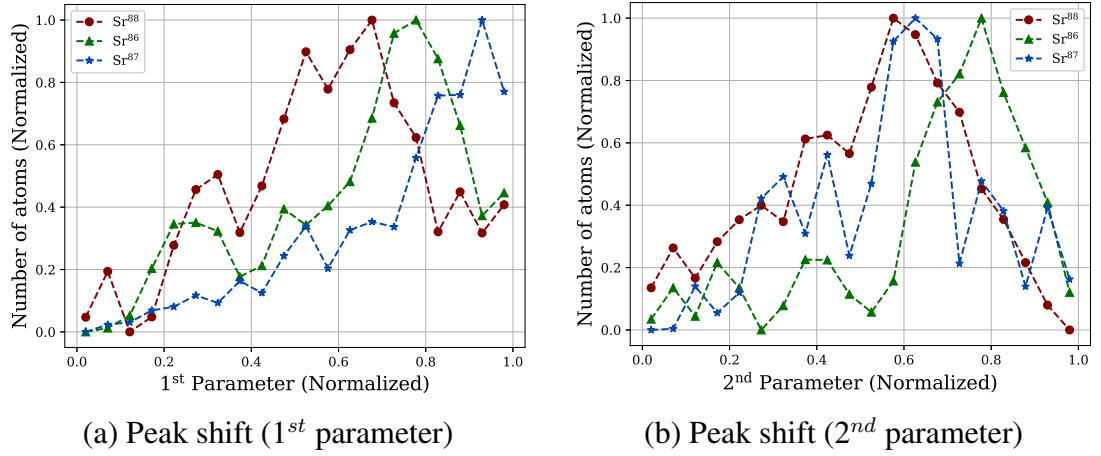


Figure 4.7: The peak-shift of different isotopes for different physical parameters. For some parameters, the shift is quite large (shown in(a)). For some parameters, the shift is not quite significant (shown in(b)).

To summarize, we have demonstrated the maximization of the number of atoms in MOT using optimization algorithms and highlighted the importance of doing so. Maximizing the number of atoms in the MOT through manual adjustments of multiple parameters is a time-consuming and error-prone process that can result in being caught in a local maxima of the parametric space. Additionally, the optimized parameters for one isotope may not be the best for other isotopes. In a versatile setup, performing manual optimization for different isotopes repeatedly for different experiments wastes significant time and does not always return the most suitable parameters. In contrast, machine learning optimization algorithms plays a crucial role in solving these problems and automating the loading of different isotopes into MOT.

4.2 Characterization of the blue MOT

4.2.1 The MOT dynamics

After optimizing the MOT, the next step involves characterizing the blue MOT to determine the number and temperature of trapped atoms. The number of atoms in the MOT can be determined from the fluorescence of the trapped atoms, using the relation:

$$N = \frac{4\pi}{d\Omega} \frac{P}{\hbar\omega\gamma_{sc}} \quad (4.1)$$

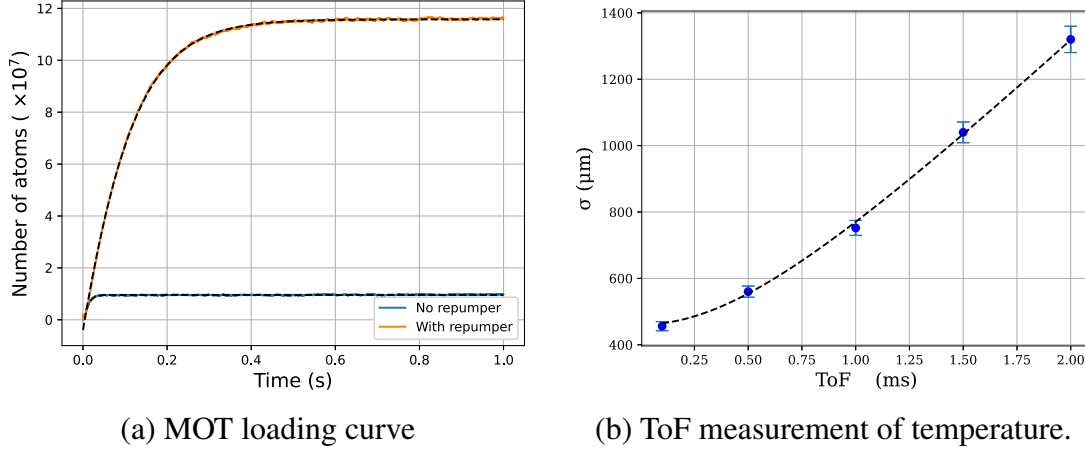


Figure 4.8: (a) shows the loading of atoms in the blue MOT with and without repumper laser. (b) shows the expansion of MOT cloud with ToF. The temperature estimated from this data is ~ 4 mK.

Here, $d\Omega$ is the solid angle subtended by the fluorescence collection aperture at the MOT center, $\langle F \rangle$ is the force on atoms in the MOT, P is the power of the MOT fluorescence ω is the angular frequency of the light, and γ_{sc} is the scattering rate which has been mentioned in section 2.2.2. This equation allows us to calculate the number of atoms in the MOT.

Experimental investigations were conducted to analyze the loading of atoms into the blue MOT, providing insights into the dynamics of MOT loading. The loading dynamics can be described by the equation:

$$\frac{dN}{dt} = L - aN - bN^2 \quad (4.2)$$

Here, L represents the rate of MOT loading, a denotes the first-order loss of atoms due to undesired collisions with particles in the imperfect vacuum environment, and b signifies the two-body loss of atoms due to atom-atom scattering. The loading curve of the blue MOT is illustrated in Figure 4.8. By fitting the experimental data (with the repumper laser) to the solution of Eq. (4.2), we determined the values of L , a , and b to be $L = 8.3 \times 10^8 \text{ s}^{-1}$, $a = 2.9 \text{ s}^{-1}$, and $b = 3.7 \times 10^{-8} \text{ s}^{-1}$.

4.2.2 The MOT temperature

The another crucial parameter is the temperature of the MOT. Temperature of the MOT is usually measured using the time of flight (TOF) method. TOF measurement is a common technique used to determine the temperature of cold atoms in a magneto-optical trap MOT. In this method, the atoms are suddenly released from the trapping potential of the MOT, allowing them to expand freely. As the atoms move outward, their velocity is inversely proportional to their mass, meaning lighter atoms move faster than heavier ones.

By measuring the spatial distribution of the expanding cloud of atoms at different times after release, the velocity distribution of the atoms can be obtained. From this velocity distribution, the temperature of the atom cloud can be calculated using the ideal gas law or Maxwell-Boltzmann distribution. The time it takes for the cloud to expand to a certain size provides information about the velocity distribution and hence the temperature of the atoms.

In the context of the MOT, the spatial distribution function of atoms (at $x, y, z = 0, 0, 0$) with zero average velocity can be described by:

$$f(\mathbf{r}, t) \propto \prod_{i=x,y,z} \exp\left(\frac{-r_i^2}{\sigma_i(t)}\right) \quad (4.3)$$

Here, $\sigma_i(t)$ represents the width of the distribution and is defined as:

$$\sigma_i^2(t) = \sigma_i^2(t_0) + \frac{k_B T}{m} (t - t_0)^2 \quad (4.4)$$

By monitoring the spatial distribution of atoms over time during their flight and fitting the data according to Eq.(4.4), we can determine the temperature of the atoms. This analysis is illustrated in FIG().

In this chapter, we have explored the process of blue MOT cooling applied to strontium atoms. The primary focus has been on optimizing the number of atoms within the blue MOT through the implementation of ML techniques, as well as the thorough characterization of the resulting blue MOT configuration. This stage prepares the atoms for the subsequent phase of cooling, known as the red MOT. The upcoming chapter will

delve into the intricacies of red MOT cooling in greater detail.

Chapter 5

An intermediate three level cooling

At this juncture in the thesis, it's evident that the conventional method of cooling alkaline earth metals involves a two-tier Doppler cooling approach to make the atoms adequately cold for various quantum experiments. For instance, for strontium atoms, the lowest achievable temperature (Doppler limited temperature) in the first stage is $760 \mu\text{K}$ (refer to Eq. 2.25). This temperature, though relatively low, is unsuitable for experiments involving quantum phenomena. Consequently, a second stage of cooling is necessary, with its Doppler-limited temperature (proportional to the linewidth of the optical transition) reaching lower than the single photon recoil temperature (460 nK) [110]. Therefore, the two stages of cooling of alkaline earth atoms are respectively known as the initial MOT cooling phase and the narrow-line MOT cooling phase [8, 111, 112]. Nevertheless, due to the considerable difference between the line-widths of these MOT transitions, the inter-MOT atom transfer process becomes susceptible to a significant loss of atoms. This is attributed to the following reason:

The temperature and, consequently, the velocity distribution of atoms within a MOT is directly proportional to the linewidth of the corresponding MOT transition. Moreover, the number of atoms interacting with light within a MOT is dependent upon both the linewidth of the transition and the laser detuning. For a fixed light detuning, the lower the linewidth, the lower is the number of interacting atoms inside a MOT [113]. Hence, during the transfer of atoms from the initial MOT with a broader linewidth to the narrow-

