

Ultracold atoms in 1-D optical lattices: experiments towards quantum chaos and atom interferometry

A thesis

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Doctor of Philosophy

by:

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To my **Family**

Certificate

Certified that the work incorporated in the thesis entitled **“Ultracold atoms in 1-D optical lattices: experiments towards quantum chaos and atom interferometry”** submitted by **Sumit Sarkar** was carried out by the candidate, under my supervision. The work presented here or any part of it has not been included in any other thesis submitted previously for the award of any degree or diploma from any other university or institution.

Date: April 11, 2019

(Supervisor)

Declaration

I declare that this written submission entitled “**Ultracold atoms in 1-D optical lattices: experiments towards quantum chaos and atom interferometry**” represents my ideas in my own words and where others’ ideas have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

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(Sumit Sarkar)

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Abstract

The present thesis aims at addressing two major problems using the interaction of ultracold atoms with 1-D lattice. The initial goal was to set up a cross-optical dipole to generate a Bose-einstein condensate. This would provide a basic platform in order to realize the problems of our interest. An existing experimental set up capable of producing laser-cooled atoms was used to realize the Bose-Einstein Condensation in an optical dipole trap. The problems addressed using this system are as follows:

- (a) Demonstration of non-exponential quantum decoherence in a Levy kicked rotor system. and
- (b) Realization of diffraction of an atom laser in Raman-Nath regime.

A major part of this thesis was to demonstrate experimentally the non-exponential decoherence in a Levy kicked rotor system. This experiment addresses one of the fundamental problems in quantum mechanics, known as decoherence. In general, quantum systems lose coherence exponentially fast upon interacting with the environment, and this puts a limit on some technological applications using Quantum systems, for e.g. on the Quantum gate-operation time and efficiency in Quantum information processing and precision metrology. In this work, we used an atom optics delta-kicked rotor subjected to non-stationary Levy noise in the kicking sequence and demonstrated slower than exponential decoherence manifested in the form of sub-diffusion in the mean energy growth of the rotor. To realize the system experimentally using cold atoms, we used a cold cloud of atoms having Maxwell-Boltzmann distribution in momentum space and made it to interact with a pulsed 1-D optical lattice to realize the kicking sequence. By controlling the kicking sequence, we studied the decay of coherence in the system. This work is extremely relevant in the context of understanding the dynamics in disordered nonlinear lattices and diffusion in chaotic quantum systems. This work will also be helpful in understanding fundamental problems related to quantum mechanics and non-linear dynamics of chaotic systems.

Another principal objective of this thesis work was to generate a slow coherent beam of ultracold atoms (known as atom laser) from the reservoir of BEC and demonstrate the diffraction of an atom laser. We form the atom laser by outcoupling a quasi-continuous beam of coherent atoms from a reservoir of ^{87}Rb BEC by spilling method. The flux of the outcoupled atoms is controlled to achieve a continuous wave atom laser lasting up

to 400 ms. The diffraction of this atom laser in Raman-Nath regime is realized using a grating formed by a standing wave of far-detuned laser light. By controlling the interaction of the atom laser and light-grating, the strength of the diffraction into various orders is precisely controlled, and momentum up to $\pm 18\hbar k$ is imparted. Such diffraction would allow for further construction of an atom-interferometer to probe changes in physical environments continuously up to a few hundred milliseconds. This would provide a novel platform to construct atom interferometers which can provide measurement with high bandwidth and free from aliasing effects.

List of publications

- Non-exponential decoherence and subdiffusion in atom-optics kicked rotor; **Sumit Sarkar**, Sanku Paul, Chetan Vishwakarma, Sunil Kumar, Gunjan Verma, M. Sainath, Umakant D. Rapol, and M. S. Santhanam, Physical Review Letters, **118**, 174101 (2017).
- Diffraction of an atom laser in the Raman-Nath regime; **Sumit Sarkar**, Jay Mangaonkar, Chetan Vishwakarma, and Umakant D. Rapol, Physical Review A, **98**, 043625 (2018).
- Bose-Einstein condensation in an electro-pneumatically transformed quadrupole-Ioffe magnetic trap; Sunil Kumar, **Sumit Sarkar**, Gunjan Verma, Chetan Vishwakarma, Md Noaman, and Umakant Rapol, New Journal of Physics, **17**(2), 023062 (2015).
- A simple atomic beam oven with a metal thermal break; Chetan Vishwakarma, Jay Mangaonkar, Kushal Patel, Gunjan Verma, **Sumit Sarkar**, and Umakant Rapol, *Manuscript under review*.

List of Figures

1.1	Optical lattice potentials: (a) 1-D optical lattice formed by a pair of counterpropagating laser beams. Atoms are confined in an array of 2-dimensional disks, (b) 2-D optical lattice formed by superimposing a pair of orthogonal 1-D lattices. The atoms are confined in an array of 1-dimensional tubes, (c) 3-D optical lattice formed by superimposing three orthogonal 1-D optical lattices. In this case, atoms are confined in a cubic array of potential wells.	3
1.2	Atoms in optical lattices: Atoms sitting in the potential wells of one plane of a 3-D optical lattice similar to an egg-crate.	5
1.3	Optical lattices with triangular and honeycomb symmetry: (a) A possible scheme to realize an optical lattice with triangular symmetry. Three linearly polarized laser beams are aligned in a plane with their polarization vector perpendicular to their plane. The angle between the consecutive laser beams are kept $\pi/3$. The potential minimas are formed with triangular symmetry in space. (b) A possible schematic to realize a honeycomb symmetry in an optical lattice. The geometry of the laser beams remains same as the previous case, but the polarizations are kept in the plane as shown in figure. Every alternate minima possesses opposite circular polarization of light.	6

1.4	Band structure of a 1-D optical lattice: Band structure of 1-D optical lattice in reduced zone scheme. (a) Energy dispersion curve for a free particle ($V_0 = 0$); and (b), (c), (d) correspond to energy dispersion for $V_0 = 3E_R$, $V_0 = 6E_R$, $V_0 = 20E_R$ respectively.	7
1.5	Sisyphus cooling of ^{87}Rb atoms: Two counterpropagating laser beams in $\text{lin} \perp \text{lin}$ configuration exhibit polarization gradient over space. Atoms in degenerate ground state travels the potential hill and loses its kinetic energy. Then it decays back to the other ground state via an excited state.	9
1.6	Schematic of a Mach-Zehnder atom interferometer: Atoms pass through three pulsed gratings used in $\pi/2 - \pi - \pi/2$ configuration resulting in an overall phase accumulation, that depends on the effective enclosed area of the interferometer.	18
2.1	Condensate fraction: Condensate fraction vs temperature for an ideal gas in a harmonic trap given in Eq. 2.10	27
2.2	Doppler force: The Doppler force on an atom in a counter-propagating pair of laser beams as a function of the velocity of the atom for: $I/I_{\text{sat}} = 0.1$ and $\Delta = -\Gamma$. The blue dashed curve shows the force for the effective blue detuned laser, the red dotted curve shows the force due to the effective red-detuned laser and the curve in solid line shows the sum of these two forces and shows the net force is dissipative.	33
2.3	Magneto-optical trap (MOT): Configuration of the quadrupolar magnetic field along with the desired polarizations of laser beams in a MOT. The black arrows indicate the direction of current, and the notation on each beam (σ^+/σ^-) indicates the type of transition it drives in the trapped atoms.	36
2.4	MOT operation: Illustration of the operating principle of a MOT in a $J = 0,1$ system. 'z' can be considered as distance from the trap center along the direction of any pair of laser beams shown in fig. 2.3. Dotted lines show the spatial modulation of energy of the Zeeman sub-levels due to the quadrupolar magnetic field. Dashed line represents the equivalent energy level addressed by the laser frequency.	37

2.5	Focused gaussian laser beam: Variation of intensity of a focused gaussian laser beam. Variation of beam waist and divergence of the beam is shown with increasing distance from the focus. θ is the divergence of the beam.	43
2.6	Combined potential: Variation of potential energy as a function of vertical distance z from the trap center. (a) potential due to individual contribution. Green dashed curve represents the magnetic potential. Blue dotted curve represents the potential due to gravity. Solid red curve shows a nearly harmonic potential due to crossed optical dipole trap. (b) Black dashed curve represents the combined potential. In both the cases, -ve z -axis represents the direction of gravity as shown by the arrow.	45
2.7	Atom laser vs. optical laser: Analogy between the outcoupling mechanism of optical laser and atom laser. Atom trap is similar to the optical cavity. BEC serves as the macroscopic state from which an atom laser is outcoupled.	46
2.8	Poincare section of kicked rotor: Poincare section of kicked rotor with varying kick strength. The corresponding values of kick strengths are (from the left): $k = 0$, $k = 0.5$, $k = 0.9$, and $k = 5.5$. For $k > 5.5$ the phase space becomes totally chaotic. Here ρ is the scaled momentum and θ represents the scaled position.	52
3.1	Oven chamber: Rb oven-side chamber in the experiment.	67
3.2	Vacuum assembly: An image of the experimental setup of vacuum assembly.	68
3.3	Vacuum chamber layout: Schematic of vacuum chamber assembly used in the experiment.	70
3.4	Laser system: Laser frequencies with reference to the ^{87}Rb transition lines.	71
3.5	Laser table: Photograph showing the laser table with lasers and other optics used in the experiment.	72
3.6	AOM controller: Schematic of AOM control circuit.	74
3.7	Magnetic coils assembly: Schematic of magnetic coils assembly showing the quadrupole coils and shimming coils.	75

3.8	Quadrupole coils winding: Photograph displays the winding of one of the quadrupole coils. The spacer ensures sufficient space for the flow of water used to cool down the coil assembly.	76
3.9	Quadrupole coil assembly: Image displays the quadrupolar coil assembly along with the shimming coils used in the experiment	76
3.10	MOT schematic: Schematic representation of the Zeeman slower laser beam and the magneto-optical trapping laser beams with retroreflected geometry using $\lambda/4$ and mirror. The mirror and $\lambda/4$ plate is not shown in the vertical path.	77
3.11	MOT: Photograph showing the fluorescence of atomic cloud trapped in the MOT at the center of the quadrupole coils. The photograph has been taken by putting a black aluminium foil in the background to enhance the contrast of the photograph.	78
3.12	Dipole trap laser : The photograph of Ytterbium linearly polarised single frequency CW laser controller from IPG Photonics.	80
3.13	Dipole trap laser collimator : The photograph of the collimator connected at the output of the fiber laser.	82
3.14	Optical dipole trap-laser table layout : A schematic diagram of preparation of two independent laser beam for a crossed optical dipole trap. The undeflected zeroth order beam from the AOM is not shown in the figure. The figure does not also display the change in the beam waist of the laser over the distance in order to simplify the diagram.	83
3.15	Laser beam preparation for crossed optical dipole trap : The photograph of the experimental setup to extract laser beams for crossed optical dipole trap.	84
3.16	Beam profile of laser beam 1 : Spatial beam profile of the laser after passing through 110 MHz AOM and the final telescope. (a) and (b) are 2-D and 3-D image respectively of the beam profile as recorded by the profilometer. (c) and (d) represent respectively the horizontal and vertical cross-section plot of the intensity of the laser beam. The solid lines are the measured data, and the dashed lines represent the Gaussian fit.	85

3.17	Beam profile of laser beam 2 : Spatial beam profile of the laser after passing through 120 MHz AOM and the final telescope. (a) and (b) are 2-D and 3-D image respectively of the beam profile as recorded by the profilometer. (c) and (d) represent respectively the horizontal and vertical cross-section plot of the intensity of the laser beam. The solid lines are the measured data, and the dashed lines represent the Gaussian fit.	85
3.18	Optical dipole trap potential : Trapping potential formed by a single far detuned focused Gaussian laser beam with a beam waist of $35\ \mu\text{m}$ at the focus and varying powers (a) 5.5 W, (b) 4.5 W, (c) 3.5 W, and (d) 2.5 W.	86
3.19	Dipole trap alignment: Photograph displays the side view of the focusing lens and probe. The probe is perpendicular to the length of the glass cell.	87
3.20	Angle between the dipole trap beams: Image displays the angles between the pair of dipole trap beams along with the orientation of MOT and probe beam	87
3.21	Optical dipole trap geometry : Top view of the schematic of the position and orientation of all the laser beam in horizontal plane with respect to the quadrupole field center is shown. The bottom quadrupole coil and the horizontal cross-section of the glass cell are shown for reference. M_1 and M_2 are the MOT beams present in the horizontal plane, and D_1 and D_2 are the dipole trap beams. 'P' is the probe beam. 'BD' represents beam dump used to block the high power laser beam for safety.	88
3.22	Experimental sequence: The five major part of the sequence in order. (1) the loading of a MOT, (2) MOT stabilization, (3) compression of the MOT, (4) depumping of atoms into the ground state, and (5) evaporative cooling. For the sake of simplicity the last step consists of probing the atoms is not shown here. Image not to scale.	89

3.23	Loading a crossed dipole trap: Absorption image of the cold atoms trapped in the crossed optical dipole trap (bright crossed lines with high optical density) and comparatively hotter atoms falling under gravity (diffused image with lower optical density).	90
3.24	Evaporative cooling mechanism: Cooling of atoms with reducing trap depth. (a) Deeper potential with high confinement, (b) After reducing the depth of the potential adiabatically, atoms near the bottom of the trap start cooling.	93
3.25	Absorption imaging: Experimental setup used for absorption imaging. The path between the imaging lens and camera is covered using tubes to block stray lights to enter into the camera.	96
3.26	Observation of Bose-Einstein Condensation (BEC): Typical absorption images of the ultracold cloud of atoms taken after 10 ms of expansion under gravity. (a) Thermal cloud at a temperature about of $1\ \mu\text{K}$, (b) bimodal density distribution at temperature 330 nK showing the emergence of a sharp parabolic peak at the center with a condensate fraction of $\sim 25\%$. (c) Almost pure condensate at the final stage of evaporation; the condensate fraction is 95%. The lower set of curves is the horizontal cross-section plots of the images above. Dashed lines represent the Gaussian fit to the thermal atoms.	97
3.27	Evolution of condensate fraction: Typical growth of condensate fraction as a function of the temperature of the atomic cloud.	98
3.28	Cooling rate during evaporation: Decrease in mean temperature of the atomic cloud after different stages of evaporation. Symbols represent experimental data. Dashed line is guide to eye.	99
3.29	Evolution of phase space density: Increase in phase space density with evaporation. The PSD is plotted against the final power of single beam in each case. Symbols represent experimental data. Dashed line is guide to eye.	100
3.30	Loss of atom no during evaporative cooling: Number of atoms remaining in the trap after different stages of evaporation. Symbols represent experimental data. Dashed line is guide to eye.	101

3.31	Spin purity of BEC: Deflection of the condensate in one particula direction with increasing field gradient from (a) to (d). The field of view of each frame is $0.8 \text{ mm} \times 0.4 \text{ mm}$. (a) represents the case with zero magnetic field.	102
3.32	Lifetime of BEC: Decay of number of atoms in BEC with time. An exponential profile is fit to the experimentally measured data. The half-life of the atoms are found to be 15 seconds.	103
3.33	Measurement of trap frequency along horizontal direction: Oscillation of center of mass of the atomic cloud in horizontal direction. . . .	103
3.34	Measurement of trap frequency along vertical direction: Oscillation of center of mass of the atomic cloud in horizontal direction.	104
4.1	Experimental schematic: Experimental schematic shows the 1-D optical lattice and the absorption imaging system below the lattice. Off-resonant laser light after passing through an acousto-optic modulator, single-mode fibre and beam expansion creates a 1-D standing wave. . .	110
4.2	Dynamical localization: Suppression of normal diffusion of energy in a quantum kicked rotor. Solid line represents the classical energy growth. Symbols represent experimentally measured energy of a quantum kicked rotor. Dashed line is a guide to eye. The error in the estimation of energy is about $\pm 5\%$ of the measured value, which has not been shown in the graph.	112
4.3	localization: Momentum distribution of atomic cloud (a) Red – without kicks, (b) Blue – after 90 kicks. For each case, symbols represent the experimental data. Initial momentum distribution shows a gaussian wavepacket. Solid red line represents a gaussian fit to the experimental data. Momentum distribution after 90 kicks shows exponential bevavior. Solid blue line is an exponential fit to the data. The decay constant comes out to be 5 which is nearly equal to the localization length in momentum units.	113

4.4	Momentum spread of atomic cloud with and without kicks: Absorption image of atomic cloud with and without kicks. Due to the kicking process, the width of the atomic cloud has increased along the direction of the kicking lattice. The extension of the atomic cloud perpendicular to the direction of lattice remains unchanged.	113
4.5	Kicks statistics: Schematic representation of appearance of kicks for different Lévy exponents in comparison to a periodically kicked standard kicked rotor.	114
4.6	Effect of Lévy noise: Measured mean energy with $K \approx 5.8$ for periodic kick sequence (circles) and for $\alpha = 0.75$ (triangles). Dashed lines are the analytical results and solid lines are from numerical calculations. Error bars represent the standard deviation of energy measurement over 14 Lévy noise realizations. Black solid line is the simulated energy growth for classical kicked rotor with $\alpha = 0.75$. Inset shows the classical stroboscopic section for $\alpha = 0.75$. Momentum profiles corresponding to labels (a,b,c) are shown in Fig. refalpha75momprof. Vertical line marks the break-time t_b	115
4.7	Momentum subdiffusion: (a,b,c) Experimentally obtained momentum profiles corresponding to labels (a,b,c), respectively, in fig. 4.6. The insets show the optical density of the absorption images from which momentum profiles have been extracted. Solid lines in (b,c) are obtained from quantum simulation. Dashed lines in (b) are shown as a guide to the eye.	115
4.8	Subdiffusion in energy: Mean energy growth for various values of Lévi exponent α . Experimental data (symbols) matched against numerical simulations of AOKR (solid lines) and theoretical results (dotted lines) in Eqs.4.7-4.8.	119
4.9	Evolution of momentum distribution: (a) Experimentally measured (symbol) and simulated (dashed line) momentum profiles at times $t = 5 \approx 1.5t_b$ (left panel), $t = 15 \approx 4t_b$ (middle) and $t = 66 \approx 20t_b$ (right) for Lévy exponents (a-c) $\alpha = 0.25$, (d-f) $\alpha = 0.5$ and (g-i) $\alpha = 2$	120

4.10	Decoherence: Experimentally measured (symbol) and numerically simulated (dashed line) occupation probability $f(p = 0)$ against time in log-log plot. (Inset) $f(0)$ for numerically simulated AOKR with stationary amplitude noise with noise strengths of 16% and 25%. All the solid lines (black, green, blue, red) are to guide the eye.	121
5.1	Michealson interferometer: Schematic of Michealson interferometer using optical laser. The movement of one mirror by quarter of the wavelength of the laser changes the dark fringe into a bright one.	124
5.2	Individual trapping potentials at the end of evaporation: Individual trapping potential formed by optical dipole trap (solid red line), quadrupolar magnetic field (dashed green line) and gravity (dotted blue line) along the vertical direction. The trap profiles are numerically calculated with the parameters used at the end of the evaporation process. The dipole trap center is aligned about a beam-waist below the quadrupole trap center. Arrow shows the direction of gravity.	125
5.3	Adiabatic change in trap depth of combined potential: Change of combined potential with an adiabatic decrease in the total power of two crossed dipole trap laser beam: (a) 10 mW, (b) 8 mW, and (c) 6 mW. The effective trapdepths are (a) 220 nK, (b) 100 nK, and (c) 10 nK. The dashed line shows the potential without the dipole trap. Arrow shows the direction of gravity.	126
5.4	Characterization of tunneling: (a) Loss of atoms from the trap with trap depth much higher than the temperature than BEC. The solid line shows exponential fit to the experimental data. The decay constant is found to be ~ 12 s. (b) Loss of atoms from the trap with trap depth lowered to just equal to the temperature of atoms. An initial fast decay shows tunneling with decay constant calculated from exponential fit as 189 ms. The later slowed decay shows natural loss of atoms with a decay constant ~ 10 s.	127

- 5.5 **Acceleration of atom laser:** Absorption images of atom laser at different time (t) after the start of outcoupling: (a) $t = 0$ ms, (b) $t = 10$ ms, (c) $t = 17$ ms, (d) $t = 30$ ms, and (e) $t = 40$ ms. The field of view of each image is $6 \text{ mm} \times 1.2 \text{ mm}$. Color scale represents optical density. . 128
- 5.6 **Collimation of atom laser:** Left panel: a typical absorption image of an atom laser. Two horizontal dashed white lines are drawn to mark the region from which many transverse cross-sections of the atom laser have been selected at arbitrary places, and Thomas-Fermi distribution is fit to these profiles to measure the transverse spread of the atom laser. Color scale represents optical density. Right upper panel: Shows a typical fit to Thomas-Fermi distribution to the cross-section of an atom laser. Right bottom panel: A histogram showing the variation of Thomas-Fermi radius of an atom laser over the selected region. The dashed line is a Gaussian fit to the histogram with mean $18.1 \mu\text{m}$ and standard deviation of $1.7 \mu\text{m}$ 129
- 5.7 **Lifetime and flux of atomlaser:** Left panel: No. of atoms in the reservoir as a function of time for three different outcoupling rates. Atomic flux for the three cases are: (a) 1.39×10^6 atoms/s, (b) 2.91×10^5 atoms/s, (c) 1.71×10^5 atoms/s. Right panel: Images of the atom laser corresponding to the three flux rates in the left panel. These images are taken at (a) 40 ms, (b) 200 ms and (c) 300 ms after the start of outcoupling. Color scale represents optical density. Although in the experiment, the atom laser falls vertically along the axis of gravity, due to a relative tilt between the planes of the final imaging lens and the camera sensor, the absorption images of our atom laser look slanted towards left. 131
- 5.8 **Schematic of experimental setup to realize the diffraction of an atom laser:** Experimental schematic used for demonstrating the diffraction of an atom laser using a 1-D optical lattice and an atom laser. 132

- 5.9 **Near field and far field diffraction:** Absorption images showing diffraction of an atom laser as a function of varying Fresnel number: (a) without grating, (b) $F_n = 110$, (c) $F_n = 2.9$, (d) $F_n = 0.464$, (e) $F_n = 0.181$, (f) $F_n = 0.116$, (g) $F_n = 0.029$, (h) $F_n = 0.0186$, (i) $F_n = 0.0129$, (j) $F_n = 0.0072$, (k) $F_n = 0.0032$, and (l) $F_n = 0.0018$. The field of view of each image is $6 \text{ mm} \times 1.2 \text{ mm}$. For (b) and (c), $F_n > 1$ implies Fresnel diffraction regime. Whereas, for (g) - (l), $F_n \ll 1$ implies Fraunhofer regime. The resolution of our imaging system is not enough to resolve the wavefronts in near-field regime. Color scale represents optical density. 134
- 5.10 **Typical Kapitza-Dirac diffraction of an atom laser:** The image in the upper panel shows a typical time of flight absorption image of the atom laser diffraction. The red curve drawn beside the image is the Gaussian intensity profile of the standing wave of light that forms the diffraction grating with which the atom laser interacts. The lower panel shows the normalized population in different diffraction orders (up to $\pm 18 \hbar k$). Circles represent experimental data whereas the solid line shows the numerical simulation. 136
- 5.11 **Kapitza-Dirac diffraction of an atom laser with varying strength:** Normalized population in different diffraction orders with varying interaction time. (a) $\tau = 0.15 \mu s$, (b) $\tau = 0.3 \mu s$, (c) $\tau = 0.4 \mu s$ and (d) $\tau = 0.5 \mu s$, keeping the peak Rabi frequency and detuning constant. Black circles represent experimental data. Solid line represent numerical simulation for (a) $\theta = 2.64$, (b) $\theta = 4.29$, (c) $\theta = 6.62$, (d) $\theta = 8.23$. Insets show corresponding time of flight absorption images of the diffracted atom laser. The dimension of each image frame is $\sim 1.85 \text{ mm} \times 1.54 \text{ mm}$. Color scale represents the optical density. 137

List of Tables

2.1	Changed units for AOKR system.	55
2.2	Scaling of variables for AOKR system to derive dimensionless variables.	56
3.1	Summary of the evaporative cooling parameters for achieving Bose-Einstein condensation.	92

Contents

Declaration	vii
Acknowledgments	ix
Abstract	xiii
List of publications	xv
List of figures	xvii
List of tables	xxix
1 Introduction	1
1.1 Atoms in optical lattices	1
1.2 Optical lattice potentials	3
1.3 Physics with atoms in optical lattice	7
1.3.1 Cooling and trapping of atoms in an optical lattice	8
1.3.2 Quantum simulations	11
1.3.3 Quantum information processing	16
1.3.4 Atom interferometry and inertial sensing	17
1.4 Objective of the thesis	19
1.5 Organization of the thesis	20

2	Theoretical background	23
2.1	Bose-Einstein Condensation	23
2.1.1	Bose-Einstein Condensation: a phase transition from statistical physics approach	25
2.1.2	Bose Einstein condensation inside a trap	27
2.2	Laser Cooling of Alkali atoms	29
2.2.1	Two level system in electro-magnetic radiation	30
2.2.2	Doppler cooling	32
2.3	Magneto-optical trap (MOT)	35
2.4	Magnetic trapping of neutral atoms	38
2.4.1	Spherical Quadrupole trap	39
2.5	Optical dipole trap	40
2.5.1	Optical dipole force	40
2.5.2	Optical dipole trapping with a two-level atomic system	40
2.5.3	Dipole forces on a multi-level atom	41
2.5.4	Atoms in a focused gaussian beam	42
2.5.5	Crossed optical dipole trap	44
2.5.6	Effect of gravitational potential and quadrupolar magnetic trap .	45
2.6	Atomlaser	46
2.6.1	Outcoupling schemes	47
2.6.2	Properties of atom laser	48
2.7	Atom optics kicked rotor (AOKR)	49
2.7.1	Chaos in a system	49
2.7.2	kicked rotor: classical and quantum dynamics	50
2.7.3	Atom optics kicked rotor (AOKR)	55
2.8	Atom interferometry	56
2.8.1	Bragg diffraction	59
2.8.2	Kapitza-Dirac diffraction	62
2.9	Summary	64
3	Experimental methods and apparatus	65
3.1	Overview of the BEC apparatus	65
3.2	Rb effusive oven	66

3.3	Vacuum assembly	68
3.4	Laser System for cooling and repumping	69
3.5	Magnetic field coils	74
3.6	Loading and characterization of MOT	77
3.7	Compressed magneto-optical trap (CMOT)	79
3.8	Optical dipole trap	80
3.8.1	Optical dipole trap Laser	81
3.8.2	Preparation of laser beams for a crossed-optical dipole trap . . .	81
3.8.3	Beam profile	84
3.8.4	Trap characteristics	87
3.8.5	Alignment of the dipole trap	87
3.8.6	Loading a crossed optical dipole trap	89
3.9	Evaporative cooling	91
3.10	Imaging System	93
3.10.1	Absorption imaging	93
3.10.2	Analysis of an absorption image of BEC	95
3.11	Observation of BEC and its characterization.	97
3.11.1	Phase transition	97
3.11.2	Spin-purity	100
3.11.3	Lifetime of BEC	100
3.11.4	Measurement of trapping frequency	101
3.12	Summary	104

4 Demonstration of non-exponential decoherence and energy subdiffusion in

	AOKR system	107
4.1	Definition of the problem	108
4.1.1	The basic model:	109
4.2	Experimental schematic and procedure	110
4.3	Standard AOKR	111
4.4	Realization of a Lévy kicked rotor	114
4.4.1	Kick sequence:	114
4.4.2	Effect of Lévy noise:	116
4.4.3	A brief theoretical model:	117

4.4.4	Subdiffusive energy growth:	118
4.4.5	Non-exponential decoherence:	120
4.5	Conclusions	122
5	Kapitza-Dirac diffraction of an atom laser	123
5.1	Definition of the problem	123
5.2	Atom laser generation	125
5.3	Deceleration of atom laser	128
5.4	Characterization of atom laser	130
5.4.1	collimation	130
5.4.2	Flux and lifetime	130
5.4.3	Diffraction of an atom laser	132
5.4.4	Near-field diffraction	133
5.5	Realization of Kapitza-Dirac diffraction	133
5.6	Conclusions	137
6	Conclusion and outlook	138
6.1	Summary	138
6.2	Future outlook:	140
	Bibliography	141

Chapter 1

Introduction

1.1 Atoms in optical lattices

Light-matter interaction started as one of the frontier research areas in the last century. It began with the study of radiation pressure and its effect on particles. Later, it gradually evolved through the investigation of the interaction between different forms of light and matter. One such research pursued by today's science community is the interaction between an optical lattice and neutral atoms [1]. At present, this is one of the most frequently used tools in the field of atomic physics and quantum optics. The fact that this system offers a novel platform for addressing a bunch of exciting physics problem has made this tool very popular among researchers. An optical lattice is formed by two or more overlapping laser beams to produce a periodic modulation of light intensity in space. Neutral atoms, while interacting with such form of an electro-magnetic field via their dipole moment, feel a regular array of quantum-mechanical potential wells over the space. Such periodic array or potentials can be used to probe several interesting physics problems starting from cooling and trapping of atoms to quantum simulations [2], quantum information processing [3], atom-interferometry [4], inertial sensing [5–7], and metrology [8,9].

The earlier research in this field was started by P. N. Lebedev and A. Einstein. The Russian scientist Pyotr Lebedev was the first person to experimentally demonstrate [10] the effect of radiation pressure on molecules and atoms. In the similar time frame, Einstein introduced the quantum mechanical nature of radiation [11]. He explained that the

existence of light pressure is caused by the directivity of the process of stimulated and spontaneous emission or absorption. After 1960, the creation of lasers brought a revolution in the research area of light-matter interaction. A bunch of exciting proposals such as creating compression and rarefaction in atomic gases by light [12], isotopes separation by the deflection of atomic beams using lasers [12], creating a high-resolution velocity analyzer [13], etc. were reported. Along with these proposals, it was also proposed for the first time to restrict the motion of the atoms by placing them at the nodes and antinodes of the standing waves of laser [14, 15]. The motivation for such trapping was to eliminate the Doppler broadening of the atomic spectral lines. The theoretical investigation of the atomic motion in a resonant and non-resonant standing light wave has been presented in Ref. [16, 17]. The first proposal of localizing a two-level atomic system near the nodes of a blue-detuned 1-D (one dimensional) optical lattice was given by Lethokov *et al.* [18]. In 1987, Salomon *et al.* [19] first reported the experimental observation of channeling of the atoms along such nodal lines. A separate development in the same time-frame was focused on setting up an atom interferometer employing the light-matter interaction. Since diffraction and interference of particles like neutron and electron were already realized, similar techniques were being applied on neutral atoms. The first experimental realization on a deflection of an atomic beam using a standing wave of light was reported by Arimondo *et al.* [20] in the year 1979. Although there were several reports on such diffraction/deflection of atoms using standing waves, the first report where diffraction in Kapitza-Dirac regime was studied intensively was by Philip *et al.* [21, 22]. Later a similar configuration was used to demonstrate a three-grating atom interferometer with three optical lattices playing the role of diffraction-gratings.

During the last decade of the previous century, rapid development has occurred in the field of atomic physics. The innovation of laser-cooling [23] techniques made it possible to cool down atoms to a temperature close to absolute zero and produce a coherent state of matter known as Bose-Einstein condensate (BEC) [24]. During this period, many experimental realizations on setting up an atom interferometer using optical lattice [25] as well as periodic nano-structures [26] were reported, which will be discussed in details in section 1.3. Zoller and Cirac proposed to use atoms in the optical lattice as a platform to study many-body physics problems [27]. Following this proposal, several theoretical studies were reported to use the same system for Quantum information

processing and quantum computing. The recent development of powerful tools like quantum gas microscopy has made it possible to probe atoms in each single lattice site. This motivates to study several interesting spin systems in condensed matter physics. The spatial symmetry of the potentials of the optical lattice has been exploited to study several Floquet systems. Several quantum mechanical systems starting from the area of high energy physics to non-linear dynamics can be simulated experimentally using a group of atoms in 1-D, 2-D or 3-D optical lattices. A few relevant systems among all the several interesting modern techniques involving optical lattice will be discussed in the later sections of this chapter.

1.2 Optical lattice potentials

Various systems can be realized by a combination of atoms and optical lattices. let us first consider a simplified interaction model involving a single atom and an optical lattice.

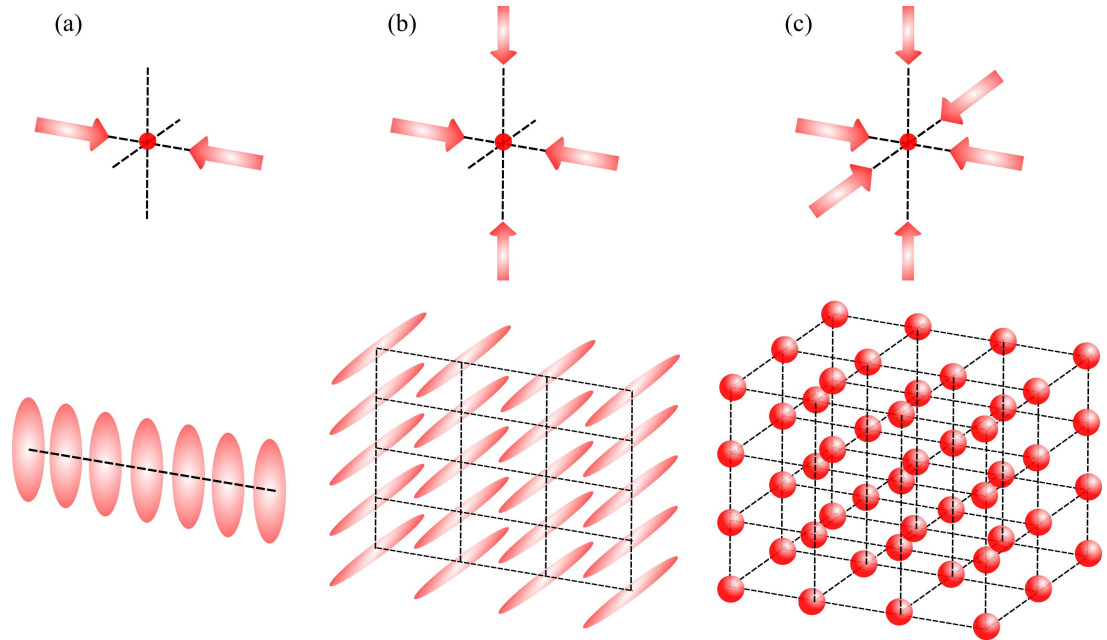


Figure 1.1: Optical lattice potentials: (a) 1-D optical lattice formed by a pair of counterpropagating laser beams. Atoms are confined in an array of 2-dimensional disks, (b) 2-D optical lattice formed by superimposing a pair of orthogonal 1-D lattices. The atoms are confined in an array of 1-dimensional tubes, (c) 3-D optical lattice formed by superimposing three orthogonal 1-D optical lattices. In this case, atoms are confined in a cubic array of potential wells.

An atom possesses a time-dependent induced dipole moment while interacting with the oscillating electric field of the laser $\mathbf{E}(r, t)$, such that [28]-

$$\mathbf{d}_i^{(\pm)}(t) = \sum_{j=x,y,z} \alpha_{ij}(\omega_L) \mathbf{E}_j^{(\pm)}(r, t) \quad (1.1)$$

where, $\mathbf{d}_i^{(\pm)}$ denotes the complex values of induced dipole moment with positive and negative frequency component and ω_L is the laser frequency. α_{ij} represents the polarizability tensor which implies that the direction of the induced dipole moment is not always same as that of the electric field. The polarizability is in general a function of the laser frequency and the energy of the excited states of the atom. If one of these states is very close to the resonance then the polarizability becomes inversely proportional to the detuning $\Delta = \omega_L - \omega_a$, where, ω_a is the atomic resonance frequency. Therefore, the electronic energy undergoes a shift $\Delta E = \mathbf{d} \cdot \mathbf{E}$, similar to the AC stark effect. The expression for such energy shift can be given as [28],

$$\Delta E(r, t) = -2 \sum_{i,j=x,y,z} \text{Re} [\alpha_{ij}(\omega_L) \mathbf{E}_i^-(r, t) \mathbf{E}_j^+(r, t)] \quad (1.2)$$

If the polarizability is considered as an isotropic scalar quantity, then the expression reduces to the simplified form given by:

$$\Delta E(r, t) = -2 \times \text{Re}(\alpha) I(r, t) \propto \frac{I(r, t)}{\Delta} \quad (1.3)$$

As a consequence of the energy shift, the atoms face an optical potential $V_{opt}(r) \propto \frac{I(r, t)}{\Delta}$. Since a 1-D optical lattice is formed by a pair of counterpropagating overlapping laser beams, the resultant electric field varies sinusoidally over the space which gives the form of the potential as

$$V(x) = V_0 \sin^2 kx \quad (1.4)$$

where $k = 2\pi/\lambda$, λ being the wavelength of the lattice lasers and V_0 is the depth of the potential. Optical lattices can be realized in one, two and three dimensions with suitable geometries (fig. 1.1). A 1-D optical lattice can be realized by overlapping two counter-propagating laser beams, where the atoms feel steep potential along the direction of the lattice, and they are free to move in a plane perpendicular to the lattice – the potential

looks like a periodic array of pancakes (fig. 1.1). Similarly, in case of a 2-D optical lattice, the potential looks similar to arrays of long tubes, and for a 3-D case, the optical lattice looks similar to periodic arrays of harmonic potential wells. Fig. 1.2 shows that atoms loaded into an optical lattices looks similar to an egg crate or periodic structure of a crystal. An optical lattice can be used to exploit different geometries with desired periodicity. A couple of such schemes used to prepare an optical lattice with triangular or honeycomb symmetry [29] are shown in fig. 1.3.

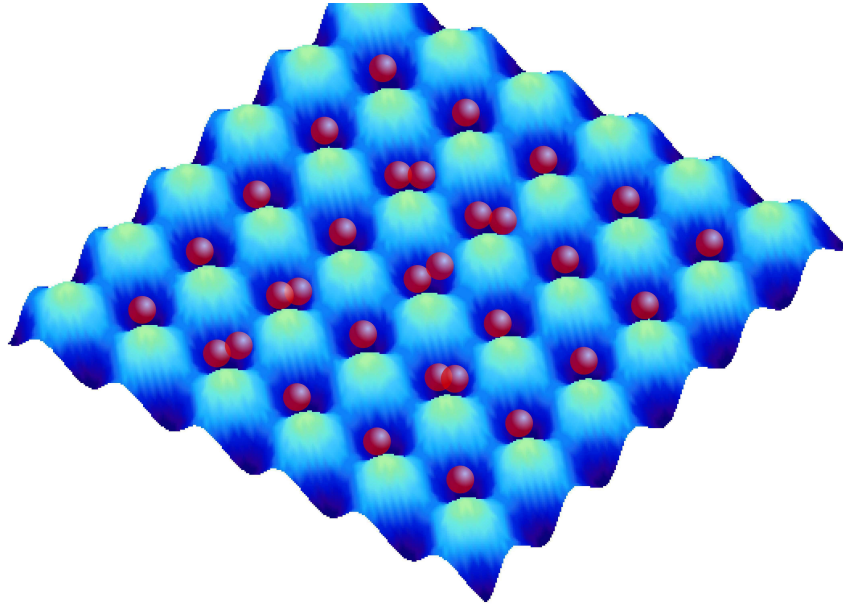


Figure 1.2: Atoms in optical lattices: Atoms sitting in the potential wells of one plane of a 3-D optical lattice similar to an egg-crate.

Since an atom in optical lattice resembles to that of an electron in a periodic array of potentials, we can extract a similar band structure for allowed energies in case of such systems. For an atom in one-dimensional optical lattice, the Schrödinger equation can be written as

$$\left[\frac{p^2}{2m} + V(x) \right] \Phi_q^{(n)}(x) = E_q^{(n)} \Phi_q^{(n)}(x) \quad (1.5)$$

The possible solutions to this equation are the Bloch functions which are the product of a plane wave and a periodic function, having same periodicity as the potential:

$$\Phi_q^{(n)}(x) = e^{iqx} u_q^{(n)}(x) \quad \text{with,} \quad u_q^{(n)}(x + d) = u_q^{(n)}(x) \quad (1.6)$$

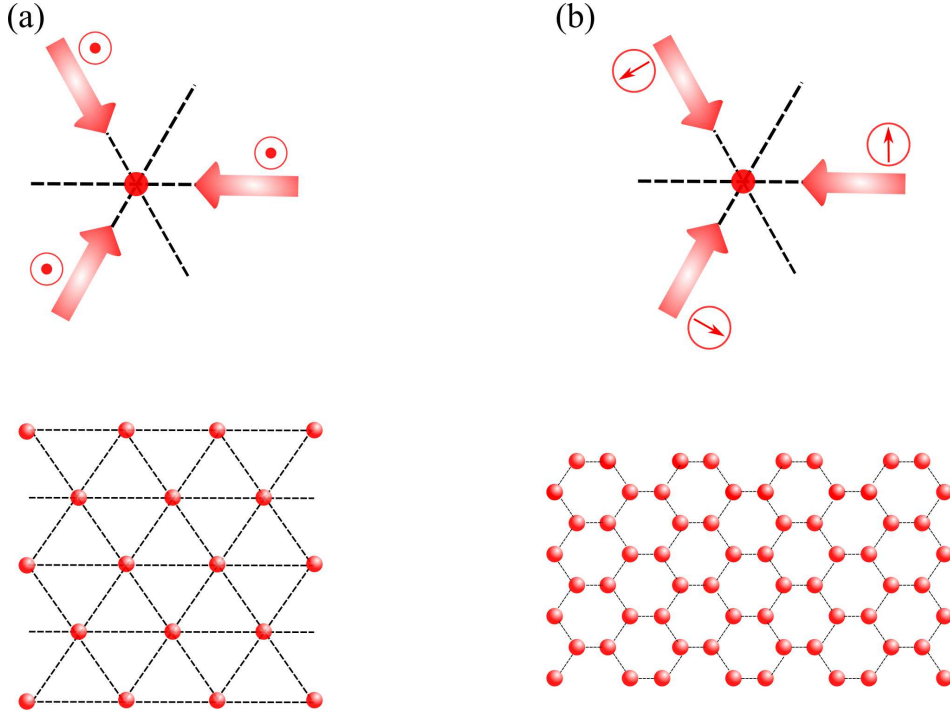


Figure 1.3: Optical lattices with triangular and honeycomb symmetry: (a) A possible scheme to realize an optical lattice with triangular symmetry. Three linearly polarized laser beams are aligned in a plane with their polarization vector perpendicular to their plane. The angle between the consecutive laser beams are kept $\pi/3$. The potential minimas are formed with triangular symmetry in space. (b) A possible schematic to realize a honeycomb symmetry in an optical lattice. The geometry of the laser beams remains same as the previous case, but the polarizations are kept in the plane as shown in figure. Every alternate minima possesses opposite circular polarization of light.

where $V(x + d) = V(x)$. Plugging eqn. 1.6 into eqn. 1.5, we get

$$\left[\frac{(p + \hbar q)^2}{2m} + V(x) \right] u_q^{(n)}(x) = E_q^{(n)} u_q^{(n)}(x) \quad (1.7)$$

where, q is the quasimomentum – a quantum number arising out of translational symmetry of the problem. Due to periodic nature of the potential, q can be confined in the first Brillouin zone i.e. $-k < q < k$; where $k = \pi/d$. In order to find out the solution to Eq. 1.7 under the tight binding approximation with $V = V_0 \sin^2 kx$, one

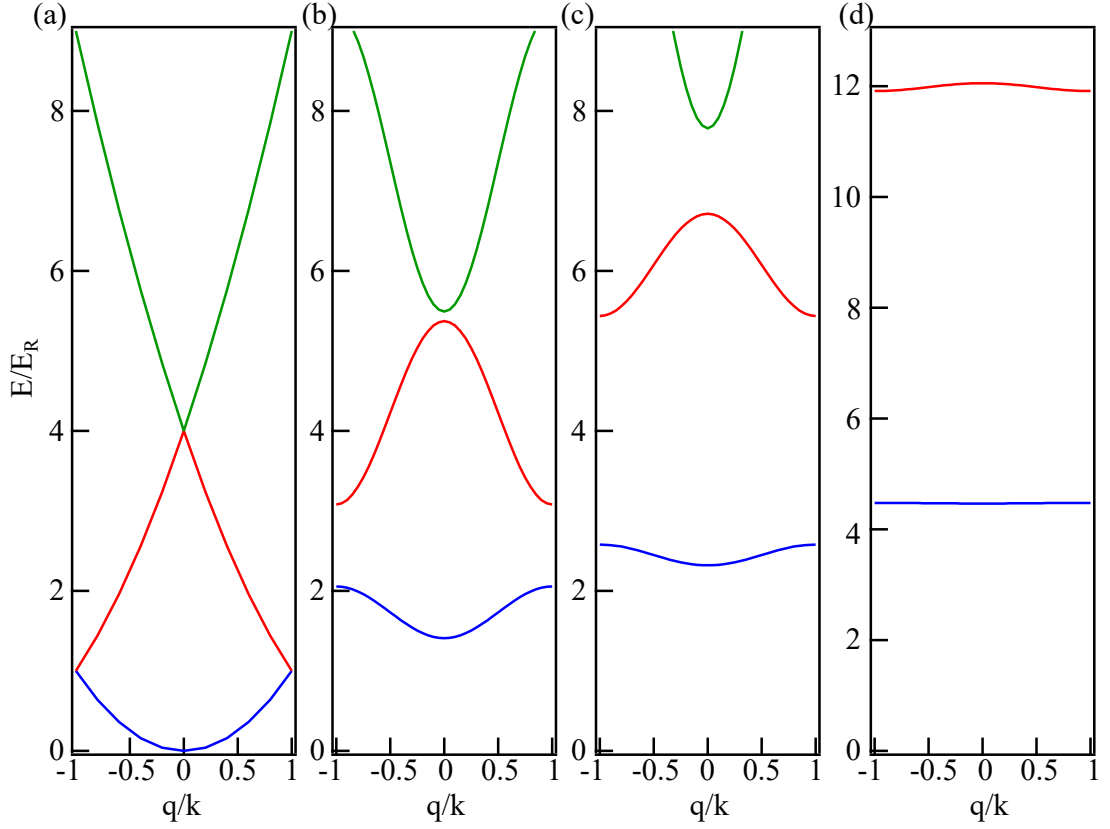


Figure 1.4: Band structure of a 1-D optical lattice: Band structure of 1-D optical lattice in reduced zone scheme. (a) Energy dispersion curve for a free particle ($V_0 = 0$); and (b), (c), (d) correspond to energy dispersion for $V_0 = 3E_R$, $V_0 = 6E_R$, $V_0 = 20E_R$ respectively.

needs to solve Mathieu equation [30]. The energy of the lowest band is given by [1]:

$$\frac{E_q}{E_R} = \sqrt{s} - 2J \cos(qd); \quad \text{where,} \quad J = \frac{4}{\sqrt{\pi}} (s)^{\frac{3}{4}} e^{-2\sqrt{s}} \quad (1.8)$$

where, $s = V_0/E_R$; $E_R = \frac{\hbar^2 k^2}{2m}$ being the recoil energy. The band structure with increasing lattice depth is shown in fig.1.4. With increasing lattice depth, the band gap increases and the bands flatten.

1.3 Physics with atoms in optical lattice

In the previous section, it has been shown that an atom in optical lattice feels similar potential like an electron in the crystal. Following Feynman's idea [31, 32] about the equivalence between physics of any two-level systems, we can easily identify that

atoms in an optical lattice can be employed as a novel platform to simulate physics of strongly correlated condensed matter systems. Various spatial geometries and even spin-dependent potentials can be realized easily with optical lattices. Using two counterpropagating laser beams with a small difference in the frequency, one can realize a traveling lattice which is useful for unidirectional momentum transfer to an atomic cloud. An atom interferometer based on the diffraction of atomic wavepackets from the optical gratings made of 1-D optical lattices offers the best inertial sensor till date. Exploiting the periodicity of the lattice, similar systems having time-periodicity such as Floquet systems can be simulated experimentally. By controlling the lattice depth and periodicity one can control the on-site interaction and tunneling of atoms through each site. Such a mechanism is useful to realize novel Mott insulator phase of matter and also provides an appropriate platform for quantum information processing and quantum computing. In the next few sections, a few interesting systems realized using the combination of atoms and optical lattices will be discussed briefly.

