Investigation of in-scale and mean flow interactions of MRG waves using spectral energetics framework

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Dedicated to Amma, Achan and Devi

Certificate

This is to certify that this dissertation entitled '**Investigation of in-scale and mean flow interactions of MRG waves using spectral energetic framework**' towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by **Bharath Krishnan U** at Indian Institute of Science Education and Research under the supervision of **Dr Sarvesh Kumar Dubey**, Assistant Professor, Centre for Atmospheric Sciences, IIT Delhi, during the academic year June 2022 to April 2023.

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Declaration

I hereby declare that the matter embodied in the report entitled '**Investigation of in-scale and mean flow interactions of MRG waves using spectral energetic framework**' are the results of the work carried out by me at the department of Earth and Climate Science, Indian Institute of Science Education and Research, Pune, under the supervision of Dr Sarvesh Kumar Dubey and the same has not been submitted elsewhere for any other degree.

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Contents

List of Figures

List of Tables

Abstract

Mixed Rossby Gravity (MRG) waves are westward propagating equatorial waves that explain a significant amount of tropical variability. There exists a strong coupling between mean background state and MRG activity and the thesis focuses on exploring the qualitative and quantitative aspects of this MRG-mean flow interaction. Scale energetics in the frequency as well as the wavenumber domain is used to study the out-of-scale (in our study, energetic interactions with mean flow) and in-scale (energy exchange within a scale) interactions of MRG over the Pacific domain and a zonal belt around the globe extending 15∘ south and north of the equator. Horizontal eddy KE over the tropical Pacific domain is found to be higher for strong MRG seasons at high and low altitudes. The in-scale energetics (baroclinic conversion) in the frequency domain indicate that the APE and KE exchange on the MRG scale is not only pivotal for its maintenance but also affects its strength. Mean flow interactions of MRG scale in frequency domain shows that MRG scale KE generation is limited to eastern Pacific during weak seasons while during strong seasons, KE flows from mean flow to MRG scale in the eastern and central Pacific as well. Scale energetic in the wavenumber domain showed no relationship with that in the frequency domain. For strong seasons, higher KE was observed in the mid-altitudes about the equator and mean flow interactions showed higher MRG scale KE generation in similar pressure levels.

1. Introduction

Equatorial waves are atmospheric disturbances trapped in the vicinity of the equator. These waves decay quickly in the meridional direction away from the equator and propagate in eastward and westward along the tropics. They are elementary components of the tropical atmosphere and therefore crucial for understanding tropical variability. They can be described as solutions of shallow water equations, trapped around the equator and propagating vertically and zonally. Such waves can be distinguished by their zonal wavenumber, frequency, meridional mode (n) and equivalent height (Wheeler and Kiladis, 1999). Rossby waves are large-scale westward propagating disturbances, driven by Earth’s rotation, seen in the midlatitude and tropical atmosphere while inertia-gravity waves are faster disturbances owing their existence to buoyancy and inertial effects. Mixed Rossby-Gravity (MRG) waves are special equatorial waves, behaving like both low-frequency Rossby waves and high frequency Inertio-Gravity (IG) waves (Matsuno, 1966). Even though westward propagating MRG waves appear continuous with n=0 eastward propagating Inertio-Gravity (EIG) waves in the wavenumber-frequency phase space, they are dynamically distinct from the latter (Matsuno, 1966; Wheeler and Kiladis, 1999; Kiladis et al., 2009).

Convectively coupled MRG waves are westward propagating disturbances observed in dynamic as well as convective fields over the tropics. They are symmetric about the equator, and exhibit a baroclinic structure in circulation(Matsuno, 1966; Wheeler and Kiladis, 1999). Other defining characteristics of MRG waves include a periodicity of 3 to 6 days, wavelength ranging from 2,000 to 10,000 km, and an eastward group velocity of ~5 ms^{-1}. Convection due to MRG waves was found to be dominant over the tropical Pacific region and during late boreal summer and fall. Observations of vertical structure indicate a west-east oriented tilt in the lower troposphere and an east-west tilt in the stratosphere (Hendon and Liebmann, 1991; Takayabu, 1994; Roundy and Frank, 2004; Chen and Huang, 2009; Au-Yeung and Tam, 2018; Suhas et al., 2020). Tropical depression (TD) - type disturbances, having similar frequency of convectively coupled MRG waves, can be distinguished, the former being confined to western Pacific and distinct in the space-time spectra. Circulation structures obtained from filtered wind data showed westward propagating symmetric gyres centred on the equator, eventually moving northwestward. Dry, fast MRG waves are dominant in the upper troposphere and lower stratosphere while those coupled with convection are observed to have shallower equivalent depths and generally propagate in the lower troposphere (Wheeler and Kiladis, 1999; Wheeler et al.,2000; Kiladis et al., 2009).

Convectively coupled MRG waves explain a significant amount of tropical variability. They are known to modulate the Madden-Julian Oscillation (MJO), tropical depressions (TD), easterly waves, tropical cyclones, and Quasi-Biennial Oscillation (QBO) (Takasuka et al., 2018). Observational and theoretical studies have shown that the vertically propagating MRG waves play a significant role in forcing semi-annual oscillation, QBO during its descending easterly phase, etc. (Lindzen and Holton, 1968; Holton and Lindzen, 1972; Plumb, 1977). Strong MRG wave forcing generated from barotropic instability at later stages of QBO, was also found to favor QBO disruption in a recent case study (Kang and Chun, 2021). MRG wave contraction generates eddy kinetic energy (EKE) which in turn helps in the initiation and propagation of MJO. Convective development of MJO is also enhanced by mid-tropospheric moistening due to shallow MRG circulations (Takasuka et al. 2018, 2019). There exists plenty of evidence showing transition of MRG waves into TD-type disturbances in Western North Pacific regions and thereby tropical cyclones. The anti-clockwise MRG gyres tend to move away from the equator and become cyclonic vorticity centers preferred for cyclogenesis. Such transition from MRG scale to other scales is considered to be facilitated by barotropic conversion (Dickinson and Molinari, 2002; Frank and Roundy, 2006; Kiladis et al.,2009). These studies also brought out a strong dependence on background mean flow and similar transitions. Studies suggest that accumulation and transfer of energy occurs from summer mean flow to synoptic scale over Western-North Pacific. For instance, the ENSO phase was found to play a significant role in MRG-TD transitions (Chen and Huang, 2009; Wu et al., 2014). EKE budget studies by Wu et al. (2014) suggests strong correlation between ENSO phase and strengthening of TD-MRG type disturbances.

Thus, convectively coupled MRG waves facilitate scale interactions between mean flow and other tropical variabilities. On detailed study on the inter-annual variability and background state of convectively coupled MRG waves, it was found out that the MRG wave amplitudes were closely related to the Pacific mean background state (stronger during La Nina like state and weaker during El Nino like state) (Suhas et al., 2020).

Several studies have documented the spatial and temporal structures of dry as well as convectively coupled MRG waves. Similarly, the domain of activity, seasonality, propagation characteristics, and vertical structure have also been investigated. Available potential energy-kinetic energy conversions and wave-mean flow interactions have been observed to influence the spatio-temporal patterns and energetics of MRG waves as well as other synoptic disturbances (Lau and Lau, 1990; Wu et al., 2014; Au-Yeung and Tam, 2018). Energetics studies on MRG waves conducted by Au-Yeung and Tam (2018) brought out the significance of barotropic conversion in its maintenance and also the role of upper level waves in activating low level waves over the tropical Pacific. In Suhas et al. (2020), strong and weak MRG seasons were identified based on MRG wave amplitudes defined from principal component analysis of OLR data. Further, seasonal composites of OLR, SST and wind anomalies provided evidence that the seasonal background states were significantly different during strong and weak seasons. Thus, there has been plenty of qualitative evidence for the strength of convectively coupled MRG waves being dependent on mean background state (of tropical Pacific) (Suhas et al., 2020).

Thus, studying the energetics of MRG waves is key in understanding the mechanism of these waves, as well as their dependency on background state. Previous findings call for additional investigation to comprehend the physical processes that underlie the role of background states in modulating the strength of convectively coupled MRG waves. Thus, there arises a need for a scale interaction study of MRG waves in a fully multiscale framework. Here, interaction of MRG scale disturbances with mean flow is inspected using a framework of scale interactions (Saltzman, 1957;

Hayashi, 1980; Krishnamurti et al.. 2016; Dubey et al., 2018). Saltzman (1957) made use of Fourier transforms along with the convolution theorem to study nonlinear kinetic energy exchanges in the wavenumber domain. Hayashi (1980) modified this approach and used the cross-spectral approach to formulate a compact framework to compute nonlinear kinetic energy and available potential energy exchanges in the frequency as well as the wavenumber domain. While Saltzman (1957) used a triple product approach entirely in the wavenumber domain, Hayashi (1980) used a grid space approach to avoid any convolution in the phase space. The across-scale and in-scale energetics are discussed in this work as a way to better comprehend the MRG.

