Exploring Quantum Effects in Inflation

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by

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Certificate

This is to certify that this dissertation entitled Exploring Quantum Effects in Inflationtowards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Elaf Ansariat Indian Institute of Science Education and Research under the supervision of Diptimoy Ghosh, Associate Professor, Physics , during the academic year 2024-2025.

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This thesis is dedicated to my parents, whose unwavering support and belief in me have been my greatest strength. Despite my unconventional path, they always stood by my side and their encouragement has guided me to this moment.

Declaration

I hereby declare that the matter embodied in the report entitled Exploring Quantum Effects in Inflation are the results of the work carried out by me at the Physics, Indian Institute of Science Education and Research, Pune, under the supervision of Diptimoy Ghosh and the same has not been submitted elsewhere for any other degree.

Elaf Ansari

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Abstract

Loop corrections in inflationary cosmology have received comparatively little attention, despite the fact that tree-level results can become unreliable when loop contributions dominate.So, in this thesis we compute the one-loop corrections to the power spectrum and bispectrum in a single-field inflationary models, focusing on leading contributions in slow-roll parameters . Although,full evaluation of all possible diagrams still remains open due to technical challenges,we made some progress in calculating and understanding higher-order correction.We also present observations arising from the derivation of the perturbation action for inflaton fields. In addition, we review various methods for computing loop corrections and share insights gained during their evaluation.Finally, we also derive some perturbative constraint on the wavefunction coefficient in deSitter using the unitarity of Bunch-Davies S-matrix under some assumptions.

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Chapter 1

Introduction

Inflation is a leading theory for solving the horizon problem,flatness problem,monopole problem and it also explains the origin of the structures in the universe [2, 3]. During this phase of accelerated expansion, quantum fluctuations were stretched to cosmic scales, leaving imprints in the cosmic microwave background (CMB) and the distribution of galaxies. These fluctuations are described by in-in correlation functions and the wavefunction, which encodes the information about the primordial perturbations [4].In this thesis,we mainly explore these objects and try to constrain or refine them to higher order.

This thesis consists of two parts. In the first half, we attempt to constrain the wavefunction coefficients of the universe using the unitarity of the S-matrix. This is a part of a broad program in theoretical cosmology, where flat-space results such as the optical theorem and cutting rules are extended to cosmology.Additionally,since S-matrix is a quantum object,violation of the constraint on wavefunction will shed some light on the nature of the perturbations.Now, By applying unitarity conditions to the Bunch-Davies S-matrix [5,6], we reproduced some results related to cosmological cutting rules [7,8] and the optical theorem [9] . However, inconsistencies in specific diagrams suggested that our inverted S-matrix and wavefunction coefficient relations were incomplete, due to missing combinatorial factors. As a result,even though we have obtained constraints on wavefunction coefficients in a perturbative and theory-dependent manner, a fully non-perturbative derivation remains an open problem.

In the second part of this thesis, we focus on studying loop corrections in single field slow-roll inflation. Although tree-level calculations have been studied extensively in cosmology [10–14], loop-level corrections remain relatively unexplored.Nonetheless,some progess has been made in understanding loops [15–19] and its IR-effects [20–22] ,particularly in proving the time-independence of curvature perturbation at superhorizon scale with loop corrections [23], which is essential for the predictability of the inflation.Researchers have also explored the loops in Effective Field Theory of Inflation(EFToI) in simple limits [1, 17] but they remain largely unexplored in the single field slow-roll model of inflation,which is in a very good agreement with the observations.

The primary difficulty in loops arises from the lack of time-translation symmetry in an expanding universe, which causes time and momentum integrals to mix in a way which complicates loop calculations.Some techniques have been proposed to simply these integrals,for example by leveraging on the dilation symmetry of the deSitter [24, 25], dispersion relation [26] and spectral decomposition [27].However,these methods are more suited for massive fields whose mode functions are Hankel functions and also it decays rapidly at late times. whereas, we are mostly interested in massless field,which has a simpler mode function and also leaves an imprint at late times [28, 29].

There are two main motivations for studying these corrections.Firstly, it may happen that after renormalisation,the loop correction may become larger than the tree level contribution, rendering the perturbation theory of inflation invalid.In addition,as previously stated,loop corrections had not been computed previously in single field slow-roll inflation,so it will be better to have some results,which can be used to analyze and conjecture the analytic structure of the loops .

Therefore,we calculate the one-loop correction to the power spectrum(2-point correlation) and bispectrum(3-point correlation).We chose bispectrum because it is the simplest non-Gaussian observable but with a rich structure due to the three momenta. To perform the loop calculations, we have explored various methods, including Mellin space, and other techniques commonly used in flat-space loop calculations [30].However,these techniques are not very useful in massless case as discussed previously.

Chapter 2

Inflation

2.1 Motivations

Inflation is a phase of the universe in which universe undergoes a rapid expansion.It was originally introduce to explain causal origin of CMB homogeneity and flatness of the universe.Later,it was also used to explain the superhorizon correlations,CMB fluctuations and galaxy structure through quantum origin. [28, 29]

One of the primary motivations for inflation is the horizon problem .The basic idea is that if we look at two points on the CMB separated enough spatially,we see that they nearly have the same temperature of 2.7K. But the hot Big-Bang model predicts that there wasn't enough time from the Big-Bang to the recombination time for this points to have exchanged information.Inflation solves this problem by shifting the Big-Bang singularity back enough in time,so that this two points can have causal contact(see fig:2.1.To be more precise,inflation doesn't really solves the homogeneity problem rather it makes it more plausible by mapping a very large CMB patch to a small homogeneous part(within two Hubble radius) which has been stretched to cosmic size due to rapid expansion.

Mathematically,comoving Hubble radius,

$$(aH)^{-1} = \frac{1}{a(t)H(t)} \tag{2.1}$$

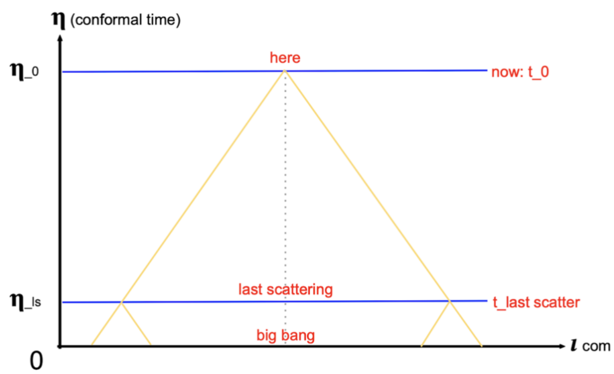


Figure 2.1: Horizon problem in hot Big-Bang model:Two far enough points having same temperature do not have an overlapping past light-cone

is a measure of causal patch and in a universe with decelerating expansion(e.g. radiation and matter dominated).This quantity decreases in the past,which implies that some regions currently observed in the CMB could never have been able to exchange information in the past. Inflation resolves this by introducing a period in which comoving Hubble radius increases in the far past,which gives the extra time needed to allow causal contact in the past,this condition also has a consequence that universe must undergo accelerated expansion ($\ddot{a} > 0$) and hence the name. Another significant motivation for inflation is the *flatness problem*. The spatial curvature parameter Ω_k evolves according to

$$\Omega_k(t) \propto \frac{1}{(aH)^2} \tag{2.2}$$

flatness problem is a fine-tuning problem that is if we measure the spatial curvature of the universe today, it looks very flat and the hot Big-Bang model tell us that in the far past,universe had to be extremely flat for this to happen,had the deviation been little larger,our universe would get very high curvature.Inflation resolves this problem by making the curvature of the past irrelevant as due to exponential expansion,everything would look locally flat in the far future.

Additionally,Inflation was also used to explain the slight inhomogeneity in the CMB,superhorizon correlation in the CMB and distribution of large structures of the universe.This theory tells us that these homogeneities are generated by small quantum fluctuations at the start of the inflation,which has been stretched to this large scale by exponential expansion,which also seeds the structure formation.

2.2 Inflation as a Scalar Field Theory

The simplest model of inflation is described by a single scalar field. The action for this field, assuming minimal coupling to gravity is

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right], \tag{2.3}$$

where ϕ is decomposed as

$$\phi=\bar{\phi}+\varphi,$$

with $\bar{\phi}$ representing the classical background component that dictates the large-scale evolution of the universe and φ represents small perturbations arising from quantum fluctuations and self-interactions. The function $V(\bar{\phi})$ describes the potential energy of the background field.

The background evolution of the universe is governed by the Friedmann equations,

$$H^2=\frac{1}{3M_{\rm Pl}^2}\left[\frac{1}{2}\dot{\bar{\phi}}^2+V(\bar{\phi})\right], \tag{2.4}$$

$$\dot{H}=-\frac{1}{2}\dot{\bar{\phi}}^2 \tag{2.5}$$

which relates the Hubble H to the energy density of the background field. The dynamics of the background field $\bar{\phi}$ follows from the covariant conservation of stress energy tensor.

$$\ddot{\bar{\phi}}+3H\dot{\bar{\phi}}+V'(\bar{\phi})=0 \tag{2.6}$$

This equation describes how the field evolves under the influence of its potential and the friction-like effect. Now,we know that the above equation won't necessarily give accelerating expansion solution.For inflation to occur,requires the condition ,

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0.$$

and to ensure a prolonged period of inflation, the background field must evolve slowly. This is captured by the slow-roll approximation, which assumes that the kinetic term of the field is much smaller than its potential energy.This condition is equivalent to demanding deSitter like expansion(quasi deSitter) as the kinetic term of deSitter is zero and potential term is non-zero(cosmological constant).We demand this condition because, first we want inflation.Second,we want to explain the scale invariance of CMB fluctuations and we also want the inflation to end.Therefore,we get

$$\dot{\bar{\phi}}^2 \ll V(\bar{\phi}), \quad |\ddot{\bar{\phi}}| \ll |3H\dot{\bar{\phi}}|.$$

These conditions lead to the introduction of slow-roll parameters:

$$\varepsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'(\bar{\phi})}{V(\bar{\phi})}\right)^2 \approx \frac{M_{\rm Pl}^2}{2}\left(\frac{\dot{\bar{\phi}}}{H}\right)^2, \tag{2.7}$$

$$\eta = M_{\rm Pl}^2 \frac{V''(\bar{\phi})}{V(\bar{\phi})} \approx \frac{M_{\rm Pl}^2}{2}\left(\frac{\dot{\bar{\phi}}}{H}\right)^2 - \frac{\ddot{\bar{\phi}}}{H\dot{\bar{\phi}}} \tag{2.8}$$

Inflation proceeds as long as $\varepsilon \ll 1$ and η ensures that the field rolls slowly enough for accelerated expansion to persist. The end of inflation occurs when $\varepsilon \sim 1$, marking the onset of the reheating phase.

Since,this condition makes the potential flat,the self interaction term generated by potential will be slow-roll suppressed.

2.3 In-In Correlation Functions and Wavefunction of the Universe

Unlike collider experiments, where initial and final states are well-defined, cosmological observations involve measurements at a single final time slice. To compute expectation values in this context, we employ the *in-in* formalism [31]. The expectation value of an operator $\mathscr{O}(t)$ is given by

$$\langle \mathscr{O}(t) \rangle = \left\langle 0 \left| \bar{T} e^{i \int_{t_0}^{t} H_I dt'} \mathscr{O}(t) T e^{-i \int_{t_0}^{t} H_I dt'} \right| 0 \right\rangle \tag{2.9}$$

Here, T and $\bar{T}$ denote time and anti-time ordering, respectively, and H_I is the interaction Hamiltonian and $|0\rangle$ is a free vacuum state. The structure of this expression gives rise to multiple types of Feynman diagrams, often called *Feynman-Witten diagrams* [15, 32],which are used to compute correlation functions in cosmology.A few important observations are in order.Since inflation lacks a time-translation Killing vector, the vacuum state is not preserved over time. As a result, even if the system starts in the vacuum at early times, particles can be produced at late times due to the time-dependent background. .As, the boundary operator $\langle \mathscr{O}(t) \rangle$ is not within the time-ordering, the propagators splits into two types: one connecting the boundary to the bulk and another connecting bulk-to-bulk interactions within the time/anti-time ordered structure. Additionally, the presence of both time and anti-time ordering leads to two types of diagrams, which are complex conjugates of each other. Consequently, it suffices to compute one diagram and extract its real or imaginary parts to obtain the full result. Lastly, in-in correlators in position space are real when the boundary operators are Hermitian and in-in correlators of momentum space operators will also be real if they also have a parity symmetry.