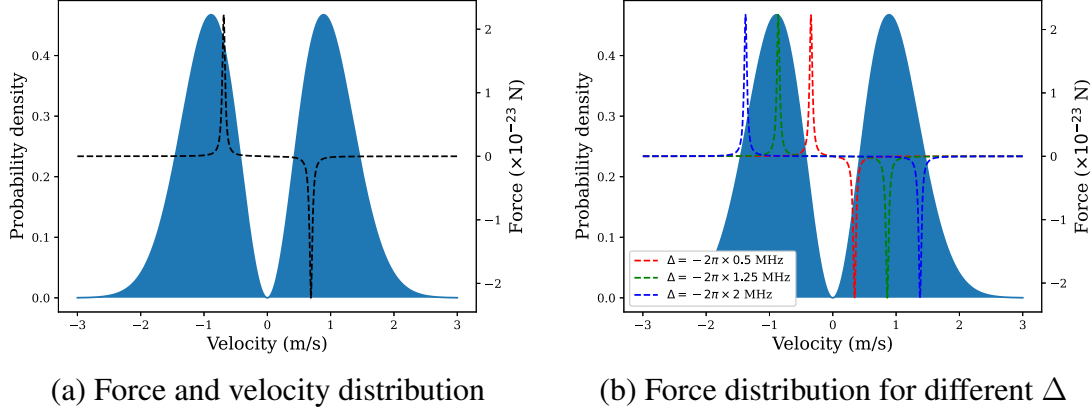


Figure 5.1: (a) illustrates the velocity distribution (Maxwell-Boltzmann distribution) of strontium atoms at 4 mK (the blue MOT temperature) alongside the force distribution of the red laser with a saturation parameter of 100 and $\Delta = 2\pi \times 1$ MHz. This depiction suggests that only a small fraction of atoms will interact with the light. (b) demonstrates the force distribution for various detunings, suggesting that a modulated detuning will enhance the interaction. However, even after the modulation, a lot of atoms remain unaddressed by the light.

line MOT, a large number of atoms do not interact with the laser that is precisely locked to the narrow-line transition of the secondary MOT (Figure. 5.1(a)). This causes a substantial loss of atoms from the trap during the transfer process, even if the laser detuning is artificially broadened through temporal modulation (Figure. 5.1(b)) [7]. The loss of atoms elevates various noise levels in different quantum experiments, including but not limited to atom shot noise in optical atomic clocks and atom interferometry ($\sim \frac{1}{\sqrt{N}}$, N —number of atoms), frequency instability in atomic clocks attributable to fluctuations in atom numbers ($\sim \frac{\delta N}{N}$) etc [32]. Additionally, the number of atoms holds significant importance in gravitational wave detection based on atom interferometry. Long-baseline GW detector based on these atoms, taking advantage of both optical clocks and atom-interferometry can detect sources in the band from 0.01Hz-3Hz [44, 114, 115]. This band is not covered by the existing terrestrial interferometers (LIGO/Virgo) and the planned LISA satellite detector. In order to maximize sensitivity of such a detector, it is crucial that we have as many cooled atoms as possible, such that their velocity spread is as small as possible.

As previously mentioned, ^{88}Sr requires to undergo a sequential two-step Doppler cooling process: the blue MOT $[(5s^2)^1S_0 \rightarrow (5s5p)^1P_1, 461 \text{ nm}]$ and the red MOT

$[(5s^2)^1S_0 \rightarrow (5s5p)^3P_1, 689 \text{ nm}]$. The linewidths of these transitions are $2\pi \times 32 \text{ MHz}$ and $2\pi \times 7.5 \text{ kHz}$ respectively [116] which results in a poor inter-MOT transfer efficiency. So far, only up to $\sim 40\%$ transfer efficiency has been achieved [7], thereby limiting the atom number available for experiments. In this chapter, we report $\sim 85\%$ transfer efficiency by employing a novel yet technically-simple three-level cooling approach for ^{88}Sr . We demonstrate that it is possible to reduce the temperature of the blue MOT significantly (from $\sim 4 \text{ mK}$ to $\sim 1 \text{ mK}$). This technique effectively narrows the Doppler-broadened velocity profile of the blue MOT, thereby facilitating efficient loading of atoms into the narrow-line MOT. Moreover, due to similar configuration of the energy levels, it might be possible to implement the same technique for the cooling of other alkaline earth metals, ensuring a high Signal-to-Noise Ratio (SNR) throughout the experimental process. It should be mentioned here that there are alternative techniques available to enhance transfer efficiency [117, 7]. One such approach involves adjusting the width of the artificial broadening of the light utilized for the narrow-line MOT transition. The transfer efficiency can also be enhanced by simply reducing the intensity of the blue MOT light. However, the technique described in this chapter offers either comparable or better transfer efficiency compared to the methods mentioned previously.

5.1 Theoretical Background

In this section, we present a simplified theoretical model of the three-level cooling of ^{88}Sr atoms.

1D equation of motion of an atom in a MOT can approximately be given by the classical Langevin equation [118, 119].

$$m \frac{dv}{dt} = -\alpha v - kx + \sqrt{D}\eta(t) \quad (5.1)$$

In Eq.(5.1) $\alpha, k = -\partial_{v,x}F(v, x)$, where F is the linear order approximation of the actual force $F_t = F_+(x, v) + F_-(x, v)$, $F_{\pm}(x, v) = \pm \hbar k_L \Gamma \rho_{ee}$, diffusion coefficient $D = \hbar^2 k_L^2 \Gamma \rho_{ee}$ and $m, v, x, k_L, \Gamma, \rho_{ee}$ respectively denote mass of the atom, velocity of the atom, position of the atom with respect to the centre of the trap, wave-vector of the MOT laser, linewidth of the MOT transition and the fraction of the atomic population

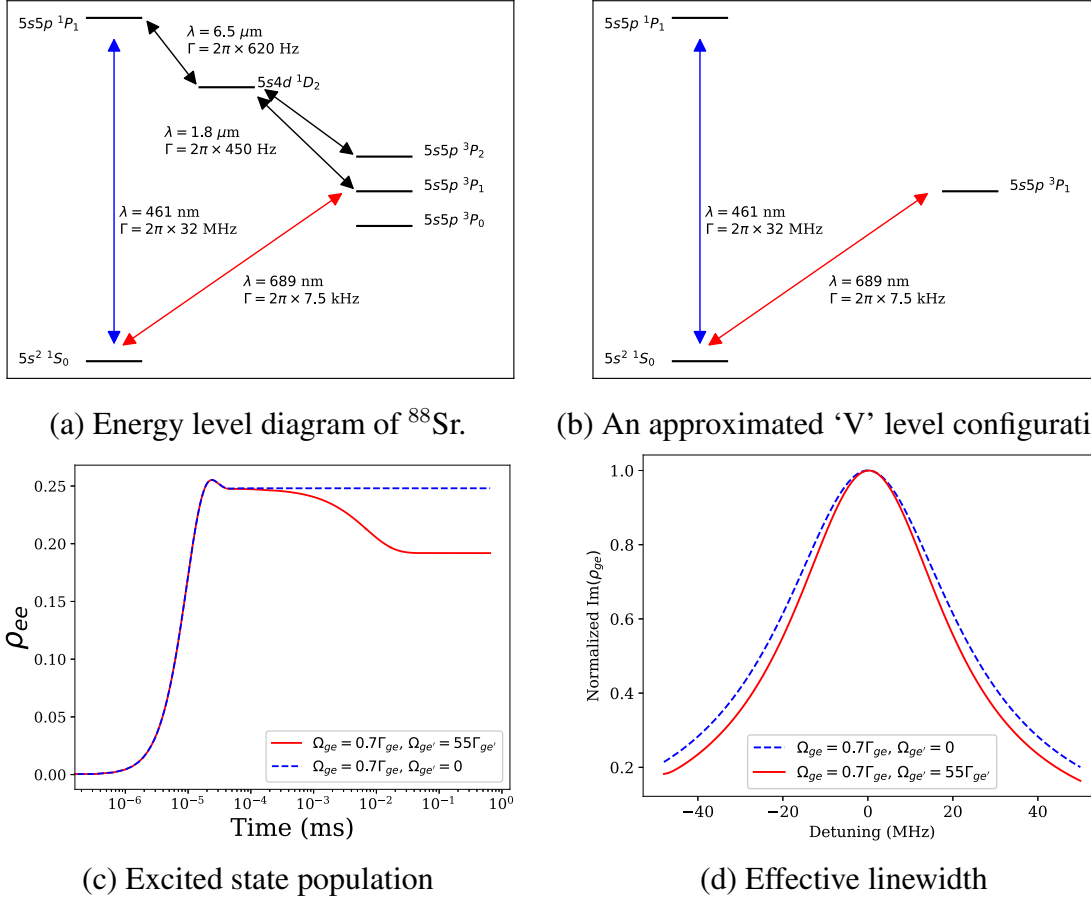


Figure 5.2: (a) and (b) provide the low lying and simplified energy level diagram energy level diagrams for ^{88}Sr . (c) illustrates the variation of ρ_{ee} of a strontium atom, both with and without the 689 nm light. (d) presents the impact of the 689 nm light on the effective linewidth of the 461 nm transition of ^{88}Sr .

in the excited state. The function $\eta(t)$, with properties $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t+\tau) \rangle = \delta(\tau)$, characterizes the fluctuating nature of random kicks on an atom due to spontaneous emission. It is important to mention that Eq.(5.1) is an approximated equation and is valid only under certain limits of position and velocity of the atoms. The limits are as follows: $|v| \ll \frac{1+s_0+4\delta^2/\Gamma^2}{8|\delta|k_L/\Gamma^2}$ and $|x| \ll \frac{\hbar|\delta|}{g\mu_B \frac{dB}{dx}}$, where s_0 - saturation parameter of light, δ - detuning of light, Γ - linewidth of MOT transition, k_L - wavevector of light, g - Landé g factor $\frac{dB}{dx}$ - magnetic field gradient of the MOT.

From Eq.(5.1), it can be shown that as $t \rightarrow \infty$, the temperature (T) can be given by $T = \frac{D}{k_B \alpha}$, where k_B is the Boltzman constant. It is evident that by manipulating ρ_{ee} one

can control the temperature of the MOT. ρ_{ee} can be obtained by solving the following open quantum optical Bloch equation (Eq. 5.2) [120, 121].