1.3.1 Cooling and trapping of atoms in an optical lattice

The 1st realization of a 2-D optical lattice was reported in 1989-1990 by Burns and Golovchenko [33] where the experiments were performed with a small dielectric sphere. Using such a scheme for atoms, the major hurdle would come from spontaneous emission which would result in momentum diffusion and heating of the atomic cloud. A possible way out was to use a far off-resonant lattice laser. The inclusion of the degeneracy of the Zeeman levels in this problem by Castin and Dalibard [34] was the crucial step towards atom trapping in a 1-D lattice. They used two laser beams with crossed linear polarization ($\text{lin} \perp \text{lin}$) [35] and explained the observation of cooling atoms to sub-Doppler temperatures by Philips [36] *et al.* It was theoretically proved that this configuration not only provides localization around the potential minima of the nodes but also cools down the atoms to the lowest vibrational levels. This mechanism is well known as Sisyphus cooling.

Sisyphus cooling

If we consider two counterpropagating beams of same frequency ω and real amplitude E_0 in a $\text{lin} \perp \text{lin}$ configuration ($\mathbf{e}_x$ and $\mathbf{e}_y$) as shown in fig. 1.5, then the resultant electric field can be identified as exhibiting a polarization gradient over space. The expression

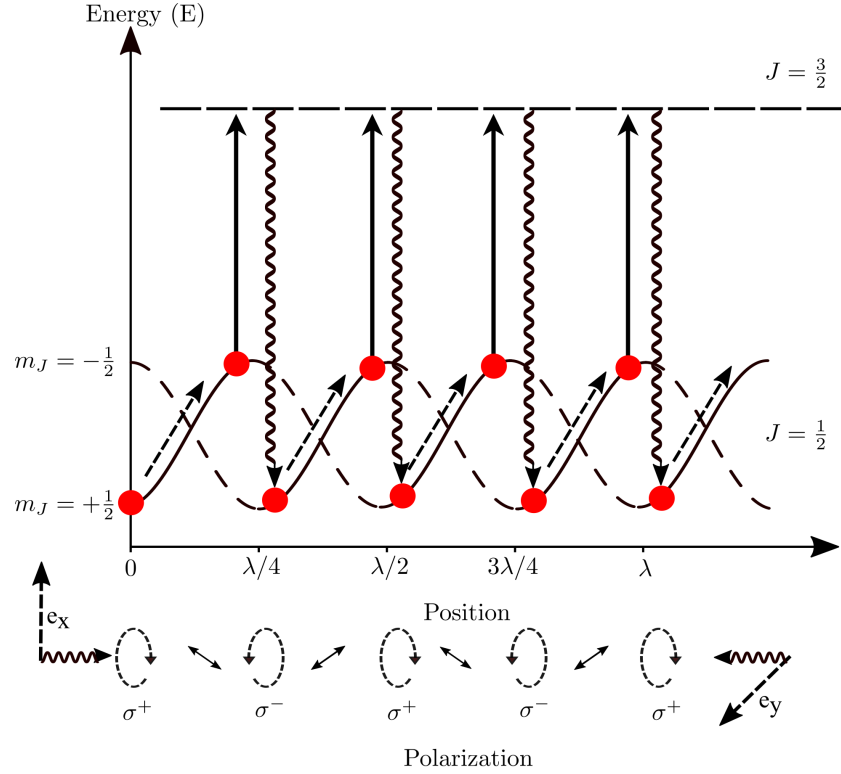


Figure 1.5: Sisyphus cooling of ^{87}Rb atoms: Two counterpropagating laser beams in lin $\perp$ lin configuration exhibit polarization gradient over space. Atoms in degenerate ground state travels the potential hill and loses its kinetic energy. Then it decays back to the other ground state via an excited state.

of the electric field can be written as:

$$\mathbf{E}(z) = \text{Re}[(E_+\mathbf{e}_+ + E_-\mathbf{e}_-)\exp(-i\omega t)] \quad (1.9)$$

where, $E_+ = i\sqrt{2}E_0 \sin kx$, $E_- = \sqrt{2}E_0 \cos kx$ and $\mathbf{e}_{\pm} = \mp \frac{\mathbf{e}_x \pm i\mathbf{e}_y}{\sqrt{2}}$. One can understand the resultant configuration as the existence of an electric field with periodic variation of σ^+ and σ^- polarization with a periodicity of $\lambda/2$, where λ is the wavelength of the laser. In order to make this mechanism work, we need the atom to possess an angular momentum number $J_g > 0$, such that it can exist in a degenerate ground state labeled by their m_J quantum numbers. Due to different light shifts, these two ground states will show complementary modulation of energy level in space, i.e., the maxima for one level will occur at the position of minima of the other and vice-versa as shown in fig. 1.5. The atom sitting at the minimum of one sub-level will move towards

the maximum point and thereby lose its kinetic energy. At the maximum point, it finds the orthogonal polarization of light which excites the atom to the nearer excited state. However, the atom then decays to the ground state of the other hyperfine level due to the selection rule. In the whole process, the kinetic energy of the atom reduces leaving it into the same ground state energy most of the time. Thus many such operational cycles are capable of cooling down the atomic cloud below doppler limit. Such cooling mechanism has also been shown to work with $\sigma^+ - \sigma^-$ configuration of light [36].

Localization of atoms: Lamb-Dicke regime

The very first stage of trapping of atoms can be described by the signature of localization. Earlier in 1991, in an experiment carried out in the laboratories of Claude Cohen-Tanoudji and Christophe Salomon, a narrow Raman line was observed from atoms in a 1-D optical lattice which had a frequency closed to the expected frequency of the oscillation of atoms in the lattice. Later it was understood that it was due to the confinement of atoms within a small distance [37] as compared to the wavelength of the light (Lamb-Dicke regime) [38]. In another independent experiment carried out by the group of Phillips [39], more impressive results were found, and the difference between such line with Rayleigh line was understood.

To understand the phenomenon, let us consider the example with $J_g = 1/2 \rightarrow J_e = 3/2$ and $\text{lin} \perp \text{lin}$ configuration of optical lattice. Due to the light shift, the optical potentials for the two Zeeman states will be given as

$$U_{\pm}(z) = \frac{U_0}{2} (-2 \pm \cos 2kx) \quad \text{with} \quad U_0 = -\frac{\hbar\Delta}{3} \frac{\Omega_1^2}{\Delta^2 + \Gamma^2/4} \quad (1.10)$$

where the resonant Rabi frequency $\Omega_1 = dE_0/\hbar$. For the atoms located near the potential minima, the potential can be expanded in a Taylor series, and the oscillation frequency can be given as $\Omega_{\nu} = \frac{\sqrt{(4E_R U_0)}}{\hbar}$. The vibrational levels with quantum number $n \ll \hbar\Omega_{\nu}/E_R$ are said to be in the Lamb-Dicke regime because the extension of such wavefunction is much less than the wavelength of light.

Recent progress on trapping and cooling of atoms using an optical lattice has resulted in remarkably fast production of Bose-Einstein condensation (BEC). In a recent article [40], experimental realization of cooling ^{87}Rb atoms to BEC within 300 ms has been reported using tight confinement of 2-D optical lattice and Raman sideband cool-

ing. Such a process has been shown to be capable of losing a minimum number of atoms to increase the phase space densities that are required to produce a BEC.

1.3.2 Quantum simulations

Quantum simulations use a simpler and experimentally more accessible quantum system to mimic the dynamics of another quantum system of interest. The concept of the quantum simulator was first introduced by Feynman [32] and was based on reducing the complexity of the classical simulation of quantum systems. As an example, we can consider a problem with a system of N spin $1/2$ particle. The dimensions of Hilbert space and eventually the number of independent coefficients required to describe the dynamics is of the order of 2^N . This problem can be solved easily by using another two-level quantum system instead of a classical computer. The modern concept of the quantum simulator is to use a simpler quantum system which is capable of mimicking the Hamiltonian of the system of interest and offers more freedom regarding tunability and realization. Ultracold atoms in optical lattice offer a novel platform to simulate different quantum systems from various research fields as diverse as condensed matter physics, high energy physics, non-linear Floquet system, and quantum chemistry [41]. Assisted by high-resolution microscopy technique capable of probing atoms in a single lattice site [42], an optical lattice has become the best simulator of many-body physics governed by Bose-Hubbard [43] or Fermi-Hubbard [44] models. The following sections are dedicated to describing the optical lattice-based quantum simulation in a bunch of selected physical systems from various research fields of interest.

Simulation of Condensed matter systems

It has already been shown that ultracold atoms when interacting with an optical lattice feel a periodic potential similar to the one felt by electrons in a solid crystal lattice. Besides, a system with cold atoms in optical lattice offers huge advantages on controlling the lattice parameters starting from the lattice depth to the lattice geometry, dimensionality, and disorder. Moreover, such systems can be engineered to introduce an artificial magnetic field stronger than even that which can be realized in a conventional condensed matter experiment [45]. The progress of this particular branch of physics is so overwhelming that summarizing all the interesting experimental results would not fit into the scope of this work. Therefore, the discussion below will be limited to a few

relevant experimental observations.

The study of such systems started with the proposal of Jaksch and Zoller [46]. The interaction of an optical lattice with bosons can be written as

$$H = -J \sum_{\langle ij \rangle} \mathbf{a}_i^\dagger \mathbf{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) \quad (1.11)$$

where, $\mathbf{a}_i^\dagger$ and $\mathbf{a}_i$ are the creation and annihilation operator respectively at i^{th} site and $\hat{n}_i(\hat{n}_i - 1)$ being the number operator at i^{th} site. The Hamiltonian is known as Bose-Hubbard Hamiltonian. The first term in the Hamiltonian corresponds to coupling between nearest neighbors via tunneling where the strength of the coupling is given by J . The second term represents the on-site energy with strength U . In the limit of $U/J \ll 1$, one can observe the superfluid state of matter using a BEC in a 3-D optical lattice. Whereas, an adiabatic increase of the lattice depth can lead to the opposite extreme limit of $U/J \gg 1$ dominated by large interaction over kinetic energy. In this regime, the system possesses a strongly correlated phase - known as Mott insulator. In this state, the number of atoms in each lattice site is fixed, and therefore the system suffers from large uncertainty of phase over different lattice sites. The phase randomization manifests a hazy interference pattern due to the overlapping of wavepackets from different lattice sites, given a suitable time of free evolution. In contrary, the superfluid state is defined by certain phase correlations and thereby shows regular interference pattern under similar experimental condition. The transition between these two states is reversible and was first realized experimentally by the group of I. Bloch [47].

Another exciting result realized using ultracold atoms in an optical lattice is the Higgs mode in a 2-D Bose-Hubbard system [48–50]. Such modes can be identified as the oscillation of the order parameter of the superfluid phase in the vicinity of the superfluid to Mott insulator transition and can be understood by relativistic field theory approach. These modes need a very clean excitation method to be observed and hence made it inaccessible in traditional condensed matter system. In recent experiments, [51] on low-dimensional materials evidence of the existence of such modes have been found.

Simulation of quantum magnetism, e.g., the Ising model is of fundamental interest. Such systems have been realized in a tilted lattice by mapping the population difference between two neighboring sites on the two spin states of interest [52]. These systems

have been employed to study the phase transition from a paramagnet to an Ising anti-ferromagnet. Non-equilibrium dynamics of Ising chains has been studied by quenching the parameters of the corresponding Hamiltonian near the phase transition [53]. A two-component boson in an optical lattice with suppressed tunneling has been used to realize the Heisenberg model in order to understand the mechanism of superexchange coupling [54, 55]. In an experiment [56], a two-component bosonic behavior is simulated by choosing any two Zeeman levels from the manifold of all hyperfine levels of an atom. Such spin dynamics have been realized with ^{87}Rb atoms in the 1-D optical lattice. Similar experimental platform clubbed with high-resolution microscopic technique has been useful to study spin transport, spin dynamics, etc [57].

Optical lattice can be made spin sensitive or spin dependent by choosing appropriate polarization of light. The most important problem realized through such systems is the famous Fermi-Hubbard model which is very helpful in the context of high-temperature superconductors. Interaction in such systems is tuned using Feshbach resonances [58]. The fermionic quantum gas microscope has become a powerful tool to manipulate the occupation in each lattice site in a Mott insulator state. This allows simulating anti-ferromagnetic spin-correlation with a temperature below the energy scale of superexchange [59–61].

Investigation of topological phases of matter is of great interest at present. Such topological phases arise out of the strong external magnetic field or via a spin-orbit coupling. Such states are characterized by non-local topological invariant known as Chern number. As the neutral atoms do not possess any charge and it is apparent to be the main hurdle in simulating such systems with cold atoms. The remedy is to map the Hamiltonian using cold atoms in optical lattice and simulate the Aharonov-Bohm effect [62]. An atom performing a closed loop motion in an optical lattice picks up an extra phase which is similar to that of electrons and hence depends on the topology of the system. One way to study such systems is to reduce the tunneling by applying a strong magnetic field gradient and later restore the tunneling in a controlled way via a pair of laser beams. Such an experimental setup has been implemented to realize the famous Hofstadter model [63]. Another way to realize such a model is via shaking of the lattice on a circular trajectory and use the resulting time-independent Floquet Hamiltonian [64] to emulate the desired system. Such systems have been used in combination with a trian-

gular lattice [65] and a hexagonal lattice structure [66] to realize the famous Haldane model.

Disordered systems and chaos: Atom Optic Kicked Rotor

Disorder and chaos are closely related concepts. In classical physics, chaos arises through non-linearity in the Hamiltonian. Whereas, in the quantum world, the dynamics are governed by Schrödinger equation which is linear. So quantum mechanically chaotic system refers to those quantum systems whose classical counterpart is chaotic. One such chaotic system is a kicked rotor. The detailed theoretical background of such systems will be discussed in the next chapter. A summary of recent scientific interest and its connection with optical lattice will be motivated in this section.

Anderson model [67] is an example of a quantum system investigated in the presence of disorder. In 1-D, this model predicts the localization of electronic wavefunction, and in 3-D, the system undergoes a second order phase transition from delocalized (metal) to localized (insulator) state [67]. Such transitions are known as metal-insulator transitions. A kicked rotor is a periodically driven system where chaotic nature can be observed beyond a certain value of the stochastic parameter. The quantum mechanical Hamiltonian of a kicked rotor system can be mapped into the Anderson model very easily, and the system is found to suppress classical diffusion of momentum leading to a localized state similar to that of Anderson model. Such localization is known as dynamical localization [68]. Here the role of the disorder is played by the chaos in the system. A kicked rotor model can be realized experimentally by kicking a cloud of cold atoms using a pulsed 1-D optical lattice. The very first experimental demonstration of such a system was done by the group of Mark Raizen [69]. Later several experimental results have been reported showing dynamical localization in 1-D and Anderson like metal-insulator transition in 3-D. The breaking of symmetry in the kicking sequence is being exploited to realize the model in 3-D with the same 1-D lattice. The concept behind such experiment is also to kick the atomic cloud with a pulsed 1-D optical lattice, but the kicking process must possess three incommensurate frequencies. The initial localized momentum state of atoms is found to diffuse with the increasing strength (ϵ) of asymmetry and the strength of such kicks ' K '. In a parameter space of ϵ - K ; one can observe the system behaving like an Anderson-metal-insulator transition. Several experiments have been carried out to destroy the dynamical localization of the

system by introducing noise into the system by spontaneous emission [70] and in the time periodicity [71] of the kicks. The spontaneous emission rate in such systems is controlled by changing the detuning of the lattice laser. Such experiments address the fundamental physics problem of decoherence. Decoherence can be explained simply by the decay of the system from a particular desired quantum state. In such experiments, a localized momentum state represents the superposition of the initial quantum states where the system is being prepared. Introducing spontaneous emission results in subdiffusion in energy and decay of the system from such well-defined momentum state. In this thesis, we also present a similar experiment to address a more critical and fundamental aspect of decoherence using such systems (see chapter 4).

Recently, a few theoretical proposals have been reported, where they show the capability of such a system in mimicking the integer and fractional quantum Hall effect out of chaos [72]. The proposal relies on using a spinor condensate and mimics a spin-orbit type interaction in order to show a quantized behavior of the system in parameter space. Although such systems cannot provide all the details of the Hall effect, the simplicity of the system makes it more appealing.

Another interesting regime of Atom optics kicked rotor (AOKR) is the resonant case. It has been shown in ref. [73] that if the experiment is performed with a coherent cloud of atoms, then the growth of the mean energy of atoms become quadratic with time if the resonance condition is satisfied. The resonance condition in such cases can be achieved for a particular value of the kicking period known as Talbot time [74]. It has been proposed in ref. [75] that keeping the kicking period fixed at Talbot time and applying a set of kicks followed by a similar set of time reversal kicks, one can realize an atom interferometer. The fidelity of merging depends on the purity of the initial momentum state of the wave packet and the accuracy in timing sequence to maintain the Talbot time. Such a system shows a distinct interference pattern with a small deviation in time and has been shown to be capable of measuring time with an accuracy of 3 ns in $\sim 66 \mu\text{s}$. Sub-Fourier characteristics of such systems have also been studied [76]. Recently, experimental demonstration of uni-directional ratchet type motion has been reported using a similar system [77]. Thus the AOKR serves as an apt test-bed for probing quantum chaos and dynamics of disordered systems.

1.3.3 Quantum information processing

The properties that a system needs to possess in order to qualify as a test-bed for quantum information processing (QIP) are given by so-called DiVincenzo criteria [78]:

- (i) the system has to be scalable with well-defined qubits,
- (ii) the system could be initializable to a simple fiducial state,
- (iii) the system should have long coherence time as compared to the gate operation time,
- (iv) the system should have a set of universal quantum gates,
- (v) The system should have a qubit specific measurement.

An optical lattice loaded with atoms offers a large number of qubits which is an advantageous property for a quantum computer from the perspective of scalability. On the other hand, the spatial difference between atoms sitting in two neighboring sites is $\sim \lambda/2$, which makes it very difficult to address them individually. In the last decade, many experiments have been reported where single atom addressability has been achieved [79, 80]. Although the neutral atoms possess long coherence time and do not decay if prepared in the ground state, bringing them into any superposition state or entangled state make the state fragile regarding decoherence. Therefore adequate error correction is needed to correct for the systematic errors. One of the computational approaches with optical lattice is the one-way quantum computation [81]. The model relies on creating a large multipartite state capable of producing an entangled state. Another approach for quantum computation with atoms in an optical lattice is the topological quantum information processing which is based on the hidden topological order present in the system and was first introduced by Kitaev [82]. The very first proposal for the realization of a quantum circuit using optical lattice was made by Jaksch *et al.* [83]. It was suggested to introduce a controlled collision in conjugation with the time-dependent potential to increase the interaction beyond the nearest neighbor limit. In an experiment performed in National Institute of Standards and Technology (NIST) realization of SWAP gate has been reported [84]. Several proposals have been made to use dipolar interaction or Rydberg atoms to achieve faster gate operation [85]. Proposals to reduce the decoherence in such systems have also been reported [86]. The last important approach is the topological quantum computing which provides a built-in mechanism for error correction. Since it is based on the topological order which is fault-tolerant for quantum computation and quantum memories, several proposals have been reported to perform QIP with

such systems [87].

1.3.4 Atom interferometry and inertial sensing

Atom interferometry [88] is one of the most useful innovations of the last century in the field of precision metrology [89, 90] and inertial sensing [5]. An atom interferometer is a replica of the optical interferometer with the role of the light replaced by a matter-wave. The technical realization of such interferometers is done by coherent manipulation of either kinetic or the internal degrees of freedom of quantum matter like atoms, molecules, etc. A photon-based optical interferometer uses metallic or dielectric surfaces to split and recombine light. Whereas in an atom interferometer, the manipulation of matter-waves can be realized using several tools, e.g., a nano-fabricated grating [91], a spatially periodic optical potentials [92], a sequence of microwave pulses of specified duration [93], optical Raman pulses, etc. Unlike photons, atoms or molecules possess several physical properties (e.g., mass, polarizability, magnetic moment, etc.) which enables them to interact with the various physical environments (e.g., magnetic field, electric field, gravitational field, etc.) making the interferometer sensitive to the physical quantities of our interest. Because of such high sensitivity, desired de Broglie wavelength and higher flexibility offered by atom interferometers, it has become an apt candidate for precision measurements in both fundamental science and commercial applications. Since atoms have more simplified energy structure compared to the molecules, it is very easy to address and manipulate atoms using electric, magnetic or electromagnetic fields. Thus matter-wave interferometers based on atoms have become more useful and popular. Therefore, we will focus only on the atom-based matter wave interferometers and henceforth we will use the phrase “atom-interferometer” (AI) to indicate such interferometers only. AIs have already been successfully used to measure gravitational constant [94–98], gravity gradients [99], fine-structure constant [100–102], earth’s rotation vector [6, 103–105], atomic polarizability [106], etc. and have been proposed to be used as a potential candidate to detect gravitational waves [107, 108], to test the general relativistic corrections to motion under Newtonian gravity [109–111] and to test antimatter equivalence principle [112, 113], etc. Unlike all the above-mentioned successful measurements performed in a laboratories, commercial applications related to navigation and geodesy requires more compact and robust experimental setups. In the context of commercial applications, six axes of inertial measurements have been

demonstrated [7] using a single apparatus. Precision rotation measurement has been demonstrated [5] within a sensor head volume less than $1/8 \text{ m}^3$, although commercially available sensors are of smaller sizes. In the last decade, research on improving the robustness of such interferometers has gained impetus. Several tests of survival of such interferometers under significant shocks are parts of ongoing research work throughout the world. During a parabolic flight of an airplane, such interferometers have been tested to measure gravity under the influence of a noisy environmental background [98]. Effect on such AIs by dropping it from the top of 100-meter tall tower has also been studied [114]. Continuing in this direction, it would be interesting to study the performance of such interferometers when put in a rocket [115] or satellite [116] leading to prolonged free-fall time of the order of a few minutes but under strong vibrations.

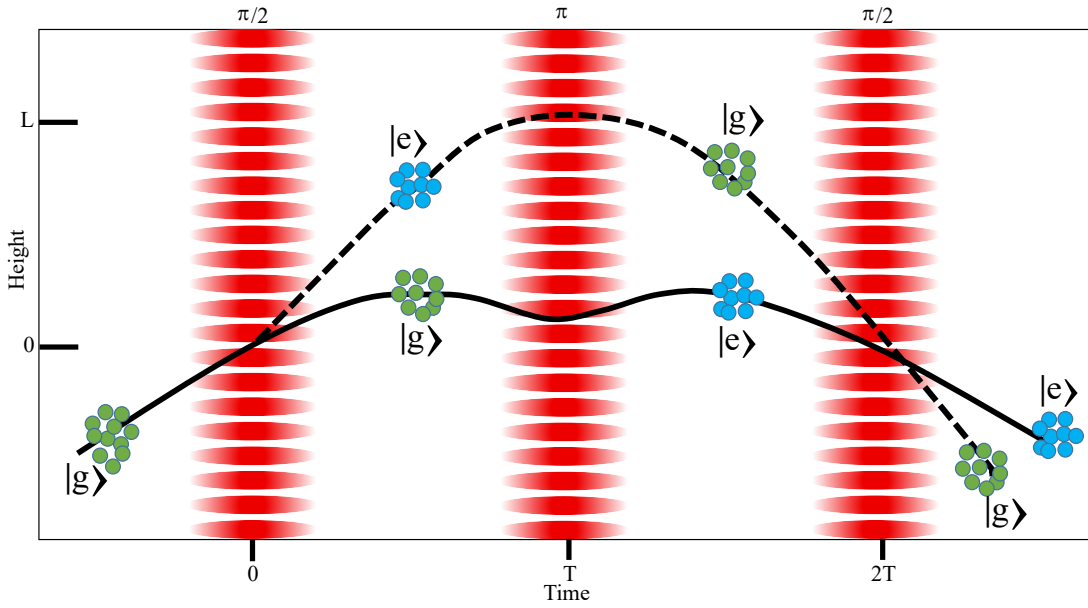


Figure 1.6: Schematic of a Mach-Zehnder atom interferometer: Atoms pass through three pulsed gratings used in $\pi/2 - \pi - \pi/2$ configuration resulting in an overall phase accumulation, that depends on the effective enclosed area of the interferometer.

The earlier realizations of an AI were with thermal beams of atoms or cold atomic samples. Replacing the thermal beam of atoms with an ultracold sample of coherent atoms like BEC, the sensitivity, and the signal-to-noise ratio have been improved by nearly three orders of magnitude [4]. In an AI, the role of beam splitters and mirrors are realized by multiple optical gratings which are nothing but 1-D optical lattices. The earlier realizations of such gratings were done by nano-fabricated periodic structures

which suffer from dephasing due to the interaction of atoms with such structures and thereby lack of high contrast at the output of the interferometer. Later, it was proposed that a periodic occurrence of light intensity maxima and minima can be used to diffract the atoms. Thus a 1-D optical lattice serves the purpose of the grating for neutral atoms. Depending on the technique used, such interferometers can broadly be divided into two groups - a) Bragg interferometry, where the splitting of wavepackets is realized by manipulating the momentum states of the wavepackets, and b) Raman interferometry, where the splitting is done by coherent control of the internal state of the atoms or molecules. Fig. 1.6 represents the schematic of a typical Mach-Zehnder atom interferometer realized in the time domain. Three pulsed grating acts as a sequence of $\pi/2$ - π - $\pi/2$ pulses (fig. 1.6). The first $\pi/2$ pulse creates an equal population of atoms in the two chosen internal states. Due to the light force, they feel different forces and take different trajectories as shown in fig. 1.6. The second π pulse comes after a time T and reverses the state of the atoms in each arm of the interferometer. This also leads to a change in the physical trajectories of the atomic samples. At time $2T$, the final $\pi/2$ pulse is applied to recombine the atoms. The manifestation of the phase difference can be observed as a change in the population in each internal state of the atoms at each output port of the interferometer. Such interferometers are also known as Raman interferometers. In such interferometers, the atoms in two arms of the interferometer differ by both internal state and momentum. The total interrogation time is the time which the atomic sample spends in the interferometer is given by $2T$. Detailed theoretical background of AIs has been provided in the next chapter.

1.4 Objective of the thesis

The present thesis aims at addressing two major problems using the interaction of ultra-cold atoms with 1-D lattice. The problems are as follows:

1. Demonstration of non-exponential quantum decoherence in a Lévy kicked rotor system.
2. Realization of diffraction of an atom laser in Raman-Nath regime.

A major part of this thesis was to demonstrate the non-exponential decoherence experimentally in a Lévy kicked rotor system. This experiment addresses one of the funda-

mental problems in quantum mechanics, known as decoherence. In general, quantum systems lose coherence exponentially fast upon interacting with the environment, and this puts a limitation on some technological applications using Quantum systems, e.g., on the Quantum gate-operation time and efficiency in Quantum information processing and precision metrology. In this work, we used an atom optics delta-kicked rotor subjected to non-stationary Lévy noise in the kicking sequence and demonstrated slower than exponential decoherence manifested in the form of sub-diffusion in the mean energy growth of the rotor. To realize the system experimentally using cold atoms, we used a cold cloud of atoms having Maxwell-Boltzmann velocity distribution and made it interact with a pulsed 1-D optical lattice to realize the kicking sequence. By controlling the kicking sequence, we studied the decay of coherence in the system. This work is relevant in the context of understanding the dynamics in disordered nonlinear lattices and diffusion in chaotic quantum systems. This work will also be helpful in understanding fundamental problems related to quantum mechanics and non-linear dynamics of chaotic systems.

Another principal objective of this thesis work was to generate a slow coherent beam of ultracold atoms (known as atom laser) from the reservoir of a BEC and demonstrate the diffraction of an atom laser. We form the atom laser by outcoupling a quasi-continuous beam of coherent atoms from a reservoir of ^{87}Rb BEC by using the ‘spilling’ method. The flux of the outcoupled atoms is controlled to achieve a continuous wave atom laser lasting up to 400 ms. The diffraction of this atom laser in the Raman-Nath regime is realized using a grating formed by a standing wave of far-detuned laser light. By controlling the interaction of the atom laser and light-grating, the strength of the diffraction into various orders is precisely controlled, and momentum up to $\pm 18\hbar k$ is imparted. Such diffraction would further allow for a construction of an atom-interferometer to probe changes in physical environments continuously up to a few hundred milliseconds. This would provide a novel platform to construct atom interferometers which can provide measurement with high bandwidth and free from aliasing effects.

1.5 Organization of the thesis

- Chapter 2 provides a theoretical background required for the work presented in

this thesis. This chapter includes a brief description of light-matter interaction, laser cooling and trapping of atoms using light-forces and a brief overview of the statistical theory of Bose-Einstein condensation. Later, in this chapter, a detailed theory of AOKR system and atom interferometry have been discussed which is mostly relevant to understanding this work.

- Chapter 3 presents a detailed description of the experimental apparatus to produce ^{87}Rb BEC in a crossed-optical dipole trap. This chapter describes the experimental setup, characterisation of the trap, and finally, the characterization of the phase transition to BEC.
- Chapter 4 reports the experimental details for the realization of a standard AOKR system. This chapter also summarizes the demonstration of non-exponential decoherence in an AOKR system. The experimental results and discussions of this work have been represented in this chapter.
- Chapter 5 provides the experimental scheme for generating an atom laser, characterization of the atom laser and finally the results of the realization of diffraction of an atom laser in the Raman-Nath regime.
- Chapter 6 summarizes and presents the future outlook for this work.

Chapter 2

Theoretical background

In this chapter, we briefly discuss the relevant theoretical concepts necessary to understand the work presented in this dissertation. The first part of this chapter outlines the theory of trapped Bose gases, basics of laser cooling and trapping, techniques for trapping and cooling down atoms in a crossed optical dipole trap. The later sections of this chapter are dedicated to the theory of two particular systems of our principal interest—the Atom Optics Kicked Rotor (AOKR) and Atom Interferometry (AI).

2.1 Bose-Einstein Condensation

Let us consider a system of N -particles represented by a wave function $\psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$. If the system consists of N identical particles, then exchanging the position of two particles will not change the probability density $|\psi|^2$. Therefore, such an operation of swapping two particles will result in

$$\psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_i, \dots, \mathbf{x}_j, \dots, \mathbf{x}_N) = e^{i\theta} \psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_j, \dots, \mathbf{x}_i, \dots, \mathbf{x}_N) \quad (2.1)$$

where, $e^{i\theta}$ is the phase factor. The equation is valid for any possible pair of values of i and j . To understand the origin of such phase, we can think of doing the same swapping operation twice with the same pair of particles which would result in ending up with the same wavefunction. Therefore, the phase gathered during such operation is given by $(e^{i\theta})^2 = 1$. As a consequence, it is quite obvious that $e^{i\theta} = \pm 1$. For particles, where the wavefunction under such an operation picks up a $+1$ sign due to phase change

are known as Bosons – the wavefunction is symmetric under position interchange, and they obey Bose-Einstein statistics [117–119]. On the contrary, for the case, where the wavefunction picks up a -1 sign due to the Wavefunction being anti-symmetric, are known as fermions and they obey Fermi-Dirac statistics. The above argument is true for three or more dimensions. Based on the assumptions of (a) Lorentz invariance, (b) causality and (c) unitarity, it can be concluded that bosons are particles with integer spin whereas, fermions are particles with half-integer spin.

The anti-symmetric wavefunction of fermions prohibits identical fermions from occupying the same quantum state [120, 121] and leads to the well-known Pauli exclusion principle [122]. On the other hand, there is no limit on the number of Bosons occupying the same quantum state [123]. As a consequence, fermions placed in a quantum mechanical potential well will occupy different energy levels even at lowest possible energy configuration. While for bosons, there can be any number of particles occupying the same ground state. These laws explain the physics of electronic transport in solids, Superconductivity [124], Superfluidity [125, 126] etc. This thesis work has been done using ^{87}Rb which is bosonic in nature. So in the remaining part of this section, we will examine the ground state of bosons at the lowest energy configuration.

Let us consider a bosonic system with chemical potential μ and the energy E . Following the Bose-Einstein statistics, the probability $f(\epsilon)$ of occupation of a state with energy ϵ can be given by:

$$f(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_B T} - 1} \quad (2.2)$$

where T is the absolute temperature of the system, and k_B is the Boltzmann's constant.

For a bosonic system, at very low temperature and high densities, the ground state possesses a macroscopic population. This phase transition is 2nd order and leads to a completely new phase of matter known as Bose-Einstein condensation. Such a transition is characterized by a parameter called phase space density (ρ). From the theoretical considerations, the condition for achieving BEC is given by:

$$\rho = n\lambda_{dB}^3 \simeq 1 \quad (2.3)$$

where n being the number density of the particles and λ_{dB} is the thermal de Broglie wavelength. For a system at temperature T , the thermal de Broglie wavelength is written

as:

$$\lambda_{dB} = \frac{h}{\sqrt{2\pi m k_B T}} \quad (2.4)$$

BEC is a very special phase, where the mean inter-particle distance is much less than the individual de Broglie wavelength of the particles. Therefore, in this phase, the associated wavefunctions of each particle overlap leading to the formation of a common macroscopic giant many-body wavefunction where particles lose their individual entity and behave as a single coherent wave.

2.1.1 Bose-Einstein Condensation: a phase transition from statistical physics approach

In a laboratory, a BEC is realised inside a trap. In this section, we will consider a simple model for the trap and explore the statistical properties for a BEC in such a trap. Considering the range of trap-geometries that are experimentally feasible, we choose a three dimensional harmonic potential. The potential energy in such a 3-D harmonic trap can be written as :

$$U(\mathbf{r}) = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \quad (2.5)$$

where ω_i s are the trapping frequencies in each direction. The energy eigenvalues for such a potential are-

$$\epsilon(n_x, n_y, n_z) = \left(n_x + \frac{1}{2}\right)\hbar\omega_x + \left(n_y + \frac{1}{2}\right)\hbar\omega_y + \left(n_z + \frac{1}{2}\right)\hbar\omega_z \quad (2.6)$$

The number of particles at energy ϵ and at a given temperature T is given by -

$$N_i(\epsilon_i) = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} - 1} \quad (2.7)$$

where, μ denotes the chemical potential and k_B is the Boltzmann constant. Since, the total number of particles (N) and total energy (E) are conserved, we can write these as

—

$$N = \sum_i N_i(\epsilon_i) \quad (2.8a)$$

$$E = \sum_i \epsilon_i N_i(\epsilon_i) \quad (2.8b)$$

For convenience, we should modify Eq. 2.7 and rewrite by identifying the contributions from the ground state and rest of the energy levels separately.

$$N = N_0 + N_{th} = N_0 + \sum_{i \neq 0} \frac{1}{e^{(\epsilon_i - \mu)/k_B T} - 1} \quad (2.9)$$

In the limit of large volume ($V \rightarrow \infty$), where the energy difference between adjacent energy levels is much less than $k_B T$, the discrete sum can be replaced by an integral -

$$N = N_0 + \int_0^\infty \frac{dn_x dn_y dn_z}{\exp\left(\frac{\hbar(\omega_x n_x + \omega_y n_y + \omega_z n_z)}{k_B T}\right) - 1} = N_0 + N_{th} \quad (2.10)$$

The integration can be solved by introducing a variable change $\omega_i n_i / k_B T = \tilde{n}_i$. The exact value of the integral is given by

$$N - N_0 = \zeta(3) \left(\frac{k_B T}{\hbar \bar{\omega}} \right)^3 \quad (2.11)$$

where ζ denotes the Riemann-zeta function and $\bar{\omega}$ is the geometric mean of the trap frequencies, given by $(\omega_x \omega_y \omega_z)^{1/3}$. The transition occurs at the critical temperature T_c^0 where $N_0 \rightarrow 0$ and this temperature is given by

$$k_B T_c^0 = \hbar \bar{\omega} \left(\frac{N}{\zeta(3)} \right)^{1/3} \quad (2.12)$$

The critical temperature can be achieved in the thermodynamic limit $N \rightarrow \infty$ and $\hbar \bar{\omega} \rightarrow 0$ keeping the product $N \bar{\omega}^3$ constant. Below the critical temperature $T < T_c$ the condensate fraction is given by-

$$\frac{N_0}{N} = 1 - \zeta(3) \left(\frac{k_B}{\hbar \bar{\omega}} \right)^3 \frac{T^3}{N} = 1 - \left(\frac{T}{T_c} \right)^3 \quad (2.13)$$

BEC occurs at high densities and low temperatures, and the phase space density increases beyond ~ 2.6 . The growth of condensate fraction with decreasing temperature has been shown in fig. 2.1.

For an interacting Bose gas, the specific heat shows a discontinuity at the beginning of condensation [119], which implies that such transition is a second order phase transi-

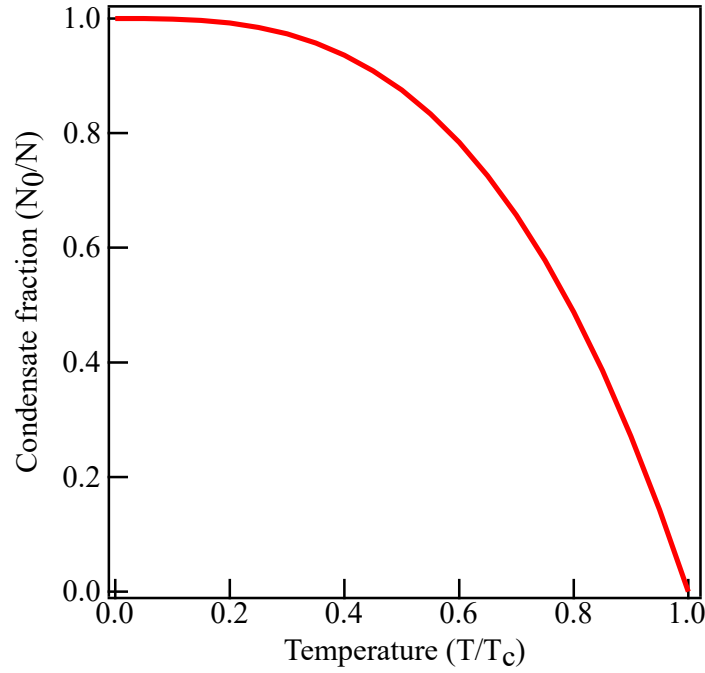


Figure 2.1: Condensate fraction: Condensate fraction vs temperature for an ideal gas in a harmonic trap given in Eq. 2.10

tion. A typical example of discontinuity of specific heat is λ -point of liquid He^4 , which showed that superfluidity of liquid He^4 [119, 125] and it is analogous to Bose-Einstein condensation.

2.1.2 Bose Einstein condensation inside a trap

In the previous section, we have learnt about the phase transition of a BEC. Once a Bose gas has gone through the phase transition and reached the phase of BEC, we are interested to know about its wavefunction. All other relevant dynamics can be extracted using the wavefunction. In this section we will solve the Schrödinger equation for a non-interacting Bose gas in a harmonic potential and derive the wavefunction of BEC under some realistic approximation. The schrödinger equation can be written as [126, 127]:

$$i\hbar \frac{\partial \Phi(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{ext}(\mathbf{r}) + g|\Phi(\mathbf{r}, t)|^2 \right] \Phi(\mathbf{r}, t) \quad (2.14)$$

Where the three terms on the right hand side of the equation are kinetic energy term, external applied potential and mean field energy term considering the nearest neighbour

interaction respectively. g is the strength of the interaction and is defined as:

$$g = \frac{4\pi\hbar^2 a_s}{m} \quad (2.15)$$

where, a_s is the s-wave scattering length. Eq. 2.14 is the well known Gross-Pitaevskii (GP) equation. The ground state wavefunction of the condensate $\Phi(\mathbf{r}, t)$ can be written as:

$$\Phi(\mathbf{r}, t) = \phi(\mathbf{r}) e^{-i\mu t/\hbar} \quad (2.16)$$

where, μ is the chemical potential and $\phi(\mathbf{r})$ is real, and is normalized to the total number of particles $N = \int |\phi(\mathbf{r})|^2 d\mathbf{r}$. Plugging Eq. 2.16 into Eq. 2.14, the GP equation modifies to:

$$\left[-\frac{\hbar^2 \nabla^2}{2m} + V_{ext}(\mathbf{r}) + g|\phi(\mathbf{r}, t)|^2 \right] \phi(\mathbf{r}) = \mu \phi(\mathbf{r}) \quad (2.17)$$

For an external harmonic trapping potential, the GP equation can be written as:

$$\left[-\frac{\hbar^2 \nabla^2}{2m} + \frac{m}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + g|\phi(\mathbf{r}, t)|^2 \right] \phi(\mathbf{r}) = \mu \phi(\mathbf{r}) \quad (2.18)$$

In the harmonic trapping potential the interaction energy E_{int} is of order $\frac{N^2 |a_s|}{a_{ho}^3}$ and the kinetic energy E_{kin} is proportional to $\frac{N}{a_{ho}^2}$. Hence, the ratio of interaction energy to kinetic energy can be given as:

$$\frac{E_{int}}{E_{kin}} \propto \frac{N |a_s|}{a_{ho}} \quad (2.19)$$

where, $a_{ho} = \sqrt{\frac{\hbar}{m\bar{\omega}}}$ is the mean harmonic oscillator length of the trapping potential. Therefore, with increasing number of particles, the interaction energy term dominates over the kinetic energy. Hence, considering a system with a large number of particle N , the kinetic energy term $-\frac{\hbar^2 \nabla^2}{2m}$ in Eq. 2.18 can be neglected. This approximation is called the Thomas-Fermi approximation. Under this approximation, Eq. 2.18 becomes:

$$\frac{m}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + g|\phi(\mathbf{r}, t)|^2 = \mu \quad (2.20)$$

By solving the above equation, we get the density distribution of condensate:

$$n(\mathbf{r}) = |\phi(\mathbf{r})|^2 = \frac{\mu}{g} \left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2} \right) \quad (2.21)$$

where, R_i is the width of condensate in the i^{th} direction and given by:

$$R_i = \sqrt{\frac{2\mu}{m\omega_i^2}} \quad (2.22)$$

For a harmonic potential with oscillation frequencies $\omega_x = \omega_y \equiv \omega_r$ and ω_z , Eq. 2.21 modifies to

$$n(\mathbf{r}) = |\phi(\mathbf{r})|^2 = \frac{\mu}{g} \left(1 - \frac{x^2}{\alpha^2} - \frac{y^2}{\alpha^2} - \frac{z^2}{\beta^2} \right) \quad (2.23)$$

where, $\alpha = \sqrt{\frac{2\mu}{m\omega_r^2}}$ and $\beta = \sqrt{\frac{2\mu}{m\omega_z^2}}$. Eq. 2.20 represents an inverted parabola with sharp edges at $r = \pm\alpha$ and $z = \pm\beta$. The chemical potential can be calculated by normalizing the ground state wavefunction:

$$N = \int |\phi(\mathbf{r})|^2 d^3r \quad (2.24)$$

$$= \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} \left(1 - \frac{x^2}{\alpha^2} - \frac{y^2}{\alpha^2} - \frac{z^2}{\beta^2} \right) dx dy dz \quad (2.25)$$

Solving the above equation, we get:

$$N = \frac{\mu}{g} \frac{8\pi}{15} \beta \alpha^2 \quad (2.26)$$

and hence, the chemical potential μ can be given as:

$$\mu = \frac{1}{2} (15 N a \hbar^2 m^{1/2} \omega_r^2 \omega_z)^{2/5} \quad (2.27)$$

2.2 Laser Cooling of Alkali atoms

As seen in the previous section, BEC can be achieved at very low temperatures and high densities. Laser cooling has been the most successful technique for cooling atoms down to several hundred micro Kelvins. To reach low temperatures and high densities from

conventional sources of atoms (background vapor, thermal beams etc.), several steps of cooling are needed. In this section, we will discuss the basic methods of such cooling starting with a simplified model of a two-level atomic system under the influence of an electro-magnetic radiation.

2.2.1 Two level system in electro-magnetic radiation

An introduction to the origin and early development of light-matter interaction is given in the previous chapter. In this section, we will discuss the theory of interaction of an atom (two-level system) with an electro-magnetic field in the context of laser cooling and magneto-optical trapping. We will take a semi-classical approach to develop the background theory.

In the semi-classical model, the energy of an atom is considered to be quantized, and light as a classical oscillating electric field [128]. Coupling of such light with a two-level atom induces a population oscillation in the system between the ground and excited states at a frequency, known as “Rabi frequency”. In addition to that, the bare atomic energy levels of the atoms are modified in the presence of interaction energy term in the Hamiltonian. This shift is known as “light shift ” or AC Stark shift and has been described in detail in the previous chapter. For $\Delta \gg \Omega, \Gamma$, light shift [129, 130] is given by :

$$\delta E_{light} = \frac{\hbar \Omega^2}{4\Delta} \quad (2.28)$$

where ‘ Ω ’ is the Rabi frequency of the population oscillation between ground and excited states of the atom and $\Delta = \omega_L - \omega_0$ is the detuning of light from the atomic resonance frequency ω_0 . Using the dipole approximation and $|\Delta| \gg \Gamma$ (Γ being the natural linewidth of the transition of upper level or excited state of the two-level atomic system), for a red-detuned light ($\Delta < 0$), the sign of the light shift is positive for the upper level while it is negative for the ground state. In the presence of light, there are two types of radiation forces [131] on atoms. The first one is the *dissipative force* which involves spontaneous emission and can be considered as the scattering force for the atoms. The second type of force is the *dipole force* which arises from the gradient in the light intensity.

In the presence of an electro-magnetic field, the radiation pressure produced by scattering force on the atoms can be understood based on the principle of conservation of

momentum. Einstein proposed that, light is quantized with each quanta or photon having energy $E = h\nu$ and momentum $\mathbf{p} = \hbar\mathbf{k}$ where, ' ν ' is the frequency and $|\mathbf{k}| = 2\pi/\lambda$ is the wave vector of the light having wavelength λ . When an atom is placed in a beam of near-resonant light it absorbs a photon, and not only does it makes a transition to the excited state but also gets a recoil momentum kick in the opposite direction of the incident beam. Subsequently, the atom decays back to the ground state by spontaneous emission of a photon and again gets a recoil momentum kick in the opposite direction of the emitted photon. The spontaneous emission of a photon is a random process and has an equal probability of emission in all directions. Therefore, averaging over many absorption and emission cycles the net momentum kick on the atom is *zero* due to emitted photons. When the light beam is present, after many scattering events, the change in momentum or the net force on the atom is produced in the direction of the propagating light beam due to the unidirectional property of absorption of photons. Hence, the scattering force $\mathbf{F}_s$ on an atom in the presence of the light beam is given by the rate at which momentum is imparted to the atom:

$$\mathbf{F}_s = \hbar\mathbf{k}R_s \quad (2.29)$$

where, R_s is the scattering rate of photons by the atom, and given is by:

$$R_s = \frac{\Gamma}{2} \left(\frac{\Omega^2/2}{\Delta^2 + \Omega^2/2 + \Gamma^2/4} \right) = \frac{\Gamma}{2} \left(\frac{I/I_{sat}}{1 + I/I_{sat} + 4\Delta^2/\Gamma^2} \right) \quad (2.30)$$

where, I is the intensity of light and I_{sat} is the saturation intensity of the transition which is defined as, the intensity at which the transition is power broadened by a factor of $\sqrt{2}$ and is given by:

$$I_{sat} = \frac{h\nu}{2\tau (3\lambda^2/2\pi)} \quad (2.31)$$

where, Δ is the detuning of the light beam from the atomic resonance, Γ is the natural linewidth of the transition, τ is the lifetime of the excited state, and ν is the atomic transition frequency.

From Eq. 2.30, it is quite evident that the scattering force is Lorentzian [130] in nature. The linewidth of scattering rate can be modified by changing the intensity of light beam such that, I/I_{sat} is not negligible. For zero detuning and intensity I much

larger than I_{sat} , the scattering force experienced by the atom is maximum and is given by $\hbar k\Gamma/2$. In particular, for ^{87}Rb the natural linewidth (FWHM) of $5^2\text{S}_{1/2} \rightarrow 5^2\text{P}_{3/2}$ transition is $\Gamma = 2\pi \times 6.1$ MHz and saturation intensity is 1.6 mW/cm^2 [132]. For the realization of significant scattering force, the linewidth of the laser has to be less than the natural linewidth of the transition. In 1975, after the discovery of lasers, Hänsch and Schawlow [133] and independently Wineland and Dehmelt [134] proposed that narrow-band tunable lasers could be used to cool atoms and ions respectively.

