MATHZINE

"Mathematics is
the queen of all sciences."

2023
3rd Edition

FROM THE DESIGNERS

As the Mathzine returns for its 3rd edition after a 3 year hiatus, the cover had to be special. We decided to use text to image generators and settled on Adobe Firefly. Our captivating cover artwork celebrates the majestic influence of mathematics, portraying it as the undeniable queen that guides and shapes the very fabric of science. As you flip through these pages, you will step into a mesmerizing realm where intricate equations dance like royalty, and numerical symphonies rule the universe.

Maths Club presents

MATHZINE

2023 Edition

From the Editor's desk

The 3rd edition of Mathzine, Maths Club's Annual Magazine, brings you a unique blend of Mathematics, Science, Art and life experiences. It's a cumulative collection of works of bright minds in cutting-edge research, along with entertaining recreational content and golden nuggets of learnings one can draw from the day-to-day life of mathematicians at IISER Pune.

Mathzine started in the year 2018. After its 2019 edition, it had an unforeseen three year break due to the pandemic, but this year Mathzine blooms afresh once again! Among other things, the 2023 edition highlights the Maths Club events that took place during the academic year 2022-23.

Mathzine has been woven keeping in mind a general audience from various backgrounds; but at the same time, designed to have a special appeal to the eyes of Mathematics enthusiasts. It narrates a unique tapestry, providing a means to voice out the ideas of individuals, bringing out their creativity, and spotlighting Mathematics at IISER Pune.

We are grateful to the volunteers who till the last minute painstakingly edited and designed the magazine.

Also, as a bonus, celebrating the new edition of Mathzine, we have interesting Easter eggs embedded in the magazine for the readers to discover. The first few individuals to find these and write to the Maths Club will get a chance to grab amazing prizes!

Happy Reading!

– Mathzine Editorial Team

Pillars of support

Maths Club extends its deepest gratitude to the Mathematics Department of IISER Pune, with special thanks to the Chair of Mathematics Department Dr Mainak Poddar and other members who facilitated the publication of Mathzine. We also wish to thank all the faculties who took time out of their busy schedules and contributed to Mathzine by giving us an opportunity to interview them. Also we would like to thank the entire team Mathzine for their diligent work. Last but not the least, we would like to thank all the Mathzine contributors who decorated the pages with words of knowledge, fun and made Mathzine a happy read!

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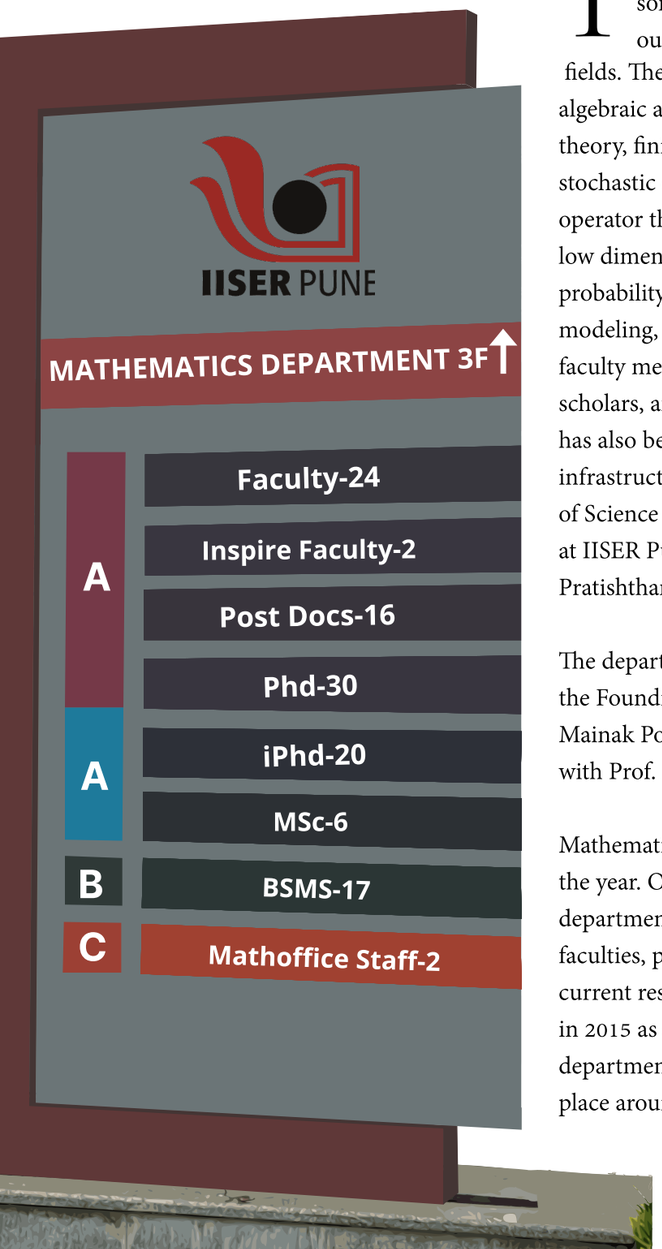
about the

Math Department

-Vivek T..A
-Gurjot Singh
-Jigyas Baruah



The mathematics department of IISER Pune has some of the most renowned mathematicians of our country, each with expertise in their respective fields. Their research spans a wide range of topics like algebraic and analytic number theory, representation theory, finite groups, algebraic K-theory, partial and stochastic differential equations, harmonic analysis, operator theory, algebraic geometry, differential geometry, low dimensional topology, cryptography, coding theory, probability theory, mathematical finance, statistical modeling, and more. The department currently has 25 faculty members, 16 postdoctoral fellows, 30 PhD scholars, and 20 Integrated PhD scholars. The department has also been awarded funds to expand research infrastructure under the FIST Program of the Department of Science and Technology. The mathematics department at IISER Pune also has an MoU with the Bhaskaracharya Pratishthana, Pune.

The department of was formed with Prof. A. Raghuram as the Founding Chair, and has been growing since. Prof. Mainak Poddar is currently the Chair of the department, with Prof. Anup Biswas as the Deputy Chair.

Mathematics department organizes several events round the year. One of the most prominent events of the department, the Mathematics Symposium is a place where faculties, post-doctorates, and students discuss their current research in mathematics. The symposium started in 2015 as a medium for people in the mathematics department to share what everyone is working on. It takes place around the time of International Day

Prof.Mainak Poddar
Professor and Chair, Mathematics

"I am very proud of the quality of work being done by the students, postdoctoral fellows and the faculty. Also, I'm very encouraged by the recognition being obtained, especially by the younger faculty members, like Swarna Jayanti fellowships and Associateship of the Indian Academy of Sciences, etc."

the two events have become almost synonymous with each other.
The latest symposium was hosted on 23rd and 24th March, 2023, where people of the mathematics department presented their ongoing research through talks in the Madhava Hall in the Main Building of the college campus. The discussions had a wide variety of topics, covering many aspects of mathematics, giving everyone in the symposium a chance to expand their horizons of knowledge.

IISER Pune also participated in The Indian Women and Mathematics (IWM) Annual Conference. The Indian Women and Mathematics (IWM) Annual Conferences aim to bring together women mathematicians, students, college and university teachers and early career researchers working at the frontiers of mathematics to exchange mathematical ideas and share their experiences. The conference was held at IISER Pune during December 27 to 29, 2022 with Prof. Anisa Chorwadwala, Assistant Professor in the Department of Mathematics, as the convener of the event. Distinguished speakers such as Prof. Nutan Limaye (IIT Bombay and IT University of Copenhagen) and Prof. Kavita Ramananan (Brown University) were guest speakers in this conference.
The mathematics department of IISER Pune has always been a source of great support for the Maths Club. The people in the department have engaged a lot in the club activities, be it through delivering talks in the midst of their already busy schedules, helping in inviting professors from outside for talks, or providing support in whatever way needed.

The club acknowledges that it would be very difficult for us to smoothly organize events in the grand scale that we have done without the support of the mathematics department, and is always grateful for such unending support.

We would like to take this opportunity to thank the entire mathematics department for their continuous support for the Maths Club throughout the year, and wish to continue this healthy relationship in the coming years as well.

"If you don't know history, then you don't know anything. You are a leaf that doesn't know it is part of a tree."

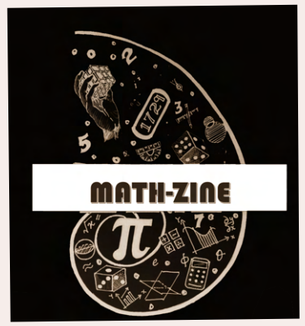
-Michael Crichton
Author and filmmaker

A brief history of Maths Club

You have been witnessing some of our events. But have you wondered how the club started off? Read on!

The club began by helping in the organization of one of its flagship events - celebrating π day, also known as mathematics day amongst the faculty. It was in January 2014 when the club was formed with the idea that it would help organize the annual Math Day celebration with support from the Math department and would also organize various other math related activities such as talks, exhibits and games, movie screenings, and any other event which presents the elegance of mathematics to people.

Math day celebration started in 2012 but that year and the year after, the Math department organized it. The club was inaugurated in March 2014 with Prof. Steven Spallone as the faculty coordinator of the club, and it simultaneously celebrated Math Day as its first event. It featured lots of new and fun activities such as the Treasure Hunt, Prime Number Bee, playing with hexaflexagons and various



mathematical games, Rubik's cube solving competitions, along with a skit and a Women in Math poster competition. The chief guest for the event was Prof. M. S. Narasimhan FRS (Fellow of Royal Society). Math Day 2015 introduced python programming competitions and merchandise that included t-shirts and coffee mugs. Math Day 2016 featured Dr. Harish Chandra's bust unveiling, a public lecture on his works by Prof. A Raghuram and a video lecture by Robert Langlands. Math day 2017 featured Integration Bee for the first time, in which integrals were hidden throughout LHC, and participants had to find and solve them.

Math day 2018 featured the release of the first edition of Mathzine – a mathematical magazine. This year's Math day had a panel discussion 'Bridging the Gap', which aimed to address important topics such as the general perception of mathematics and ways to improve mathematics communication. The most frequent event of the club is talks. The club has organized numerous talks with topics in pure math and about the applications of math in other disciplines given both by students and faculty at IISER Pune and outside. Some of the eminent speakers featured were: Prof. MS Raghunathan, recipient of Shanti Swarup Bhatnagar award; Prof. K.B. Athreya, co-author of the popular text 'Measure theory and probability theory'; Prof. Neena Gupta, recipient of Ramanujan Prize; Prof. Sudeshna Sinha, Fellow of the World Academy of Sciences; Prof. Ramanujam, recipient of Indira Gandhi Prize for popularization of science; and Prof. Ravi Kulkarni, former director of HRI.

The club also organized special events such as the Fields Medal and Abel Prize symposia in which the works of Field medalists and Abel prize winners is discussed in a way that makes it more approachable to a wider audience.

This year, two new events were introduced: Let's Talk Math which allows one to explore fascinating topics of mathematics through informal discussions rather than a formal talk,and the MS Thesis talk series which try to answer common questions like 'How is it to do an MS thesis?', 'What does writing a paper feel like?', 'What are the topics and exciting areas of research out there?' which are in the minds of BS-MS students.

I hope you enjoyed reading about our past events! Looking forward to witnessing your participation in future ones!

-Heerak Sharma

A year's story in a few words....

Maths Club? So, a bunch of nerds doing Maths together? Certainly not. Maths Club is a vibrant platform where Mathematics meets fun, games, movies, art, sciences and literally everything.

The academic year began with the Integration Bee, with over 50 participants battling for the crown of 'Grand integrator'. The traditional Student's Talk sessions were continued, with talks by Arjun Ranganathan, Sreedev M and Karthik Vasisht. The talks ranged from discrete mathematics to geometry and algebra and gave a fresh perspective on the subjects and illustrated the journey of stepping into research. Let's Talk Math, a newly introduced event, facilitated engaging discussions about various topics in mathematics. Two of these sessions were conducted and moderated by Varun Shah and Sreedev M. Merch design contest was held twice this year, with the winning designs printed on t-shirts and sold.

On the occasion of National Mathematics Day (22nd December), the birthday of Srinivasa Ramanujan, a talk commemorating his life and works was given by Surya Teja Gavva, a PhD student at Rutgers University, New Jersey, USA.

During the winter break, adding to the festivity, the movie 'N is a Number' was screened online. The Sudoku competition, also conducted during the vacation, was exceptionally well received, with over 150 participants comprising students, faculty and administrative staff participating enthusiastically, culminating in 18 winners!

Various stalls and maths expositions were organised throughout the year as stand alone Maths Club events or as collaborations. Notable ones are Game Night and fossil day stalls in collaboration with Karavaan'22, and stalls on National Science Day in collaboration with SAC (Science Activity centre). An enthralling magic show was also performed by Asif Mohammed as a part of Karavaan'22. Innovative maths demonstrations were organised as a part of Sci-con in collaboration with Mimamsa'22.

A hybrid mode lecture by Prof. Vivek Mohan Mallick was organised as part of a collaboration with the Grothendieck School of Thought.

As a part of the May 12 initiative, an online talk by Dr Cristiana De Filippis on her life and work was delivered, and the movie Olga Ladyzhenskaya was screened.

The Pi Day was a spectacular success, with outreach activities for school students, insightful talks by distinguished mathematicians, heated panel discussion, a breathtaking juggling performance, a thrilling treasure hunt and many more events.

The Maths club is eager to conduct many such events in the coming years and hopes to continue receiving the same support and enthusiasm from the IISER community!

-Padmapriya S

Chasing Dreams is like solving a complex math equation: It's easier to proceed step by step than guessing the outcome.

-Suvradip Maity
BSMS IISER Kolkata

Joining hands

"Alone we can do so little; together we can do so much."

– Helen Keller

Maths club has always believed in the spirit of working together. Thus, the club has organized many events in collaboration with other clubs round the year.

First, the club organized a talk on Triangulated Categories in collaboration with the Grothendieck School of Thoughts which is a modern instructional school in India which organizes hybrid mode lecture series on advanced topics. The plan of this school is to become a delocalized system, so that when a speaker gives lectures from an institute, the audience on campus can enjoy it offline there, while other participants can see it live on zoom. The lecture was delivered by Prof. Vivek Mohan Mallick offline and it was live streamed online enabling wider reach to the interested.

Then came π day 2023 – one of Maths Club's club's flagship events for the year – which was loaded with collaborations: Erdős quiz with the Quiz club which brought together the realms of mathematics and popular culture; 'Month of Symmetries' with Pixels in which we received some breathtaking photos; a panel discussion 'Bridging the Gap' with Literary Club, a well-received discussion about the importance of higher mathematics; and the movie Pi, which revolves around themes such as mathematics, mysticism and religion, screened with Kaleidoscope.

Later, Maths club, in collaboration with the math department, organized a talk and a movie screening celebrating women in mathematics at IISER Pune, as part of the May 12 initiative.

to rise together

Then on the occasion of national science day 2023, in collaboration with SAC, the club organized exhibits aimed for the general public. Demonstration of Galton's board and various shapes of soap films piqued the interest of the audience.

During this year, the club put up exhibits in Mimamsa's SciCon. From the geometry of the double pendulum to the mathematical structure in chemical crystals, the display was an enriching experience especially for the school students among the crowd. Cryptarithms and Cryptography problems left the audience pondering. As usual, a lot of chocolates were distributed to make the experience delightful.

The talk was given by Dr. Cristiana De Filippis and was about her life and research work; the movie was a short documentary about the Russian Mathematician Olga Ladyzhenskaya.

Maths Club has always believed that anyone can enjoy the beauty of mathematics and therefore never leaves an opportunity to exhibit the elegance of mathematics. In accordance with this, the club put up exhibits during some of the institute's biggest events: Karavaan, Mimamsa and national science day.
During Karavaan 2022, the club put up some enchanting exhibits: simulating mathematical models, various proofs of the Pythagoras Theorem and more. The club also organized a game evening – an evening filled with mathematical games which entertained the crowd. Tiling problems, Josephus problem, Numberle and many more fun puzzles left the crowd intrigued. As usual, the winners went away with tasty chocolates.

I would like to thank all the people involved, as their support made these events better in all regards.

-Heerak Sharma

Thomae's Function in some way tells us about the importance of rationality. Being irrational gets you nothing but being rational brings you everything.

-Suvradip Maity
BSMS IISER Kolkata

We answer her quizzes, she answers our questions

Interviewed by: Yash Karampuri

Edited by: Shreyas Pai

"She worked as an Assistant Professor of Practice at NYU-ECNU Institute of Mathematical Sciences, New York University, Shanghai, and presently is a faculty at IISER Pune".

1. How has your academic journey been? Could you share it briefly?

I started my academic journey without knowing what 'academia' really is. Still, I was fascinated to be scientist and knew I would do Physics or Mathematics very early. This was mainly because, from an early age, my parents would give me mathematical problems as puzzles making me curious andkeeping me interested in the subject.

The stepping stone towards academia was International Mathematical Olympiads (IMO). My parents encouraged me to give the Olympiads, and though I couldn't make it to the top six at national level, it has helped me a lot in academia.I got introduced to topics like Enumerative Combinatorics, Diophantine equations, Elementary NumberTheory, Higher level Euclidean Geometry and many more which weren't a part of the school curriculum.The whole process of preparation helped me inculcate patience, zeal and the will to not give up, due to which I can work on research problems for months. This got instilled because the solutions to the Olympiad problems weren't available, and thus, my peer group and I would sit together, discuss and crack the question making us self-trained.

Prof. Moumanti with her doctoral advisor Prof. Joel Spencer, after receiving the Harold Grad Memorial Prize

The journey continued with me entering ISI Kolkata, which had a challenging course curriculum and my M.Stat. dissertation with Krishanu Maulik was the first experience of research I ever had. Here, I worked on Heavy-Tailed Distribution which is helpful in Ruin Probability. I enjoyed reading the related research papers and working on related problems. I was grateful to get Prof. Joel Spencer as my supervisor during my PhD. He is a great mentor and still guides me in academia. Determination, eagerness and excitement are required to work on the chosen problem. To sustain this, a good, kind mentor is needed. His guidance helped me complete my thesis and do two back-to-back post-docs. This also persuaded me to serve in institutions abroad and now in IISER Pune. I am happy to be a researcher at this institution. I am also thankful to my friends who helped, supported and inspired me in this journey, making it memorable.

"In a burnout situation, I would suggest not fighting it out."

2. Could you please explain your research briefly?

During my PhD, I worked on topics related to mathematical logic and random graph, where properties of graphs can be expressed in terms of mathematical logic like first-order logic, etc. The main question being what given a graph, what is the probability that the graph has a certain property? This question can become a little clumsy if solved by looking at properties individually. Thus, probabilities of properties that can be grouped into a single mathematical logic are to be studied.

Now, let's consider a random graph sequence, say a graph having n vertices and between each pair of vertices, there is an edge which occurs with probabilities p(n) and these events (an edge is drawn between two vertices) is independent of such other events. Now, this can be seen as a sequence as n varies. One of the questions considering first-order logic that can be asked is, " Is there a limit to this probability as $n \to \infty$?" and if these limits turn out to be 0 or 1, then now we are dealing with 0-1 laws. Nowadays, I work on two-player combinatorial games. I consider them perfect information games as they tell the game's purpose and result. Chances come from the randomness of the premise of the game and not from the randomness of which player wins. The importance of such games lies in how they are connected with different objects.The game I am very interested in and have studied a lot is the percolation game. Percolation is an essential topic in physics. It deals with an infinite network, a sink and a source. Sink and source are some subsets of thet set of vertices. The network has edges, each open with a certain probability, and these events occur independently. Consider a liquid let to flow from the source to the sink, the question being, "Is there an open path through which the fluid can flow?". A tougher version of the problem would be when we consider the strategies used by both players to win and not settle for a draw (find some open path). For a given network (infinite), it is also not known whether percolation happens, i.e., whether there are infinite open paths which we call percolation cluster. If we solve the game, i.e., say we find the probability of a draw, then we can find a bound on the threshold (probabilities with which an edge is open/ closed) beyond which percolation happens. This is one such instance where combinatorial games are helpful. Combinatorial game is a central theme where I can connect all different areas like Statistical Mechanics, Percolation theory, Automata theory, etc., with games that I study. Another area that I am keen to get into is Stochastic processes, where at every

3. Have you found any mathematical course/field challenging?

Yes, there are several of them. Many are those that are not my areas of interest. The topic that I would love to learn and felt hard is Stochastic Calculus, as I found the content technically heavy. The mistake I make is focusing on every detail of the topic, thus losing the bigger picture, which is a common mistake. If learnt/ taught in a way where the bigger picture and its application are given importance, then it is not hard. While learning, I got lost in the technicalities that I couldn't keep my enthusiasm for learning the topic; hence, I hope I don't repeat this in the future.

Prof. Moumanti sketched this at the time when the Amazon rainforests were experiencing a lot of wildfires.

4. How do you spend your leisure time?

I sometimes sketch in my leisure time, but I would not like to do them always as they take a lot of time, and one must be enthusiastic about finishing the sketch before starting it. Other than that, I sing. I used to learn Classical music, but now I sing on my own or sometimes pick some songs, new ones may be of a different language. I read quite a lot. I love reading epic fantasy books 'and dream of having bookcases full of epic fantasies!'. I also love cooking and running. Running and working out is the only time I don't think of math. One can call all this leisure, but I do it whenever I would like doing them.