Available potential energy (APE) is a notion that has significant application in the study of atmospheric energetics. Conversion between kinetic energy (KE) and APE can be perceived as a necessary condition for energy conversion (Lorenz, 1955; Pfeffer, 1957). Scale interaction studies in frequency or wavenumber domain, which includes in-scale and across-scale kinetic energy and available potential energy exchanges, can help us prepare a complete energy budget. In-scale processes refer to energy transfers corresponding to quadratic nonlinearity and explain energy exchange within the same scale. It thus describes processes like generation and transfer of energy from one form to another. Across-scale or out-of-scale energetics correspond to cubic or higher order nonlinearity and explain exchange of kinetic or available potential energy between different scales of motion (Krishnamurti et al., 2016, 2017; Dubey et al., 2018). Scale energetics have been used before in the frequency domain for studying energetics of MJO, hurricanes, etc. and have helped to understand their interaction with synoptic and other scales (Krishnamurti et al., 2003, 2017; Dubey et al., 2018). Energetics for the Northern hemisphere was examined in the frequency domain by Sheng and Hayashi (1990). Krishnamurti et al. (2016) uses the framework to distinguish and understand the energetics of hurricanes and MJO and to study the various factors behind their mechanism and maintenance. Dubey et al. (2018) employs this scale energetic framework in the frequency domain along with DYNAMO observations to study the convective features and maintenance of MJO. These studies could successfully explain and quantify the energy exchanges between various scales of motion. A recent study by

Prasanth et al. (2020) utilises the framework in the wavenumber domain to study the energetics of asymmetries of tropical cyclones (TC) to see their impact in the intensity changes of TC. The current study includes scale energetic approaches for MRG scale in the frequency domain, as well as in the wavenumber domain. In our study, we are using a co-spectra method, as described in Hayashi (1980). This method allows us to study energetics from a regional domain which helps us to calculate local contributions to the scale of interest.

In our study, we try to quantitatively distinguish MRG weak and strong seasons in terms of kinetic energy, mean flow interactions and in-scale energy conversions. We try to check the distribution of sources and sinks of MRG scale KE during weak and strong seasons. The relative contribution of baroclinic and mean flow interactions towards MRG scale is investigated over the Pacific domain. The altitude-wise contribution is also looked upon. Further, mean flow interactions are studied in the wavenumber domain as well. Longitude and latitude wise KE and energy conversion distribution are also calculated and visualised over the MRG strong domain. We here try to address these questions using a spectral framework proposed by Hayashi (1980). This method has been used to study MJO, hurricanes, etc. but this is a novel attempt on MRG waves. The method is employed both in the frequency and wavenumber domains in our study.

The following section describes the methodology and the data used. Scale energetics in the frequency and wavenumber domain are explained along with their brief formulation. Following sections summarise the observations made from this study. Section 3.1 compares the power spectra for meridional wind for strong and weak seasons. Section 3.2 describes some results on the kinetic energy, in-scale energetics and mean flow interactions in the frequency domain. The section 3.3 summarises the studies conducted in the wavenumber domain. Final section lists out the conclusions of the results and their implications.

2. Data and Methodology

Convectively coupled MRG waves are inspected using daily outgoing longwave radiation (OLR) data obtained from the National Oceanic and Atmospheric Administration (NOAA), with 2.5° x 2.5° horizontal resolution, for the period 1 January 1979 to 31 December 2019 (Liebmann and Smith, 1996). The European Center for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-5) 6 hourly (analyses at 0000, 0600, 1200 and 1800 UTC), 0.25° x 0.25° horizontal resolution temperature, vertical pressure velocity, meridional and zonal wind data at 22 pressure levels (from 100 hPa to 1000 hPa) for 5 weak and 5 strong MRG seasons (August to November months) are used to study the energetics of MRG scale flow.

MRG waves were seen to show strong seasonality with nearly 50% activity during August to November (boreal fall and summer). 5 strong and 5 weak seasons between 1979 and 2019, identified by Suhas et al. (2020) are used in this study. The spatial domain used is the Pacific domain where the MRG waves are more active (Suhas et al., 2020). The MRG convective activity was also noted to be the strongest in the troposphere, hence we concentrate on energetic studies between pressure levels 100 hPa and 1000 hPa.

Strong MRG seasons	Weak MRG seasons
1981	1988
1995	1990
2000	2002
2014	2011
2017	2015

Table 1*: 5 years each of strong and weak MRG activity classified by Suhas et al. (2020). The MRG waves were isolated by an approach that combines principal component (PC) analysis and wavenumber-frequency filtering. MRG seasonal amplitudes were defined from the variance of MRG wave indices and thereby 5 strong and 5 weak seasons were identified between 1979 and 2019 (Suhas et al., 2020).*

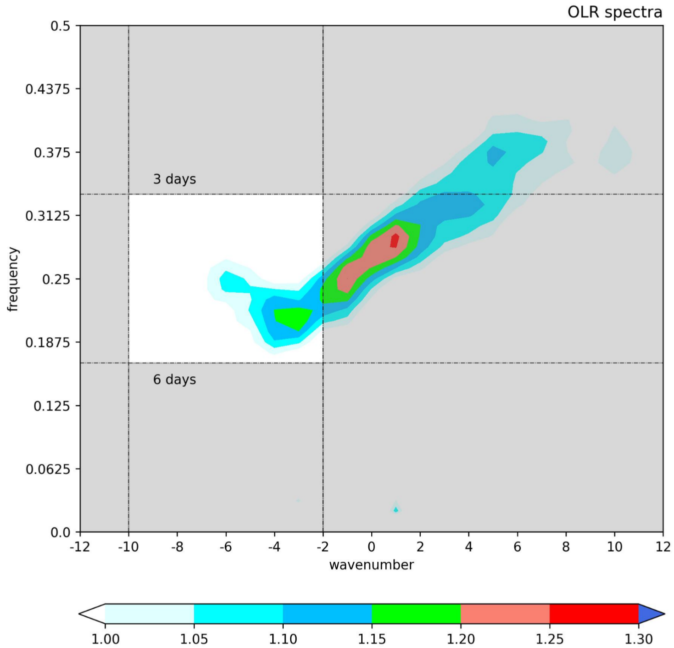

Figure 1*: The wavenumber-frequency power spectra of antisymmetric component of OLR for the years 1979-2019, normalised by a smooth red background. Spectra is latitudinally averaged between 25 S and 25 N. Statistically significant peaks corresponding to MRG scale motion are left unshaded (period: 3 to 6 days and wavenumbers: -2 to -10). Frequency is in cycles per day (cpd).*

MRG waves in convection were identified by wavenumber-frequency power spectral analysis of daily OLR data from 1979 to 2019 (Wheeler and Kiladis, 1999; Suhas et al., 2020). OLR data centred around the equator, is separated into symmetric and anti-symmetric components and Fourier transformed in longitude and time. The spectra is normalised by the smooth red background noise to bring out the significant peaks. MRG spectral power peaks can be observed in the antisymmetric OLR spectra. This implies that the convection corresponding to MRG activity is antisymmetric about the equator. Further, from figure 1, we can note that the highest spectral power related to westward travelling mixed Rossby Gravity waves is from wavenumbers -2 to -10 and period 3 to 6 days. These wavenumber and frequency limits are employed in our study involving the energetics of MRG scale flow. We use wavenumbers from 2 to 10 in our wavenumber domain studies, where we analyse energetics of westward as well as eastward propagating disturbances of those scales.

2.1 Scale interactions in the frequency domain

Hayashi (1980) calculated total kinetic energy interactions of a scale of interest with all other scales. The modified approach adopted in our study helps us to compute energy exchanges between two scales of our choice. A cross bi-spectral methodology, very similar to co-spectra, is employed here to explore the scale energetics in the frequency domain. Below mentioned are the equations and procedure used for in-scale energetics: energy exchange between eddy Available Potential Energy (APE) and eddy Kinetic Energy (KE) for a scale n ⟨*A(n).K(n)*⟩ and out-of-scale energetics for KE: eddy KE exchange between mean flow and scale n ⟨*K(0).K(n)*⟩. To study the interaction between 2 different scales, Hayashi extended the work of Saltzman (1957) on scale interactions in the wavenumber domain.

Any variable can be discrete Fourier transformed, i.e. for any variable u,

$$u_n = \sum_{m=0}^{M-1} U_m \, e^{2\pi \, i \, m n / N}$$

Fourier coefficient for frequency mode *m* is denoted by U_m, *N* is the number of data points and *M* is the Nyquist frequency given by $(1/2\tau)$ where τ is the sampling period.

Sample frequency co-spectra (Jenkins and Watts, 1968) can be defined as twice the real part of their cross-spectrum:

$$P_n(u, v) = 2\Re(U_n^* V_n)$$

where asterisk denotes complex conjugate. For Nyquist frequency and n=0, co-spectra should be halved. If a variable v is a product of time series b(t) and c(t), then the convolution theorem defines its Fourier transform as:

$$V_n = 2 \sum_m B_{n-m} C_m$$

where B and C are Fourier transforms of b and c respectively. Substituting this in the previous equation gives:

$$P_n(u, bc) = \Re \sum_m 4U_n^* B_{n-m} C_m = \sum_m R_{n,m}(u, bc)$$

where

$$R_{n,m}(a, b, c) = 4\Re(A_n B_{n-m} C_m)$$

denotes frequency cross bi-spectra of time series a(t), b(t), and c(t).

Given u(t) and v(t), i.e. the zonal and meridional wind velocities, the horizontal kinetic energy per unit mass can be calculated by:

$$K(n) = \frac{1}{2}[P_n(u, u) + P_n(v, v)]$$

Spectral KE equation in the frequency domain can be obtained by taking the co-spectra of zonal and meridional momentum equations and adding them. The resulting equation can be expressed compactly as:

$$0 = \langle K \cdot K(n) \rangle + \langle A(n) \cdot K(n) \rangle + D(n) + F(n)$$

where the first term is the non-linear kinetic energy spectra, the second term denotes the baroclinic conversion, *D(n)* denotes the frictional dissipation and *F(n)* is the convergence of energy flux by pressure gradient force.