2.2.2 Doppler cooling

A classical ensemble of atoms follows Maxwell-Boltzmann velocity distribution. Following the equipartition theorem of energy, for an atom of mass m and temperature T , the relation between thermal energy and the mechanical energy can be given by:

$$\frac{3}{2}k_B T = \frac{1}{2}mv_{rms}^2 \quad (2.32)$$

where k_B is the Boltzmann constant, m is the atomic mass and v_{rms} is the root-mean-square velocity of the atom at temperature T . Therefore, to reduce the temperature of a cloud of atoms, we should reduce the average velocity of the atomic ensemble.

The basic tool for laser cooling is the scattering force given in Eq. 2.30. Since, for a laser which is red-detuned with respect to atomic transition, the scattering force becomes dissipative, hence is used to reduce the average velocity of a group of atoms.

Such a cooling relies on the scattering force, which in turn is a function of atomic velocity due to Doppler effect. In case of a moving atom, the Doppler shift in the frequency of laser experienced by the atom in its frame of reference is given by:

$$\omega_D = \mathbf{k} \cdot \mathbf{v} \quad (2.33)$$

Hence, the frequency of the laser in the frame of reference of an atom is:

$$\omega'_L = \omega_L - \omega_D = \omega_L + \mathbf{k} \cdot \mathbf{v} = \omega_0 + (\Delta + \mathbf{k} \cdot \mathbf{v}) \quad (2.34)$$

The term $(\Delta + \mathbf{k} \cdot \mathbf{v})$ is the effective detuning as seen by a moving atom.

From Eq. 2.34, it is evident that if the detuning Δ of a laser is equal and opposite to Doppler shift ω_D , the atom will see the laser at resonance frequency ω_0 . In this case,

the scattering rate is maximum leading to the maximum probability of absorption of a photon by the atom. For an atom moving towards the laser beam, a red-detuned beam is used such that the Doppler shift reduces the effective detuning and thereby increases the probability of absorbing a photon.

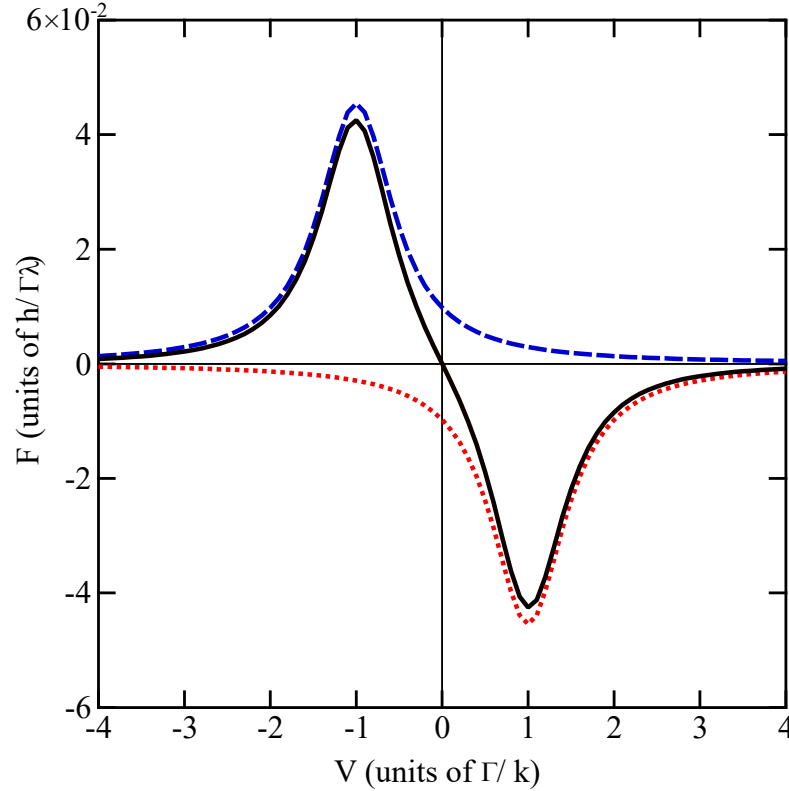


Figure 2.2: Doppler force: The Doppler force on an atom in a counter-propagating pair of laser beams as a function of the velocity of the atom for: $I/I_{sat} = 0.1$ and $\Delta = -\Gamma$. The blue dashed curve shows the force for the effective blue detuned laser, the red dotted curve shows the force due to the effective red-detuned laser and the curve in solid line shows the sum of these two forces and shows the net force is dissipative.

Now, let us consider an atom moving in two red-detuned, counter-propagating laser beams. An atom trying to move towards one such laser beam sees the light towards resonance and absorbs photons from the laser beam. On each absorption of a photon, the atom gets a momentum kick of magnitude $\hbar k$ in the direction opposite to its motion. Hence, the combination of two counterpropagating lasers always tries to reduce the velocity. When the atom decays to ground state due to spontaneous emission, it emits photons in random directions and therefore averaging over many such absorption and

emission cycles, the net momentum on the atom averages to zero. To conclude, atoms remain in a dynamical equilibrium where the force produced by the laser beams tries to minimize its velocity in both directions of propagation along the laser beams.

For low intensities $I/I_{sat} \ll 1$, the total force on the atom by two such counter-propagating laser beams in 1-D is the sum of force applied by each beam and given by [130]:

$$\mathbf{F}_{1D} = \hbar \mathbf{k} \frac{\Gamma}{2} \left[\frac{I/I_{sat}}{1 + I/I_{sat} + 4(\Delta - \mathbf{k} \cdot \mathbf{v})^2/\Gamma^2} \right] - \hbar \mathbf{k} \frac{\Gamma}{2} \left[\frac{I/I_{sat}}{1 + I/I_{sat} + 4(\Delta + \mathbf{k} \cdot \mathbf{v})^2/\Gamma^2} \right] \quad (2.35)$$

The nature of the force is shown in Fig. 2.2. It is clear that the force is in the opposite direction of the velocity of atoms. In the low-velocity region, the force has a linear dependence on the velocity of the atoms which can be seen from Eq. 2.35. For small velocities $kv \ll \Delta$, Eq. 2.35 can be approximated by:

$$\mathbf{F}_{1D} \simeq -\beta \mathbf{v} \quad (2.36)$$

where,

$$\beta = -4\hbar k^2 \frac{I}{I_{sat}} \frac{I/I_{sat}}{[1 + I/I_{sat} + 4\Delta^2/\Gamma^2]^2} \quad (2.37)$$

Therefore, such force $\mathbf{F}_{1D}$ is dissipative like that of a particle in a viscous medium with damping constant β .

The limit of Doppler cooling comes because of its dependence on the absorption of photons. Due to the stochastic nature of the absorption/emission process, the atom performs a random walk in the momentum space leading to heating caused by momentum diffusion. Hence, the lowest temperature possible to achieve by such cooling occurs when this heating rate becomes equal to the cooling rate. In the low intensity limit $I/I_{sat} \ll 1$ and detuning $\Delta = -\Gamma/2$, this minimum temperature T_D is given by [129, 130]:

$$k_B T_D = \frac{\hbar \Gamma}{2} \quad (2.38)$$

Where T_D is called as the Doppler limited temperature. Using the above framework, for ^{87}Rb , with natural linewidth $\Gamma = 2\pi \times 6.1$ MHz, the calculated theoretical Doppler temperature limit is $T_D = 146\mu\text{K}$.

The above model describes the cooling mechanism of atoms in one dimension that is, along the axis of propagation of laser beams. This model can easily be extended to 2-D and 3-D by introducing extra pairs of counterpropagating beams along the relevant dimensions. This configuration in 3-D is known as “optical molasses” because the dynamics of atoms in such light fields are analogous to that of particles in a viscous fluid. In 1985, Steven Chu reported the first experimental demonstration of optical molasses [135].

2.3 Magneto-optical trap (MOT)

The optical molasses configuration described in the previous section indeed provides slowing and cooling of atoms but cannot confine the atoms in space. If the atoms in the optical molasses diffuse out of the region of overlap of the laser beams, they get lost from the cooling cycle. The typical time scale of such dynamical confinement of atoms in optical molasses is of the order of ~ 100 ms. Therefore, a spatially dependent restoring force is needed to confine the atoms in space which can be achieved by superimposing a spherical quadrupolar magnetic field with the optical molasses configuration. Such a magnetic field can be produced by a pair of anti-Helmholtz current carrying coils. The magnetic field lifts the degeneracy of the Zeeman levels of atoms, resulting in a spatially varying transition frequency and radiative force. Choosing the appropriate polarizations (fig. 2.3) of laser beams decided by the direction of the magnetic field, a restoring force can be achieved. In a MOT, the center of the quadrupolar magnetic field is overlapped with the intersection of six molasses beams, as illustrated in Fig. 2.3. For the operation of MOT, the optical molasses laser beams are used in the $\sigma^+ - \sigma^-$ configuration.

Let us consider a simple one-dimensional model in order to understand the origin of the restoring force in the MOT. We assume the atom to possess $J = 0$ angular momentum in the ground state and $J = 1$ angular momentum in the excited state. Doppler shift is neglected, considering the atom at rest. Due to the presence of the magnetic field, Zeeman levels of the excited state with $J = 1$ split into $m_J = 0, \pm 1$ sublevels. The magnetic field is zero at the center and increases linearly with the position z . Hence the energies of magnetic sub-levels also increase or decrease linearly in space as illustrated in fig. 2.4. In the presence of circularly polarized beams, σ^+ beam drives

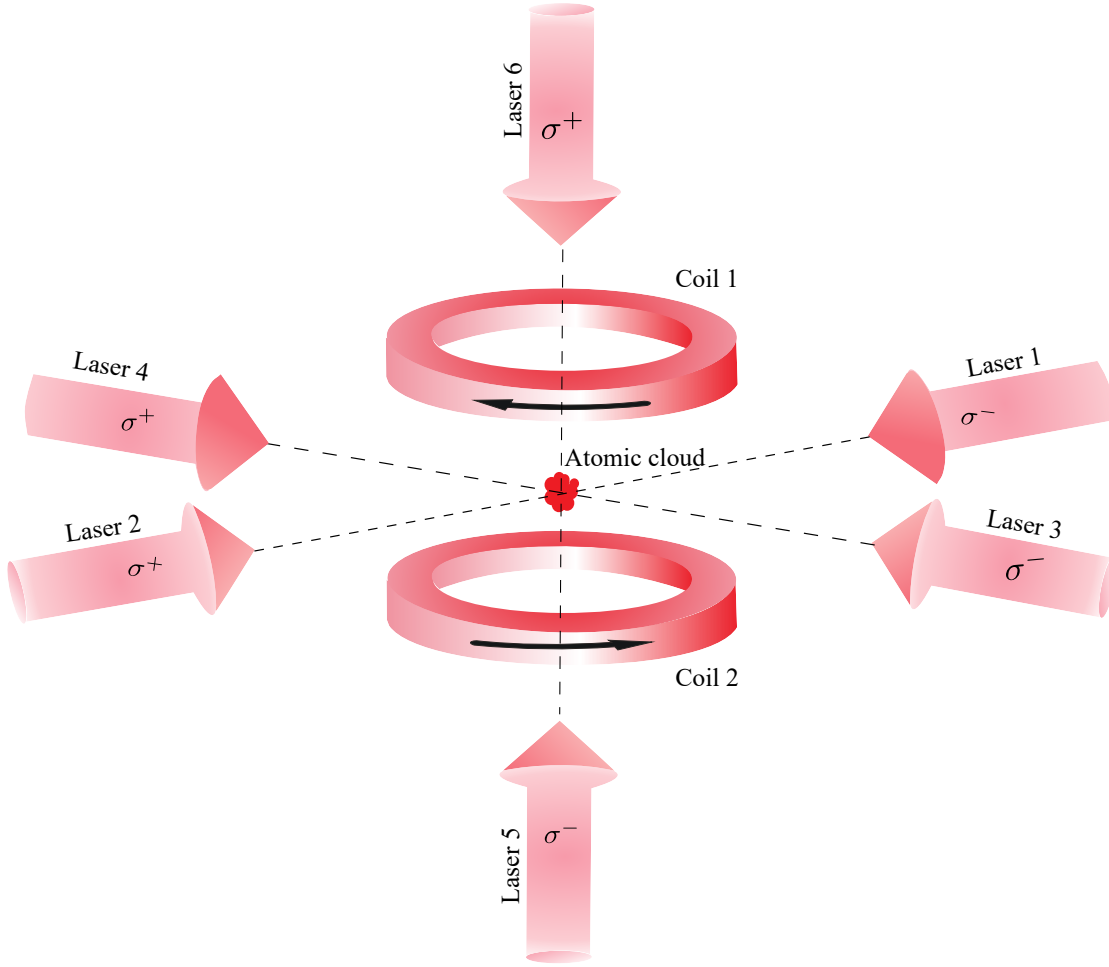


Figure 2.3: Magneto-optical trap (MOT): Configuration of the quadrupolar magnetic field along with the desired polarizations of laser beams in a MOT. The black arrows indicate the direction of current, and the notation on each beam (σ^+/σ^-) indicates the type of transition it drives in the trapped atoms.

the $m_J = 0 \rightarrow m'_J = 1$ transition and σ^- beam drives the $m_J = 0 \rightarrow m'_J = -1$ transition. The allowed transitions are governed by the selection rules $\Delta m_J = \pm 1$. Let us consider a case in which frequency of both the counterpropagating beams of a molasses is kept red-detuned to the atomic transition. If an atom moves to the right from the center ($z > 0$), atom sees the σ^- beam (which drives the $m_J = 0 \rightarrow m'_J = -1$) to be closer to atomic resonance for $m'_J = -1$ level. This results in the increase of scattering rate. Due to the absorption of photons, the laser beam opposes the motion and hence, net momentum pushes the atom back towards the center. Similarly, when atom moves to left from the center ($z < 0$), it absorbs more photon from σ^+ light

as it turns out that $m'_J = -1$ possesses the lowest energy in this case. Hence, in this manner, it behaves as a restoring force which opposes the motion of atoms and is always directed towards the center. The net force on the atom is the sum of dissipative force and the restoring force and is given by:

$$\mathbf{F}_{MOT} = -\beta\mathbf{v} - \kappa z \quad (2.39)$$

where the first term is the dissipative force given by Eq. 2.36 and the second term corresponds to restoring force having a spring constant κ given by:

$$\kappa = \frac{g_F \mu_B m_F}{\hbar k} \beta \frac{dB}{dz} z \quad (2.40)$$

where $\frac{dB}{dz}$ is magnetic field gradient, g_F is Lande g-factor, m_F is the magnetic sublevel of the corresponding F level, and μ_B is the Bohr magneton.

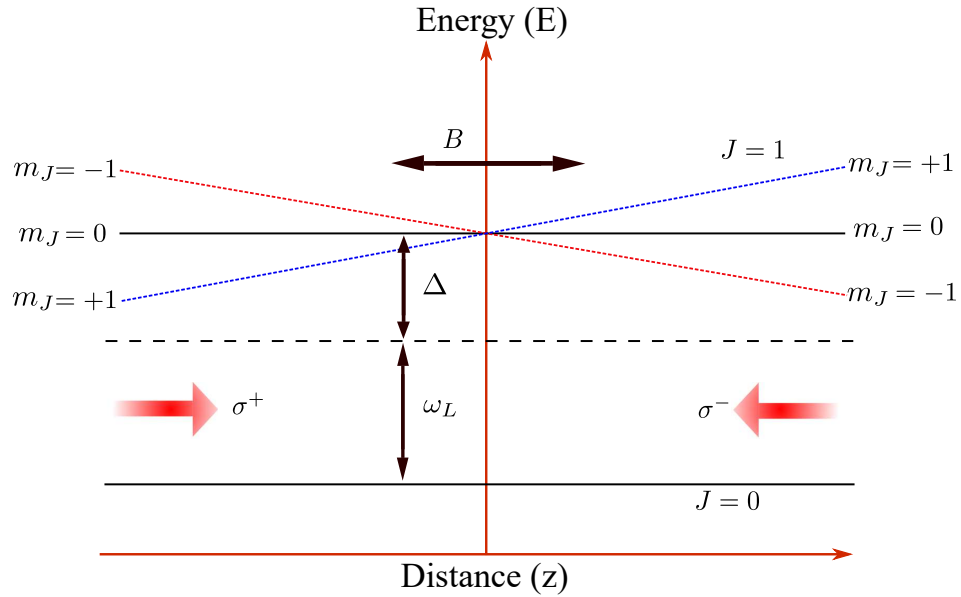


Figure 2.4: MOT operation: Illustration of the operating principle of a MOT in a $J = 0,1$ system. ‘z’ can be considered as distance from the trap center along the direction of any pair of laser beams shown in fig. 2.3. Dotted lines show the spatial modulation of energy of the Zeeman sublevels due to the quadrupolar magnetic field. Dashed line represents the equivalent energy level addressed by the laser frequency.

This model can be extended to 2-D and 3-D configuration using atomic systems having non zero angular momentum for ground state resulting in the realization of a 2-

D or a 3-D MOT. The number of trapped atoms in MOT can be estimated by measuring the amount of fluorescence of the trapped atoms using a light collection device like photodiode or photomultiplier tube. This power of the fluorescence P is given by:

$$P = \frac{hc}{\lambda} \frac{\Omega}{4\pi} N R_s \quad (2.41)$$

where N is the number of atoms R_s is the scattering rate given in Eq. 2.18. Ω is the solid angle subtended by the collection lens.

2.4 Magnetic trapping of neutral atoms

Laser cooling works on the principle of photon scattering. It turns out that closer the frequency of the laser is to the atomic resonance frequency, more the probability of scattering. However, this also induces heating. Hence the laser cooling can be employed as a primary cooling stage while it cannot produce temperatures needed to reach quantum degeneracy for all the atomic species. Also, laser cooling is not sufficient to confine the atoms in a strong potential which is much needed for further manipulation. Hence, we need a strong trapping potential. Here, we consider the quadrupolar magnetic field to create such desired trap.

The basic principle of magnetic trapping is based on the interaction between the magnetic moment μ of an atom and the magnetic field strength $B(\mathbf{r})$. Magnetic trapping potential is a manifestation of the energy modulation due to Zeeman effect. Hence, the potential experienced by a magnetic dipole in the presence of an inhomogeneous magnetic field is given by:

$$U(F, m_F) = -g_F m_F \mu_B |\mathbf{B}| \quad (2.42)$$

where g_F is the Lande g factor. It is straightforward to see which Zeeman sublevel can produce a potential well for trapping. For example, in case of ^{87}Rb , $|F = 2, m_F = 2\rangle$, $|F = 2, m_F = 1\rangle$ and $|F = 1, m_F = -1\rangle$ are the three low-field seeking states. Hence atoms can be trapped in one of these states.

2.4.1 Spherical Quadrupole trap

One of the simplest magnetic traps is a spherical quadrupole trap which is realized using a pair of current-carrying coils in “anti-Helmholtz” configuration, in which, the direction of currents in the coils is opposite to each other. This trap gives an axially symmetric magnetic field about the axis and varies linearly over distance near the center of the trap. The magnetic field near the center of the trap is given by:

$$\mathbf{B} = B'_z z \mathbf{e}_z + \frac{1}{2} B'_x x \mathbf{e}_x + \frac{1}{2} B'_y y \mathbf{e}_y \quad (2.43)$$

where the axial and radial gradients are related by:

$$B'_z = \frac{\partial B}{\partial z} = 2 \frac{\partial B}{\partial r} = 2B'_r \quad (2.44)$$

The spherical quadrupole magnetic field has a minimum value of the magnetic field equals to zero, which occurs at the center of the trap. The magnetic moment of the atoms always precesses about the local magnetic field at Larmor frequency ω_{Larmor} , which is given by:

$$\omega_{Larmor} = \frac{g_F m_F \mu_F}{\hbar} |\mathbf{B}| \quad (2.45)$$

In a magnetic trap, the local magnetic field as seen by the atoms is always changing as the atom moves in the trap. If this change of the local magnetic field is adiabatic with respect to Larmor frequency, only then can the atom get trapped. Thus the condition required for trapping is that the Larmor frequency should be much larger than the variation rate of the angle ϕ of the magnetic field realized in the atomic frame, i.e.:

$$\frac{d\phi}{dt} \ll \omega_{Larmor} = \frac{g_F m_F \mu_F}{\hbar} |\mathbf{B}| \quad (2.46)$$

This adiabatic condition ensures that the atom remains in the same quantum state relative to the instantaneous magnetic field direction.

For spherical quadrupole trap, when the temperature is very low, and densities are high, the adiabatic condition is violated because at the center, where the magnetic field is zero ($|\mathbf{B}| = 0$). When atoms come near the trap center, the separation of the Zeeman sub-levels becomes very small, and the atom has a very high probability to undergo

transitions into untrapped states, known as “Majorana spin flips” [136], resulting in the loss of atoms from the trap. The effective rate of Majorana spin flips is given by $\sqrt{2\hbar v/\pi\mu B'}$. Hence, in order to eliminate such loss of atoms, we need a potential with the non-zero minimum. Several schemes can be used viz. Ioffe-Pritchard (IP) trap, time-averaged orbiting potential (TOP) trap [137], etc. to design such a trap by the magnetic field.

2.5 Optical dipole trap

Instead of a magnetic trap for trapping atoms, in this thesis we use a far-detuned optical trap for further trapping and cooling. This section is dedicated to the discussion of such an optical dipole trap.

2.5.1 Optical dipole force

The first realization of the trapping of microparticles using a highly focused laser beam was reported [12] by A. Ashkin in the year 1970. Later, it was proposed to use such a trap for neutral atoms [138]. Such trapping potentials are conservative in nature and hence cannot be employed for laser cooling. Moreover, the lasers used for the construction of such traps are far detuned from the atomic resonance frequency which reduces the scattering rate. As a result, the heating due to spontaneous emission also reduces. The dynamics of atoms in such a trap has been well-studied in Ref. [139, 140] and the first experiment to use such a trap for cooling atoms down to quantum degeneracy was reported by M. D. Barrett in 2001 [141]. In the following sections, we will first discuss the theory of such trapping by considering a two-level atomic system. This idea will be extended for a real multi-level atomic system. Finally, we will discuss about the crossed optical dipole trap and the combined potential formed by such a trap along with the quadrupolar magnetic trap and gravitational potential.

2.5.2 Optical dipole trapping with a two-level atomic system

Let us consider a two-level atomic system in a linearly polarised laser field given by $\mathbf{E}(r, t) = \mathbf{E}_0 \exp(-i\omega t) + \text{c.c.}$, where ω is the frequency of the laser and $\mathbf{E}_0$ is the amplitude of the laser. Such an oscillating electric field induces an oscillating dipole moment $\mathbf{P}$ in the atom. $\mathbf{P}$ is proportional to the amplitude of the electric field and is

given by:

$$\mathbf{P} = \alpha \mathbf{E}_0 \quad (2.47)$$

where, α is the complex polarizability of the atom. This induced dipole upon interaction with the electric field gives rise to a potential of the form,

$$U_{dip} = -\frac{1}{2} \langle \mathbf{P} \cdot \mathbf{E}_0 \rangle = -\frac{1}{2\epsilon_0 c} \text{Re}(\alpha) I \quad (2.48)$$

Where, ϵ_0 is the permittivity of free space, c is the speed of light in vacuum and I is the intensity of light given by $I = 2\epsilon_0 c |\mathbf{E}_0|^2$. The dipolar force can easily be derived by calculating the gradient of the energy given in Eq. 2.48. Thus the dipolar force is given by:

$$F_{dip}(r) = -\nabla U_{dip}(r) = \frac{1}{2\epsilon_0 c} \text{Re}(\alpha) \nabla I(r) \quad (2.49)$$

In order to know the effective potential, it is necessary to calculate the scattering rate. The scattering rate can be calculated using the imaginary part of the polarizability and is given by:

$$\Gamma_{sc}(r) = \frac{1}{\hbar \epsilon_0 c} \text{Im}(\alpha) I(r) \quad (2.50)$$

In order to replace α by physically realizable quantities, we consider the atom as a simple classical oscillator with eigenfrequency ω_0 . Considering low scattering rates, Eq. 2.49 and Eq. 2.50 simplify to [142]:

$$U_{dip}(r) = -\frac{3\pi c^2}{2\omega_0^3} \left(\frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) I(r) \quad (2.51)$$

$$\Gamma_{sc}(r) = \frac{3\pi c^2}{2\hbar \omega_0^3} \left(\frac{\omega}{\omega_0} \right)^3 \left(\frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right)^2 I(r) \quad (2.52)$$

Introducing $\Delta = \omega - \omega_0$, the detuning of the laser from the atomic resonance frequency, we can notice that the optical potential U_{dip} scales as I/Δ , whereas, the scattering Γ_{sc} scales as I/Δ^2 . Therefore a far-detuned laser with high intensity is desired for realization of such a trap.

2.5.3 Dipole forces on a multi-level atom

In the preceeding section, a simplified model with a two-level system has been considered to derive the basic formulation of an optical dipole trap. In a practical experimental

scenario, the atoms possess many energy levels. Therefore, it is necessary to consider all the energy levels of the atom and incorporate the necessary modifications to the trapping potential. The basic scaling of potential energy and scattering rate will still be applicable. So, to reduce our efforts, we consider the dressed state picture of the atom in the presence of the laser field. With the help of second order time-independent non-degenerate perturbation theory. We can write the correction or shift in the energy levels due to a perturbing Hamiltonian H_{opt} as [142]:

$$\Delta E_i = \sum_{j \neq i} \frac{|\langle j | H_{opt} | i \rangle|^2}{\epsilon_i - \epsilon_j} \quad (2.53)$$

where, ϵ_i s denote the energy of the dressed levels. For example, if the atom is in ground state and the field containing n photon in the existing energy level, then the total energy of the system will be $\epsilon_i = n\hbar\omega$. On the other hand, if the atom stays at excited state then the total energy of the system will be $\epsilon_j = \hbar\omega_0 + (n-1)\hbar\omega = n\hbar\omega - \hbar\Delta_{ij}$. Recalling that the perturbed Hamiltonian to be equal to $H_{opt} = -\mu \cdot \mathbf{E}$, we can find the energy shifts as:

$$\Delta E = \pm \frac{|\langle e | \mu | g \rangle|^2}{\Delta} |\mathbf{E}|^2 = \pm \frac{3\pi c^2 \Gamma}{2\omega_0^3 \Delta} I \quad (2.54)$$

It is clear that the energy shift is opposite in sign for the ground and excited state. Since, the shift in the energy comes out to be a function of intensity, a laser beam with gaussian intensity distribution will create a nearly harmonic trapping potential for the atoms in ground state. Eq. 2.54 can be extended for a multilevel case, as:

$$\Delta E_i = \frac{3\pi c^2 \Gamma}{2\omega_0^3} \sum_j \frac{c_{ij}^2}{\Delta_{ij}} I \quad (2.55)$$

where, c_{ij} s are the dipole matrix elements between the energy levels.

2.5.4 Atoms in a focused gaussian beam

We have derived the optical dipole trap potential as a function of intensity of the laser. In this section, we will assume the laser to possess gaussian intensity distribution (which is common to most experimental setup) and characterize the trap.

Let us consider a focused gaussian laser beam (fig. 2.5) with power P and propagating

along the z -axis. For such a laser beam the intensity distribution is given by :

$$I(r, z) = \frac{2P}{\pi\omega^2(z)} \exp\left[-\frac{2r^2}{\omega^2(z)}\right] \quad (2.56)$$

where, r is the radial co-ordinate and $\omega(z)$ denotes the beamwaist, defined as the distance from the center along the radial direction, where the intensity decreases to $1/e^2$ of the maximum value. The variation of beam-waist as a function of distance from the focus can be given as:

$$\omega(z) = \omega_D \sqrt{1 + \left(\frac{z}{z_R}\right)^2} \quad (2.57)$$

where, ω_D is the beam waist at the focus of the laser beam and $z_R = \pi\omega_D^2/\lambda$ is the Rayleigh range signifying the distance scale upto which the effect of the focus can be realized.

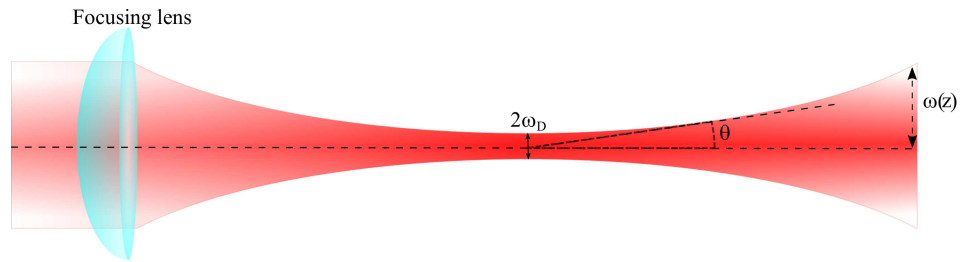


Figure 2.5: Focused gaussian laser beam: Variation of intensity of a focused gaussian laser beam. Variation of beam waist and divergence of the beam is shown with increasing distance from the focus. θ is the divergence of the beam.

To find out the profile of the optical potential due to a focused gaussian laser beam we can simply put the intensity profile of Eq. 2.56 into the Eq. 2.51. Since, we are interested in finding out the trapping frequencies, the net potential energy function is Taylor expanded around the minima and the harmonic term is used for further analysis. Such harmonic potential has the form given by:

$$U(r, z) = -U_0 \left[1 - 2 \left(\frac{r}{\omega_D} \right)^2 - \left(\frac{z}{z_R} \right)^2 \right] \quad (2.58)$$

Hence, near the bottom of the trap, the trapping frequencies can be given as:

$$\omega_r = \sqrt{\frac{4U_0}{m\omega_D^2}} \quad \text{and,} \quad \omega_z = \sqrt{\frac{2U_0}{mz_R^2}} \quad (2.59)$$

It is evident from Eq. 2.59 that the radial frequency is much higher than the axial frequency, leading to a higher confinement along the radial direction of the laser beam and weaker confinement along the axial direction which is the direction of the propagation of the beam. Due to such weaker confinement along the direction of the propagation of the beam, a single optical dipole trap is not suitable to perform further cooling of atoms in it. The solution is to use another similar optical beam ideally perpendicular to the previous laser beam. Such a configuration is known as crossed optical dipole trap.

2.5.5 Crossed optical dipole trap

High elastic two-body collision rate is the key to achieve an efficient cooling in any trap. A trap with higher trap frequency and tighter confinement is always desired to achieve high collision rate for required thermalization of the atomic cloud during the evaporative cooling step [143]. Such cooling is done by continuously reducing the trap depth and allowing the higher energy atoms to leave the trap, allowing the rest of the atoms to rethermalize and achieve a lower mean temperature. Therefore, a crossed optical dipole trap serves as the best platform to achieve a faster and efficient cooling. The polarizations of the two laser beams are chosen to be orthogonal to each other to avoid any interference between the laser beams. The net intensity, considering the two beams propagating along x-direction and y-direction respectively, can be written as:

$$I(x, y, z) = \frac{2P_x}{\pi\omega^2(x)} \exp\left[-2\frac{y^2 + z^2}{\omega^2(x)}\right] + \frac{2P_y}{\pi\omega^2(y)} \exp\left[-2\frac{x^2 + z^2}{\omega^2(y)}\right] \quad (2.60)$$

Hence, following the similar methodology, the trapping frequencies can be found as [144]:

$$\begin{aligned} \omega_x &= \sqrt{-\frac{8U_0}{\pi m} \left(\frac{P_y}{\omega_{D,y}^4} + \frac{P_x}{2\omega_{D,x}^2 x_R^2} \right)} \\ \omega_y &= \sqrt{-\frac{8U_0}{\pi m} \left(\frac{P_x}{\omega_{D,x}^4} + \frac{P_y}{2\omega_{D,y}^2 y_R^2} \right)} \\ \omega_z &= \sqrt{-\frac{8U_0}{\pi m} \left(\frac{P_y}{\omega_{D,y}^4} + \frac{P_x}{\omega_{D,x}^4} \right)} \end{aligned} \quad (2.61)$$

In practice, due to the presence of gravity along the vertical direction, the optical dipole trap potential gets modified in the vertical direction. In this thesis work, a quadrupolar magnetic field has also been employed during the experiment to overcome the effect of gravity. In the next section, we discuss the profile of the net trap because of the combined effect of all these potentials.

2.5.6 Effect of gravitational potential and quadrupolar magnetic trap

The gravitational potential can be written as mgz , where z is the coordinate in the vertical direction or the direction of gravity. We assume the trap center to be at $z = 0$. Therefore the magnetic potential will look like a $\beta|z|$ function, β being the axial gradient of the quadrupolar magnetic field. We consider the case where the magnetic field gradient is less than the gravity gradient by a small amount. The combined form of the potential has been shown in fig. 2.6 (Image not to scale). In an experiment, the

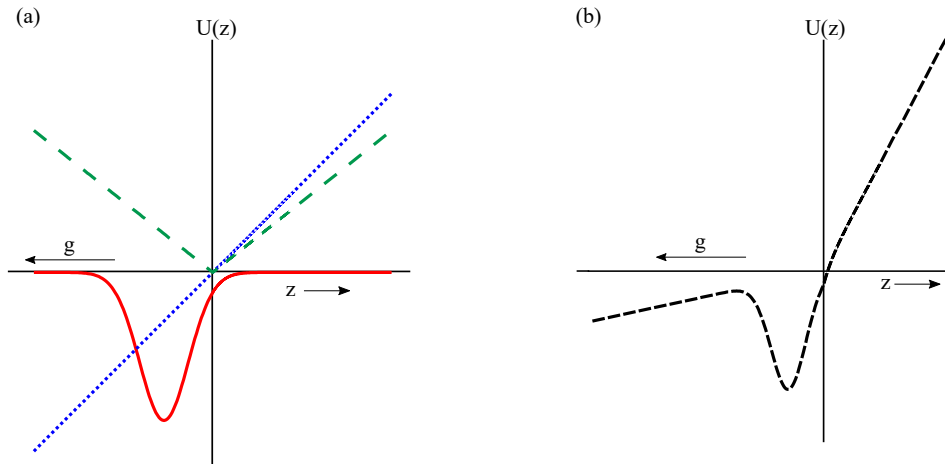


Figure 2.6: Combined potential: Variation of potential energy as a function of vertical distance z from the trap center. (a) potential due to individual contribution. Green dashed curve represents the magnetic potential. Blue dotted curve represents the potential due to gravity. Solid red curve shows a nearly harmonic potential due to crossed optical dipole trap. (b) Black dashed curve represents the combined potential. In both the cases, -ve z -axis represents the direction of gravity as shown by the arrow.

magnetic field gradient can be controlled by controlling the current in the electromagnetic coils. Therefore, changing the gradient of the magnetic trap will definitely change the nature of the combined trap. We have considered the case where the magnetic field

gradient is less than the gravity gradient because this is consistent with the experimental parameters used in this thesis work.

In this work, we have used such a trap to extract a coherent flow of ultracold atoms known as atom laser. A brief overview of an atom laser is given in the next section.

2.6 Atomlaser

An atom laser is an analog of an optical laser. To understand the properties and advantage of an atom laser, let us first discuss the properties of laser light. A detailed definition of an atom laser is given by Wiseman [145]. A laser is a device which can be well approximated by a classical wave with fixed amplitude and phase. Unfortunately, this definition precludes some specific types of lasers, e.g., pulsed lasers. Without loss of generality, a laser can be defined as a guided or freely propagating beam of bosons out-coupled from one or more macroscopically occupied trapped modes [146]. This definition encompasses various types of lasers such as a pulsed laser, q-switched laser, multimode continuous wave (CW) laser, etc. From the perspective of origin, it can be

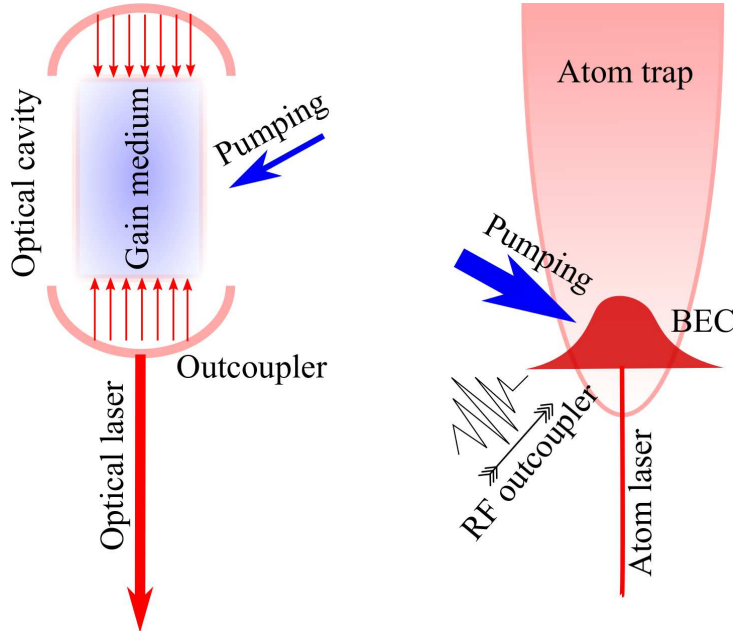


Figure 2.7: Atom laser vs. optical laser: Analogy between the outcoupling mechanism of optical laser and atom laser. Atom trap is similar to the optical cavity. BEC serves as the macroscopic state from which an atom laser is outcoupled.

shown easily that the outcoupling of an atom laser is analogous to that of an optical

laser. An optical laser is outcoupled from a cavity containing a gain medium. Similarly, an atom laser is outcoupled from a BEC produced in an atom trap. Frequently practiced outcoupling procedures will be discussed in details in the following paragraph.

2.6.1 Outcoupling schemes

The most common scheme for outcoupling of coherent beam of atoms from a BEC is the radio frequency (RF) coupling. Such a scheme can be realized when BEC is formed in a magnetic trap. For a BEC formed in a magnetic trap, all the atoms stay in a single Zeeman hyperfine level. For the other hyperfine level, a trapping potential is not formed. So obviously, atoms in those hyperfine levels cannot be trapped using the magnetic field configuration. The RF coupling scheme works on this principle. If the BEC is formed in a particular Zeeman hyperfine level, then by applying RF pulses, atoms undergo a transition to other (untrapped) hyperfine levels resulting in making the atom leave the trap. The untrapped atoms start coming out of the trap and form the atom laser. Realization of such a scheme has been reported in Ref. [147]. These type of atom lasers fall freely under gravity and normally are quasi-continuous.

Another important out-coupling scheme is using Raman pulses. Raman pulses are realized by a pair of phase-locked laser beams. An atom absorbs a photon from one laser and emits it in the direction of the other laser via stimulated emission. Therefore, the net effect of such pulses results in a momentum kick in a particular direction. By changing the angle between the laser beams, one can control the direction of the imparted momentum along with the magnitude of it. The maximum possible momentum transfer on the atoms is $2\hbar k$, where 'k' being the wave-vector of light. Raman out-coupling scheme has been realized in Ref. [148]. The atom lasers formed in this case can be outcoupled in any preferred direction instead of gravity, and they are pulsed in nature. In Ref. [148], Hagley *et. al* demonstrated a quasi-continuous laser using Raman pulses at ~ 1 KHz.

Next in the queue is a bunch of schemes realized in case of an optical trap. Since optical traps support all the Zeeman hyperfine levels, hence an RF out-coupling scheme is of no use in such cases. Different techniques have been implemented so far to outcouple atom laser from an optical trap. The first realization of outcoupling from an optical trap was by introducing a high magnetic gradient [149] after formation of BEC. Due to the state-selection of the magnetic trap, atoms leaked out of the trap leading to the generation of an atom laser. Later, a comparatively easier scheme was realized by reducing the

trap depth continuously [150] and thereby allowing the atoms to come out till the whole trap is emptied. Such an atom laser is continuous in nature but suffers from the poor spacial mode. The last one in the row is using a guided push beam [151]. Such scheme has been realized in hybrid (magnetic-optical) traps. Initial evaporation of atoms is done in a magnetic trap. At the end of cooling, a strongly focused laser beam is placed in the center of the trap to evaporate faster. However, the atoms are loosely bound in the direction of the beam and eventually come out in that direction. Such atom lasers have extremely high collimation due to their guided nature. Later, generation of atom laser has also been reported [152] as a result of interference of many matter-wave leaking from the sites of an optical lattice. The following section is dedicated to the discussion of the special properties of the atom lasers.

2.6.2 Properties of atom laser

Let us recall the special properties offered by a conventional optical laser. A conventional laser is a continuous source of coherent waves. It possesses a fair amount of collimation up to certain distance. Using proper optical elements, lasers with desired clean output mode can be achieved. The practical criteria of spatial mode can be given by the divergence of the beam. A more rigorous criterion for the mode of a laser is given by the measure of quality factor M^2 . For an atom laser, the expression of M^2 can be given as [146]:

$$M^2 = \frac{2}{\hbar} \Delta x \Delta p_x \quad (2.62)$$

where Δx is the width of the beam at the waist and Δp_x is the momentum spread in the longitudinal direction. For an ideal Gaussian beam, $M^2 = 1$.

Spatial mode of an atom laser is experimentally characterized by the divergence of the beam. An atom laser out-coupled via RF coupling suffers from high divergence. The reason behind such divergence is that out-coupled atoms correspond to the untrapped hyperfine Zeeman level for which the magnetic field creates a potential hill. Therefore, the outcoupled atoms gain a transverse momentum while traveling downhill. This results in a high divergence for such atom lasers. On the contrary, atom lasers out-coupled from optical traps see the same trapping potential and thereby giving rise to a highly collimated source of atoms. The minimum divergence can be achieved by out-coupling via guided beam.

The brightness of the atom laser depends on the flux of the atoms. An atom laser plays the role of an optical laser in an atom interferometer setup. Therefore, for a high sensitivity limited by quantum projection noise [153], a large amount of flux is required. Since the number of atoms in a BEC is limited by the trap parameters, increasing the flux results in a reduction of the operation time. In experiments, a typical value of the flux of an atom laser that can be achieved is $\sim 10^5 - 10^6$ atoms per second. Current research on atom laser is focused on realizing a continuously operable atom laser decoupled from a continuous reservoir of coherent atoms [154].

The last and most important property of a laser is its coherence. It is the property which is accountable for high brightness and spectral density. For an atom laser, the first order correlation was studied in Ref. [155]. Such experiments are based on interferometric technique and show a high contrast in fringes indicating a good phase-correlation in the atom laser. A second-order correlation was studied in Ref. [156]. In this experiment, a freely falling atom laser was made to pass through a cavity, kept below the source of the atom laser. By detecting the presence of the atom in the cavity a second order correlation function was measured.

Atom lasers suffer from less wavefront aberration and offer a high degree of collimation in comparison to a BEC. Because of all these properties, an atom laser can be thought of as an appropriate candidate for atom interferometry and inertial sensing.

2.7 Atom optics kicked rotor (AOKR)

One major part of this dissertation is dedicated to addressing the study of decoherence. We have used AOKR as a test-bed to study decoherence. In this section, we give a brief introduction about AOKR system. AOKR is an example of Hamiltonian chaos. Therefore, we will first introduce the concept of chaos and then come back to AOKR system where chaos plays a major role in the dynamics of the system.

2.7.1 Chaos in a system

Chaos describes random behavior in a deterministic system where the separation of nearby trajectories in phase-space grows exponentially with time. Such behavior can be seen in some non-linear systems. To be specific, if a system is non-integrable, then we might find chaotic behavior in the system. The only way left to find out the dynamics

of such a system is via numerical computation. Due to the non-linear nature of the system, any small change in the initial condition of the system ends in a totally different final state. In order to get the desired trajectory, corresponding to a large time, one needs to iterate the solution many times. From a practical standpoint, limited by the precision of the calculation or bytes, the final result differs when solved using different processors [157]. Therefore, the behavior of the system becomes unpredictable for such a long time. Such systems are called classically chaotic. The idea of quantum chaos is more complicated and controversial [158]. The most accepted definition of a quantum-mechanical chaotic system is a system whose classical counterpart shows chaos. We can find the existence of chaos in a range of physical systems, e.g., the field of mechanics, electrical engineering, laser physics, fluid flows, biological systems, etc. In the following section, we will discuss the role of chaos in a kicked rotor system.

2.7.2 kicked rotor: classical and quantum dynamics

A classical kicked rotor can be defined as a rotor kicked periodically. The Hamiltonian of such a system can be given as:

$$H = \frac{\rho^2}{2} + K \cos \theta \sum_{n=-\infty}^{\infty} \delta(\tau - nT) \quad (2.63)$$

where, the mass of the particle has been considered to be unity. The first term represents the kinetic energy with ρ being the angular velocity. θ represents the azimuthal co-ordinate. K is the stochasticity parameter, known as kick-strength. As the name suggests, K defines the amount of the energy provided to the system at every kick. The δ -function in the last term depicts a periodic kicking process with period T . The evolution of the dynamical quantities of the system can be found out by the following standard map introduced first by Chirikov [159],

$$\theta_{n+1} = \theta_n + \rho_{n+1} \quad (2.64a)$$

$$\rho_{n+1} = \rho_n + K \sin \theta_n \quad (2.64b)$$

Where θ_n and ρ_n are azimuthal position and angular velocity respectively after every n^{th} kick. The phase-space diagram of such Hamiltonian depends on the kick strength

K . Fig. 2.8 displays the phase-space diagram for different values of K . When $K = 0$, the phase-space diagram resembles to that one of a free particle. With small values of K , e.g. $K \sim 0.5$ the phase space shows bounded regions along with local chaotic signature. For $K \sim 0.9$, even the largest KAM tori breaks indicating the onset of a global chaos. When the value of K increases beyond 5, the regular regions wash out leading to an unbounded motion [158]. The right most panel in fig. 2.8 shows the chaotic phase-space trajectories with $K = 5.5$.

To estimate the diffusion in the average energy of the system, Eq. 2.64 is iterated many times. After N such cycles, the momentum can be calculated and hence the energy diffusion can be identified as the square of the increase in momentum $E = (\Delta\rho)^2$. The growth of energy with N kicks can be written as:

$$\langle(\rho_N - \rho_0)^2\rangle = K^2 \sum_{n=0}^{N-1} \langle \sin^2 \theta_n \rangle + K^2 \sum_{n \neq n'}^{N-1} \langle \sin \theta_n \sin \theta_{n'} \rangle \quad (2.65)$$

Since, for large values of K , the correlation between the kicks in the azimuthal coordinate θ is negligible, then the second term in the right hand side of Eq. 2.65 equals to zero and Eq. 2.65 reduces to:

$$\langle(\rho_N - \rho_0)^2\rangle = \frac{K^2}{2} N \quad (2.66)$$

Hence, the mean energy grows linearly with number of kicks:

$$\langle(\rho_N - \rho_0)^2\rangle = DN \quad \text{with, } D = \frac{K^2}{2} \quad (2.67)$$

Quantum dynamics and dynamical localization

A kicked rotor shows striking difference in energy diffusion when treated quantum mechanically. The quantum mechanical model can be understood as a particle undergoing a series of diffractions. Between two consecutive diffractions, the particle undergoes a free evolution. The particle interacts with each diffractor for a very small time which modifies the transverse motion of the particle. As a consequence of the modification of the transverse momentum, an interference between all possible paths through the diffractor takes place. The final state is decided by the interaction time with the diffrac-

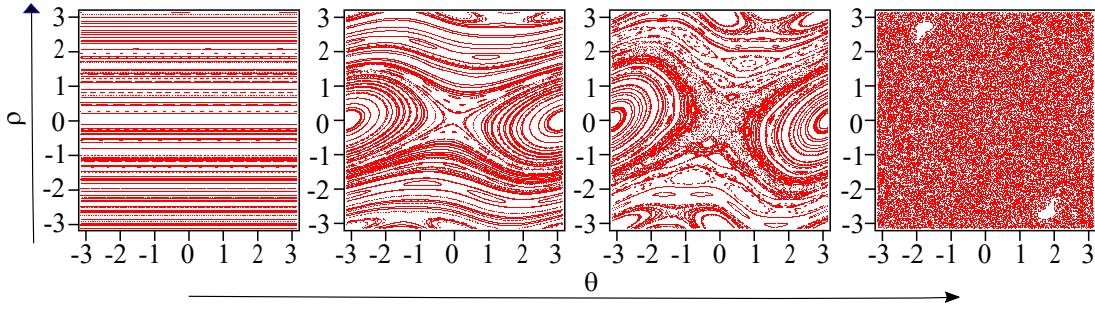


Figure 2.8: Poincaré section of kicked rotor: Poincaré section of kicked rotor with varying kick strength. The corresponding values of kick strengths are (from the left): $k = 0$, $k = 0.5$, $k = 0.9$, and $k = 5.5$. For $k > 5.5$ the phase space becomes totally chaotic. Here ρ is the scaled momentum and θ represents the scaled position.

tor along with the free evolution time.