5. Have you ever tried interdisciplinary research?

I would not say so. Though I work on Statistical Mechanics and Percolation theory, which is a part of physics, the way mathematicians look at it differs from how physicists would look. The work I did on these topics was math-heavy. Hence, I would say I haven't yet done interdisciplinary research. I talk to my colleagues/friends working in these areas, irrespective of how much I understand; I find it very advanced. But, I am always interested in these topics and would help my fellow researchers in any way possible.

6. We realise that research requires a lot of patience. What was the longest time for which you were stuck on a problem?

It is not often the case where one works on only one research problem. Researchers would be involved in multiple projects, and I involve myself in many. The amount of time spent on a project is not uniform, i.e., one spends more time on a project and less on others; thus, different projects are prioritised at different times. This way, completing a research project would take at least two years. It also depends on the area, and it's also not the case that applied math takes less time than pure math, but it's only that people working in applied areas are under a lot of pressure to publish papers, which in the case of pure math, is not that intense. Researchers will be uncertain about where one is leading when working on a problem, how accurate their paper is, whether it will be accepted, etc. Still, the love for the subject would enable one to hold on and proceed.

7. Were you always into academia? Have you ever thought about going for industry jobs?

No, I haven't done anything besides academia or considered going to industry. Academia has now become a part of my identity. I would choose to teach in a school rather than goindustry because I don't know how to be in an industry and wouldn't survive either."There is a funny story from my childhood, when I read Robin Crusoe and decided I would be a sailor"

8. We have seen you put much effort into courses and your research! What do you do when you feel burned out?

In a burnout situation, I would suggest not fighting it out. I haven't gained success doing that. Fighting burnout, i.e., refusing that one is tired and needs a break and not showing kindness to oneself, will not benefit one as it will not heal the burnout one is facing. During this phase, try to think of the stage before where one was very productive and try accepting that this is also a phase of life/career. It doesn't mean that one should give up altogether.Try to focus on another project very different from the current focus topic. One can make notes for class (of course, one would teach) or try finding research articles that can be given to one's students to read. Going for a walk would also help. It is also a time to do creative hob- bies, something innovative that your field of interest doesn't allow. I

7 used to sketch, and sketching would take a lot of time (≈ 12 hours), diverting me completely from academia. Bringing all this would give a sense of fulfilment. Going for a trip/trek or even just planning a trip itself would require a lot of energy and focus that stops bothering one, "Oh! I have not been able to focus on my research." Now, it depends on what would help an individual, but one shouldn't be punishing oneself by being harsh to oneself.

9. Do you think mathematical research had an impact on your daily life? Do you approach other challenges in a mathematical/logical way?

Very much. I don't think there is anything that I don't approach in that way (mathematically/logically). It is unconsciously embedded in my way of thinking or approaching day-to-day challenges.

10. Combinatorics requires out-of-the-box thinking (most of the time). How do you get new ideas?

If one asks me if I ever truly got a completely novel idea, I would find it hard to say "Yes". Most of the time, I would develop many things from scratch, but some central ideas form the subject's core. For example, the way to go about combinatorial games is recurrence relations which are sometimes easy to derive. When they are complex, we would see a bigger space and derive the relation there. These would surely give new findings, but not new techniques.

11. Before joining IISER, you were a professor at NYU Shanghai. How is the work culture over there different from IISER?

People in China are extremely disciplined, and they work very, very hard. Compared to other countries, I feel China is a strict regime. I used to work for long hours in my department. I used to come very early to the office and leave very late. That's because I loved the ambience of the department. I liked my office so much that I used to go to my office on weekends and paint. Working long didn't feel like I was forcing myself. Talking about work culture among faculty members there and here at IISER Pune depends on the faculty in question. I work equally hard here, but it's not always in the department, though I also like the department and my office here. Here, I also have the privilege of being on campus and going to my apartment doesn't disconnect me from my office. In Shanghai, I used to stay far from my office (≈ 45 minutes of walk from the office), thus preferred staying longer in the department. Other faculty's work culture depends on the faculty themselves. I know some faculty members here who work very hard compared to those in Shanghai. Hence, I haven't found any difference as such.

12. Have you tried applying mathematics in real life? Could you elaborate with some examples?

The kind of research I do doesn't render much directly to real life problems. They (topics of research) are all connected, but indirectly. My research links with Experimental physicists (who can apply their research to day-to-day life problems). Thus, in that way, I can say I am dealing with real life problems, but surely not directly. I am interested in understanding how math (or science in general) is related to art. I read some books related to this, and it has fascinated me.

13. You have completed your undergrad and graduation in the field of statistics. What inspired you to study pure math?

ISI Kolkata has a more mathematical curriculum. If one wants, one can shift more and more towards mathematics. For example, in M.Stat., both years had more electives. Thus, besides the compulsory statistics courses, students could choose the courses they liked. These helped in building a strong foundation in mathematics. Once I started learning Bayesian Statistics, I didn't feel good about it as it didn't fascinate me. I also learned from my seniors that research in statistics revolved around Bayesian Statistics, and I didn't want to do that. That's why I moved away from statistics though now I feel like returning to it. Moreover, there was a huge fascination with Probability, particularly one connected with Combinatorics. Thus, I chose this as my area of research.

14. As you said earlier, there are uncertainties in research, and there will be downfalls. What would you suggest doing during these failures? What advice would you like to give to your students?

In academia, especially research, there is a lot of uncertainty. Thus, falling would be a part of it. For example, rejections from journals, rejection of one's application, stumbling etc., will all be there. I am holding on because of my love for the subject. One must love what they do; if it isn't the case, one will get questions such as "Why I am bearing all doing these and bearing all the miseries"? Just like I said for burnouts, rejections also would demotivate one, which would make one's day less bright than before, but it's going to be for a few days, and it's perfectly OK and doesn't mean it is the end of the world. From my experience, I would say that life opens open up new doorways.
In my case, getting an offer from NYU Shanghai was precisely that. I say so as I wasn't much productive during my post-doc as I went through severe depression during that time. After moving to Shanghai, things started becoming better. One has to take these leaps of faith if one truly cares about the subject. If one feels they can do something better than this, this means one doesn't have a fierce passion for the subject; if not, then the obvious question would be, "What else do I do except this?" this also helps one not to lose hope and keep trying while healing from the situation. So, it's two things. Firstly, one must have patience with life, and second, one must have faith in new doorways opening. Life will open doorways only if one keeps trying. Failures are a stepping stone; thus, they are considered constructive criticisms. For example, a research paper of mine got rejected even though I was in touch with the journal and the editor for about nine months. Though the rejection upset me, I realised afterwards that the reviewer had given a detailed review of six pages that helped me improve my paper. Though publishing a paper takes a long time, one must not think this is a waste of time, as it would not lead to happiness, and that's not the ultimate goal. I don't know about other occupations, but for a researcher, it is so important that they hold on to the little happiness they get in the journey.

Another piece of advice I would like to give is not to compare oneself with others. In a country like India, where there are fewer opportunities for several people, one can quickly get into comparing that I am not as good as that individual etc. Comparing oneself and being jealous doesn't lead to anything good; instead, accept these facts. I advise people to compare themselves with themselves the previous day. This will surely help one to grow. Lastly, I would like to point out that students must know when to ask for help and from whom. Not doing so may deteriorate one's mental health. Do reach out for help without hesitation, as it is okay if one is dealing with depression and anxiety. Don't isolate yourself from the world; if done, it would lead to a more severe problem than just a mental issue. Build a support system with friends, family members and professors who will encourage you to seek help and would help and support you. Physical activities would keep one fit and help in dealing with these problems. These are the few suggestions that I would give.

15. Do you think the participation of women in mathematics or science, in general, is lesser than men? What are your views on this?

Yes, if one sees the gender ratios. But it also depends on which place or country one is looking at. For example, these ratios are much better in Scandinavian countries. In general, I feel women face quite a bit of challenge, mostly due to unconscious bias, which creates obstacles for women. I am privileged to get supportive parents and be in a progressive family. Thus, I have faced fewer challenges than many women as I had a strong support system. From the beginning of a child's life, I feel that she should be in an environment where she never needs to apologise for voicing her opinion. Thus, it's a society-wide issue where many things are interconnected. At some point, I would want to motivate more women into the system, as my mother is presently doing. She works in an organisation which helps train women who come to Kolkata from rural backgrounds and make them capable of being economically independent.

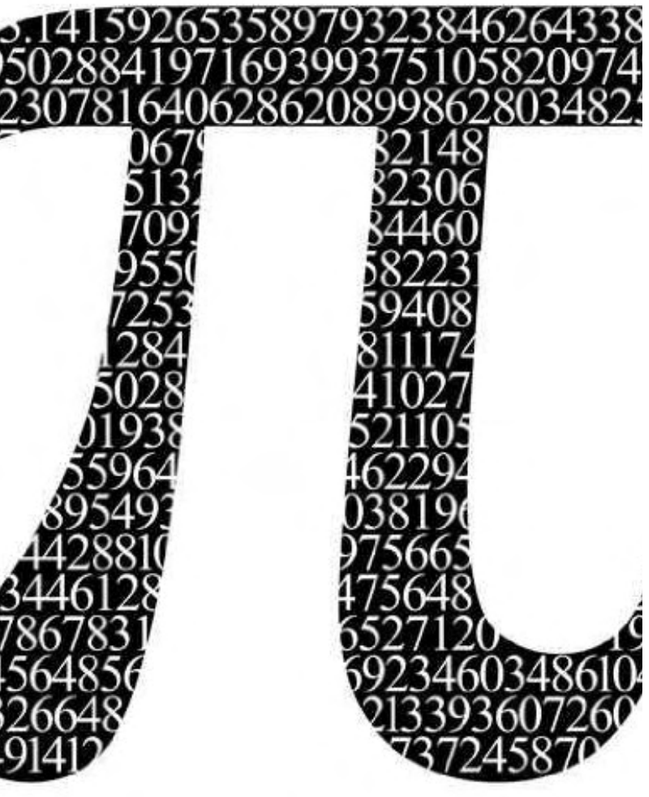
"Failures are a stepping stone; thus, they are considered constructive criticisms."

Pi Day

A Celebration Of The Beauty Of Mathematics

"Beauty is the first test: there is no permanent place in this world for ugly mathematics"

-GH Hardy,
A Mathematician's Apology



π - known to everybody as the ratio of the circumference of a circle to its diameter, has held the fascination of mathematicians and physicists across millennia. From describing a circle to appearing in the formula governing the motion of a pendulum, it has applications in such diverse areas as geometry, number theory, statistics and mechanics. What is a better day to depict the beauty of various branches of mathematics than the day celebrating this omnipresent number, the 14th of March? (written 3/14 in US style, it signifies the decimal approximation to two digits of this irrational number.)

Pi Day is the most important Maths club event of the calendar year. Typically, the celebrations are held during the weekend closest to the 14th of March. This year, the 14th of March was Tuesday, so we decided to make the event a grand five-day affair, with most of the events being held on the 11th and 12th of March and having an event each on the following three days.

The preparations started in early January with a meeting of the coordinators and core team members. A tentative plan was laid out, with suggestions of various competitions and events and popular maths communicators to be invited , who could highlight the beauty of various aspects of mathematics. This plan was presented to Prof. Anisa Chorwadwala, Chair, CCA and Prof. Mainak Poddar, Chair, Dept. of Mathematics, who enthusiastically supported our proposals and made valuable suggestions.

With the groundwork laid, the organisational work began. I will mention a few essential organisational tasks. Searching for guest speakers was one: Prof. R. Ramanujam, visiting faculty at Azim Premji University, and Prof. Ravi Kulkarni, Distinguished professor at Bhaskaracharya Pratishthan, graciously accepted our invitation; another was to invite school students to Pi day stalls, and I am glad to mention that schools across Pune turned up for the

While all the background work was proceeding, π Day was drawing closer. It was the last week of February, and still, a lot of work remained. The most important among them was to set up questions for the Pi Day treasure hunt. The treasure hunt is quite popular, and in the weeks leading up to π day, many seniors walked up to me and mentioned that they were looking forward to it. The expectations were high. A treasure hunt team was soon set up, which gracefully rose to meet the challenge.

The week leading up to Pi Day was hectic. While coordinators were bustling about making last-minute preparations, the treasure hunt team was roaming the campus in search of good clues and working day and night to come up with exciting ways to present the competition. The day before the start of the celebrations was filled with anxiety and excitement. We were exhausted from the week-long work but could barely rest before the big day when the audience would judge two months' worth of effort.

3.11

Festivities started with an inauguration ceremony in the morning in the NLH (New Lecture Hall). The event began with a short address by Prof. Mainak Poddar, thanking everybody who helped organise Pi Day, followed by cake cutting. Professors, volunteers and core team members heartily participated in the event, and Pi Day got off to a great start.

Following this, stalls were presented in the LHC (Lecture Hall complex)—the stalls aimed to showcase the beauty of mathematics to school students in a fun and interactive way. Over 150 students from various schools in Pune accompanied by their teachers had come to witness our volunteers present fascinating exhibits on numerous topics of maths: **Galton board**, which demonstrates the central limit theorem using a vertical board with rows of pegs in which beads are to be dropped to observe the same, presented by Gurjot Singh and Jigyas Baruah; **Nim game** - a combinatorial game, in which two players take turns removing objects from distinct heaps, by Sushrutha H S; the myriad of shapes created by **soap films** and its importance in the study of minimal surfaces, by Sreedev M; **Origami**, it's interesting properties and connection to mathematics, by Yuktha J G; simulation of **dartboard method**, in which darts thrown to a circle inscribed in a square would estimate the value of π, by Harshal Chawre; and lastly **pure rolling**, in which two flat gears attached to a board were rotated on a smooth surface, demonstrating the associated (quite unintuitive) mathematics, by Atul Prakash and Praneet Kumar Patra.

Prof. Mainak Poddar at the inauguration

Students at the stalls

Students at the stalls

Later, a talk for the students by Prof. Tejas Kalelkar titled 'Euler, Plato and Balloons' explored beautiful identities in maths, such as the famous Euler identity

$$e^{i\pi} = -1$$

and the Euler characteristic,

$$F - V + E = 2$$

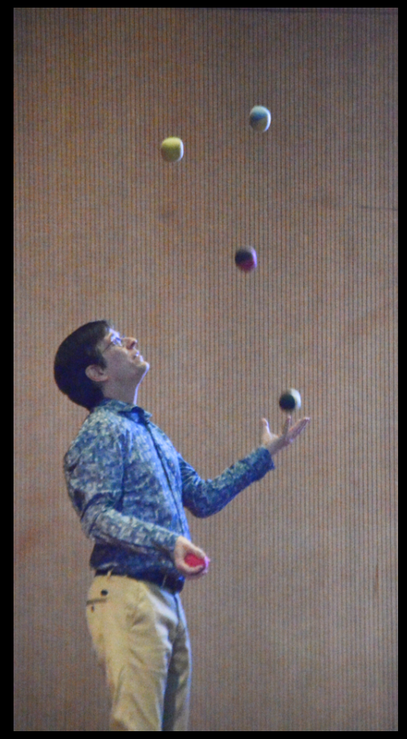
the various properties of platonic solids. With beautiful illustrations and some Sheldon references, he impressed the students with the beauty of mathematics.

Post lunch, there was a fun-filled jugglology lecture by Prof. Steven Spallone in the CV Raman Auditorium. While showcasing his breathtaking juggling skills, he spoke about the systematic study of juggling patterns. With the use of the siteswap notation, which specifies the number of beats when the object is thrown again (0 for an empty hand, 1 for a throw from one hand to the other, 2 for a momentary hold and so on), and used this to show that certain patterns are non-jugglable! More surprisingly, he provided a mathematical approach to analysing whether a sequence of throws is jugglable, leaving the audience mesmerised.

The school students and their teachers who attended the event thoroughly enjoyed their visit. Hear from a teacher about their experience:

> "Its a totally positive feedback from the teaching staff of my school, Dwarka School. So many things which are from beyond the textbooks you have given today and I can hope that all our students will start showing more interest in Science, Technology, Engineering and Mathematics. Thank you! It was a fruitful day."

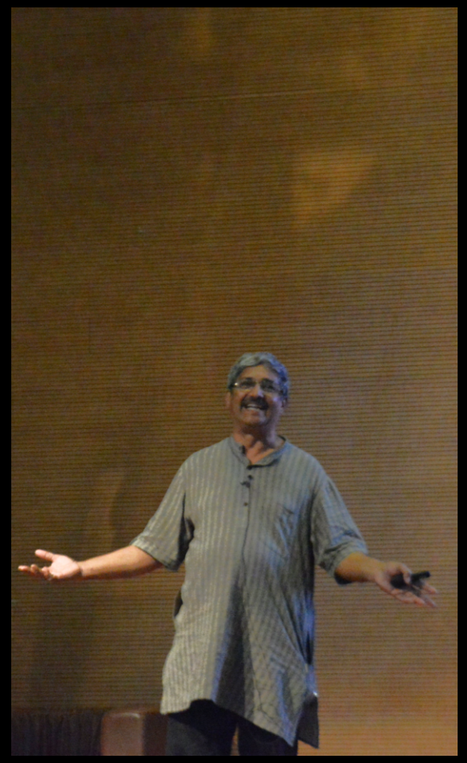
Prof Tejas Kalelkar during his talk



Prof Steven Spallone showing his juggling skills

Later, Prof. Ramanujam, a retired professor at IMSC Chennai, talked about the importance of logic in everyday life. A popular mathematics and science communicator, he is a recipient of the Indira Gandhi Prize for popularisation of science. The title of the talk, 'Why should we learn **logic**, and why should we learn logic?' is a testament to his wit and skill as a communicator. Professors and students thronged to the CV Raman auditorium to attend the talk of this captivating speaker. Starting with some riddles involving logical thinking, he managed to hold the attention of this diverse audience by delving into the importance of logic in security and voting protocols and its relation to computation. The question and answers session went on for an hour past the talk, and he answered many more questions from the audience after the talk's conclusion! It was with satisfaction that we watched him patiently answer all the questions with a smile.

Then came the much-awaited treasure hunt. The competition was to be held overnight. Over 50 teams, comprising three members each, gathered eagerly at the amphitheatre for the hunt at about 10:30 pm. In a mark of ingenuity, the treasure hunt team devised two supporting plots leading to the same places. Each participating team was given a sheet of paper with detailed instructions. The questions would be issued by the telegram bot. Each of the mathematical questions, which followed a plot, had to be solved, which would lead them to a unique location on campus. The numerical answer had to be entered to proceed to the next question. But, why go to the location indicated in the question at all? Because the participants would then have to click a photograph of the specific object hinted at in the question on the



Dr R Ramanujan, retired professor IMSc, Chennai

server, and these photographs would be checked to ensure that they had completed the hunt. There were ten questions in total. General instructions were soon given, and at 11 pm, the hunt began.

While many teams dashed off to a place they thought was hinted at in the question, other teams sat pondering and trying to solve it. As I took a long walk around the campus that night, seeing some of my friends running around the campus while others were sitting exhausted or thinking about a question, I knew we had lived up to the expectations. It filled me with joy and pride about the treasure hunt team.

The hunt ended at around 3:30 in the morning, with around ten teams finishing almost simultaneously. However, the first place was claimed jointly by team **Chonkey Donkeys** comprising Ishaan Chaudhury, Deep Shah and Ameya Kherde; and team **Vanwal** comprising Vignesh Pai, Yash Gupta and Vansh Aggarwal. Team **PSBKTA** comprising Prachi Saini, Bittu Kumar and Tushar Arora, were declared second.

The participants found the questions interesting and the hunt an enthralling competition. Let's hear a participant recount their experience:

> The treasure hunt was definitely a night to remember, credits to the Maths Club team. The communication of answers and clues through telegram was quite innovative and clean, great job to whoever thought of that.
>
> The questions were mathematical, but at the same time also required some imagination to understand the clues well. They made us work hard for the next clue, but it was fun. Some questions had multiple solutions, I remember sitting on the sidewalk with my laptop solving a question computationally. Definitely something I've never seen in a treasure hunt before.
>
> I especially liked the rules of the treasure hunt. The fact that we had to explain our answers in the end made sure that we solved each problem logically. There was also a good policy on hints, it allowed us to leverage our knowledge and lack of knowledge in a balanced way.
>
> The discussions with other participants and the treasure hunt teams after the event were also fruitful. Maths Club has done an amazing job of combining the thrill of a treasure hunt and the fun of doing mathematics into a well organized event. I can't wait to help organize it next year.

3.12

Glimpse of the panel discussion

In the morning, Kunal Arora and Visakh Narayanan, PhD students and Karthik Vasisht, a 5th-year BS-MS student, presented posters, visually explaining to the audience vastly different areas of maths. Kunal presented a poster about category theory, Karthik presented about 'Simplicial homotopy', while Visakh presented a table of classification of knots and briefly introduced knot theory.

In the afternoon, the stalls presented on the previous day were put up again briefly so that the people who could not see the stalls before could witness them.

Later in the day, a panel discussion titled 'Bridging the Gap' was organised in collaboration with the Literary Club. The discussion addressed vital topics such as mathematical pedagogy and ways to improve mathematical curiosity among students. The panellists were from a variety of backgrounds. On the panel were three professors: Prof. Debargha Banerjee, Dept. of Mathematics, Prof. Suneeta Varadarajan, Dept. of Physics and Prof. MS Madhusudhan, Dept. of Biology; three BS-MS students: Aniketh Sivakumar and Krishna Girish, 4th year, Arun Ravi, 5th year; and two moderators: Shweta Shrothri, PhD in Biology and KS Dhruva, iPhD in Physics.