The wavefunction of the universe provides a fundamental description of late-time cosmological evolution and is closely related to in-in correlation functions. Similar to the wavefunction in quantum mechanics, it is defined as

$$\Psi[\phi(x)] = \langle \phi(x) | 0 \rangle \tag{2.10}$$

This wavefunction allows us to express expectation values of field correlations as

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \int \mathscr{D}\phi \, \phi(x_1) \cdots \phi(x_n) |\Psi[\phi]|^2 \tag{2.11}$$

where the functional integral runs over all possible field configurations. This formulation highlights that the wavefunction encodes probabilities for different field values at late times, making it a useful tool for computing in-in correlators.

An equivalent representation can be obtained using the path integral formalism, where the wavefunction is expressed as

$$\Psi[\phi] = \int_{\Phi(-\infty^+)=0}^{\Phi(0)=\phi} \mathscr{D}\Phi\, e^{iS[\Phi]} \tag{2.12}$$

To simplify calculations, it is common to parameterize the wavefunction in an exponential form, where the coefficients of different terms are referred to as *wavefunction coefficients*:

$$\Psi[\phi] = \exp\left(-\frac{1}{2}\int d^3x_1 d^3x_2\, \Psi_2(x_1,x_2)\,\phi(x_1)\phi(x_2) + \sum_{n=3}^{\infty}\int d^3x_1\cdots d^3x_n\, \Psi_n(x)\,\phi(x_1)\cdots\phi(x_n)\right) \tag{2.13}$$

This expansion can also be rewritten in momentum space as

$$\int d^3x_1\cdots d^3x_n\, \Psi_n(x)\,\phi(x_1)\cdots\phi(x_n) = \int \frac{d^3k_1\cdots d^3k_n}{(2\pi)^{3n}}\, \Psi_n(k)\,\phi_{k_1}\cdots\phi_{k_n} \tag{2.14}$$

2.4 de Sitter Space

Having introduced the fundamental objects of our study, we now focus on de Sitter (dS) spacetime [33], which serves as a useful approximation when computing in-in correlators at leading order in the slow-roll parameter.

Definition and Geometry

De Sitter space is a maximally symmetric solution to Einstein's equations with a positive cosmological constant Λ. In four-dimensional spacetime ($3+1$ dimensions), it can be described as a hyperboloid embedded in a five-dimensional Minkowski space:

$$-X_0^2+X_1^2+X_2^2+X_3^2+X_4^2=H^{-2}$$

where H is the Hubble parameter, related to the cosmological constant by

$$H^2=\frac{\Lambda}{3M_{\mathrm{Pl}}^2} \tag{2.15}$$

Physically, global de Sitter space represents a universe undergoing contraction and then accelerated expansion.

Flat Slicing Coordinates

Since,we are interested in inflation,we focus only on its acceleration phase.A convenient way to describe it is through *flat slicing coordinates*(also known as Expanding Poincaire patch), in which the metric takes the form

$$ds^2=-dt^2+e^{2Ht}d\mathbf{x}^2 \tag{2.16}$$

By introducing conformal time η, defined as

$$\eta=-\frac{e^{-Ht}}{H}, \quad \text{where } \eta\in(-\infty,0) \tag{2.17}$$

the metric transforms into

$$ds^2=\frac{1}{H^2\eta^2}\left(-d\eta^2+d\mathbf{x}^2\right) \tag{2.18}$$

This representation makes the dilation isometry explicit.

Symmetries of de Sitter Space

As,deSitter space is one of the maximally symmetric space,it possesses ten isometries four dimensions,which forms the group $SO(4,1)$ as evident from its higher dimensional embedding definition . These include translations(3) , rotations(3) in three spatial dimensions, dilations(1), and special conformal transformations(3) also called boost. These symmetries strongly constrain the behavior of correlation functions and wavefunction coefficients, playing an important role in inflationary cosmology.

2.4.1 Quantum Field Theory in Rigid de Sitter Space

We treat de Sitter space as a classical background while studying quantum fields propagating within it. This *rigid* de Sitter approximation allows us to analyze the behavior of fluctuations without considering backreaction on the metric.

The scalar field φ is minimally coupled to flat deSitter background and we will be interested mainly in massless field as the massive fields decay at late times.this leads to the equation of motion

$$(a\varphi)'' - \frac{2}{\eta}(a\varphi)' - \nabla^2 a\varphi = 0 \tag{2.19}$$

where a is a deSitter scale factor.

Mode Expansion and Solutions

To analyze quantum fluctuations, we expand the field into Fourier modes:

The mode functions satisfy

$$(a\varphi_k)'' - \frac{2}{\eta}(a\varphi_k)' + k^2(a\varphi_k) = 0 \tag{2.20}$$

$$\phi_k^+(\eta) = \frac{H}{\sqrt{2k^3}}(1 - ik\eta)e^{ik\eta} \tag{2.21}$$

where $\phi_k^+(\eta)$ is a positive frequency mode and negative mode will be its conjugate.

This solution is obtained by recognizing that at early times, all physical modes were well within the Hubble radius which characterizes the local curvature and expansion rate was also slow compared to later times. In this regime, the modes should behave as Minkowski space solutions. Imposing this condition determines the solution upto a phase.

Chapter 3

deSitter S-matrix

3.1 S-matrix

S-matrix is a mathematical object which describes the relation between initial and final states.It is difficult to define S-matrix in deSitter [34].This is due to the following reasons:firstly,deSitter space is infested with IR-divergence due to exponential expansion,secondly,due to causal structure of deSitter which has the spacelike past and future boundaries [33],which implies for a given observer that there are regions of space from which one cannot get an any information,hence one cannot access the complete in/out states(this is strictly not a problem,but it tells us that the S-matrix is a not basic object of the theory).Thirdly,since deSitter doesn't have global time-translation symmetry,there is no concept of energy conservation.Thus,there are no stable asymtotic states.Lastly,due to deSitter Isometry,we have a family of deSitter invariant vacuums,hence we don't have a natural choice for an actual vacuum.However,some reasons can be given for Bunch-Davies vacuum [35].

Despite all this difficulties there has been an effort to define S-matrix in various patches of deSitter [5, 6, 34, 36].They mitigate these problems by restricting to IR finite interactions.

Pimentel and Melville recently defined an S-matrix for massive scalar field which has a natural relation with wavefunction coefficient and in-in correlator [5, 6](It is also valid for massless field with certain interactions,otherwise it gives rise to divergences.).This S-matrix is useful because,it has a simple crossing relation which interchanges in and out states and it is related to wavefunction coefficient and in-in correlator.So on using the optical theorem of S-matrix one can put a constrain on the observables of interest i.e. wavefunction coefficient and in-in correlator.

They have defined the in and out states as the states which behaves like a free states(states of the free scalar theory in rigid deSitter) in the far past and far future respectively.

$$\lim_{\tau\to-\infty(0)} \langle\alpha|\hat{O}(\tau)|0,-\infty\rangle_{\text{in(out)}} = \lim_{\tau\to-\infty(0)} \langle\alpha|\hat{O}(\tau)|0,-\infty\rangle \tag{3.1}$$

Concretely,if we choose the interaction picture and Heisenberg picture coinciding time in the far past,we can define the in states as the excitation of Bunch-Davies vacuum,while the out states are defined as an evolution of these free states by interacting hamiltonian in the far future.Note,it is essential to choose the coinciding time to be around the far past,where the Hamiltonian will be nearly time independent , which is required to project the interacting vacuum to free vacuum [1].

$$\begin{aligned} |n,-\infty\rangle_{\text{in}} &= \hat{a}_n^\dagger \dots \hat{a}_1^\dagger|0\rangle \\ \langle n,-\infty|_{\text{out}} &= \langle 0|\hat{a}_1\dots\hat{a}_n U_{\text{I}}(0,-\infty) \end{aligned} \tag{3.2}$$

Finally,Bunch-Davies S-matrix is defined as overlap of in and out states

$$S_{n'\to n} \equiv {}_{\text{out}}\langle n,-\infty|\,|n',-\infty\rangle_{\text{in}} \tag{3.3}$$

Now,we can easily derive,the LSZ formula for this S-matrix by following the same steps as we do in flat space [37],and one will see the parallel between flat and deSitter S-matrix(hardly surprising,as we followed the same steps in both derivation),It will be just mode functions for putting external legs on-shell,free equation of motion operators for amputating the external legs and time order correlator of the fields.,We can also analytically continue it for all energies and we will see that as we flip the sign of the energies ,we can make in states into out states owing to Hankel function property.

As a side note,due to definition of Bunch Davies S-matrix,it follows certain properties such as there is no particle production in a free theory,but on switching the interactions,we can get particles out of nothing,as there is no energy conservation in deSitter.

3.2 Wavefunction and S-matrix Relation

Moving ahead,we can extract the generalised wavefunction coefficient(this has both connected and disconnected diagrams for a given kinematics) from the wavefunction and wavefunction coefficient relation.

$$\psi_n(\tau) = \frac{\langle \phi = 0|\hat{\Pi}_{k_1} \ldots \hat{\Pi}_{k_n}|0_{\text{out}}\rangle}{\langle \phi = 0|0_{\text{out}}\rangle} = (-i)^n \frac{1}{\Psi[0]} \frac{\delta^n \Psi[\phi]}{\delta \phi_{k_1} \ldots \delta \phi_{k_n}}\bigg|_{\phi=0} \tag{3.4}$$

Now on inserting the complete set of in-in states we generate S-matrix times some coefficient,

$$\lim_{\tau \to 0} \psi_n(\tau) = \sum_j B^n_j S_{0\to j} \tag{3.5}$$

This coefficients can be computed by expanding the field eigenstates in terms of out states and by writing the canonical momentum in terms of creation and annihilation operators.

$$|\phi = 0\rangle = \sum_n \int d\Pi_n \, C^{\text{out}}_n(k_1, k_2, \ldots, k_n)|k_1, k_2, \ldots, k_n\rangle_{\text{out}} \tag{3.6}$$

C_n can also be found out by acting the field operator written in terms of creation and annihilation operators.

Pimentel and Melville obtained the following result:

$$\lim_{\tau \to 0} \psi_{k_1 \ldots k_n}(\tau) =$$
$$\sum_{j=0}^{\infty} \int dq_1 \ldots dq_j \, dq'_1 \ldots dq'_j \prod_{\ell=1}^{j} P_{q_\ell q'_\ell}(\tau)(-1)^j \frac{S_{0\to k_1 \ldots k_n q_1 q'_1 \ldots q_j q'_j}}{\left(\prod_{b=1}^{n} (-\tau)^{d/2} f^+(k_b \tau) \prod_{c=1}^{j} (-\tau)^d f^+(q_c \tau) f^+(q'_c \tau)\right)} \tag{3.7}$$

Here, P's and f's represent the power spectrum(free two point function in momentum space) and mode functions(Hankel function's) for the Bunch-Davies vacuum; for further details, please refer to [5]. Note that this relation also seems to have cutting rules engrained in them as higher point functions[right hand side] seems to be related to lower point functions[LHS].

However,we think that this relation is incorrect or more precisely it holds only for tree-level diagrams with some assumptions.

3.3 Some Results and Some Problems

We can see the problem with the eqn:3.7 in two ways.Obvious way is to directly check if the LHS and RHS matches using their feynmann rules.

Another way is that if we assume this relation is indeed correct,then we can do the following and obtain a simple non-perturbative result.After closely examining this equation , we observe that it constitutes an alternating series in the variable j. Thus, we will reconsider the aforementioned relation, now incorporating two additional external momentum. We will integrate this equation over these extra momenta and subsequently add both equations together after multiplying the latter by the power spectrum. This leads to the following expression:

$$S_{0\to k_1\ldots k_n} = \prod_{b=1}^{n}(-\tau)^{d/2} f^{+}(k_b\tau)\left[\psi_{k_1\ldots k_n}(\tau) + \int_{qq'} P_{qq'}(\tau)\psi_{k_1\ldots k_n qq'}(\tau)\right] \tag{3.8}$$

after using the optical theorem of S-matrix

$$S_{0\to n} + S^{*}_{n\to 0} = -\sum_{I} S_{0\to I}S^{*}_{n\to I} \tag{3.9}$$

and exchange relations(changing the sign of energies to interchange in and out states)

we expect that this should generate the cutting rules for wavefunction coefficient.

Coincidentally,eqn:3.8 does reproduces the cutting rules [7,8] for four-point ϕ^3 tree level exchange and it also reproduces cosmological optical theorem [9]. For example, S-matrix formulation provides a single line derivation of contact cosmological optical theorem ,as the right-hand side of the S-matrix optical theorem does not contribute to the first order.we get

$$\boxed{\psi_n(k_a, \hat{k}_a\cdot\hat{k}_b) + \psi_n(-k_a, \hat{k}_a\cdot\hat{k}_b)^{*} = 0} \tag{3.10}$$

Note that the optical cosmological theorem has been previously derived through alternative methods [9],but S-matrix formulation provides a quicker way.