Assuming a periodic kicking sequence, we can identify the process as a Floquet system and the corresponding Floquet operator can be written as [158]:

$$F = \exp\left(-\frac{i}{\hbar}K \cos \theta\right) \exp\left(-\frac{i}{\hbar}\frac{\rho^2 T}{2}\right) \quad (2.68)$$

where, ρ^2 can be identified as a quantum mechanical operator given by $-\hbar^2 \partial^2 / \partial \theta^2$. The floquet operator can be identified as a multiplication of two unitary operators. The first one denotes the phase gathered during the kicking process, whereas, the second is the phase gathered during free evolution. The problem can be solved in the eigenbasis of ρ . The eigenstates are given by:

$$|n\rangle = \frac{1}{\sqrt{2\pi}} e^{in\theta} \quad (2.69)$$

Therefore, the transition from the state $|n\rangle$ to $|m\rangle$ can be given just by pasting the floquet operator between the two state vectors:

$$\begin{aligned} F_{nm} &= \langle n | F | m \rangle \\ &= \frac{1}{2\pi} \int_0^{2\pi} e^{-in\theta} \exp\left(-\frac{i}{\hbar}K \cos \theta\right) \exp\left(-\frac{i}{\hbar}\frac{\rho^2 T}{2}\right) e^{im\theta} d\theta \\ &= \exp\left(-\frac{i}{\hbar}\frac{m^2 T}{2}\right) \frac{1}{2\pi} \int_0^{2\pi} e^{-in\theta} \exp\left(-\frac{i}{\hbar}K \cos \theta\right) e^{(m-n)\theta} d\theta \\ &= \exp\left(-\frac{i}{\hbar}\frac{m^2 T}{2}\right) i^{m-n} J_{m-n}\left(\frac{K}{\hbar}\right) \end{aligned} \quad (2.70)$$

where, J_{m-n} represents the Bessel functions. From here, the final state after N kick can be calculated using an initial state. For a given initial gaussian wavepacket, the final solution after a large number of kicks shows exponential profile. The mean energy of the system shows a linear diffusion as predicted by classical dynamics only upto a certain number of kicks and then the energy saturates indicating curtail of any diffusion. This phenomenon is known as dynamical localization [158]. This behavior can be understood by the following sequence of arguments. The energy of the system diffuses linearly following classical laws until the discrete nature of the Floquet states are not resolved. Since Floquet states form a complete basis, the initial state of atom can be written as a superposition of the floquet states. Therefore, the diffusion is limited by the values of floquet states that give rise the initial wavefunction. Until this point is reached, the energy grows linearly with kicks, after that the diffusion stops, leading to a localized state in momentum basis. The localized wavefunction is given by: $|\psi(\rho/\hbar_s)|^2 \sim \exp(|\rho/\hbar_s|/\zeta)$; where, $\hbar_s$ is the scaled Planck constant of the system and ζ is the localization length given by $\zeta = \frac{K^2}{\hbar_s^2}$.

Dynamical localization is similar to Anderson localization in a crystal with lattice defect. Let us recall that for Anderson model in 1-D, the Schrödinger equation can be written as:

$$\sum_{k \neq n} W_{nk} a_k + E_n^{(0)} a_n = E a_n \quad (2.71)$$

where,

$$E_n^{(0)} = \int \psi_0^*(x - nL) \left[-\frac{\hbar^2}{2m_e} \frac{d^2}{dx^2} + V(x) \right] \psi_0(x - nL) dx \quad (2.72a)$$

$$W_{nk} = \int \psi_0^*(x - nL) \left[-\frac{\hbar^2}{2m_e} \frac{d^2}{dx^2} + V(x) \right] \psi_0(x - kL) dx \quad (2.72b)$$

Here, ψ_0 is the single atom eigenfunction, m_e is the mass of electron, $V(x)$ is the periodic lattice potential, L is the period of lattice and a_k s are the probability amplitude of the single atom wavefunctions when the electronic wavefunction is expressed as a linear combination of these eigenfunctions ($\psi(x) = \sum_n a_n \psi_0(x - nL)$). The term $E_n^{(0)}$ represents onsite energy, whereas, W_{nk} is the hopping between nearest sites.

Now, we need to map the kicked rotor Hamiltonian into the Anderson model to show the analogy between the localizations. For this purpose, let us rewrite the Hamiltonian

of kicked rotor as:

$$H = P(\rho) + V(\theta) \sum_n \delta(t - n) \quad (2.73)$$

where, $P(\rho) = \rho^2/2$ and $V(\theta) = K \cos \theta$. Then the modified floquet operator can be written as:

$$F = e^{-iV(\theta)} e^{-iP(\rho)} \quad (2.74)$$

The transition probabilities can be given as:

$$F_{nm} = e^{-iP(m)} J_{n-m} \quad \text{where,} \quad J_n = \frac{1}{2\pi} \int_0^{2\pi} e^{-iV(\theta)} e^{in\theta} d\theta \quad (2.75)$$

Let us assume, $A = \{a_{i1}\}$ be the column matrix representing the eigenvector of F with eigenphase ϕ . Then the eigenvalue equation can be written as [158]:

$$FA = e^{-i\phi} A = e^{-iV(\theta)} e^{-iP(\rho)} A \quad (2.76)$$

Let us recall the identity:

$$e^{-iV(\theta)} = \frac{1 - i \tan\left(\frac{V(\theta)}{2}\right)}{1 + i \tan\left(\frac{V(\theta)}{2}\right)} = \frac{1 + iW(\theta)}{1 - iW(\theta)} \quad \text{where,} \quad W(\theta) = -\tan\left(\frac{V(\theta)}{2}\right) \quad (2.77)$$

Using the above identity, the eigenvalue equation (Eq. 2.76) can be rewritten as:

$$\tan\left(\frac{\phi - P}{2}\right) \bar{A} + W \bar{A} = 0 \quad \text{with,} \quad \bar{A} = (e^{i(\phi - P)} + 1) A \quad (2.78)$$

Eq. 2.78 can be rewritten as:

$$\sum_{k \neq n} W_{nk} \bar{a}_k + E_n^{(0)} \bar{a}_n = E \bar{a}_n \quad (2.79)$$

With,

$$W_n = \frac{1}{2\pi} \int_0^{2\pi} W(\theta) e^{-in\theta} d\theta \quad (2.80a)$$

$$E_n^{(0)} = \tan\left(\frac{\phi - P}{2}\right) \quad (2.80b)$$

$$E = -W_0 \quad (2.80c)$$

2.7.3 Atom optics kicked rotor (AOKR)

An AOKR is an experimental platform to study the dynamics of a kicked rotor system by simulating its Hamiltonian using atom-optics. We have already seen that the interaction between an 1-D optical lattice and an atomic cloud with effective two-level system can be written as:

$$H = \frac{p^2}{2m} + V_0 \cos(2k_L x) \quad \text{where,} \quad V_0 = \frac{\hbar \Omega_0^2}{8\Delta} \quad (2.81)$$

The first term in Eq. 2.81 is the kinetic energy term and the second term is due to the interaction. Ω_0 represents the effective Rabi frequency and $\Delta = \omega_L - \omega_a$ being the detuning of the lattice laser beam from the atomic resonance frequency. In order to identify the equivalence of this Hamiltonian with that of a kicked rotor, we need to introduce the following units [160]:

$t_u \equiv T$	unit of time	T being the time period of kicking sequence.
$d_u \equiv \frac{1}{2k_L} = \frac{1}{2\pi} \frac{\lambda}{2}$	unit of distance	$\frac{\lambda}{2}$ is the period of the standing wave.
$M_u \equiv M_{atom}$	unit of mass	M_{atom} being the mass of the atom.
$p_u \equiv M_u d_u / t_u$	unit of momentum	derived using above units.
$E_u \equiv p_u^2 / M_u$	unit of energy	derived using above units.
$\hbar_u \equiv d_u p_u$	unit of phase space volume	derived using above units.

Table 2.1: Changed units for AOKR system.

To work with these new units, we must introduce a scaling to make them dimensionless. Hence, the list of this scaling is given below [160]:

τ	θ	μ	ρ	E	K	H	$\hbar_s$
t/t_u	$x/d_u = 2k_L x$	$M/M_u = 1$	p/p_u	E/E_u	V_0/E_u	H/E_u	$\hbar/\hbar_u = 8\omega_R t_u$

Table 2.2: Scaling of variables for AOKR system to derive dimensionless variables.

With the above scaling, Eq. 2.81 reduces to a kicked rotor Hamiltonian. To implement the periodic nature in the kicking process, the lattice is switched on periodically with the value of on-time negligible compared to the off-time.

The experimental details for realizing AOKR system is given in chapter 4.

2.8 Atom interferometry

Another important part of this thesis is the realization of Kapitza-Dirac diffraction using an atom laser. Such diffraction is one of the main tools towards the realization of atom interferometers. In section 1.3, an introduction about atom interferometry was given. In this section, a detailed working scheme of a typical light-pulse atom interferometer will be presented. In this connection, the theory of Kapitza-Dirac diffraction will be discussed in detail.

The working principle of such interferometers can be understood from the interaction picture of a two-level system with such pulse sequences. Since we are interested only in the population of two internal states (ground and excited state), let us denote the amplitude of the corresponding states by $c_g(t)$ and $c_e(t)$ at time t . The interaction between the atoms with such pulses can be written as [161]

$$M_{\Omega t} = \begin{pmatrix} \left(\cos\left(\frac{\Omega_f t}{2}\right) - i\frac{\delta}{\Omega_f} \sin\left(\frac{\Omega_f t}{2}\right) \right) e^{\frac{i\delta t}{2}} & -i\frac{\Omega}{\Omega_f} e^{-i\phi} e^{\frac{i\delta t}{2}} \sin\left(\frac{\Omega_f t}{2}\right) \\ -i\frac{\Omega}{\Omega_f} e^{-i\phi} e^{\frac{-i\delta t}{2}} \sin\left(\frac{\Omega_f t}{2}\right) & \left(\cos\left(\frac{\Omega_f t}{2}\right) + i\frac{\delta}{\Omega_f} \sin\left(\frac{\Omega_f t}{2}\right) \right) e^{\frac{-i\delta t}{2}} \end{pmatrix} \quad (2.82)$$

where, $\Omega_f = (\Omega^2 + \delta^2)^{\frac{1}{2}}$ is the effective Rabi frequency, Ω being the on-resonant Rabi frequency and δ is the detuning from the transition. ϕ is the phase of the laser at the place of the atom at $t = 0$. The evolution of the population can therefore be written as -

$$\begin{pmatrix} c_g(t) \\ c_e(t) \end{pmatrix} = M_{\Omega t} \begin{pmatrix} c_g(0) \\ c_e(0) \end{pmatrix} \quad (2.83)$$

Hence, for a $\pi/2$ and a π pulse, the transformation matrix will be given as

$$M_{\pi/2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -ie^{-i\phi} \\ -ie^{i\phi} & 1 \end{pmatrix} \quad \text{and} \quad M_{\pi} = \begin{pmatrix} 0 & -ie^{-i\phi} \\ -ie^{i\phi} & 0 \end{pmatrix} \quad (2.84)$$

It is quite clear that if we start with all the atoms in ground state ($c_g(0) = 1, c_e(0) = 0$), a $\pi/2$ pulse will create a superposition of equal number of atoms in both the states and therefore can be considered as an analog of the beamsplitter. Similarly, a π pulse will transfer the total population into the excited state and therefore can be considered as an analog of a mirror. But in both the cases, an extra phase factor of ϕ will be introduced because of the laser phase. Since the phase of the laser depends on the position of the atoms, the effect of the operation of π or $\pi/2$ pulses effectively records the position of the atoms via the phase factor. Thus each pulse can effectively measure the position. Extending this concept, we can immediately understand that two of such pulses separated by a well known time can, therefore, be employed to measure velocity and three of such pulses in a sequence have the potential to measure acceleration. Further, four or many pulses can be employed to measure the acceleration gradients.

When an atom interacts with a photon, the momentum of the photon is transferred into the atoms. Therefore, depending upon an absorption of a photon or an absorption followed by a stimulated emission, an atom can gain a maximum momentum of $2\hbar k$ in a direction. Since the atoms in the two arms of Raman interferometers are physically separated, they feel different phase of the laser. This differential laser phase can be denoted by $\Delta\phi_{laser}$. Another source for the two arms to gather different phase ($\Delta\phi_{prop}$) is due to propagation via different path, this phase can be attributed as the variation of different force fields over two independent trajectories. Other source of the differential phase is called separation phase ($\Delta\phi_{sep}$), which is due to the imperfection of the two trajectories caused by the asymmetries like gravity gradients, rotation, timing asymmetries, etc. For a Mach-Zehnder interferometer, the differential phase introduced by the laser is $\Delta\phi_{laser} = \phi_1 - 2\phi_2 + \phi_3$, where ϕ_1 and ϕ_3 are the phase gathered at the first and last $\pi/2$ pulse and ϕ_2 is the phase accumulated at the position of π pulse.

The differential propagation phase can be found by calculating the classical action along

the trajectories added with the contribution from their internal state energy E .

$$\Delta\phi_{prop} = \frac{1}{\hbar} \sum_{upper} \int_{t_i}^{t_f} (L - E) dt - \frac{1}{\hbar} \sum_{lower} \int_{t_i}^{t_f} (L - E) dt \quad (2.85)$$

where, L is the classical Lagrangian due to the force fields.

The separation phase depends on the position difference between the two arms at the final beamsplitter pulse, $\Delta x = x_{upper} - x_{lower}$ and the canonical momentum p given by $p = \frac{1}{2} \left(|\nabla_{\dot{x}} L|_{upper} - |\nabla_{\dot{x}} L|_{lower} \right)$. The separation phase is given by

$$\Delta\phi_{sep} = -\frac{1}{\hbar} (p \cdot \Delta x) \quad (2.86)$$

Finally, the total phase accumulated over the interferometer can be written as [161]

$$\Delta\phi = \Delta\phi_{laser} + \Delta\phi_{prop} + \Delta\phi_{sep} \quad (2.87)$$

Hence the probability of finding the atom in ground state at the output of the interferometer can be written as a function of the total accumulated phase as

$$|c_g|^2 = \left| \frac{1}{2} (1 + e^{-i\Delta\phi}) \right|^2 = \frac{1}{2} (1 + \cos \Delta\phi) \quad (2.88)$$

To measure any physical quantities such as, magnetic field, earth rotation or gravitational acceleration, etc., we need to write the effective Lagrangian and calculate their contribution in the differential phase. For example, the Lagrangian due to the combined effect of all the above-mentioned physical quantities can be written as

$$L = \frac{1}{2} \left[\frac{p}{m} + \Omega_r \times (r + R_E) \right]^2 - m\Phi(r + R_E) \pm \frac{1}{2} \hbar \alpha |B|^2 \quad (2.89)$$

where, the first term in the right hand side of the equation is the total energy due to the momentum of the atomic wavepacket and the coriolis force. The second term is the gravitational potential and the last term is due to magnetic field. Ω_r is the rotational speed of the earth, R_E is the radius of the earth and α is the sensitivity of the atoms to the magnetic field. The effective Lagrangian can be expanded and contribution to differential phase from each term can be calculated.

2.8.1 Bragg diffraction

The heart of atom interferometry is the transfer of momentum into the atomic wavepacket. Bragg scattering is the simplest method to realize such momentum transfer. In this section, we will discuss elaborately about such diffraction. Let us consider a grating formed by a standing wave of light. The standing wave can be written as a superposition of two linearly polarised counter-propagating laser beams along the z -axis :

$$\vec{E}(z, t) = E_0 f(t) \sin(kz - \omega t) \hat{e} + E_0 f(t) \sin(kz + \omega t) \hat{e} \quad (2.90)$$

The process involves transfer of momentum through absorption and stimulated emission. To understand the mechanism of the process, we will use the momentum space as our basis of description. The effect of absorption of N photons followed by stimulated emission of N photons results in N -th order diffraction. The state undergoes a change described as $|g, -N\hbar k\rangle \rightarrow |g, +N\hbar k\rangle$. The excited state remains unpopulated throughout the process as long as the temporal envelope of the electric field $f(t)$ does not possess any strong component near the frequency equal to the detuning.

Let us now calculate the population distribution by calculating the probability of the atom to be found in a diffraction order. We will derive this formalism for the 1st order diffraction and then generalize it for N -th order diffraction. Let us consider a transition $|g, -\hbar k\rangle \rightarrow |g, +\hbar k\rangle$ to calculate the probability $p_1^{Bragg}(\tau)$ of the first order Bragg diffraction with the interaction time between the atom and the electric field being τ . The assumption that we make to simplify the derivation is that $f(t)$ has a square profile, which means the switching on and off time of the lattice is negligible compared to τ . Recalling that the atom-light interaction is given by $-\mu \cdot \mathbf{E}$, the interaction Hamiltonian can be written as:

$$H_{int}(t) = -ie^{-i\omega t} \frac{\hbar\Omega_R}{2} (|e, 0\rangle \langle g, -1| - |e, 0\rangle \langle g, +1|) + h.c. \quad (2.91)$$

where, $\Omega_R = \mu E_0 / \hbar$ is the single photon Rabi frequency. The first term in the equation represents the absorption from a beam whereas the hermitian conjugate term represents the stimulated emission. Since μ can be written as the dipole matrix element $\langle e | q\vec{r} | g \rangle$, q is the electronic charge. Without loss of generality, we assume μ and Ω to be positive

real valued quantity. Hence, the total Hamiltonian can be written as:

$$H_0(t) = \hbar\omega_0 |e, 0\rangle \langle e, 0| + \hbar\omega_{rec} (|g, -1\rangle \langle g, -1| + |g, +1\rangle \langle g, +1|) \quad (2.92)$$

where, ω_{rec} is the single photon recoil frequency and the recoil energy is given by $E_{rec} = \hbar\omega_{rec} = \frac{\hbar^2 k^2}{2m}$.

Assume the solution of Eq. 2.92 to be

$$|\psi(t)\rangle = c_{-1}(t) e^{-i\omega_{rec}t} |g, -1\rangle + c_0(t) e^{-i\omega_0 t} |e, 0\rangle + c_{+1} e^{-i\omega_{rec}t} |g, +1\rangle \quad (2.93)$$

Substituting the solution in Eq. 2.92, we get three coupled first order differential equation governing the dynamics of the population [162]:

$$\dot{c}_{\pm 1}(t) = \mp \frac{\Omega_R}{2} e^{i\Delta t} c_0(t) \quad (2.94a)$$

$$\dot{c}_0(t) = \frac{\Omega_R}{2} e^{-i\Delta t} (c_{+1}(t) - c_{-1}(t)) \quad (2.94b)$$

where, $\Delta = (\omega - \omega_0) + \omega_{rec}$. The usual practice to solve such coupled equation is to differentiate both side of one equation and substitute the derivative from the other equation. Following the treatment, we differentiate Eq. 2.94a and substitute using Eq. 2.94b to get the required decoupled differential equation:

$$\ddot{c}_{\pm 1}(t) - i\Delta \dot{c}_{\pm 1}(t) + \frac{\Omega_R^2}{4} (c_{\pm 1}(t) - c_{\mp 1}(t)) = 0 \quad (2.95)$$

To reach to a definite solution, we use the following set of initial conditions:

$$c_{-1}(t=0) = 1 \quad (2.96a)$$

$$c_0(t=0) = 0 \implies \dot{c}_{\pm 1}(t=0) = 0 \quad (2.96b)$$

$$c_{+1}(t=0) = 0 \quad (2.96c)$$

Applying the initial conditions, we reach upto the following final solutions for the three coefficients [162]:

$$c_{-1}(t) = e^{-i\frac{\Omega_R^2}{4\Delta}t} \cos\left(\frac{\Omega_R^2}{4\Delta}t\right) \quad (2.97a)$$

$$c_{+1}(t) = ie^{-i\frac{\Omega_R^2}{4\Delta}t} \sin\left(\frac{\Omega_R^2}{4\Delta}t\right) \quad (2.97b)$$

$$c_0(t) = -i\frac{\Omega_R}{2\Delta}e^{-i\Delta t}e^{-i\frac{\Omega_R^2}{4\Delta}t} \quad (2.97c)$$

It is very clear from Eq. 2.97 that the two probability amplitudes $c_{-1}(t)$ and $c_{+1}(t)$ are orthogonal to each other and oscillates sinusoidally. Hence, the transition probability from $|g, -\hbar k\rangle \rightarrow |g, +\hbar k\rangle$ will be given by:

$$p_1^{Bragg}(\tau) = |c_{+1}(\tau)|^2 = \sin^2\left(\frac{\Omega_R^2}{4\Delta}\tau\right) \quad (2.98)$$

Therefore, the system performs an oscillation between two momentum states similar to Rabi oscillation and such oscillations have been observed for atoms [163] and neutrons [164].

Extending this idea, we can certainly identify that a N -th order Bragg process is basically a $2N$ -photon Raman process with $2N-1$ intermediate states. Therefore, the $2N$ -photon Rabi frequency is given by:

$$\Omega_R^{(2N)} = \frac{(\Omega_R)^{2N}}{2^{2N-1}\Delta_1\Delta_2\cdots\Delta_{2N-1}} \quad (2.99)$$

where, N -th order Bragg detuning can be written as:

$$\Delta_m = \begin{cases} (\omega - \omega_0) + (2Nm - m^2)\omega_{rec}; & \text{n odd} \\ (2Nm - m^2)\omega_{rec}; & \text{n even} \end{cases} \quad (2.100)$$

The condition that a system must satisfy to undergo Bragg diffraction can be given from the consideration of not populating the prohibited transitions. For a N -th order Bragg process, such condition on the interaction time can be given as:

$$\tau \gg \frac{\pi}{2(N-1)\omega_{rec}} \quad (2.101)$$

The upper bound of the interaction time comes from the consideration of spontaneous emission. With increasing interaction time the amount of spontaneous emission increases. Therefore, τ should be chosen in such a way that the number of spontaneous

emission event per atom (N_s) during this time period can be neglected. The value of N_s for such process can be given by:

$$N_s = |c_0(t)|^2 \Gamma \tau = \frac{\Omega_R^2}{4\Delta^2} \Gamma \tau \quad (2.102)$$

where, Γ is the natural decay rate of the excited rate. From Eq. 2.102, the upper bound on the interaction time can be given as:

$$\tau \ll \frac{4\Delta^2}{\Omega_R^2 \Gamma} \quad (2.103)$$

Such a Bragg process was first realised at MIT [163] using a standing wave of near-resonant laser light.

2.8.2 Kapitza-Dirac diffraction

This section will be dedicated to a very special type of diffraction which is widely used in realizing modern atom interferometers.

Scattering of electron from a standing wave of light was predicted [165] by Kapitza and Dirac in the year 1933. In this type of scattering, the motion of the particle can be neglected during the scattering process. This diffraction is known as Kapitza-Dirac scattering. The condition when the kinetic energy term in the Hamiltonian is neglected, is known as Raman-Nath approximation [166]. In order to satisfy this condition, the interaction time is chosen to be very small. The upper limit of the interaction time is given by $\tau \ll 1/\omega_{rec}$. The condition implies that the atom does not even get sufficient time to move due to recoil momentum.

In order to find out the population distribution, we consider the standing wave potential as:

$$U(z, t) = \frac{\hbar \Omega_R^2}{\delta} f^2(t) \sin^2(kz) \quad (2.104)$$

where, $\delta = \omega - \omega_0$ is the detuning of the lattice laser from the atomic resonance frequency. Unlike the Bragg diffraction, in this case the kinetic energy is neglected, which leads to addition of a phase factor in the initial atomic wavefunction. If $|\psi_0\rangle$ is assumed to be the atomic wavefunction before the interaction, then the wavefunction after

interaction will be given by:

$$|\psi\rangle = |\psi_0\rangle e^{-\frac{i}{\hbar} \int dt' U(x, t')} = |\psi_0\rangle e^{-i \frac{\Omega_R^2}{2\delta} \tau} e^{i \frac{\Omega_R^2}{2\delta} \tau \cos(2kz)} \quad (2.105)$$

where, $\tau = \int dt' f^2(t')$ an integration of the pulse. To simplify Eq. 2.105, we will recall the following identity of Bessel functions:

$$e^{i\alpha \cos(\beta)} = \sum_{n=-\infty}^{\infty} i^n J_n(\alpha) e^{in\beta} \quad (2.106)$$

where, J_n is the n^{th} order Bessel function of the 1st kind. Using the above identity, Eq. 2.105 can be rewritten as:

$$|\psi\rangle = |\psi_0\rangle e^{-i \frac{\Omega_R^2}{2\delta} \tau} \sum_{n=-\infty}^{\infty} i^n J_n\left(\frac{\Omega_R^2 \tau}{2\delta}\right) e^{i2nkz} = e^{-i \frac{\Omega_R^2}{2\delta} \tau} \sum_{n=-\infty}^{\infty} i^n J_n\left(\frac{\Omega_R^2 \tau}{2\delta}\right) |g, 2n\hbar k\rangle \quad (2.107)$$

where, we can identify the position space representation of the momentum states as $|g, p\rangle = M e^{ipz/\hbar}$; M being the normalization factor. Hence, the population in n^{th} diffracted order can be written as [162]

$$P_n = J_n^2(\theta); \quad n = 0, \pm 1, \pm 2, \dots \quad (2.108)$$

where, θ defines the strength of the interaction and can be expressed as a function of the peak Rabi frequency (Ω_0), detuning of the laser (δ) and the interaction time (τ) as:

$$\theta = \frac{\Omega_R^2 \tau}{2\delta} \quad (2.109)$$

The condition $\tau \ll \Omega_{rec}$ is not a sufficient condition for Kapitza-Dirac diffraction. In order to provide a more rigid bound, we have to impose the condition that the interaction time is much less than a quarter of the oscillation time period ($\tau \ll \tau_{osc}/4$) of the atoms in the trap. Using Eq. 2.104, the curvature of the trap is found to be $\partial^2 U / \partial z^2 = 2\hbar\Omega_R^2 k^2 / |\delta|$. Hence, the oscillation frequency and time period are given by [162]:

$$\omega_{osc} = 2\omega_R \left(\frac{\omega_{rec}}{|\delta|} \right)^{1/2} \quad \text{and,} \quad \tau_{osc} = \frac{\pi}{\omega_R} \left(\frac{|\delta|}{\omega_{rec}} \right)^{1/2} \quad (2.110)$$

2.9 Summary

This chapter covered most of the theoretical background that is used for the work carried out in this thesis. In the following chapters, we will present some of the experimental results.

Chapter 3

Experimental methods and apparatus

This chapter presents the details of the experimental setup used to produce ultracold atoms and BEC for the experiments reported in this thesis. A major part of the experimental system is the one used in the work done by Sunil Kumar [167] which has been modified significantly for the production of ultracold atoms using optical dipole trap.

3.1 Overview of the BEC apparatus

The detailed experimental setup and procedure to produce BEC in a crossed optical dipole trap is presented in this chapter. Our experimental sequence for achieving a ^{87}Rb BEC consists of four sections - (a) loading of atoms in MOT (magneto-optical trap), (b) compression of the atomic cloud, (c) transfer of the atoms in a crossed optical dipole trap, and (d) evaporation of atoms in the crossed optical dipole trap.

We load the MOT with nearly 5×10^7 atoms in 5 seconds using a Zeeman slower [168]. Atoms are then compressed by increasing the detuning of the MOT beams and increasing the quadrupolar magnetic field gradient. The compression stage produces an atomic cloud with very high density and low temperature ($\sim 50\mu\text{K}$). After the compression, the atoms are pumped into $|F = 1\rangle$ sub-level of the ground state by turning off the repumper for nearly 1 ms. At this stage, a maximum number of atoms are transferred into the crossed optical dipole trap. Our crossed optical dipole trap consists of two laser beams of wavelength 1064 nm and separated by 10 MHz in frequency. The two beams have a beam waist $\sim \phi = 70 \mu\text{m}$ which produces an optical trap with a trap depth $\sim 330 \mu\text{K}$. After this stage, we perform evaporative cooling of atoms in ~ 6 sec by ex-

ponentially ramping down the power of the dipole beams in a piecewise linear fashion. Such cooling reduces the temperature of the atomic cloud and finally produces a BEC with $\sim 6 \times 10^4$ atoms having a temperature ~ 67 nk. For further analysis, the absorption image of the BEC is taken using the standard time-of-flight technique. Throughout the experiment, the synchronization and precise timing of the experimental sequence are achieved by communicating with all the instruments through computer controlled programs and LabView interface.

3.2 Rb effusive oven

In our experiment, a thermal beam of ^{87}Rb atoms is generated by using an effusive atomic source. Here, we briefly describe different parts of the oven and their functioning. A more detailed description can be found in ref. [167].

In the experiment, a glass ampoule containing ~ 5 gm of Rb is used as a reservoir. The ampoule is kept in a copper container connected to the vacuum system. This container is heated to $\sim 80^\circ \text{C}$ to produce sufficient flux. The Rb container is connected to a big elbow which is operated at a temperature of 120°C . During the experiment, the temperature at both the elbow and copper container are maintained using a PID controller. The other end of the elbow is connected externally to the rest of the vacuum assembly through a thermal isolator made of glass capable of enduring high thermal gradient. The Rb source is internally connected to the next six-way cross vacuum part of the system through a long tube, which works as a beam collimator due to its small diameter to length ratio. This six-way cross vacuum part is connected to an ion pump having a pumping speed ~ 70 l/s. Since Rb has a high vapor pressure even at room temperature, the pressure in this part of the system is high and reads as $\sim 4 \times 10^{-7}$ Torr by monitoring the current of the ion pump. Due to such a high pressure, the ion pump was never found to be working efficiently and needed to be heated twice a year or so to improve the performance. Inside the six-way cross, there is a copper cup with a small opening located at the center of it. This cup plays a major role in controlling the collimation as well as the pressure in the chamber. Due to its geometry and alignment, the cup works as a good collimator by blocking the atoms with large divergence from the desired path. This copper cup is connected to a cold block kept outside the vacuum chamber through high current feedthroughs (~ 600 A) acting as thermal conductor from

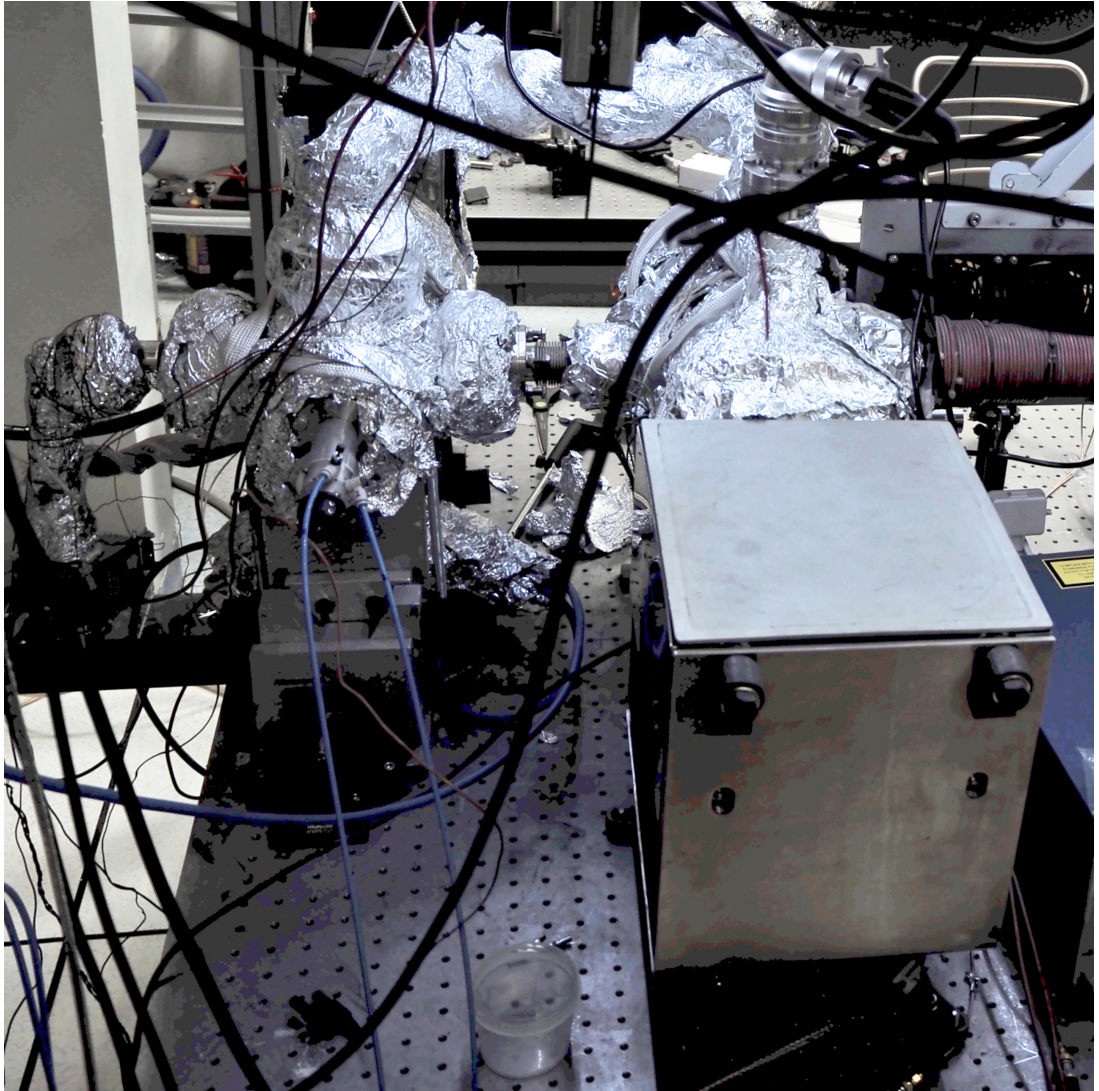


Figure 3.1: Oven chamber: Rb oven-side chamber in the experiment.

inside to the outside of the vacuum system. The cold block is cooled by a peltier down to $\sim -25^\circ \text{C}$. Due to the low temperature of the cold cup, atoms are not allowed to go to the next chamber as they stick very easily to the cup leading to improvement of the vacuum level of the system. Finally, this chamber is connected to the next chamber of the system through a differential pumping tube which is very much needed as the pressure in both the sides of this tube are different. The next chamber is connected to an 80 cm long Zeeman slower tube. Hence a highly collimated beam of atoms produced by the oven enters into the next part of the vacuum assembly for further cooling process to

take place using Zeeman slower. The working principle of this part will be described in the following section. A labeled diagram of the oven is displayed in 3.3

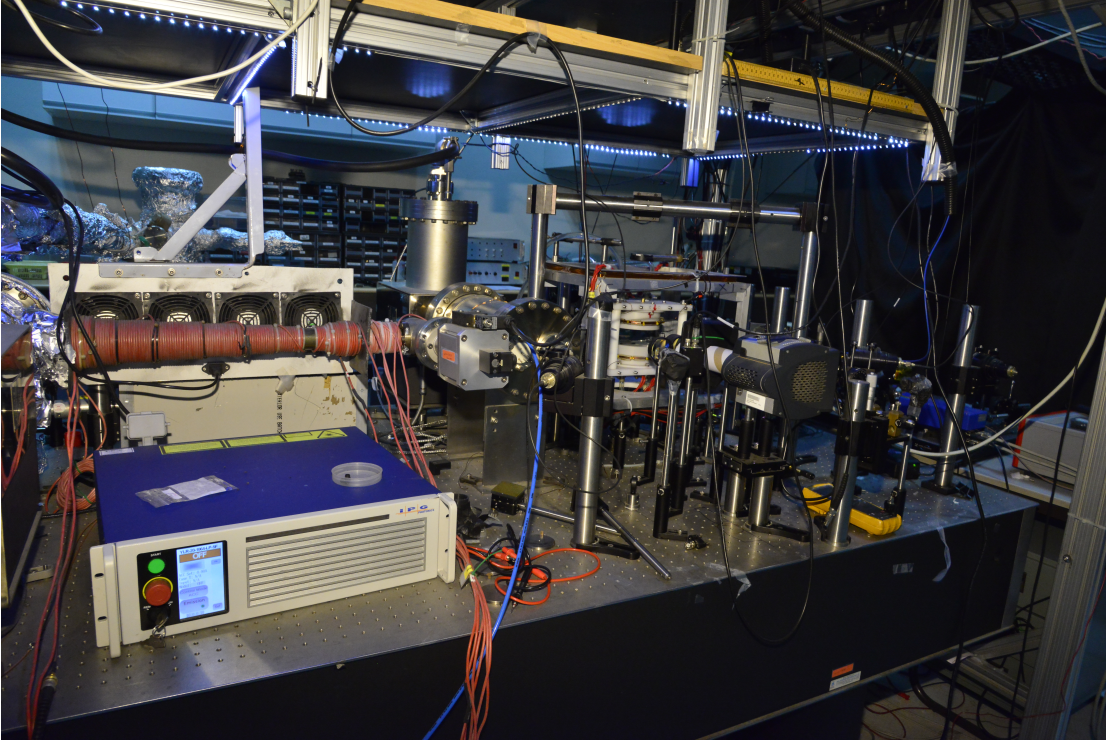


Figure 3.2: Vacuum assembly: An image of the experimental setup of vacuum assembly.

3.3 Vacuum assembly

A high vacuum level ($< 10^{-11}$ Torr) lies at the heart of any cold atom experiment. The concept of cooling atoms relies on reducing the kinetic energy of the system through elastic collisions between the trapped atoms. Collisions play a major role in trapping and cooling atoms. In some cases collisions are unwanted, and sometimes they are the most desired in the process. For example, if we want to cool down a cloud of Rb atoms by evaporative cooling, then the cooling efficiency depends on the rate of inelastic collision between the trapped atoms. Whereas, during the process, we want the atoms to stay inside the trap. Hence, any collision between an untrapped atom with a trapped atom will lead to a transfer of energy of the untrapped atom to the trapped one. This will lead to loss of atoms from the trap. As the atoms get free, if their kinetic energy is more than the potential energy of the trap they are lost from the trap. Such collisions are known

as in-elastic collisions, and an ideal environment for cold atom experiment should not have any such collision. Therefore, in laboratory experiments, an ultra-high vacuum is desired in the experimental chamber to minimize such collision rate and maximize the lifetime of the atoms in the trapping potential.

Fig. 3.3 displays a schematic layout of the complete vacuum system used in our experiment. The system consists of three main parts – (a) the oven, (b) the Zeeman slower and (c) the science chamber. The description of the oven has been given in the previous section. The conceptual idea of our Zeeman slower has been adopted from [168] with minor modifications. A detailed description of our Zeeman slower can be found in ref. [169]. The Zeeman slower addresses atoms with maximum velocity ~ 250 m/s and slows down atoms to ~ 25 m/s. Therefore, atoms coming out of Zeeman slower tube possess velocity well within the capture velocity of the MOT (~ 35 m/s for our system) [170].

The next part in the row is the science chamber which is the quartz cell. The quartz cell has a thickness of ~ 4 mm and the external dimension of $39 \times 39 \times 150$ cm. The quartz cell is attached to the rest of the assembly through a graded quartz-to-metal seal connected to a CF35 stainless steel flange. This cell provides maximum access to light for achieving a large cloud of atoms in the MOT and alignment of optical dipole trapping beams. To maintain a good vacuum level, two more ion pumps with capacity ~ 150 l/s and a non-evaporable getter (NEG) pump (CapaciTorr D400-2) are connected to the system as shown in fig. 3.3. A Bayard-Alpert vacuum gauge is connected close to the NEG pump (not shown in fig. 3.3) to monitor the health of the vacuum whenever needed. In addition, two Ti-sublimation filaments are installed in the vacuum assembly and are operated regularly to maintain the vacuum level. The pressure near the science chamber is $< 10^{-11}$ Torr. A detailed description of the bake-out procedure of the system can be found in ref. [167].

3.4 Laser System for cooling and repumping

Lasers plays crucial role in cooling and trapping atoms. Therefore, an important part of building a cold atom setup is generation, stabilization, and locking of lasers. Here, we briefly discuss our laser system used for the BEC experiment. A more detailed descrip-

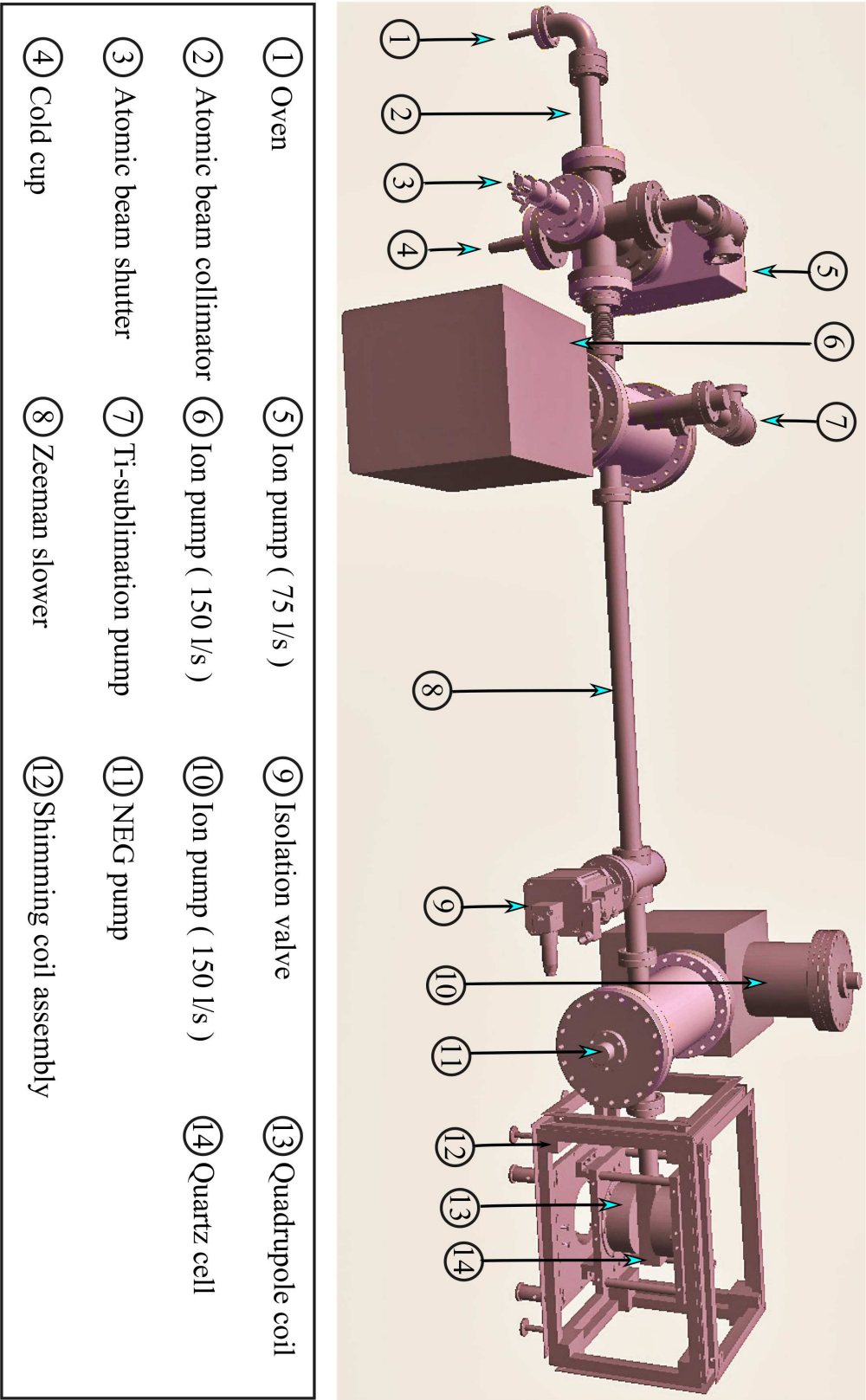


Figure 3.3: Vacuum chamber layout: Schematic of vacuum chamber assembly used in the experiment.

tion can be found in ref. [167].

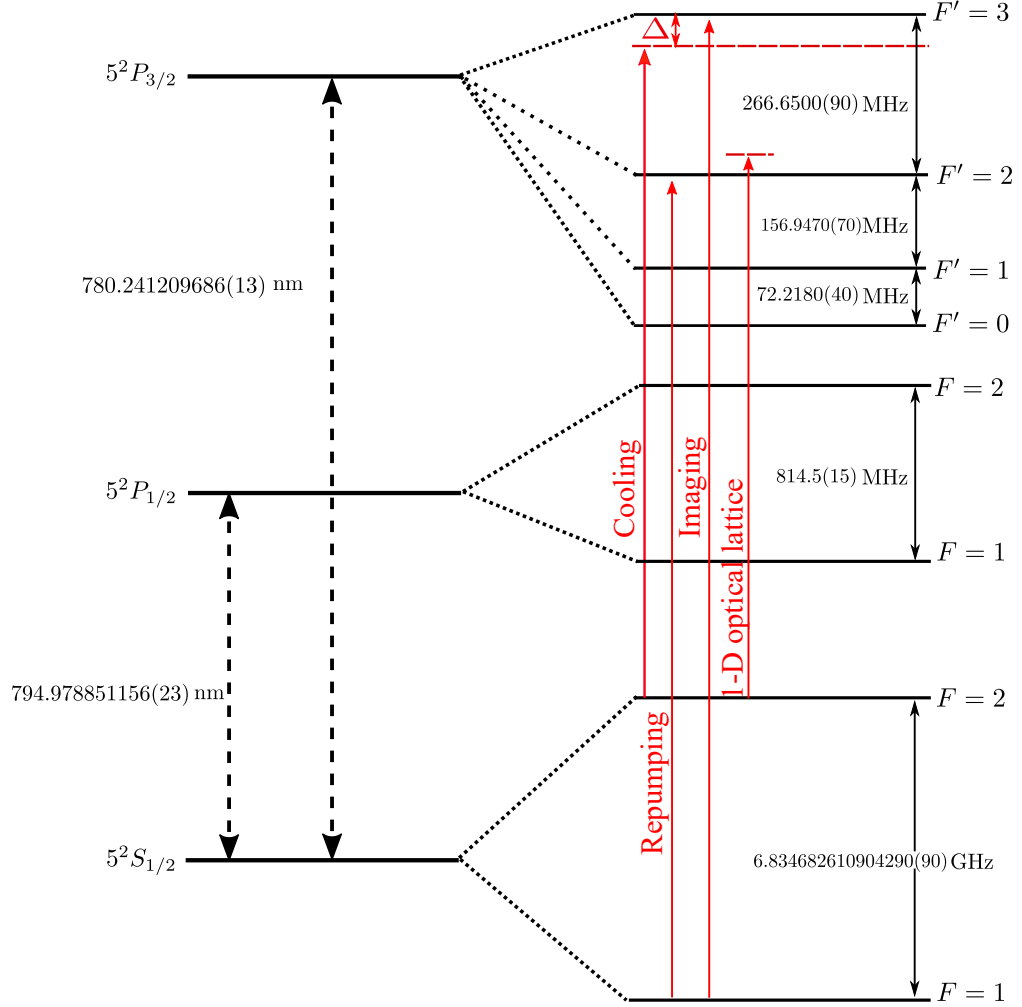


Figure 3.4: Laser system: Laser frequencies with reference to the ^{87}Rb transition lines.