They presented their views about various topics, such as the beauty of mathematics, the ever-growing importance of mathematics in other disciplines of science and the difference in the research culture of mathematics compared to other fields of science. They passionately debated topics such as improving mathematics teaching at the school level, the importance and appropriateness of popular maths videos as a tool of mathematics communication, the general perception of mathematics as difficult, and career opportunities in mathematics. Their insightful remarks and views were very informative and eye-opening. Prof. Mainak Poddar, who was present in the audience, later addressed many questions and took feedback from the audience about the IISER mathematics curriculum.

We were all relieved and overjoyed with so many events having gone smoothly and successfully. However, some anxiety remained about the remaining events in the following three days.

Glimpse of the panel discussion

3.13, 3.14 & 3.15

On the 13th, the Erdős quiz was held, named so after one of the most prolific mathematicians and collaborators, Paul Erdős, dubbed 'The Mathematical Nomad', owing to his unique lifestyle of travelling around the world working. The quiz was a unique blend of popular culture with mathematics.

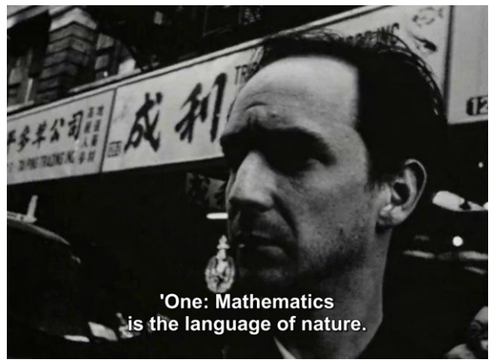
Krishna Girish and Amogh Rakesh secured the first place, while Varun Shah and Aditya Pujari took home the second.

Prof Ravi Kulkarni giving his talk

The next evening, Prof. Ravi Kulkarni, former director of Harish Chandra Research Institute and former chair of the Mathematics department at IIT Bombay, gave a talk titled 'Philosophy of Mathematics', in which he aimed to define mathematics. Starting with the meaning of philosophy, epistemology and ontology, he talked about the epistemology of mathematics and previous attempts at a definition of mathematics, finally bringing the audience to his fresh outlook on the age-old question.

This was followed by a prize distribution ceremony of all the competitions held by the Maths club throughout the year: Sudoku contest, Erdős quiz and Treasure hunt, with Prof. Kulkarni awarding the winners.

The following night, a psychological thriller, Pi was screened in collaboration with Kaleidoscope. It explores many themes like mathematics, mysticism and religion. With this, the almost week-long celebration of mathematics in different forms ended.



Clip from the movie Pi

In addition, in collaboration with us, Pixels announced March as the 'Month of Symmetries' as part of the 'Month of X' initiative. We received some beautiful entries.

The Maths department, delighted by the efforts of the Maths club in organising Pi Day, invited us to the Maths Day symposium dinner, which was held shortly after the conclusion of Pi Day. It was a delightful evening, where we chatted with professors, ate, and danced heartily after an exciting but tiring week of organisation.

Credits - Amoghvarsha A V

Thank You!

The success of any event rests on the efforts and support of many people. Maths club is particularly grateful to Prof. Mainak Poddar for his constant support and help with many of the organisational difficulties that arose; Math office staff: Mr. Yogesh Kolap and Ms. Suvarna Bharadwaj, for help with the logistics; Mr. Ashok Rupner from SAC (Science Activity Centre) for assisting with coordination during the exhibition of stalls to students; the clubs that collaborated with us during the event: Literary club, Quiz club, Kaleidoscope and Pixels; the volunteers who passionately explained their stalls and posters; the speakers who gave insightful talks; the panellists who provided their invaluable views about an important topic; Amoghavarsha A V and Amruthamshu A K for capturing some memorable photographs; Astha Gupta, Adhiraj Sundar and Anuvind Pramod for recording the talks; our core team for assisting with organisational work; our design team for making spellbinding posters; housekeeping staff for their much-needed help; and technical staff of CV Raman, especially Mr. Narendra Khandekar for the technical assistance provided during the talk.

Special thanks should go to the treasure hunt team: Sreedev M, Jetharam Bhambhu, Shreyas Pai, Rudresh Gade, Racchit Gupta and Narayanan U. They worked tirelessly, roaming the campus in the middle of the night to find interesting clues, debating with one another about the ease of solving a question, and trying to perfect the plots. It was delightful to see their efforts well appreciated by the IISER community. It was a great experience to work with them, and they too had fun organising the competition. Shall we hear from them about their experience?

> “It started on a wonderful tone, because of the amazing crowd which was twice what we expected. We had two separate paths through telegram with two separate story lines and they were assigned randomly.
>
> Many of the participants surprised us by working through both the location password question and the problem in a clever manner. This in the end looked so beautiful because it made the treasure hunt much more accessible and inviting, throughout the five hours.
>
> It had a fun and dreamy story ..it had inviting cute math problems.. and the best part was it made us stay a large part of IISER Pune an entire night to discuss math and roam around our beautiful campus.
>
> We are deeply indebted to the maths club for giving us such an unforgettable night!”

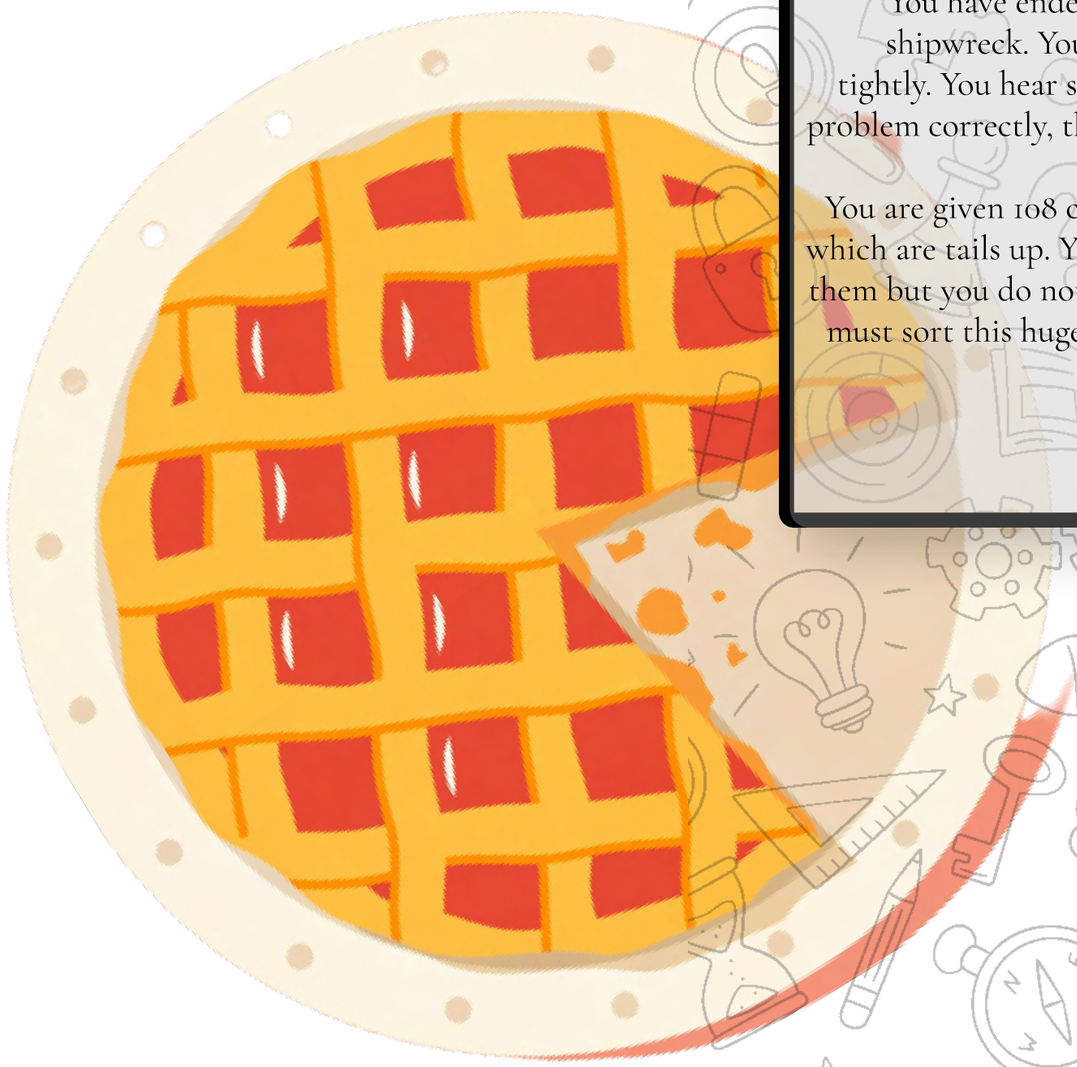
Last but not least, thanks go to you, the audience. Your constant support and interest in the activities of the Maths Club, and your enthusiastic participation during π Day, have motivated Maths Club to continue presenting to you more such exciting events.

Organising this event has given me a sense of fulfilment and joy that I will never forget. I am looking forward to witnessing grander π days in the future.

I am sure you are looking forward to the next Pi day too!

Stay tuned!

-Prasanna Bhat



Fiddle With The Riddles

1. Toss it

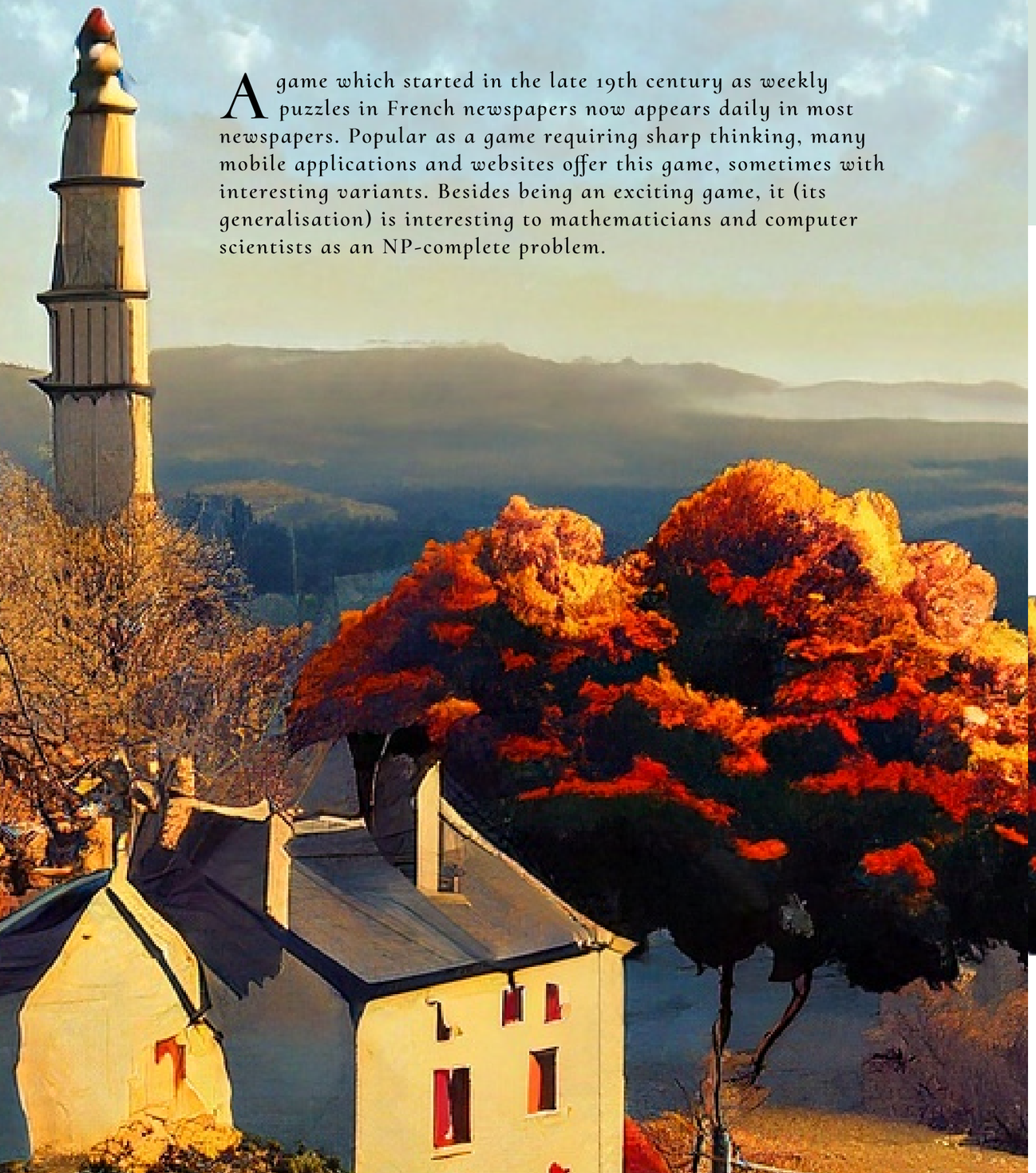
You have ended up on the mysterious island of Finlocip after your shipwreck. You find yourself sitting in front of a desk, blindfolded tightly. You hear someone's voice who narrates a problem. If you solve the problem correctly, then your blindfold will be removed, and you will be freed.

You are given 108 coins that are heads up and an unknown number of coins which are tails up. You can only count the number of coins, flip them and sort them but you do not know which ones are heads up or which are tails up. You must sort this huge pile of coins into 2 piles containing an equal number of heads.

How do you do this?

The Christmas Present!

A game which started in the late 19th century as weekly puzzles in French newspapers now appears daily in most newspapers. Popular as a game requiring sharp thinking, many mobile applications and websites offer this game, sometimes with interesting variants. Besides being an exciting game, it (its generalisation) is interesting to mathematicians and computer scientists as an NP-complete problem.



Yes, I am referring to the Sudoku.

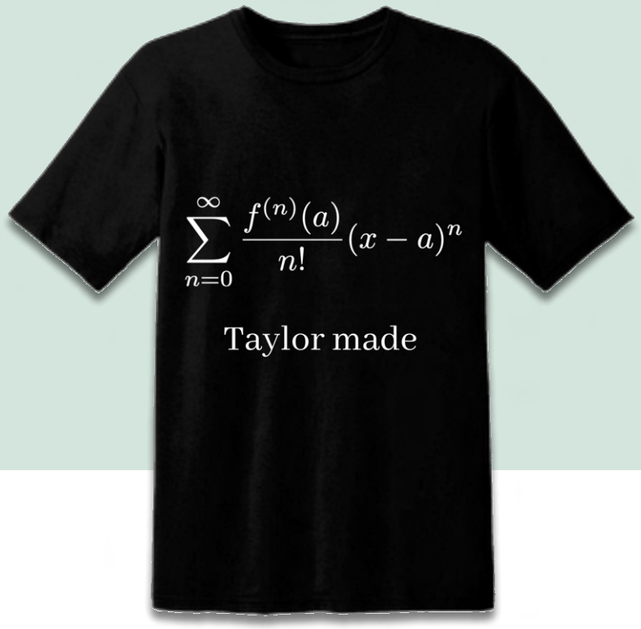
In early December last year, just before returning home for the winter vacation, the coordinators and core team members chalked out the plans for the month. Various suggestions were brought up, one among them being a week-long Sudoku contest, a suggestion made by Yuktha JG. It was decided that the event would be held in the last week of December, and an email announcing it was soon sent

"If age is a number, Sudoku is the life."

Details such as scoring policy, tiebreaker rules, etc. were disclosed only a few days before the competition started. Deciding most of the rules was easy-a question would be sent every day in the morning. We would stop accepting responses for that puzzle precisely a day later, and the score for the day's Sudoku would be mentioned along with the puzzle, based on the difficulty level. However, deciding on the tiebreaker policy was tricky, especially as we expected good participation considering the game's popularity. We learnt about a new variant called the Killer Sudoku in which there is a 'cage' - a group of cells with a dotted line whose sum is shown as a small number above, and the remaining rules of Sudoku stay the same. We decided to use this variant to resolve ties. We soon finished setting the questions, and awaited the start of the competition anxiously.

The first puzzle was sent on the morning of Christmas. Needless to say, there were a few organisational difficulties, which were soon resolved. The next day, when we opened the Google form to evaluate the responses, we were immensely pleased that there were a staggering 120 participants comprising BS-MS and PhD students, postdocs, faculty members and non-teaching staff. The enthusiasm continued throughout the week, though the number of participants dipped slightly.

The week of the competition has been one of the busiest during my tenure as coordinator. There were emails requesting an extension of the deadline, queries about the Killer Sudoku and claims that some of the Sudoku puzzles did not have a unique solution or were just wrong. Ultimately, despite our best efforts to break ties, 18 participants claimed the top spot!

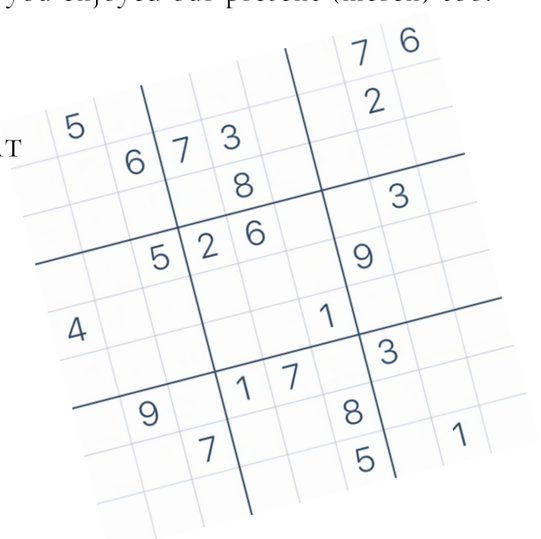


They were awarded customised merch by Prof. Ravi Kulkarni, who had been invited as a guest speaker during the Pi Day celebrations.

The zeal of the IISER community about this competition has filled me with a sense of joy and fulfillment as a coordinator. As with most events, the credit for helping ensure the smooth conduct of the competition goes to many individuals. On behalf of the Maths Club, I would like to especially thank Mr. Ranjan Sahu from the academic office for forwarding all the emails that had to be sent to everyone.

Thank you for giving the club a big present by your enthusiastic participation! Hope you enjoyed our present (merch) too!

- Prasanna Bhat



Formal Math, Informal Chat

Prof. Chandrasheel Bhagwat is an Associate Professor in the Department of Mathematics. He completed his Master of Mathematics from Indian Statistical Institute, Bangalore and received PhD in Mathematics from Tata Institute of Fundamental Research, Mumbai. This interview was taken by Sanchit Nayak. Below is an excerpt from the interview:

Interviewed by: Sanchit Nayak
Edited by: Padmapriya S & Nishkal Rao

1. What made you realize you want to be a mathematician?
I can trace my passion for mathematics to my school days, around 7th and 8th standard. The mathematics I learnt in school, the books I read outside the school curriculum, and discussions on these with my parents and grandparents must have fueled my interest. At that time, one doesn't know what an academic life of a mathematician is. Instead, one is often interested in solving puzzles and intrigued by the kind of mathematics used in these puzzles. So that was my first encounter with mathematics. Developing a passion for mathematics is a slow and long journey. Because the subject is very abstract, it is difficult to excite someone about mathematics. Also, the passion for mathematics is not sustainable; we have seen cases where one is interested and likes mathematics, but later on, they shift to another discipline. Over many years, the students I taught and the colleagues I worked and collaborated with have contributed to sustaining my love for the subject.

2. What were the challenges you encountered while pursuing a career in mathematics, like concerns from parents, the abstract nature of the subject,potential financial considerations, and the time required to become proficient in the field?
In my personal experience, I was fortunate enough to receive great support from my family, who granted me the freedom to pursue my passion and nurtured my interests. However, given the inherent characteristics of a career in fundamental science research, which is often perceived as financially less stable, I consider myself truly fortunate that my family not only comprehended my chosen path but also exhibited remarkable patience throughout my journey.

3. Could you provide a simplified explanation of your research?
Broadly speaking, my research lies in algebra. Specifically, my research focuses on representation theory and number theory.

One class of problems I investigate involves special values of L-functions For instance, the famous Riemann zeta function ζ(s) can be thought of as a prototype L-function. It poses questions about the existence of special values. For example, the sum of the reciprocals of the squares of natural numbers:

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$$

, which can be regarded as the value of the Riemann zeta function at s = 2. We explore the intriguing relationship between π and the arithmetic object (such as integers) from which we started. This investigation allows us to gain deeper insights into the arithmetic object through the lens of its associated special value. We also examine functions defined on integers and their characters and other L- functions arising from geometric and analytic constructions. This encompasses the study of how these functions and L-functions are related to the underlying arithmetic objects and their properties. The broader idea is to elucidate how the special values of these functions provide valuable information about the objects themselves. Another aspect of my research revolves around the representation of certain groups. In this context, we explore the symmetries exhibited by objects and delve into how symmetry relates to the internal structure of these objects. An example is considering the notes produced by a drum as the eigenvalues of specific differential operators. The eigenvalues, governed by the drum's symmetrical characteristics, unveil intricate equations and relationships that reveal insights into the drum's underlying properties.

4. Have you personally ever experienced that looking at a problem from the perspective of a field helped in solving a problem from a different field?
Considering the actual research problem I work on, it is difficult to pinpoint exact instances where a particular result from a very different-looking field came to help. These things can happen at a subconscious level.