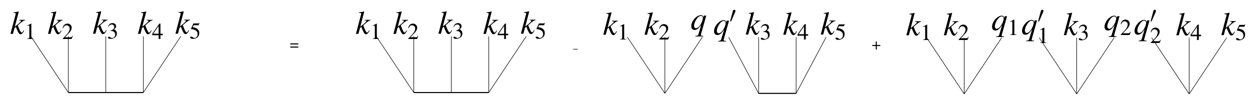
However,eqn:3.8 fails to reproduce five-point tree level in ϕ^3 theory.This is expected because from the cutting rules 5-point tree level will also be related to diagrams that have two power spectrum insertions and these element is clearly absent in the eqn:3.8.

Nonetheless,if we assume that if all q's are treated equally in the original result,which means while finding combinatorial factors we don't distinguish all the q's, then eqn:3.7 works for tree level diagrams.However,note that the eqn:3.8 generally will not hold , but rather will be modified by having more power spectrum terms.This happens because in order to derive that equation we had treated k's and q's to be at equal footing,which we noted is not a correct way.

In short,the above prescription works for tree level,but it will fail again at loop level.Now,we will concretely illustrate why the wavefunction coefficient and S-matrix relation is true for tree level with some prescription but it fails at loop:

Tree-Level:we have schematically wrote eqn:3.7 and carefully calculated its combinatorial factors

$$3!^4 x\frac{5!}{2!^3}(G-K^2)(G-K^2) = 3!^4 x\frac{5!}{2!^3}(G^2) \;-\; 3!^4 x\frac{5!}{2!^2}(GK^2) \;+\; 3!^4 x\frac{5!}{2!^3}(K^4)$$



$$\psi_5 = S_5 - \int P_{qq'} S_{5,qq'} + \int P_{q_1 q_1'} \int P_{q_2 q_2'} S_{5,q_1 q_1', q_2 q_2'}$$

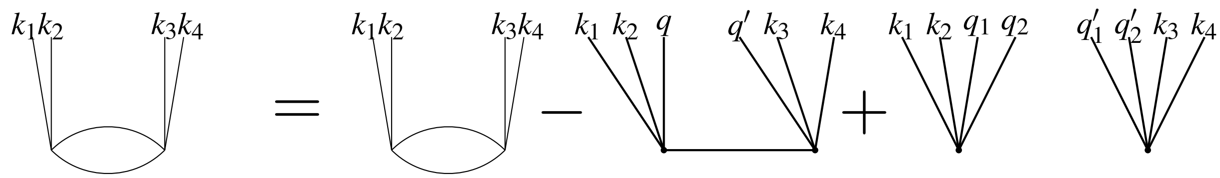
Here, $x = \frac{\lambda^3}{3!3!^3}$ and the rest of combinatorial factors account for permutations of different vertices and contractions within each vertex ($3!^4$), and summing over all possible channels for a given set of external momenta (fractions involving 5! and 2!).
G refers to the Feynman propagator, K refers to the bulk-boundary propagator and thus $G-K^2$ schematically refers to bulk-bulk propagator.

To reiterate , We assumed the prescription that all internal momenta (q's) are to be considered identical/indistinguishable as far as the combinatorial factors of the diagram are concerned. This prescription seems correct to us at tree level in general, as we just illustrated by the relation between ψ and S for double-exchange diagram here. If q's were to be permuted as well, we would have gotten extra factors of 2! in the corresponding last 2 terms on RHS of the above equation, which would clearly be problematic.

As a side note,one may have a doubt that diagrammatically,LHS and RHS are nearly equal,so why do we have extra diagrams.This can be explained by noting that the bulk-bulk progragator for wavefunction coefficient and S-matrix are different and these extra diagrams take this into account.

Loop-level: Tree-level prescription fails for loop

$$4!^2 y\frac{4!}{2!^3}(G-K^2)(G-K^2) = 4!^2 y\frac{4!}{2!^3}(G^2) \quad - \quad 2!4!^2 y\frac{4!}{2!^3}(GK^2) \quad + \quad 2!4!^2 y\frac{4!}{2!^3}(K^4)$$



Here, y = $\frac{\lambda^2}{2!4!^2}$. Our prescription clearly fails to work at 1-loop as illustrated here. In particular, the combinatorial factors of K^4 terms on RHS and LHS don't match

To summarise,if we use S-matrix and wavefunction coefficient with our prescription for tree level diagrams,we have been able to reproduce cosmological optical theorem and the cosmological cutting rules.but this prescription doesn't work for loops

Chapter 4

Perturbations in Inflation

4.1 Cosmological perturbations

The most general form of a metric perturbation in a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime can be written as:

$$ds^2 = a^2(\eta)\left[-(1+2A)\,d\eta^2 + 2B_i\,dx^i\,d\eta + (\delta_{ij}+2E_{ij})\,dx^i\,dx^j\right]$$

Due to rotational symmetry of the background,it is convenient(they don't mix at first order) to decompose the perturbation variables as scalar,vector and tensor of SO(2)(SVT decomposition).For example,

$$E_{ij} = C\delta_{ij} + \nabla_{\langle i}\nabla_{j\rangle}E \underbrace{+\nabla_{(i}\tilde{E}_{j)}}_{\text{vector}} + \underbrace{\tilde{E}_{ij}}_{\text{tensor}}$$

Now,as Einstein Gravity is a gauge theory,all these perturbations are not independent,for instance we can eliminate some of these variables by small coordinate transformations.This procedure is called gauge fixing and it also give rise to many gauges.As we know that the functional form of these perturbations are coordinate system dependent(as metric is coordinate dependent),it will be useful to define variables,which doesn't change their values on coordinate transformation and they are called gauge invariant perturbations.

For instance, :

$$\zeta = -C + \frac{1}{3}\nabla^2 E - H(v+B)$$

is gauge invariant,eventhough its constituents are not.

H is hubble, v is related to scalar part of momentum density perturbation and B is scalar part of B_i [28]

4.2 ADM Formalism

ADM formalism is an alternative formulation of Einstein Gravity [38],wherein spacetime is foliated by hypersurfaces.It is useful for Hamiltonian formulation of General Relativity and hence in defining the energy of the spacetime as we can take hypersurfaces to be spacelike which gives a preferred time direction.This formalism is also useful in making non-dynamical variables clearer,these variables lapse(N) and shift(N_i) are related to the how we choose to slice the spacetime,hence these are non-dynamical.

In this approach,the metric decomposes as :

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

where h_{ij} is a metric of the hypersurface.

4.3 Non-Gaussinities in Single Field Inflation

We place the quantum scalar field (inflaton) in a quasi-de Sitter background and analyze its dynamics [4]. Mathematically, we minimally couple the field to gravity. The matter perturbation induces a slight backreaction on gravity, which in turn affects the inflaton's evolution. This feedback loop generates higher-point effective interactions for the inflaton field.In order to obtain the action of the fluctuations,we employ ADM formalism and we can trade the constraint for the fluctuations using the constraint equation.

Now in ADM formalism,Ricci scalar can be decompose into the Ricci-scalar of the hypersurface,extrinsic curvature of the surface and its normal.

$$R = R^{(3)} - M^2 + M_{ij}M^{ij} + 2\nabla_\mu \left(n^\nu \nabla_\nu n^\mu - n^\mu \nabla_\nu n^\nu\right) \tag{4.1}$$

where $n^\mu$is the unit normal to the spatial hypersurface. The last term is a total divergence, which does not contribute in the action integral. Therefore,we can write the action of gravity and matter as

$$S = \frac{1}{2}\int \sqrt{h}\left(NR^{(3)} + N^{-1}(K_{ij}K^{ij} - K^2) + N^{-1}\left(\dot{\phi} - N^i\partial_i\phi\right)^2 - Nh^{ij}\partial_i\phi\partial_j\phi - 2NV\right) \tag{4.2}$$

where ${}^{(3)}R$ and $M_i j = N^{-1}K_{ij}$ are Ricci scalar and extrinsic curvature of the spatial slice and V is the potential of the inflaton.Note that reduce planck mass($M_{\mathrm{Pl}} = \frac{1}{\sqrt{8\pi G}}$) has been set to one for convenience but it will be reintroduce later. K_{ij} can be written in terms of the metric and normal of the hypersurface as

$$K_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right), \quad K = K_i^i. \tag{4.3}$$

We will obtain the following Hamiltonian and momentum constraint on extremizing this action with shift and lapse function,which will help us in swapping constraints with fluctuations.

$$-\nabla_i\left[N^{-1}(K_j^i - \delta_j^i K)\right] + N^{-1}(\dot{\phi} - N^i\partial_i\phi)\partial_i\phi = 0 \tag{4.4}$$

$$R^{(3)} - 2V - N^{-2}(K_{ij}K^{ij} - K^2) - N^{-2}(\dot{\phi} - N^i\partial_i\phi)^2 - h^{ij}\partial_i\phi\partial_j\phi = 0 \tag{4.5}$$

For cosmological perturbations, one expands h_{ij} around a background spacetime in different gauges.

In the comoving gauge or curvature perturbation gauge: metric becomes,

$$ds^2 = -N^2dt^2 + a^2(t)e^{2\zeta}e^{\gamma_{ij}}(dx^i + N^i dt)(dx^j + N^j dt) \tag{4.6}$$

where ζ represents the curvature perturbation and γ_{ij} represents tensor perturbations,$a(t) = e^{\rho(t)}$,$H = \dot{\rho}$ and $\det(e^{\gamma_{ij}}) = 1$.

It has been shown that curvature perturbation becomes constant on exiting the Hubble radius [23] during inflation which is important in connecting the inflationary predictions to actual observations as we don't know much physics of the reheating time.Curvature perturbation can also be directly related to temperature fluctuations in the CMB.

Moreover,in this gauge,we expect the action to be proportional to slow-roll parameters as curvature perturbation is a gauge mode in deSitter($\varepsilon = 0$).There is a little subtlety here as ζ is not well-defined in deSitter but we can still define a scalar perturbation in the spatial metric which will be a gauge-dependent quantity and the action will be independent of this quantity.

Simplifying the action in this gauge is Algebraically involved,because after solving the constraint in terms of ζ and graviton,we have to substitute it back into the action and have to do lots of integration byparts into order to bring the action in the form where each term is proportional to the some slow-roll parameters.

Now,we will write down the observations,which we made while dealing with this calculations:

- Zeroth order action will only contribute to the boundary term as background field satisfies equation of motion(2.4,2.5,2.6) and first order action in ζ will be absent as, again background satisfies equation of motion and hence action for ζ begins from second order.
- Whenever,we see Hubble in the denominator without any background scalar field,we should try to do the integration by parts, as this will generate slow roll parameter and simultaneously ,we should also try to remove all temporal derivatives on ζ from all such terms as this will terminate the integration byparts cycles.However,the last prescription won't always work,for instance,look at the second term of the quartic action given below.
- Taylor expansion $\frac{1}{N}$ will generate terms like$(1 - x + x^2 + \cdots)$,due to alternating signs,some terms accompanying it will cancel each other.
- Terms having only ζ, $\dot{\zeta}$ and no spatial derivative of $zeta$ must have atleast two $\dot{\zeta}$ for it to

become constant at superhorizon scale,where spatial derivative term is negligible.

- Each spatial derivative term will have a inverse of scale factor in addition to the scale factor from the measure,this is due to the following isometry of the background flat quasi de-Sitter space($a(t) \to a(t)/\lambda, x^i \to \lambda x^i$

In the flat gauge:

metric becomes,

$$ds^2 = -N^2 dt^2 + a^2(t) e^{\tilde{\gamma}_{ij}} (dx^i + N^i dt)(dx^j + N^j dt) \tag{4.7}$$

It turns out it is easier to get the action in the flat gauge.This is because,first of all due to flatness spatial ricci scalar is zero and extrinsic is also very simplified.

However,in this gauge scalar field fluctuation evolves in superhorizon limit,hence it is not directly useful for relating it to the actual observation.But as this gauge choices are just coordinate choices we can relate various variables of different gauges by coordinate transformation.To do so,we know how does the full metric transforms under coordinate change ,in order to get the transformation rules of fluctuation,we take the background(we decompose the full metric as background and its fluctuations) to be invariant under small coordinate change while we dump the entire change to the fluctuations.

for example,gauge transformation,upto linear order gives

$$\zeta = -\frac{\dot{\rho}}{\dot{\phi}} \varphi \tag{4.8}$$

Note that since the above gauge transformation has been derived perturbatively,it will be necessary to derive the above relation to appropriate order in fluctuations depending upon the order of action of interest.For example,if we are interested in getting the ζ action from φ action upto quartic order,it will be necessary to have this relation correct upto cubic order,as this cubic terms can generate quartic term from the quadratic action,if we don't do it,we will be missing some terms.