2.1.1 In-scale energetics

In-scale energetics in the frequency domain or the energy exchange between eddy available potential energy and eddy kinetic energy for scale n is calculated using,

$$\langle A(n).K(n) \rangle = -P_n(\alpha, \omega),$$

where $P_n(\alpha, \omega)$ is the co-spectrum of pressure velocity ω and specific volume α. Here, we should select a scale of interest by assigning a range of values to n in the frequency domain. We should calculate the above term for all the frequencies of interest and add them up to get the gross baroclinic conversion for MRG scale flow (period ranging from 3 to 6 days).

2.1.2 Out-of-scale energetics

In our study, we are calculating the eddy kinetic energy transfer between MRG scale and mean flow. The nonlinear kinetic energy transfer spectra can be written as:

$$\langle K.K(n)\rangle = \langle K(0).K(n)\rangle + \sum_{m(m\neq 0)} \langle K(m).K(n)\rangle$$

$\langle K(m).K(n)\rangle$ is the nonlinear KE exchange between frequency n and with some other frequency m. Co-spectra puts a constraint on the possible values for m, i.e.

$$n = (n \pm m) \mp m$$

$\langle K(0).K(n)\rangle$ is the nonlinear kinetic energy transfer between frequency n and the mean flow or the mean-wave interaction (Hayashi, 1980; Sheng and Hayashi, 1990). We have the KE transfer to MRG scale from all scales (including mean flow):

$$\langle K.K(n)\rangle = \sum_m - \Bigg[R_{n,m}\left(u, u, \frac{u_\lambda}{a\cos\phi}\right) + R_{n,m}\left(u, v, \frac{u_\phi}{a}\right) + R_{n,m}(u, w, u_p) + R_{n,m}\left(v, u, \frac{v_\lambda}{a\cos\phi}\right) + R_{n,m}\left(v, v, \frac{v_\phi}{a}\right) + R_{n,m}(v, w, v_p) - \frac{\tan\phi}{a}(R_{n,m}(u, u, v) - R_{n,m}(v, u, u))\Bigg]$$

R denotes the frequency cross bi-spectra, subscripts denote derivatives with respect to respective variables. We can also calculate the nonlinear KE transfer between MRG scale and all scales (excluding mean flow):

$$\langle k(m).K(n)\rangle = -\Bigg[R_{n,m}\left(u, u', \frac{u'_\lambda}{a\cos\phi}\right) + R_{n,m}\left(u, v', \frac{u'_\phi}{a}\right) + R_{n,m}(u, w', u'_p) + R_{n,m}\left(v, u', \frac{v'_\lambda}{a\cos\phi}\right) + R_{n,m}\left(v, v', \frac{v'_\phi}{a}\right) + R_{n,m}(v, w', v'_p) - \frac{\tan\phi}{a}(R_{n,m}(u, u', v') - R_{n,m}(v, u', u'))\Bigg]$$

where primes denote deviation from mean flow.

Nonlinear KE interactions between MRG scale and mean flow, $\langle K(0).K(n)\rangle$ can be calculated by subtracting $\langle K(m).K(n)\rangle$ from $\langle K.K(n)\rangle$. An important feature of the co-spectra method in the frequency domain is that we can study energy contributions to our scale of interest from a regional domain. This helps us to calculate local contributions for the energy exchange. While mass integrating the energetics terms over the global domain, there are divergence of flux terms that vanishes and reduces the computational cost.

2.2 Scale interactions in the wavenumber domain

Scale energetics in the wavenumber domain is similar to that in the frequency domain except here we utilise a decomposition according to spatial scales. The formulation of various energetics terms are similar to that discussed in the previous section. This framework was introduced by Saltzman (1957) to study global energetics of disturbances of different spatial scales. Since we are doing Fourier transforms along longitude alone, we face some limitations in filtering the MRG wave. We are able to study disturbances with the spatial scales of MRG waves but not the direction of propagation. Here, we study energetics of disturbances with wavenumber 2 to 10 but cannot distinguish between westward and eastward propagating. We can later observe that in our case, spectral peaks corresponding to eastward propagating perturbations with similar spatial scale to that of MRG are much weaker than the spectral peaks of westward propagating disturbances.

3. Results and Discussion

3.1 Power Spectral Analysis

Spectral analysis of the meridional component of wind velocity at 850 hPa and 200 hPa was performed to investigate its space-time properties in the western Pacific. Meridional wind data around the equator across the globe for the MRG season, i.e. August to November months, was used for the power spectral analysis. The spectra was obtained by averaging the meridional wind spectra from 15 S to 15 N. The final spectra was obtained by averaging the seasonal power spectra for all strong and weak years. From figure 2, we can point out that the strongest activities are found in the symmetric spectrum around wavenumbers -2 to -10 and period 3 to 6 days. This pattern was observed irrespective of the altitude or season. We can see that symmetric v wind spectra at 850 hPa and 200 hPa are consistent with the antisymmetric OLR spectrum. This suggests some coupling between circulation and convection associated with these waves. So spectral studies of wind velocities can be used to study MRG scale disturbances.

Spectral power confined between wavenumbers -10 to -2 and period 3 to 6 days of meridional wind spectra was considered to be of MRG scale. At 850 hPa, we can see that more spectral peaks were observed for strong seasons. The spectral power was also seen to be more spread in the phase space. Similar features were also observed for the strong and weak v wind spectra at 200 hPa. While comparing the power spectra with altitude, we can see more prominent peaks at lower altitudes (850 hPa) than those at higher altitudes (200 hPa). No significant spectral peaks were observed in the anti-symmetric spectra of meridional wind. The widespread spectral peaks imply MRG activity of various spatial and temporal scales during strong seasons, while for weak seasons, the diversity of the spatial and time scales of MRG activity is lesser. More spectral peaks suggest strong circulation features of different wavenumber and frequency.

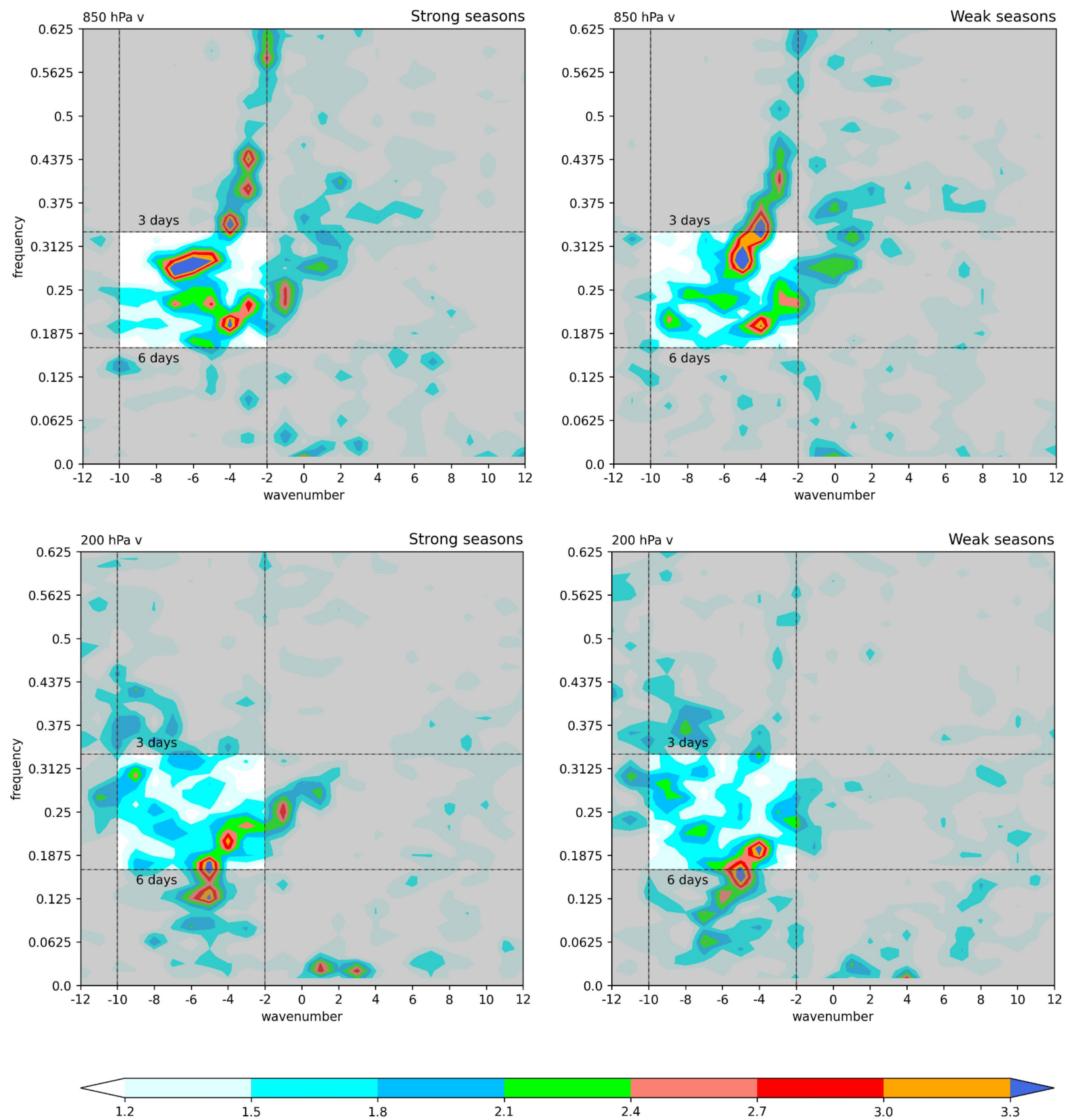

Figure 2*: The antisymmetric wavenumber-frequency power spectra of meridional wind for weak and strong seasons, at 850 hPa (top panel) and 200 hPa (bottom panel), normalized by smooth red background. The area left unshaded is the MRG wave domain (period: 3 to 6 days and wavenumber: -2 to -10). Frequency is in cycles per day(cpd). Spectra is latitudinally averaged from 15 S to 15N. Statistically insignificant peaks are shown for the strong (left) and weak (right) seasons.*

3.2 Frequency domain

3.2.1 Kinetic energy

Quantifying and studying the kinetic energy (KE) of the scale of interest is important to understand the energetics of that scale. It is also essential to understand the variation of the magnitude and distribution of KE over the three dimensional space. Here we make use of the meridional and zonal wind to calculate the horizontal KE of MRG scale flow over the MRG active domain. The period of MRG waves is 3 to 6 days, most active over the tropical Pacific region and the MRG season being August to November months (Suhas et al., 2020).