Fig. 3.4 displays the cooling and repumping scheme for ^{87}Rb atoms along with the relevant energy level diagram. For trapping, cooling and repumping, we address the D2 transition line $5^2S_{1/2} \rightarrow 5^2P_{3/2}$ of ^{87}Rb . For initial trapping and cooling in MOT, we use $|F = 2\rangle$ as the ground state. The cooling laser is red-detuned from the transition $|F = 2\rangle \rightarrow |F' = 3\rangle$. During this process, an atom goes through several cycles of cooling and loses its kinetic energy. Since the cooling laser is a little detuned from $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition, it has a finite probability to end up in populating $|F' = 2\rangle$ level. In this case, the atom may decay back to $|F = 1\rangle$ ground state and goes

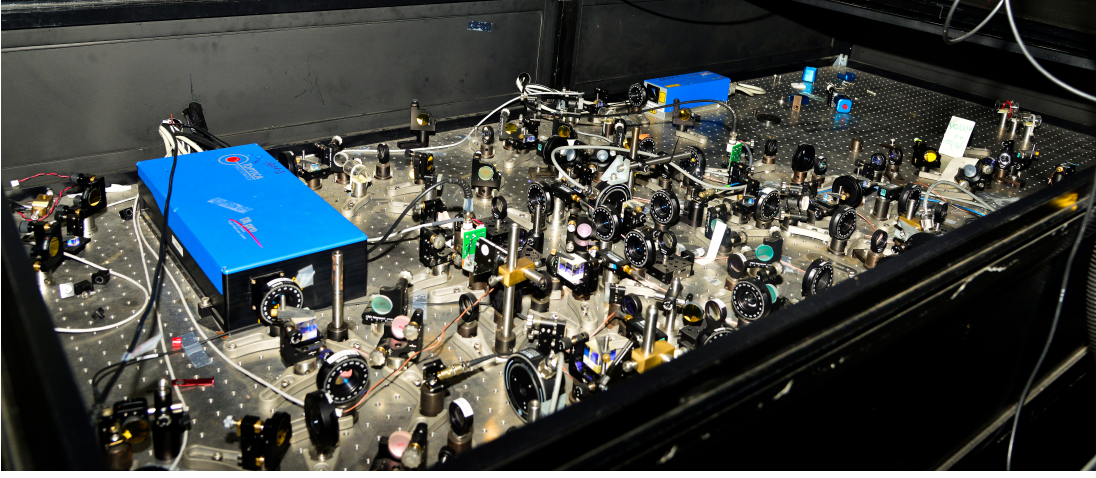


Figure 3.5: Laser table: Photograph showing the laser table with lasers and other optics used in the experiment.

out of the cooling cycle. This process hinders the cooling mechanism. Therefore, to ensure that an atom always remain in the cooling cycle, another laser is introduced to operate between $|F = 1\rangle \rightarrow |F' = 2\rangle$ to repump the atoms from $|F = 1\rangle$ level. This laser is thus named as repumper laser. It should be noted that, in the absence of the repumper, eventually all the atoms are accumulated in $|F = 1\rangle$ which is much needed for trapping the atoms in an optical dipole trap and will be explained in detail in the following section. Since the final cooling of atoms is performed in an optical dipole trap in $|F = 1\rangle$ ground state, while imaging, the atoms are needed to be brought back to $|F = 2\rangle$ level by using the repumper. For absorption imaging purpose, the probe laser is kept nearly resonant to $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition to ensure maximum absorption rate.

We use extended cavity diode lasers (ECDL) which employ an optical feedback from a grating and thus provide an output with narrow linewidth ($\sim$ few 10s of KHz). The lasers used in the experiment, have Littrow configuration and offers a range of frequencies controlled either by the current of the laser diode or by the angle of the grating. A piezoelectric transducer is used to precisely control the angle of the grating. The base of the laser is maintained at a constant temperature using a peltier cooler driven by the output of a PID controller.

A commercially available diode-laser system (Toptica make, Model: TA-pro) is used for cooling and imaging. The center frequency of operation of the diode is 780 nm with

a linewidth ~ 100 KHz. The laser uses a tapered amplifier and a seed laser and produces power maximum up to ~ 1.5 W. The laser has two outputs. The main output is used for deriving the beams for MOT, Zeeman slowing and imaging whereas, the other output is used for spectroscopy which is needed to lock the frequency of the laser to the atomic resonant frequency. We use another diode laser (Toptica DL 100 design) to extract the beams for repumping during MOT loading and imaging.

The first step towards locking the lasers to atomic frequency is to generate the reference frequencies. We perform standard saturation absorption spectroscopy (SAS) for this purpose. SAS consists of two overlapping counterpropagating laser beams and one free laser beam passing through a 10 mm thick Rb vapor cell. The stronger beam among the two overlapping beams saturates the transition for the other beam. The signal from this weaker laser when subtracted from the Doppler-broadened signal derived from the free propagating beam, a doppler free reference of the atomic resonance is achieved. To generate the complete energy spectrum of our interest the laser frequency is scanned over the region of our interest. As a result, the hyperfine transitions and their crossover peaks which appear at a frequency exactly halfway between the hyperfine transitions [130, 171, 172] are achieved precisely.

We use commercially available locking module of Toptica Digilock 110 for locking the lasers. This module is based on fast FPGA technology and is integrated to a scan generator, PID controller and frequency modulation controller. The module comes with a user-friendly computer interface based on Labview programming language. For locking the laser, an error signal is needed to be generated. The frequency modulation module serves this purpose. The laser frequency is modulated by modulating the angle of the grating through the PZT. The modulation frequency in our experiment is about 200 KHz. The Digilock module generates the error signal and feeds it to the PID module. The output of one of the PIDs is connected internally to the PZT which controls the grating. The output of another PID is connected to directly control the laser current for fast feedback. The main laser (TA Pro) is locked at the atomic transition $|F = 2\rangle \rightarrow |F' = 2\rangle$, while the repumping laser is locked at atomic transition $|F = 1\rangle \rightarrow |F' = 1\rangle$.

The complete experimental sequence consists of different steps, and in many stages, a frequency sweep or precise turning on/off operations are much needed. Therefore, precise control over the laser frequency and output is well desired without sacrificing

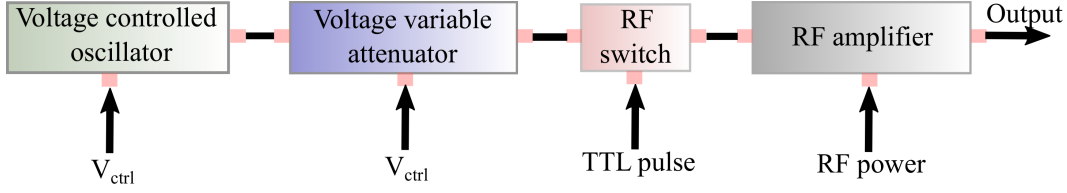


Figure 3.6: AOM controller: Schematic of AOM control circuit.

the laser lock-in mechanism. The solution is achieved by using Acousto-optic modulators (AOM). The working principle of AOMs relies on the acousto-optic effect caused by driving a piezoelectric transducer by RF signal. Inside the AOM, there is an optically transparent medium which on applying RF power displays a periodic change of refractive index over the space analogous to a grating. Thus by controlling the RF power to the AOMs, a well-controlled frequency shift, as well as the direction of the laser beam, can be achieved. In the absence of the RF power, the transparent medium acts normally leading to an undeflected laser output with the same power as the input. Thus a high-speed RF switch can control the switching of the laser output as desired. In our experiment, AOMs from IntraAction Corp. (model no. ATM-801A2, ATM-1101A2) have been used. These AOMs consists of a TeO_2 (Tellurium dioxide) crystal as an acousto-optic material. The electronic control circuit required to drive an AOM is shown in fig. 3.6. The circuit consists of a voltage-controlled oscillator (VCO) (Mini-Circuits, ZX95-100+ or ZX95-200+), a voltage-variable attenuator (Mini-Circuits, ZX73-2500+), an RF switch (Mini-Circuits, ZYSWA-2-50DR) and a power amplifier (Mini-Circuits, ZHL-1-2W). The voltage controlled oscillator produces a signal with a desired frequency controlled by its input voltage. The output of the oscillator is fed to the attenuator which controls the amplitude of the signal depending on the control voltage provided. This signal is then given to the input of a 2W RF amplifier via an RF switch. The RF switch is controlled by external TTL pulses. Thus an AOM provides fast control of the amplitude, frequency, and switching of a laser beam without hindering the laser lockin process.

3.5 Magnetic field coils

This section describes the design and construction of the magnetic trap needed for the experiment. For trapping atoms in MOT, we need a quadrupolar magnetic field which

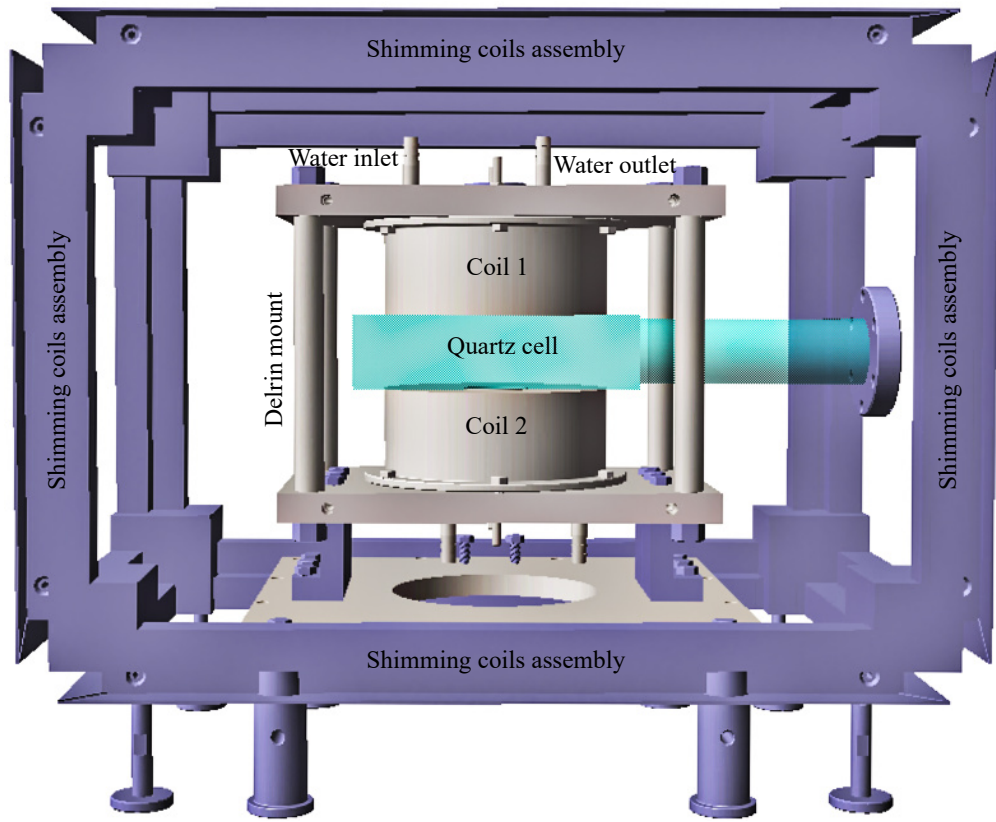


Figure 3.7: Magnetic coils assembly: Schematic of magnetic coils assembly showing the quadrupole coils and shimming coils.

can be assumed to vary linearly around the trap center. Fig. 3.7 shows the schematic of the quadrupole coil assembly around the quartz cell. The direction of the current in the two coils are kept in anti-Helmholtz configuration. Each coil is wound with 17 AWG enameled magnet wire and has 15 layers, with 19 turns in each layer. The coils produce a magnetic field gradient of $13 \text{ G-cm}^{-1}\text{-A}^{-1}$ in the axial direction and $6.5 \text{ G-cm}^{-1}\text{-A}^{-1}$ in the radial direction. Both the coils are separated by 40 mm, which is nearly equal to the dimension of the quartz coil. The coils are water cooled to prevent heating. Fig. 3.8 shows the spacing between each layer made using 1 mm thick spacer capable of sustaining high temperature. Such channels between two consecutive layer allow flowing cold water ($\sim 12^\circ\text{C}$) through the coils. Each coil is enclosed in a cylindrical water-tight Poly-oxymethylene plastic (Delrin) box. Fig. 3.9 shows the image of the quadrupole coils arrangement used in the experiment. Further protection is given by introducing a flowmeter at the outlet of the quadrupole coil assembly, which produces

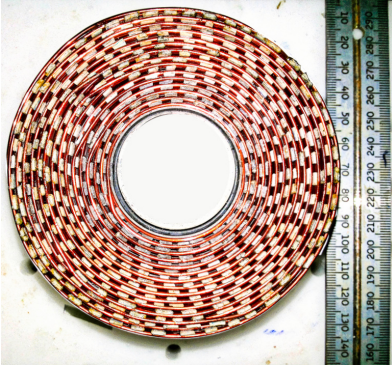


Figure 3.8: Quadrupole coils winding: Photograph displays the winding of one of the quadrupole coils. The spacer ensures sufficient space for the flow of water used to cool down the coil assembly.

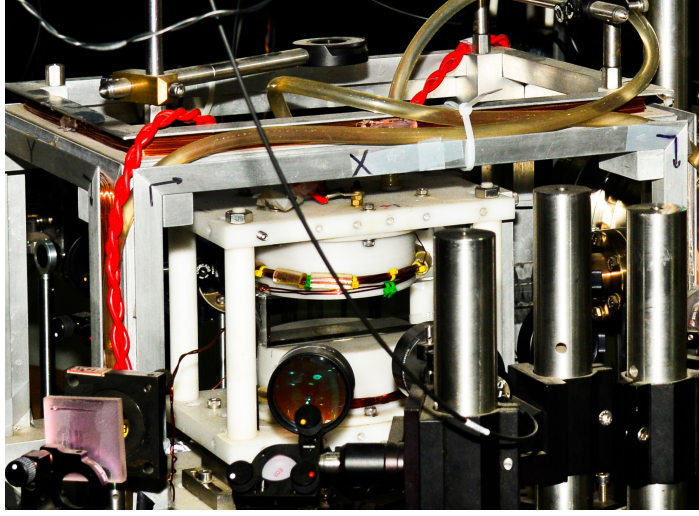


Figure 3.9: Quadrupole coil assembly: Image displays the quadrupole coil assembly along with the shimming coils used in the experiment

a TTL pulse for the measured flowrate going below a certain set value. This TTL pulse is feed to the interlock of the power supply driving the quadrupole coils which does not flow any current in case the interlock reads a high voltage.

The quadrupole coils are driven by a power supply (Delta Electronika, model SM60-100) capable of delivering 6000 W power. A fast-switching ($< 200 \mu s$) for the inductive load provided by the quadrupole coils ($\sim 3 \text{ mH}$) is realized by an IGBT (Make EU-PEC, model no BSM300GB120DLC) connected in series with the coils. The proper alignment of MOT needs the quadrupolar magnetic field center to overlap with the center of the MOT laser beams. Sometimes this cannot be achieved just by geometrical constraints. The reason behind this is the magnetic field due to earth and other stray magnetic field which shifts the effective center of the quadrupolar field. Therefore, to get rid of such unwanted magnetic field, three pairs of Helmholtz coils are introduced around the quadrupole coil assembly with each having an axis perpendicular to the same of others. These coils are named as shimming coils and are shown in fig. 3.7. Each pair of such coils is independently driven by power supplies capable of producing a magnetic field $\sim 2 \text{ G}$ at the center. Since the quadrupole coils and the shimming coils are driven by separate power supplies, the switching off-time for these coils are not synchronous and create heating of the atomic cloud by producing a sudden jerk in the trap center

while switching. Therefore, loading a MOT without the shimming coils are found to be more effective in our experiment. This has been achieved by manually adjusting the position of the quadrupole coil assembly such that the combined magnetic field center matched the center of the MOT laser beams. Although this process is very tedious, it is more preferable in comparison to using the shimming coils.

3.6 Loading and characterization of MOT

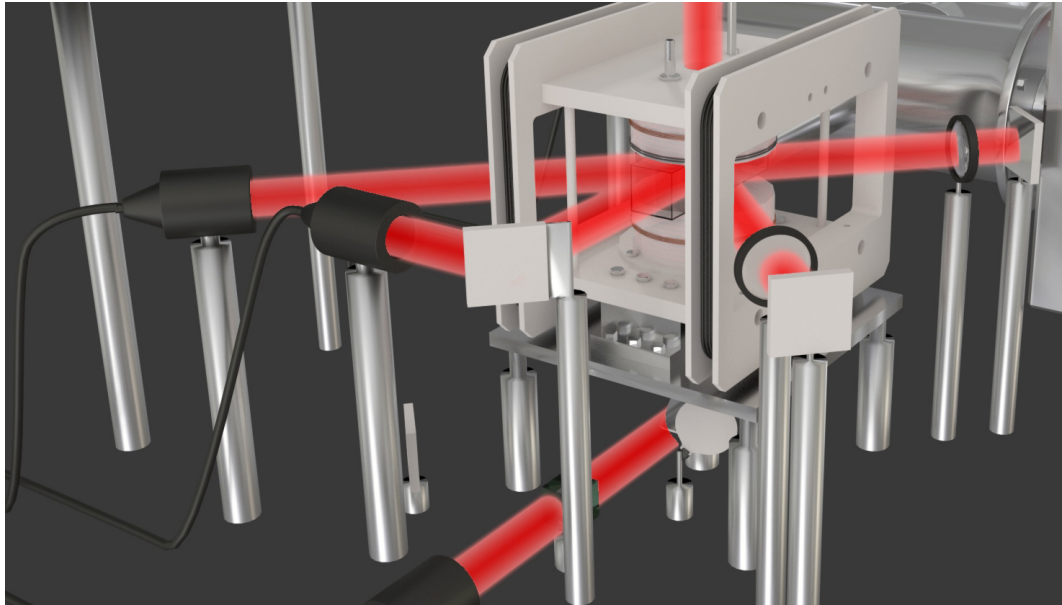


Figure 3.10: MOT schematic: Schematic representation of the Zeeman slower laser beam and the magneto-optical trapping laser beams with retroreflected geometry using $\lambda/4$ and mirror. The mirror and $\lambda/4$ plate is not shown in the vertical path.

In our experiment, we load a 3-D MOT from the flux of atoms slowed down by Zeeman slower. In order to realize the MOT geometry, we use two horizontal laser beams perpendicular to each other and another laser beam perpendicular to this horizontal plane. Each laser beam is extracted from large collimators (Shäfter + Kirchoff GmbH) with a beam output having a $1/e^2$ diameter of 22 mm. Such collimators come with in-built quarter waveplate which allows generating a circularly polarised light out of a linearly polarised light. This is needed, since the optical fibers connected to the collimators are polarization maintaining and supports a particular linear polarization. The laser beam from this three collimators is made to overlap at the center of the quadrupo-

lar field. In order to complete the setup, the light from each collimator, after passing through the quartz cell is made to pass through a quarter wave plate and then retro-reflected by dielectric mirrors. The schematic of MOT with all the lasers is shown in fig. 3.10. Each beam in the MOT has a maximum power ~ 20 mW with detuning $\sim -2\Gamma$ from $5^2S_{1/2}|F=2\rangle \rightarrow 5^2P_{3/2}|F'=2\rangle$ transition. $\Gamma = 6.1$ MHz is the natural linewidth of the excited state. The repumper laser beam is overlapped with the Zeeman

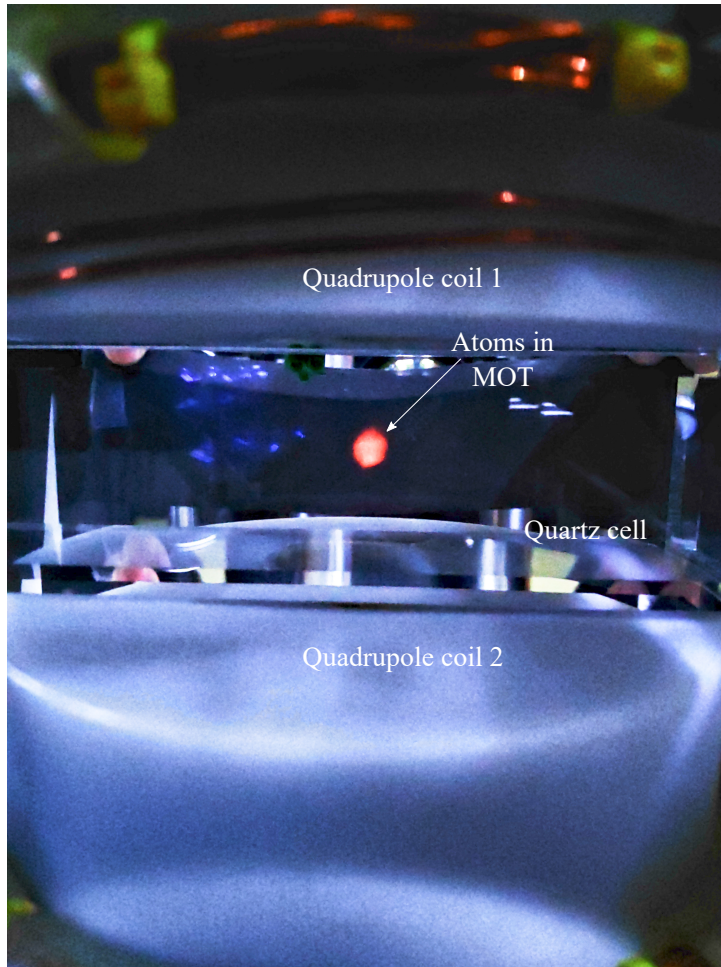


Figure 3.11: MOT: Photograph showing the fluorescence of atomic cloud trapped in the MOT at the center of the quadrupole coils. The photograph has been taken by putting a black aluminium foil in the background to enhance the contrast of the photograph.

slower beam (ZS beam). The repumper serves both the purpose of a Zeeman slower repumper (ZSR) and MOT repumper. The maximum powers in the ZS beam and repumper beams are ~ 20 mW and ~ 24 mW respectively. The ZS beam is detuned by $\sim$

-20Γ from $5^2S_{1/2}|F=2\rangle \rightarrow 5^2P_{3/2}|F'=2\rangle$ atomic transition, whereas, the repumper laser beam is detuned by $\sim -1.5 \Gamma$ from the $5^2S_{1/2}|F=1\rangle \rightarrow 5^2P_{3/2}|F'=1\rangle$ transition line. The ZS beam is focused at the outlet of the differential pumping tube to ensure maximum interaction with the atomic cloud.

The axial field of the quadrupolar magnetic field is kept at $\sim 13 \text{ G cm}^{-1}$ during loading of the MOT. The maximum number of atoms that we could capture in MOT is $\sim 1 \times 10^9$. Fig. 3.11 shows a real photograph of the fluorescence of atoms trapped in MOT. The number of atoms in MOT is preliminarily estimated by collecting the fluorescence in a photo-diode using a plano-convex lens. A more detailed procedure regarding the alignment of MOT beams can be found in ref. [167].

3.7 Compressed magneto-optical trap (CMOT)

A very big MOT with a large number of atoms is always desired. However, the cooling in MOT is limited due to photon scattering. Therefore, for further cooling, the atoms are transferred into a magnetic trap or to a far-detuned optical dipole trap. The transfer efficiency depends on the dimension of the atomic cloud and the effective volume of the trap. In our experiment, we use an optical trap for further cooling of atoms. Optical traps possess very high frequency and small trap volume. Therefore, a smaller atomic cloud is more preferred for satisfactory transfer efficiency. To serve the purpose, we compress the atomic cloud after MOT. This increases the density of the atomic cloud without losing a significant number of atoms. This technique was first demonstrated by Petrich *et al.* [173] and is known as compressed MOT.

The CMOT stage is realized in general by increasing the detuning of the MOT laser beams along with the increase in the quadrupolar field gradient. However, in our experiment, the quadrupole field gradient is increased moderately to prevent the atoms to stay at high gradient where they possess more energy leading to an increase in the temperature of the atomic cloud. Therefore, we compress the atomic cloud by increasing the detuning of the MOT beams to nearly -12Γ and the quadrupole field gradient to 22 G cm^{-1} in 40 ms. The temperature of the atomic cloud is measured after this step to be $\sim 50 \mu\text{K}$.

3.8 Optical dipole trap

One of the major steps in the experiment is the designing and implementation of an optical dipole trap. All the hardware related to the dipole trap has been done in this thesis.

Since ^{87}Rb atoms cannot be cooled to quantum degeneracy only by laser cooling. Therefore other cooling mechanisms are needed to be implemented for further cooling. The first step towards this is to turn off the MOT laser beams and transfer the atoms into a purely magnetic, purely optical, or a hybrid (magnetic+optical) trap. In our experiment, we chose a hybrid crossed-optical dipole trap. The role of the magnetic field is to provide an opposite potential against the vertical gravitational sag. Whereas, the evaporative cooling and trapping of atoms are mainly performed by the optical trap. In this section, we will discuss in detail about the trapping scheme, trap characteristics and cooling of atoms in this trap down to quantum degeneracy.

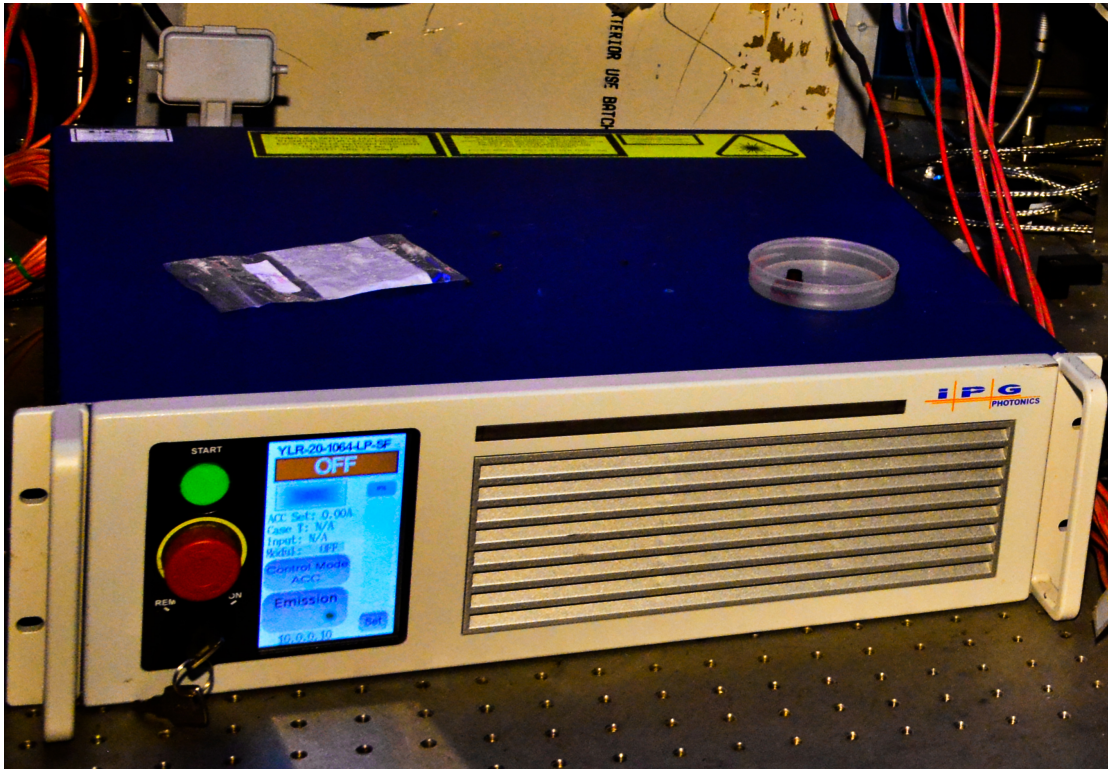


Figure 3.12: Dipole trap laser : The photograph of Ytterbium linearly polarised single frequency CW laser controller from IPG Photonics.

3.8.1 Optical dipole trap Laser

To set up a dipole trap, we use a continuous wave (CW) Ytterbium fiber laser (model: YLR-20-LP-SF) from IPG Photonics. The output of the laser is linearly polarized. The central wavelength of the laser is 1064 ± 0.5 nm. The laser offers a linewidth of ~ 15 KHz. The output of the laser is coupled to a single mode optical fiber, which in turn is connected to an optical isolator with a collimator for producing a good quality gaussian beam. At the output of the fiber, a maximum power of 20 W can be achieved. The output has a very low power fluctuation $\sim 0.5\%$ as measured over 8 hours. The typical beam diameter of the output is ~ 2 mm with a beam quality of $M^2 = 1.08$. The laser consumes ~ 200 W of electrical power and does not need any external water cooling. The image of the laser controller is shown in fig. 3.12. The controller provides a user-friendly touchscreen display for setting the required parameters and monitoring the laser temperature. In addition, the laser comes with a RS232 connector at the back which can be employed for remote control through a sequence of logical programming. The image of the final collimator is shown in fig. 3.13. The collimator has an isolator at the output, which can be rotated to maximize the output power of the laser by monitoring the feedback coming out of the window provided in the collimator. The long-term performance of the laser was not satisfactory, and the output power of the laser degraded to 14 W over a couple of year of operation without affecting the polarization and beam quality.

3.8.2 Preparation of laser beams for a crossed-optical dipole trap

It has been discussed in the previous chapter that a single optical dipole trap cannot provide strong confinement from all the direction. Therefore, the usual practice is to introduce another laser beam preferably in the perpendicular direction to the propagation of the previous laser beam. Since the focii of these laser beams should overlap, these beams should possess different frequencies in order to avoid interference. Again, we need independent control on the amplitude of these laser beams for a further cooling mechanism to take place. Fig. 3.14 shows the schematic layout of how the two laser beams can be extracted in order to fulfill the criteria mentioned above. The laser beam coming out of the collimator of the 20 W laser is divided into two parts using a combination of a half waveplate and a polarizing beam splitter (PBS) cube. Such a scheme

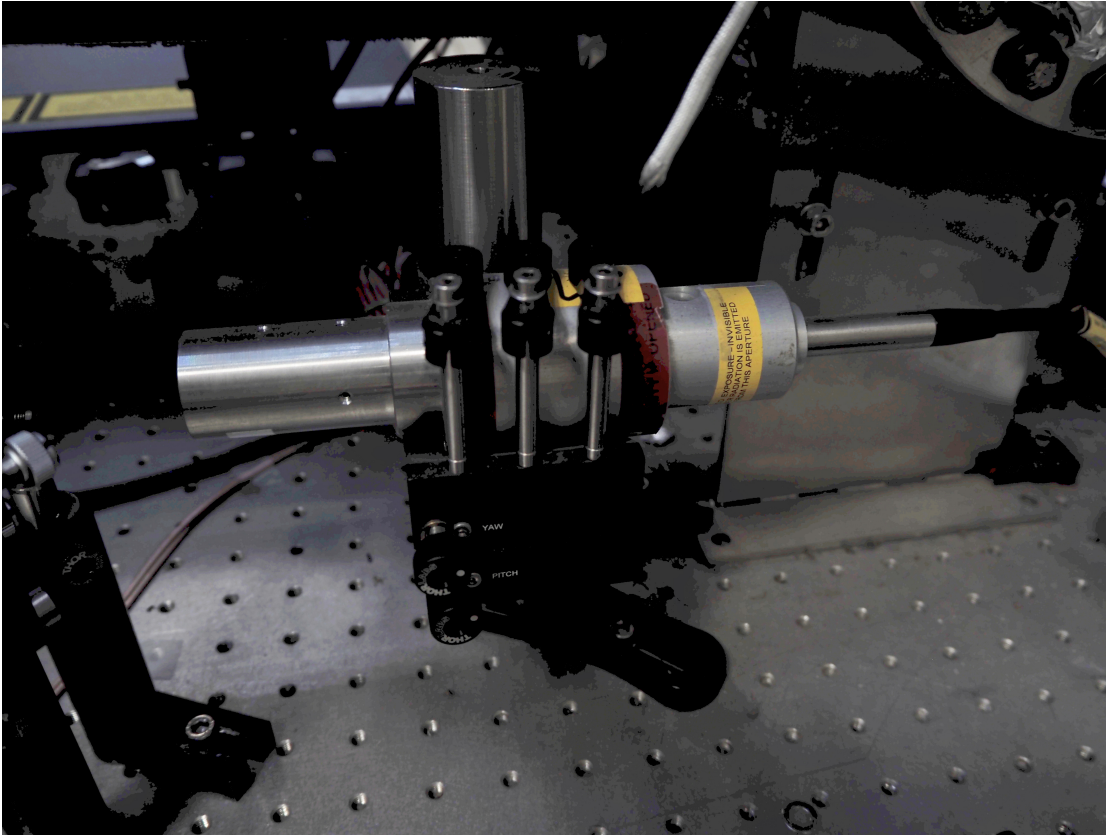


Figure 3.13: Dipole trap laser collimator : The photograph of the collimator connected at the output of the fiber laser.

is preferred over using a single beam splitter since in the latter case we can only have two beams from the beam splitter with equal power. Whereas, the combination of a half-waveplate and a PBS cube allows to separate the power in any proportion in the two paths. Each laser beam coming out of the PBS cube are passed through an AOM. The AOMs used in the experiment are operated at their center frequencies 110 MHz and 120 MHz respectively. The 1st order diffracted beam from the AOM is used in each case for further experiment. Thus the two laser beams used in the experiment differ in frequency by 10 MHz and hence avoid interference in the experiment. The power of both the laser beam can be controlled separately by controlling the RF power supplied to the AOMs. In addition, the AOMs are used for fast switching of the two laser beams. Since the active aperture of the AOMs is smaller compared to the natural beam diameter of the laser coming out of the collimator, a telescope is introduced in order to reduce the beam size and achieve a high efficiency in the diffracted order from the AOMs. The

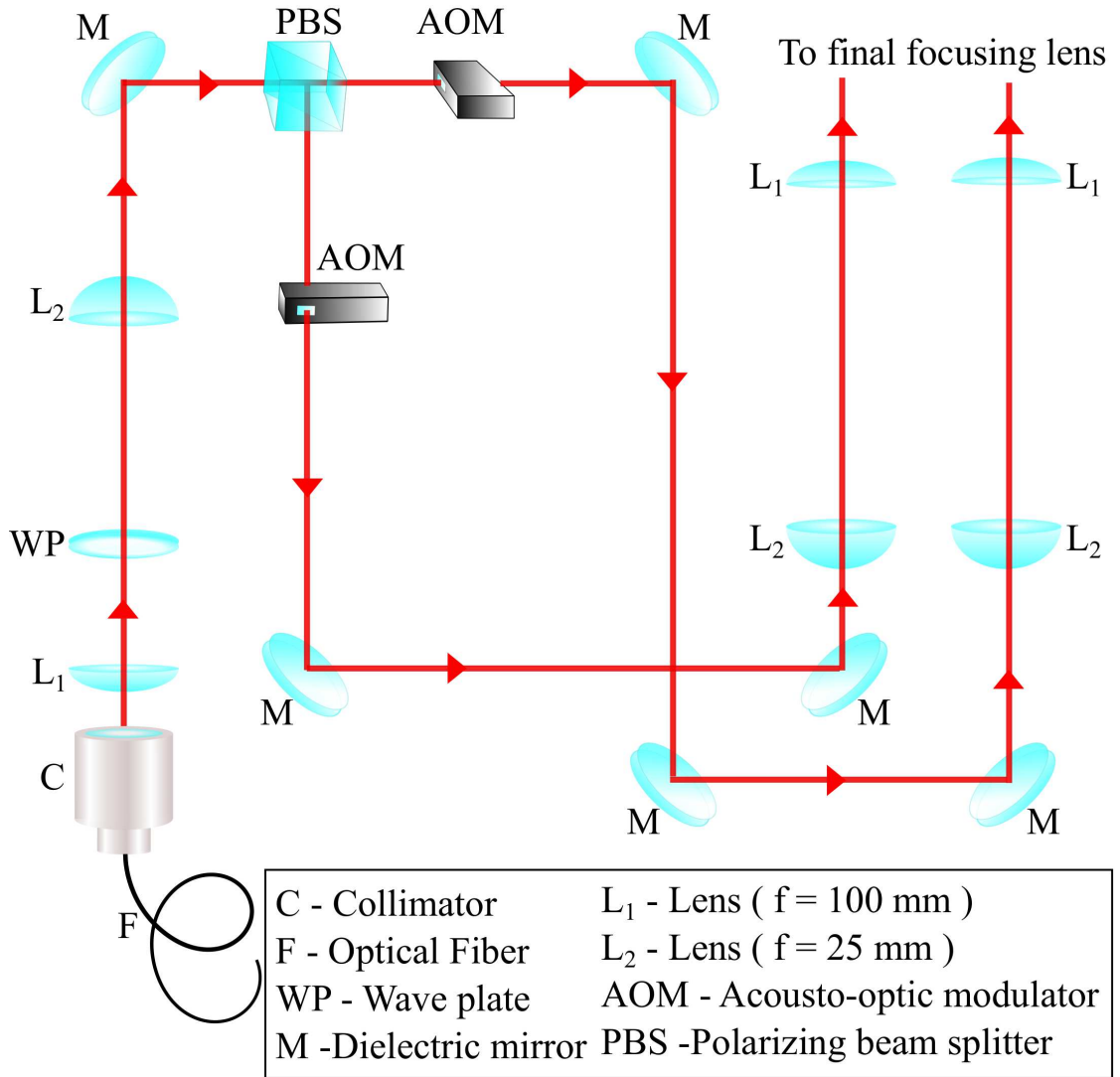


Figure 3.14: Optical dipole trap-laser table layout : A schematic diagram of preparation of two independent laser beam for a crossed optical dipole trap. The undeflected zeroth order beam from the AOM is not shown in the figure. The figure does not also display the change in the beam waist of the laser over the distance in order to simplify the diagram.

AOMs used in the experiment are not suitable for high power operation and comes with a smaller active aperture. Therefore a maximum of 70% efficiency has been achieved in the diffracted order of the AOMs. A laser beam with a large diameter is always desired before going through the final lens. This is because a tight focus requires a bigger beam. Therefore, after the AOM, another telescope is used in the path of each beam to increase the beam diameter once again. After the two final telescopes, we achieve two

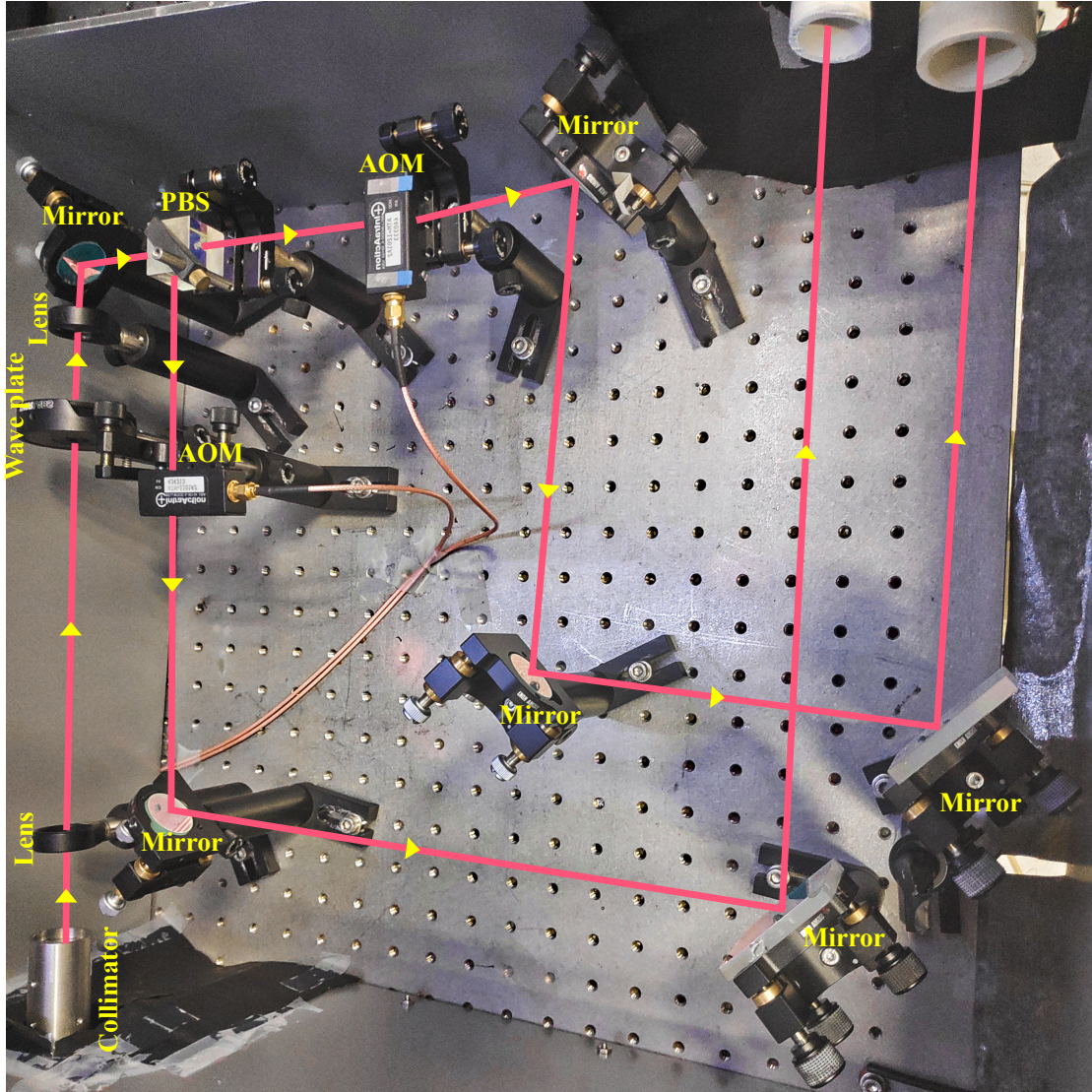


Figure 3.15: Laser beam preparation for crossed optical dipole trap : The photograph of the experimental setup to extract laser beams for crossed optical dipole trap.

beams with 5.5 W power in each of them. In the next section, we will discuss about the spatial profile of these beams.

3.8.3 Beam profile

The spatial profile of the beam plays a major role in deciding the dynamics of the atoms in the trap. As we have already seen in the previous chapter that, an optical dipole trap potential depends on the intensity distribution of the laser beam. Therefore, any small

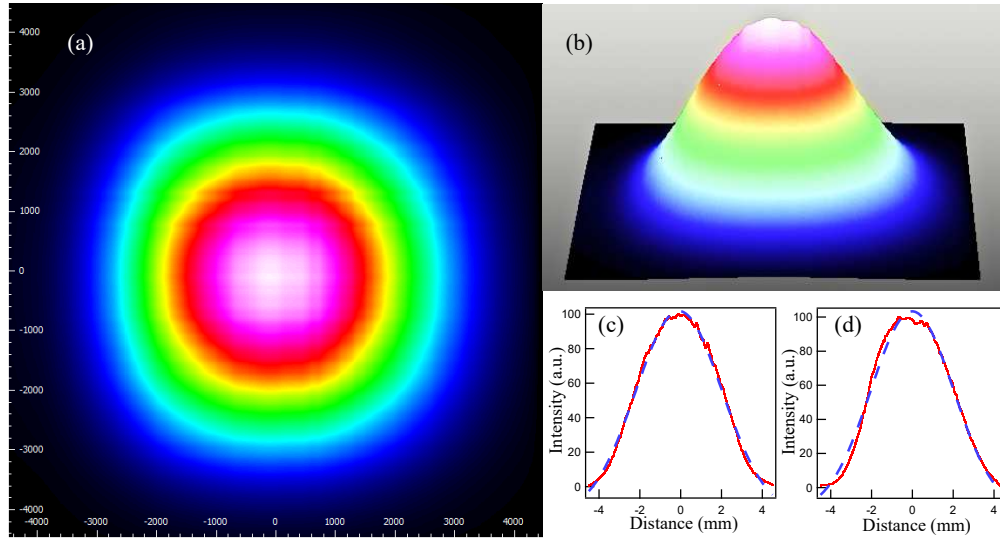


Figure 3.16: Beam profile of laser beam 1 : Spatial beam profile of the laser after passing through 110 MHz AOM and the final telescope. (a) and (b) are 2-D and 3-D image respectively of the beam profile as recorded by the profilometer. (c) and (d) represent respectively the horizontal and vertical cross-section plot of the intensity of the laser beam. The solid lines are the measured data, and the dashed lines represent the Gaussian fit.

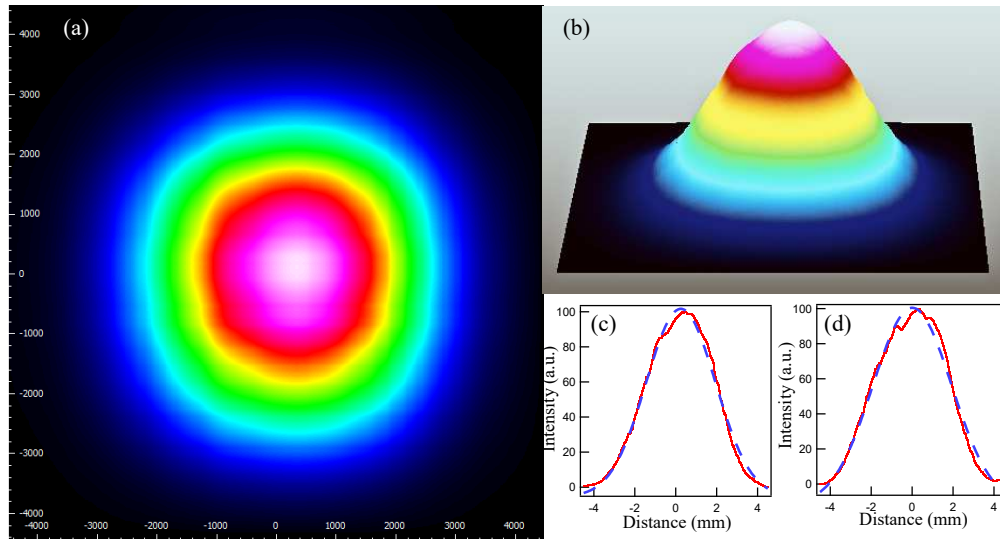


Figure 3.17: Beam profile of laser beam 2 : Spatial beam profile of the laser after passing through 120 MHz AOM and the final telescope. (a) and (b) are 2-D and 3-D image respectively of the beam profile as recorded by the profilometer. (c) and (d) represent respectively the horizontal and vertical cross-section plot of the intensity of the laser beam. The solid lines are the measured data, and the dashed lines represent the Gaussian fit.

change in the spatial profile of the intensity of the light modifies the trap geometry drastically at the focus, which in turns affects the dynamics of the atoms in the trap. In our experiment, we use a scanning-slit optical beam profiler (model: Thorlabs, BP209-VIS/M) to characterize the spatial mode of the two dipole trap lasers. The profiler is capable of working in the wavelength range of 200-1100 nm. It offers two different modes of operation - (a) scanning slit and (b) knife edge to measure the intensity profile of the laser. Fig. 3.16 and fig. 3.17 represent the 2-D and 3-D plot of the intensity distribution of the two laser beam used for trapping. The profiles are measured just before the final focusing lens in each case. In addition, the figures display the intensity plot of the two laser beams along the horizontal and vertical direction of the beam perpendicular to the direction of propagation. Both the beams have minor ellipticity with the ratio of two diameters being $\sim 0.95\%$. Any dust particle sticking to the optical elements kept in the path of the beam scrambles the beam profile leading to inefficient operation of the trap. To serve this purpose and also for the sake of safety the path of the dipole trapping laser beams are covered by long tubes of polyvinyl chloride (PVC).

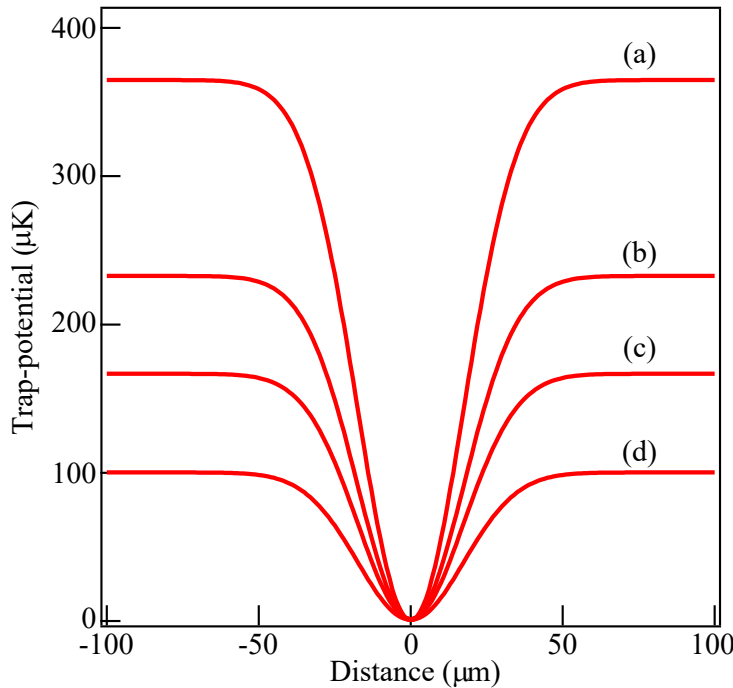


Figure 3.18: Optical dipole trap potential : Trapping potential formed by a single far detuned focused Gaussian laser beam with a beam waist of $35 \mu\text{m}$ at the focus and varying powers (a) 5.5 W, (b) 4.5 W, (c) 3.5 W, and (d) 2.5 W.