There are no hard boundaries; there is always a grey area. Mathematics is a very organic and fluid entity, and one has to swim around and know different things. It is difficult to stay in one area of mathematics without learning anything at all. Maybe it is possible for some exceptional areas in mathematics, but in general, mathematicians have to use insights from other fields. On the other end, for a complete layperson, the connections may not be evident; this is the nature of mathematics.

5. If there is a certain result or standard way of proving a class of problems when we look at alike but new problems, we tend to think along similar lines. So is that how solving research problems, in general, starts?
Yes, most of the time. Unless it is a completely new area and a new paradigm of looking at something. Often there are some motivating problems which have been solved. Whereas in the case of Ramanujan, who hadn't studied traditional mathematics in the way most of his contemporaries had. Without looking at different templates, he could create results. But for others, most of the time, we stand on other giants' shoulders. Though there are certain templates to look at, one should be creative and informed. One should keep an eye out for such results across different fields.
On the other end, there are instances where, if not as extreme as Ramanujan, people have created works on their own without looking at a different template. So both these happen.

6. Data science and powerful computational tools have influenced almost all of science and mathematics. Since you are also teaching a few data science courses here, how do you feel data science ties in with all these?
Let's take the example of number theory, where conjectures are often guesses about the existence or non-existence of something; it is impossible to verify every example by hand. With increased computing power, one can hunt for a possible counter example and disprove a conjecture. But there is no way to prove something only using computing power. One can argue that at some point in time, Artificial Intelligence will prove, which is a different way to look at it. Just with data and computing, one can only get more patterns and insights to prove analytically.
Data science is a vast and rapidly evolving concept. One major part of data science is statistical thinking. Since my BSc days, I have studied statistics; though I haven't done any research in statistics or used serious statistics in my research, I am always passionate about learning the subject on my own. I have done my Master's at Indian Statistical Institute, so some of my courses there were in this area. That has motivated me. Out of interest, I decided to teach the basic compulsory course for all BS-MS students, 'Probability and Statistics', and I certainly enjoyed it. Similarly, there are advanced courses in statistics and statistical inference that also I taught. My love for statistics and my love for teaching are combined. I cannot integrate my actual research with these kinds of courses because these are different areas. But just as a subject, mathematical thinking is involved there, slightly different, non-conventional mathematical thinking, which is called probabilistic thinking. Although these are traditionally more applied subjects, when introduced to the subject, a student must see both the rigorous part of the course as well as how it is applied. If they miss one of them, then their education won't be complete. So I try to keep the balance between both these aspects of the subject in my teaching. The previous semester, my colleague from data science and I developed a new joint course in mathematics and data science. My contribution was to introduce the mathematical concept behind the Bayesian theory of inference.

7. Given your strong passion for teaching, I would like to inquire about how you effectively balance your teaching responsibilities with your research endeavors.
Balancing teaching and research can be challenging due to demanding situations and time constraints.I allocate my time and energy appropriately between teaching and research to manage these responsibilities effectively. For example, when preparing to teach a course with a lecture in two days, my focus is primarily on course preparation, separate from research activities This localized approach is

commonly employed by my colleagues as well. However, it is important to note that allocating an equal amount of time to teaching and research is virtually impossible in the long run. I dedicate more time to teaching during the semester while compromising slightly on research. During breaks between semesters, I prioritize research and exploration. The guiding principles of institutions like IISER, which aim to bridge the gap between research and education, are immensely helpful. Although finding the optimal strategy for balance is challenging, I am grateful for having minimal teaching responsibilities compared to an average college, allowing me to focus more on research. In mathematics, teaching indirectly influences research and vice versa. Concepts from my research naturally influence my curriculum, fostering an exchange of ideas. Although publishing results in mathematics can be infrequent and challenging, assignments and homework serve as research problems that cultivate innovative thinking in students, preparing them for future research pursuits. This reciprocal interaction contributes to the growth of individuals as they apply knowledge gained from teaching to their research endeavours.

8. In addition to your academic pursuits of teaching and research, could you share any other hobbies or interests that you engage in?

I find great pleasure in reading. In the past, I used to immerse myself in science fiction novels and works in Marathi (my mother tongue) and English. Another hobby that has been with me since childhood is chess. Although I have not played the game professionally, my enthusiasm for it remains undiminished. Recollecting a delightful incident that intertwines my academic life and chess during the International Congress of Mathematicians (ICM) held in India in 2010, Vishwanathan Anand, the world chess champion at the time, participated in an event alongside the ICM attendees. I had the incredible opportunity to play a game of chess with Anand and managed to sustain the match for 40 moves.

Prof. Bhagwat with Viswanathan Anand in ICM 2010.

To commemorate this experience, I received a chessboard signed by him —a cherished memory indeed. In addition, I also derive enjoyment from watching cricket and movies.

9. Do number theorists tend to have a preferred/ favourite number? If so, could you also mention what your favourite number is?

I cannot say whether all number theorists have a favourite number or if it is the same number for most mathematicians, but for me, it is 'π'. π holds numerous fascinating properties and connections that have captivated mathematicians for centuries. Also, √ π is often underrated compared to π. Although its geometric significance might not be immediately evident, √ π manifests itself in various contexts. For instance, it prominently appears in the Gaussian distribution, widely used in big data analysis and statistics, in analytic number theory and other fields. Indeed, it is true that mathematicians from various disciplines often develop a fondness for specific mathematical objects. Among those, the group $SL_2(\mathbb{R})$ holds a special place in my mathematical journey. This group exhibits a captivating charm due to its rich properties and elegant structure.

Exploring the Enigmatic Tapestry of Kashmir's Mathematical Heritage and its Prodigal Role in the Mathematics Revolution

Krishna Saproo

Testled amidst the majestic splendor of the Himalayas, the land of Kashmir unveils a hidden treasure trove: a remarkable legacy of mathematical brilliance. This article delves into the intricate layers of Kashmir's enigmatic mathematical heritage, intricately interwoven with the global mathematics revolution. From ancient Sanskrit texts to groundbreaking discoveries, Kashmiri mathematicians have made significant contributions to the tapestry of mathematical knowledge. Join us on this exploration as we unravel the mystique surrounding Kashmir's profound role in the realm of mathematics.

Vaṭeśvara:

Vaṭeśvara was a 10th-century Indian mathematician and astronomer from Kashmir. He was born in 880 and authored the Vaṭeśvara-siddhānta, a treatise on astronomy and applied mathematics, at the age of 24. The work criticized Brahmagupta and defended Aryabhata I.

Al-Biruni referred to Vaṭeśvara's works, particularly the Karaṇasāra, noting that the author was the son of Mihdatta who belonged to Nagarapura (also given as Anandapura, which is now Vadnagar).

The Vaṭeśvara-siddhānta is a significant work in the history of mathematics. The work presents a number of new mathematical ideas, including the use of trigonometric functions to solve astronomical problems. The work also presents a number of new mathematical techniques, including the use of series to approximate functions. The work was written in Sanskrit and is divided into 15 chapters. It has chapters devoted to astronomy and applied mathematics. The work was widely used in India and other parts of Asia for centuries. The work was also translated into Arabic and Persian. The Vaṭeśvara-siddhānta is a valuable source of information about Indian astronomy and mathematics. The work is a testament to the mathematical and astronomical skills of Vaṭeśvara and his contemporaries.

Sanskrit Texts as Vessels of Wisdom:

The land of Kashmir is home to a rich and vibrant mathematical tradition, dating back to the 11th century. This tradition is preserved in a vast corpus of Sanskrit texts, which offer a glimpse into the intellectual and mathematical prowess of the Kashmiri people.

These texts cover a wide range of mathematical topics, including number theory, algebra, arithmetic, geometry, and trigonometry. They also contain insights into the history of mathematics, as well as the philosophical and religious underpinnings of Kashmiri mathematics. The Vateshwara Siddhanta is a prime example of this.

Another important Sanskrit text on mathematics is the Siddhanta Shiromani, or "Crown of the Siddhantas". This text was written by Bhaskara II in the 12th century, and it is considered to be one of the most comprehensive works on mathematics in the Indian tradition. The Siddhanta Shiromani covers a wide range of topics, including arithmetic, geometry, trigonometry, and astronomy. This was adored by Kashmiri mathematicians.

The Sanskrit texts on mathematics from Kashmir represent a significant contribution to the history of mathematics.

They provide a glimpse into the intellectual and mathematical prowess of the Kashmiri people, and they offer insights into the development of mathematics in the Indian tradition.

In addition to their mathematical content, these texts also provide a valuable insight into the philosophical and religious underpinnings of Kashmiri mathematics. For example, the Bija Ganita is prefaced with a verse from the Bhagavad Gita, which emphasizes the importance of knowledge and understanding. This verse suggests that Kashmiri mathematicians saw mathematics as a path to spiritual enlightenment. The Sanskrit texts on mathematics from Kashmir are a valuable resource for scholars of mathematics, history, and religion.

The Lingering Influence on the Global Stage:
Like ripples on a serene lake, the mathematical reverberations of Kashmir extended far beyond its geographical confines. A vibrant nexus of cross-cultural exchanges blossomed, attracting scholars from distant lands to Kashmir's intellectual sanctuary. In the embrace of Kashmir's luminous mathematical minds, these scholars absorbed knowledge, spirit, and innovation, carrying their newfound enlightenment back to their homelands. This intercontinental symbiosis of ideas nurtured the burgeoning mathematics revolution, positioning Kashmir as a seminal catalyst in the global mathematical discourse.

Resurrecting the Dormant Glory:

In the twilight of history, valiant endeavors have emerged to revive and resurrect Kashmir's dormant mathematical glory. Organizations and institutions, driven by the ardor of intellectual preservation, spearhead initiatives to safeguard ancient manuscripts, unravel the mysteries of untranslated texts, and undertake extensive research on Kashmiri mathematics. Such resolute efforts not only shed light on the past achievements of Kashmir's mathematical virtuosos but also ignite the ember of inspiration within a new generation of Kashmiri mathematicians, urging them to explore, unravel, and contribute to the ever-evolving frontiers of mathematical excellence.

A Kashmiri Poet's Ode to Mathematics:

"The language of numbers is sweeter than honey,
Its melody more pleasing than the song of the nightingale.
Its beauty is more resplendent than the sun and moon,
Its light more radiant than the stars.
Mathematics is the key to all knowledge,
The gateway to all wisdom.
It is the language of the universe,
The blueprint of creation.
I bow down to mathematics,
The queen of all sciences,
May she bless me with her wisdom,
And guide me on the path of truth."
- Lal Ded (14th century Kashmiri poet)

Conclusion
Kashmir's mathematical heritage stands as a testament to the region's intellectual prowess and its significant contribution to the broader mathematics revolution. From the pioneering works of Mahavira Acharya to the groundbreaking algebraic discoveries of Bhaskara, Kashmiri mathematicians have left an indelible mark on the field of mathematics.

By recognizing and revitalizing this rich legacy, Kashmir has the potential to once again become a hub for mathematical scholarship and research. Through the preservation of ancient manuscripts, translation of texts, and the promotion of mathematical education, Kashmir can reclaim its rightful place in the global mathematics community and inspire future generations to continue the tradition of mathematical excellence.

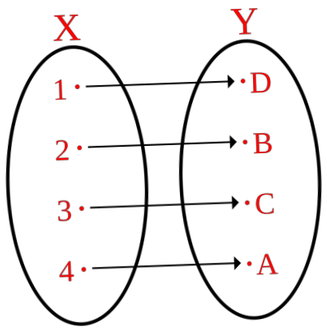
NEW EVENTS

Let's Talk Maths

Usual ways of math communication present math in its pure, raw form directly. But these do not do a good job at inculcating enthusiasm and motivation within students.
This year, we introduced 'Let's Talk Math' an initiative which aims to present mathematics in the form of discussions rather than formal talks – motivating ideas through healthy discussions.Through this, we hope to create a platform for exchange of interesting mathematical ideas, and hope to facilitate interactions about various areas of mathematics and of active mathematical research. This also provides an opportunity for those who are interested in presenting some piece of math which they have read or worked on for quite some time to present it in a more informal way than a typical talk. We hope that this connects math people together and establishes a better math community in IISER Pune.
Two Let's Talk Math sessions were organized this year moderated by:

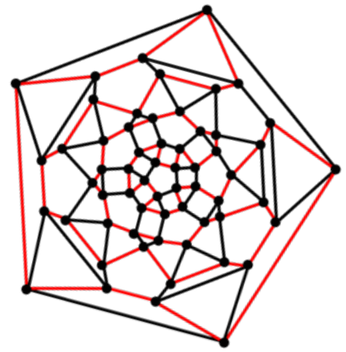
1. Sreedev M
2. Varun Shah

Read on to get a better perspective about this new and unique initiative, and learn some exciting mathematics along the way!



Bijective Combinatorics

-Shreyas Pai



Let's Talk Math is a new series of talks introduced by the Maths Club of IISER Pune. Thanks to the coordinators of the Club for implementing this amazing idea where people from the IISER community could come together to discuss a topic that was chosen for the discussionby the moderator of the talk, who would start the discussion and put together all the ideasthat came up in the session. This time the talk was on 'Bijective combinatorics'

Combinatorics is an exciting branch of mathematics which mainly deals with counting. Usually, from a given set of objects, we want to count the number of objects that satisfy a particular property. But, Counting is sometimes easy, sometimes hard. It may be hard to count what is required directly. We may find another set of objects that is easy to count with a one-to-one correspondence to the original set of objects we aimed to count. This is what bijective combinatorics is all about. So, I was very excited to attend this session. The moderator of the session was Varun. I had heard that he was a good math communicator, and it was indeed the case as he kept all of us engaged in the discussion of bijective proofs, and none of was felt lost during the session. He motivated us throughout the session to think about the various problems he presented and to come up with proof for those problems. He started with some basic concepts of combinatorics, and then it was a problem-solving session!

The concepts covered were as follows. A bijection is a one-to-one map between two sets; thus, the two sets have the same number of elements, which allows us to convert a problematic counting problem into an easy counting problem. And then he put up a couple of problems, which were:

If p is a prime and a is a positive integer, then show that $a^p - a$ is divisible by p.

and

A composition of n is a sequence $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k)$ of positive integers such that $\sum_{i=1}^{k} \alpha_i = n$ for some k. What is the number of compositions of n?

The idea that we came up with to tackle the first problem, a famous theorem named Fermat's little theorem combinatorially, was to think about the term $a^p - a$. Before that, let us assume that $'a'$ is not divisible by $'p'$ as $'a'$ being a multiple of $'p'$ holds trivially. Suppose we had a collection of beads of $'a'$ different colours and aimed to make a necklace of $'p'$ beads so that all beads are not of the same colour. The number of ways of doing it was the term mentioned above. We aim to partition this into sets $'p'$elements to complete the proof.

Observe that the necklaces we are considering have at least two different colours. If we consider one such necklace, we can rotate this necklace $'p'$ times and get other permutations of the necklace, which we have counted in the term $a^p - a$ For example, if we denote $x_1, x_2, \ldots, x_a$. $'a'$ to be the different colours we chose, then the necklace $\underbrace{x_1 x_1 x_1 \cdots x_1}_{p-1 \text{ times}} x_2$ has different permutations-

$(\underbrace{x_1 x_1 x_1 \cdots x_1}_{p-1 \text{ times}} x_2),\ (\underbrace{x_1 x_1 x_1 \cdots x_1}_{p-2 \text{ times}} x_2 x_1),\ (\underbrace{x_1 x_1 x_1 \cdots x_1}_{p-3 \text{ times}} x_2 x_1 x_1), \ldots, (x_2 \underbrace{x_1 x_1 \cdots x_1}_{p-1 \text{ times}}).$

Now, one may be thinking that it may so happen that distinct permutation (linear) for a particular combination of colours may be less than $'p'$; if that is the case, then there must be a sequence of colours that is repeating in the given permutation; thus, the length of that smaller repeating unit must divide the length of the necklace which cannot happen as the length of the necklace is a prime number. We can group all the necklaces which differ from one another by rotation, and each of these groups has exactly $'p'$ elements. Thus, the number $a^p - a$ is divisible by $'p'$.

For the next problem, we came up with the idea of splitting n as follows:

$$\underbrace{1\square 1\square 1 \cdots \square 1}_{n \text{ times}}$$

where, in the box, we either write a `,' or a `+'. The number of 1s separated by a `+' are to be added together to form a positive integer ( $\alpha_i$ ), whereas `,' would indicate the `,' in the composition mentioned in the problem. Thus, the number of ways of splitting $n$ into $k$ is the same as counting the sequences containing `+'s and `,'s of length $n-1$, the same as looking at all the binary digits of length $n-1$. Thus, this number is 2^{n-1} , hence the answer.

The latter problem deals with the idea of partition, where we are partitioning (or breaking) a given number into smaller numbers.

The order in which numbers are written is not important in a partition. A problem related to it was-

Prove that the number of partitions of n with the largest part being k is equal to number of partitions of n with exactly k parts.

This was the main content of the discussion. Then, he briefed us on some concepts and gave us some problems.
A graph is a collection of vertices and the edges connecting these vertices. There is a special kind of graph called trees; these don't have loops, where a loop is an edge whose endpoints are the same vertex. If every node has at most two children nodes, then such a tree is called a binary tree. We could label these vertices of the binary tree, and if we number them such that each vertex is smaller than its children, then such labelling is called increasing. Thus, one could ask the following question-

How many such increasing binary tree of n vertices exist?
The answer is $n!(nfactorial)$.
A permutation, as we all know, is a rearrangement of given sets of objects. One familiar with group theory knows that a permutation can be represented as a cycle, representing a bijective map. A question that comes up is something like-

Let σ be a random permutation of the set $\{1, 2, \ldots, n\}, n \geq 2$ .What is the probability that 1 and 2 belong to the same cycle?

Catalan numbers are another critical class of numbers that help count the number of valid bracketing strings of letters or triangulation of a polygon (i.e., dividing a polygon into triangular pieces). Lastly, he talked about some higher mathematics concepts in matrices, colouring problems and representation theory as part of the motivation.
It was truly a wonderful session, with the take-home message being that bijective proofs are cleaner and neater. It provided more insights which helped generalise the given problem to get a more interesting one. Moreover, from the session, we all felt that this way of proving a given equation or finding an answer is aesthetically more pleasing that doing a bunch of algebra.
I would like to thank Varun for this unique session and for the time and effort he put into preparing for the same.

MS Talks

How does it feel to write an MS thesis? What are the kind of open problems that are commonly taken up by a MS thesis student? These are some questions that commonly arise in the mind of every undergraduate at IISER. To address them, Maths club organised a new event 'MS Talk series', in which 5th year students presented their thesis work in a manner accessible to a broad audience. While their work enthralled the audience, they also briefly spoke about their experiences during the time, which attempted to distance the fear that one has about stepping into research. Below is a list of the illustrious speakers during the inaugural edition of the event.

1. Mihir
2. Aravintakshan
3. Karthik Vasisht
4. Ajaykrishnan
5. Shruthi
6. Anand Tadipatri
7. Hrishikesh V
8. Ipsa Bezberua
9. Sreedev M
10. Anirudh Rachuri

Read on to know more about their work..

P, NP and Finding Nemo

By Ajay Krishnan

Hey doc, heard you recently joined the CS department. Could you tell me a little about what you work on?

Sure, let's see. Have you seen Finding Nemo?

Ummm... Sure, yeah. A bit confused by that question, but yeah, I have seen the film.

Awesome. Imagine yourself to be a tiny fish frantically looking for directions to "P. Sherman, 42 Wallaby Way, Sydney". Since time is of the essence, you and your friend collect various pieces of information which might help find the way. Unfortunately, many of them end up contradicting each other and you are left with no choice but to ignore a few. Can you figure out the smallest number of hints which can be ignored to make the remaining ones consistent? Equally importantly, how quickly can you do it?
The above is an example of a computational problem, you have a question, which has to be answered on a given input using some calculations. The magical set of instructions which allow you to solve any input instance of a problem is called an algorithm. The question that people like me are typically interested in, is the amount of resources such as time and space required to run an algorithm on various instances.

So, can we do it? Can we solve the problem that you mentioned quickly?

Well, the solving part is straightforward enough. Any reasonable computational problem can be brute forced. We just make a list of all possible solutions and check each of them, one by one. But as you can imagine, the list might be very long. The quick part on the other hand, gets a bit more complicated. To begin with, how do you quantify the speed of an algorithm? We very well can't use computers since the speed of execution is dependent on the hardware. The mathematical way out is to quantify it in terms of input size. Say for example, we have the earlier problem with n pieces of information. Can we solve it using just n many basic operations, such as addition, multiplication, logical AND, logical OR and so on? How about n2 many? If we can do so with poly(n) many operations, then we consider that algorithm quick.

I see. So, is there a quick algorithm for this one? Or for any problem for that matter?

Well, here I am going to have to resort to one of my favorites - "I don't know". I don't think so, and an overwhelming majority thinks the same for this question, but no one has any idea how to prove this belief either. So, anything could be possible.
The question you just asked has been formally stated before. Allow me to explain intuitively. Let us denote by P, the set of computational problems, which can be solved quickly. These are problems for which there exists an algorithm, which takes input instances of size n and returns the correct solution in poly(n) time. Some common problems in P involve searching within a list of numbers or finding the maximum matching in a graph.
Alongside this, let us denote by NP, the set of problems whose solutions can be verified quickly. These are problems whose solutions can easily be checked, say for example, whether an element is the largest in a list of n numbers, which can be done using at most n-1 comparisons. The question is whether P = NP or not. So far nobody knows, even though most people believe it to not be the case. Now, the reason is hard to get into, so I shall refer you to a book named "The Golden Ticket - P, NP and the search for the impossible" by Lance Fortnow.