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{1}{i\hbar} [\hat{H}, \hat{\rho}] - \frac{1}{2} \{\hat{\Gamma}, \hat{\rho}\} + \hat{C} \quad (5.2)$$

In Eq.(5.2) $[\cdot, \cdot]$ and $\{\cdot, \cdot\}$ represent the commutator and the anti-commutator of the variables. $\hat{\rho}$ is the density matrix and $\hat{H}$ is the Hamiltonian of light atom interaction. $\hat{C}$ has been introduced to ensure unitary time evolution of the density operator. By implementing the rotating wave approximation (RWA) and successively performing an appropriate unitary transformation that makes $\hat{H}$ independent of time, one can write the following equations [120, 121] for a three-level ‘V’ system with a ground state $|g\rangle$, and a pair of excited states $|e'\rangle$ and $|e\rangle$.

$$\hat{H} = \hbar \begin{pmatrix} 0 & \Omega_{ge}/2 & \Omega_{ge'}/2 \\ \Omega_{ge}^*/2 & -\Delta_{ge} & 0 \\ \Omega_{ge'}^*/2 & 0 & -\Delta_{ge'} \end{pmatrix} \quad (5.3)$$

$$\hat{\Gamma} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Gamma_{ge} & 0 \\ 0 & 0 & \Gamma_{ge'} \end{pmatrix} \quad (5.4)$$

$$\hat{\rho} = \begin{pmatrix} \rho_{gg} & \rho_{ge} & \rho_{ge'} \\ \rho_{eg} & \rho_{ee} & \rho_{ee'} \\ \rho_{e'g} & \rho_{e'e} & \rho_{e'e'} \end{pmatrix} \quad (5.5)$$

In Equation (5.3), Ω_{ge} and $\Omega_{ge'}$ represent the Rabi frequencies for light-assisted transitions $|g\rangle \rightarrow |e\rangle$ and $|g\rangle \rightarrow |e'\rangle$, respectively. The detunings of these transitions are denoted by Δ_{ge} and $\Delta_{ge'}$ and they include contributions from both manual detuning ($\delta = \omega_L - \omega_0$, ω_L - laser frequency, ω_0 - resonant frequency of the atom) and Doppler detuning ($\Delta_D = -k_L v$, k_L - wavevector of the light, v - velocity of the atom). When light associated with the $|g\rangle \rightarrow |e'\rangle$ transition is switched off i.e., $\Omega_{ge'} = 0$, these equations elucidate the behavior of a conventional two-level system, where the evolution of the density matrix is solely influenced by Ω_{ge} . However, by adjusting the value of $\Omega_{ge'}$, the dynamics of the system can be manipulated to reduce the value of ρ_{ee} and subsequently decreasing the temperature of the atoms (Figure. 5.2(c)).

An alternative explanation is as follows:

The Doppler theory provides a prediction for the temperature of atoms within a MOT, expressed by the equation [122]:

$$T = \frac{\hbar \Gamma_{ge}^2}{8k_B |\Delta_{ge}|} \left(1 + \frac{2\Omega_{ge}^2}{\Gamma_{ge}^2} + \frac{4\Delta_{ge}^2}{\Gamma_{ge}^2} \right) \quad (5.6)$$

By setting $\Delta_{ge} = -n\Gamma_{ge}$ and employing the low power approximation, this equation simplifies to

$$T \approx \frac{\hbar \Gamma_{ge}}{8k_B} \left(4n + \frac{1}{n} \right) \quad (5.7)$$

This indicates that the temperature of two-level atoms trapped in a MOT is directly proportional to the linewidth of the cooling transition, provided that the detuning of light is appropriately adjusted. However, when it comes to calculating the temperature of three-level atoms interacting with two distinct lights in a MOT, the task becomes intricate. Nevertheless, a reasonable approximation can be achieved using Eq. 5.7, especially when one of the transition linewidths of the atom is significantly smaller than the other. In such cases, Γ_{ge} can be substituted with an effective value of Γ_{ge} (Γ_{ge}^{eff}) in the presence of second light, allowing for the derivation of an approximate temperature value. Γ_{ge}^{eff} can be determined by extracting the width of the steady-state solution of $\text{Im}(\rho_{ge})$. The impact of the second light on Γ_{ge}^{eff} is illustrated in Figure.5.2(d) revealing that, at a specific intensity of the first light, the second light facilitates additional cooling of the atoms.

The Hamiltonian mentioned in Eq. (5.3) has an Eigenvector with Eigenvalue 0 indicating the presence of a ‘dark state’ and consequently the occurrence of destructive interference responsible for Electromagnetically Induced Transparency (EIT) [123]. Atoms in the dark state do not participate in the optical transitions and may play a crucial role in atom cooling. However, this dark state only manifests when $\Delta_{ge} = \Delta_{ge'} = 0$. In a MOT, such a condition is feasible if the following equation admits a solution.

$$\begin{pmatrix} \pm k_{ge} & \pm Z_{ge} \\ \pm k_{ge'} & \pm Z_{ge'} \end{pmatrix} \begin{pmatrix} v \\ x \end{pmatrix} = \begin{pmatrix} \delta_{ge} \\ \delta_{ge'} \end{pmatrix}. \quad (5.8)$$

Here, $k_{ge,ge'}$ represents the wavevectors of the corresponding transitions, $Z_{ge,ge'}$ signifies the Zeeman detuning per unit length in the MOT, and $\delta_{ge,ge'}$ represents the manual detuning of the lights. v and x denote the velocity and position of the atom, respectively.

The matrix in Eq. (5.8) is typically nonsingular. Consequently, v and x assume specific values, indicating that only atoms from particular regions of the classical phase-space are likely to exhibit EIT. For these atoms to be within a MOT, v should be sufficiently low and x should be smaller than the size of the MOT. Furthermore, for a significant contribution from EIT, the fraction of atoms in the dark state ($\frac{|\Omega_{ge'}|^2}{|\Omega_{ge}|^2 + |\Omega_{ge'}|^2}$) should be high.

Taking into account all these factors, it has been determined that in our system, with the chosen parameters, the contribution from EIT is not predominant. However, for certain parametric values, the EIT phenomenon may dominate and significantly influence the cooling mechanism.

To study the behavior of three-level atoms within the MOT, it becomes crucial to explore a multi-level system as the magnetic field within the MOT splits the excited levels into Zeeman sublevels [124]. For instance, in case of alkaline earth metals ($|g\rangle = |^1S_0\rangle$, $|e\rangle = |^1P_1\rangle$ and $|e'\rangle = |^3P_1\rangle$), one should ideally consider a 7 level system including a pair of extra levels for each of the excited states. However, in the context of a one dimensional (1D) MOT, the utilization of σ_+ and σ_- polarized light eliminates the possibilities of certain transitions, namely $|^1S_0\rangle \rightarrow |^1P_1, m_f = 0\rangle$ and $|^1S_0\rangle \rightarrow |^3P_1, m_f = 0\rangle$ (m_f - the component of angular momentum along the quantization axis). Consequently, the system simplifies to a 5 level system. It is noteworthy that the Zeeman splitting in a MOT is inhomogeneous and is given by $\Delta_Z = \mu_B m_f g \frac{dB}{dx} x$, with μ_B representing the Bohr magneton, g representing the Landé g-factor of the state, and $\frac{dB}{dx}$ denoting the magnetic field gradient. This Zeeman splitting contributes to the overall detuning of the light, alongside δ and Δ_D . With these considerations in hand, one can find out the steady-state solution of ρ through the solution of Eq.(5.2) under the condition of $\frac{\partial \rho}{\partial t} = 0$ [125], consequently calculating the temperature of the MOT.

One can also explore the dynamics of an atom in a MOT, as governed by Eq. 5.2, over a sufficiently extended interval of time. By employing the ergodicity hypothesis, it is possible to deduce the velocity distribution of a large ensemble of atoms within

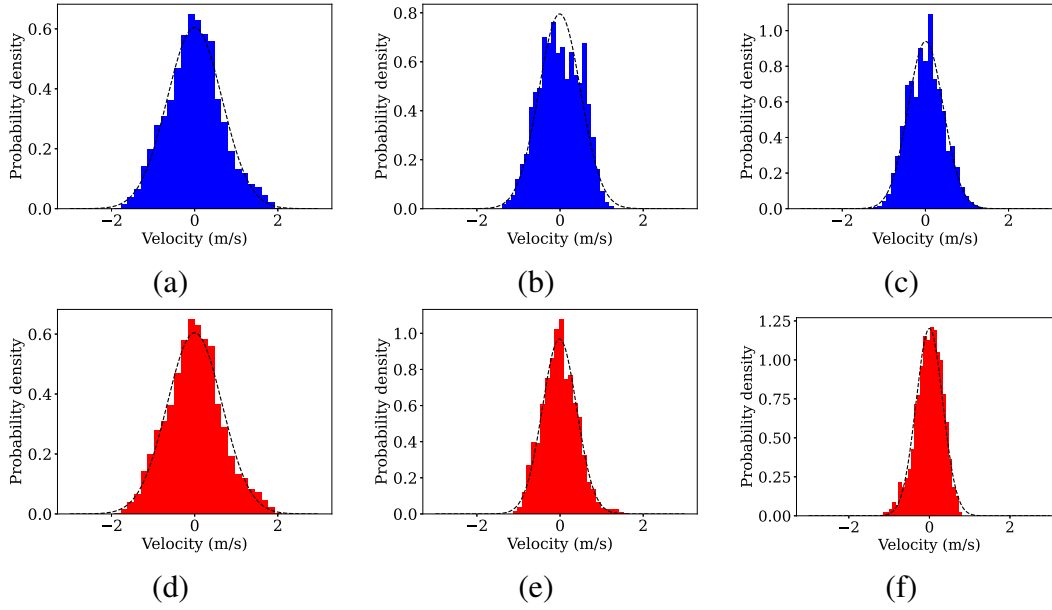


Figure 5.3: Evolution of the velocity distribution of strontium atoms at different instants. This figure displays the simulated velocity distribution of ^{88}Sr atoms within a MOT with fitted normal distribution curves. In a series of three consecutive 10-millisecond intervals, the saturation parameter of the blue light is systematically reduced from 1.5 to 0.3 and then to 0.06. Figures (a), (b), and (c) illustrate the velocity distribution of atoms as the blue light is progressively reduced without the introduction of red light. Whereas, panels (d), (e), and (f) depict the velocity distribution under the same decreasing blue light conditions, but with the addition of an intense red light (saturation parameter - 6000) after the initial 10 milliseconds.

the MOT. The standard deviation of this distribution serves as an indicator of the temperature of the atoms. In the next section, it will be shown that the implementation of three-level cooling has the notable effect of reducing the temperature of the atoms.