4.3.1 what order of N's

Here,we will see ,why in order to get the fluctuation's action upto mth order we only need to solve the constraint upto $\lceil \frac{m-1}{2} \rceil$. [39]

For example,in order to get the quadratic and cubic action we only need to solve the constraints at first order in fluctuation and to get the quartic and quintic action,we just need to solve it upto second order.

To show this,we evaluate the exact action(which means,when we substitute the actual non-perturbative constraint solution)by taylor expanding it around the perturbative constraint solution.

$$S_{\text{exact}} - S_{\text{approx}} = \int \left(N_{\text{exact}} - N^{(n)}\right) \frac{\delta \mathscr{L}}{\delta N} \left(h, N^{(n)}, N^{i(n)}\right) + \left(N^i_{\text{exact}} - N^{i(n)}\right) \frac{\delta \mathscr{L}}{\delta N^i} \left(h, N^{(n)}, N^{i(n)}\right) + \cdots \tag{4.9}$$

where

$$N_{\text{exact}} = N^{(n)} + O(\varphi^{n+1}), \quad N^i_{\text{exact}} = N^{i(n)} + O(\varphi^{n+1}) \tag{4.10}$$

Now,we can see that the error we are making in the action,when we neglect$O(\varphi^{n+1})$ is $O(\varphi^{2n+2})$ as from eqn:4.10 and the fact that error in the constraint equation is also of $O(\varphi^{n+1})$ when we solve the constraint upto $O(\varphi^n)$

So,we have proven the claim.

4.3.2 Perturbation Actions

After solving the constraint in terms of fluctuations,we obtain the following actions in flat gauge.

$$S_2 = \frac{1}{2} \int dt\, d^3x \left[e^{3\rho} \dot{\varphi}^2 - e^{\rho} (\partial \varphi)^2\right]. \tag{4.11}$$

The cubic action [4] is

$$S_3 = \int dt d^3x e^{3\rho} \left(-\frac{\dot{\bar{\phi}}}{4\dot{\rho}} \varphi\dot{\varphi}^2 - e^{-2\rho} \frac{\dot{\bar{\phi}}}{4\dot{\rho}} \dot{\varphi}(\partial\varphi)^2 - \varphi\dot{\partial}_i\chi\partial_i\varphi \right). \tag{4.12}$$

and the quartic action [40–43] will be

$$S_4 = \int dt\, d^3x\, e^{3\rho} \frac{1}{4} \left[\frac{\partial_i}{\partial^2}(\dot{\varphi}\partial_i\varphi) \frac{\partial_j}{\partial^2}(\dot{\varphi}\partial_j\varphi) - \frac{1}{\dot{\rho}} \frac{\partial_j}{\partial^2}(\dot{\varphi}\partial_j\varphi) \left(\dot{\varphi}^2 + e^{-2\rho}\partial_i\varphi\partial_i\varphi \right) + 4\dot{\varphi}\partial_i\varphi \frac{1}{\partial^2}(\dot{\varphi}\partial_i\varphi) \right]. \tag{4.13}$$

Since,we have the constraint till second order,we state the quintic action just for completeness:

$$\begin{aligned} S_5 = \frac{1}{2} \int dt\, d^3x \Bigg(& e^{3\rho} 18(\dot{\rho})^2 (N^{(2)})^2 N^{(1)} - N^{(1)} \left(\frac{(\partial_i (N^j)^{(2)})^2 + (\partial_i (N^j)^{(2)})(\partial_j (N^i)^{(2)}) - 2(\partial_i (N^i)^{(2)})^2}{2} \right) \\ & + 4\dot{\rho}(\partial_i (N^i)^{(1)})(N^{(2)})^2 - 2\dot{\varphi}^2 (N^{(1)})(N^{(2)}) - 2\dot{\varphi}\dot{\bar{\phi}}(N^{(2)})^2 + 2\dot{\varphi}(N^i)^{(2)}\partial_i\varphi(N^{(1)}) \\ & + 2\dot{\varphi}(N^i)^{(1)}\partial_i\varphi(N^{(2)}) + 2\dot{\varphi}(N^i)^{(2)}\partial_i\varphi(N^{(1)}) \Bigg) \end{aligned} \tag{4.14}$$

where to leading order in slow-roll parameters and after SVT decomposition,we get

$$N^{(0)} = 1, \tag{4.15}$$

$$N^{(1)} = \frac{\dot{\bar{\phi}}}{2\dot{\rho}} \varphi, \tag{4.16}$$

$$N^{(2)} = \frac{1}{2\dot{\rho}} \frac{1}{\partial^2} \partial_i(\dot{\varphi}\partial_i\varphi), \tag{4.17}$$

$$N_i^{(n)} = N_i^{T(n)} + \partial_i\psi^{(n)}, \tag{4.18}$$

$$N_i^{T(0)} = N_i^{T(1)} = 0, \tag{4.19}$$

$$N_i^{T(2)} = 2e^{2\rho} \frac{1}{\partial^2} \left(\frac{\partial_i\partial_j}{\partial^2} - \delta_{ij} \right) (\dot{\varphi}\partial_j\varphi), \tag{4.20}$$

$$\psi^{(1)} = \frac{1}{\partial^2} \left[\frac{\dot{\bar{\phi}}^2}{2\dot{\rho}^2} \frac{d}{dt} \left(-\frac{\dot{\rho}}{\dot{\bar{\phi}}} \varphi \right) \right], \tag{4.21}$$

$$\psi^{(2)} = -\frac{e^{2\rho}}{4\dot{\rho}} \frac{1}{\partial^2} \left(\frac{6\dot{\rho}}{\partial^2} \partial_i(\dot{\varphi}\partial_i\varphi) + \dot{\varphi}^2 + e^{-2\rho t}\partial_i\varphi\partial_i\varphi \right). \tag{4.22}$$

One feature to observe is that the leading order contributions of quadratic and quartic are slow-roll independent and hence it doesn't vanish in the deSitter limit while the leading contribution from cubic and quintic action are proportional to $\sqrt{\varepsilon}$,so this begs the question is this a general feature. We think this trend will hold due to the following reasons Let's look at the momentum constraint more closely,

$$-\nabla_i \left[N^{-1}(K^i_j - \delta^i_j K)\right] + N^{-1}(\dot{\phi} - N^i\partial_i\phi)\partial_i\phi = 0$$

schematically this will become,

$$\dot{\rho}\partial_j N^{(n)} + N^{-1}(\dot{\bar{\bar{\phi}}} + \dot{\varphi})\partial_j\varphi + \cdots = 0$$

where dots represent remaining terms.First,we know that $N^{(1)}$ is of order $\sqrt{\varepsilon}$ and the leading order of $N^{(2)}$ in slow roll is ε independent.Now, can $N^{(3)}$ be slow-roll independent? the answer is no.Because for this to happen,we have to avoid background field term as this generates slow-roll parameter,but from the second term we are forced to choose $N^{(1)}\dot{\varphi}\partial_j\varphi$,which has a slow roll dependence(we are solving perturbatively).Following this logic will support the claim.Moving forward,we need to substitute it into the action and follow similar reasoning to arrive at the conclusion.Note,it may happen that this leading order contribution may cancel by parts or with each other,*so the more precise statement is that odd action vanishes in deSitter,while even order action will survive*

4.3.3 Hamiltonian

As,we have the Lagrangian,the next step is to obtain the Hamiltonian in the interaction picture so as to proceed to the loop calculation.But,given that the Lagrangian has higher power of temporal derivative term,we need to be careful to get the interaction Hamiltonian in the interaction picture. Consider a toy Lagrangian having higher power of temporal term,

$$L = \frac{\dot{q}^2}{2} + \lambda(\dot{q})^n \tag{4.23}$$

the canonical momentum is

$$p = \dot{q} + n\lambda(\dot{q})^{n-1} \tag{4.24}$$

and the Hamiltonian ,

$$H = (\dot{q} + n\lambda(\dot{q})^{n-1})\dot{q} - \frac{\dot{q}^2}{2} - \lambda(\dot{q})^n$$

Moving forward,

$$H = n\lambda(\dot{q})^n + \frac{\dot{q}^2}{2} - \lambda(\dot{q})^n$$

The important step is to eqn:4.24,which will generate infinite terms as a function of momentum after taylor expansion. To find the actual interaction terms,we declare $p := \dot{q}$ whose rationale will be explained later,for now let's proceed.

we can invert it by making an ansatz,

$$\dot{q} = p + a_2 p^2 + \dots \tag{4.25}$$

After substituting it in 4.24,we will get all a's to be zero until $a_{n-1} = -n\lambda$ and higher a's will be more suppressed in terms of coupling.

Finally,

$$H = n\lambda(p)^n + \frac{(p^2 - 2n\lambda p^n)}{2} - \lambda p^n + O(\lambda^2)$$

So on declaring $p := \dot{q}$,we obtain,

$$H = \frac{\dot{q}^2}{2} - \lambda(\dot{q})^n + O(\lambda^2) \tag{4.26}$$

First thing to note is that to leading order in coupling,we have obtained the usual answer,i.e we flip the sign of potential to get the Hamiltonian from the Lagrangian.

Let's us now explain,why this process needs to be done in order to obtain the interaction terms in terms of velocity.This can be illustrated with a particle in a constant magnetic field.

Consider,the Lagrangian

$$L = m\frac{\dot{v}^2}{2} + A(t)v \tag{4.27}$$

Notice that this is not exactly the particle in a magnetic field,as there is no magnetic field in 0-

dimensions,but the idea is the same.

It's Hamiltonian in terms of the velocity is

$$H = m\frac{\dot{v}^2}{2} \tag{4.28}$$

We can now see why it is necessary to write the Hamiltonian in terms of canonical momentum in order to make the interaction obvious.Otherwise, it just looks like the Hamiltonian of a free particle and *the inversion(eqn:4.25)and substitution $p := v$ separates the actual free part from the actual interaction part.*

In our case,performing the similar but more involved analysis will give us,$H_I = -L_I$,one thing to note is that,cubic terms will also generate additional quartic terms in interacting Hamiltonian which is absent in the Lagrangian,however this terms are more slow-roll suppressed,hence we neglect them.

4.4 Loops in inflation

This loop calculation is going to be general,i.e. this analysis is true for any potential that allows slow-roll inflation.Because of this,we don't exactly know the mode functions,but since this is a slow-roll ,we take ε to be nearly constant.Additionally, to leading order in slow-roll,we can also approximate the mode function to be a mode function of massless scalar field in deSitter [44],which is due to the flatness of the potential.Eventhough,slow-roll parameters and Hubble are assumed to be constant,they do evolve a little during inflation.So to obtain a more accurate answer we set these quantities at the horizon crossing of the sum of the energies.Henceforth,it will be assumed as such.This is because, the loop integrals mostly peak at the horizon crossing(at early times,modes oscillates very rapidly and at late times integral dies of due to scale factors).

For example,free two point function is

$$< \tilde{f}_k(\eta)\tilde{f}^*_k(\eta) >= (2\pi)^3\delta^3(\tilde{k}+\tilde{k}')\frac{H^2}{2k^3}(1+k^2\eta^2)$$
$$\text{for superhorizon modes this becomes } (2\pi)^3\delta^3(\tilde{k}+\tilde{k}')\frac{H^2}{2k^3}, \quad \text{for } k\eta \ll 1$$

Before diving into the computation it will be better to understand to understand some basics of these computations and what information can be extracted without evaluating the integrals.

4.4.1 Regularisation and Renormalisation

In quantum field theory, loop calculations are often divergent.Therefore,we need to do regulate these integrals in order to make sense of the theory.So,we regulate this integrals by putting a cut-off.But,this introduces an unknown parameter. As we expect this theory to predict physical results,we can trade this unknown cutoff by equating this answer to physically measurable quantity at some energy.This way we can eliminate the unknown cutoff for a measurable result and this procedure is called renormalisation.

Now,we say that a theory is renormalisable,if we have to perform this procedure finite number of times to tame the divergence for any physical processes.Otherwise,it is called non-renormalisable.Non-renormalisable theory is also not a problem as we know that every theory is EFT.Therefore, we still have to add only finite number of counter-terms(for cancelling divergence) to get a physical answer at a required accuracy.