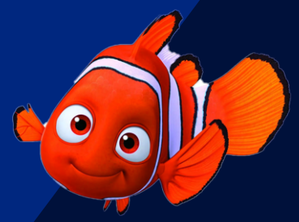
But suffice it to say, if P did equal NP, then any problem whose solution you can verify, you can solve. Which would imply leaps of progress in anything and everything. Medicine, Architecture, Engineering, you name it. Hence the disbelief. That, alongside the fact that people have been unable to find quick solutions for many of these problems for nearly centuries now. Anyways, I digress.
Do you have any more questions?

Yeah, just one more. If P does not equal NP like people believe, then doesn't that imply that there are problems which are inherently hard, ones that cannot be quickly solved? What do we do then?

Great question. The thing is, P not being equal to NP only implies that there cannot be a fast algorithm which correctly solves every possible input. We could compromise on one (or more) of these criteria and work with the remaining. We could look for quick algorithms which can work with all instances and find solutions which come close to the correct one, say a solution of size 2k if the correct solution has size k. We could look for algorithms which produce the correct solution for all instances slowly, yet faster than brute force. Or we could look for algorithms which give the correct solutions quickly, but only for input instances with some special properties. There are loopholes, we just have to be okay with making compromises

Interesting. I'd like to learn more about the thing you did during your Master's, but I am running late for a meeting. Will it be okay if I mailed you?

Absolutely. My institute mail id is firstname.lastname@students.iiserpune.ac.in, so you won't have trouble finding out. For that matter anyone listening in on this conversation (turns to look at you, the reader) can also feel free to



Interactive theorem provers
History, Ideas and Future

By Anand Rao Tadipatri

Broadly speaking, interactive theorem provers or proof assistants are a kind of mathematical technology for constructing and verifying mathematical proofs. The verification of the proof is done automatically according to the rules of the underlying mathematical foundation, while the construction is done interactively with inputs from the user. Interactive theorem provers can be regarded as tools for mathematical reasoning, playing a different role than other mathematical software built for purposes such as visualisation, symbolic computation, statistical analysis or document generation. With expressive syntax and the facility to build up proofs interactively at a high level, modern proof assistants are beginning to make the large- scale formalisation of mathematics practical [Geu09]. There has been a tremendous amount of recent progress on formalising important mathematical results, leading to the development of large libraries of formal mathematics such as Lean's mathlib [Com20], Isabelle's Archive of Formal Proofs [MFV21], Coq's Mathematical Components libraries [MT17], the Mizar Mathematical Library [NK09] and the Agda-UniMath library [RBP+].
Notable formalisations include those of the four-colour problem [Gon+08], the Jordan curve theorem [Hal07], the prime number theorem [Har09], the Kepler conjecture [Hal+17], the odd order theorem [Gon+13], the central theorem of condensed mathematics [Sch22] and sphere eversion [MDN22].
Interactive theorem provers and related technology have immediate uses for formalising of mathematics, formal verification of software and hardware, organising mathematical knowledge into vast libraries and as educational tools for learning and creating mathematics. As the technology continues to rapidly develop, there are many more potential uses for interactive theorem provers beyond formalisation which can together transform interactive theorem provers into valuable tools for mathematical research.
Interactive proof documents generated automatically from formal proofs, as demonstrated in Patrick Massot's recent IPAM talk or at the following web-page https://www.imo.universite-paris-saclay.fr/~patrick.massot/Examples/ContinuousFrom.html, allow one to expand and collapse proofs at any level of depth, not just the one set by the author.

Interactive games in Lean, such as Kevin Buzzard's Natural Number Game, are wonderful educational tools. Interactive theorem provers can also be integrated with dynamic visualisation tools, as seen in 1 to complement formal deductive proofs. Several proof assistants are also full-fledged general purpose programming languages, making them the ideal tools for proved algorithms – algorithms intertwined with formal proofs – which can be used, for instance, to create libraries of verified symbolic algebra. Most importantly, recent advances in machine learning – particularly large language models – open up a vast range of possibilities for integration of AI and formal reasoning, for example in developing tools for automatic translation of mathematical statements into a formal language (see 2).

In conclusion, interactive theorem provers are powerful tools that hold great promise in changing the way we do mathematics. By integrating with other kinds of mathematical software, interactive theorem provers may become indispensable tools for mathematicians to experiment, discover, create, automate, verify, communicate and understand mathematics.
The next major steps towards this vision will likely involve a strong mathematical component – advancing foundations to bring formalism closer to mathematical practice, developing strong automation and decision procedures, and reformulating and organising mathematical theories in a way that aligns with both intuition and formalism. Advances in computer science and artificial intelligence will no doubt play a big role too, with collaboration tools, programming language design and automatic program synthesis heavily influencing the design and use of interactive theorem provers. Overall, interactive theorem provers point towards a future of fruitful mathematical collaborations between humans and computers.

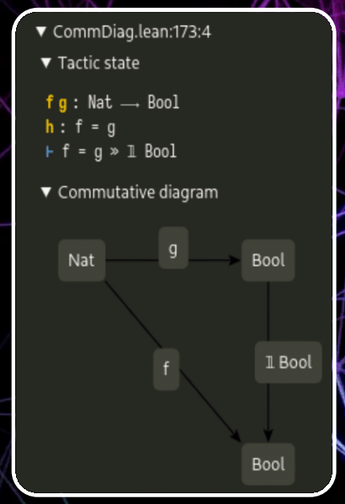


Figure 1: Automatic rendering of a commutative diagram from the goal state using Lean4 widgets

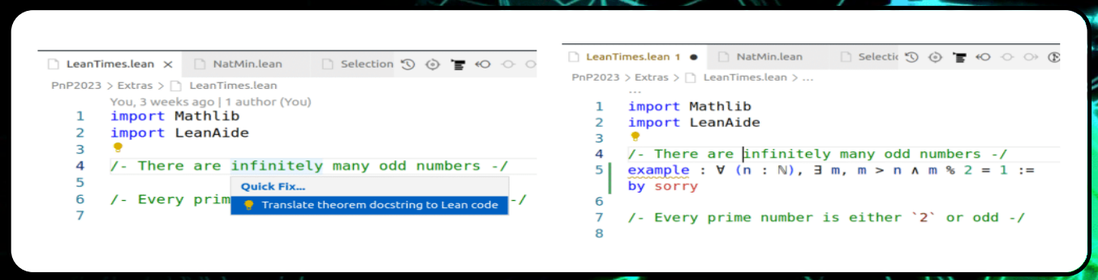


Figure 2: A screenshot of LeanAide in action, see this youtube video https://youtu.be/_IZe3fBZLBR0

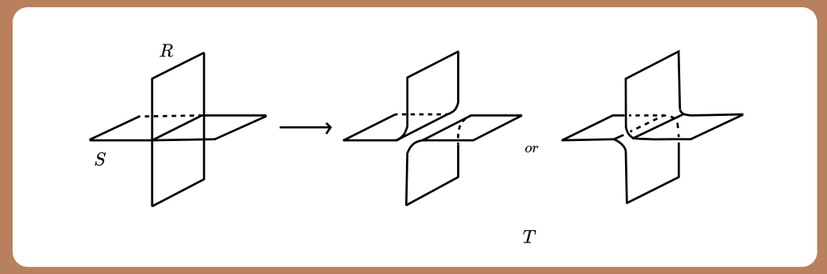
Simplicial Complexes on Seifert Surfaces of Links

By Ipsa Bezbarua

A **knot** is an embedding of $\mathbb{S}^1$ in $\mathbb{R}^3$, while a **link** is the embedding of several disjoint $\mathbb{S}^1$ in $\mathbb{R}^3$. The study of knots and links is called **knot theory**. Over the last century, it has developed into an active area of research in the field of topology. This is partly due to its connections with several other areas of science and technology.

One of the fundamental structures studied in knot theory is a compact connected surface whose boundary is the knot under consideration. Such a surface is called a **Seifert surface**. If we relax the connectedness criterion in this definition, we obtain what is called a **spanning surface**. Two spanning surfaces are said to be **equivalent** if they can be isotoped into each other, which is a technical term for saying that they can be deformed into each other without cutting. The set of all equivalence classes of spanning surfaces for a given link L is denoted by $\mathscr{S}(L)$.

We call a surface **incompressible** if it is not a sphere or a disc, and it satisfies the following condition: if a closed curve on the surface bounds a disc in the ambient 3-manifold, then it must also bound a disc on the surface. A spanning surface is called **minimal genus** if its Euler characteristic is the highest amongst all spanning surfaces for a given link, meaning it has the minimal number of "1-dimensional holes" amongst all possible spanning surfaces for the link. We denote the set of equivalence classes of all incompressible spanning surfaces for the link L by $\mathscr{IS}(L)$ and the set of equivalence classes of minimal genus spanning surfaces for L by $\mathscr{MS}(L)$.
In my thesis (carried out under the supervision of Prof Jennifer Schultens of the University of California Davis), I worked with two simplicial complexes for a given link - the **incompressible complex** and the **Kakimizu complex**. These complexes were defined by Osamu Kakimizu. The incompressible complex, denoted by $\mathcal{IS}(L)$ has vertex set $\mathscr{IS}(L)$ such that the vertices $\sigma_0, \ldots, \sigma_k$ span a k-simplex whenever they have disjoint representatives.
The Kakimizu complex, denoted by $\mathcal{MS}(L)$ is defined the same way, except using $\mathscr{MS}(L)$ instead of $\mathscr{IS}(L)$. Kakimizu showed that these complexes are **connected**.



Performing the **cut-and-paste operation** (see figure) along two minimal genus spanning surfaces gives a new minimal genus spanning surface. However, this does not hold when performing the operation along two incompressible spanning surfaces. As cut-and-paste is a very simple process, the Kakimizu complex became more widely studied than the incompressible complex.

The Kakimizu complex is known to have several exciting exotic properties. For instance, Jessica Banks showed that the twisted Whitehead double of the trefoil knot is **locally infinite** (has a vertex adjacent to a countably infinite number of other vertices). Further, she also showed that a locally infinite Kakimizu complex can only come from the satellite of a torus knot, a cable knot or a connected sum, with winding number zero. Jennifer Schultens showed that the Kakimizu complex is **simply connected**, and later, in collaboration with Piotr Przytycki, that it is also **contractible**. Further, Jesse Johnson, Roberto Pelayo and Robin Wilson showed that the Kakimizu complex is also **quasi-isometric to** $\mathbb{Z}^n$.

A possible future direction from this point onwards would be to extend these properties to the incompressible complex. One can also explore the potential of a novel way of classifying knots using these simplicial complexes, as Banks' results seem to imply that this direction could be promising.

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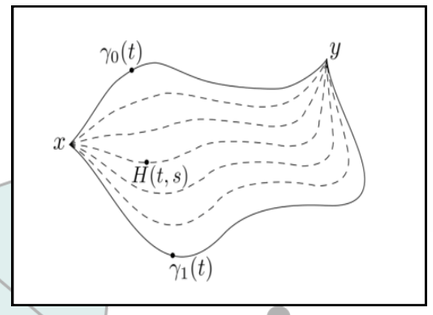
Algebraic Topology and Simplicial Sets

By Karthik Suraj Vasisht

It is quite difficult to classify topological spaces up to homeomorphism. The question arises whether there is a weaker condition one may impose which leads to a more tractable classification theory. One possible situation is to view two topological spaces as equivalent if they have the same *homotopy type*. A *homotopy* between two maps $f, g : X \to Y$ is a continuous map $F : X \times I \to Y$ such that $F(x, 0) = f(x)$ and $F(x, 1) = g(x)$. This can be thought of as a movie. At each , we have a map F_t that can be thought of as a snapshot of the movie. Now, a map $f : X \to Y$ is called a *homotopy equivalence* if it has a *homotopy inverse*, that is, a map g in the opposite direction such that both $f \circ g$ and $g \circ f$ are homotopic to their respective identities. In this case, the topological spaces are said to have the same *homotopy type*.

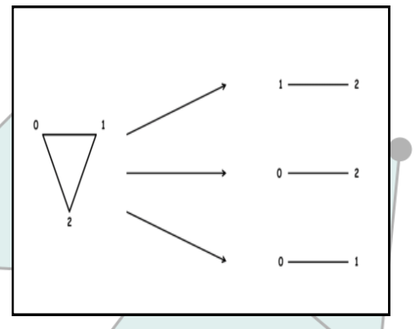


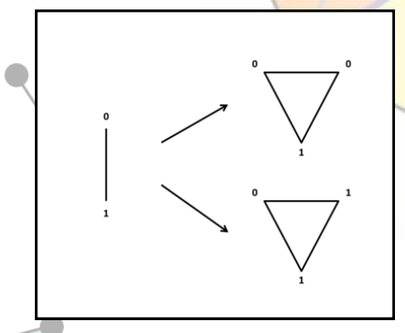
A discussion of homotopy type is incomplete without the discussion of fundamental groups. Loosely speaking, fundamental groups tell us how many "types of loops" can be drawn in a space. Fundamental groups are one of many groups known as homotopy groups which are instrumental in characterising the homotopy type of a topological space.

Theorem : Let $(X, x_0), (Y, y_0)$ be pointed topological spaces and let $f : X \to Y$ be a basepoint-preserving homotopy equivalence. Then, f induces an isomorphism on all homotopy groups.

The above theorem reduces the problem of classifying topological spaces to that of computing fundamental groups and finding which groups are in some sense ”admissible”. However, it is too early to dust off our shoulders. Computing homotopy groups is no easy task. In fact, we know very little about the homotopy groups of spheres, one of the simplest classes of spaces known to mathematicians. (See this ncatlab page https://ncatlab.org/nlab/show/homotopy+groups+of+spheres.)
Topological spaces are not very easy to work with. Our imagination aids us till the third dimension, but beyond that many spaces are hard to visualise. To understand them we often resort to roundabout methods, such as computing homotopy groups. These practical difficulties motivate the introduction of a new structure which can be related to topological spaces but provides an easy way to manipulate it. Simplicial sets are one such structure.

Definition 0.1. A *simplicial set K* is a graded set indexed by $\mathbb{N} \cup \{0\}$ along with maps $\partial_i : K_n \to K_{n-1}$ (face maps) and $s_i : K_n \to K_{n+1}$ (degeneracy maps), 0 ≤ i ≤ n which satisfy the certain properties.
Put simply, a simplicial set is a collection of *n*-simplices, for each *n* ≥ 0, along with two types of maps, called face and degeneracy maps that relate the different simplices in the simplicial set. The 0-simplices are often called vertices, the 1-simplices edges, the 2-simplices faces, and so on. The face maps contain information on how to glue simplices together, while degeneracy maps tell us how to collapse multiple simplices into one.





Just like for topological spaces, we can define homotopies and homotopy groups for simplicial sets as well.

Definition 0.2. Let *K* be a simplicial set. Two *n*-simplices *x, x′* are said to be *homotopic*, denoted $x \sim x'$, if $\partial_i x = \partial_i x'$ for all 0 ≤ i ≤ n and there exists $y \in K_{n+1}$ such that:
(i) $\partial_n y = x, \partial_{n+1} y = x'$
(ii) $\partial_i y = s_{n-1}\partial_i y = \partial_i x$

One way to think about the above definition would be to relate it to the topological definition by observing the case of a path homotopy. In this case, the whole movie of the path deforming appears to be filling in a two-dimensional space, effectively a 2-simplex. The initial and final paths are both “faces” of this filled in region. Can you guess what the third face is?
Things become more interesting when we relate these abstract objects to topological spaces. To any simplicial set, we can associate a topological space called its *geometric realization*. This involves collapsing and gluing the simplices based on the face and degeneracy maps as mentioned earlier. Interestingly, the geometric realization of a simplicial set has the same homotopy type as the simplicial set itself. To elaborate, we may also define simplicial homotopy groups for certain nice simplicial sets. (For a definition, see [May67, Definition 3.6].)
The homotopy groups of these nice simplicial sets coincide with the homotopy groups of the simplicial set’s geometric realization. Thus, we may choose to do all our tedious computations using simplicial sets, a more manipulable structure, instead of topological spaces.
All in all, simplicial sets have proven to be useful in a number of fields such as stable homotopy theory and higher category theory. They provide an alternative category to do homotopy theory in, and in many cases offer advantages that the category of topological spaces does not.

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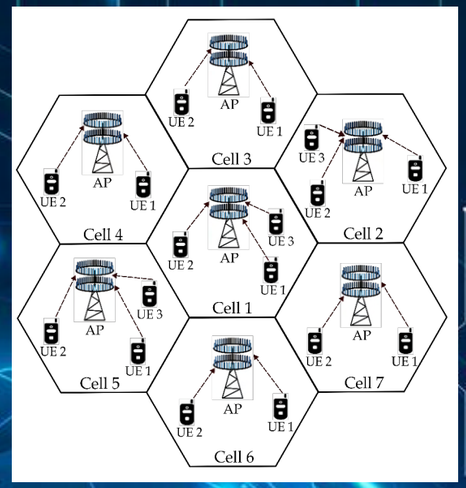
Moving towards 6G networks!

By Shruthi Prusty

Of course, that's just the marketing strategy here. Still, our problem of focus makes sense only in the context of wireless networks. Wireless communication is enabling billions of people to connect to each other and the internet, transforming every sector of the economy, and building the foundations for powerful new technologies that hold great promise to improve lives at an unprecedented rate and scale. The current technology for wireless communication of voice and data is cellular networks, in which the coverage area is divided up into non overlapping cells and we have a single access point (AP) coordinating transmissions amongst user devices/equipment (UEs) within its cell (see Figure 1). Cellular networks are categorised into 'generations' to portray the improvements of a current generation over previous generations. 1G to 3G networks prioritized only telephone communications but afterwards data transmission became an added requirement in the network and a dominant feature in 4G and 5G networks.

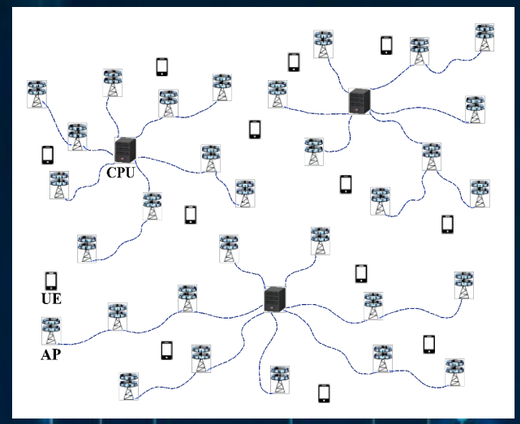
Cellular networks face a number of challenges which are inherent to the type of network architecture, including interference between cells. Presently, the growing demand for high-speed data transmissions and broader network coverage, as we gradually move to an era of machine-to-machine communications (with over 50 billion connected devices predicted in 2050), accentuates the limitations of cellular networks. This demand has necessitated the ongoing research for more robust technologies in future wireless networks (6G) using a different architecture called cell-free massive MIMO (CF-mMIMO). As the name implies, cell boundaries are removed and all UEs can be served by all APs in the network (see Figure 2). The goal of this network is to reduce inter-cell interferences, improve connectivity amongst users and enhance network reliability.



Cellular network architecture

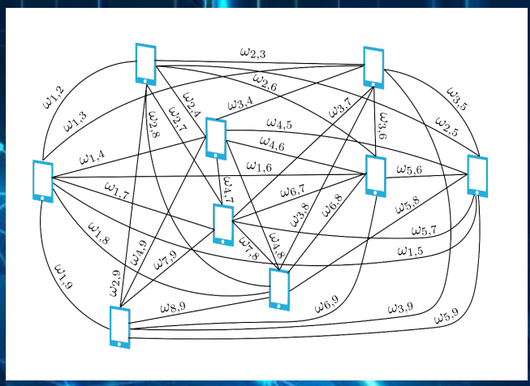
The CF-mMIMO architecture is usually visualized as a graph, with the UEs forming the vertices and the edges depicting the interference between the UEs (see Figure 3). CF-mMIMO is accompanied by many challenges of its own, one of which is efficiently allocating limited resources. A major resource allocation problem in wireless networks, namely the Pilot Assignment (PA) problem, has been mapped to several well-known graph theory problems in the proposed CF-mMIMO architecture, such as the graph vertex colouring problem and the max-k-cut problem.

These problems fall in the class of NP-hard problems. Thus, heuristic graph algorithms have been employed to find reasonable solutions to PA. Despite all the research effort that has been put into solving PA in cell-free massive MIMO systems, the Pilot Assignment problem has received very little attention from a theoretical computer science perspective. In particular, an explicit proof of the computational hardness of this problem is yet to be seen. Further, assuming the problem is indeed hard to solve, which justifies the various heuristic algorithms being employed to arrive at a reasonably close-to-optimal solution, there are no theoretical guarantees on how well one can approximate the optimal solution in a reasonable amount of time.



Cell-free network architecture

In our work, we make several breakthroughs in this direction. We not only study the inherent hardness of the Pilot Assignment problem, but also find provable guarantees on the quality of achievable solutions (approximability) of the problem. We further present approximation lower bounds for special cases of this problem, where there are exactly two or three pilots in the system. The big picture is that we fully characterize the relationship between a well-studied classical graph problem and the Pilot Assignment problem, thus shedding light on a new direction for future work on heuristic algorithms for PA.