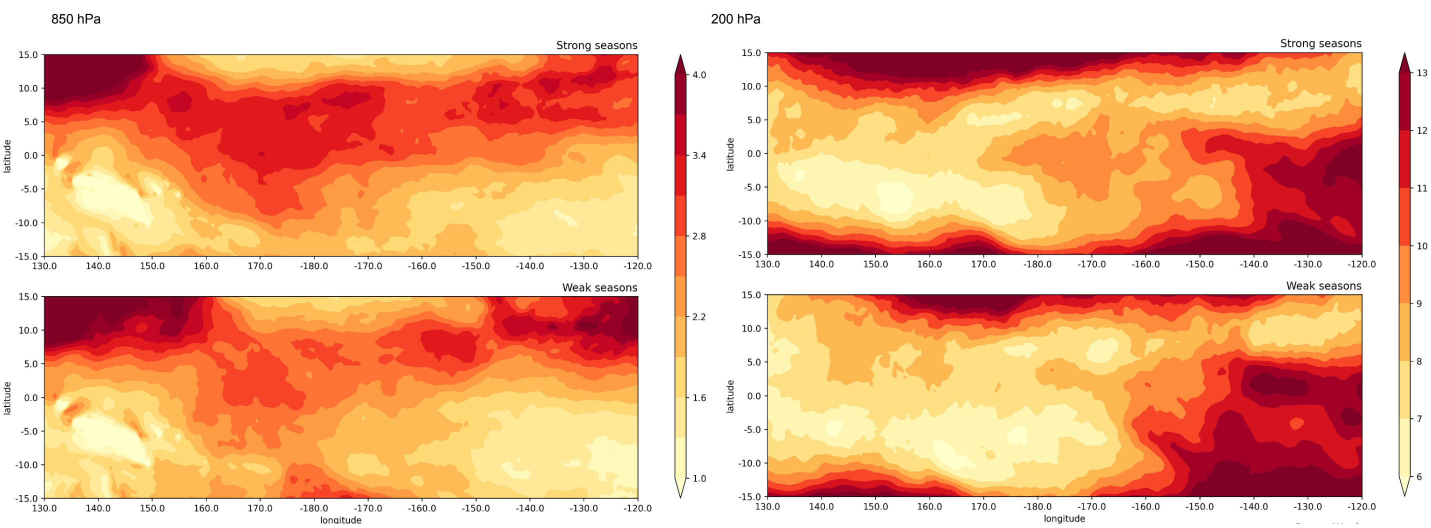

***Figure 3**: Eddy horizontal kinetic energy per unit mass patterns of MRG scale at 850 hPa (left panel) and 200 hPa (right) for strong (top panel) and weak (bottom) seasons over tropical Pacific domain (130 E to 120 W and 15 S to 15 N). Composite seasonal KE summed over all MRG scale frequencies. KE measured in J m kg^{-1}.*

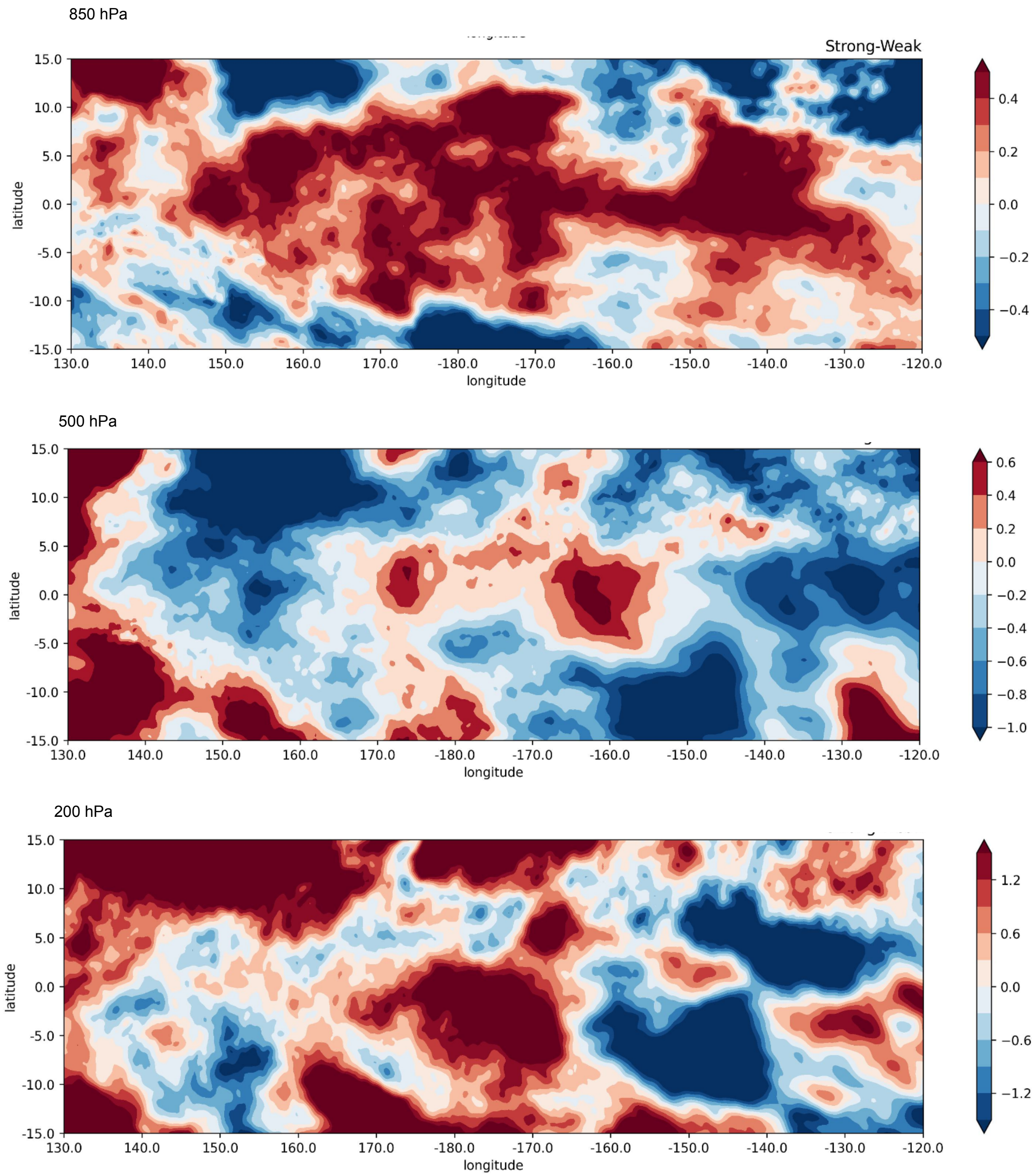

Figure 4*: Strong-Weak plots of MRG scale eddy horizontal KE per unit mass at 850 hPa (top), 500 hPa (middle), and 200 hPa (bottom) over tropical Pacific domain (130 E to 120 W and 15 S to 15 N). Composite seasonal KE summed over all MRG frequencies are obtained for strong and weak seasons and their difference is calculated at each altitude. KE measured in J m kg^{-1}.*

Strong-weak plots of MRG scale KE are calculated and plotted at different pressure levels in figure 4. Upon comparison, we can see that the KE of MRG scale at 850 hPa is higher for strong seasons over the entire tropical Pacific domain. It can be seen that for strong seasons, the central Pacific region has nearly 20% higher KE than that for weak seasons. This result is as expected as the strong MRG seasons are supposed to have higher MRG scale horizontal KE. We can see that the KE pattern is similar for strong and weak seasons. At 200 hPa, the KE is higher for strong seasons in the central Pacific by about 20%, while in the eastern Pacific the KE of MRG scale for strong seasons is less than that for weak seasons by about 10%. Similar to 850 hPa, the pattern of KE is similar for both strong and weak seasons. While comparing the KE at 850 hPa and 200 hPa, we can see that the magnitude is higher at 200 hPa. At higher altitudes, due to lesser frictional dissipation the wind is stronger which can explain the horizontal KE being nearly double that at lower altitudes. Further, we can also observe KE peaks away from the equator at higher altitudes all over the Pacific domain. At 500 hPa, it was interesting to note that the horizontal KE for strong seasons was less than that for weak seasons in most parts of the domain except central Pacific. Studying the in-scale or across-scale energy contributions might explain why this pattern is observed.