Dimensional Regularisation

Regulating integrals by introducing cutoff seems more natural,but it is problematic for gauge theories,as this procedure breaks the gauge invariance,which ultimately leading to answers,which doesn't agree with the experiment.

So,we often regulate the integral by analytically continuing the integral at different dimension where the integral is finite.

For example,

$$\int_0^\infty dx \frac{1}{\sqrt{x+1}}$$

is divergent.But in lower dimension which is equivalent to saying that power of x is more in the denominator than the numerator for example

$$\int_0^\infty dx \frac{1}{(x+1)^2}$$

is finite.In dimreg,we do this process continuously to analytically continue it to our four dimension.

4.4.2 Scaleless Integrals

We will be encountering loop integrals which won't be injecting external kinematic variables in the loop and it also doesn't have a mass scale.This integral will be of the form

$$\int x^\alpha d^d x \tag{4.29}$$

Now,this integral has to be regularise to zero.The peculiarity of this integral is that it is either UV or IR divergent in any dimension unlike the integral presented above which diverges for higher dimension and converges in the lower dimension.To regulate this,we break this integral into two parts,such that one has UV and other has IR divergence.Now,we can regulate this, integral in their respective dimensions where each will be evaluated to zero.Another way is to note that integral value shouldn't vary under change of variables.Therefore,

$$\int x^\alpha d^d x = \lambda^{\alpha+d} \int x'^\alpha d^d x' \tag{4.30}$$

has to be zero.

4.4.3 Can We Determine If a Diagram Vanishes before Integration?

We know that due to the structure of the In-In correlator [15, 32],we either have to take a real or imaginary part depending upon the coupling order of a feynmann-witten diagram to obtain the final answer.So ,we can ask,if it is possible to know if a given diagram evaluates to real or imaginary number without calculating the integral.So,we won't have to evaluate those diagrams; which,say, evaluates to real,but we have to take its imaginary part to obtain the final answer.Unfortunately,the answer is no.Our claim is that,this diagrams are either imaginary or real in deSitter when time integral doesn't give any divergence,but not both and it is imaginary when number of interaction vertex is odd and it is real when number of interaction vertex.Therefore,we cannot wash our hands from computing any of these diagrams,as the answer will be real when we have to take its real part and imaginary when we have to take its imaginary part .In order to show this,we have to note that each time variable from massless scalar mode function has an imaginary number accompanying it,so let's absorb the imaginary number into the time variable,i.e. we do the wick rotation, and since we have even power of scale factor present in the measure we will not be getting any imaginary number from this part.So the only part that will give us an imaginary number will be the $d\eta$ which is dependent upon the number of interaction vertices.Hence,this integral will be either real or imaginary,which is what we wanted to prove.

4.4.4 Power Counting

Power Counting For Momenta

We note that the action exhibits an approximate scale symmetry in a quasi-deSitter background and an exact scale symmetry in the pure de Sitter case.

In either case, each spatial derivative is accompanied by an inverse scale factor, in addition to the scale factors coming from the measure.

In de Sitter space, scale-invariant interactions and the vacuum impose constraints on the n-point function. Since the measure itself is scale-invariant, the scaling dimension of scalar field is zero. Consequently, correlation functions in position space remain unchanged under rescaling. Mathematically, this translates to:

$$\langle \varphi(x_1)\varphi(x_2)\dots\varphi(x_n)\rangle = \langle \varphi(\lambda x_1)\varphi(\lambda x_2)\dots\varphi(\lambda x_n)\rangle. \tag{4.31}$$

Here, the isometry requires scaling of both the time and the space. However, since we measure correlations at late times ($\eta = 0$), this effectively reduces to a spatial scaling symmetry at late times.

As a result, the Fourier transform of this correlation function (at late times) satisfies:

$$\boxed{\langle \varphi(\mathbf{k}_1)\varphi(\mathbf{k}_2)\dots\varphi(\mathbf{k}_n)\rangle \sim \frac{1}{k^{3(n-1)}}} \tag{4.32}$$

where k represents any combination of the momenta.

For instance, this implies:

$$\langle \varphi(\mathbf{k})\varphi(-\mathbf{k})\rangle \sim \frac{1}{k^3}$$

and

$$\langle \varphi(\mathbf{k}_1)\varphi(\mathbf{k}_2)\varphi(\mathbf{k}_3)\rangle \sim \frac{1}{k^6}.$$

This allows us to extract the overall momentum dependence from our loop calculations

Power Counting for Hubble and Planck Mass

Let us do the Power Counting for the n-point inflaton function in terms of the Hubble scale H and the reduced Planck mass M_{pl}.

Each external leg and internal line contribute a factor of H^2. While each interaction vertex contributes H^{-2} due to the approximate scale invariance of the action. Additionally, each vertex contributes a factor of $\frac{1}{M_{pl}^{n_i-2}}$, where n_i represents the number of fields at that vertex. This result follows from absorbing M_{pl} from the gravitational sector into the matter sector and canonically normalizing the inflaton field, which introduces a factor of M_{pl}^{-2}.

Thus, for an n-point inflaton function, the overall power of H and M_{pl} is:

$$\boxed{H^n\left(\frac{H}{M_{pl}}\right)^{\sum_i (n_i-2)V_i}} \tag{4.33}$$

where:

- n_i is the number of fields at the interaction vertex.

- V_i is the number of vertices corresponding to n_i.

For example, consider a diagram4.2(b) where:

$$n_3 = 3, \quad V_3 = 1, \quad n_4 = 4, \quad V_4 = 1.$$

Substituting these values, we obtain the overall factor:

$$H^3 \left(\frac{H}{M_{pl}} \right)^3 .$$

Similarly, we can apply this procedure to the n-point function of curvature perturbations. This yields:

$$\left(\frac{H}{M_{pl}} \right)^{n+L-1} \tag{4.34}$$

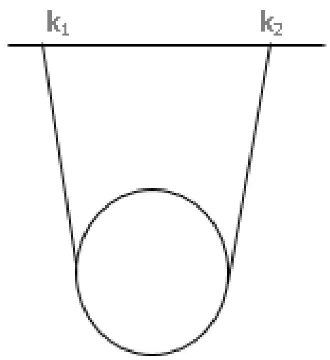
where L is the total number of loops in the diagram.

Key insight: Due to the approximate scale symmetry of the action, we can determine the power counting without explicitly knowing the interaction vertices. Normally, deriving this result would be challenging because the n-point function involves multiple dimensionful quantities—H, M_{pl}, and external momenta. However, while H and M_{pl} are dimensionful, they do not scale like momentum. By leveraging scale invariance, which dictates the momentum dependence, we can systematically determine the powers of all relevant quantities.

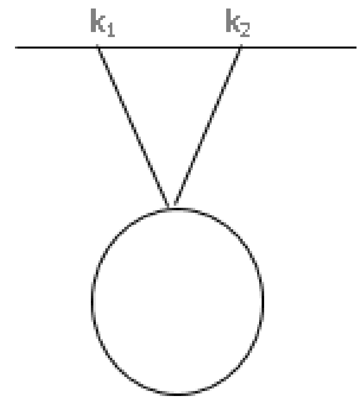
4.4.5 Diagrams

In this subsection,we will list all the diagramatic structures present in one-loop correction of power spectrum and bispectrum in inflation.We also determine the total number of diagrams(having different interactions) we need to compute.

Firstly,power spectrum gives the following diagram skeletons.



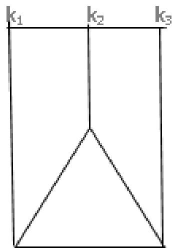
(a) Along with this, there will be one more figure where one vertex is above the horizontal line.



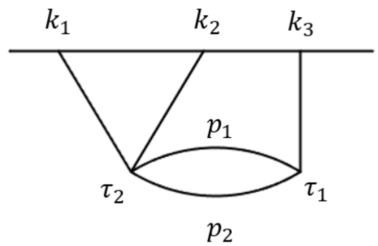
(b) Scaleless diagram

Figure 4.1: Power spectrum at one-loop

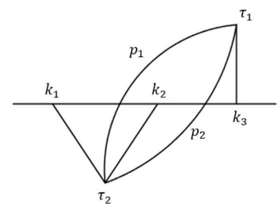
But since there are many different interaction vertex, we will have to evaluate various integrals for various interaction vertex. Moreover ,we get different kind of contractions for a fixed interaction vertex(as interaction is mixture of fields and their derivatives and hence we can't just multiply it by permutation factor to get the final answer).Taking all this into account we have to evaluate 102 distinct integrals(which also includes various contractions) in Fig:4.1(a)[This also includes the diagram mentioned in the caption.].When we do the calculation in dimreg Fig:4.1(b) doesn't contribute by scaleless argument for some interaction vertex,while for others integrand itself is divergent,for example,inverse laplacian term in quartic vertex blows off when it contracts within the loop. In cutoff regularisation scaleless diagrams also contributes.



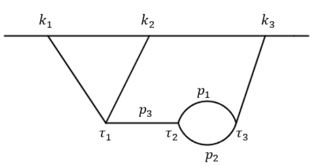
(a) Along with this, there will be four more figures where one or two vertex is above the horizontal line



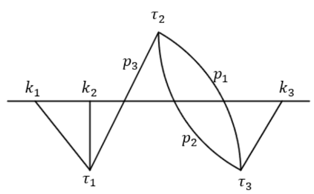
(b)



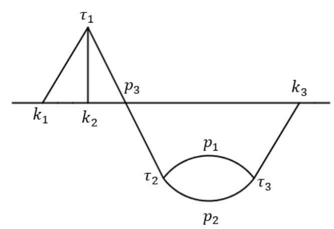
(c)



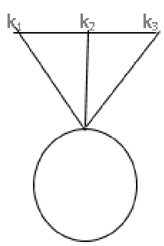
(d)



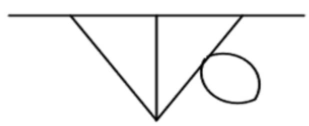
(e)



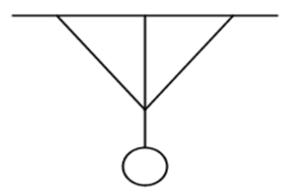
(f)



(g)



(h)



(i)

Figure 4.2: Bispectrum at one-loop(some figures are taken from [1])

Similarly,for Fig :4.2,we can try to get a rough estimate.For example,consider the interactions

$$\frac{1}{H}\frac{\partial_j}{\partial^2}(\dot{\varphi}\partial_j\varphi)\dot{\varphi}^2 \text{ and } (-\frac{\dot{\bar{\phi}}}{4\dot{\rho}}\varphi\dot{\varphi}^2)$$

interaction.We get 9 distinct contractions for the given interaction vertex of the diagram in fig:4.2(b).So,we have to evaluate atleast $8*4*3*2 = 192$ distinct contractions , as we have four kind of different quartic interactions and three kind of cubic interactions and two different diagrams: time ordered(b) and unordered(c).This is a lower bound because,the interactions we have considered have repetitions in time derivative.For instance if we have considered $4\dot{\varphi}\partial_i\varphi\frac{1}{\partial^2}(\dot{\varphi}\partial_i\varphi)$ and $\varphi\dot{\partial}_i\chi\partial_i\varphi$,we will get much larger answer.Now for fig:4.2(d,e,f),we can see that it has a subdiagram which is a power spectrum,so this will alteast contribute 2(different contraction for a given cubic vertex) $*$ 3(three kind of diagrams) $*$ 56(power spectrum) $= 336$.Finally,for triangle diagram,even if we consider the all fields to be same at a given vertex,we will atleast have to evaluate $3^3 * 5 = 135$ due to three different vertices and time and antitime ordered stuff.

In summary,we have to evaluate 102 distinct contractions to get the complete power spectrum at one-loop and atleast 663 distinct contractions to get the complete bispectrum.Note this integrals are distinct from each other and hence needs to be evaluted separately.

4.4.6 Methods to Compute Integrals

Mellin-Barnes Representation [30]:The idea of this method is to make the sum in the denominator as a ratio of this terms,this makes it easier to perform integral over this terms at the cost of introducing a Gamma functions and their integral.Mathematically,

The Mellin-Barnes representation for the function $(A+B)^{-\lambda}$ is given by:

$$\frac{1}{(A+B)^\lambda} = \frac{1}{\Gamma(\lambda)}\int_{c-i\infty}^{c+i\infty}\frac{dz}{2\pi i}\Gamma(-z)\Gamma(\lambda+z)A^zB^{-\lambda-z} \tag{4.35}$$

where:

- The contour is chosen such that it separates the poles of $\Gamma(-z)$ (at $z = 0,1,2,\ldots$) from the poles of $\Gamma(\lambda+z)$ (at $z = -\lambda, -\lambda-1, -\lambda-2, \ldots$).
- A valid choice for the contour is c such that $-\lambda < c < 0$, ensuring that the integral converges.