Approximations:

1. The numerical results of the three-level cooling presented in this chapter are computed under the assumption that the MOT is 1D. It is important to note that a quantum mechanical analysis of a three-dimensional (3D) MOT might yield different numerical outcomes. However, it is assumed that, in both scenarios, the solutions will exhibit qualitative similarities.
2. The proposed three-level cooling scheme is based on the ‘V’ shaped energy level configuration with $|g\rangle = |^1S_0\rangle$, $|e\rangle = |^1P_1\rangle$, and $|e'\rangle = |^3P_1\rangle$. While in practice, the scenario does not precisely emulate the ‘V’ scheme of the three-level cool-

ing as the transition $|^1P_1\rangle$ has a small decay to $|^3P_1\rangle$ via $|^1D_2\rangle$. Contributions from these transitions can be neglected when $(\Gamma_{^1P_1 \rightarrow ^1D_2}, \Gamma_{^1D_2 \rightarrow ^3P_1}) \ll \Gamma_{^1S_0 \rightarrow ^1P_1}$, $\Gamma_{^1S_0 \rightarrow ^3P_1}$ holds true. This approximation is valid for all alkaline earth metals, suggesting that the proposed three-level cooling approach is likely applicable to other elements within this metal group as well. For instance, from the energy level diagram in Figure(5.2(a)), it becomes evident that this approximation remains justifiable for ^{88}Sr atoms. As a result, one can simplify this diagram as the one depicted in Figure(5.2(b)).

3. When calculating the temperature of three-level atoms in the MOT, one should ideally consider the diffusion coefficients arising from both transitions ($D_{^1S_0 \rightarrow ^1P_1}$ and $D_{^1S_0 \rightarrow ^3P_1}$). However, given that $D \propto \Gamma \rho_{ee}$ and $\Gamma_{^1S_0 \rightarrow ^3P_1} \ll \Gamma_{^1S_0 \rightarrow ^1P_1}$, the impact of $D_{^1S_0 \rightarrow ^3P_1}$ can be neglected, unless the population in $|^3P_1\rangle$ is ‘sufficiently larger’ than that in $|^1P_1\rangle$.

5.2 Results and discussions

In case of ^{88}Sr , a temperature reduction within the blue MOT can be achieved through the utilization of $(5s^2)^1S_0 \rightarrow (5s5p)^3P_1$ transition. Herein, we designate the states $(5s^2)^1S_0$, $(5s5p)^1P_1$, and $(5s5p)^3P_1$ as $|g\rangle$, $|e\rangle$, and $|e'\rangle$ respectively. Pertaining to our system, the respective parameters are as follows: $\Gamma_{ge} = 2\pi \times 32$ MHz, $\Gamma_{ge'} = 2\pi \times 7.5$ kHz, $\lambda_{ge} = 461$ nm (blue laser), and $\lambda_{ge'} = 689$ nm (red laser). The Rabi frequency can be expressed as $\Omega = \sqrt{\frac{s}{2}}\Gamma$, where $s = I/I_{sat}$, with I signifying the light intensity and I_{sat} representing the saturation intensity of the transition. In the outlined scheme, we have $I_{sat,ge} = 42$ mW/cm² and $I_{sat,ge'} = 3$ μ W/cm² [126]. Notably, the Rabi frequencies are characterized in terms of the saturation parameters s_{ge} and $s_{ge'}$.

We conduct a numerical analysis of the steady-state temperature of atoms within a MOT, exploring the impact of varying intensities of 461 nm and 689 nm lasers. We observe that the standard deviation of the velocity distribution, i.e., the temperature of the atoms in the blue MOT, was significantly lower than that of a conventional MOT when the intensity of the blue laser was low and the intensity of the red laser was high (Figure.(5.3)). It is crucial to highlight that, while the primary cause of the temperature decrease can be attributed to the attenuation of blue light intensity (refer to Eq.5.6),

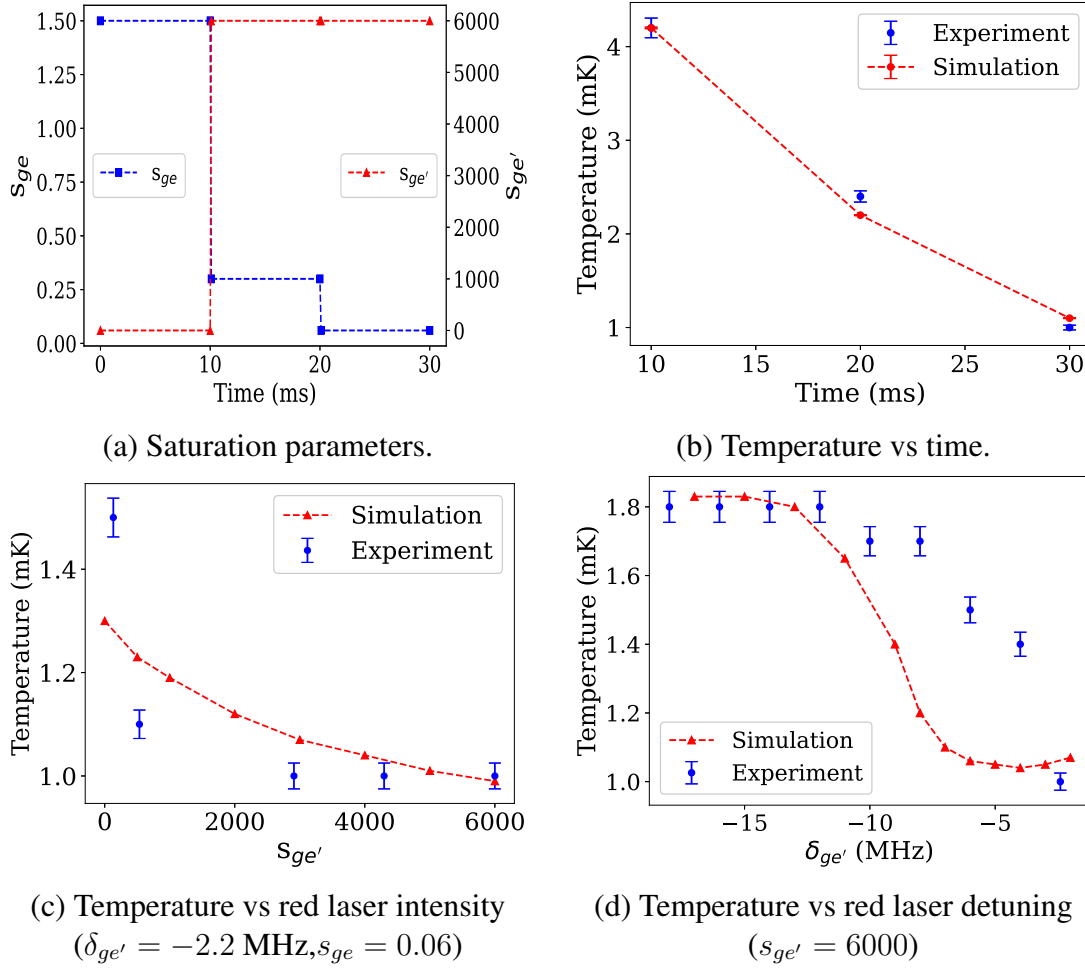


Figure 5.4: (a) displays the saturation parameters of the red and the blue laser at different time. (b) shows that how MOT temperature goes down with three-level cooling. (c) and (d) illustrate the temperature variations of MOT with different intensities and detunings of the red laser.

an additional cooling effect resulted from the presence of red light, as depicted in Figure.5.4(c). While it may appear that the impact of red light is small, our experimental observations (Figure.5.5(a)) suggest that it is substantial enough to increase the loading efficiency significantly (from $\sim 75\%$ to $\sim 85\%$). We also calculated the temperature of atoms in the MOT as a function of laser detunings. We found that the temperature was minimum when the red laser was red detuned by ~ 2 MHz (Figure.5.4(d)).

We applied this technique in our experimental setup to effectively lower the temperature of the blue MOT. Following the initial conventional blue MOT cooling phase, we turned on the 689 nm laser while gradually attenuating the power of the 461 nm laser

over a period of 30 ms by controlling the RF power of an AOM (Acousto-Optic Modulator) configured in a double-pass arrangement. We systematically lowered the power of the blue laser over a sequence of three discrete time intervals, each spanning 10 ms (Figure. 5.4(a)). The saturation parameter values corresponding to the blue laser for these intervals were set at 1.5, 0.3, and 0.06, respectively. Simultaneously, the saturation parameter values for the red laser were set at 0, 6000, and 6000, respectively. The magnetic field gradient is maintained at 50 Gauss/cm throughout the sequence. Using the *Time of Flight* (ToF) method, we measured the rate of expansion of the atomic cloud and inferred that the temperature of the blue MOT was reduced substantially due to the effect of three-level cooling (Figure. 5.4(b)). It is worth mentioning here that a gradual reduction in the power of the blue laser is necessary as an abrupt decrement in laser power causes a sudden reduction in the MOT force leading to the loss of significant number of atoms.