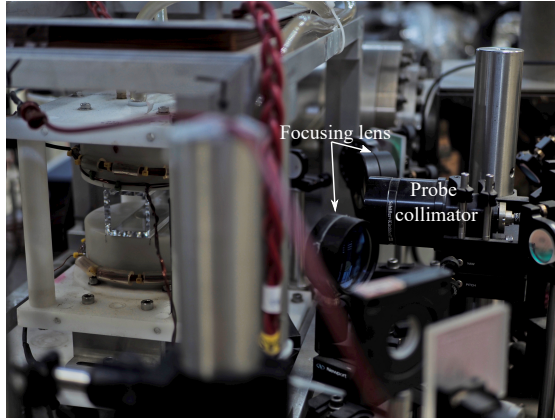


Figure 3.19: Dipole trap alignment: Photograph displays the side view of the focusing lens and probe. The probe is perpendicular to the length of the glass cell.

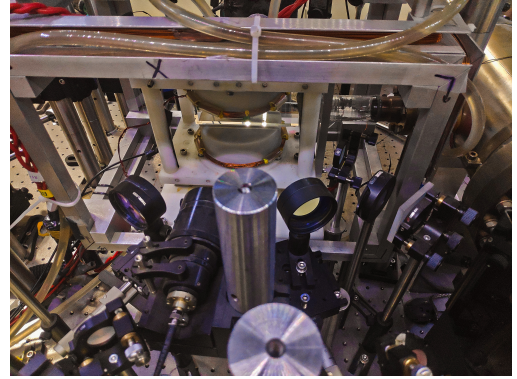


Figure 3.20: Angle between the dipole trap beams: Image displays the angles between the pair of dipole trap beams along with the orientation of MOT and probe beam

3.8.4 Trap characteristics

The diameter of the laser beams is calculated from Gaussian fit to the intensity profile measured by the profilometer. The average diameter of the beams before the final focussing lens is measured to be ~ 3.8 mm and ~ 4 mm. The beam waist diameter at the focus can be given by

$$2\omega_D = \frac{4\lambda}{\pi} \frac{f}{D} \quad (3.1)$$

where λ is the wavelength of the light, f is the focal length of the focusing lens and D being the diameter of the beam before the focusing lens. From Eq. 3.1, the beam waist at the focus for the laser beams used in our experiment comes out to be $2\omega_D \approx 70 \mu\text{m}$. Fig. 3.18 shows the trapping potential due to a single beam with the measured beam waist and varying power in the range of 1.5 W - 5.5 W. The trap depth is found to be linearly proportional to the power and inversely proportional to the square of the beam waist. Since the potential is not harmonic; the trapping frequencies cannot be calculated directly. In the later section, the method of measuring trapping frequencies are discussed in detail.

3.8.5 Alignment of the dipole trap

The dipole beams are focused at the center of the quadrupolar trap using a Plano-convex achromatic doublet ($f = 200$ mm). After the final telescope, each laser beam passes

through a periscope arrangement before entering the final focusing lens. The pair of mirrors used to construct the periscope helps in adjusting the height of the laser beam as well as the direction of propagation. The last mirror before the focusing lens is used to align the beam into the atomic cloud. Each focusing lens is mounted over a translator stage capable of a precise movement of 0.1 mm. Fig. 3.21 shows the top view of the schematic showing the relative angles between the different laser beams. Fig. 3.19 and fig. 3.20 display the photograph of the experimental setup showing the relative alignment of the dipole beams with respect to the MOT and probe laser beams.

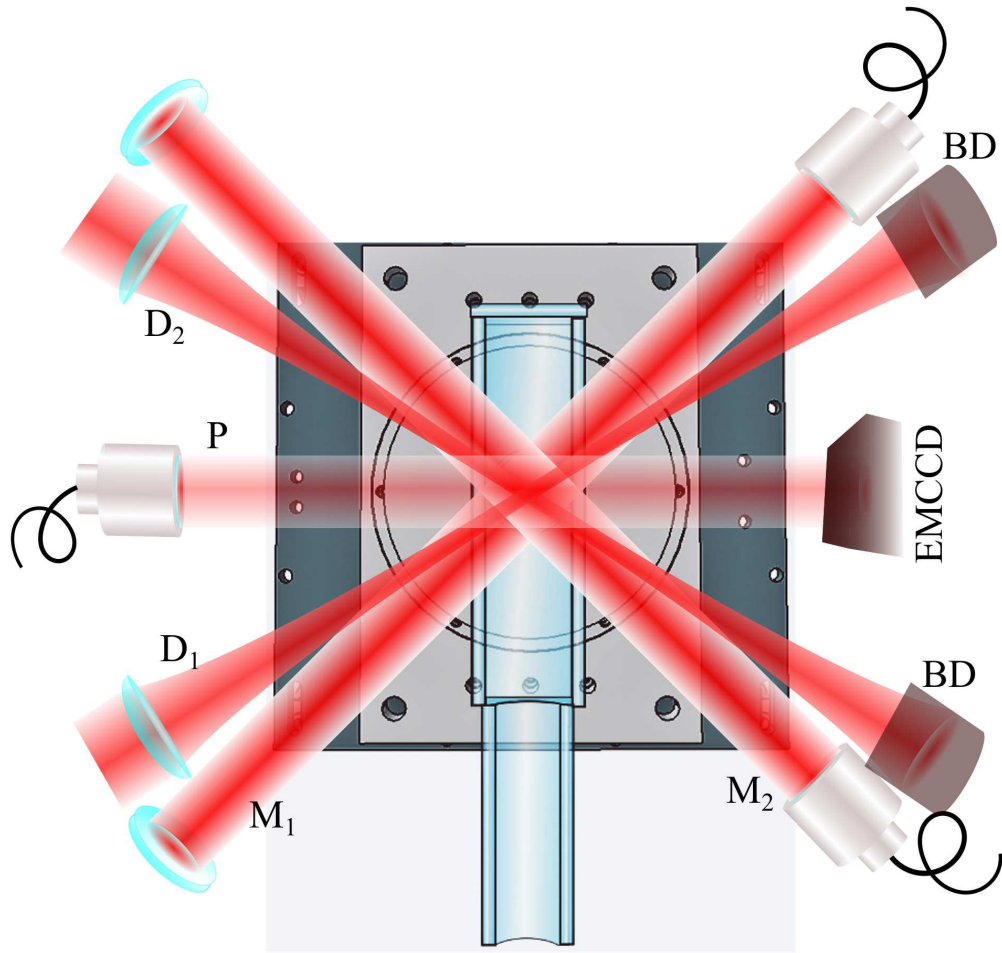


Figure 3.21: Optical dipole trap geometry : Top view of the schematic of the position and orientation of all the laser beam in horizontal plane with respect to the quadrupole field center is shown. The bottom quadrupole coil and the horizontal cross-section of the glass cell are shown for reference. M_1 and M_2 are the MOT beams present in the horizontal plane, and D_1 and D_2 are the dipole trap beams. 'P' is the probe beam. 'BD' represents beam dump used to block the high power laser beam for safety.

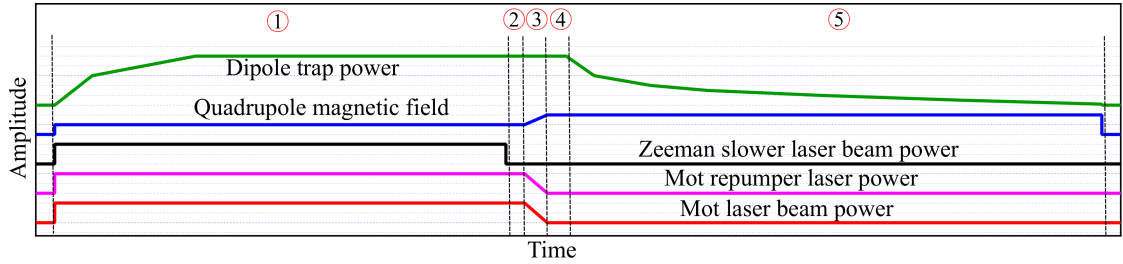


Figure 3.22: Experimental sequence: The five major part of the sequence in order. (1) the loading of a MOT, (2) MOT stabilization, (3) compression of the MOT, (4) depumping of atoms into the ground state, and (5) evaporative cooling. For the sake of simplicity the last step consists of probing the atoms is not shown here. Image not to scale.

3.8.6 Loading a crossed optical dipole trap

The trapping state of ^{87}Rb atom in an optical dipole trap can be chosen to be either $|F = 1\rangle$ state or $|F = 2\rangle$ state. Since $|F = 1\rangle$ is lower in energy, due to collision between the atoms, they make transition from $|F = 2\rangle$ to $|F = 1\rangle$ state. This is known as two-body collision loss. Therefore, it is preferred to choose $|F = 1\rangle$ as the trapping state. Since all the cooling cycles run between $|F = 2\rangle \rightarrow |F' = 3\rangle$ along with the repumper on and tuned to $|F = 1\rangle \rightarrow |F' = 2\rangle$, the atoms do not populate $|F = 1\rangle$ state. In order to transfer the atom into the dipole trap, the atoms are depumped to $|F = 1\rangle$ state by turning off the repumper. The complete experimental sequence is shown in fig. 3.22. The whole sequence can be broken into six major steps:

- loading of a MOT,
- MOT stabilization,
- Compression of the MOT,
- Depumping the atoms and loading the dipole trap
- Evaporative cooling, and
- Absorption imaging

The loading of a MOT has been described in detail in the previous section. We load atoms in MOT for 5 sec. During the first second the power in the optical dipole beams are ramped up to 4 W. In the remaining 4 seconds, the power in these beams is increased

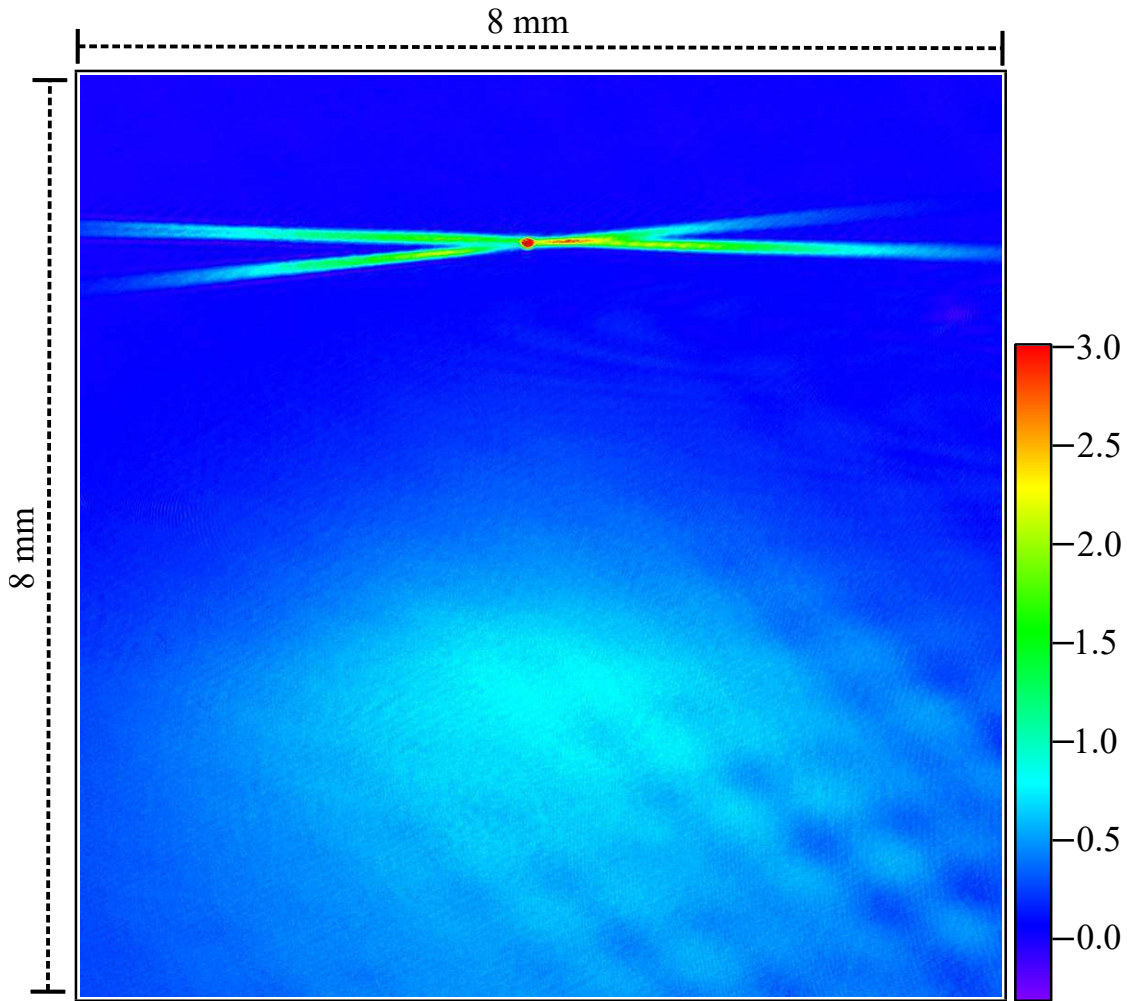


Figure 3.23: Loading a crossed dipole trap: Absorption image of the cold atoms trapped in the crossed optical dipole trap (bright crossed lines with high optical density) and comparatively hotter atoms falling under gravity (diffused image with lower optical density).

up to maximum possible value ~ 5.5 W. The dipole beam centers are aligned about a beam-waist below the magnetic trap center in order to have a mode-matching of the two traps which ensures a maximum transfer of atoms.

Once the MOT loading is done, we turn off the ZS beam and the atomic beam shutter. This step lasts for 10 ms. During this step, the atoms in the MOT stabilizes, so we call it MOT stabilization step.

The next one in the row is the compression of the MOT which also has been discussed in details in the previous section. During this step, the detuning of the MOT beams is increased to $\sim -12 \Gamma$, and axial quadrupolar field gradient is increased up to 22 Gcm^{-1} . The powers in MOT and MOT repumper beams are reduced to nominal value gradually. The step lasts for 40 ms and produces a very cold atomic cloud with high density.

Once we create a small dense atomic cloud by compressing the MOT, the powers in both MOT and MOT repumper beams are made zero for ~ 1 ms to allow the atoms to populate $|F = 1\rangle$ state. During this time, the cold atoms are transferred into the optical dipole trap, and the hotter ones fall under gravity. Fig. 3.23 displays the absorption image of such trapping of atoms.

Once the atoms are transferred, they are cooled down by evaporative cooling and then imaged using standard time-of-flight absorption imaging technique which will be discussed in the next sections.

3.9 Evaporative cooling

The basic idea underlying evaporative cooling is as simple as cooling down a cup of tea. The difference being the nature of the process. In any cold atom experiment, the atomic cloud is forced to be cooled down. From this perspective, the process is a forced evaporative cooling. In a trap, the velocity distribution of the atoms can be well-described by the Maxwell-Boltzmann (M-B) velocity distribution. Therefore, the atomic cloud possesses mean kinetic energy, and hence a mean temperature can be defined for the group of atoms. In order to cool down such a group of atom, the hotter atoms are thrown out of the trap, resulting in modifying the velocity distribution by cutting the tail of the M-B distribution. Under these circumstances, the atomic cloud rethermalizes and generate a modified M-B distribution with lower average kinetic energy and hence lower mean temperature. The only catch in the whole scheme lies in the process of thermalization.

It is much desired that a particle leaving the trap will take away the energy and the rest of the particles in the trap will exchange energy to redistribute the energy in order to modify the M-B distribution. This process depends on the rate of inelastic collision in the trap, which in turns is a function of the confinement produced by the trap. Therefore a trap with high trapping frequency is much preferred for efficient evaporative cooling. For a typical optical dipole trap, the trapping frequencies are in the range of a few KHz which is reasonably very tight compared to magnetic traps and therefore allows faster evaporation. In the context of evaporation in an optical dipole trap, it should be noted that the trapping frequencies decrease with the power of the laser power. A shallow trap ends up with a slow evaporation rate leading to the loss of a large number of atoms. Fig. 3.24 shows an initial deep trap with hot atoms. To throw away the hotter particles, the trap depth has been reduced adiabatically which resulted in less number of atoms in a comparatively shallow trap but with colder atoms left in the trap. With decreasing trap depth, the temperature of the atomic cloud reduces, but due to a decrease in trapping frequency, the evaporation takes longer.

In our experiment, we decrease the power in both the dipole laser beams in piecewise linear functions until the final temperature of the atomic cloud reaches well below the critical temperature to produce a BEC. The evaporation sequence is given in the following table:

Stage	time(s)	Beam 1 start power (W)	Beam 1 end power (W)	Beam 2 start power (W)	Beam 2 end power (W)
1	0.3	5.5	3	5.5	3
2	0.5	3	0.9	3	0.9
3	0.7	0.9	0.5	0.9	0.5
4	1.2	0.5	0.2	0.5	0.2
5	1.5	0.2	0.02	0.2	0.02
6	1.5	0.02	0.005	0.02	0.005

Table 3.1: Summary of the evaporative cooling parameters for achieving Bose-Einstein condensation.

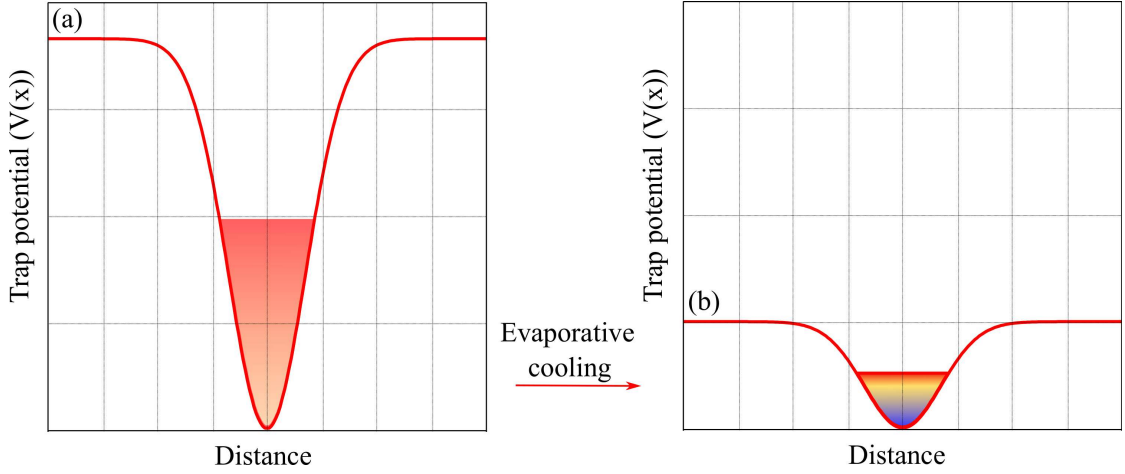


Figure 3.24: Evaporative cooling mechanism: Cooling of atoms with reducing trap depth. (a) Deeper potential with high confinement, (b) After reducing the depth of the potential adiabatically, atoms near the bottom of the trap start cooling.

3.10 Imaging System

3.10.1 Absorption imaging

To find the spatial extension, density and temperature of the atomic cloud, we use standard time-of flight absorption imaging. In this method, the trap is suddenly turned off and the atoms are allowed to fall freely under gravity for some time. After the free evolution, a resonant laser beam is shone upon the atoms and the shadow of the atomic distribution is collected on a CCD camera (ANDOR-iXon3). On further processing of this image, one can find out the above-mentioned parameters used to characterize the atomic sample. A detailed analysis of the image processing is given below.

The absorption imaging works on the principle of Lambert-Beer's law. Assuming the probe beam to be on resonant with the atomic resonant frequency, the intensity distribution of the probe beam after interacting the atomic cloud can be written as:

$$I(x, y) = I_0(x, y) \exp \left[-\sigma \int n(x, y, z) dz \right] \quad (3.2)$$

Where, $n(x, y, z)$ is volume density, which has been integrated along the propagation direction of probe beam in order to take care of the projection of the column density. σ

is the resonant cross section and given as:

$$\sigma = \frac{3\lambda^2}{3\pi} \quad (3.3)$$

where λ being the wavelength of the probe laser. A series of three images are taken in order to map the density of the atomic cloud. The first two images are taken in the presence of the probe light, but one with the shadow of the atoms $I(x, y)$ and other bearing only the intensity distribution of probe $I_0(x, y)$. The third one is the reference image $I_{ref}(x, y)$ which is taken without light to find out the background noise level and is employed to compensate for the constant background noise level. The optical density $D(x, y)$ can be calculated as:

$$D(x, y) = \ln \left[\frac{I(x, y) - I_{ref}(x, y)}{I_0(x, y) - I_{ref}(x, y)} \right] = -\sigma \int n(x, y, z) dz \quad (3.4)$$

Hence, the number of atoms can be calculated as:

$$n = -\sigma \int \int n(x, y) dx dy = \frac{1}{\sigma} \int \int D(x, y) dx dy \quad (3.5)$$

In order to calculate the temperature of the atomic cloud, we should recall that the velocity distribution of the atoms is governed by their temperature. Therefore, if the trapping potential is turned off and the atomic cloud is allowed to expand freely while falling under gravity, the spread of the atomic cloud over time will change because of its temperature. In general, the spatial extension of a group of atom in the trap can be well approximated by a Gaussian distribution. Since M-B distribution is also Gaussian, these two functions convolve in order to give rise another Gaussian function which decides the spatial distribution of the atoms after a time evolution. Let us recall that if two gaussian convolve to produce another Gaussian, then the variance of the two convolving gaussian add up to give the variance of the new Gaussian function. Let us assume that the momentum distribution of the atoms is given by:

$$\phi(p) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp \left(-\frac{p^2}{2\sigma_p^2} \right) \quad (3.6)$$

Hence, after time-of-flight t , we can write-

$$\sigma_x^2(t) = \sigma_x^2(t=0) + \frac{\sigma_p^2}{m} t^2 \quad (3.7)$$

Where, σ_x and σ_p represent the gaussian width of the position and momentum distribution respectively. The width of the position distributions can be calculated by fitting two Gaussian functions to the images taken by the camera at time ' t ' and ' $t = 0$ '. This can be used to measure the width of the momentum distribution. Hence the temperature can be given as:

$$T = \frac{\sigma_p^2}{mk_B} \quad (3.8)$$

Where, ' m ' is the mass of atom and ' k_B ' is the Boltzmann constant.

3.10.2 Analysis of an absorption image of BEC

The density distribution of a thermal cloud inside the trap can be well approximated by a 2-D gaussian function given by:

$$n_{2D}^{thermal}(x, y) = n_{2D_{peak}}^{thermal}(x, y) \exp\left(-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2}\right) \quad (3.9)$$

Where, $n_{2D_{peak}}^{thermal}$ being the peak density in the integrated distribution and σ_x and σ_y are the widths of the atomic distribution in X and Y direction respectively.

The dynamics of a BEC is governed by Bose-Einstein statistics. Therefore, the spatial distribution of a condensate follows Thomas-Fermi distribution. which is parabolic in nature. Hence, the column integrated density can be given as:

$$n_{2D}^{BEC}(x, y) = n_{2D_{peak}}^{BEC} \left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2}\right)^{\frac{3}{2}} \quad (3.10)$$

Where, $n_{2D}^{BEC}(x, y)$ is the peak density in the integrated distribution and R_x and R_z are the Thomas-Fermi radii with the column integration performed along Z direction.

In case of a condensate, after a free evolution, the density distribution remains parabolic but the condensate aspect ratio inverts according to scaling relations governed by the ratios of the trapping frequencies [174].

In our experiment, the imaging system consists of a single lens geometry in which an

achromatic doublet (focal length $f = 200$ mm THORLABS) lens is kept $\sim 2f$ distance from the atomic cloud. The chip or image plane of a CCD camera is placed in the other side of the lens at $2f$ distance. The probe laser beam is sent through a collimator (Shäfter + Kirchoff GmbH) having $1/e^2$ diameter of 22 mm. The probe beam is circularly polarized. The magnification factor of the imaging system in our experiment is found to be ~ 1.3 . To optimize the sharpness of the image, the camera is installed on a linear translational stage which allow precise movement of the camera along the axis of the probe.

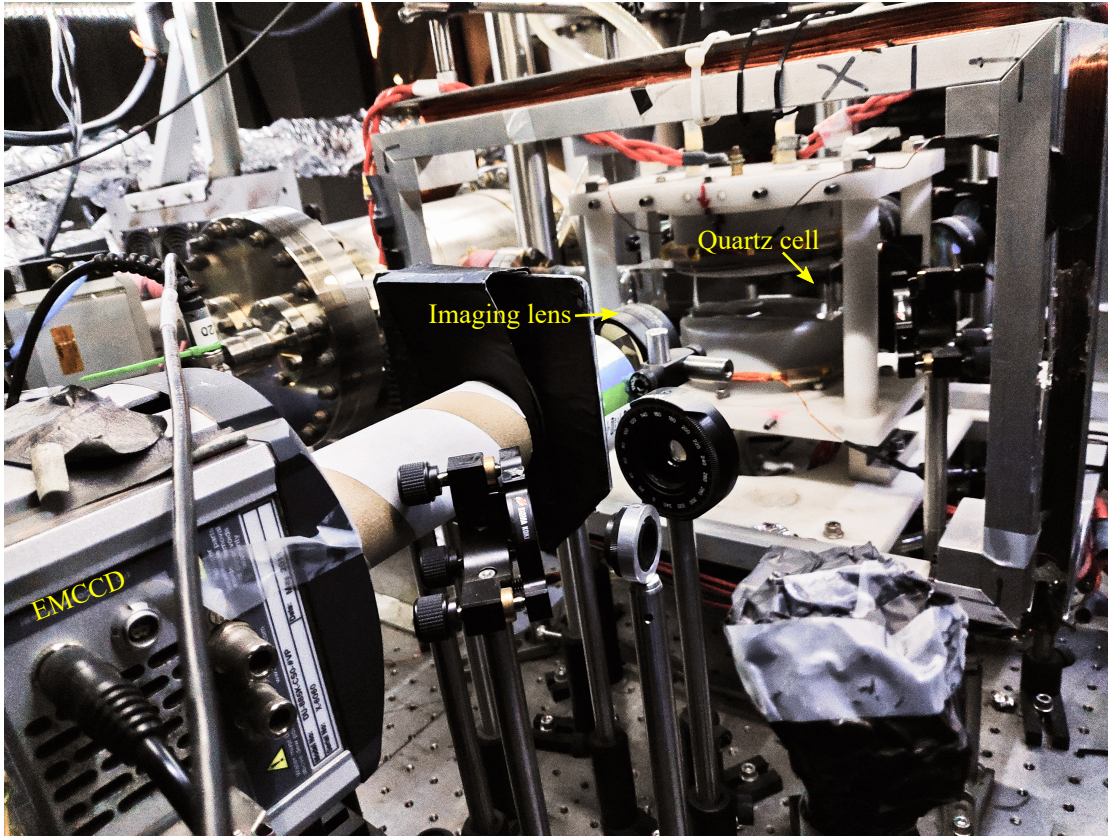


Figure 3.25: Absorption imaging: Experimental setup used for absorption imaging. The path between the imaging lens and camera is covered using tubes to block stray lights to enter into the camera.

3.11 Observation of BEC and its characterization.

3.11.1 Phase transition

During the process of forced evaporation, the temperature of the atomic cloud keeps reducing and with the temperature going below the critical temperature, a condensate fraction appears at the center of the atomic cloud along with the Gaussian distributed thermal atoms. More evaporation leads to a diminishing of the Gaussian background and emergence of a parabolic distribution. With time-of-flight images, the transition can be well detected by the evolution of a hazy atomic cloud into shining small pixels in the camera. Fig. 3.26 shows such a phase transition observed using the crossed dipole trap in our experiment.

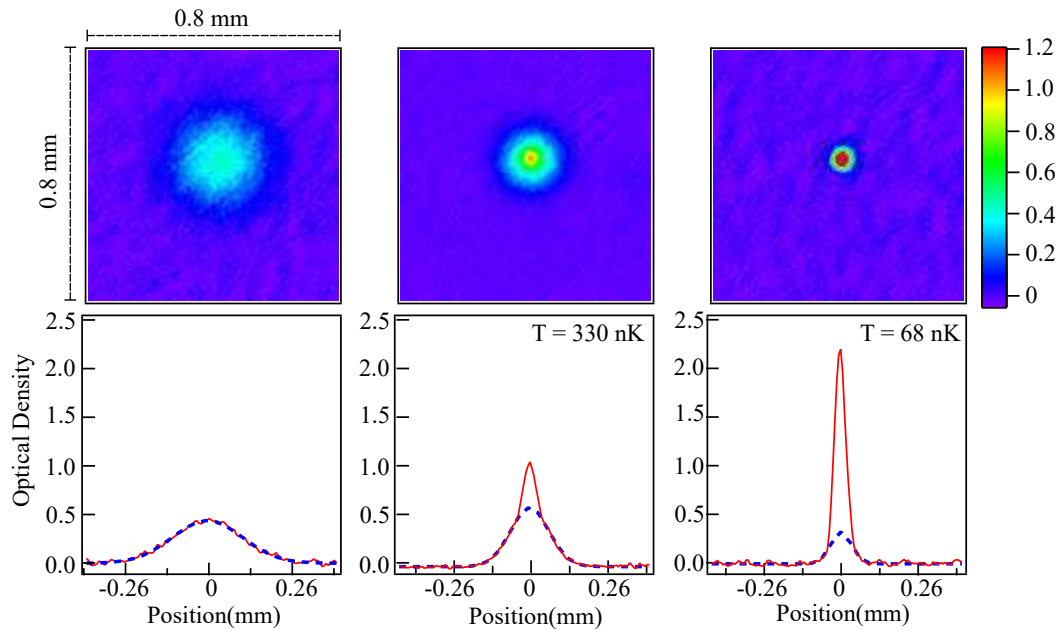


Figure 3.26: Observation of Bose-Einstein Condensation (BEC): Typical absorption images of the ultracold cloud of atoms taken after 10 ms of expansion under gravity. (a) Thermal cloud at a temperature about of $1 \mu\text{K}$, (b) bimodal density distribution at temperature 330 nK showing the emergence of a sharp parabolic peak at the center with a condensate fraction of $\sim 25\%$. (c) Almost pure condensate at the final stage of evaporation; the condensate fraction is 95%. The lower set of curves is the horizontal cross-section plots of the images above. Dashed lines represent the Gaussian fit to the thermal atoms.

The phase transition normally happens very fast making it difficult to monitor several parameters during the transition. Hence, to study the phase transition properly, at

the final stage of evaporation we truncate the process at different powers in the two crossed dipole beams very near to the bottom of the trap. The resolution of such transaction is made well-controlled and high enough to observe a proper phase transition. A very good parameter of our interest is the condensate fraction. The evolution of the condensate fraction is shown in fig. 3.27. In each case, the number in the condensate is measured by fitting a parabolic function to the condensate portion for each horizontal cross-section. Whereas the temperature is estimated by fitting a gaussian curve to the thermal background. Therefore, the temperature of the condensate fraction can be assumed to be lower than the estimated temperature. The experimental data is fitted to the well-known function and shown in fig. 3.27. The critical temperature found from the fit

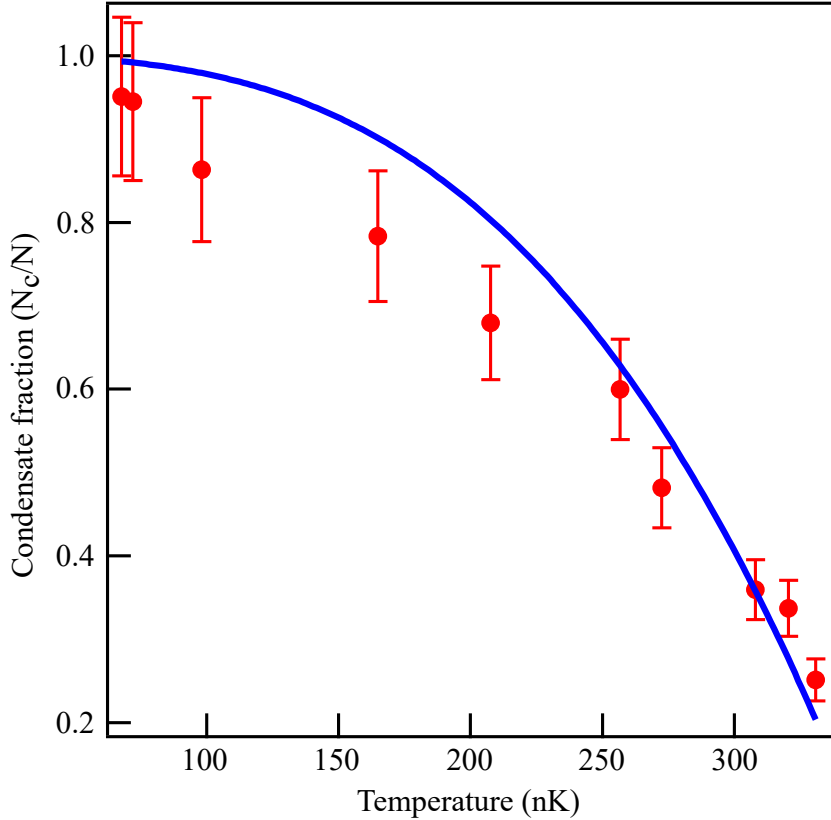


Figure 3.27: Evolution of condensate fraction: Typical growth of condensate fraction as a function of the temperature of the atomic cloud.

is ~ 350 nK, which agrees with the experimental observation. In order to characterize the evaporation process, we study the evolution of phase-space density, loss of atom number and a simultaneous decrease in temperature over the phase transition period.

For a better resolution during the phase transition, the evaporation process is truncated near to the bottom of the trap with high resolution in the final power in each dipole beam. Fig. 3.28 shows a fast drop-off in temperature in the initial stages of evaporation in contrary to a gradual decrease in temperature during the phase transition. This is because with decreasing power of the dipole trap, the trap frequency decreases which in turn reduces the collision rate making the evaporation less efficient. The evolution of phase space density is shown in fig. 3.29. The PSD shows a very sharp increase over the phase transition period. Higher values of PSD suffers from a larger error in estimation, due to uncertainty in the calculation of temperature. Well below the phase transition, the temperature of the atomic cloud is measured from the evolution of a Gaussian background made of thermal atoms. Whereas, the temperature of BEC stays even below to that estimated temperature. This leads to an underestimation in the calculation of PSD. To characterize the rate of loss of atom, the number of atom remaining after each evaporation stage is shown in fig. 3.30. We start with 2×10^6 atoms in the crossed dipole trap. Throughout the evaporation, we lose two order of magnitude in a number of atoms and end up with a BEC of $\sim 6 \times 10^4$ atoms.

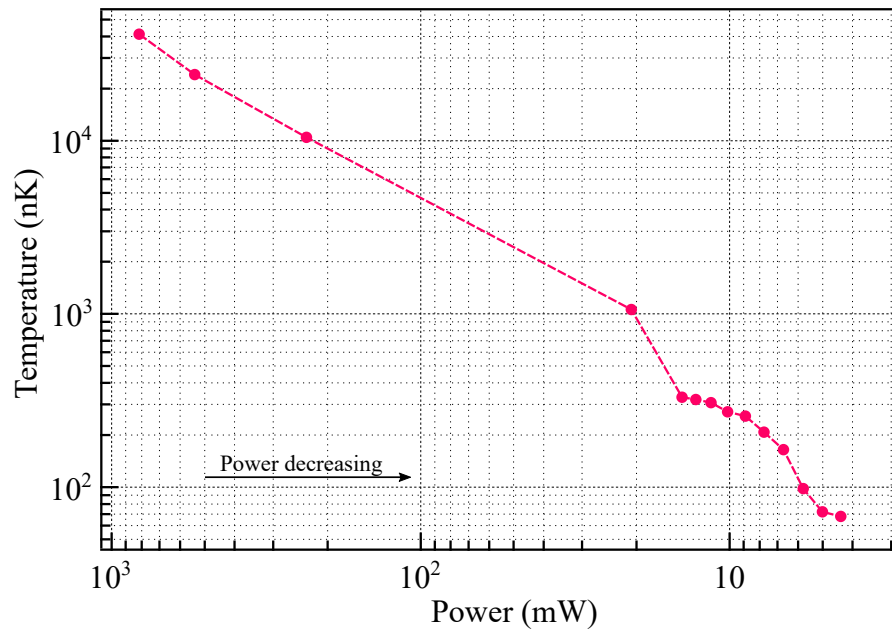


Figure 3.28: Cooling rate during evaporation: Decrease in mean temperature of the atomic cloud after different stages of evaporation. Symbols represent experimental data. Dashed line is guide to eye.

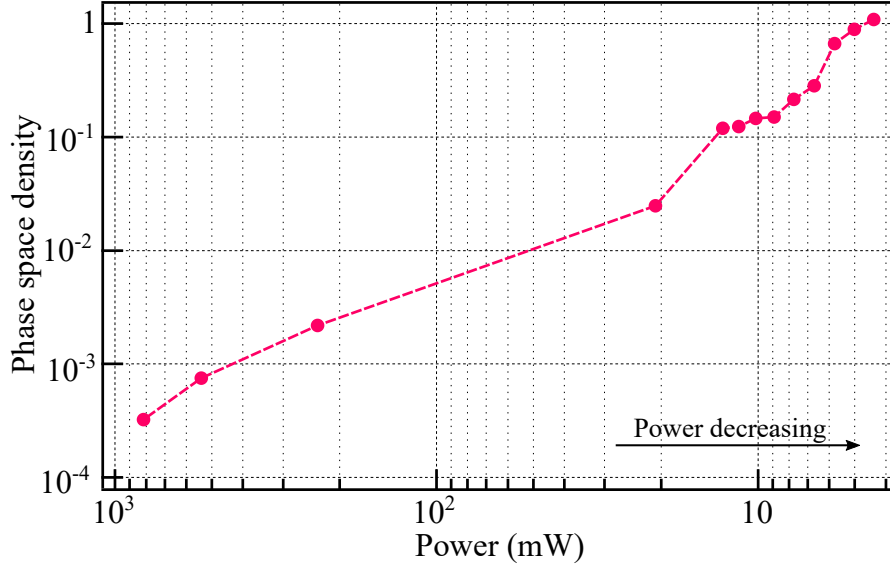


Figure 3.29: Evolution of phase space density: Increase in phase space density with evaporation. The PSD is plotted against the final power of single beam in each case. Symbols represent experimental data. Dashed line is guide to eye.

3.11.2 Spin-purity

We use a hybrid trap (quadrupolar magnetic field and crossed optical dipole trap) for evaporative cooling purpose. Although the optical dipole trap is spin independent, the magnetic field creates a trapping potential only for $m_F = -1$ hyperfine level. Therefore, the atoms trapped in our experiment are in the purely single hyperfine level. To ensure the state of the atoms, we perform a Stern Gerlach experiment [175]. The atoms are allowed to fall under gravity freely. During this time an inhomogeneous magnetic field is applied. Any presence of spin mixture would have resulted in the separation of different parts of the atomic cloud having different spins. However, on the application of the magnetic field, the whole atomic cloud is found to deflect in one single direction. With increasing magnetic field gradient the value of deflection increased. The results are shown in fig. 3.31. This concludes that the atoms in BEC are in a single hyperfine state.

3.11.3 Lifetime of BEC

The optical trap suffers from less fluctuation compared to the magnetic trap. Magnetic trap suffers from fluctuation due to noise in the current which results in heating and

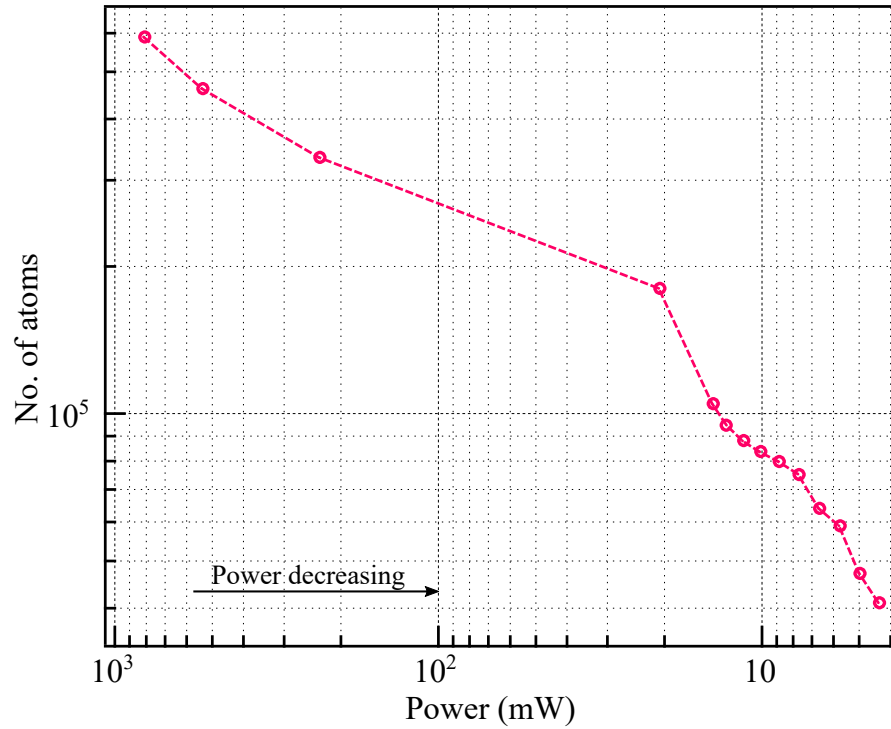


Figure 3.30: Loss of atom no during evaporative cooling: Number of atoms remaining in the trap after different stages of evaporation. Symbols represent experimental data. Dashed line is guide to eye.

loss of atoms from the trap. In comparison, such noise is absent in optical traps. In our experiment, the number of atoms in BEC decays to half of the initial value in 15 seconds (see fig. 3.32).

3.11.4 Measurement of trapping frequency

It has been already discussed that trapping frequency of a dipole trap depends on the trap depth. Also the trapping potential formed by the crossed dipole trap is not harmonic. But, one can always expand the functional form of such potential around the minimum and find the trap frequency from the harmonic term. In order to experimentally measure the trapping frequencies, the trap depth are increased adiabatically by increasing the power in each beam upto 25 mW in 1 second. Using the shimmming coils, a sudden magnetic jerk is given to the atomic cloud over a period of 5 ms. After that, the atoms are kept in the trap for sometime and then all the traps are turned off. The horizontal and vertical positions of the atomic cloud is recorded after 10 ms time of flight. The

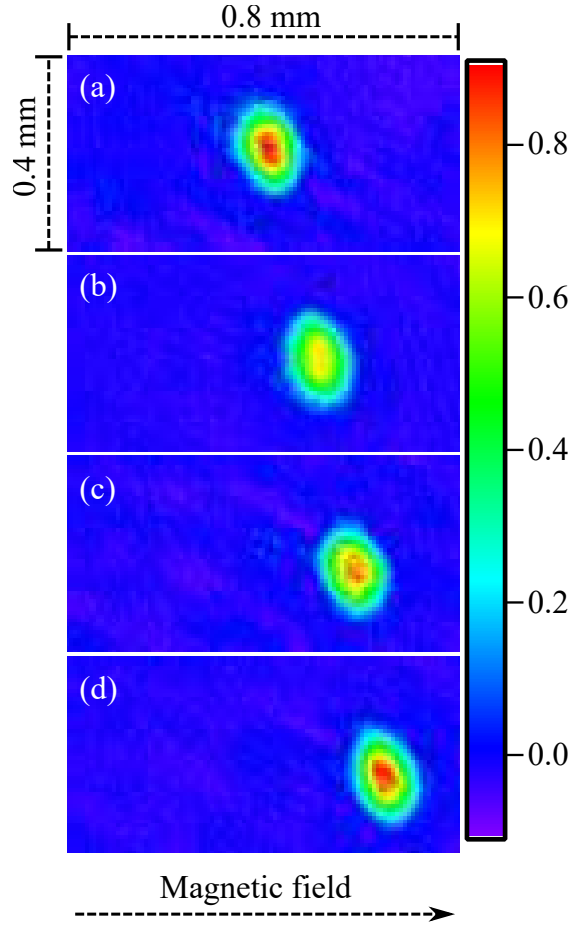


Figure 3.31: Spin purity of BEC: Deflection of the condensate in one particular direction with increasing field gradient from (a) to (d). The field of view of each frame is $0.8 \text{ mm} \times 0.4 \text{ mm}$. (a) represents the case with zero magnetic field.

results are shown in fig. 3.33 and fig. 3.34. Since the atoms perform a decayed simple harmonic motion in the trap, the time of flight image essentially maps the velocity of the atoms in trap. Let us assume the motion of the atoms in the trap can be described as:

$$x = Ae^{-bt} \sin(2\pi ft) \quad (3.11)$$

where, 'x' is the position, 'f' is the frequency of oscillation and 'b' is the damping parameter. Then the velocity can be given by:

$$v = \frac{dx}{dt} = Ae^{-bt} \left(2\pi f \cos(2\pi ft) - b \sin(2\pi ft) \right) \quad (3.12)$$

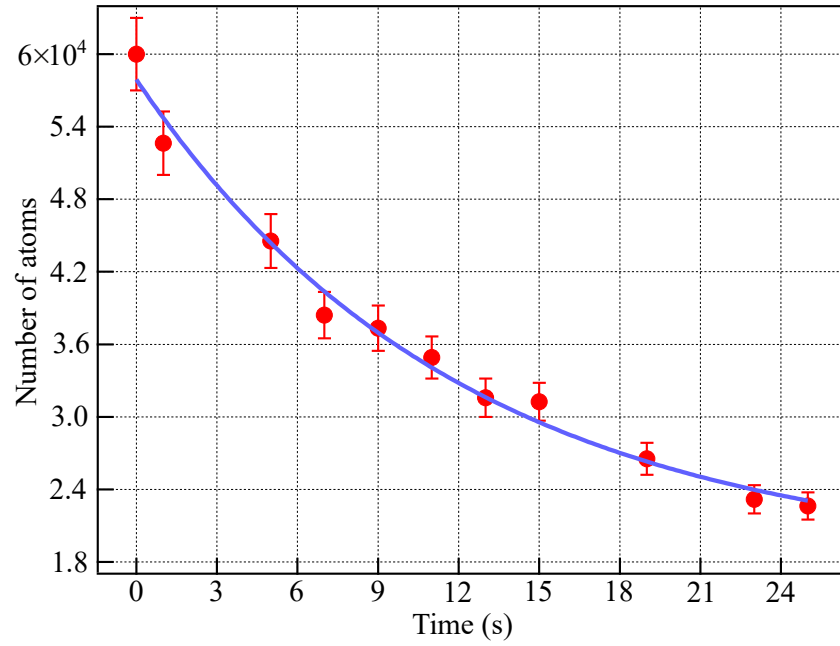


Figure 3.32: Lifetime of BEC: Decay of number of atoms in BEC with time. An exponential profile is fit to the experimentally measured data. The half-life of the atoms are found to be 15 seconds.

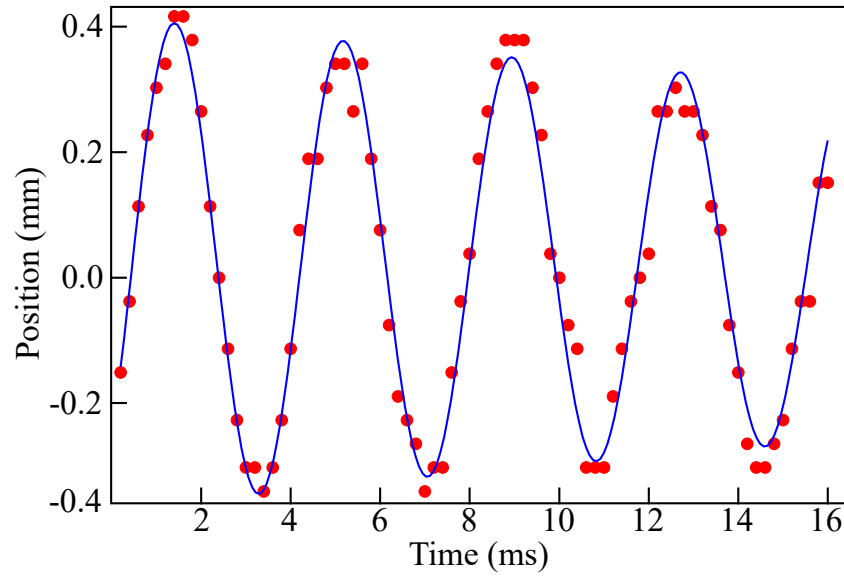


Figure 3.33: Measurement of trap frequency along horizontal direction: Oscillation of center of mass of the atomic cloud in horizontal direction.