KE variations with latitude and longitude can be seen in figure 5. Here, the KE was mass integrated over western tropical Pacific from 130 E to 170 W. At 850 hPa, we can see that the KE is higher for strong seasons at all latitudes from 10 S to 10 N, while at 200 hPa the KE for strong season is less than that of weak seasons around the equator while away from the equator, strong seasons have higher MRG scale KE. The distribution of KE with latitude can also be noted to be different, with a more or less symmetric distribution at 200 hPa and steadily declining distribution at 850 hPa. In figure 5b, the mass integrated KE from 10 S to 10 N was calculated over the western tropical Pacific. At 850 hPa, strong seasons showed higher KE for all longitudes. At 200 hPa, western domain showed lower KE for strong seasons while towards the central Pacific, the KE for strong seasons tends to increase. The central Pacific has higher KE than western Pacific for strong as well as weak seasons at both pressure levels.

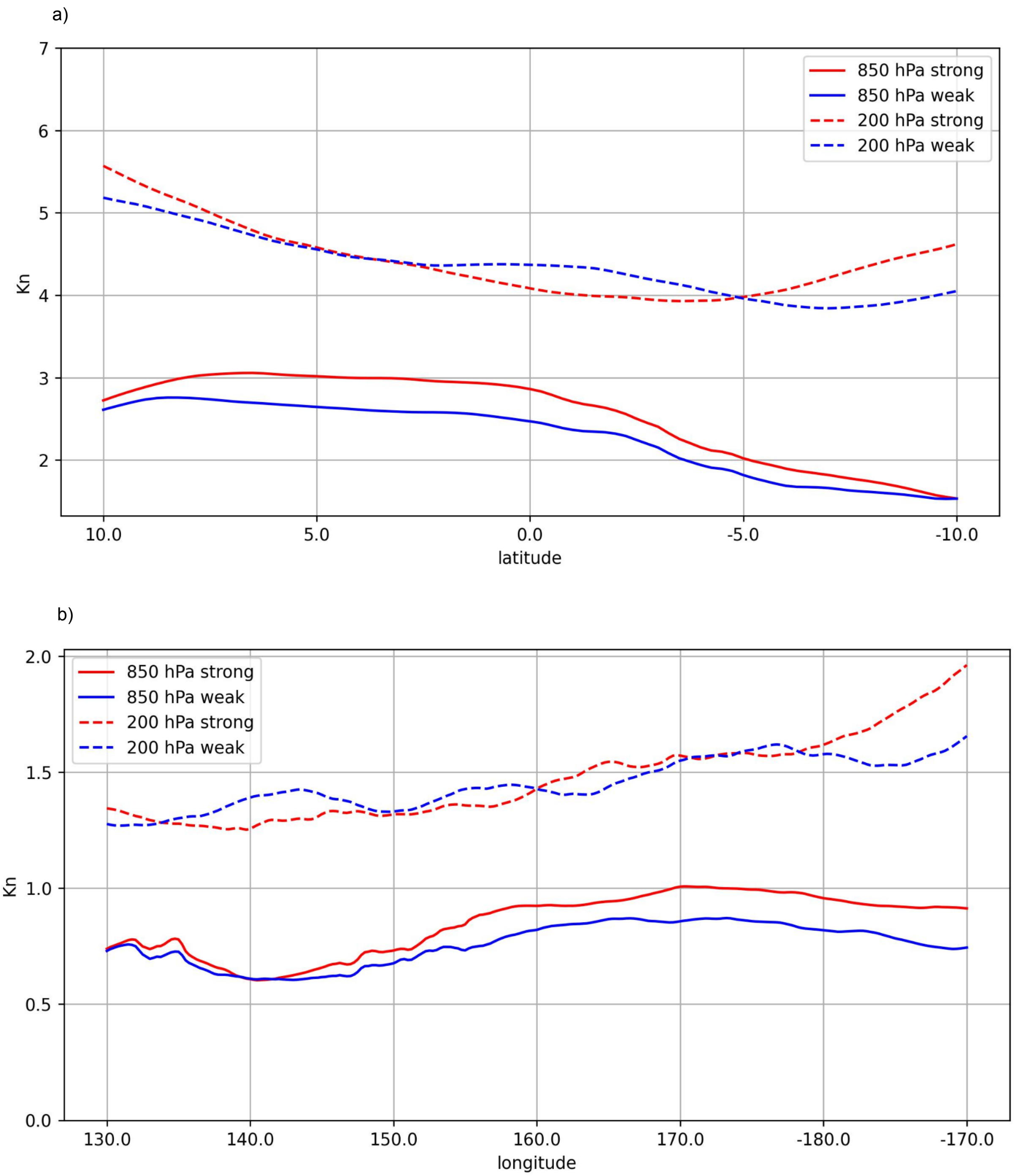

Figure 5*: Eddy horizontal KE for MRG scale per unit mass at 850 hPa and 200 hPa over western tropical Pacific integrated along a) longitudes from 130 E to 170 W and b) latitudes from 10 S to 10 N. Composite seasonal KE summed over all MRG frequencies obtained for strong and weak seasons. KE measured in J m kg^{-1}.*

3.2.2 In-scale processes

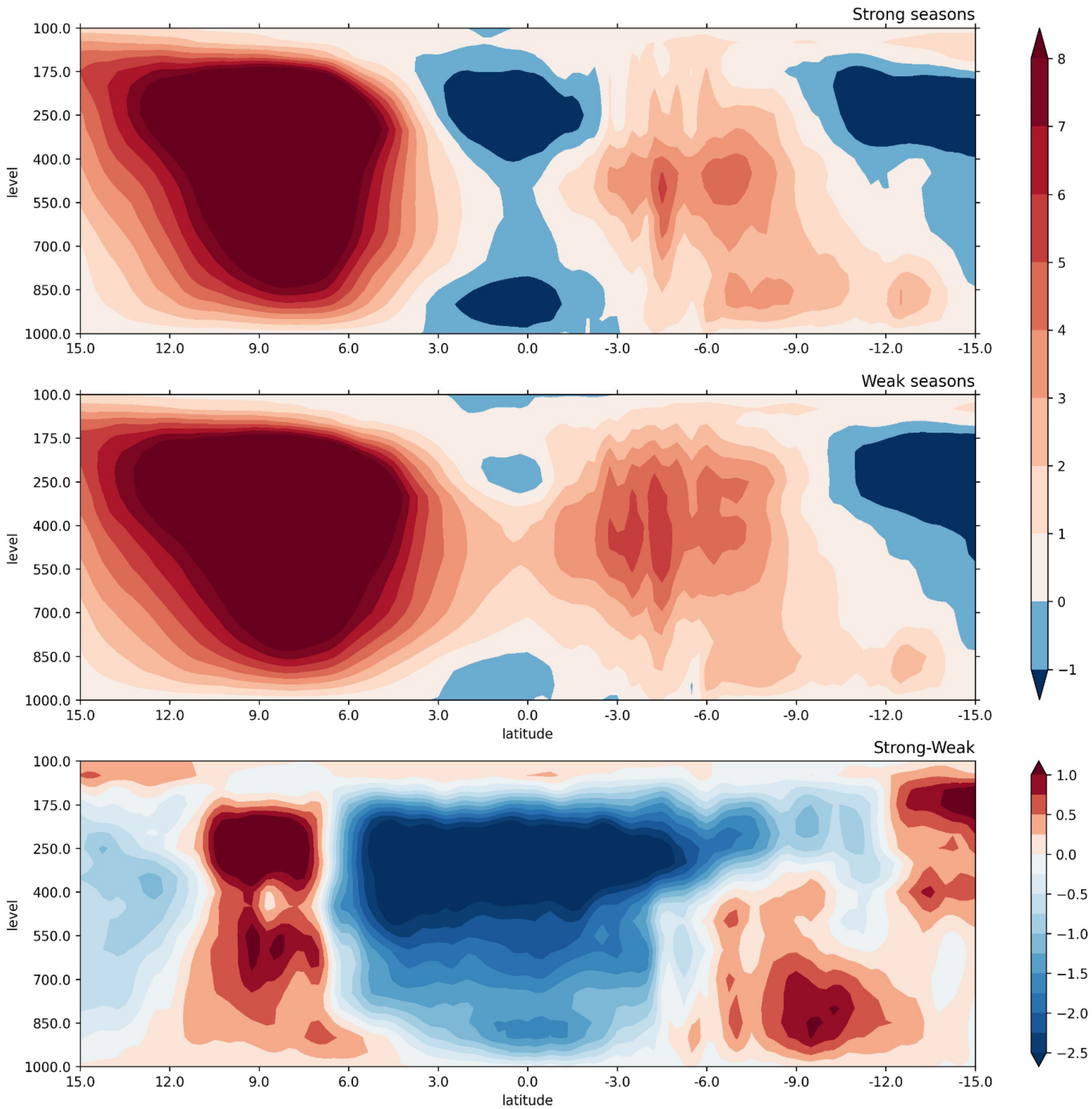

***Figure 6**: Baroclinic energy exchange between eddy APE and eddy KE ($A(n).K(n)$) for MRG scale, in the frequency domain, zonally integrated over western tropical Pacific (130 E to 170 W) for strong seasons (top), weak seasons (middle) and their difference (bottom).The spectra summed over all MRG frequencies. Energy transfer measured in W m kg^{-1}. The levels are in hPa.*

Figure 6 shows the in-scale energy exchange between Available Potential Energy (APE) and KE for MRG scale. Baroclinic energy transfer between APE and KE is integrated over the tropical western Pacific (130 E to 170 W). We can see clear differences between the baroclinic conversion during strong and weak seasons. Strong source of KE was observed north of the equator for the strong as well as weak seasons. A weaker source of KE was observed towards the south of the equator. Around the equator, a sink of KE was observed at low and high altitudes. Stronger sink was observed for strong MRG seasons which implies stronger conversion of MRG scale KE to APE. From the strong-weak plots, we can see that the source north of the equator is stronger for strong seasons. We can further observe that the source of kinetic energy is also stronger towards the south of the equator for strong seasons. When examining the distribution of baroclinic conversion with altitude, we can note the most variation between strong and weak seasons around 500 hPa. Here, for weak seasons, the sink of energy disappears and instead a relatively strong source of KE is present. Next, we compare the baroclinic conversion at 500 hPa for weak and strong seasons.