Furthermore, we investigated the influence of the red laser's intensity while maintaining a constant blue laser intensity. The reduction in the red laser's intensity led to an attenuation of the three-level cooling effect, subsequently resulting an increment in the MOT's temperature (see Figure. 5.4(b) for illustration). Similarly, we explored the detuning effect of the red laser (refer to Figure. 5.4(c)) on the temperature. We found that the temperature reached its minimum point at a detuning slightly below zero, at which the maximum fraction of the atoms were in resonance with the red light. It is worth mentioning that all the data found from the experiments are in good agreement with the theoretical predictions of the approximated model we described in one of the earlier sections.

Implementing the aforementioned technique, we have achieved a significant decrease of temperature of atoms in the MOT – more than 75% (Figure. 5.4(b)). Since a lower temperature of the atomic cloud leads to the compression of the atomic velocity distribution, a greater number of atoms in the blue MOT interacted with the red laser leading to a higher inter MOT transfer efficiency (see Figure. 5.5(a) and 5.4(b) for illustration). Our observations indicated a significantly enhanced transfer efficiency compared to the conventional approach. Employing the three-level cooling method, we achieved a transfer efficiency exceeding 85%, a notable improvement compared to 39%

attained without utilizing the three-level cooling.

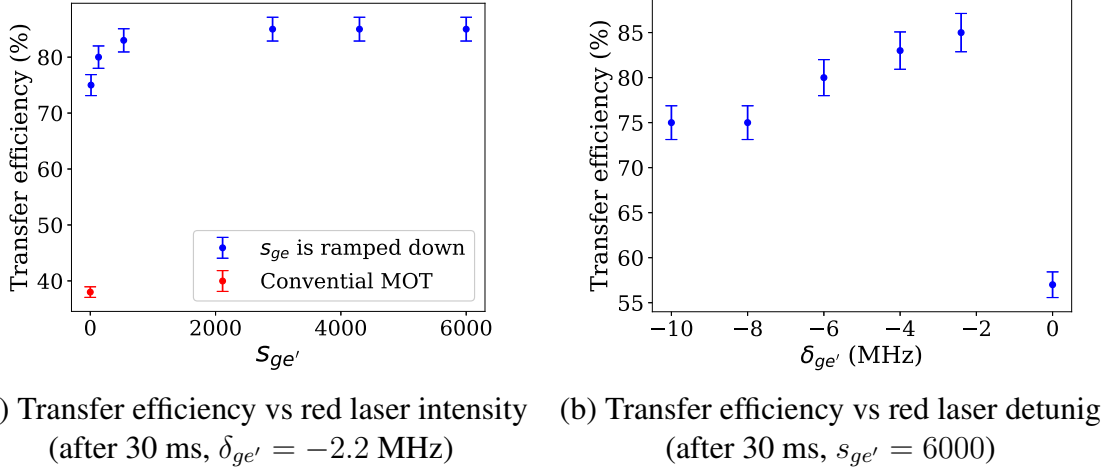


Figure 5.5: (a) and (b) show the transfer efficiency for different intensities and detunings of the red laser. The three-level cooling method doubles the transfer efficiency compared to the conventional method, which typically achieves an efficiency of around 40%. [7, 8].

It is imperative to mention here that our numerical results and experimental findings only qualitatively align with the values derived from the theoretical predictions of the approximate 1D model. The differences between the experimental and theoretical results can be attributed to various factors, including but not limited to:

1. The numerical computations were conducted based on the assumption of a one-dimensional (1D) MOT. Introducing a three-dimensional (3D) MOT solution could yield minor differences in numerical values.
2. Imperfections in the magnetic field profile of the MOT and inaccuracies in field measurements in the actual experimental setup may significantly impact the results. This influence is particularly notable as it directly influences detuning through the Zeeman shift. The first-order Zeeman shifts for the $5s5p^1P_1$ and $5s5p^3P_1$ levels are approximately 1.5 MHz/Gauss and 2 MHz/Gauss, respectively. Hence, even a deviation of 2 Gauss in the magnetic field can affect temperature measurements by $\pm 100 \mu\text{K}$.

To summarize, in this chapter an innovative three-level cooling approach for strontium atoms has been demonstrated. This technique significantly reduced the tempera-

ture of the blue MOT, leading to a notable enhancement in the efficiency of inter-MOT atom transfer. This method can be approximately modeled with the ‘V’ scheme of energy level configuration, Our experimental results were consistent with the qualitative expectations of the theoretical predictions from this approximate model.

Nevertheless, this cooling method may be applied across all the elements belonging to the alkaline earth metal group as they share similar level structures. By applying this approach, we can effectively amplify the inter-MOT loading efficiency for these elements, consequently elevating the SNR for various applications.

Chapter 6

The red MOT

Following the successful implementation of the novel intermediate cooling technique, the next pivotal phase in our laser cooling experiments with strontium atoms is the conventional red MOT cooling. As previously discussed, the temperature of the strontium atoms after the initial blue MOT cooling stage is not yet optimal for many quantum experiments.

Even with the remarkable temperature reduction achieved through our innovative three-level cooling method detailed earlier, the atoms still remain at a state that limits their utility in precise quantum operations. This realization underscores the critical need for a subsequent cooling stage to further bring down the temperature of our atomic ensemble.

This second stage of cooling is achieved through the utilization of a narrow transition line, a technique known as the red MOT cooling. The narrow transition line, due to its significantly low Doppler limited temperature, offers a refined method to cool the strontium atoms to the desired state of ultra-low temperature, facilitating their manipulation and control for a variety of quantum applications.

The process of red MOT cooling consists of two stages: the broadband red MOT and the single-frequency red MOT. The broadband red MOT is a crucial precursor to the single-frequency red MOT, as explained in the preceding chapter.

6.1 Broadband Red MOT

After the intermediate cooling, the temperature of the atoms is reduced to approximately 1 mK. At this temperature, the most probable velocity of the atoms is about 0.5 m/s, corresponding to a Doppler shift of around $2\pi \times 1$ MHz. However, the red laser's linewidth is considerably narrower, below $2\pi \times 7.5$ kHz. To maximize the interaction between the light and the atoms, the laser detuning needs to be modulated over a suitably broad range around $-2\pi \times 1$ MHz. In our experiment, we employed a detuning modulation in the range between $-2\pi \times 1$ MHz and $-2\pi \times 3$ MHz at a modulation frequency frequency of $-2\pi \times 50$ kHz. This modulation artificially broadens the laser linewidth and creates a comb-like distribution of the force (see Figure 5.2). Additionally, a high saturation parameter (2000) was maintained to ensure efficient atom capture through power broadening of the resonance peak.

Subsequent to the broadband red MOT, the temperature of the atoms in our experiment was measured to be approximately $54 \mu\text{K}$ using the time-of-flight method described in Section 4.2.2. At this stage, we turned off the modulation of the red light and proceeded to the single-frequency operation for further cooling.

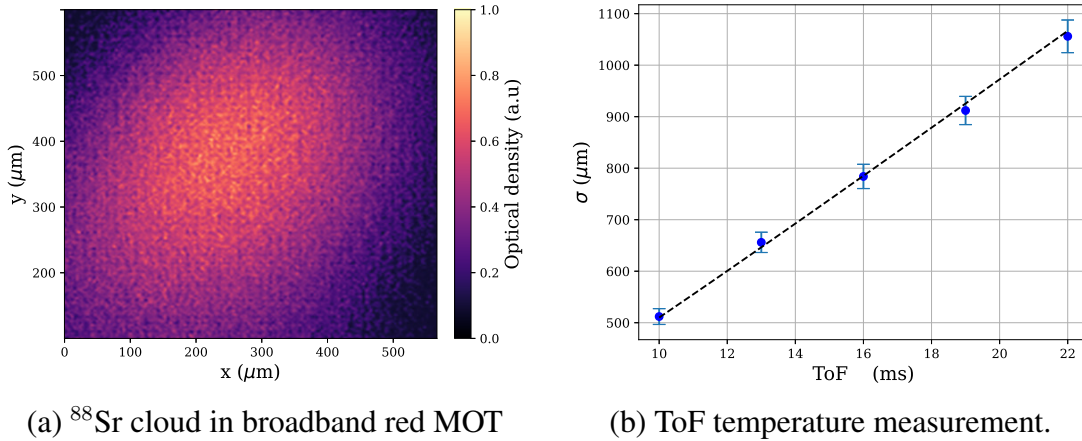
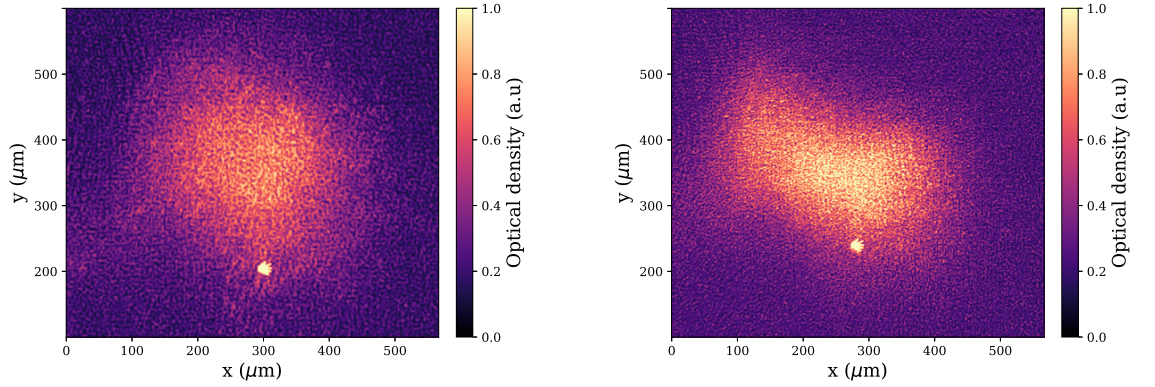


Figure 6.1: Broadband red MOT(a) In situ ^{88}Sr cloud in broadband red MOT. (b) Time of flight temperature measurement of the broadband red MOT. The number of atoms in this stage is $\sim 8 \times 10^7$.