This identity is like doing a taylor expansion,for example:

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots$$

This is expected because this integral involves Gamma functions,which has poles at integers,so by residue theorem we will obtain the above identity.

This method could have been useful as our interaction terms involves inverse laplacian,which will generate a term having sum of internal and external momenta in denominator.However,I did not find this method useful.Eventhough, this makes the integral over momenta calculable,but it also introduces Extra Gamma functions and an integral,and this integral is incalculable because it involves Gamma functions and Generalised Hypergeometric functions.

Schwinger parametrisation:As stated,momenta in the denominators are problematic in evaluating the loop integrals in dimreg,we can also try

$$\frac{1}{(A+B)^n} = \frac{1}{\Gamma(n)} \int_0^\infty dt\, t^{n-1} e^{-t(A+B)}, \quad \text{for } A+B > 0. \tag{4.36}$$

Despite the fact that,this again brings the momenta under the exponential and can be separated by the property of exponential,momentum integral generates Bessel,Struve and trignometric functions,so final integral over t is incalculable.As,for proving the above identity,it comes from just

$$\frac{1}{A} = \int_0^\infty dt\, e^{-tA}$$

Fractional derivative [45]:We will often encounter, Integral of the form

$$\int dx x^\delta e^{ax} \ldots$$

due to the scale factor,where δ is dimreg parameter. This can be rewritten as

$$(\partial_a)^\delta \int e^{ax} \ldots$$

,this is much easier to compute and after that we taylor expand the fractional derivative to get the final answer.This can use to compute time integrals without taylor expanding the mode function(eqn:4.37)

Differential equation for Integrals [46]:The idea of this method is to convert the integral of interest _by differentiation w.r.t external kinematics_ to master integrals and the linear combination of master integrals produces large class of integral of interest.Now,the problem reduces to computing this master integrals which are computed by differentiating it with eternal kinematics and now as these master integrals span the space of integrals,we will obtain a linear differential equation for master integrals,which is easier to solve and the boundary condition is fixed by computing the master integral for kinematics which makes it computable.

Mellin transform of Hankel function [24, 25]:This method is useful for computing the loop integral for massive fields as it involves expressing Hankel functions as contour integral having Gamma function while the time and momentum integral decouples which is easier to evaluate.

Cutoff regularisation: This method has been employed in computing loop correction for simple interaction in EFToI [1, 17].However,this doesn't work in our case as it eventhough simplifies the integrand,it still generates very complicated conditional statement after angular integral.

Finally,there are some other methods such as spectral decomposition [26, 27],which uses tree level result and spectral density to compute the loop.Moreover,sometimes(when cubic vertex is present) it is useful to to use Convolution theorem,which decouples a nested integral into two separate integrals and decomposing the Dirac delta functions [47] in terms of Spherical harmonics,this allows to one to evaluate the angular integral.However,the last method is useful in the case,when we have lots of momentum contraction.

4.4.7 Loops Computation

Having discussed the possible techniques,now we will directly jump into computing this integrals in dimreg and cutoff.

We will be using deSitter approximation and use the massless mode functions of scalar in deSitter for reasons explained before.

However,even the massless mode functions are complicated in higher dimension,we analytically continue [1, 45] the mass as well in order to keep the mode-function simpler. This complexity comes from the scale factor which makes the mode function dimension dependent,unlike flat-space.More precisely, In d dimensions, the mode functions for massive fields are given by the Hankel functions $H_\nu^{(1)}(-k\tau)$, where

$$\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}.$$

We apply a modified version of dimensional regularization by analytically continuing the mass to d dimensions, using the substitution

$$\frac{m^2}{H^2} \to \left(\frac{d^2}{4} - \frac{9}{4}\right).$$

after this,the positive energy mode function becomes(other mode function will be its conjugate)

$$f_k^-(\tau) = (-H\tau)^{\delta/2} \frac{H}{\sqrt{2k^3}}(1+ik\tau)e^{-ik\tau}$$

Since,we don't expect IR-divergence coming from time as our interactions are good interactions [15] [16].In other words,as explained before,time integral converges at early time due to rapid oscillations and at late times by due to scale factors.we can perform,

$$f_k^-(\tau) = (-H\tau)^{\delta/2} \frac{H}{\sqrt{2k^3}}(1+ik\tau)e^{-ik\tau} = \left(1 + \frac{\delta}{2}\log(-H\tau)\right) \frac{H}{\sqrt{2k^3}}(1+ik\tau)e^{-ik\tau} + O(\delta^2), \tag{4.37}$$

$$a(\tau)^\delta = 1 - \delta \log(-H\tau) + O(\delta^2). \tag{4.38}$$

In our calculations, we treat the external mode functions at $\tau = 0$ and the momentum-conserving delta function $\delta^d\left(\sum_i \vec{k}_i\right)$ as three-dimensional, since the counterterms precisely cancel the corrections arising from these terms [17].

The general expression for one loop correction to bispectrum $I(k_1, k_2, k_3)$ is given by:

$$I(k_1,k_2,k_3) = \delta^3\left(\sum_i \vec{k}_i\right) \mu^{-\sum_i n_i \delta} \int d^d\vec{p}_1\, d^d\vec{p}_2\, Q(p_1, p_2, \{k_i\}, \{\tau_i\}) \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \times \prod_i \left[(-H\tau_i)^{n_i\delta} d\tau_i\right]. \tag{4.39}$$

where,μ is a scale , introduce to keep the dimension of coupling constant across dimensions.After expanding the last term,we get

$$I(k_1,k_2,k_3) = \delta^3\left(\sum_i \vec{k}_i\right)\mu^{-\sum_i n_i\delta}\int d^d\vec{p}_1 d^d\vec{p}_2\, Q(p_1,p_2,\{k_i\},\{\tau_i\})\delta^d(\vec{p}_1+\vec{p}_2-\vec{k}_3)$$
$$\times\left(1+\sum_i n_i\delta\log(-H\tau_i)+O(\delta^2)\right)\prod_i d\tau_i. \tag{4.40}$$

For making our computation simpler,we make the integral dimensionless by dividing every momenta and Hubble within the integral by one of the external energy(denoting scaled external energy by x_i and internal by $q's$),multiplying time by one of the external energy(denoted by η_i),this makes them dimensionless(we are working in natural units).We can integrate out delta function,so,we will be left with magnitude and angular integral over internal momenta,after doing change of variables for angular integral[see Appendix A of [1] for more details],we get

$$I_1(k_1,k_2,k_3) = (overall\,factor)\Bigg[\delta^3\left(\sum_i \vec{k}_i\right)S_{d-2}(1+\delta\log k_m+O(\delta^2))(1-\sum_i n_i\delta\log(\mu)+O(\delta^2))$$
$$\times\int_1^\infty dq_+\int_{-1}^1 dq_-\, 2^{-\delta-2}\left(q_+^2-q_-^2\right)\left(-\left(q_-^2-1\right)\left(q_+^2-1+\varepsilon\right)\right)^{\delta/2}Q(q_1,q_2,x_i,\eta_i)$$
$$\times\left(1+\sum_i n_i\delta\log(-H\eta_i)+O(\delta^2)\right)\prod_i d\eta_i\Bigg] \tag{4.41}$$

- overall factor for power spectrum is $\frac{H^2}{k_m^3}\left(\frac{H}{M_{pl}}\right)^2$
- overall factor for bispectrum is $\frac{H^3}{k_m^6}\left(\frac{H}{M_{pl}}\right)^3$
- k_m is an external energy with respect to which we have scaled all our variables.
- $q_1 = \frac{1}{2}(q_- + q_+), q_2 = \frac{1}{2}(q_+ - q_-)$
- S_{d-2} is a surface area of $d-2$-unit sphere
- ε is a regulator,which disappears on taking the limit $\varepsilon\to 0$ after integral evaluation.

We split the above integral into two parts: one which has $\log(-H\eta)$(let's call this J_2) and other J_1 and evaluate them separately [32]

Now,I will list a few things about this integrals,before explaining each of them.

1. $J_2 = (\log(\frac{H}{\sum k}) - EulerGamma)J_1 + \text{finite parts}$
2. Coefficient of $\log(\frac{1}{\mu})$ is equal to Coefficient $\log(-H\eta)$.This has a consequence that we always have $\log(\frac{H}{\mu})$
3. Unordered integral(interaction vertex are above and below the horizontal line)are more convergent than time ordered integral.
4. Had there been no loop integral,we will have a sum of energies in the denominator instead of getting,for instance $k_1 - k_2 - k_3$ in the denominator.
5. We will always have more power of q_+ than q_-in the denominator.So,when we don't have q_-in the denominator,q_- integral will simply be a beta function as obvious from its limits.This makes the loop integral calculable.On the other hand,when q_- is present in the denominator, its integral evaluates to be Hypergeometric functions in q_+,this makes any further integration impossible(however some terms can be integrable,see the hypergeometric section in [48])
6. We also see that we will be left with dimensional full logs,unless $\sum_i n_i = 1$ after using point 1 and 2.
7. We seems to be getting only rational polynomials of external energies as the coefficient of divergent term.

Now,we will give the reasons for each of the above points.For the first point,we should been expecting this because J_2 is nearly equal to J_1 except the log term,so after integration byparts we should be getting the result of point one.However,it is more subtle than this ,let's illustrate this with an example,Consider,

$$\int_{-\infty}^{0} \exp(ax)\, dx = \frac{1}{a}$$

and

$$\int_{-\infty}^{0} \log(x)\exp(ax)\, dx = -\frac{\log(a) + \gamma - i\pi}{a}$$

we get euler-mascheroni constant and some other terms because when we do the byparts,it will also generate Exponential integral and after using its asymtotic expansion,we get euler-mascheroni constant,the required log term and the divergent term.However,this divergent term is cancelled by the other term of the byparts at late time.Another way to predict the behaviour of J_2 is to note that since it contains polynomial and decreasing exponential(after wick rotation),it will peak somewhere in the middle which will be around the horizon crossing and since log evolves slowly,it will just take the value at the horizon crossing.

Point two can be explained by noting that each vertex of nth-order contributes $(-H\eta)^{\frac{n\delta}{2}}$,while scale factor at each vertex gives $(-H\eta)^{-\delta}$ and finally since the action should be dimensionaless(natural units),the dimension of the coupling changes by $\mu^{-(\frac{n\delta}{2}-1)}$.

Now,let us understand the third point.In time ordered integrals,when we do the inner time integral,the internal energy vanishes from the exponent as the innermost time integral ends at the next time integral variable.This has a consequence that when we do the other time integral,it won't generate any internal energy term in the denominator which help us in taming the UV divergence.In contrast,for the unordered case,since each time integral are independent,so each will generate internal energy in the denominator for each time integral.

For point four,The apparent folded poles $(k_1 - k_2 - k_3)$ vanish after a Taylor expansion, as expected. Vacuum evolution should only generate a total energy pole. Had it been the evolution of a particle state, genuine folded poles would have appeared, analogous to how energy conservation in Minkowski space translates into poles in de Sitter.

Finally,the fifth point can be easily explained by observing that exponentials at each vertex will have internal energiesq_1, q_2 with the same sign and so the time integral will generate q_+ 's in the denominator,while the rest of the terms give equal number of q_-'s and q_+'s,due to their definition.

Power Spectrum

Now,we will enlist the regularise answer when both the interaction vertex in Fig4.1(a)[time ordered] are $\varphi\dot{\varphi}^2$ for the following contractions:

External legs has no temporal derivative contraction:

$$-\frac{H^2}{k_1^3}\left(\frac{H}{M_{pl}}\right)^2\frac{\dot{\phi}^2}{8\dot{\rho}^2}\left(-\frac{1}{1536\delta}+\frac{\frac{683}{15}-8\log(4)}{24576}-\frac{1}{1536}(\log\frac{H}{\mu}-\gamma-\log(2))+\{k_1\leftrightarrow k_2\}\right)\delta^3\left(\sum_i \vec{k}_i\right)$$

When one of the external leg has no temporal derivative and other has one temporal:

$$-\frac{H^2}{k_1^3}\left(\frac{H}{M_{pl}}\right)^2\frac{\dot{\phi}^2}{4\dot{\rho}^2}\left(\frac{11}{3072\delta}+\frac{44\log(4)-\frac{436}{3}}{24576}+\frac{11}{3072}(\log\frac{H}{\mu}-\gamma-\log(2))+\{k_1\leftrightarrow k_2\}\right)\delta^3\left(\sum_i\vec{k}_i\right)$$

When each internal line has one temporal derivative ,but there is no contraction between temporal derivative term,we obtain:

$$-\frac{H^2}{k_1^3}\left(\frac{H}{M_{pl}}\right)^2\frac{\dot{\phi}^2}{4\dot{\rho}^2}\left(-\frac{5}{768\delta}-\frac{5(\log(64)-5)}{4608}-\frac{5}{768}(\log\frac{H}{\mu}-\gamma-\log(2))+\{k_1\leftrightarrow k_2\}\right)\delta^3\left(\sum_i\vec{k}_i\right)$$

We can see that we do not have any dimensional unphysical log as expected for power spectrum.