Therefore, after time of flight t_e , the displacement can be given by :

$$y = vt_e = At_e e^{-bt} \left(2\pi f \cos(2\pi ft) - b \sin(2\pi ft) \right) \quad (3.13)$$

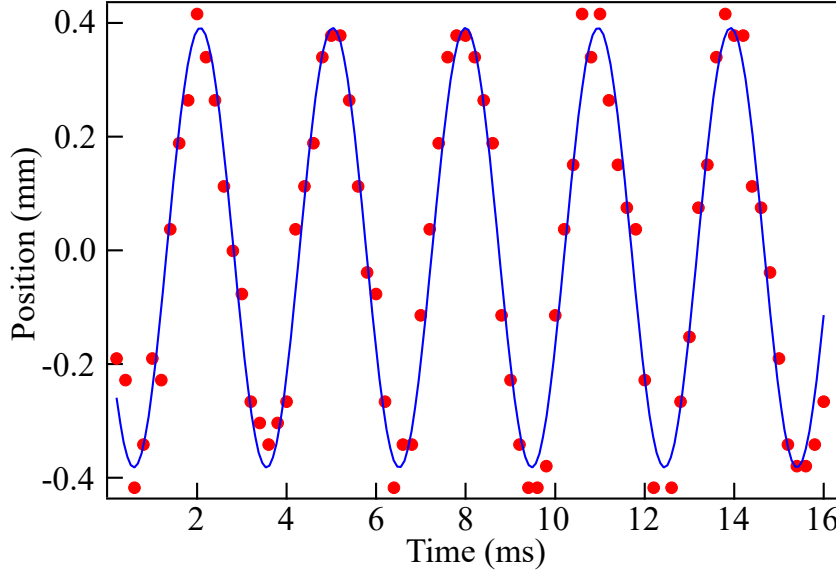


Figure 3.34: Measurement of trap frequency along vertical direction: Oscillation of center of mass of the atomic cloud in horizontal direction.

The experimental data represented in fig. 3.33 and fig. 3.34 are fit to Eq. 3.13. The trap frequencies in the horizontal and vertical direction are measured respectively $\omega_x = 2\pi \times 265$ Hz and $\omega_y = 2\pi \times 338$ Hz respectively.

3.12 Summary

To summarize, a detailed description of the experimental set up to produce BEC in a crossed-optical dipole trap is presented in this chapter. In order to provide a complete overview, experimental schematics and even real photograph of the experimental set up are shown in many cases. This chapter starts with a brief description of the design of the oven and its role in producing a well-collimated thermal beam of atoms. This section is followed by the description of laser cooling and trapping of atoms in MOT. Later, a detailed description of the design and characterization of a crossed optical dipole trap is provided. Also, the experimental sequence to transfer of atoms into the optical dipole trap is shown. After this section, a description of the evaporative cooling process in the dipole trap has been described. Finally, a brief description of the absorption imaging process and experimental results showing the phase transition from thermal atomic cloud to BEC are provided. In order to characterize the phase transition, variation of experimentally measured properties of the atomic cloud (e.g., number of atoms, mean

temperature, and phase space density) over the experimental sequence are also displayed at the end of the chapter.

Chapter 4

Demonstration of non-exponential decoherence and energy subdiffusion in AOKR system

This chapter has been published as:

“Non-exponential decoherence and subdiffusion in atom-optics kicked rotor”

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This work has been done in collaboration with the group of Prof. M. S. Santhanam. The numerical simulations presented in this chapter is done by Sanku Paul.

In this chapter, we describe the experimental demonstration of non-exponential decoherence manifested by a subdiffusive growth of energy in an AOKR system following a kick sequence that obeys Lévy statistics. The first section of this chapter is dedicated to define the problem and also to provide the current state of the art of the relevant research area. In the later sections, we will first describe the experimental schematic to realize AOKR followed by the experimental results of the standard kicked rotor to characterize the system. In the end, we will represent the experimental results of a demonstration of non-exponential decoherence in an AOKR system.

4.1 Definition of the problem

Quantum systems undergo decoherence due to unavoidable interaction with the environment. As a consequence, the macroscopic quantum superpositions are strongly suppressed, and classical behavior emerges from the quantum regime [176, 177]. The physics at the borderline of classical and quantum regimes are not well understood yet and continues to attract attention [177]. Quantum systems coupled with the environment lose their coherence exponentially fast [178]. This decoherence is of the form $\exp(-t/t_c)$, where t_c is the coherence time that depends on the system parameters and the strength of coupling to the environment. In many applications, e.g., in quantum computers and emerging quantum technologies [179], it is necessary to sustain quantum coherence for longer times which might be possible by tuning t_c .

The experimental developments in quantum control and reservoir engineering [180–182] in the last two decades have allowed direct observation of decoherence dynamics. The experiments using single ions in harmonic traps coupled to an engineered reservoir of random electric fields [183], dispersively coupled atom and field in a cavity [184] and localized quantum states of ultracold atoms kicked by noisy pulsed optical lattices [185, 186] have provided evidence for exponential decay of quantum coherence. These experiments allow tuning the coherence time t_c by changing a system parameter or the coupling to the environment.

An alternative approach [187] to prolong t_c is to explore non-exponential or a relatively slow coherence decay rates of the form $t^{-\alpha}$, where $\alpha > 0$ is the exponent. Such non-exponential decoherence has been theoretically shown for quantum kicked rotor [188, 189] and dissipative quantum two-level system [190] influenced by non-stationary noise. A novel feature in these is that they encompass a regime in which the mean waiting time diverges and the quantum system does not complete its transition to the classical regime. One challenge in the experimental realization of non-exponential decoherence is that it will require quantum systems to be sensitive to non-stationary noise within the coherence time scales. In this work, experimental realization of non-exponential decoherence in the atom-optics kicked rotor (AOKR) system is presented using an unusual form of non-stationary timing noise.

The standard AOKR system – cold atoms periodically pulsed (kicked) by the electromagnetic fields – corresponds to a fundamental model of Hamiltonian chaos [191]. In

this system, classically chaotic dynamics leads to unbounded, diffusive mean energy growth whereas its quantized version suppresses the energy growth due to destructive quantum interferences. The resulting dynamical localization (DL) is a phase coherent effect analogous to the Anderson localization in disordered periodic lattices [67, 192, 193] and was experimentally observed in one- [194], two- [195] and three-dimensions [196]. With its unambiguous and distinct signatures of energy growth in the classical and quantum regimes, AOKR is a suitable test-bed to study decoherence.

4.1.1 The basic model:

In AOKR, DL can be destroyed by inducing decoherence through (i) spontaneous emission of the atoms [185, 197], or (ii) addition of noise in the amplitude of the kicks [186] or in the periodicity of the kick [71] or in the phase of the periodic kicks [198]. In contrast to these approaches, we induce decoherence by suppressing kicks entirely at certain time instants dictated by the value of waiting time τ between successive kicks drawn from Lévy distribution $w(\tau) = \alpha \Gamma(\tau) \Gamma(\alpha + 1) / \Gamma(\tau + \alpha + 1)$, where α is the Lévy exponent [189]. For $0 < \alpha < 1$, this represents a non-stationary noise with diverging mean waiting time $\langle \tau \rangle$. We show through experiment and theory that this scenario leads to non-exponential decoherence rates and manifests as sub-diffusive quantum mean energy growth. The theoretical work related to this work was done in collaboration with the group of M. S. Santhanam. For details see Ref. [199]. Besides quantum decoherence, these results are relevant in the general context of transport and diffusion in chaotic quantum systems [200–202] and disordered nonlinear lattices [203].

The dimensionless Hamiltonian for AOKR, i.e., two-level atoms in a pulsed standing wave of near-resonant light, subjected to Lévy noise is given by

$$H = \frac{p^2}{2} + K \cos x \sum_{n=1}^N (1 - g_n) \phi_{sq}(t - n). \quad (4.1)$$

where g_n is a telegraph stochastic process that randomly switches between 0 and 1 [204]. If $g_n = 1$, then no kick acts at that instant and if $g_n = 0$ then a scaled kick strength of amplitude K acts at that time instant. The waiting time between the successive occurrences of 0 is taken from $w(\tau)$. The rectangular pulse $\phi_{sq}(\cdot)$ is of unit amplitude and K determines whether the dynamics is chaotic, regular or mixed. The noise-free limit,

with $g_n = 0$ for all n , represents the standard AOKR. In this limit, the classical system is integrable for $K = 0$. If $0 < K < 1$, the dynamics is mixed as the regular and chaotic regions coexist in phase space. When $K > 1$, the system becomes increasingly chaotic. For $K \approx 5.8$ used in this work, the classical phase space is largely chaotic over the experimentally accessible energy range.

4.2 Experimental schematic and procedure

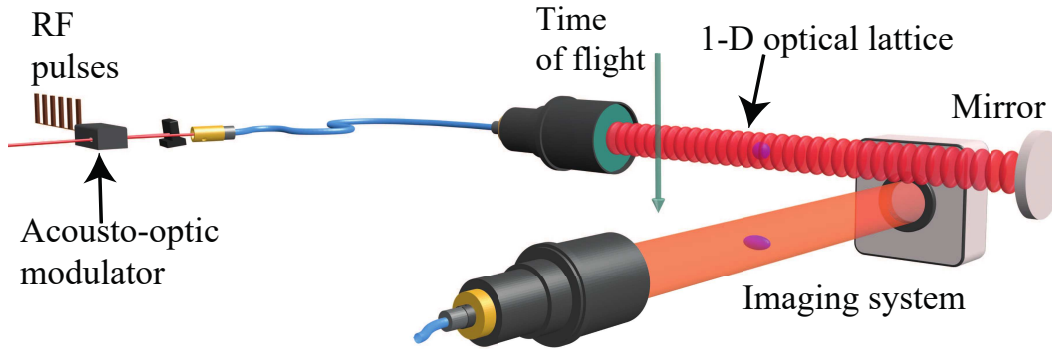


Figure 4.1: Experimental schematic: Experimental schematic shows the 1-D optical lattice and the absorption imaging system below the lattice. Off-resonant laser light after passing through an acousto-optic modulator, single-mode fibre and beam expansion creates a 1-D standing wave.

In order to realize a standard AOKR system experimentally, we first load the ^{87}Rb crossed optical dipole trap. These atoms are then cooled by forced evaporative cooling down to $\sim 3 \mu\text{K}$ in 3 seconds by lowering the intensity of the trapping beams. The final power in the trapping beams is 100 mW at this stage. We ensure that the temperature of the atomic cloud is much higher than the critical temperature and the momentum distribution of the atomic cloud is still governed by Maxwell-Boltzmann statistics. Therefore, the velocity distribution of the atomic cloud remains Gaussian. Since such experiments need a gaussian wavepacket of small width, we try to end the evaporation process at that temperature which is just higher than the critical temperature.

The Atom-Optic Kicked-Rotor (AOKR) system is simulated in the following manner: The periodic optical potential needed for the AOKR system is generated by a 1-D standing wave created from a laser operating near 780 nm and detuned by ~ -6.8 GHz from the $5S_{1/2} |F = 1\rangle \rightarrow 5P_{3/2} |F = 2\rangle$ transition. The laser beam is generated from

the same laser source that is used to trap atoms in the MOT. This laser beam is passed through an AOM and sent to the experiment chamber through single mode optical fibers. The intensity, pulse-width and duty cycle of the light pulses is controlled by AOMs that is driven by an RF amplifier and in turn controlled by a computer. The lattice is pulsed by switching the RF through a pulse generator that drives the digital switch for the RF. The 1-D standing wave is created in the horizontal plane, and it goes through the center of the intersection of the dipole trap. The beam-waist diameter of the lattice laser beam is $700\ \mu\text{m}$. The vertical position of this 1-d lattice beam is lower by less than the beam-waist diameter from the trap center. The minimum pulse width that can be achieved in our experiment is $\sim 40\ \text{ns}$. For the periodically kicked AOKR, the on-time of the pulse is $220\ \text{ns}$, and the off-time is $10.6\ \mu\text{s}$ such that the kicking period is $T = 11.02\ \mu\text{s}$. For the experimental parameters used in this work, the kick strength is $K \approx 5.8$ with 10% uncertainty. The scaled Planck's constant is $\hbar_s = 8\omega_r T$, where $\omega_r \approx 24\ \text{KHz}$ is the recoil frequency of the lattice beam. This yields $\hbar_s \approx 2.09$. The probe beam is made almost perpendicular to the 1-D optical lattice (fig. 4.1) to measure the expected momentum distribution of the cloud along the direction of the lattice. After the application of the pulses, the cloud is allowed to expand freely for $4\ \text{ms}$. Then an absorption image of the atomic cloud is taken from which the momentum distribution of the cloud is measured.

4.3 Standard AOKR

The first goal towards characterizing the system was to observe the dynamical localization, a typical characteristic of AOKR experiment. While the atomic cloud is falling freely, it is kicked by turning on the optical lattice momentarily. We perform the experiment for a sequence of 90 periodic kicks, which takes $\sim 1\ \text{ms}$. During this time period, the atomic cloud travels negligible distance along the direction of gravity. Since the lattice laser has a sufficiently large beam waist, the curvature of the Gaussian envelope of the lattice beam seen by the atomic cloud can be neglected safely. The on-time of the lattice being very small, the spontaneous emission per kick is estimated to be less than 1%.

Momentum distribution of the atomic cloud is calculated from the time-of-flight absorption image of the atomic cloud. Each momentum distribution is generated by averaging

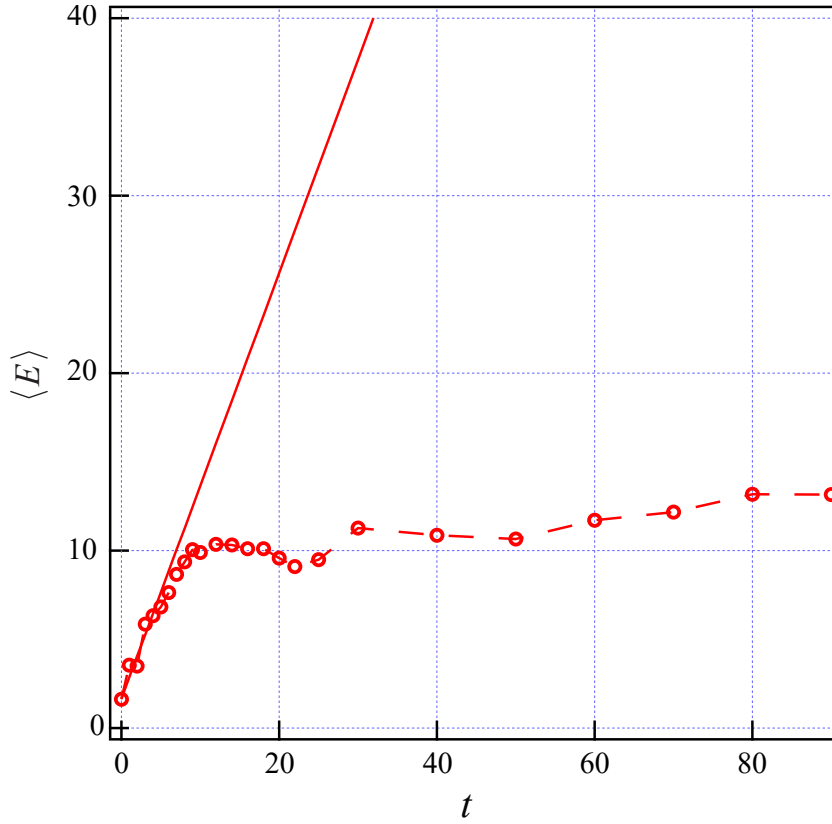


Figure 4.2: Dynamical localization: Suppression of normal diffusion of energy in a quantum kicked rotor. Solid line represents the classical energy growth. Symbols represent experimentally measured energy of a quantum kicked rotor. Dashed line is a guide to eye. The error in the estimation of energy is about $\pm 5\%$ of the measured value, which has not been shown in the graph.

over three or more similar images. In all the measurement, a set of ten images is taken. Images that have pronounced and random interference pattern due to the imaging system are discarded, and the remaining images are averaged. In the averaged image, a further summation of the pixel intensities is done along the direction perpendicular to the 1-D lattice. This results in a 1-D data of intensity vs. pixel position. We ensure that the baseline of the data is set to zero to avoid errors in calculating the average energy gained by the atoms in the direction of the 1-D optical lattice.

Fig. 4.2 represents the suppression of normal diffusion of energy in case of a periodic quantum kicked rotor. The solid line in fig. 4.2 represents the classical energy growth. Whereas, the experimental data shows that the mean energy of a quantum kicked rotor initially increases following the classical energy growth rate and then saturates. Fig. 4.3

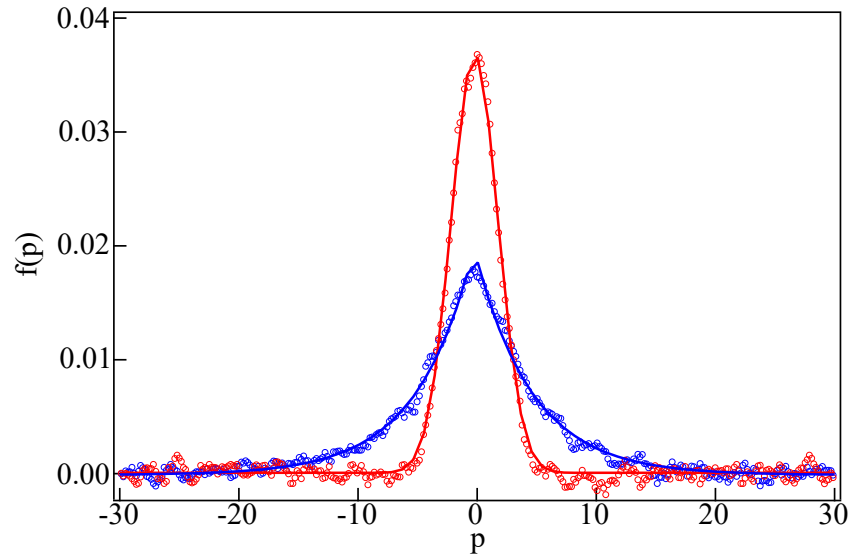


Figure 4.3: localization: Momentum distribution of atomic cloud (a) Red – without kicks, (b) Blue – after 90 kicks. For each case, symbols represent the experimental data. Initial momentum distribution shows a gaussian wavepacket. Solid red line represents a gaussian fit to the experimental data. Momentum distribution after 90 kicks shows exponential behavior. Solid blue line is an exponential fit to the data. The decay constant comes out to be 5 which is nearly equal to the localization length in momentum units.

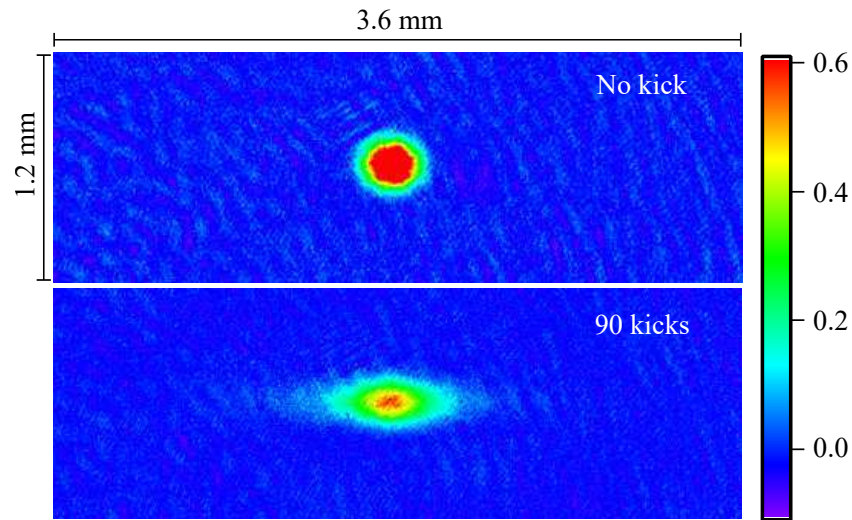


Figure 4.4: Momentum spread of atomic cloud with and without kicks: Absorption image of atomic cloud with and without kicks. Due to the kicking process, the width of the atomic cloud has increased along the direction of the kicking lattice. The extension of the atomic cloud perpendicular to the direction of lattice remains unchanged.

represents the normalized momentum distribution of the atomic cloud (a) without any kick, and (b) after 90 kicks. The initial wavepacket shows a Gaussian nature with small width $\sim 2\hbar k$. The momentum distribution of the atomic cloud after 90 kicks shows an exponential nature with characteristic decay constant ≈ 5 , which is nearly equal to the localization length of the energy growth as measured from fig. 4.2. Fig. 4.4 displays the absorption image of the atomic cloud without any kick and after 90 kicks. The atomic cloud shows a significant increase in width along the direction of the lattice, keeping the extension in the other direction unaffected.

4.4 Realization of a Lévy kicked rotor

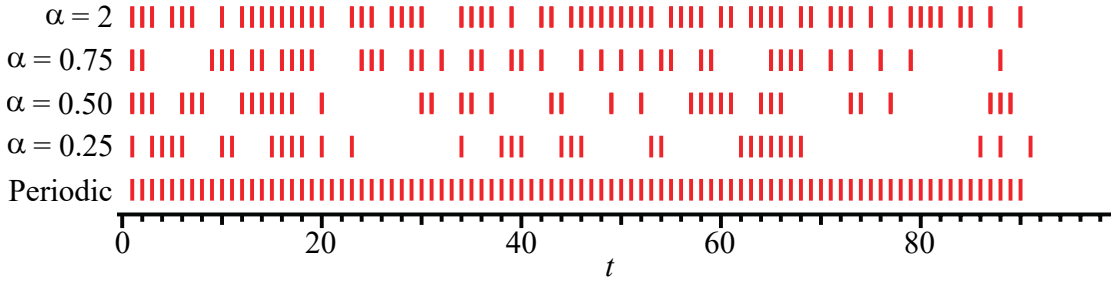


Figure 4.5: Kicks statistics: Schematic representation of appearance of kicks for different Lévy exponents in comparison to a periodically kicked standard kicked rotor.

4.4.1 Kick sequence:

Fig. 4.5 displays periodic kick sequence, a crucial ingredient necessary to maintain the quantum coherence and dynamical localization. We induce decoherence by using a kick sequence with waiting time, a random integral multiple of T , drawn from Lévy distribution $w(\tau)$ with exponent α . Asymptotically $w(\tau)$ decays as $\tau^{-\alpha-1}$. The number of kicks actually imparted in any finite time interval is a random variable, and its mean is dependent on α (see fig. 4.5). This represents the source of noise for the periodically kicked AOKR. When α is large, the number of kicks is large too. Properties of Lévy distribution relevant for our purposes are reviewed in Ref. [189]. As shown in fig. 4.1, the radio-frequency applied to the acousto-optic modulator (AOM) is pulsed in accordance with $w(\tau)$, thereby pulsing the lattice beam which acts on the cold atomic cloud during pulse on-times. The momentum distribution of the cloud is measured after a fixed time-of-flight by absorption imaging similar to a standard kicked rotor problem.

The experimental data reported in this work, for each value of α , are averaged over 14 realizations of Lévy distributed waiting times.

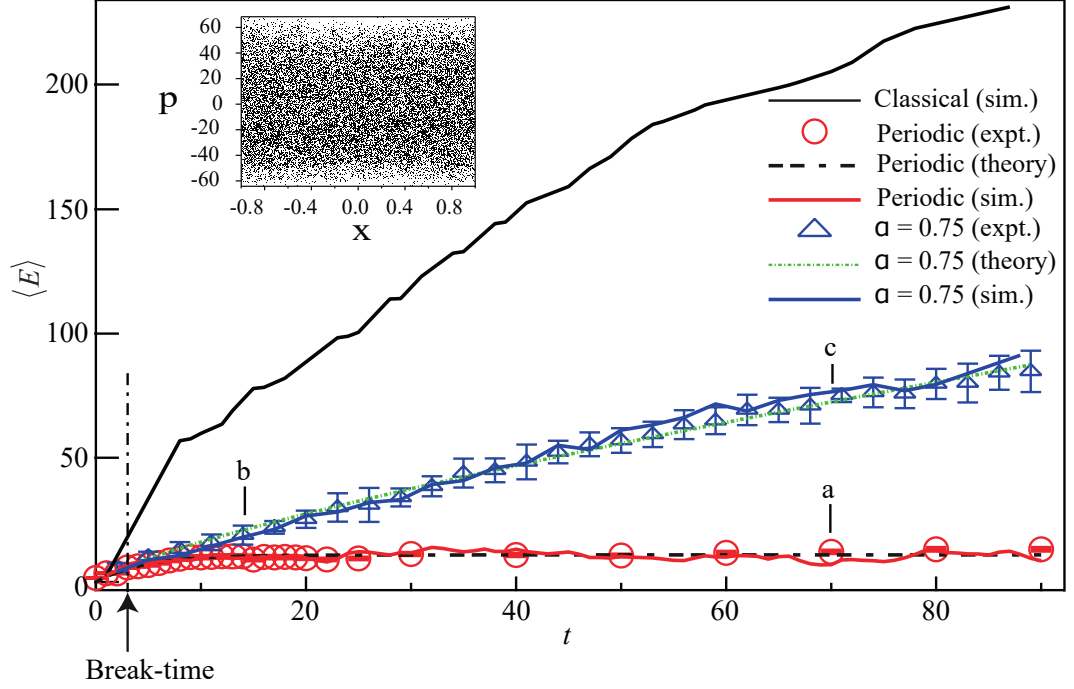


Figure 4.6: Effect of Lévy noise: Measured mean energy with $K \approx 5.8$ for periodic kick sequence (circles) and for $\alpha = 0.75$ (triangles). Dashed lines are the analytical results and solid lines are from numerical calculations. Error bars represent the standard deviation of energy measurement over 14 Lévy noise realizations. Black solid line is the simulated energy growth for classical kicked rotor with $\alpha = 0.75$. Inset shows the classical stroboscopic section for $\alpha = 0.75$. Momentum profiles corresponding to labels (a,b,c) are shown in Fig. refalpha75momprof. Vertical line marks the break-time t_b .

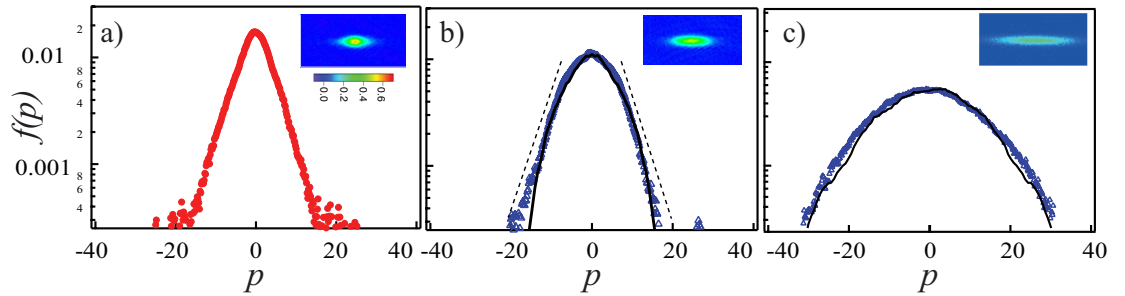


Figure 4.7: Momentum subdiffusion: (a,b,c) Experimentally obtained momentum profiles corresponding to labels (a,b,c), respectively, in fig. 4.6. The insets show the optical density of the absorption images from which momentum profiles have been extracted. Solid lines in (b,c) are obtained from quantum simulation. Dashed lines in (b) are shown as a guide to the eye.

4.4.2 Effect of Lévy noise:

To compare with the experimental results, we performed numerical simulations of the noisy AOKR system in Eq. 4.1, by replacing the square pulses with delta kicks and the parameters were closely matched with the experimental ones. The corresponding Floquet operator is repeatedly applied to an initial state $\psi(x, 0)$, taken to be of Gaussian form. The waiting time obtained from $w(\tau)$ with exponent α is incorporated into the Floquet operator. The simulation results represent an average over 900 realizations of Levy distributed waiting times.

In figs. 4.6 and 4.7, we illustrate the central results for the case $\alpha = 0.75$. The classical stroboscopic section (fig. 4.6) obtained by simulations for the AOKR in Eq. 4.1 with Lévy exponent $\alpha = 0.75$ shows that the phase space is largely chaotic for momenta up to $p = 60$. Corresponding energy growth displayed as solid (black) line is consistent with approximately quasi-linear growth proportional to t . On the other hand, the measured mean energy $\langle E \rangle$ for $\alpha = 0.75$ (see fig. 4.6), averaged over Lévy noise, displays sub-diffusive growth. This is in good agreement with the quantum simulations (blue, solid line) and also with the theoretical result (green, dashed line) $\langle E \rangle \approx A_0 t + A_1 t^\alpha$ (see Eq. 4.7). Regression performed on the observed $\langle E \rangle$ with this analytical form gives $A_0 = 0.123$ and $A_1 = 2.6(2)$ and $\alpha = 0.75(2)$. For stationary noise imposed on the kicking sequence, exponential decoherence would have resulted in the experimental data (blue triangles) having a similar trend as the classical mean energy growth (solid black line). At $\alpha = 0.75$, the noise is non-stationary and the signature of slower than exponential decoherence rate comes from this observation that AOKR with Lévy distributed inter-kick intervals display sub-diffusion rather than normal diffusion expected under conditions of complete decoherence. In a related theoretical work on the kicked rotor with amplitude noise applied with Lévy distributed waiting times [188, 189, 205], subdiffusion resulting from non-exponential decoherence was noted. We emphasize that the observed sub-diffusion in this experiment does not arise from spontaneous emission of photons (at less than 1% per kick) or the amplitude noise (about 1%), both of which are highly suppressed. This is confirmed by the fact that noise-free AOKR with periodic kicks displays energy saturation and DL (red circles in fig. 4.6) beyond break-time $t_b \approx 3$, in agreement with the established results [194, 206].

Fig. 4.7(a,b,c) displays, respectively, the measured momentum profile $f(p)$ corre-

sponding to the labels (a,b,c) indicated in fig. 4.6. For periodically kicked AOKR, $f(p)$ at $t = 70 \approx 20t_b$ obtained from the absorption image shown in fig. 4.7(a) displays expected exponential profile. The momentum distribution for $\alpha = 0.75$ at $t = 14 \approx 4t_b$ shown in fig. 4.7(b), extracted from the corresponding absorption image, maintains a reasonable exponential profile indicating that coherence is preserved. However, at $t = 70 \approx 20t_b$ the momentum distribution in fig. 4.7(c) is well approximated by a Gaussian profile implying a loss of coherence. The numerically simulated momentum profile is also in good agreement with the measured profiles.

4.4.3 A brief theoretical model:

The starting point for theoretical analysis is the modified delta-kicked rotor Hamiltonian of the form

$$H = \frac{p^2}{2} + K \cos x \sum_n (1 - g_n) \delta(t - n). \quad (4.2)$$

As before, we take the waiting time between successive kicks from Lévy distribution $w(\tau)$ characterised by exponent α . The corresponding Floquet operator is

$$\hat{F}(K) = e^{-\frac{i}{2\hbar_s} p^2} e^{-\frac{i}{\hbar_s} K \cos x} e^{-\frac{i}{\hbar_s} K'_n \cos x}, \quad (4.3)$$

where $K'_n = K g_n$. Following Ref. [207] and its extension to non-perturbative regime in Ref. [188, 189], we relate the decoherence factor to the survival probability of quasi-energy eigenstates $|s\rangle$ of the Floquet operator $\hat{U} = e^{-\frac{i}{2\hbar_s} p^2} e^{-\frac{i}{\hbar_s} K \cos x}$ to obtain an expression for $\langle p^2 \rangle(t)$. An important ingredient is the use of random phase approximation to estimate the contribution $\langle s' | e^{iK'_n \cos x / \hbar_s} | s \rangle$ when the quasi-energy state responds to noise by transiting from state $|s\rangle$ to $|s'\rangle$. For our noise scenario, we obtain the decoherence factor $D(t, 0)$ for the quantum evolution from $n = 0$ until $n = t$ to be,

$$D(t, 0) \sim e^{-(1-q^2)t} E_\alpha \left\{ (1 - q^2) \frac{\sin \pi \alpha}{\pi \alpha} t^\alpha \right\}, \quad (\alpha < 1), \quad (4.4)$$

$$D(t, 0) \sim e^{-(1-q^2)(1-1/\bar{\tau})t}, \quad (\alpha > 1), \quad (4.5)$$

where $\bar{\tau}$ is the mean waiting time between kicks, the function $q(K'_t/\hbar_s)$ is given by,

$$q(K'_t/\hbar_s) = 1 - \frac{K'^2}{2\hbar_s^2} \cos^2 x + \frac{K'^4}{4!\hbar_s^4} \cos^4 x + \dots, \quad (4.6)$$

and $E_\alpha(\cdot)$ is the Mittag-Leffler function [208]. The decoherence factor is non-exponential for $\alpha < 1$ and is exponential for $\alpha > 1$. Using the results in Eqs. 4.4-4.5, for $\alpha < 1$ and $t \gg 1$, we obtain the mean energy growth as

$$\langle E \rangle \approx A_0 t + A_1 t^\alpha, \quad (4.7)$$

and for $\alpha > 1$ and $t \gg 1$, we get

$$\langle E \rangle \approx A_2 t. \quad (4.8)$$

where the constants A_0 , A_1 and A_2 depend on the break-time t_b for the corresponding standard kicked rotor system, $\hbar_s$ and α . These results are in good agreement with the experimental data and simulations presented in figs. 4.6 and 4.8. In order to compare the analytical results for $\langle E \rangle$ with the experimental data, we use A_0 , A_1 and A_2 as fitting parameter. In Refs. [185, 186], sub-diffusive mean energy growth appears as the signature of decoherence due to spontaneous emission (SE) or amplitude noise (AN). We emphasize that the observed sub-diffusion in this work results from the non-exponential decohering effect of Lévy noise as opposed to SE or AN (which are in any case suppressed) because the best fit value of α is close to that used in generating kick sequences with Lévy noise, as predicted by Eqs. 4.7-4.8. Such physical relevance for the sub-diffusive growth exponent is absent in case of SE or AN induced decoherence.

4.4.4 Subdiffusive energy growth:

A broader picture of the results in fig. 4.8 displays the experimentally measured mean energy growth for values of Lévy exponent $\alpha = 0.25, 0.5, 0.75$ and 2.0. For comparison, this figure also shows the observed $\langle E \rangle$ for AOKR with random kicks or stationary timing noise (STN), i.e., the kicking period is $T + \delta$, where $-\Delta \leq \delta \leq \Delta$ is a uniformly distributed random variable with noise strength $\Delta \approx 20\%$. The quantum simulations performed for the AOKR system using experimental parameters agree with

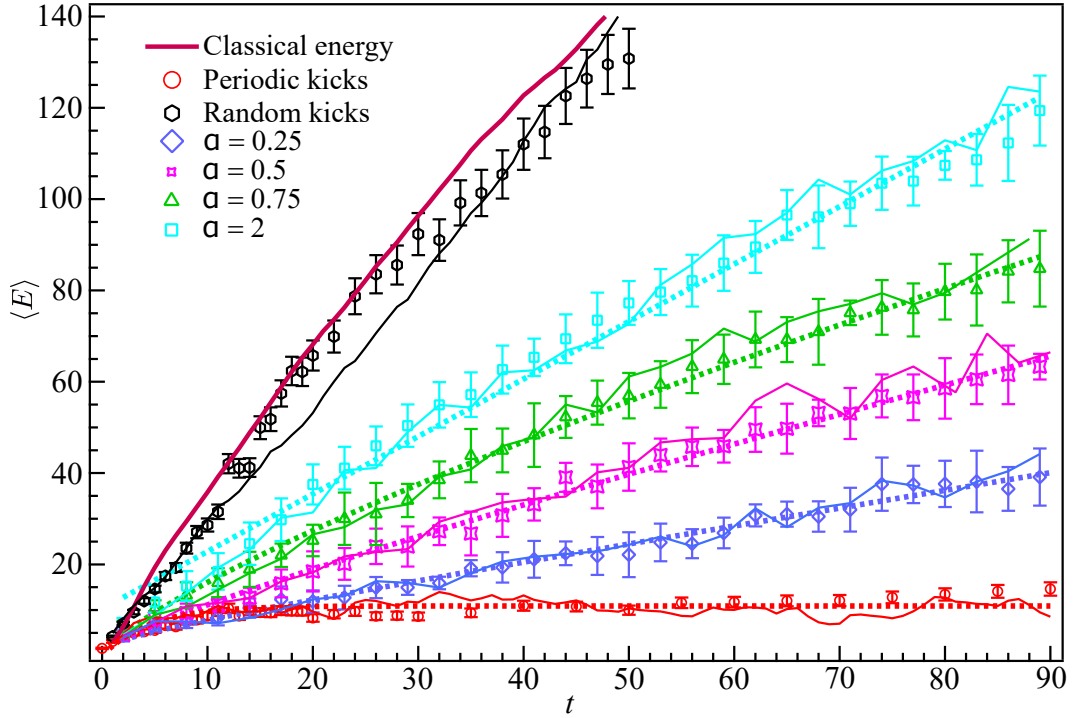


Figure 4.8: Subdiffusion in energy: Mean energy growth for various values of Lévy exponent α . Experimental data (symbols) matched against numerical simulations of AOKR (solid lines) and theoretical results (dotted lines) in Eqs.4.7-4.8.

the measurements. For $\alpha < 1$, subdiffusion is clearly visible, implying non-exponential decoherence. In contrast, for STN, approximately normal diffusion is indicative of exponential decoherence. In fig. 4.8, the experimental results are also compared with the analytical results in Eq. 4.7-4.8 and we obtain a good agreement between the two. In all these cases, subdiffusion of quantum mean energy growth is highly pronounced for $\alpha < 1$, the regime in which $\langle \tau \rangle$ diverges as well. This differs considerably from normal diffusion exhibited by AOKR with STN. The classical stroboscopic plots corresponding to parameters used in fig. 4.8 are found to have predominantly chaotic features for the energies accessed by these experiments. It is indeed surprising that the AOKR system 'feels' the Lévy distributed waiting times in few tens of kicks and clearly distinguishes it from the case of STN through different energy growth profiles.

The momentum profiles extracted from absorption images, shown in fig. 4.9, provide further evidence for slower decoherence. For $\alpha = 0.25$, fig. 4.9(a,b) display $f(p)$ at $t = 5$ and $t = 15$ respectively. Approximate linear decay from the peak at $p = 0$ (in

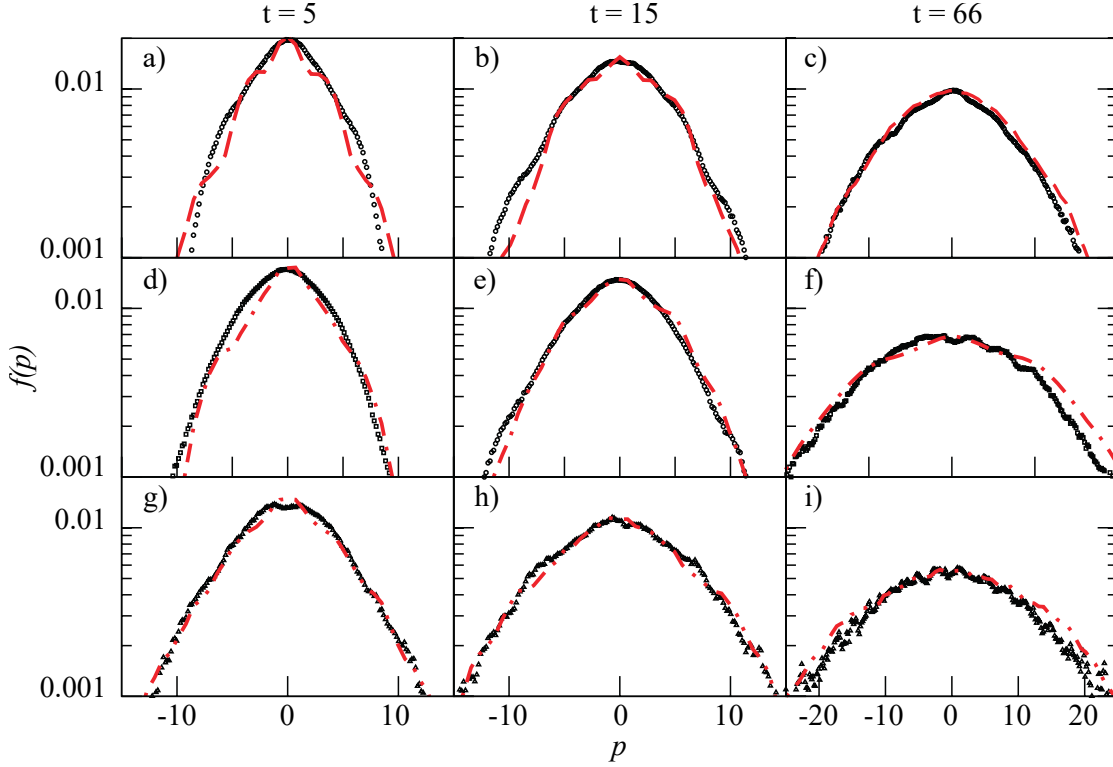


Figure 4.9: Evolution of momentum distribution: (a) Experimentally measured (symbol) and simulated (dashed line) momentum profiles at times $t = 5 \approx 1.5t_b$ (left panel), $t = 15 \approx 4t_b$ (middle) and $t = 66 \approx 20t_b$ (right) for Lévy exponents (a-c) $\alpha = 0.25$, (d-f) $\alpha = 0.5$ and (g-i) $\alpha = 2$.

semi-log axes) indicates that the coherences are preserved. Indeed, it is preserved for almost up to $t \approx 8t_b$, where t_b is the break-time. A similar conclusion can be drawn for $\alpha = 0.5$ case displayed in fig. 4.9(d-e). In all these cases, the simulation results represented as red dashed lines agree with the measured profiles. In contrast, if $\alpha = 2$, Gaussian nature of the profile is evident in fig. 4.9(g-i) and is marked by absence of coherence even for $t = 5$. For long times of order $t \geq 20t_b$, coherence of the kicked atomic cloud has decayed for all α as seen in fig. 4.9(c,f,i).

4.4.5 Non-exponential decoherence:

The measured momentum profiles provide further evidence for slower decoherence (fig. 4.9) but do not provide access to phase information. Instead, decay of occupation probability of zero momentum state $f(p = 0)$ provides an alternative approach to measure coherence [209]. We display measured $f(0)$ in fig. 4.10 for various noise scenarios.

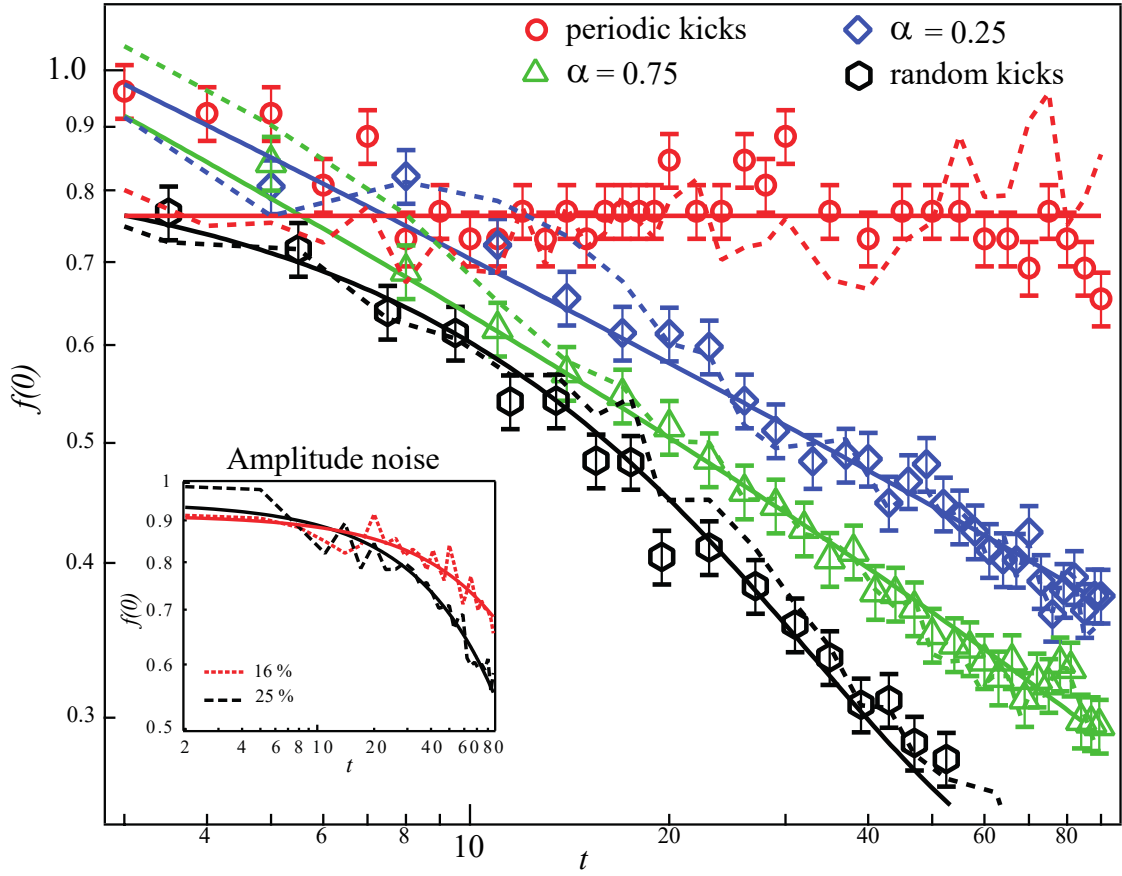


Figure 4.10: Decoherence: Experimentally measured (symbol) and numerically simulated (dashed line) occupation probability $f(p = 0)$ against time in log-log plot. (Inset) $f(0)$ for numerically simulated AOKR with stationary amplitude noise with noise strengths of 16% and 25%. All the solid lines (black, green, blue, red) are to guide the eye.

The decay of $f(0)$ for STN is well represented by an exponential form while for Lévy kicks with $\alpha < 1$ the decay is relatively slow, approximately linear in a log-log plot. In contrast, for noise-free periodically kicked AOKR, as expected, measured $f(0)$ lacks pronounced decay implying a negligible loss of coherence. The numerical AOKR simulations (dashed lines) display good agreement with the measurements. Finally, note that for the decoherence due to *stationary* amplitude noise in Ref. [186], numerically simulated $f(0)$ displays an exponential decay (inset of fig. 4.10) though associated with subdiffusive growth for $\langle E \rangle$. Thus, in spite of energy growth being qualitatively similar for stationary and non-stationary noise in AOKR, they can be distinguished by their qualitatively different coherence decays.

4.5 Conclusions

In summary, an experimental realization of slower than exponential decoherence in a noisy atom-optics kicked rotor system is presented in which the waiting times τ between subsequent kicks are chosen from Lévy distribution characterized by the exponent α . For $0 < \alpha < 1$, the noise induced in AOKR is nonstationary and the accompanying slower decoherence manifests as subdiffusive quantum mean energy growth. Remarkably, the AOKR system can 'feel' the non-stationary Lévy distributed kick sequence in a few tens of kicks. By tuning α , mean coherence time can be prolonged, and it is possible to access the regime of non-exponential decoherence rates in experiments. The analytical expressions obtained for subdiffusive energy growth are in good agreement with the experimental and simulation results of AOKR system.

Chapter 5

Kapitza-Dirac diffraction of an atom laser

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“Diffraction of an atom laser in the Raman-Nath regime”

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In this chapter, we describe the experimental demonstration of the Kapitza-Dirac diffraction of an atom laser. We will first define the problem. Later we discuss about the generation and characterization of the atom laser. In the end, we describe the experimental procedure to realize the diffraction of an atom laser.