(a) single red MOT cloud (sat. param =1000) (b) single red MOT cloud (sat. param =100)

Figure 6.2: Single frequency red MOT ^{88}Sr cloud within the single-frequency red MOT for different saturation parameters. The distortion observed in the cloud's shape is attributed to gravitational effects, as the force of gravity becomes comparable to the force of light at this stage. The number of atoms in this stage is $\sim 1 \times 10^7$.

6.2 Single-Frequency Red MOT

The detuning parameter is of significant importance in the single-frequency red MOT. During the blue MOT and intermediate cooling stages, the magnetic field gradient was maintained at 50 Gauss/cm. However, for the single-frequency cooling to commence, the Zeeman detuning corresponding to the narrow-line transition at 1 mm away from the center was approximately $2\pi \times 2$ MHz. This detuning was not ideal due to the low temperature of the atoms after the broadband red MOT. Therefore, the magnetic field gradient was gradually decreased from 50 Gauss/cm to 3 Gauss/cm, while simultaneously reducing the laser detuning to around $2\pi \times 200$ kHz. Additionally, the intensity of the red light was decreased to achieve a saturation parameter of 100 to reduce the force on atoms due to light. The variation of parametric values can be found in Figure(6.3).

Following the completion of the single-frequency red MOT cooling process, the temperature of the atoms was measured once again. This stage of cooling brought the temperature down to approximately $6 \mu\text{K}$.

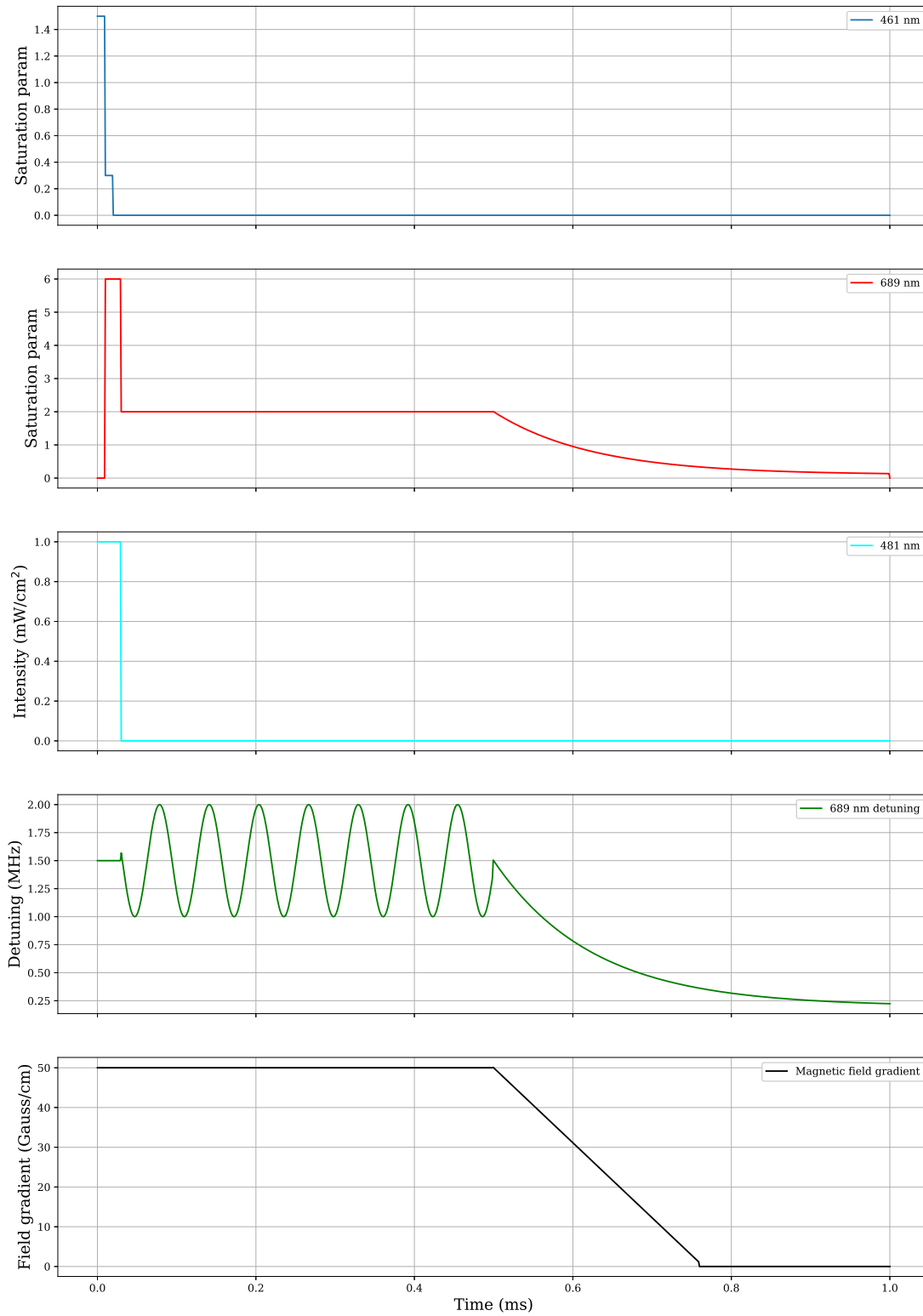


Figure 6.3: Timing diagram: The timing sequence used for the loading and cooling atoms in the red MOT.

Chapter 7

Conclusion and Outlook

7.1 Summary

The thesis was designed to explore techniques for enhancing the Signal-to-Noise Ratio (SNR) in experiments utilizing laser-cooled strontium atoms, renowned for their highly restricted dipole transitions and wide-ranging applications. The quantity of atoms plays a pivotal role in amplifying SNR, thereby suppressing various noise sources such as atom shot noise and frequency instability in atomic clocks. The increase in atom numbers ensures a higher SNR, proving invaluable for a diverse range of experimental setups.”

Detailed in this thesis is the experimental setup designed for the cooling and trapping of ^{88}Sr atoms. The process involves generating an atomic beam, slowing it down with a Zeeman slower, and subsequently loading it into the Magneto Optical Trap (MOT). We achieve a loading of approximately 10^7 ^{88}Sr atoms (without the use of a repumper laser) in the blue MOT, operating in the standard $\sigma^+ - \sigma^-$ configuration with a magnetic field gradient of approximately 50 Gauss/cm in the quadrupole coils. Additionally, we employ transverse cooling to improve the collimation of the atomic beam before loading into the blue MOT. After the completion of the blue MOT cooling stage which helps to achieve a temperature of ~ 1 mK, the strontium atoms were subjected to a subsequent cooling phase known as the red MOT cooling. This phase further reduces the temperature of the atoms to approximately $1\text{ }\mu\text{K}$, enabling the manifestation of quantum effects

while suppressing thermal decoherence.

To improve the overall Signal-to-Noise Ratio (SNR) of the experiment, efforts were made to increase the number of atoms in the following two stages:

1. Optimization of the parameters during the blue MOT cooling process.
2. Implementation of a novel three-level cooling technique during the transfer of atoms from the blue MOT to the red MOT.

We have demonstrated the optimization of the number of atoms in the Magneto Optical Trap (MOT) using optimization algorithms during the blue MOT cooling and emphasized its significance. Manually adjusting multiple parameters to maximize the MOT atom number is a time-consuming and error-prone process that can lead to getting stuck in local maxima of the parameter space. Moreover, optimized parameters for one isotope may not be optimal for others. In a versatile setup, repetitive manual optimizations for different isotopes in various experiments consume valuable time and may not yield the most suitable parameters. In contrast, machine learning optimization algorithms are crucial in addressing these challenges and automating the loading of different isotopes into the MOT.

During the transfer of atoms from the blue MOT to the red MOT, we implemented a novel three-level cooling approach for strontium atoms. This technique significantly reduced the temperature of the blue MOT, resulting in a noticeable improvement in the efficiency of inter-MOT atom transfer. This method can be approximated by the 'V' scheme of energy level configuration. Our experimental results aligned with the qualitative expectations from this approximate model. However, this cooling approach may be applicable to all elements in the alkaline earth metal group due to their similar level structures. Implementing this strategy can effectively enhance the inter-MOT loading efficiency for these elements, thereby boosting the Signal-to-Noise Ratio (SNR) for various applications.

Subsequently, we conducted conventional red MOT cooling experiments on ^{88}Sr atoms and achieved a temperature of approximately $6\ \mu\text{K}$ with an increased number of atoms. These laser-cooled atoms can be utilized in various quantum experiments such as quantum information processing and the development of high precision optical atomic

clocks. The enhanced number of laser-cooled atoms will prove advantageous as it helps mitigate various quantum noises arising from the limited number of atoms.

7.2 Future work

An area of cold experiments that remains relatively unexplored is the creation of a quantum network using the atom-plasmon system. While this system shares similarities with the atom-cavity setup, it presents unique advantages. Our setup allows for the establishment of a quantum network node utilizing the atom-plasmon system. Moreover, this system shows promise for the development of precise optical atomic clocks, serving purposes in fundamental research and time-keeping applications.