In figure 7, we can see significant differences in magnitude as well as regional distribution of baroclinic conversion for strong and weak seasons. For both seasons, we can see a source of KE in the western Pacific while in the eastern Pacific a weaker sink is observed about the equator. Towards the north of the equator, a source of MRG scale KE is observed all across the Pacific domain for both seasons. The difference is observed mainly in the central Pacific. For strong seasons, the sink is seen to be more spread towards the central Pacific while for weak seasons, the sink of KE is limited to the eastern domain and a weak source is present in the central region. From the strong-weak plots, we can see that in the western Pacific, the source of MRG scale KE is stronger in strong seasons by about 30%. On further investigation, we can see that the sink located in the eastern Pacific is weaker for strong seasons. From the strong-weak plots, the sink for strong seasons is found to be weaker than that for weak seasons by roughly 30%. This source-sink distribution was also found to be most prominent around 500 hPa. This might explain why MRG scale KE was less for strong seasons just around the central Pacific at 500 hPa.

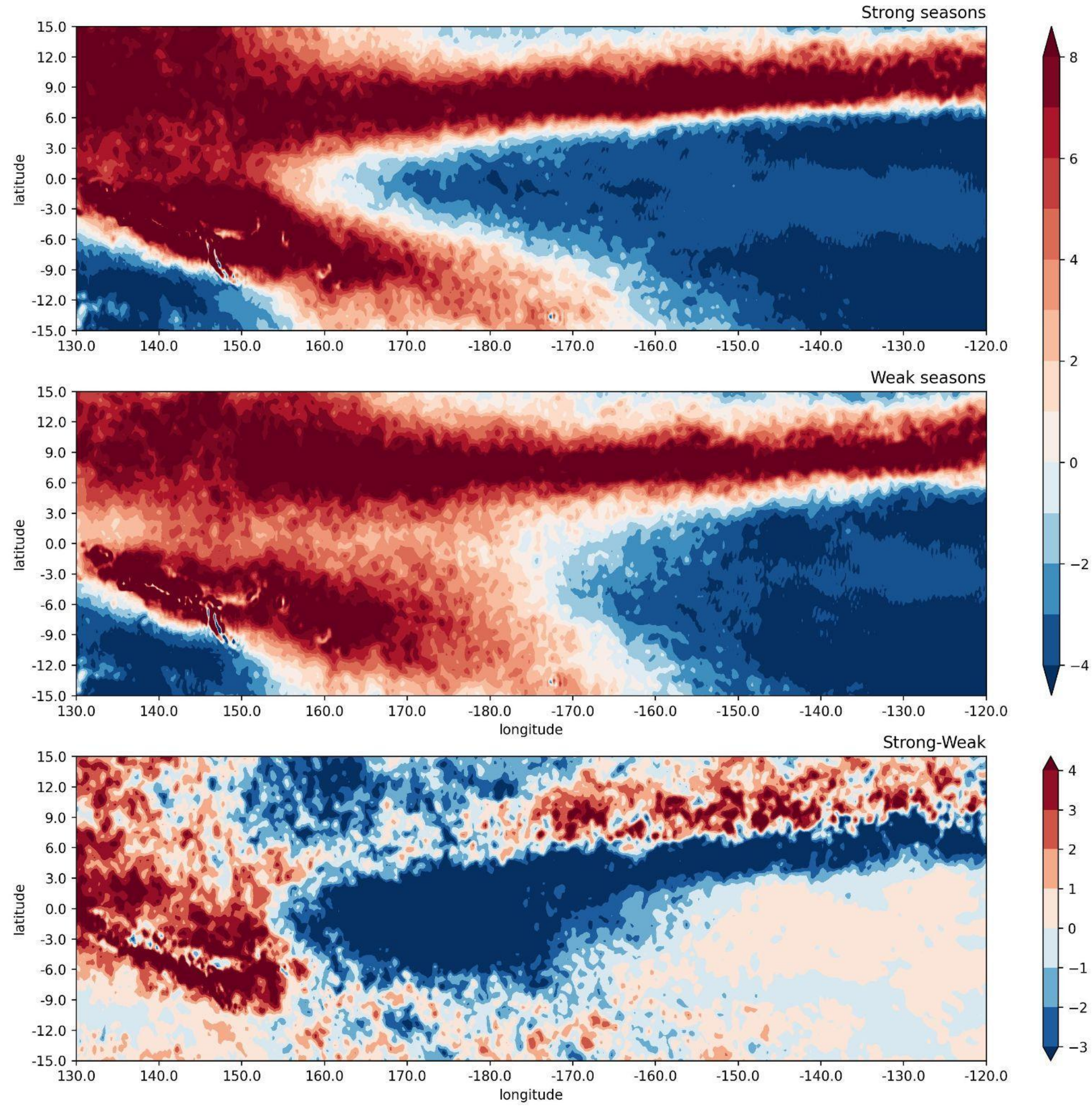

Figure 7*: Baroclinic energy transfer between eddy APE and eddy KE ($A(n).K(n)$) for MRG scale (in the frequency domain), at 500 hPa over the tropical Pacific domain (130E to 120 W and 15 S to 15 N) for strong seasons (top), weak seasons (middle) and their difference (bottom). Spectra obtained by summing over all MRG scale frequencies and averaging for the strong and weak seasons. Energy conversion measured in W m kg^{-1}.*

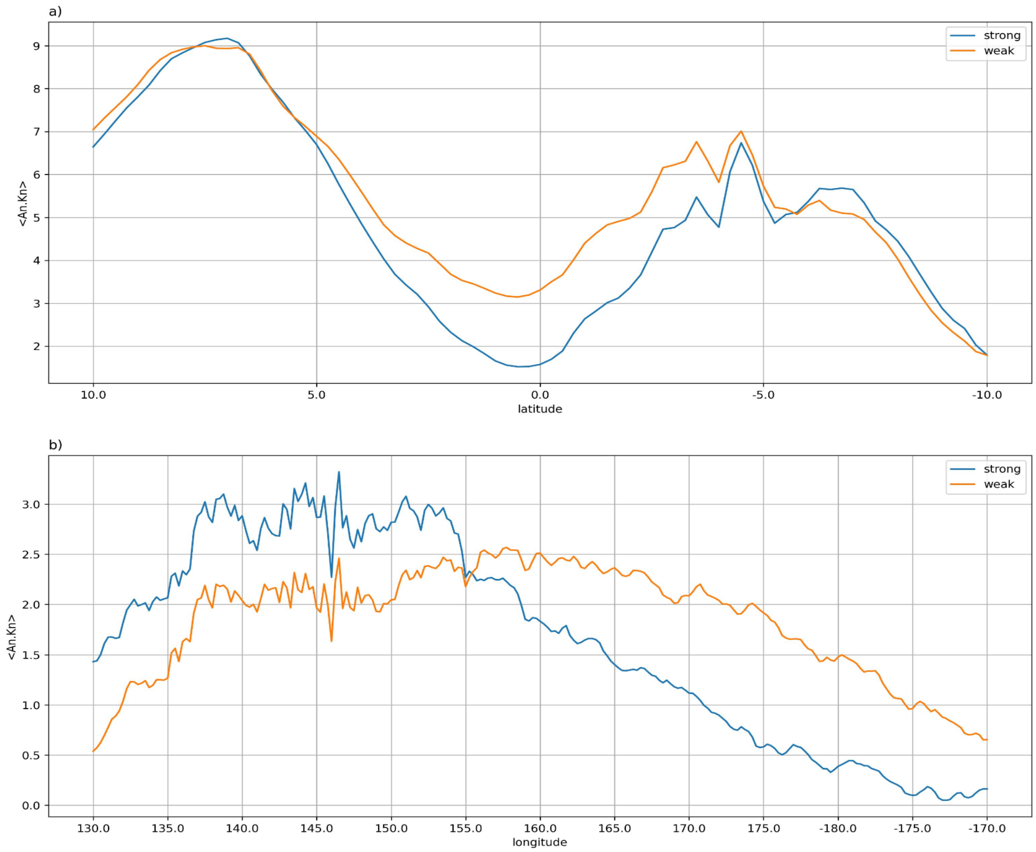

Figure 8*: Baroclinic energy transfer between eddy APE and eddy KE ($A(n).K(n)$) for MRG scale in the frequency domain, at 500 hPa, integrated along a) longitudes over the western tropical Pacific, i.e. 130 E to 170 W and b) latitudes from 10 S to 10 N for strong and weak seasons. Energy conversion measured in W m kg^{-1}.*

In figure 8, we can see the latitudinal and longitudinal variations of the baroclinic conversion for MRG scale. We can see that APE to KE conversion is slightly higher about the equator for weak seasons while away from the equator, higher MRG scale KE is generated for strong seasons. It is interesting to note that for strong seasons, KE generation is higher in the western Pacific as observed before while it is lower in the eastern Pacific. We know that during a La-Nina like state, due to high SST over western Pacific, there will be more convective activity and APE generation. This might explain the higher APE to KE conversion in the western Pacific during strong seasons. Similar reasoning can be used for the higher KE generation over eastern Pacific during weak seasons. An El Nino like state causes high SST over eastern pacific, thereby higher convective activity and higher APE generation.