5.1 Definition of the problem

A brief overview of the atom interferometers [210] is given in the first couple of chapters. However, despite the versatile applications and overwhelming success in inertial sensing and metrology, all such interferometers cannot be employed to monitor a slowly varying physical process continuously. Therefore, repeated measurements are done in order to study such processes. In case of realization of such interferometers, the duty cycle of the measurement is primarily limited by the wavepacket preparation time. The generation of a BEC in an experiment can take from 1 second to a few 10s of seconds

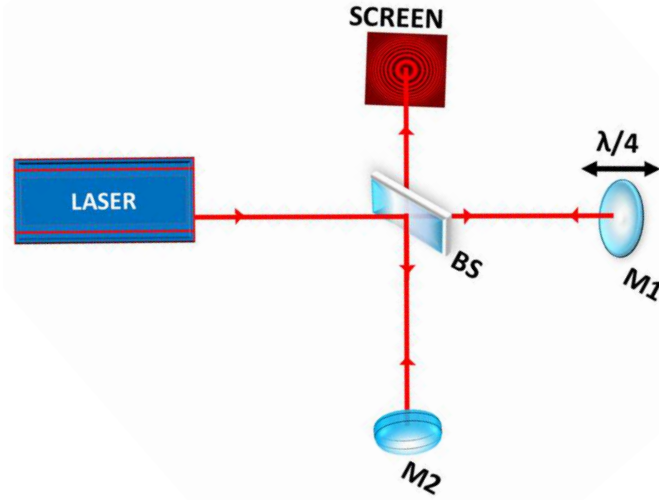


Figure 5.1: Michealson interferometer: Schematic of Michealson interferometer using optical laser. The movement of one mirror by quarter of the wavelength of the laser changes the dark fringe into a bright one.

depending upon the experimental procedure. One solution to this problem is to directly employ thermal atoms to reduce the time needed for cooling. However, in such cases, the resolution in the measurement of phase change is poor and reads $\sim 10^{-3}$. Therefore, as per the current state of the art of atom interferometers, there is no reported experiment where any physical process has been monitored continuously. There have been considerable efforts in the construction of a zero dead-time atom interferometer [211, 212] based on Raman pulses where an atomic cloud is always present in the interferometer region. Despite zero-dead time operation, the atomic sample does not interact with the diffraction grating continuously for such interferometers. Therefore, the measurements with such systems are also limited in bandwidth.

In order to find a better solution, let us look back at the basic setup of Michealson interferometer (fig. 5.1). In an atom interferometer, the roles of mirrors and beamsplitter are played by 1-D optical lattices. In case of an optical interferometer, laser plays a crucial role. The resolution of measurement of length offered by such interferometers is given by $\lambda/4$, λ is the wavelength of the laser. Also, the laser offers a high degree of coherence, collimation, and continuity. In an atom interferometer, the role of an optical laser is played by a BEC. A BEC has a thermal de Broglie wavelength comparable to the wavelength of a laser. A BEC possesses a high degree of coherence, but it lacks continuity and suffers from wavefront aberration [146]. An ideal replacement of an op-

tical laser can be thought of as a state of matter offering a similar coherence like BEC but continuous in nature. Such state of matter is known as atom laser. An atom laser is a well-collimated source (limited by natural divergence similar to an optical laser) of coherent matter wave outcoupled from a reservoir of BEC. An atom laser can be operated continuously until the reservoir is emptied. The flux and lifetime of an atom laser are limited by the number of atoms in a reservoir. Therefore, an atom laser can outperform a BEC in case of continuous operation of an atom interferometer. In this chapter, we discuss the demonstration of diffraction of an atom laser in the Raman-Nath regime, which is the first step towards setting up an atom interferometer using an atom laser.

5.2 Atom laser generation

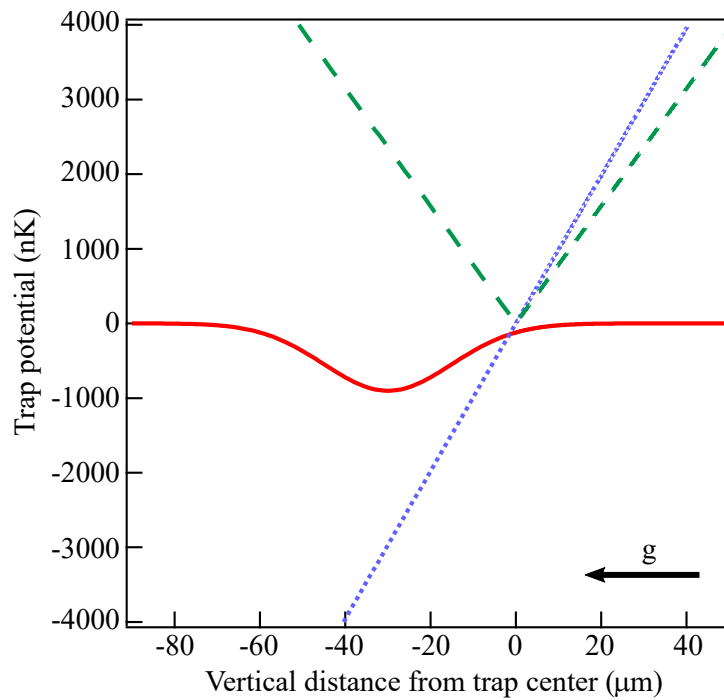


Figure 5.2: Individual trapping potentials at the end of evaporation: Individual trapping potential formed by optical dipole trap (solid red line), quadrupolar magnetic field (dashed green line) and gravity (dotted blue line) along the vertical direction. The trap profiles are numerically calculated with the parameters used at the end of the evaporation process. The dipole trap center is aligned about a beam-waist below the quadrupole trap center. Arrow shows the direction of gravity.

We choose the spilling method for the generation of our atom laser. Following the experimental sequence given in chapter 3, we produce a BEC in a quadrupole assisted

crossed optical dipole trap. At this stage, the trapping potential along the vertical direction formed by crossed optical dipole trap, magnetic trap, and gravitational field are shown in fig. 5.2. The total power in the two laser beams of crossed optical dipole trap is 10 mW at this stage. The combined potential formed by these three force-fields is given in fig. 5.3. The final potential shows an effective trap depth of ~ 220 nK, which is sufficient to hold the BEC. In order to outcouple a steady flow of coherent atoms, the powers in the crossed optical dipole trap beams are lowered adiabatically over 800 ms. The adiabatic decrease in the trap depth with reducing power in dipole trap beams are shown in fig. 5.3.

Due to the lowering of the trap depth, the atoms start spilling out of the trap. Since

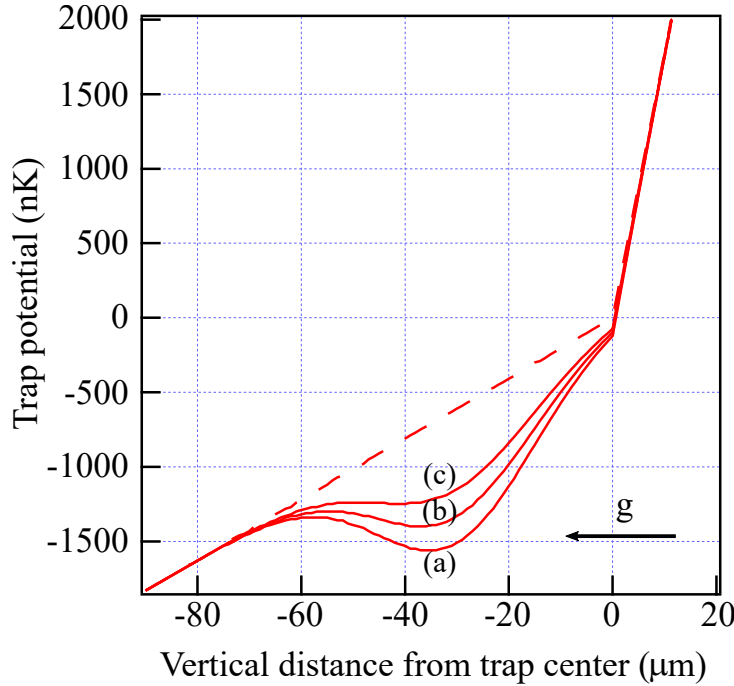


Figure 5.3: Adiabatic change in trap depth of combined potential: Change of combined potential with an adiabatic decrease in the total power of two crossed dipole trap laser beam: (a) 10 mW, (b) 8 mW, and (c) 6 mW. The effective trap depths are (a) 220 nK, (b) 100 nK, and (c) 10 nK. The dashed line shows the potential without the dipole trap. Arrow shows the direction of gravity.

the process of reducing the trap depth is adiabatic, and the trap is completely filled with atoms, a considerable amount of tunneling is also expected. The contribution of spilling and tunneling in the outcoupled flux could not be characterized. In order to ensure that a significant amount of tunneling occurs during the process, instead of reducing the trap

depth to zero, it is lowered by a certain amount and kept constant. The loss of atoms are monitored at this point and is compared with the loss of atom when BEC is kept in an optical potential with sufficient depth. The result is summarized in fig. 5.4. The normal decay of atoms from the trap with sufficiently large trap depth (trap depth much more than the temperature of BEC) is shown in fig. 5.4 (a). The solid line indicates an exponential fit to the experimental data. The decay constant found from the fit is ~ 12 s. Whereas, as the bottom is lowered and kept just above the temperature of the atomic cloud, initial atom loss shows faster decay due to tunneling. The decay constant is found to be ~ 189 ms which is very small compared to the normal trap loss. After the initial loss, when the temperature of the remaining atoms again becomes smaller compared to the set depth of trap, the decay process slows down, leading to a higher decay constant ~ 10 s as shown in fig. 5.4 (b).

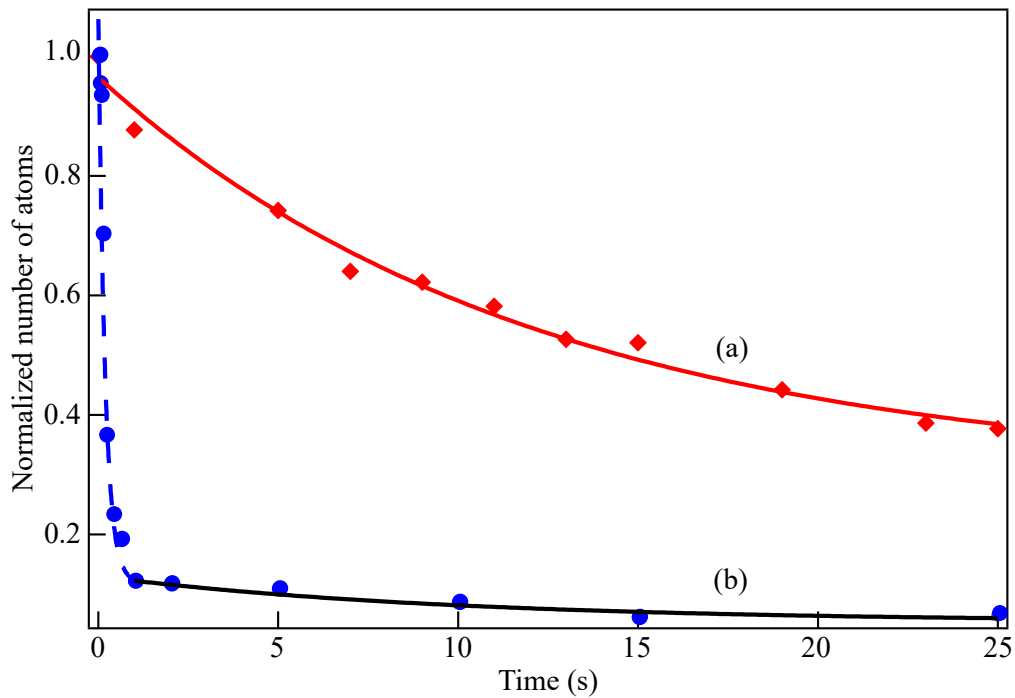


Figure 5.4: Characterization of tunneling: (a) Loss of atoms from the trap with trap depth much higher than the temperature than BEC. The solid line shows exponential fit to the experimental data. The decay constant is found to be ~ 12 s. (b) Loss of atoms from the trap with trap depth lowered to just equal to the temperature of atoms. An initial fast decay shows tunneling with decay constant calculated from exponential fit as 189 ms. The later slowed decay shows natural loss of atoms with a decay constant ~ 10 s.

5.3 Deceleration of atom laser

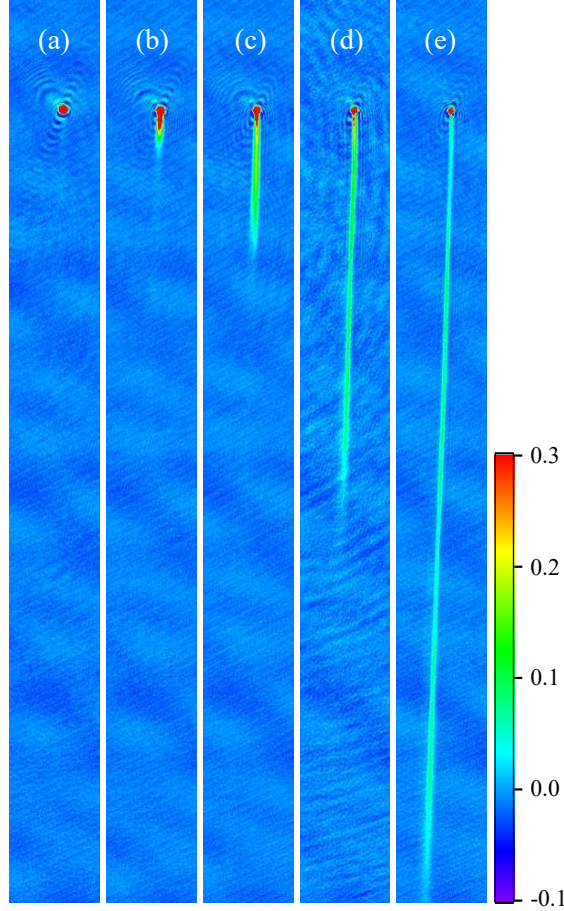


Figure 5.5: Acceleration of atom laser: Absorption images of atom laser at different time (t) after the start of outcoupling: (a) $t = 0$ ms, (b) $t = 10$ ms, (c) $t = 17$ ms, (d) $t = 30$ ms, and (e) $t = 40$ ms. The field of view of each image is $6 \text{ mm} \times 1.2 \text{ mm}$. Color scale represents optical density.

It is very clear from fig. 5.2 that the trapping potentials formed by gravity and magnetic field oppose each other below the trap center. The gradient of the potential formed by the magnetic field is less than the gradient of gravitational potential by a small amount. Therefore effective acceleration along the direction of gravity is calculated to be $\approx g/5$, where, ‘ g ’ is the gravitational acceleration. This shows that the atom laser slows down sufficiently making it apt for further manipulation. Fig. 5.5 displays a series of absorption images of an atom laser at different time after the start of the outcoupling process. The effective acceleration measured experimentally is $\approx g/2$.

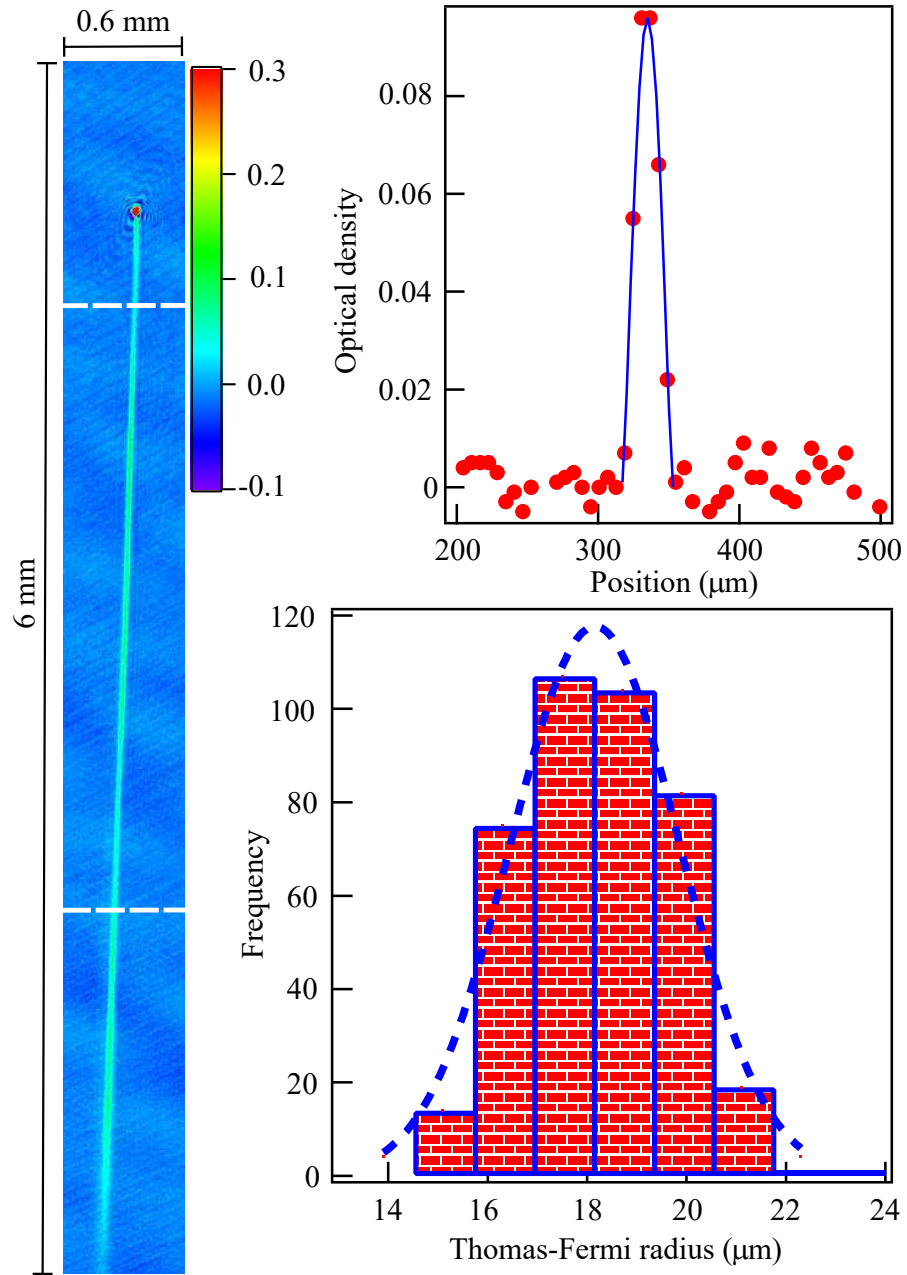


Figure 5.6: Collimation of atom laser: Left panel: a typical absorption image of an atom laser. Two horizontal dashed white lines are drawn to mark the region from which many transverse cross-sections of the atom laser have been selected at arbitrary places, and Thomas-Fermi distribution is fit to these profiles to measure the transverse spread of the atom laser. Color scale represents optical density. Right upper panel: Shows a typical fit to Thomas-Fermi distribution to the cross-section of an atom laser. Right bottom panel: A histogram showing the variation of Thomas-Fermi radius of an atom laser over the selected region. The dashed line is a Gaussian fit to the histogram with mean $18.1 \mu\text{m}$ and standard deviation of $1.7 \mu\text{m}$.

5.4 Characterization of atom laser

5.4.1 collimation

A crucial parameter to characterize an atom laser is its divergence. Optical laser suffers from natural divergence. Therefore, it will be a good practice to study the collimation of the atom laser. The divergence of an atom laser depends on several facts: (a) the geometrical shape of the trap, (b) the outcoupling mechanism, and (c) momentum spread of the BEC which acts as a reservoir. We consider a typical absorption image of an atom laser to find out its divergence (see fig. 5.6). We choose transverse cross-sections of the atom laser at random positions of the atom laser. Each such cross-section is fit to Thomas-Fermi distribution to find out the characteristic radius of the atom laser. The result of such 400 fits is represented in a histogram in fig. 5.6. The mean and standard deviation of the distribution, as measured by a Gaussian fit, are $18.1 \mu\text{m}$ and $1.7 \mu\text{m}$ respectively. This shows $< 10\%$ variation of the width of the atom laser over $\sim 4 \text{ mm}$ distance, implying high collimation of the atom laser beam.

5.4.2 Flux and lifetime

Flux can be defined as the number of particles passing through an area per unit time. In the case of an atom laser, the number of outcoupled atoms per unit time gives the estimate of the flux. In our experiment, the flux of an atom laser can be controlled by controlling the speed of lowering the trap depth of the potential. We ramp down the power of two laser beams of optical dipole trap linearly. Fig. 5.7 summarizes the result realized for three distinct flux rate. In order to measure the rate of outcoupled atoms, we monitor the number of atoms left in the trap. The decrease in such number for three different flux rate is shown in fig. 5.7. The corresponding flux rates come out as (a) $1.39 \times 10^6 \text{ atoms/s}$, (b) $2.91 \times 10^5 \text{ atoms/s}$, and (c) $1.71 \times 10^5 \text{ atoms/s}$.

The atoms are outcoupled from a reservoir of BEC. For a given trap geometry, the number of atoms in a BEC is constant and finite. Therefore, a gradual outcoupling of atoms results in emptying the trap. The time till an atom laser can be outcoupled from a reservoir is known as its lifetime. It is very obvious that more the outcoupling flux rate lesser is the lifetime of the atom laser. The lifetimes of the three outcoupling rates shown in fig. 5.7 are (a) 50 ms, (b) 250 ms, and (c) 400 ms respectively. The right panel of fig.

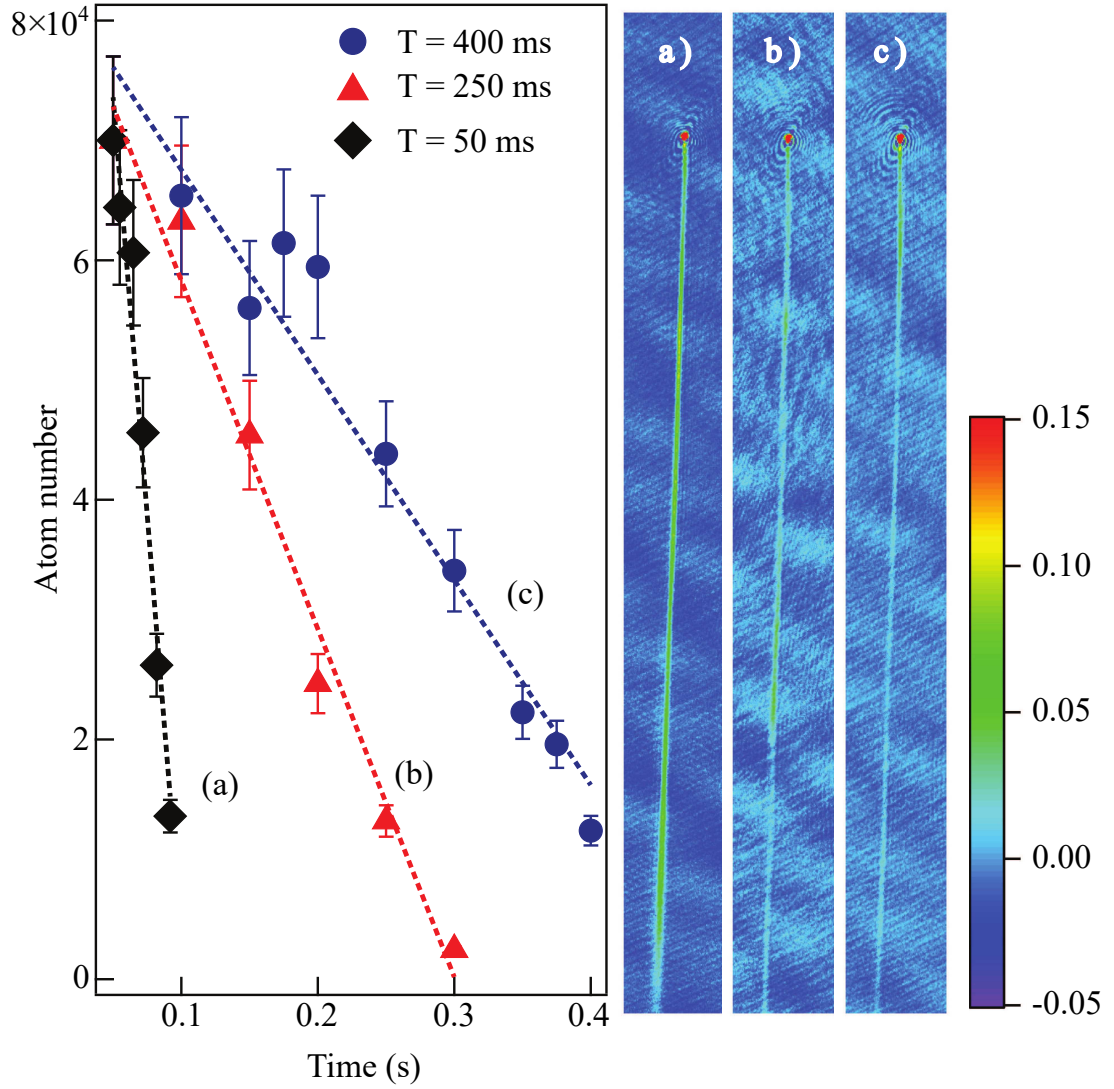


Figure 5.7: Lifetime and flux of atomlaser: Left panel: No. of atoms in the reservoir as a function of time for three different outcoupling rates. Atomic flux for the three cases are: (a) 1.39×10^6 atoms/s, (b) 2.91×10^5 atoms/s, (c) 1.71×10^5 atoms/s. Right panel: Images of the atom laser corresponding to the three flux rates in the left panel. These images are taken at (a) 40 ms, (b) 200 ms and (c) 300 ms after the start of outcoupling. Color scale represents optical density. Although in the experiment, the atom laser falls vertically along the axis of gravity, due to a relative tilt between the planes of the final imaging lens and the camera sensor, the absorption images of our atom laser look slanted towards left.

5.7 displays three absorption images of the atom laser with the flux mentioned above rate. The images have been taken at (a) 40 ms, (b) 200 ms and (c) 300 ms respectively after the start of the outcoupling process.

5.4.3 Diffraction of an atom laser

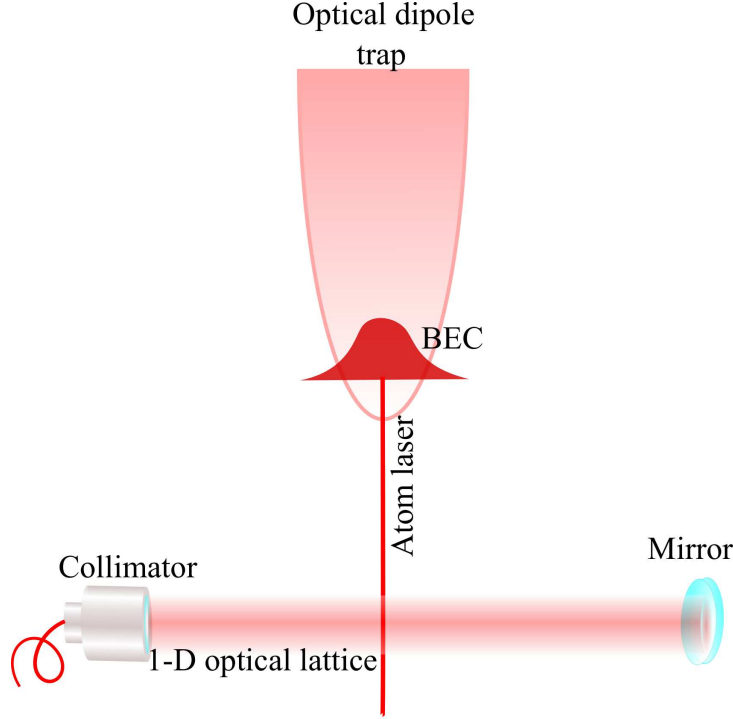


Figure 5.8: Schematic of experimental setup to realize the diffraction of an atom laser: Experimental schematic used for demonstrating the diffraction of an atom laser using a 1-D optical lattice and an atom laser.

In this section, we represent the experimental procedure to demonstrate the diffraction of an atom laser. The schematic of the setup to realize such diffraction is shown in fig. 5.8. It is shown in chapter 2 that a 1-D optical lattice works as a diffractor to the neutral atoms. In this experiment, we use a laser which is detuned by ~ -6.8 GHz from $|F = 1\rangle \rightarrow |F' = 2\rangle$ transition to set up the optical lattice. The lattice is made of a well-collimated beam of waist $\phi = 720 \mu\text{m}$. The lattice is set up 3 mm below the center of the trap. The switching of the lattice laser is controlled by an AOM. Once the atom laser is outcoupled and it reaches the position of the lattice beam, the lattice is turned on for a short time τ . During this time, the lattice interacts with the atom laser. Once the interaction is over, a time of free evolution T_{TOF} is given for the diffracted wavefronts to resolve. After the free evolution, an absorption image is taken using a probe laser beam nearly perpendicular to the lattice.

5.4.4 Near-field diffraction

There are two regimes of diffractions in general, (a) Fresnel diffraction and (b) Fraunhofer diffraction. Fresnel diffraction is known as the near-field diffraction pattern. Such diffraction can be observed when viewed very close to the diffraction grating. On contrary, Fraunhofer diffraction is known as the far-field diffraction pattern. Such diffraction pattern is observed at a distance far away from the diffraction grating. In order to characterize the length scale to distinguish between near-field and far-field diffraction, Augustin-Jean Fresnel introduced a dimensionless number F_n , known as Fresnel number. The Fresnel number is given by:

$$F_n = \frac{a^2}{L\lambda} \quad (5.1)$$

where a is the dimension of the aperture, L is the distance between the diffractor and screen and λ is the wavelength of light. A value of $F_n \gg 1$ corresponds to Fresnel diffraction. In case of an atom laser, the aperture is taken to be the periodicity of the grating; wavelength is taken as the de Broglie wavelength of the atom laser and distance is varied by changing the time of evolution after the interaction between the atom laser and the diffraction grating of the 1-D lattice. A series of absorption images of such diffraction is taken by varying the evolution time (see fig. 5.9). Fig. 5.9 shows the evolution of wavepackets after diffraction through the transition from near-field to far-field regimes. In each case, the corresponding Fresnel number has been calculated using the experimental parameters.

5.5 Realization of Kapitza-Dirac diffraction

In order to realize Kapitza-Dirac diffraction, we give a long evolution time after the interaction between the grating and the atom laser. Fig. 5.10 shows a typical time of flight absorption image of the diffraction of an atom laser for $\tau = 0.5 \mu s$, Where τ being the interaction time. A red Gaussian envelope is shown in fig. 5.10 to schematically represent the overlapping region of the atom laser with the diffraction grating formed by the lattice. Due to the Gaussian profile of the optical lattice beam, different sections of the atom laser see a different intensity along the vertical direction. So the strength of the diffraction is maximum at the center of the lattice beam and falls as we move away

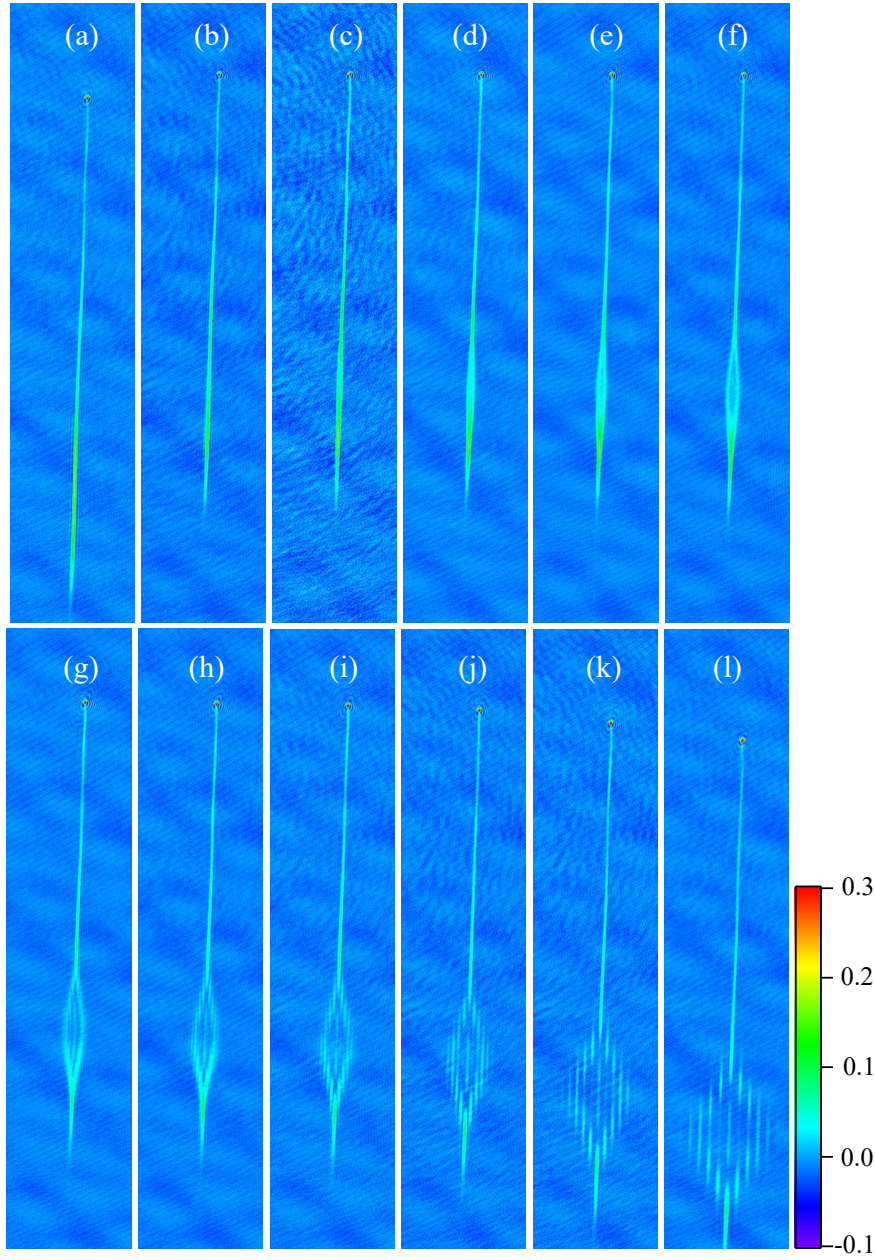


Figure 5.9: Near field and far field diffraction: Absorption images showing diffraction of an atom laser as a function of varying Fresnel number: (a) without grating, (b) $F_n = 110$, (c) $F_n = 2.9$, (d) $F_n = 0.464$, (e) $F_n = 0.181$, (f) $F_n = 0.116$, (g) $F_n = 0.029$, (h) $F_n = 0.0186$, (i) $F_n = 0.0129$, (j) $F_n = 0.0072$, (k) $F_n = 0.0032$, and (l) $F_n = 0.0018$. The field of view of each image is $6 \text{ mm} \times 1.2 \text{ mm}$. For (b) and (c), $F_n > 1$ implies Fresnel diffraction regime. Whereas, for (g) - (l), $F_n \ll 1$ implies Fraunhofer regime. The resolution of our imaging system is not enough to resolve the wavefronts in near-field regime. Color scale represents optical density.

from the center of the laser beam. The bottom panel in fig. 5.10 shows the normalized population in different momentum eigenstates due to the diffraction of an atom laser. To find out the population in the diffraction orders, we choose the section of the image where the effect of the grating can be observed. The size of the interaction region was found to be $\sim 2\omega_d$, which includes $\sim 95\%$ of the total power of the lattice beam. To find out the normalized population from the selected region, we do a column integration of the optical density and divide it by the maximum population. In the lower panel of fig. 5.10, the black circles represent the experimental data of the normalized population for the diffraction pattern shown in the upper panel of the same figure. Whereas, the solid lines display the normalized population calculated numerically.

A broader picture of the interaction of the atom laser with the light-grating has been displayed in fig. 5.11. In this figure, we have represented diffraction of an atom laser by varying the interaction time. For our experimental parameters, the recoil frequency is found to be $\omega_{rec} = 23.68$ KHz. Therefore, to satisfy the Raman-Nath condition for a thin grating, the interaction time has to be less than $42 \mu s$. Also, the condition for transition to classical oscillation regime can be found using Eq. 2.110. For the 1-D lattice used in our experiment, the peak value of traveling wave Rabi frequency is ~ 470 MHz. Hence, the upper limit of the interaction time to observe Kapitza-Dirac diffraction is given by $\tau_{osc}/4 \sim 0.9 \mu s$. The values of the interaction times chosen in our experiment are (a) $\tau = 0.15 \mu s$, (b) $\tau = 0.3 \mu s$, (c) $\tau = 0.4 \mu s$ and (d) $\tau = 0.5 \mu s$. Fig. 5.11 displays how the distribution of the population in different momentum eigenstates evolves with increasing value of the interaction time. For the maximum value of interaction time ($\tau = 0.5 \mu s$) chosen in this experiment, we have been able to transfer maximum momenta of $\pm 18\hbar k$ to the atoms. In fig. 5.11, the black circles represent the experimental data for the normalized population in different diffraction orders. The solid lines represent the numerically simulated population in diffracted orders for the values of θ approximately equal to that used in the experiment. The population in various diffraction orders are calculated numerically by convoluting the Thomas-Fermi distribution with Eq. 2.108. The average Thomas-Fermi radius of the atom laser used to demonstrate diffraction is found to be $\sim 23 \mu m$. The values of θ for which we found best fit to the diffracted population agrees with the values of θ used in the experiment within $\pm 10\%$ uncertainty.

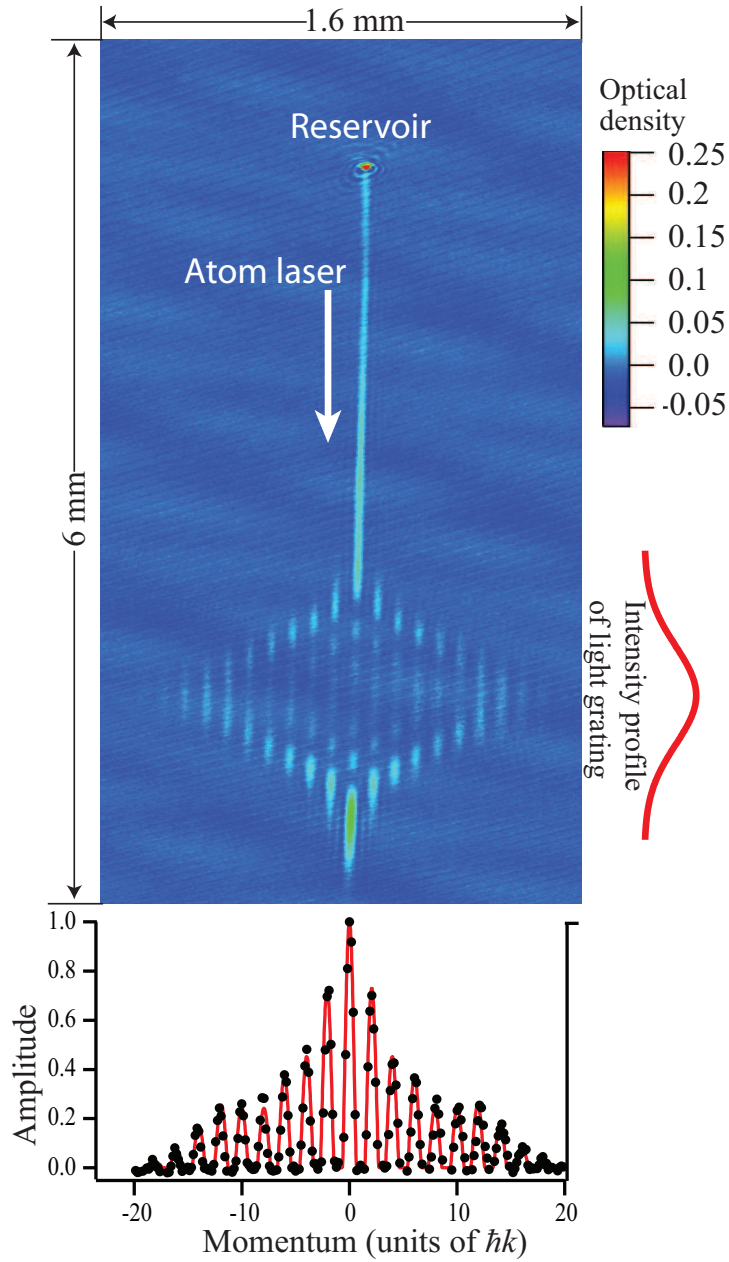


Figure 5.10: Typical Kapitza-Dirac diffraction of an atom laser: The image in the upper panel shows a typical time of flight absorption image of the atom laser diffraction. The red curve drawn beside the image is the Gaussian intensity profile of the standing wave of light that forms the diffraction grating with which the atom laser interacts. The lower panel shows the normalized population in different diffraction orders (up to $\pm 18 \hbar k$). Circles represent experimental data whereas the solid line shows the numerical simulation.

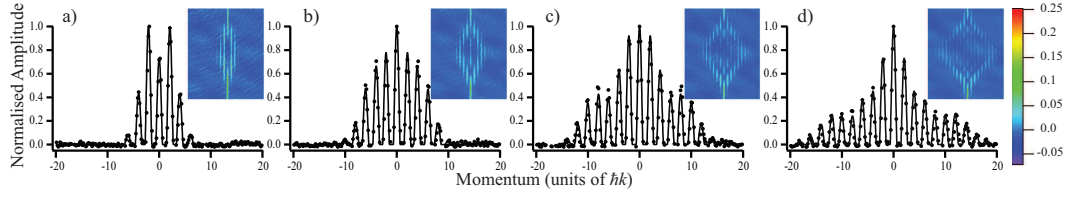


Figure 5.11: Kapitza-Dirac diffraction of an atom laser with varying strength: Normalized population in different diffraction orders with varying interaction time. (a) $\tau = 0.15 \mu s$, (b) $\tau = 0.3 \mu s$, (c) $\tau = 0.4 \mu s$ and (d) $\tau = 0.5 \mu s$, keeping the peak Rabi frequency and detuning constant. Black circles represent experimental data. Solid line represent numerical simulation for (a) $\theta = 2.64$, (b) $\theta = 4.29$, (c) $\theta = 6.62$, (d) $\theta = 8.23$. Insets show corresponding time of flight absorption images of the diffracted atom laser. The dimension of each image frame is $\sim 1.85 \text{ mm} \times 1.54 \text{ mm}$. Color scale represents the optical density.

5.6 Conclusions

In conclusion, we have demonstrated diffraction of a slowed down CW atom laser in Kapitza-Dirac limit. The atom laser can be outcoupled with well-controlled flux rate. In our experiment, the atom laser could be extracted up to 400 ms for the slowest out-coupling rate used. Since, for a constant outcoupling rate the time for which one can extract atom laser is proportional to the initial atom number in the reservoir, improving the number of atoms in the BEC will essentially allow us to use the atom laser with a high flux for a longer time. We have also numerically simulated the population distribution in the diffracted orders. Our experimental results are in good agreement with the numerical simulations which includes the effect of the Gaussian intensity profile of the grating lattice. The extension of this work is to realize a three-grating interferometer using atom laser which can be used for quasi-continuous probing of physical quantities with high bandwidth and high data rate. Since the population in the different momentum states can be controlled precisely; such diffraction can also be used to construct an interferometer by selectively using two wavepackets with large and opposite momentum. In order to use the advantage of high flux and long-time operation simultaneously, we propose to produce BEC on an atom chip with a fast production rate similar to that shown in Refs. [213–215] and have multiple reservoirs on a chip that operates in a synchronized manner to produce an effective perpetual source of atom lasers that can be integrated with this diffraction effect for realizing a continuously operable interferometer.

Chapter 6

Conclusion and outlook

6.1 Summary

The principal aim of this thesis was to study interesting physical systems using interaction between atoms and 1-D optical lattice. The work can be divided broadly into three parts. The first stage involves setting up an experimental platform to achieve ^{87}Rb BEC starting with a laser-cooled atomic cloud. To serve this purpose, a crossed optical dipole trap was set up. The trap is capable of cooling atoms down to quantum degeneracy within ~ 6 seconds. Evaporative cooling of atoms in the trap produces a BEC with 6×10^4 number of atoms and a temperature of ≈ 68 nK. Characterization of the phase transition was presented in this context.

The first problem that has been realized using this system is an atom optics kicked rotor. In order to simulate a kicked rotor, we needed a Gaussian wave packet which is given energy by means of sequential kicks via a potential. A cold thermal cloud above the critical temperature was used to serve the purpose of a Gaussian wave packet. The population distribution of a cold atomic sample in momentum space is governed by Maxwell-Boltzmann statistics. Therefore, in a momentum-space picture, such a cold atomic cloud can imitate the behavior of a Gaussian wave packet. A 1-D optical lattice was set up just below the center of the optical dipole trap in order to provide energy to the atomic cloud. The optical lattice was switched on sequentially to make the atomic cloud interact with it and hence gain momentum in the direction of the lattice. The lattice is turned on periodically by using an AOM. This led us to realize the quan-

tum mechanical model of a periodically kicked rotor. As a consequence, we observed saturation of energy and an exponential momentum distribution indicating the famous dynamical localization.

Later this system has been used to engineer an environment and the interaction of the system with it such that the system loses its coherence slower than an exponential decay. In order to demonstrate the non-exponential decoherence, we chose the off time between two consecutive kicks from a Lévy distribution. Under such non-stationary noisy condition, the system was found to see the statistics in the kick sequence and showed a sub-diffusive energy growth. A detailed study of the system showed that the subdiffusion in energy was a manifestation of non-exponential decay of coherence.

Decoherence is a fundamental physics problem of interest especially in the field of quantum information processing and quantum computing. A prolonged coherence is much desired in such systems. Therefore, our study of the demonstration of a slower decay of coherence is definitely a considerable contribution to the field. Also, the results are helpful in understanding the dynamics of the chaotic system and transport phenomenon in the non-linear regime.

Another problem realized using the same experimental setup was to demonstrate the diffraction of an atom laser. The first step to realize the problem was to generate an atom laser. Using the combined potential formed by the crossed optical dipole trap, quadrupolar magnetic trap, and the gravitational field, a slow atom laser is extracted. In our experiment, the atom laser could be extracted for a maximum of ~ 400 ms continuously. In order to use this atom laser for further manipulation, we characterize the flux and the collimation of the atom laser. It has been shown that by controlling the speed of lowering the trap depth can control the flux of the atom laser. In the end, we align the 1-D optical lattice 3 mm below the trap center. When the atom laser is outcoupled, it interacts with the optical lattice. It is shown that the depending upon the interaction time, the strength of the diffraction is modulated and diffraction of atom laser up to momentum $\pm 18 \hbar k$ has been realized. The population distribution in different momentum eigenstates is shown to follow the diffraction pattern in the Kapitza-Dirac limit.

6.2 Future outlook:

Both the problems studied in this thesis work are of fundamental research interest. One of the problems addresses a fundamental quantum mechanics problem, namely decoherence. Whereas, the other provides a new platform for building a continuously operable atom interferometer. Below, we list the possible extension of the work presented in this thesis.

- **Realization of a continuously operable atom interferometer:**

An interferometer can be constructed using three gratings. Such an interferometer is capable of inertial sensing and measuring fundamental constants. The advantage that this work can provide is the idea of continuous operation of the interferometer. An extended version of this work will be to produce many BEC sequentially and using an atom chip-based trap to extract an atom laser, in such a manner that there is a quasi-continuous stream of atom lasers. In this way, an effective prolonged CW atom laser can be realized.

- **Non-monotonous energy growth in kicked rotor**

In this thesis work, we have studied energy diffusion of a Lévy kicked rotor up to a value of the Lévy exponent $\alpha = 4$. One can notice that, in the limit of $\alpha \rightarrow \infty$, the distribution of kicks resembles the periodic kicks. Hence, we can expect the existence of some value of α for which the system will show maximum diffusion. It will be interesting to repeat the experiment with higher values of α and demonstrate a non-monotonous energy diffusion in the system.

- **Spin orbit coupling out of chaos:**

AOKR is an apt test-bed to study Anderson localization. Exploring the symmetry breaking in the temporal domain, one can simulate physics of higher dimension using the system. Also, a spinor BEC can be employed to interact with such lattice in 2-D, which can mimic the Hamiltonian of spin-orbit coupling potential. Although, such a system will not be able to possess all the features of such interaction, but will definitely provide a simplified platform to study such problems.

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