7.2.1 Quantum nodes with atom-plasmon system

In a quantum network, quantum information undergoes processing and storage within interconnected quantum nodes through *quantum channels*. These channels facilitate the transfer of quantum information between nodes with high fidelity, using *flying qubits*, thereby distributing entanglement across the network. A quantum network with n nodes and c channels offer a state space of dimension 2^{cn} . Therefore, even with comparatively lower processing capabilities of individual processors, a quantum network has the potential to outperform classical computer clusters. Thus, a quantum network has the potential to tackle the scalability challenges of individual quantum processors while simultaneously addressing issues related to error correlations.

Quantum networks also exhibit diverse applications, extending into the domains of quantum metrology and secure quantum communication, two pivotal aspects of quantum technology. Quantum network can also be utilized in simulating quantum many body problems. Considering the nodes of a quantum network as distinct components of a physical system, communication protocols can be designed to simulate a quantum many-body system. Certain classical problems, such as the classical percolation problem, can be mapped to the extended entanglement within a quantum network and thoroughly investigated.

Hence, a quantum network holds significant importance both for the advancement of fundamental physics and technological development. It presents numerous benefits for

the progress of quantum technology, particularly in addressing size scalability concerns and error correlation issues.

In an atom-plasmon quantum network node, atoms are trapped using the near-field optical potential of plasmonic nanostructures, acting as static qubits for quantum information processing. Photons, on the other hand, function as the flying qubits, carrying the processed information to other nodes. The trapping of atoms is achieved through the exploitation of localized surface plasmon resonance. When illuminated with light of a suitable wavelength, the electrons in the nanostructures begin to resonate, creating a narrow distribution of high electric field. This results in a trapping force given by:

$$\langle \mathbf{F} \rangle = -\hbar\delta \frac{s(\mathbf{r})}{1 + s(\mathbf{r})} \frac{\nabla\Omega(\mathbf{r})}{\Omega(\mathbf{r})} \quad (7.1)$$

This expression corresponds to the first term of Eq.(2.12). Due to the high intensity of the electric field, the first term dominates over the second term. In the case of red-detuned light ($\delta < 0$), to the first order, the force is given by $\langle \mathbf{F} \rangle \propto -\mathbf{r}$, which serves as the trapping force.

The atom-plasmon system can be characterized by the following three parameters.

- The atom-photon coupling parameter g_0
- The non-resonant decay rate of the atom γ
- The photon decay rate of the atom plasmon resonant system κ

In a system consisting of a single atom and a single photon, we observe slowly decaying Rabi oscillations between $|g\rangle \otimes |1\rangle$ and $|e\rangle \otimes |0\rangle$ ($|g\rangle, |e\rangle$ representing the ground and excited states of the atom, and $|0\rangle, |1\rangle$ representing the vacuum state and single photon state) if $g_0 \gg \gamma, \kappa$. This parameter regime is referred to as the strong coupling regime. Through the exploitation of this strong coupling between atoms and cavity photons, it becomes feasible to implement controlled-phase (CZ) gates and various other types of quantum gates.

For instance, the robust interaction between an atom and a cavity mode can produce the required conditional phase shift for CZ gates. Additionally, the strong coupling regime ensures a high Purcell factor, leading to an increased decay rate of atoms into

the plasmonic modes that can be subsequently outcoupled to dielectric waveguides. Consequently, processed quantum information can be effectively distributed across the network.

Our system enables the production of laser-cooled strontium atoms which can be loaded into optical tweezers. Utilizing light-assisted pairwise collision of atoms, it becomes possible to trap individual atoms within the tweezers. These isolated atoms can then be positioned in proximity to precisely engineered arrays of plasmonic nanostructures, facilitating the creation of a node within a quantum network.

7.2.2 Optical atomic clock

One of the intriguing possibilities that our system offers for future exploration is the development of a high-precision optical atomic clock. The fundamental principle of a clock lies in the counting of periodic oscillations produced by oscillators. Throughout history, there has been an evolution in the selection of stable oscillators for constructing accurate timepieces. These choices span from celestial motions, such as solar or planetary movements, to the ubiquitous quartz oscillators found in most of our everyday electronic devices.

The quality factor Q of an oscillator is defined as $Q = \nu/\delta\nu$, where ν represents the frequency of the oscillator and $\delta\nu$ is the error in the frequency measurement. For instance, a quartz crystal clock at a temperature of 30°C typically possesses a quality factor of $\sim 3 \times 10^5$. This implies that the quartz clock would exhibit an error of approximately 0.3 seconds per day. While this level of accuracy may seem adequate, the demands of the modern world necessitate much higher precision for applications such as navigation and positioning systems.

The atomic standard of time

Quantum mechanics, emerging in the early part of the last century, opened the door to a deeper understanding of atomic structure, allowing us to harness the properties of atoms as oscillators. This breakthrough paved the way for the development of the atomic clock, representing the epitome of precision in timekeeping today. Cesium atoms play a central role in many atomic clocks, where the microwave frequency required to prompt a transition between two energy states is meticulously determined. This frequency serves

as the gold standard against which all other timepieces are measured. The exceptional precision and accuracy of this microwave transition in cesium atoms led to its acceptance as the basis for the redefinition of the SI second in 1967. Hence, the SI second was defined as "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of cesium-133 atoms." The Q factor of atomic clocks is approximately 10^{16} , translating to an astonishingly minute error of about 1 second in roughly 100 million years. This extraordinary level of precision places atomic clocks among the most accurate timepieces ever created by humanity, marking a significant milestone in our quest for measuring time with unparalleled accuracy.

The optical standard of time

The remarkable precision of atomic clocks can be enhanced if we consider employing optical transitions instead of microwave transitions. These clocks are known as optical atomic clocks. The key to their enhanced precision lies in the higher frequency of optical transitions, which increases the numerator of the Q factor. Additionally, to achieve exceptional accuracy, the denominator ($\delta\nu$) of the Q factor should be minimized. The alkaline earth metals offer such transitions, providing Q factors surpassing $\sim 10^{18}$. This extraordinary level of precision translates to an error of merely 1 second over a period of approximately 10 billion years. To put it into perspective, if a clock had started ticking at the moment of the Big Bang, it would have deviated by only 1 second to this day.

The applications of optical atomic clocks extend across a wide spectrum of technological advancements and fundamental scientific pursuits. These clocks, with their unrivaled precision, are crucial in fields such as global positioning systems (GPS), telecommunication networks, and satellite-based navigation. Their precision also plays a pivotal role in fundamental physics research, enabling tests of fundamental constants and theories, including the general theory of relativity and the standard model.

In the realm of optical atomic clocks, a primary challenge arises from the need to count the oscillations provided by the atomic oscillators at a unprecedented rate. Optical transitions operate at frequencies around 100 THz, far beyond the counting capabilities of conventional electronics. The breakthrough came with the development of the optical

comb, a revolutionary tool that enables the precise counting of these oscillations.

An optical comb typically utilizes a mode-locked femtosecond laser that generate pulses that are temporally separated by $1/f_r$, where f_r represents the repetition rate of the laser. The frequency spectrum of such a laser is characterized by evenly spaced frequency components, each separated by the pulse repetition rate.

By employing the optical comb, the signal from the optical atomic clock can be made to beat with one of the comb's frequency components. This process effectively downconverts the high-frequency optical signal into the microwave regime. At this point, conventional electronics can be employed to accurately count the pulses, allowing for precise time measurements with optical atomic clocks.

To reduce the leading order frequency error in an optical atomic clock caused by the Doppler shift from atomic motion, it is crucial to confine the atoms' movement. This confinement is accomplished by trapping the atoms within a tight optical potential created by an optical lattice. The use of an optical lattice trap allows for the quantization of atomic motion, effectively removing the effect of motional degrees of freedom from the internal degrees of freedom of the atoms through a technique known as "resolved sideband cooling."

A key aspect for achieving a favorable SNR in an optical atomic clock is to load the optical lattice with a sufficiently large number of atoms. With typical parameters, the trap depth of an optical lattice is approximately $50\mu\text{K}$. This indicates that our experimental setup is well-equipped to produce cold atoms with temperatures significantly below the trap depth of the optical lattice. As a result, there will be a high efficiency in loading atoms into the optical lattice, ultimately enhancing the SNR eventually.

7.2.3 Other applications

The capabilities of our system extend far beyond the mentioned options, presenting a range of diverse applications. Apart from the mentioned functionalities, our system offers the potential for performing atom interferometry, a precise technique utilized in metrology for accurate measurements of gravitational forces and inertial effects. Additionally, our system enables the production of Bose-Einstein Condensates, paving the way for exploration into the realms of exotic quantum phenomena and fundamental

physics.

7.3 Key challenges

Realizing our system's future objectives requires addressing several key challenges. First, the current science chamber lacks a sufficient number of viewports to support upcoming stages, such as trapping cooled atoms in a far-detuned dipole trap while simultaneously performing clock spectroscopy. Additionally, positioning atoms near plasmonic nanostructures requires an increased number of access points. Moreover, the chamber is directly exposed to blackbody radiation from the strontium oven, introducing a small but significant perturbation to the clock transition, which compromises accuracy.

To overcome these limitations, atoms must be optically transported from the existing chamber to a second chamber. This process involves loading laser-cooled atoms into a dipole trap created by a focus-tunable lens and dynamically shifting the focus to move the atoms to the second chamber. While the process is intricate, our system is designed with the necessary provisions to support this machinery, enabling precise and efficient atom transport for advanced experimentation.

To implement an optical clock, it is better to use ^{87}Sr atoms, which offer several advantages over ^{88}Sr , as well-documented in the literature. However, working with ^{87}Sr presents challenges to its natural abundance being nearly ten times lower than that of ^{88}Sr , resulting in a lower SNR.