3.2.3 Mean flow interactions

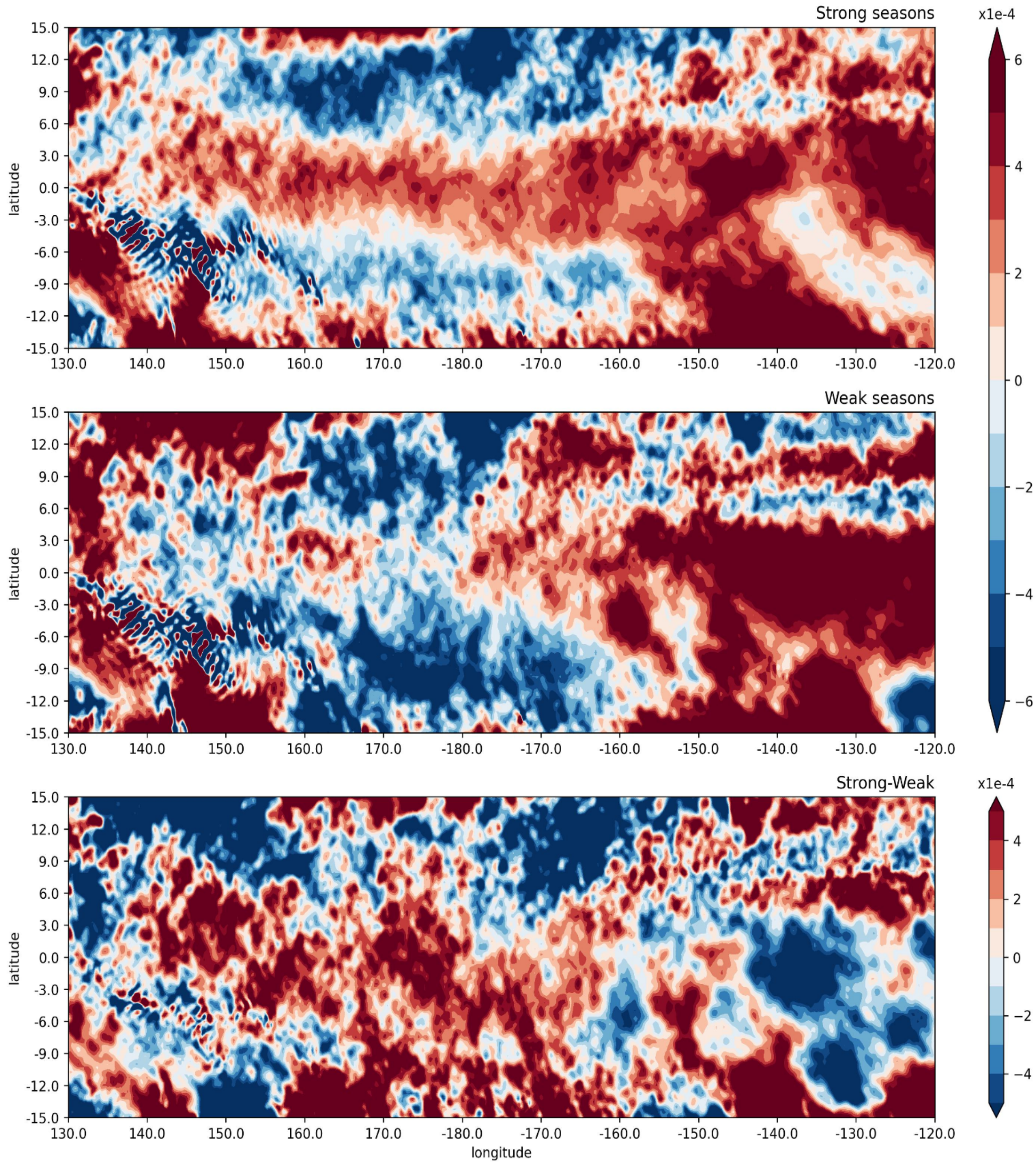

Figure 9*: Nonlinear KE exchange between MRG scale and mean flow ($K0.Kn$), in the frequency domain, vertically integrated from 100 hPa to 1000 hPa over the tropical Pacific domain (130 E to 120 W and 15 S to 15 N) for strong seasons (top), weak seasons (middle) and their difference (bottom). Energy interactions measured in W m kg^{-1}.*

In terms of across-scale energetics, we are concentrating on mean flow-MRG scale interactions in our study. Figure 9 shows vertically integrated mean flow interaction over the tropical Pacific domain. The energetics is calculated and compared for strong and weak seasons. For both strong and weak seasons, we can notice a source of MRG scale KE in the eastern Pacific. We can see that for strong seasons, the source is spread across the entire Pacific domain. The source is located about the equator and appears to have a north-westward tilt in the western Pacific. During weak seasons, a mean flow to MRG scale conversion is negligible in the central and western Pacific. Mean flow tends to get energy from MRG away from the equator in these regions.

The strong-weak plots show that the mean flow to MRG conversion in the western Pacific is stronger during weak seasons. We can also see that most of the MRG scale KE gained from mean flow is in the central and western Pacific regions. It is thus interesting to notice the influence of mean flow on MRG, i.e. when MRG scale is getting KE from mean flow over the western and central Pacific domain, it favours a stronger MRG activity. This establishes some energetics interaction between mean flow and MRG scale. In terms of magnitude, we can see that the baroclinic conversion term is the most dominant and acts as the major source/sink of MRG scale KE. Mean flow interaction values are lower than the in-scale energetics terms by nearly an order of magnitude 6, i.e. $\langle K0.Kn \rangle = 1e\text{-}6 * \langle An.Kn \rangle$. So we can see the quantitative differences between the nonlinear KE transfer between mean flow and MRG over the Pacific domain for strong and weak seasons. We can infer from the observations made that the mean flow does affect strength of MRG activity. MRG activity is enhanced when mean flow provides nonlinear KE to MRG scale in the western and central Pacific domain. Another interesting observation was that the source of energy in the eastern Pacific was weaker during strong seasons. So the energetic interactions over the central and western Pacific seem to mostly affect the strength and variability of MRG activity.

3.3 Wavenumber domain

3.3.1 Kinetic energy

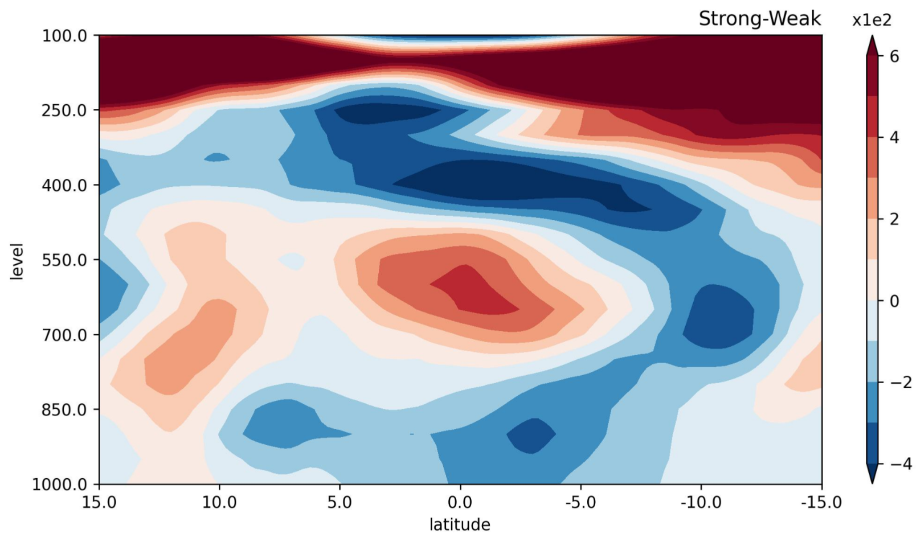

***Figure 10**: Strong-Weak plot for the MRG scale eddy KE in the wavenumber domain, summed over the MRG season (August to November months). The spectra was summed over the MRG wavenumbers, i.e. 2 to 10. Energy interaction measured in J m^{-2}. The levels are in hPa.*

We can study the major differences between the eddy KE distribution for strong and weak seasons in the wavenumber domain. From power spectral studies of meridional wind, we could confirm that most of the spectral power corresponding to MRG scale was confined between wavenumbers -2 and -10. From figure 10, we can see that the latitude-level distribution of MRG scale eddy KE is different for strong and weak seasons. For strong seasons, we can see that KE is significantly higher at high altitudes (250 hPa- 100 hPa) all over the tropical region. We can also see that the KE is higher about the equator at pressure levels around 500 hPa. The KE appears to be high for weak seasons at lower altitudes and between 400 and 250 hPa. The KE calculated here corresponds to MRG scale flow across the globe and not limited to the Pacific domain.