Bispectrum

Similarly for bispectrum with the interaction vertex $(\dot{\varphi})^2\frac{\partial_j}{\partial^2}(\dot{\varphi}\partial_j\varphi)$ and $\varphi\dot{\varphi}^2$,we have been able to compute only four of the contractions of Fig4.2(b) out of eight possible contractions.

1. When the external legs of quartic vertex has contractions with only temporal derivative(coming from the non-inverse laplacian term) and the external leg of cubic vertex is contracted with no temporal derivative term

2. When the external legs of quartic vertex has contractions with only temporal derivative(coming from the non-inverse laplacian term) and the external leg of cubic vertex is contracted with one of the temporal derivative term and but there is no contraction between temporal derivative term in the internal lines.

3. When the internal lines of quartic vertex has contractions with only temporal derivative(coming from the non-inverse laplacian term) and the external leg of cubic vertex is contracted with no temporal derivative term

4. When the internal lines of quartic vertex has contractions with only temporal derivative(coming from the non-inverse laplacian term) and the external leg of cubic vertex is contracted with one of the temporal derivative term but there is no contraction between temporal derivative term in the internal lines.

$$
\begin{aligned}
&\frac{H^3}{k_3^6}\left(\frac{H}{M_{pl}}\right)^3\frac{\dot{\bar{\phi}}}{2\dot{\rho}}\Bigg(-\frac{(-1458-4655x+3480x^2+29545x^3+47610x^4+43177x^5+24500x^6+8575x^7+1700x^8+150x^9)}{61440x_1x_2(1+x)^5\delta}\\
&\quad+\frac{1}{737280x_1x_2}\Bigg(\frac{80(19+x(126+x(165+4x(19+3x))))}{(1+x)^5}-\frac{216\log 2}{(1+x)^5}+\frac{180(15+2x(34+x(16+3x)))\log 2}{(1+x)^4}\\
&\qquad\qquad-\frac{160(1+2x)^2(56+90\log 2+5x(8+\log 512)+x^2(8+\log 512)))}{(1+x)^5}\\
&\quad-\frac{(3-6\log 2+x(6+x(47+72\log 2+x(-12+84\log 2+x(-57+x(4+3(5-2x)x)+30\log 2)))+\log 4096)}{(-1+x)^5(1+x)^8}\\
&\qquad\qquad 2(302+25x(75+x(93+38x+6x^2)))\\
&\qquad\qquad+6x^2(1+x)^3(-10+x(10+(-5+x)x))\log(1+\frac{1}{x}))\\
&\quad+\frac{80(1+2x)(44+5x(5+x))(-((-1+x)(3+x^3(37+x(-41+13x))))+6(-2+x)x^3(5+x(-5+2x))\log(\frac{1}{x}))}{(1-x^2)^5}\\
&\qquad+\frac{\begin{aligned}&2(302+25x(75+x(93+38x+6x^2)))(-((-1+x)(6+x^2(47+x(-40+11x))))\\&\qquad+6x^2(-10+x(10+(-5+x)x))\log(\tfrac{1}{x}))\end{aligned}}{(-1+x^2)^5}\\
&\qquad\quad+\frac{10x(1+x)^2((-1+x)(17+x(41+x(35+3x)))+24(1+x)^3\log(\frac{2}{1+x}))}{(-1+x^2)^5}\\
&+\frac{1}{(-1+x)^5(1+x)^8}(-49+x(-135+x(60+x(946+x(855+x(297+50x))))))(-9+x(-18+x(7+2x)(-2+x(4+x)))\\
&\qquad+\log 4096+12\log(\frac{1}{x})+12x(2+x)(2+x(2+x))\log(\frac{2}{x})+12(1+x)^4\log(\frac{x}{1+x}))\\
&+\frac{1}{(-1+x)^5(1+x)^8}(26+x(385+x(2039+5x(646+x(340+x(83+5x))))))(3+24x+9x^2-10x^3-21x^4-6x^5+x^6\\
&\qquad-2(1+x)^3(1+3x)\log 8-6(1+x)^3(1+3x)\log(\frac{1}{x})-6(1+x)^3(1+3x)\log(\frac{x}{1+x}))\\
&\qquad-\frac{1}{(1-x)^5(1+x)^8}40(1+2x)(44+5x(5+x))(9-12\log 2+x(-6+x(-12+72\log 2\\
&+x(104+96\log 2+x(2x(-69+x(23-12(-2+x)x))+9(-3+\log 16)))))-12(-2+x)x^3(1+x)^3(5+x(-5+2x))\log(\frac{x}{1+x}))\\
&\quad+\frac{\begin{aligned}&(273+x(2101+5x(1114+x(850+x(265+29x)))))((-1+x)(3+x(9+x(45+x(39+2(-1+x)^2x))))\\&\qquad+24x(1+x)^3(\log(\tfrac{1}{x})+\log(\tfrac{2x}{1+x})))\end{aligned}}{(-1+x)^5(1+x)^8}\\
&-\frac{\begin{aligned}&2(-14-35x+35x^2+210x^3+364x^4+119x^5+5x^6)(-6+x(-13+30\log 2)+x^2(3+36\log 2)+9x^3(1+\log 4)\\&\qquad+x^4(7+\log 8)+\log 512-3(1+x)^3(3+x)\log(1+x))\end{aligned}}{(-1+x)^5(1+x)^8}\\
&\quad-\frac{(-1458-4655x+3480x^2+29545x^3+47610x^4+43177x^5+24500x^6+8575x^7+1700x^8+150x^9))}{61440x_1x_2(1+x)^5}\\
&\qquad\qquad(\frac{3}{2}\log\frac{H}{\mu}+\frac{3}{2}\log\frac{k_3}{k_T}-\frac{1}{2}\log k_3-\gamma)+\{k_1\leftrightarrow k_2\leftrightarrow k_3\}\Bigg)\delta^3\left(\sum_i\vec{k}_i\right)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{H^3}{k_3^6}\left(\frac{H}{M_{pl}}\right)^3\frac{\dot{\phi}}{2\dot{\rho}}\Bigg(-\frac{(13+76x+185x^2+50x^3)}{15360x_1x_2(1+x)^5\delta}\\
&+\frac{1}{737280x_1x_2(x-1)^5(x+1)^8}\Big[-801+1917x+5147x^2+1849x^3+3830x^4+2706x^5-7002x^6\\
&-4302x^7-629x^8-3055x^9-545x^{10}+885x^{11}+132\log 2+540x\log 2+18312x^2\log 2+19176x^3\log 2\\
&+10260x^4\log 2+55500x^5\log 2+50904x^6\log 2+26784x^7\log 2+8280x^8\log 2+312x\log 4+1824x^2\log 4\\
&+6534x^3\log 4+18030x^4\log 4+27540x^5\log 4+14220x^6\log 4+11646x^7\log 4+6246x^8\log 4+480x^9\log 4\\
&-2388x\log 8-9540x^2\log 8-14640x^3\log 8-13938x^4\log 8-9230x^5\log 8-2440x^6\log 8-860x^7\log 8\\
&-730x^8\log 8-146x^9\log 8-40x^{10}\log 8+381x^4\log 16+2160x^5\log 16+3825x^6\log 16+3240x^7\log 16\\
&+1917x^8\log 16+900x^9\log 16+105x^{10}\log 16+312x^4\log 32+1824x^5\log 32+4440x^6\log 32+1200x^7\log 32\\
&-199\log 64-795x\log 64-1230x^2\log 64-3196x^3\log 64-9382x^4\log 64-14900x^5\log 64-15750x^6\log 64\\
&-11968x^7\log 64-4655x^8\log 64-645x^9\log 64+762x\log 256+2529x^2\log 256+495x^3\log 256-4590x^4\log 256\\
&-7020x^5\log 256-5715x^6\log 256-1725x^7\log 256-106\log 512-610x\log 512-1460x^2\log 512+1580x^3\log 512\\
&+5350x^4\log 512-106x^5\log 512-40x^6\log 512+146\log 4096+820x\log 4096+740x^2\log 4096+1116x^3\log 4096\\
&+2730x^4\log 4096+3804x^5\log 4096+2970x^6\log 4096+1560x^7\log 4096-30x^8\log 4096-6(1+x)^3(115+1046x\\
&+2715x^2+1950x^3-6435x^4+5238x^5+6725x^6-8090x^7+2400x^8)\log(1+x)+6(1+x)^3(159+968x+3015x^2\\
&+2280x^3-6135x^4+5404x^5+6833x^6-8140x^7+2400x^8)\log x-954\log x-8670x\log x-35256x^2\log x\\
&-79368x^3\log x-56148x^4\log x+32796x^5\log x-25920x^6\log x-128616x^7\log x-23298x^8\log x+62322x^9\log x\\
&+5640x^{10}\log x-14400x^{11}\log x+954\log(1+x)+8670x\log(1+x)+35256x^2\log(1+x)+79368x^3\log(1+x)\\
&+56148x^4\log(1+x)-32796x^5\log(1+x)+25920x^6\log(1+x)+128616x^7\log(1+x)+23298x^8\log(1+x)\\
&-62322x^9\log(1+x)-5640x^{10}\log(1+x)+14400x^{11}\log(1+x)\Big]\\
&+\frac{1}{230400\sqrt{2}(1-x^2)^5}\Big[(-1+x)(671-x(4242+x(18092+x(40617+x(25967+x(-19362+25x(-164+183x)))))))\\
&+60x^2(-65+x(-680+x(-1030+x(983+x(-349-65x+50x^2)))))\log(1/x)\Big]\\
&-\frac{(13+76x+185x^2+50x^3)}{15360x_1x_2(1+x)^5}\\
&(\frac{3}{2}\log\frac{H}{\mu}+\frac{3}{2}\log\frac{k_3}{k_T}-\frac{1}{2}\log k_3-\gamma)+\{k_1\leftrightarrow k_2\leftrightarrow k_3\}\Bigg)\delta^3\left(\sum_i\vec{k}_i\right)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{H^3}{k_3^6}\left(\frac{H}{M_{pl}}\right)^3\frac{\dot{\phi}}{4\dot{\rho}}\frac{\vec{k_1}\cdot\vec{k}_3}{k_3^2}\\
&\Bigg(-\frac{(-14-73x-59x^2+150x^3+300x^4+195x^5+45x^6-3x_1-15xx_1+450x^2x_1+450x^3x_1+225x^4x_1+45x^5x_1))}{245760x_1^3x_2(1+x)^5\delta}\\
&+\frac{1}{1474560x_1^3x_2(-1+x^2)^5}\Bigg((1+x)\Bigg(42\gamma+2385x-33\gamma x-9999x^2-465\gamma x^2-10368x^3+900\gamma x^3+59088x^4+240\gamma x^4-22902x^5\\
&-1542\gamma x^5-66750x^6+498\gamma x^6+42920x^7+900\gamma x^7+19700x^8-450\gamma x^8+13425x^9-225\gamma x^9+7281x^{10}+135\gamma x^{10}+36\log 2\\
&-18x\log 2-450x^2\log 2+360x^3\log 2+2160x^4\log 2-3996x^5\log 2+1188x^6\log 2+1800x^7\log 2-900x^8\log 2-450x^9\log 2\\
&+270x^{10}\log 2-18(-1+x)^5(-2-9x+50x^3+50x^4+15x^5)\log(-1-x)-6x^3(1220-980x-10951x^2+17019x^3-4650x^4-4800x^5\\
&+2325x^6+105x^7)\log(\frac{1}{x})+36\log x-18x\log x-450x^2\log x+360x^3\log x+2160x^4\log x-3996x^5\log x+1188x^6\log x\\
&+1800x^7\log x-900x^8\log x-450x^9\log x+270x^{10}\log x-36\log x+18x\log x-2310x^2\log x+5940x^3\log x+4620x^4\log x\\
&-3318x^5\log x-642x^6\log x+42\psi\left(\frac{7}{2}\right)-33x\psi\left(\frac{7}{2}\right)-465x^2\psi\left(\frac{7}{2}\right)+900x^3\psi\left(\frac{7}{2}\right)+240x^4\psi\left(\frac{7}{2}\right)-1542x^5\psi\left(\frac{7}{2}\right)\\
&\qquad+498x^6\psi\left(\frac{7}{2}\right)+900x^7\psi\left(\frac{7}{2}\right)-450x^8\psi\left(\frac{7}{2}\right)-225x^9\psi\left(\frac{7}{2}\right)+135x^{10}\psi\left(\frac{7}{2}\right)\Bigg)\\
&+3(-189+3\gamma-1656x-16827x^2-495\gamma x^2+26800x^3+1920\gamma x^3+39742x^4-2610\gamma x^4-64544x^5+1152\gamma x^5-14998x^6+210\gamma x^6\\
&+42280x^7+28415x^8-225\gamma x^8+25920x^9-1023x^{10}+45\gamma x^{10}-192x\log 2-30x^2\log 2+1920x^3\log 2-3300x^4\log 2+1344x^5\log 2\\
&+612x^6\log 2-450x^8\log 2+90x^{10}\log 2+\log 64-6(-1+x)^5(-1+27x+150x^2+150x^3+75x^4+15x^5)\log(1+x)\\
&-6x^3(780-8075x+1516x^2+10805x^3-8500x^4+975x^5+300x^6+135x^7)\log(\frac{1}{x})+6\log x-192x\log x-30x^2\log x+1920x^3\log x\\
&-3300x^4\log x+1344x^5\log x+612x^6\log x-450x^8\log x+90x^{10}\log x-6\log x+192x\log x+2490x^2\log x+10320x^3\log x\\
&+1230x^4\log x-1200x^5\log x-642x^6\log x+3\psi\left(\frac{7}{2}\right)-495x^2\psi\left(\frac{7}{2}\right)+1920x^3\psi\left(\frac{7}{2}\right)-2610x^4\psi\left(\frac{7}{2}\right)+1152x^5\psi\left(\frac{7}{2}\right)\\
&\qquad+210x^6\psi\left(\frac{7}{2}\right)-225x^8\psi\left(\frac{7}{2}\right)+45x^{10}\psi\left(\frac{7}{2}\right))x_1\Bigg)\\
&-\frac{(-14-73x-59x^2+150x^3+300x^4+195x^5+45x^6-3x_1-15xx_1+450x^2x_1+450x^3x_1+225x^4x_1+45x^5x_1))}{245760x_1^3x_2(1+x)^5}\\
&\qquad(\frac{3}{2}\log\frac{H}{\mu}+\frac{3}{2}\log\frac{k_3}{k_T}-\frac{1}{2}\log k_3-\gamma)+\{k_1\leftrightarrow k_2\leftrightarrow k_3\}\Bigg)\delta^3\left(\sum_i\vec{k}_i\right)
\end{aligned}
$$