Characterization of Uniform Pro-p Groups

By A P Aravintakshan

Uniform Pro-*p* groups are specialised family of pro-p groups that have extensive applications in the field of *p-adic* Lie theory, and pro-p groups in general. A major characterization of pro-p groups mentioned in [DMSS]- *A pro-p group has finite rank if and only if it admits a uniform open subgroup*.

Definition: A topological group G is *profinite* if it is Compact, Hausdorff and open subgroups of G form basis for the neighbourhoods of identity. Alternatively it is defined as the inverse limit of finite groups.
First we understand how Inverse limit of groups can have an associated topology. Let $\{(G/N) : N \in \Lambda, \pi_{NM}\}$ be an inverse system of a Group G where Λ is a directed set of a family of open normal subgroups of G, and the maps coming from natural epimorphisms $\pi_{NM} : G/N \to G/M$ where N ≤ M. Since G/N are finite groups, then by giving them discrete topology, and the product $\prod_{G/N}$ the product topology, the inverse limit Gˆ becomes a topological group with the induced topology.

Some main examples of Profinite Groups include:
• Any finite group is profinite as it is an Inverse limit of a trivial inverse system.
• The ring of *p*-adic integers, $\mathbb{Z}_{\mathbf{p}} = \varprojlim(\mathbb{Z}/p^n\mathbb{Z})$. This also is the most fundamental example of profinite group theory,
• $GL_n(\mathbb{Z}_p)$; $SL_n(\mathbb{Z}_p)$, for n ≥ 1.
• Aut(G), The automorphism Group of a finitely generated profinite Group G is profinite.
• The Principal Congruence classes of $SL_n(\mathbb{Z}_p)$ given by $\Gamma_i = \{g \in SL_n(\mathbb{Z}_p) | g \equiv 1_n \, mod \, p^i\}$ and $GL_n(\mathbb{Z}_p)$ are pro-p Groups. They are infact powerful, and $SL_1(\mathbb{Z}_p)$ becomes uniform pro-p, when p is an odd prime.

A Group is said to be pro-p if it is defined as the inverse limit of finite p-groups. Similarly, a group is said to be *procyclic* if it is defined as a limit of finite cyclic groups. Further in my thesis, we look to understand different families of pro-p groups through examples,with more focus on the families of uniform pro-p groups and powerful pro-p groups.
Definition Let G be a pro-p group. G is said to be *powerful* if $G/\bar{G^p}$ is abelian when p is odd, or $G/\bar{G^4}$ is abelian when p = 2.
Definition A pro-p group is said to be Unifomrly powerful or Uniform, if it is finitely generated , powerful and $|G_i : G_{i+1}| = |G : G_2|$.

Some major results in the theory of Uniformly powerful pro-p groups are as follows:

Theorem 1 A finitely generated powerful pro-p group is uniform if and only if it is torsion free.

Theorem 2 If G is a uniform pro-p group and G = $\overline{\langle a_1, \dots a_d \rangle}$. Then the mapping $(\lambda_1, \dots, \lambda_d) \mapsto a_1^{\lambda_1}, \dots, a_d^{\lambda_d}$ from $\mathbb{Z}_p^d$ onto G is a homeomorphism.

This "co-ordinate" like result on the structure of a Uniform pro-p group allows us to build unique additive and bracket structures that have crucial implications:

Definition: For $x, y \in G$, we define the additive structure + on a Uniform group G as follows :

$$x + y := \lim_{n\to\infty} x +_n y \text{ where } x +_n y := (x^{p^n} y^{p^n})^{p^{-n}}$$

Theorem 3 Let G be a uniform pro-p group of dimension d, and let $\{a_1, \dots, a_d\}$ be a topological generating set for G. Then, with the operations defined above, (G, +) is a free $\mathbb{Z}_p$-module on the basis $\{a_1, \dots, a_d\}$.

Definition: The bracket operation (,) for a uniform pro-p group G (for $x, y \in G$ and $n \in \mathbb{N}$ is defined as follows:

$$(x, y) = \lim_{n\to\infty} (x, y)_n, \text{ where } (x, y)_n = [x^{p^n}, y^{p^n}]^{p^{-2n}}$$

Theorem 4: With the operation (,) , the $\mathbb{Z}_p$ module (G, +) becomes a Lie algebra over $\mathbb{Z}_p$.

With all this theory, we delved into current research trends in the areas of Uniform pro-p Groups and it's characterization, The following conjecture from the paper "Characterization of Uniform pro-p Groups" by Benjamin Klopsch and Ilir Snopce (2012) [BI] was our main focus.

Conjecture: Let G be a torsion-free pro-p group of finite rank. Then G is uniform if and only if its minimal number of generators is equal to the dimension of G as a p-adic manifold, i.e., d(G) = dim(G). In particular, the statement is true whenever G is solvable or p = dim(G).
Now we know that if G is uniform then d(G) = dim(G). Thus, we aim at getting partial results of the converse in this paper. One major result we understood from the paper was:

Proposition: Suppose that p ≥ 5 and let G be a solvable pro-p group of finite rank such that Π(G) = 1. If d(G) = dim(G) then G is uniform.

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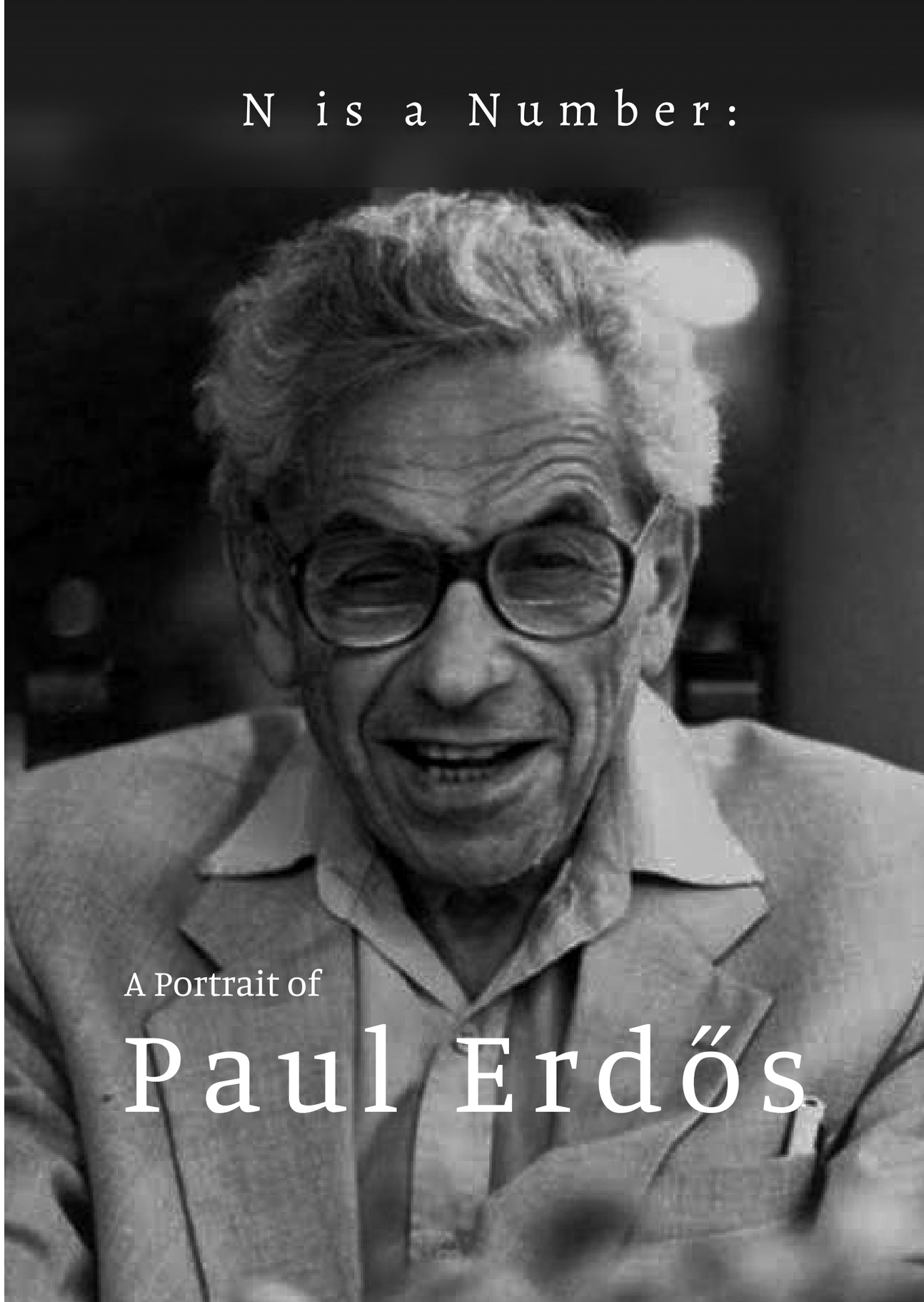


"God may not play dice with the universe, but something strange is going on with the prime numbers.
—*Paul Erdős*

On December 11, the Maths Club of IISER Pune screened an hour-long documentary, 'N is a Number: A Portrait of Paul Erdős'. This inspiring documentary describes the life and work of the well-known Hungarian mathematician Paul Erdős. It was directed by George Paul Csicsery and released in 1993. An insightful discussion followed the engaging documentary. The film won the Gold Apple Award (National Educational Film and Video Festival) and the Gold Plaque Award(Science/Nature, Documentary category, Chicago International Film Festival). Quite a large crowd assembled online to learn more about the prodigy that evening.

In the documentary, we learn about Erdős, who has published more papers and had more collaborators than anyone before him. To Erdős, mathematics is the key to the transcendental truth underlying all of reality. He had no home and had travelled constantly for more than half a century, surviving on fees from lectures and appearances. Mathematicians even have a term called the "Erdős number", which indicates how far the individual is from co-authoring a paper with Erdős. Everyone has an Erdős number of three or less. We also hear from Vera T. Sós, a Hungarian mathematician who was married to Paul Turán (one of Erdős's closest friends). She speaks of how prolific Erdős's memory is. Further, we learn about his reputation as a catalyst for mathematical cross fertilisation. Born in Budapest, Erdős grew up in a supportive environment because of his natural talent in mathematics. He proved Chebyshev's theorem when he was only 19 or 20 years old, and he was known for working simultaneously with dozens, if not hundreds, of people. He was also famous for his playful introductions and titles. Further, we learn about his reputation as a catalyst for mathematical cross fertilisation.

Born in Budapest, Erdős grew up in a supportive environment because of his natural talent in mathematics. He proved Chebyshev's theorem when he was only 19 or 20 years old, and he was known for working simultaneously with dozens, if not hundreds, of people. He was also famous for his playful introductions and titles.

Then, the narrator explains Chebyshev's theorem, which states that there is always a prime number between a number and its double. This theory was improved upon by Erdos' friend Herdish, who had a more elegant proof. Herdish and Erdos were part of a group called 'The Anonymous' in Budapest. Erdos had an incredible ability to suggest problems to his collaborators, leading them in promising directions. He cared more about the interestingness of a problem and the taste of the people solving it than the actual answer.

Later in the documentary, we learn about the collaborative nature of mathematics over cups of coffee. Prominent American mathematicians such as Ronald Graham and Fan Chung cared for Paul Erdős. He had a running joke about "the SF," (the Supreme Fascist) that he used to describe the almighty. Besides maths, he was also interested in topics from foreign policy and social issues to history and literature.

The documentary then tells us about Erdös's colleague, an eccentric Hungarian mathematician named Sidon. Known for turning towards the wall when speaking, Sidon was famous for his contributions to trigonometric series. Erdös was known for raising new problems and questions and offering money to incentivise their solution. Through the search for answers to these questions, Erdös discovered tools for opening up whole new areas of mathematics.

The video discusses Ramsey theory, a delightful branch of combinatorics.

The video then explains the mathematical details behind the theory, which left everyone in the audience in awe.

Paul had a unique style in mathematics: asking specific problems that initially seemed individual. However, they constructed mathematical theories in the long run. His work was noticed by prominent mathematicians in both England and Germany. He travelled to England in 1934. During World War II, Erdős went from school to school in the United States, not knowing what had become of his parents and friends.

In 1963, Erdős visited his mother in Hungary, where he acted as a window for Hungarian mathematicians in the West.

Erdős did not have a fixed address and travelled constantly. He spread the word about new mathematical theorems all over the world. His simple lifestyle reflected how he worked, constantly jumping from one field of mathematics to another. He discovered the probabilistic method, which uses randomness when dealing with many choices. This technique approximates the right answers well and has become an established part of mathematics.

Going ahead, the video discusses that pure mathematicians are driven to explore unknown landscapes to discover ideas and concepts. Erdős held mathematics as the closest representation of absolute truth. Through collaborations with other mathematicians, Erdős strove to continue the lineage of mathematicians before him.

Towards the end, the documentary explores the relationships in Erdős' life, particularly with his mother. He lacked romantic relationships with women, which he attributed to his inability to stand sexual pleasure. Erdős also struggled with loneliness, particularly after the death of his mother, to whom he was devoted.

Her demise left him feeling depressed and unwilling to stay in the apartment they shared in Budapest.

Finally, viewers explore his charismatic personality.

Despite his quirky demeanour, Erdős is valued for his exceptional talent in mathematics. The ultimate goal is to discover the clearest and most aesthetically beautiful proof, the "book proof".

He would always remark on seeing a very elegant mathematical proof: 'This one's from the book'. This inspired a book titled 'Proofs from the book'.

Though Erdős passed away before he could celebrate his centenary, his legacy lives on through his contributions to mathematics.

An engaging discussion followed the documentary, with participation from many spectators. The reader is encouraged to check out the following interesting theorems that came up:

- Chebyshev's and Erdős's proofs of Bertrand's postulate

- Hales-Jewett theorem

- Erdős—Kac theorem

- Erdős—Szekeres theorem

- Sylvester—Gallai theorem

- Green-Tao theorem

Apart from these, areas of mathematics related to percolation, extremal combinatorics, and Carmichael numbers were conversed upon.

The mathematicians Fan Chung and Joel Spencer were discussed too. For further reading, the following sources were suggested:

1. The High Scholer Who Solved a Prime Number Theorem

2. Teenager Solves Stubborn Riddle About Prime Number Look-Alikes

-Sumant Chopde

You do Knot know all about...

Prof. Rama Mishra

Interviewd by:
Krishna Saproo

Edited by:
Ipsa Bezbarua

Prof Rama Mishra is a Professor in the Deaprtment of Mathematics. She completed her BSc and MSc degrees from the University of Allahabad, and her PhD from the School of Mathematics, Tata Institute of Fundamental Research (TIFR), Mumbai. She held post-doctoral and visiting researcher positions at the Indian Statistical Institute (ISI) - Delhi, Osaka City University, Boise State University and the New Mexico State University, before joining IISER Pune. Prof. Mishra shares about her research interests, hobbies, views on teaching and some surprising uses of her field, knot theory. The following is an extract of the same.

1. Can you tell us about your research interests and expertise in the field of mathematics? Also, could you tell us how you got interested in this area?

I work in the field called low-dimensional topology, which deals with areas like knot theory and 3-manifolds. I am particularly concerned with developing invariants for knots in standard 3-space. Initially, I worked on defining polynomial equations representing these knots. Knot theory requires a plethora of skills from topology, analysis, algebra, combinatorics, and several other areas. At the same time, it is also very application oriented and is used even in areas like biology, for example, in understanding protein folding. In fact, Louis Kauffman has written a paper titled "Biologic", which explores the links between biology and topological invariants of knots and links. I come from a generation that did not get much exposure to different areas. I took up mathematics in my undergraduate degree at the University of Allahabad as a challenge to prove that girls, too, can do well in math. Later, while doing my Master's degree in mathematics, my professors encouraged me to pursue a PhD in abstract mathematics at the Tata Institute of Fundamental Research (TIFR), Mumbai. I enjoy the math I am doing, but it was not planned. It just happened this way.

2. Which of your published papers or articles are you most proud of?

I am very proud of my work on writing down polynomials of knots during my PhD. I had conceived the entire idea for this myself. These days I am learning sophisticated mathematics with the help of experts in my field, like Prof Louis Kauffman, who I recently visited in Chicago. I am also very proud of my recent PhD student Visakh (Narayanan). He came up with a lot of good ideas. Sometimes, younger people can open new doors to thinking.

3. What are some of the ongoing research projects you are currently involved in?

I have two major ongoing research projects. Knot theory deals with embedding a circle in a 3-sphere. Instead, we can also look at embeddings of S2 in S4 or S3 in S5. Whenever the difference in dimensions is 2, we can observe knotting.

Currently, we (my students and I) are working on 2-dimensional knots, i.e., (embeddings of) S2 in S4. This allows for visualisation, as S2 can be visualised. The other generalisation is to use some other 3-manifold instead of S3, for example, P3 (the projective 3-space). This manifold is not as simple as a sphere, and there are things like non-orientability to take care of.

4. As an editor of the journal" Knot Theory and its Applications", could you talk about your role and experience in this position?

It was a fun experience. Once, we organised a conference on knot theory and its applications in Mohali, where many people came to give talks. My role was to get the papers refereed by experts and ensure they were perfect.

KNOT THEORY SPECIAL

Have you ever attempted to solve a real-life problem using knot theory? If yes, could you share a funny or unusual example?

I have only attempted to understand protein folding using knot theory. Here I have gained some insight into how things are done. They look at DNA under a microscope and observe some knotting. But is it really knotted? Or can it be simplified? The idea is to calculate knot theoretic invariants like the Jones polynomial for this (observed) knot. The Jones polynomial for any knot equivalent to the unknot is 1. So if a given knot has a Jones polynomial not equal to 1, then it is definitely knotted. Researchers in places like MIT have found that even the protein backbone can be knotted, which was previously thought impossible. Further, only a few types of knots keep reoccurring; it is not known if (more) complicated knots can be realised in the protein backbone. So there is a lot of interdisciplinary research in knot theory. I did not get a chance to do it, but it would be fun to try it out someday. There is also a lot of interaction with physics. For example, physicists have found that there can be knotting in light. In fact, knot theory started out as an area because chemists thought atoms were different patterns of tubes of ether. So there was a project to find a table of all possible knots. Later, this was found not to be true, and they lost interest in the project, but mathematicians picked it up.

5. What are some of your hobbies and extracurricular interests?

I used to enjoy acting in plays as a kid. These days, I enjoy cooking vegetarian food. I love to listen to Indian classical music and old Hindi songs. As for movies, unless it is an art-house movie, I don't really enjoy the movies of current times.

6. How do you approach teaching mathematics to students? Do you have any particular teaching philosophies or strategies?

One of the biggest responsibilities of a teacher is to create interest in the students' mind. Simply writing definitions and stating facts will not do that. Instead, I prefer to talk more and give them ideas rather than filling the board. Most of the details are anyway in the textbook; my goal is to provide them with insight (into the subject).

7.How do you see your field of expertise evolving in the future?

Low-dimensional topology is constantly evolving. It has many connections to physics as well. It is an abstract area, and research in it takes time.

8.What advice would you give to aspiring mathematicians or students interested in pursuing a career in mathematics?

I would advise students (even those who do not intend to take up mathematics) to learn certain areas that can be directly applied in math. For example, topological data analysis is useful in biological data modelling. This is one way to get into maths. One must also remember that no one can stop us from doing something we truly want to do.

9.What would you have done if not for mathematics?

I don't know. Maybe physics (it seems like) is a fascinating area. There was very little exposure during our times. I always wanted to prove to everyone that I could take up difficult subjects and still do well. And I succeeded in this.

10.What is one piece of advice you would like to give to the younger generation?
Listen to your heart. Do what you like to do. Don't run after trends. Every career path is equally good; there is no inferior or superior choice (in this regard).

"Do you wish to bee or not bee the grand integrator?"

The 2022 edition of the Maths Club's Integration bee was our marquee event. It was definitely the highlight of my tenure as a coordinator, and according to most of our feedback, very captivating to watch as a member of the audience.

When we were selected as the coordinators of the club, one of the major goals we set for ourselves was to conduct a live integration bee on campus. Many of our volunteers were also quite enthusiastic about this idea, as they had all watched the thrilling MIT integration bee recordings. Yet, after our arrival, we put it off until the very end of our tenure. This inertia was mostly because we were concerned that organizing such a multifaceted event with multiple rounds and dozens of competitors would be a logistical challenge. But during the last couple of months of our tenure, we noticed the declining popularity of our usual events and decided that an integration bee event would be the perfect last hurrah.

The work began with a few meetings to decide on the general structure of the contest, where it was decided that we would mostly follow the MIT format, perhaps with a few small twists. We decided to stick to the usual MIT format, with a few added twists. The question making sessions were quite fun, and though they weren't very productive at first, we did slowly come up with most of our hard problems. But most of the question making was done during the last couple of days leading up to the event, when The work went quite slowly until the week prior to the actual contest, and we were a little disheartened at first when we were faced with the mountain of work to be completed. But during the final week, we held a lot of long meetings and pulled through. The last couple of days were especially rough, and we barely slept at all on the day of the event. The work really took a toll on us during the night prior to the event.

We started with 32 participants, and that number was whittled down to 16 after the prelims. The prelims mostly went smoothly, but we had a few hiccups here and there because of the rules we had set up.