3.3.2 Mean flow interactions

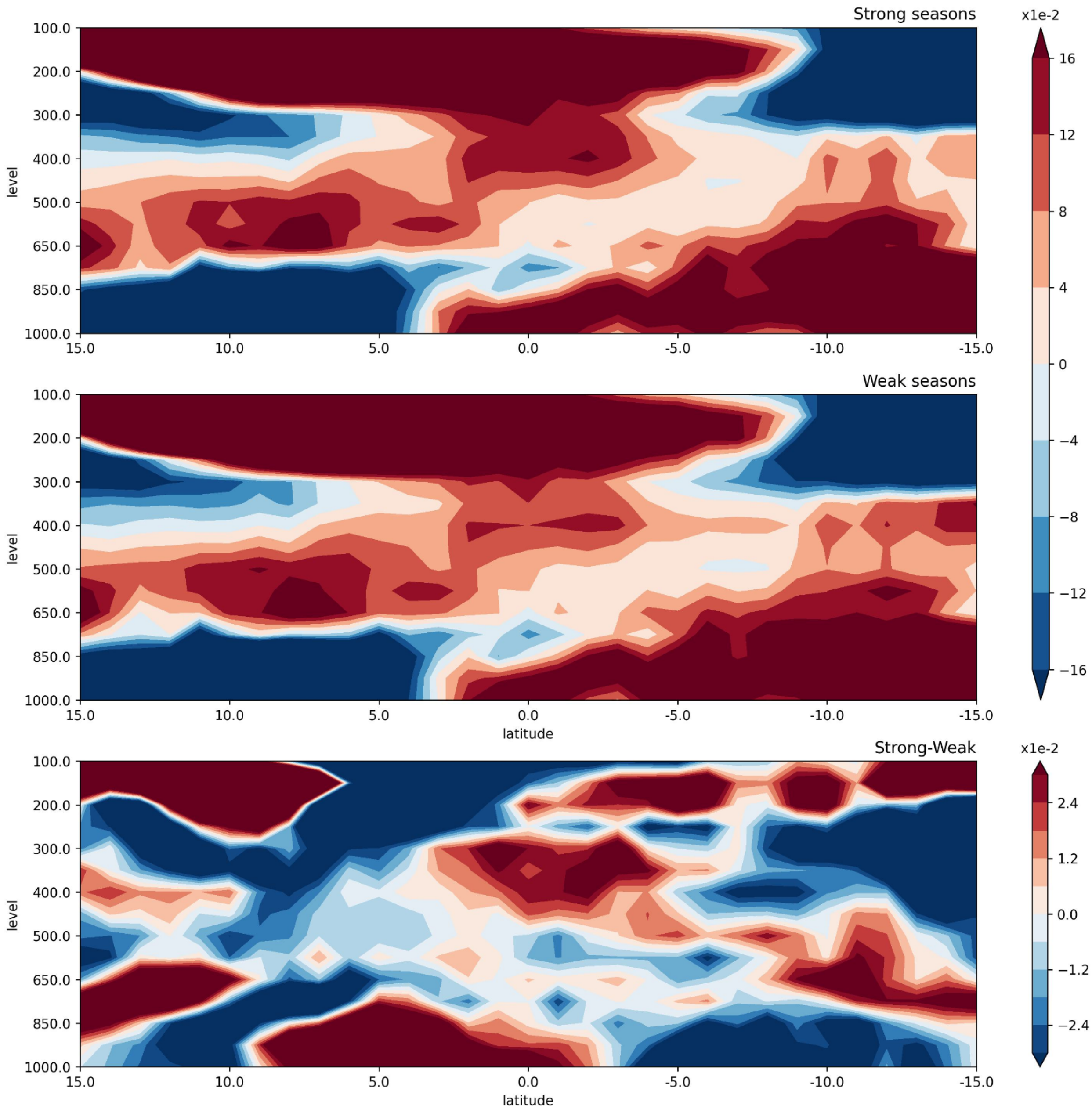

Figure 11: *Nonlinear eddy KE transfer between MRG scale and mean flow ($K0.Kn$), in the wavenumber domain, summed over the MRG season (August to November months) for strong seasons (top), weak seasons (middle) and their difference (bottom). The spectra obtained by summing over all MRG scale wavenumbers (2 to 10) and composites calculated for strong and weak seasons. Energy interaction measured in W m kg^{-1}.*

Nonlinear eddy KE transfer in the wavenumber domain between MRG scale and mean flow is shown in figure 11. The distribution of *K0.Kn* can be noted to be similar for both strong and weak seasons. However, they differ in magnitude in most parts of the domain under study. KE is transferred to the MRG scale from mean flow at almost all altitudes about the equator. At high pressure levels, we can see a source of MRG scale eddy KE in the northern hemisphere which extends into the southern hemispheric tropics. But, away from the equator, we can see that KE is getting transferred to the mean flow at high altitudes in the southern tropical region. An opposite character is observed in the KE transfer distribution at lower altitudes. A strong source is observed in the southern tropical region which extends north of the equator. Towards the northern regions away from the equator a sink of MRG scale KE is seen. In the mid tropospheric pressure levels, mean flow KE is converted to MRG scale eddy KE at almost all tropical latitudes.

Now, in the bottom panel of figure 11, we can see the magnitude differences of the sources and sinks between the strong and weak seasons. For strong seasons, we can see that the sources of MRG scale eddy KE are stronger at mid troposphere regions very close to the equator and at high altitudes in the northern tropics away from the equator. On the other hand, sources seem to be stronger for weak seasons away from the equator at mid altitudes and also at low altitudes in the southern tropical region. We can also similarly note down the regions where the sinks of MRG scale KE are stronger or weaker for strong and weak seasons. We could notice that the nonlinear KE transfer in the wavenumber is unrelated to that in the frequency domain. The distribution of the sources and sinks in the wavenumber and frequency domain appeared to show no correlation with each other.

4. Conclusions

Mixed Rossby Gravity a.k.a MRG waves are special westward propagating equatorial waves, with properties similar to that of low-frequency Rossby waves and high-frequency Inertio-Gravity waves (Matsuno, 1966). The in-scale and across-scale energetics of MRG scale flow is investigated in the frequency as well as the wavenumber domain using meridional and zonal wind data. In Suhas et al. (2020), based on MRG wave amplitude studies, 5 strong and 5 weak seasons were identified during 1979 to 2019. Further, it was observed that the background state during strong and weak MRG seasons were distinct. A La Nina like mean state over tropical Pacific favoured strong MRG activity while weak season exhibits an El Nino like mean state (Suhas et al., 2020). Several other studies have brought out a strong correlation between background mean flow and MRG activity (Chen and Huang, 2009; Wu et al., 2014). We are investigating the mean flow-MRG scale interactions during the strong and weak months using a multi-scale spectral energetic framework to look into the quantitative aspects of the interaction between MRG waves and background state. We further compare the MRG scale horizontal kinetic energy as well as the baroclinic conversion terms (in-scale energetics) for the strong and weak seasons. The spectral framework is employed in the frequency as well as the wavenumber domain.

Convective signals associated with convectively coupled MRG waves are identified using wavenumber-frequency power spectra and these signals are isolated as perturbations lying between wavenumbers -2 to -10 and time period 3 to 6 days in the anti-symmetric spectra. Spectra for meridional wind showed peaks around similar wavenumber and frequency values in the symmetric spectra which confirm the meridional wind being symmetric about the equator. Strong seasons had more spectral peaks and occupied more phase space than weak seasons implying stronger circulation and more diverse spatial and temporal scales of MRG activity. The above pattern was observed both at 850 hPa and 200 hPa. Strong peaks with a nearly 3 day period were observed at 850 hPa but not at 200 hPa for strong and weak seasons.

For frequency studies, MRG season was taken to be August to November, domain of study was over the tropical Pacific and the period of MRG waves range from 3 to 6 days (Suhas et al., 2020). KE of MRG scale at 850 hPa, was found to be higher for strong seasons as expected. KE at 200 hPa was higher than that at 850 hPa for both seasons which we can attribute to the lack of frictional dissipation at high altitudes. However at 500 hPa, the KE for strong seasons was found to be lesser than weak seasons in most parts of the domain except in the central Pacific. KE for MRG scale was found to be high for strong seasons around altitudes where the circulation features of MRG were most prominent. Further, latitude and longitude wise distribution of KE also showed higher values for strong seasons.

Baroclinic conversion for MRG scale was analysed at each altitude which showed that the maximum variation between strong and weak seasons was observed at 500 hPa. Around the equator, a source of MRG scale KE was observed for weak seasons while a sink was observed for strong seasons. At 500 hPa, KE was generated over the eastern Pacific irrespective of strength of MRG season but in the central Pacific, a sink of KE was observed for strong seasons while for weak seasons, a source was located. Away from the equator, sources of KE were located at all pressure levels on both sides of the equator. These sources were found to be stronger for strong seasons. Further, the source of KE located in the eastern Pacific was stronger for strong seasons. We can observe a net loss of MRG scale KE in the central Pacific for strong seasons, but KE is generated in the remaining domain. Next, we investigated the mean flow energy interactions of MRG scale disturbances. Here, we could see that MRG scale KE generation was higher for strong seasons all over the domain. MRG scale was getting energy from mean flow in the eastern Pacific for both seasons while in the western Pacific, energy was getting transferred from MRG scale to mean flow during weak seasons. This shows that conversion of KE to MRG scale from mean flow in the western Pacific domain favours a strong MRG season. We are thus able to study the pattern and magnitude of energy exchange between MRG scale and mean flow during strong and weak seasons.

Mean flow interactions of MRG were also investigated in the wavenumber domain. We are studying MRG scale motions in the tropics across the globe but not over the Pacific alone. Higher KE was observed for strong seasons around 500 hPa around the equator and at high altitudes. The pattern observed was unrelated to that observed in the frequency domain. Studying mean flow interaction of MRG scale also showed that MRG scale was getting more KE at altitudes around 500 hPa for strong seasons.

MRG activity over the tropical Pacific could thus be seen to be dependent on the mean background state as well as the available potential energy over the domain. The framework is very effective in the sense that it helps us calculate the energetic interactions for our scale of interest with minimal computational cost. Further detailed analysis of the complete energy budget can help us understand the initiation and maintenance of MRG activity. The framework can also be used to study MRG activity over other domains and seasons. Nonlinear energetic interactions of MRG scale with other scales rather than just the mean flow is yet to be investigated which helps us understand the quantitative interaction between MRG and those scales.

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