where $x_i=\frac{k_i}{k_3}, x=\frac{k_1+k_2}{k_3}$, ψ is digamma function, γ is euler-mascheroni constant

$$
-\frac{H^3}{k_3^6}\left(\frac{H}{M_{pl}}\right)^3\frac{\dot{\phi}}{2\dot{\rho}}\frac{\vec{k_1}\vec{k}_3}{k_3^2}\Bigg(\frac{(x(1+x)+(-1+3x)x_1)}{1024x_1^3x_2(1+x)^5\delta}+\frac{1}{73728x_1^3x_2}
$$

$$
\Bigg(-\frac{((-2+6x-6x^2+3x^7-x^3(37+24\log 2)+x^6(6-8\log 8)-6x^5(-6+\log 4096)-6x^4(1+\log 4096)+24x^3(1+x)^3\log(1+\frac{1}{x}))(x+3x_1)}{(-1+x)^5x^3(1+x)^3}
$$

$$
-\frac{18(1+x+4x_1)}{(-1+x^2)^5}\Bigg(3+9x-30x^2+30x^3+92x^4-240x^5+183x^6-47x^7+3\gamma(-1+x)^5(3+8x+10x^2)-18\log 2+42x\log 2-210x^4\log 2+378x^5\log 2
$$

$$
-252x^6\log 2+60x^7\log 2+6x^4(-35+63x-42x^2+10x^3)\log(\frac{1}{x})-9\psi\left(\frac{3}{2}\right)+21x\psi\left(\frac{3}{2}\right)-105x^4\psi\left(\frac{3}{2}\right)+189x^5\psi\left(\frac{3}{2}\right)-126x^6\psi\left(\frac{3}{2}\right)
$$

$$
+30x^7\psi\left(\frac{3}{2}\right)\Bigg)
$$

$$
+\frac{24(4+11x+7x^2+3(5+9x)x_1)}{(-1+x^2)^5}\Bigg(6x^3(-10+15x-9x^2+2x^3)\log(\frac{1}{x})+(-1+x)(-3+3\gamma(-1+x)^4(1+2x)+\log 64+x^4(41-42\log 2-21\psi\left(\frac{3}{2}\right))
$$

$$
+3\psi\left(\frac{3}{2}\right)-6x(\log 4+\psi\left(\frac{3}{2}\right))-6x^2(\log 4+\psi\left(\frac{3}{2}\right))+x^5(-13+\log 4096+6\psi\left(\frac{3}{2}\right))+x^3(-37+48\log 2+24\psi\left(\frac{3}{2}\right)))\Bigg)
$$

$$
-\frac{24(1+8x+21x^2+20x^3+6x^4+(4+26x+51x^2+21x^3)x_1)}{(-1+x)^5(1+x)^8}\Bigg(3+32x+6x^2-24x^3-17x^4+24x\log 2+72x^2\log 2+24x^4\log 2+6x^3\log 4096
$$

$$
-24x(1+x)^3\log(1+x)+24x(1+x)^3\log(x)\Bigg)
$$

$$
+\frac{(13+89x+310x^2+486x^3+315x^4+63x^5+72(1+5x+13x^2+13x^3+3x^4)x_1)}{(-1+x)^5(1+x)^8}\Bigg(14+18x+6x^2-26x^3-12x^4+12x\log 8+\log 64+10x^3\log 64
$$

$$
+3x^4\log 64+9x^2\log 256+6(1+x)^3(1+3x)\log x-6(1+x)^3(1+3x)\log(1+x)\Bigg)
$$

$$
-\frac{(-1-3x+10x^3+25x^4+23x^5+6x^6+6(1+5x+10x^2+10x^3+11x^4+3x^5)x_1)}{(-1+x)^5x^2(1+x)^8}\Bigg(-1+6x+21x^2+10x^3-9x^4-24x^5-3x^6+60x^3\log 2
$$

$$
+72x^4\log 2+18x^5\log 4+x^6\log 64+2x^2\log 512+6x^2(1+x)^3(3+x)\log x-6x^2(1+x)^3(3+x)\log(1+x)\Bigg)
$$

$$
+\frac{((1+x)(2+21x+64x^2+126x^3+78x^4+9x^5)+3(13+65x+130x^2+178x^3+93x^4+9x^5)x_1)}{(1-x)^5x(1+x)^8}\Bigg(2+15x+24x^2-14x^3-18x^4-9x^5
$$

$$
+12x\log 2+72x^3\log 2+24x^4\log 4+3x^5\log 16+6x^2\log 256+12x(1+x)^4\log x-12x(1+x)^4\log(1+x)\Bigg)
$$

$$
-\frac{1}{x^3(-1+x^2)^5}\Bigg(x(1+x)(1-x+28x^2-732x^3-2380x^4-1744x^5+4844x^6+572x^7-957x^8+369x^9
$$

$$
-24x^2(-3+4x+92x^2+193x^3+52x^4+4x^5+6x^6)\log x)
$$

$$
+6(1-19x^2-88x^3-666x^4-1522x^5-834x^6+3180x^7+281x^8-546x^9+213x^{10}-12x^3(1+26x+145x^2+260x^3+68x^4+6x^5+6x^6)\log x)x_1\Bigg)
$$

$$
+\frac{6}{(-1+x)^5(1+x)^8}\Bigg((1+x)(-15+102x+593x^2+1006x^3-81x^4-1046x^5-541x^6-30x^7+12x^8+12x\log 2+396x^2\log 2+1896x^3\log 2
$$

$$
+3432x^4\log 2+2700x^5\log 2+780x^6\log 2+12x^2(1+x)^3(35+55x+10x^2-5x^3+x^4)\log x-12x^2(1+x)^3(35+55x+10x^2-5x^3+x^4)\log(1+x))
$$

$$
+2(-24+186x+1079x^2+1857x^3-156x^4-1928x^5-981x^6-51x^7+18x^8+30x\log 2+756x^2\log 2+3468x^3\log 2+6258x^4\log 2+4950x^5\log 2
$$

$$
+1440x^6\log 2-\log 64+6x^2(1+x)^3(130+200x+35x^2-16x^3+3x^4)\log x-6x^2(1+x)^3(130+200x+35x^2-16x^3+3x^4)\log(1+x))x_1\Bigg)
$$

$$
+\frac{6(7+41x+60x^2+26x^3+2(13+63x+48x^2)x_1)}{(1+x)^5}\Bigg(4(1+x)(4+17x+13x^2+3(5+16x)x_1)
$$

$$
-\frac{1}{(-1+x)^5}(4+3\gamma(-1+x)^5+4x+27x^2-67x^3+41x^4-9x^5-6\log 2+30x\log 2-60x^2\log 2+60x^3\log 2-30x^4\log 2+x^5\log 64
$$

$$
+6x^2(-10+10x-5x^2+x^3)\log(\frac{1}{x})-3\psi\left(\frac{3}{2}\right)+15x\psi\left(\frac{3}{2}\right)-30x^2\psi\left(\frac{3}{2}\right)+30x^3\psi\left(\frac{3}{2}\right)-15x^4\psi\left(\frac{3}{2}\right)+3x^5\psi\left(\frac{3}{2}\right)\Bigg)
$$

$$
+\frac{(x(1+x)+(-1+3x)x_1)}{1024x_1^3x_2(1+x)^5}(\frac{3}{2}\log\frac{H}{\mu}+\frac{3}{2}\log\frac{k_3}{k_T}-\frac{1}{2}\log k_3-\gamma)+\{k_1\leftrightarrow k_2\leftrightarrow k_3\}\Bigg)\delta^3\left(\sum_i\vec{k}_i\right)
$$

Chapter 5

Conclusion

We have derived some perturbative constraint on wavefunction coefficients using the unitarity of Bunch-Davies S-matrix in deSitter.In particular,we have been able to reproduce some(tree-level) cutting rules and cosmological optical theorem through alternative method.But,the full potential of Bunch-Davies S-matrix can be released when we obtain a non-perturbative constraint on the wavefunction coefficient.This can be done if one is able to correct the wavefunction and S-matrix relation by carefully noting all combinatorial factors in its derivation.

Even though we have been able to evaluate only some of the contractions in the one-loop correction to the power spectrum and bispectrum due to technical challenges, we have gained a better understanding of the origin of various terms. This insight may help in addressing the challenges of loop calculations and successfully evaluating these integrals in the future. We also gained important insights, such as why unordered loops exhibit greater convergence than time-ordered loops. Additionally, we obtained dimensionful logarithms of the comoving energy, which we expect to cancel after renormalization, as their presence would break the scale isometry of the theory. Moreover, the apparent folded poles $(k_1 - k_2 - k_3)$ also cancel after a Taylor expansion. This is expected, as the evolution of the vacuum should only produce a total energy pole. Had it been the evolution of a particle state, we would have instead obtained genuine folded poles, analogous to flat-space scenarios where the energy conservation delta function in Minkowski space is translated into poles in de Sitter space.

We also identified interesting properties of the interaction terms in the inflaton action, such as the vanishing of odd interaction terms in the de Sitter limit. Furthermore,we found that due to scale symmetry, each spatial derivative term in the action will be accompanied by an inverse of scale factor and each interaction vertex will effectively contribute H^{-2}.This may help us gain a better understanding of the structure of higher order interaction vertices.

It would be interesting to evaluate bispectrum contractions where the inverse Laplacian contributes non-trivially, as we expect the addition of collinear divergences to generate higher-order poles. In such cases, we will also need to consider terms like $(\log(-H\eta))^2$. These contributions may lead to new intricate analytical structures, including dilogarithms, polylogarithms, and hypergeometric functions. Additionally, one should investigate the infrared (IR) divergences present in these calculations, which typically arise in diagrams with fewer time derivatives and non-local terms. Furthermore, the integrand of the scaleless loop involving quartic and quintic vertices itself diverges due to non-local terms, making it crucial to understand such contributions in order to achieve a complete understanding of loop effects.

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