Then came the Main event, where competitors would have to compete face-to-face. Many of the matchups had gone in unexpected directions, and when the finalists walked on to the dais, we had no idea who would emerge victorious. The finalists each won alternating rounds, which meant there was no clear winner until the very end. When the fourth problem flashed on the screen, we knew it was a little lengthy, but Tushar made quick work of it. In a very unfortunate turn of events, he forgot to substitute the limits before encircling the answer, which meant his answer was incorrect. Satyen then wrote down his answer, but we, the judges, knew it was wrong, and were quite anxious to see what he would do next. We waited with bated breath to see if he would circle it. But then, quite suddenly, he noticed the error, corrected the factor of two, and encircled it. The cheer that followed was quite unbelievable, and I don't think I'll ever forget it. Once the audience had settled down, I went to the dais and announced that our tenure had come to an end, and that Heerak, Padmapriya and Prasanna would take over as coordinators.

When I walked past the mess and the cricket stadium later that night, everyone within earshot was discussing the integration bee, which was vindication I sorely needed, and made all those sleepless nights completely worth it.

The event was (in my admittedly immodest opinion) a marvelous success, and I'd like to thank all the others responsible, namely, Heerak, Sumant and Gaurav on the question making team, as well as well as Krishna Girish, the anchor, whose magnetic presence kept the audience enthralled throughout the course of the event. I'd also like to thank all of the competitors, especially Tushar and Satyen for giving us a truly unforgettable final showdown.

By Keshav V,
Former coordinator
of the Spam Club

Fiddle With The Riddles

2. Peas

You have 10 cans of peas. The cans are open. In each can, there are 250 peas. In 9 of the 10 cans, each pea weighs one gram. In the tenth can, each pea weighs only 0.9 grams.

You do not know which can has the smaller peas and cant be figured out by the naked eye.
You are given an electronic scale. But the scale, being old, has its life left for only 1 correct measurement, after which the scale is of no use.

How can one, using only one measurement, determine which can has smaller peas?

Student Talks

So, is this just an event where people talk about exotic maths topics that are inaccessible to most? No. This event intends to give a platform to students (from 1st year BS-MS to PhD) to talk about topics that have fascinated them. They have spoken about a wide range of topics, from discrete mathematics to geometry and topology. Besides, some of the speakers have spoken about their journey of stepping into research, which has served as motivation and inspiration for others.

Our speakers for this year were:

1. *Arjun Ranganathan*
2. *Sreedev M*
3. *Karthik Vasisht*

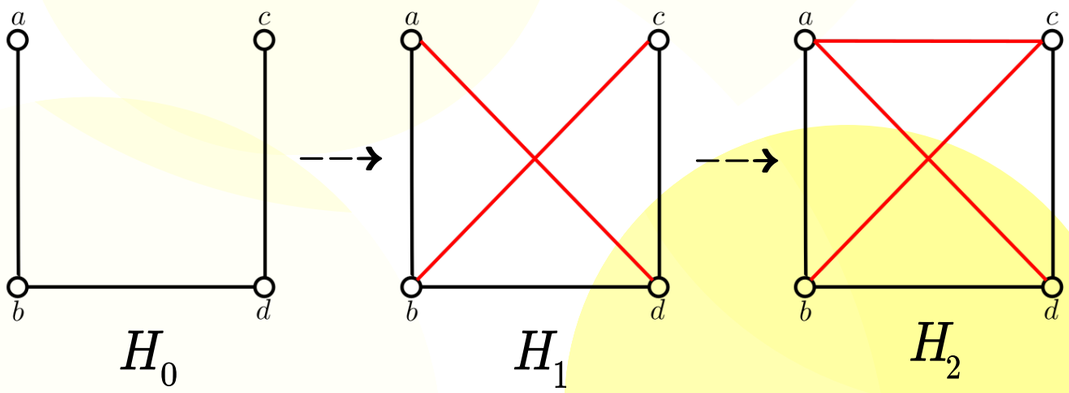
Read ahead to explore more about the topics that our speakers spoke about..

Slowing down Infection Processes on Graphs

Talk by Arjun Ranganathan

Bootstrap percolation is a model of propagation phenomena in discrete structures which originated in the statistical physics literature to study the dynamics of ferromagnetism. It has since inspired the introduction of many related models that have been the subject of intense research in sociology, computer science and, especially, mathematics.

Given a fixed graph F, the F– bootstrap process (or simply the F– process) starts with an "initially infected" graph H_0. Then, at each time step $t \geq 1$ the graph H_t is defined to contain all the edges of H_{t-1} as well as each edge e that "completes" a copy of F in H_{t-1}, i.e., $E(H_{t-1}) \cup \{e\}$ contains a new copy of F that was not present in K_3 . For instance, suppose we fix F to be K_3 – the "triangle" graph on three vertices and H_0 to be as shown. Then, the K_3 – process starting on H_0 is as depicted below.



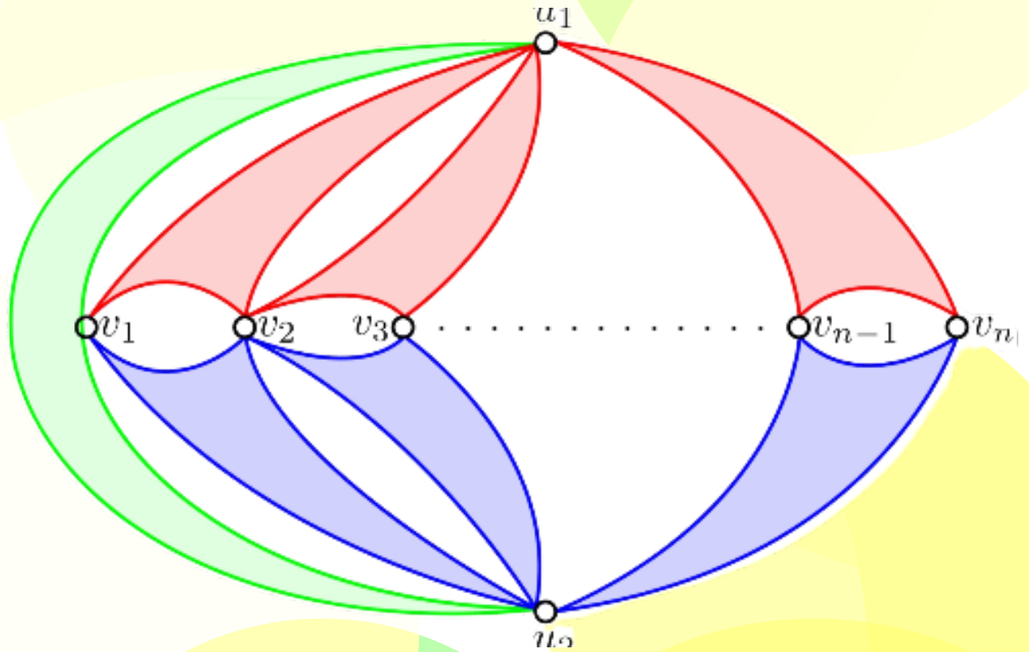
The question we focus on is trying to ascertain the "maximum running time" of the F– process – we wish to understand how long the process can run before there is no edge that can get infected any further. More specifically, given a particular graph F and fixed number of vertices n, we want to figure out the maximum possible running time of the F-process across all n-vertexed initial infections H_0 . We will restrict our study to the case of $F = K_r$ the "complete graph" on r vertices (where you have every possible edge between all the vertex pairs). In this setting, we will denote the maximum running time $M_r(n)$.

$M_r(n)$ is known to precisely be $\lceil \log_2(n-1) \rceil$. This can be seen by taking H_0 to be the "path" on n vertices and observing that the K_3 process causes the distance between any pair of vertices to halve each step. $M_4(n) = n - 3$ is the only non-trivial case where the maximum running time is exactly known. Beyond this, we only have lower bounds

$$M_5(n) \geq n^{2-O(1/\sqrt{\log n})} \quad \text{and}$$

$$M_r(n) \geq \frac{n^2}{2500}$$

Finally, we briefly discuss the case of hypergraph bootstrap percolation. A 3-uniform hypergraph consists of a vertex set V and a set of hyperedges $E \subseteq V^{(3)}$. We can further define complete 3-uniform hypergraphs K_r^3 and the corresponding bootstrap percolation processes analogously. In particular, the following example for H_0 (consisting of the red, blue and green hyperedges) shows that the maximum running time for the K_4^3 – process is at least $n - 3$.



A letter from the future

Hello wonderful person,
Yeah, it's been a long time since we last met, isn't it? Nah, you won't see me in Madhava Hall anymore..like every good thing, my IISER Pune life had to die..but am super happy, because I couldn't have looked any better, before the death of my IISER Pune persona. And I'm super glad to meet you and smile at you. Yes, especially you. So let me get to the crux of the matter. I gotta tell you about my talks. But since I'm talking about my IISER Pune persona, which is myself, it would look really narcissistic of me to write about myself. So, we are going to call my IISER Pune persona, Sree. I'm Sreedev right now. Write this down somewhere, because this is very important in the line of the story.

So the first talk Sree gave was very special and beautiful. It was a talk that focussed on minimal surfaces and their interconnectedness with other branches of mathematics. He put it in a nice story, as the aim of the talk was to tell you how to find more examples of minimal surfaces. It had a history flavor because he had pictures of all mathematicians involved, like Benedict Gross does, in his public talks. He showed a soap film-making video, defined what minimal surfaces are using Lagrange's PDE involving definition, and went from there.

The talk took its first crazy twist in ten minutes, where he said he wants to invite people to modern geometry and started giving a linear algebraic proof of why all 5 points have a unique conic passing through them. Sree just right now called me from the past, via a time machine, and told me that, he wants to specially mention Vignesh for suggesting exactly what he prepared, with maybe 200 times less amount of time.

The beauty of the proof was that he used the rank nullity theorem first, but then said, forget it, take a space where each point is a conic. That's something very crazy to do for an undergrad, to take a space of parabolas or ellipses.

With this crazy charming technique, he re-read the proof, but through the eyes of a geometer this time. After this illustration of how linear algebra played a role in a purely geometric problem, the guy shows a map where there are bridges from minimal surface theory, to complex analysis, number theory, and PDEs.

Then Sree says, that all he wants to do in this talk is to show each of those bridges. He explains what mean curvature is using pictures, and again Vignesh answers a nice question about why principal curvatures exist. At this moment, he is very jealous of Vignesh, but still, the guy goes ahead and shows how it gets to the world of PDEs.

Then he shows adjoint surfaces, an animation to go from helicoid to its adjoint surface, catenoid, and a wonderful correspondence between complex analysis and minimal surfaces. He shows the Weirstrass-Enneper representation which is a recipe to construct a minimal surface given a holomorphic map. Suddenly many functions get pictures of minimal surfaces attached to them, and he shows us a series of function picture correspondences.

This is the last twist of the talk, and according to one of the best people I've ever met, Yuv Aggarval, something he didn't see coming. Sree said, let me tell you how it is connected to number theory. He takes the famous Ramanujan sum and shows that LHS and RHS show two things about the same minimal surface. That is, the Ramanujan series just tells you, you can see a beautiful minimal surface as some other wonderful minimal surfaces stacked upon each other.

This is the end of the talk. And as you can see, Shape of Soap films_1 is an art reviewer's dream. If you are in mathematics, because of its beauty and if you think mathematicians are artists, you will find Sree's first talk to be everything you ever wanted. It was one beautiful thing after the other, crazily organized. The audience was full of his friends, and they were full of LHC101. Friends, who he met just before, and after the talk was over.

Sree is a fantastic speaker and had a crazy way of organizing his talk with a personal touch and some deliberate mistakes for jokes in between.The first talk he gave, gave him a lot of friends around him and he is super grateful for that. But the guy is still not perfect, and the talk wasn't as well, because he looked nervous in the first five minutes. Also, he didn't have many jokes, and that was good because most of the jokes he makes might offend people, and Mainak and Anisa were in the audience. But something very memorable for me is Mainak writing later in my diary,

"I haven't seen many people giving better public talks..".

But Sree chased everything he lacked in his first talk, in his second one. It was one joke after the other, and he found nothing more funnier than his own adventure to solve an open problem. The guy was open, and humble and took the audience through a roller coaster of thinking one did something, and later realizing they didn't. In this talk, mathematicians are seen as detectives and not as artists, and it was superbly ugly mathematics. More than 75 percent of the talk didn't contribute to the paper.

But the point of the talk was to normalize the stigma associated with research, to normalize himself, and to tell a very young and enthusiastic bunch of researchers, this is how it happens. He didn't paint an exaggerated picture and was happy to show his own email exchanges. And he had a crazy good habit of laughing at himself again and again and letting the audience laugh at it. It is the total opposite of his first talk, because the first talk was about him as a math-loving person, but the second one as a human trying to prove something.

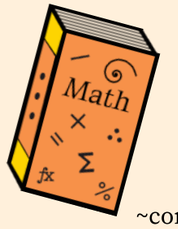
I have nothing much to say about the content of the second talk, except it is on youtube and the paper is on arXIv. The talk is something that Sree will remember maybe till the day he dies.

The organization of the talk can never be any better, and nobody saw a talk like that ever happen. So I'm gonna end this letter with a beautiful review I got about my second talk from my amazing friend, Kavya. It means the world to me, to have so many amazing friends, I only now realize how great of a time I had in IISER Pune, when I talk to other uni people. But what she wrote was, "I find in them, the excitement, exhilaration, frustration and deep satisfaction that come with working in mathematics with your heart and soul"

And that's it..that's my review of both of my talks. I wanna thank the maths club a ton for giving me such crazy opportunities, with never asking me what are you gonna present?.. the absolute freedom and the friendship I got from Heerak, Padampriya, and Prasanna is beyond amazing and means the world to me. Hope you enjoyed all the compliments I gave to myself. See you sometime, either in the gambling parts of the US or in India, or even maybe at your Nobel prize or Fields medal ceremony.

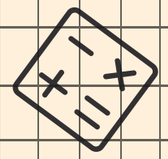
Thanks and regards
Sreedev Manikoth
(a guy who once lived in IISER Pune)

Dictionary of Definitions of Terms Commonly Used in Math Lectures



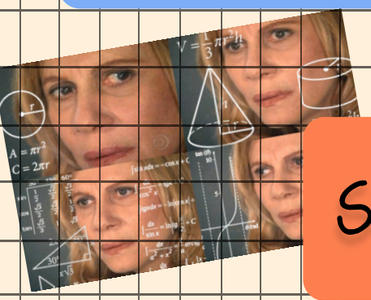
~compiled by Amruthanshu A K

Clearly I don't want to write down all the "in-between" steps.



Obviously I hope you weren't sleeping when we discussed this earlier because I refuse to repeat it.

Recall I shouldn't have to tell you this, but for those of you who erase your memory tapes after every test



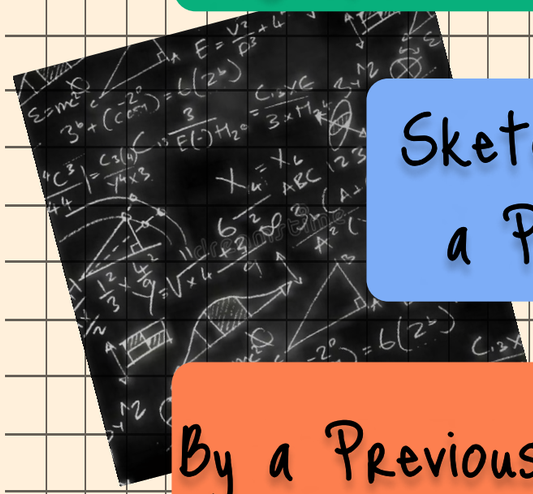
Similarly At least one line of the proof of this case is the same as before.

Trivial If I have to show you how to do this, you're in the wrong class.

WLOG Without Loss of Generality — I'm not about to do all the possible cases, so I'll do one and let you figure out the rest.

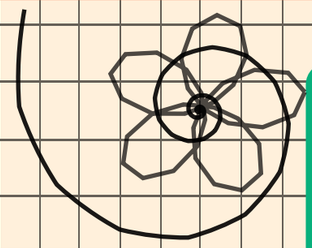
It Can Easily Be Shown Even you, in your finite wisdom, should be able to prove this without me holding your hand.

Check for Yourself This is the boring part of the proof so, you can do it on your own time.



Sketch of a Proof I couldn't verify all the details, so I'll break it down into the parts I couldn't prove.

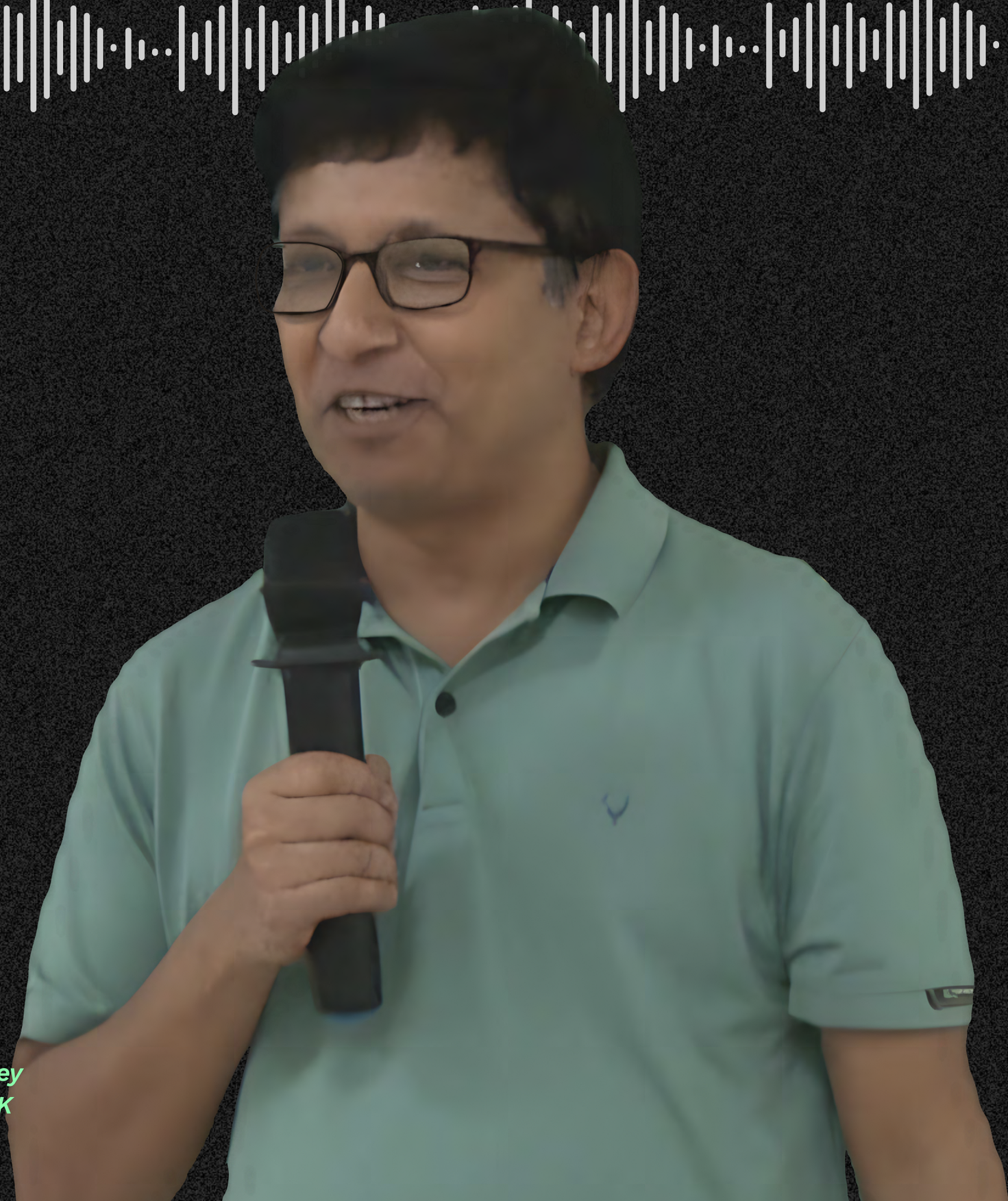
By a Previous Theorem I don't remember how it goes (coming to think of it, I'm not really sure we did this at all), but if I stated it right (or at all), then the rest of this follows.



Proof Omitted Trust me, It's TRUE.

What the Chair has to say...

Interviewed by: Ujwal Pandey
Edited by: Amruthamshu A K



Prof. Mainak Poddar, Professor and the Chair of the Department of Mathematics, pursued his Bachelor's and Master of Statistics from ISI (Indian Statistical Institute) Kolkata. Later he completed his PhD at the University of Wisconsin–Madison and held postdoctoral positions at Michigan State and the University of Waterloo.
He was a faculty at ISI Kolkata, the Universidad de Los Andes, and METU NCC before joining IISER Pune.

Interviewed by: Ujwal Pandey
Edited by: Amruthamshu A K

Read on to know more about him...

1. You are a Mathematician as well as the Chair of the Mathematics Department. Could you tell us how you handle both these aspects of your career and how the work differs from one another?