An intermediate cooling stage can also be implemented for ^{87}Sr , but this process is inherently complex due to the intricate atomic level structure arising from hyperfine splitting. The interaction of ^{87}Sr atoms with inhomogeneous magnetic field in a MOT further complicates the situation, as they hinder the efficient redistribution of atomic population among spin states via optical pumping. This might affect the performance of the intermediate cooling. This same complexity also contributes to the challenges in operating the red magneto-optical trap (MOT) for ^{87}Sr .

To successfully implement an intermediate cooling stage, a detailed analysis is required to address these issues. Additionally, it is worth investigating whether the introduction of a stirring laser, which has proven beneficial in resolving problems in the red MOT for ^{87}Sr , could also enhance the intermediate cooling stage.

Creating a quantum node with the existing setup presents significant challenges, as this area of physics is largely unexplored and lacks established procedures. To achieve this, several critical steps must be carefully addressed. First, strategies for trapping atoms near plasmonic nanostructures need to be analyzed, and numerical simulations performed to determine suitable parameters such as trap depth and trap frequency. These parameters will guide the optimization of the nanostructure's geometry.

Next, the nanostructure must be synthesized and rigorously characterized to ensure its compatibility with experimental requirements. Once validated, the nanostructure must be installed in the setup, and efforts made to position atoms near it for trapping. Verifying successful atomic trapping is particularly challenging, as the interrogation beam may drive atoms away from the trap. This requires precise experimental control to mitigate such effects.

Addressing these steps is crucial to progress toward realizing the first stage of a quantum node. In conclusion, while advancing with this setup involves overcoming several key challenges, it holds the potential to produce novel and impactful results that could benefit the broader scientific community.

Appendix A

Appendix A

This section evaluates the performance of the Zeeman slower described in Chapter 3. Under ideal conditions, the designed Zeeman slower is capable of decelerating atoms with initial velocities of up to 500 m/s, as previously noted. As the atoms approach a final velocity near zero, the probability distribution exhibits a pronounced peak around zero velocity. Within the velocity range of 0 to 500 m/s, the distribution shows no atoms, as these are decelerated by the Zeeman slower. However, atoms with velocities exceeding 500 m/s remain unaffected, as they fall outside the slower's operational range. This behavior is illustrated in Figure A.1.

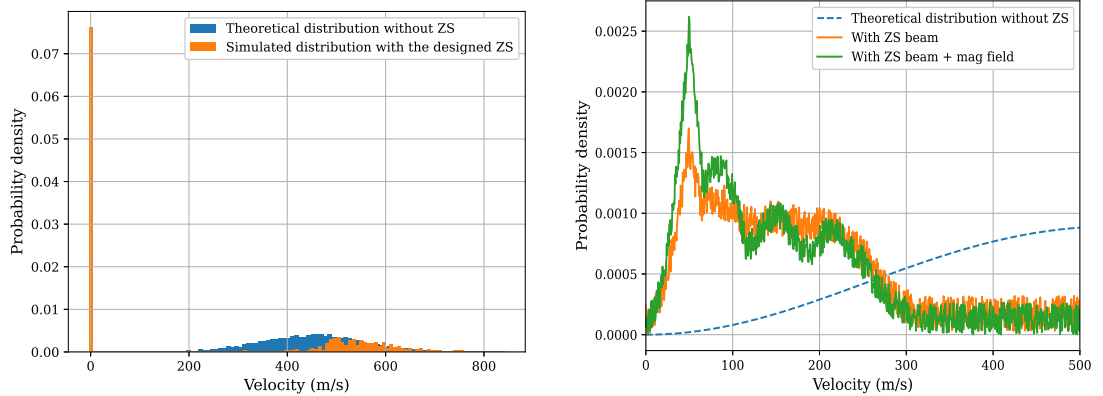


Figure A.1: Zeeman slower performance: The figure on the left hand side represents the simulated performance of the Zeeman slower where as the figure on the right hand side shows the experimental data of the performance.

Figure A.1 also presents experimental data illustrating the performance of the Zee-

man slower. This data was obtained through spectroscopy, which captures the Doppler shift and, consequently, provides information about the probability distribution of atoms across the velocity spectrum. As shown in Figure A.1, the experimental results deviate noticeably from the simulated predictions. Several factors could contribute to this discrepancy, including but not limited to the following:

The practical implementation of the Zeeman slower design is not without limitations, which affect its overall performance. One significant issue is the presence of stray magnetic fields, particularly the strong field generated by an ion pump positioned near the Zeeman slower, as shown in Figure 3.1. Additionally, due to space constraints, the Zeeman slower was designed with a shorter length, requiring a higher efficiency to achieve optimal operation - an efficiency level that may not be fully realized in practice (refer to 3.7).

The performance of the Zeeman slower is summarized in the table below. It shows the normalized number of atoms in different velocity regime.

Configuration	Velocity < 25 m/s	Velocity < 50 m/s	Velocity < 100 m/s
No ZS	1	8	63
ZS Beam	163	764	2108
ZS Beam + Mag field	168	1079	2822

Table A.1: Zeeman slower performance: Atomic population distribution at different velocity regime.

Appendix B

Appendix B

This section discusses the machine learning methods outlined in Chapter 4 for optimizing the blue MOT in more detail. As previously noted, Bayesian optimization has been implemented for the optimization procedure. Bayesian optimization is a powerful technique widely used as a machine learning tool for parameter optimization. It utilizes the celebrated Bayes' Theorem for continuous variables to optimize parameters efficiently.

As previously mentioned in Chapter 4, Bayes' Theorem states that

$$P(X|Y) \propto P(Y|X)P(X),$$

where $P(X|Y)$ is the posterior, $P(Y|X)$ is the likelihood, and $P(X)$ is the prior. Modeling $P(X)$ enables the optimization process by providing a probabilistic framework for exploring the parameter space.

During the optimization process, Bayesian optimization builds a surrogate model, whether a Gaussian Process or a neural network, to approximate the unknown objective function. The surrogate model is updated iteratively based on observations of the objective function at sampled points, with the posterior distribution $P(X|Y)$ providing a probabilistic estimate of the function across the parameter space.

In the Gaussian process method, the mapping X is modeled as

$$X \sim \mathcal{GP}(\mu(x), \sigma(x, x')),$$

where $\mu(x)$ is the mean function and $\sigma(x, x')$ is the covariance function. The key idea is that if the value of a parameter X at a specific point x is known, the values at nearby points x' are expected to be similar, with their relationship quantified by the covariance function $\sigma(x, x')$. This covariance function, often referred to as the kernel, encodes prior estimations about the smoothness and structure of the function being optimized.

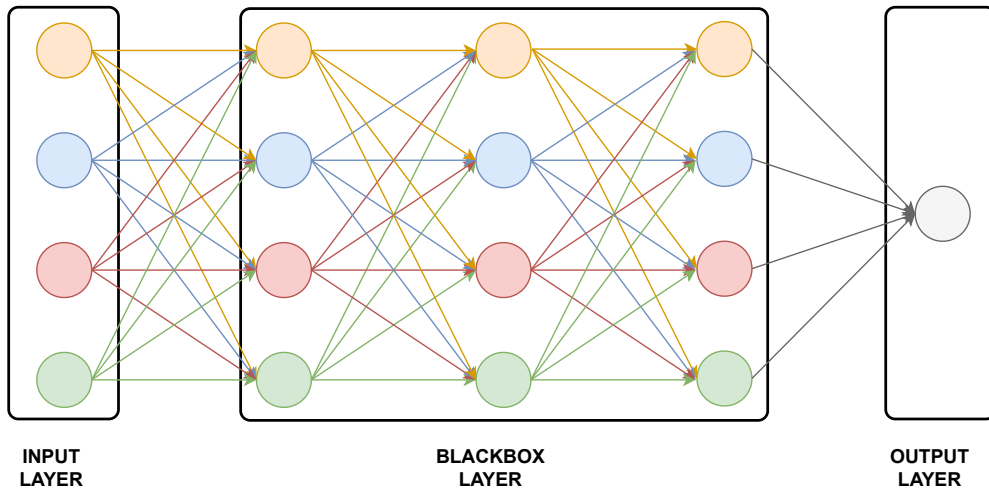


Figure B.1: Neural Network Architecture: An illustration of a neural network architecture featuring an input layer with 4 inputs, a hidden layer with 3 stages, each containing 4 neurons, and an output layer with a single output.

Alternatively, the surrogate model in Bayesian optimization can also be implemented using neural networks. Neural networks offer a flexible approach to model the mapping X , particularly in high-dimensional parameter spaces.

The architecture of the neural network for surrogate modeling often depends on the complexity of the problem and the dimensionality of the input space. Neural networks typically consist of an input layer, one or more hidden layers or black box layers, and an output layer (Figure. B.1). The hidden layers consist of multiple neurons, which are the fundamental units of computation in the network. Each neuron receives inputs from the previous layer, applies a linear transformation using weights and biases, and then passes the result through a non-linear activation function to introduce non-linearity into the model.

The number of neurons in each hidden layer determines the network's capacity to

learn complex patterns. The depth of the network, defined by the number of hidden layers, determines how hierarchical and abstract the learned representations can be. Shallow networks may fail to capture complex interactions, while excessively deep networks may suffer from increased computational cost.

Weights are parameters that define the strength of connections between neurons in adjacent layers. During training, these weights are adjusted iteratively to minimize the error between the predicted and actual outputs. Biases are additional parameters added to each neuron to shift the activation function, providing additional flexibility to the model.

The primary advantage of Bayesian optimization lies in its efficiency, requiring much less computational cost than traditional methods such as grid search or random search. By intelligently exploring the parameter space, it serves as a highly effective tool for experimental optimization.

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