Being the department chair is a difficult job, much more demanding than a professor's job. As far as a day goes, normally, I arrive at the office as early as possible and do some math first. Then, I attend to any departmental work or responsibilities. Somedays, once I start focusing on some departmental matter, the first half of a day or sometimes, the whole day would be gone. Coming back to math on those days is a bit hard. The good thing is I have a group of students and post-doc fellows to talk to and work with. I'm sure it would've been a disaster if I was by myself. When I speak and interact with them, it helps me focus too. I enjoy interacting with collaborators about math. I also find teaching helpful - it helps me focus on math. In my experience, I have found that I focus better when teaching. It helps the discipline of getting to think about math on a regular basis. But, I do take a teaching break once in a while because, at times, I feel like handling everything is quite a lot. Workshops and conferences also provide a conducive environment where you are not distracted by other duties and focus only on math. This change in the environment boosts productivity.

2. Could you briefly tell us about your current work?

I work in things related to geometry and topology, focusing on differential geometry.I work in areas which entail Symplectic structures, complex geometry involving complex varieties and manifolds, vector bundles, principal bundles etc. Many of my works describe an idea of symmetry, for example - the action of groups. These play a vital role. I started my career working on orbifolds. They are generalisation of manifolds with some extra symmetries. Although I don't work on orbifolds regularly anymore, most of the math I do has an imprint of symmetry.

3. What are some problems you would like to work on in the future?

Recently, we have been working on logarithmic connections. The theory is classical for smooth/non- singular varieties. We have been trying to extend some of this to singular varieties, and we have had some success. I feel like we have a basic theory for logarithmic connections on principal bundles over normal algebraic varieties (which contain singularity). Then we are trying to see what amount of the classical results can be recovered for this singular situation.I'm also getting interested in stacks. In algebraic geometry, I usually work on varieties. Locally, varieties can be thought of as solution to a system of polynomial equations. This set of solutions has certain geometry. But a stack has more information - there may be more information about symmetry, group actions, etc. There is a category of schemes which is broader than varieties. But stacks are even more general. This is more of a functorial approach where we can go broader. Much of this work goes back to Grothiendick, where he defined the notion of topology for a category in contrast to just topology over a space. It also has a philosophical richness to it. So, I want to explore this and solve some of the problems I have thought about in this generalised setting. I work with my postdocs on generalised complex geometry. It so happens that this provides a broader framework in which symplectic and complex structures are just examples of extreme cases of a generalised complex structure. There are some other structures here, which are neither complex nor symplectic, but it's not easy to obtain such structures or understand them. They exist and it is a slightly deeper way of looking into things - we are trying to learn and work on those things.

4. You have worked in algebraic and differential geometry. So, would you say you have an algebraic or geometric way of thinking, or is it a mix of both?

I think I have a very geometric way of thinking. The way of thinking also depends on training. I come from differential topology background. A lot of techniques in algebraic geometry were first introduced in differential geometry and then adapted to algebraic geometry. But much of this compartmentalisation of different entities in mathematics and its subcategories is artificial all of them are interlinked. They always talk to each other and exchange tools and techniques.

5. What got you interested in Mathematics? What made you realise your talent in it? Could you tell us the people who inspired you to enter this field?

When I was young, I had an interest in Physics and Mathematics. I liked to solve problems. Plane geometry was the first hurdle that I found that required some imagination. It was not easy. Even though I got comfortable with the topic later on, I struggled initially. Our town had a mathematical society. They used to organise competitions, quizzes, olympiads etc. I participated in them, and the professors there got interested in me. They invited me to visit them and encouraged me to look at many books by the Little Mathematics Library series and a number theory book by GeorgeAndrews. Those professors inspired me to learn more. My parents were very supportive of this too. My father himself was interested in math when he was young. He encouraged me and was also happy that I was going into the field of math. After 12th, I wrote an exam for ISI (Indian Statistical Institute) and got selected. There were few institutes back then for mathematics. ISI's coursework, environment and interactions with the professors really left a deep mark on my life. We had some excellent teachers, and they presented math in a very interesting way. The other thing was that they treated you with respect. The attitude was very different from high school. Here, they would almost treat us as a friend.Coming to inspiration, one doubts a lot whether they have the necessary talent. Even while doing my PhD, looking at all the beautiful theorems in the book, I wondered if I could ever come up with a theorem or a proof. When I began to solve the problems in my thesis, I gradually grew more confident, but it was a long process.

6. Other than your current field of research, what branch of math intrigues you the most?

That is a tricky question. Most of the time, I am engrossed in thinking about my area. Intriguing is a strong word. I deeply admire many branches like algebraic geometry, number theory etc. Recently there was an application of topology in data science - persistence or persistent homology where they try to find patterns in the data cloud. Topics in algebraic geometry, number theory,Langlands program etc., are complicated. But persistence homology has math at the BSMS level, but the idea to apply it in that context is very original and new. The math involved is of rings and modules, mainly the classification of finitely generated modules over a PID (principal ideal domain). Probability theory, percolation, and old classical simple problems also pique my interest. Overall, most areas of math are rich and interesting.

7. You did your bachelor's and master's in Statistics.What led you to pure math, and are you still in touch with statistics?

I had to make this decision sometime. I didn't feel that I was good at Statistics. Mathematics is highly logical, but in addition to that, Statistics requires being practical. I didn't think that I had that in me. You have to be very creative in applying the mathematics you already have while solving a problem in Statistics. That was one of the reasons to choose pure math. Also, back then, ISI didn't have a degree in math. So everyone had to learn statistics. But there was a provision for a specialisation which helped me to focus on more mathematical aspects. In the end, only a few from our batch went into math-related fields. Coming back to the question, I'm in touch with statistics because I try to read and stay in touch with new developments in data science like neural networks, etc. The concepts

used there come from statistics. I do enjoy a casual read of the application of Statistics. I have a better appreciation of statistics than I had as a student.

8. If you were not a mathematician, what would you have chosen instead?

Not work as such, but I was interested in sports. I was decent at it. I was in the school team in both cricket and football. Besides academics, that was probably my major interest.

9. What hobbies do you have?

Not many. I don't play sports anymore due to lack of time. I love listening to music. I enjoy a wide
range of genres, from Indian classical to jazz to pop and western. I used to enjoy reading books and
watching movies , but I don't do those anymore as they are heavy. They take you to a different frame of mind. It could be thought-provoking, but most of the time, it can make you lose focus which I cannot afford.

10. Which math books are your favourites and why?

There is a book by Peter Newstead - Moduli Spaces and Vector Bundles, but it is an advanced book. It would be top of my list. It describes the construction of geometric invariant theory, which I find very elegant. Then, Basic Algebraic Geometry by Shafarevich - that is what I read when I was in graduate school doing my PhD. I learnt a lot about geometry from the book by Igor. It gives so many examples. There's another book called Topology from a differentiable viewpoint by Milnor. It was like an eye-opener to the area of topology and geometry.

11. Who is your favourite mathematician?

This is hard. One of my favourites is Milnor, from whose writings I learnt much math. He has the rare ability to write out complex things concisely and approachable. I admire many people but haven't gone through their work in depth. From Indian mathematics, I think MS Narasimhan and Seshadri - who did fundamental work in algebraic and differential geometry- are my favourites. I admire Hironaka and Shigefumi Mori, who worked in birational geometry. I think I'm missing out on more people.

Though I haven't gone into much depth, the work of Maryam Mirzakhani is very exciting. Frederick Hirzebruch, who worked on signature theorem, was a great mathematician. This was a precursor to the more developed index theory by Singer and Atiyah. Riemann shouldn't be forgotten for his fundamental contributions. Last but not least, Grothendieck has a tremendous influence on the way one thinks about math.

12. In the past century, what, according to you, has been the biggest advancement in math?

In general, I think it's widely recognised that the introduction of categorical language is an important development. The three basic ingredients in math are numbers, space and logic. If you think about the theory of these things, there have been spectacular developments like the work of Godel. There is now a relationship between number theory and algebraic geometry. Now, we can see some unity in these basic entities, space and numbers. There are more developments which connect logicand topology, but I don't understand them. As I said earlier, finding the language of categories, a unifying language, is always tough. In geometry, the discovery of exotic smooth structures for manifolds, even on

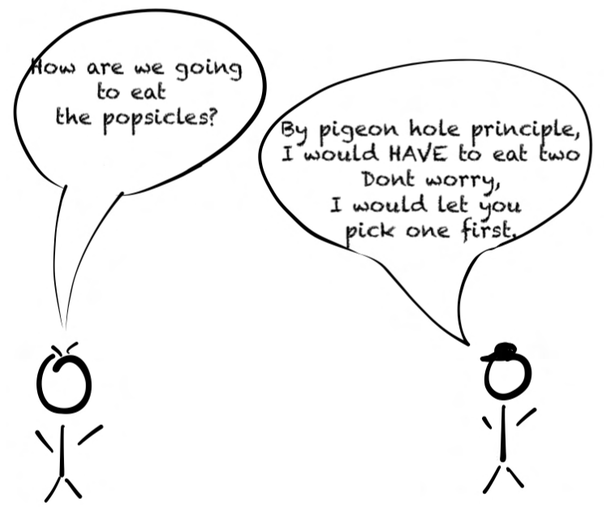
13. Which unsolved problem in mathematics would you like to see being solved in the near future?

Maybe the Riemann hypothesis - just because I think there will be very new and startling math involved in the solution. That will be very exciting. However, I must also mention the Hartshorne conjecture on vector bundles, from the late seventies, which is related to my research. A vector bundle is a family of vector spaces which is locally trivial, that is, locally a product of a vector space and an open set in the parameter space of the family. But the global geometry of a vector bundle can be very complicated. A vector space can always be decomposed into a direct sum of subspaces of rank (i.e., dimension) one. However, a vector bundle does not usually decompose into a direct sum of rank one subbundles. Interestingly, Grothendieck showed that every holomorphic vector bundle over (i.e., parametrized by) a one dimensional projective space decomposes into a direct sum of rank one subbundles. But, when the parameter space is more complicated, such a decomposition is rare.

The Hartshorne conjecture says that every holomorphic vector bundle of rank two over a projective space of dimension seven or higher decomposes into a direct sum of rank one subbundles. This is known to be equivalent to the statement that every holomorphic vector bundle of rank two over a projective space of dimension seven or higher admits a _____+ torus action that lifts the standard torus action on the projective space. Note that the second formulation bears an imprint of symmetry in the form of the torus action

14. What advice do you wish you had known as an undergraduate student?

General advice - Communication and social skills are very important. It builds your career and not just your academics.Math has become so deep and technically challenging. Very few people read through the book and understand it on their own. The more you are able to talk to people, the faster you develop intuition and understanding. Talk to peers, professors etc. - they can tell you some key ideas which would take much more effort to figure out on your own, not just in academics but in life too.



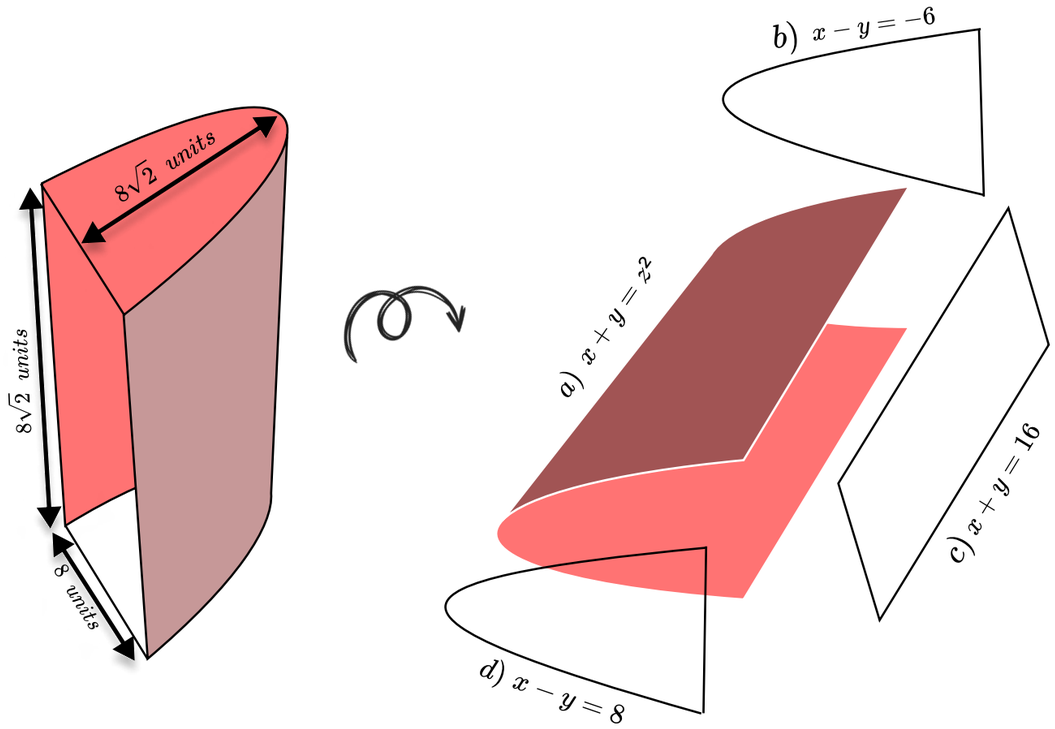
Auri

Integrating, but not really

Q Find the volume enclosed within the following, but **without direct integration**.

a) $x+y=z^2$ *b)* $x-y=-6$ *c)* $x+y=16$ *d)* $x-y=8$

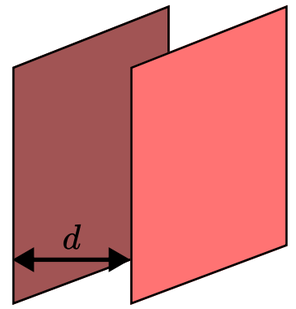
A We start by visualising what volume is represented by the 4 equations. It turns out that the volume which they enclose is a parabolic cylinder.



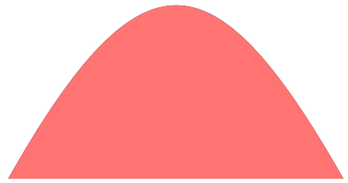
Next, we state a couple of clever results which we would use to find the required volume.

1 Given two parallel planes,
$P_1 : ax+by+cz+d_1=0$ and
$P_2 : ax+by+cz+d_2=0$,
the distance between the planes, d, is given by

$$d=\frac{|d_2-d_1|}{\sqrt{(a^2+b^2+c^2)}}$$



2 Given a rectangle and a parabola inside it as follows, the ratio of the areas of the parabola with the rectangle is always $\frac{2}{3}$.



Now, we would calculate the required volume as follows:

Using 1, the height of the parabolic cylinder is $8\sqrt{2}$ units.

Using 2, the area of the base of the parabolic cylinder is: $\frac{2}{3}\cdot\left(8\sqrt{2}\times 8\right)$

The volume of parabolic cylinder is its base area times its height:

$$\frac{2}{3}\cdot\left(8\sqrt{2}\times 8\right)\cdot 8\sqrt{2}=\frac{1792}{3}$$

-Ahona Mukherjee

NATIONAL MATHEMATICS DAY

Srinivasa Ramanujan was one of the most celebrated Indian mathematicians. With almost no formal mathematical training, he made seminal contributions to many areas of mathematics, particularly number theory, mathematical analysis and continued fractions. In 2012, the Indian government declared his birthday, 22nd December, as National Mathematical Day, in honour of this genius.

Surya Teja Gavva
Rutgers University

On this occasion, Maths club organised a talk by Surya Teja Gavva, Rutgers University, which explored the life and works of this great 20th century mathematician. We also organised an article writing competition. Read on to learn about Ramanujan's exemplary life and some of the truly inspiring articles about Indian mathematics and mathematicians that we received.

Srinivasa Ramanujan was one of the most celebrated Indian mathematicians. With almost no formal mathematical training, he made seminal contributions to many areas of mathematics, particularly number theory, mathematical analysis and continued fractions. In 2012, the Indian government declared his birthday, 22nd December, as National Mathematical Day, in honour of this genius. On National Mathematics Day, Maths club organised a talk by Surya Teja Gavva, Rutgers University, which explored the life and works of this great 20th century mathematician. Below is a summary of the talk.
Ramanujan was born on 22nd December 1887 in Pallipalayam, Erode to the couple Komalatammal and KuppuswamySrnivasa Iyengar, and had a very traditional upbringing. The family later moved to Kumbakonam where he completed his primary and secondary education.

Ever since childhood, Ramanujan exhibited a keen aptitude to mathematics. CV Rajagopalachari, a friend of Ramanujan, recounts a question asked by him during class at Kangeyam Primary School: 'Sir, if no banana is distributed to no student, will every student still get a banana?'. He was an outstanding student, and received many academic awards and scholarships throughout his schooling. He was also very religious, and spent many hours at a local temple listening to the beats of drums. He would later attribute his phenomenal mathematical discoveries to the goddess Namagiri:

> *"An equation means nothing to me unless it express a thought of god"*
>
> ~ Srinivasa Ramanujan

During This Time, He Explored A Lot Of Mathematics On His Own, And Began To Discover New Results. He Was Lent A Book On Advanced Trigonometry Titled 'Plane Trigo-Nometry' By SL Loney, Which He Mastered By Age 13 While Discovering Sophisticated Mathematical Theorems. He Later Obtained A Copy Of GS Carr's 'A Synopsis Of Elementary Results In Pure Mathematics', A Collection Of 4685 Formulae Without Proofs, Ranging Over Many Areas From Algebra To Trigonometry To Analytical Geometry And Calculus. This Book Was Just The Spark That Ramanujan Needed. In This Book He Was Introduced To Some Popular Definite Integrals, Elliptic Integrals, Hypergeometric Series, Fourier Series And Continued Fractions, All Of Which Would Hold His Fascination For The Rest Of His Life.

In the words of PV Seshu Aiyar and R.Ramachandra Rao', who would later play a major role in Ramanujan being recognised as a mathematician:
'It was this book which awakened his genius. He set himself to establish the formula given therein. As he was without the aid of other books, each solution was a piece of research so far as he was concerned.'

Ramanujan passed his Matriculation examination in 1904 and received a scholarship to study at Government Arts College in Kumbakonam owing to his mathematical brilliance. However, he got so involved with mathematics that he neglected other subjects, and thus failed most of the exams, thus losing his scholarship. He later enrolled at Pachaiyapa's college in Madras, but again focused only on mathematics and thus could not continue his higher education. He later started searching for a job to support his family. He obtained a position as a clerk at the Madras Port Trust. During his pursuit for a job, he met Prof. V Ramaswamy Aiyer, founder of the Indian Mathematical society; R. Ramachandra Rao, Nellore district collector and secretary of Indian Mathematical Society; and PV Seshu Aiyar, professor of Mathematics at Presidency college, who encouraged his mathematical pursuits. With Prof.Aiyar's help, he published his work in the Journal of Indian Mathematical Society. One of the first problems he posed in the journal was:

$$\sqrt{1+2\sqrt{1+3\sqrt{1+\dots}}}$$

the answer of which he showed to be 3 by recursively using the formula $n+2=\sqrt{1+(n+1)(n+3)}$
His first formal paper, 'Some properties of Bernoulli numbers', dealt with Bernoulli numbers, which occur as the coefficients in the formula for sum of powers of natural numbers, and have very interesting arithmetic properties.Ramachandra Rao and others soon encouraged him to write to the Eglish mathematician, GH Hardy, with some of his work attached. Hardy initially viewed the letter as a fraud, but on closer inspection, it became clear to him that it was the work of a genius. He wrote to Ramanujan expressing great interest in his work, and made necessary arrangements for Ramanujan to travel to Cambridge. Thus began one of the most fruitful and important collaborations of mathematics

Work at Cambridge:
Ramanujan and Hardy had an immensely successful collaboration, publishing over 30 papers in topics ranging from number theory to mathematical analysis. Their work opened up many new and fascinating areas of mathematics. An interesting topic that they explored was the properties of highly composite numbers.

A highly composite number is a positive integer with more divisors than any smaller positive integer. The first few of these numbers are 2,4,6,12,24. Ramanujan received the *Bachelor of arts by Research degree* for this work.

They also made important contributions to the study of *partitions*. Partition function counts the number of distinct ways of writing n as a sum of positive integers. For instance, p(4) = 5 because

$$4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1$$

They found an asymptotic formula for the partition function:

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}}$$

Their work in this area gave rise to a powerful tool in analytic number theory, called the *circle method*.
Ramanujan also studied congruence relation of these partitions and produced identities such as
(where $k \in \mathbb{N}$):

$$p(5k+4) \equiv 0 (mod\ 5)$$
$$p(7k+5) \equiv 0 (mod\ 7)$$
$$p(11k+6) \equiv 0 (mod\ 11)$$

He also made some remarkable conjectures. One example is the Ramanujan-Nagell equation, which states that the equation $x^2 + 7 = 2^n$ has solutions in natural numbers n and x when $n = 3, 4, 5, 7$ and 15. Another is a set of conjectures about the Ramanujan Tau function, which was instrumental in the proof of Fermat's last theorem.

Ramanujan produced some fascinating identities. Chief among them was this monstrous looking formula for π:

$$\frac{1}{\pi} = 2\frac{\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{k!(396)^{4k}}$$

Only the first term of this series gives π to an accuracy of six decimal places, and the next term improves the accuracy by a further 8 decimal places! He gave many other such rapidly convergent series, now called the Ramanujan-Sato series, which today form the basis of the fastest algorithms for calculating π.
These are just a few of his many invaluable contributions to mathematics. In recognition of his achievements, he was elected a *Fellow of the Royal Society*, becoming only the second Indian to be admitted, and one of the youngest fellows in the history of the society. He was also the first Indian to be elected a *Fellow of Trinity College, Cambridge*.

Return to India:

When at Cambridge, he faced several hardships such as ill health, racism and the unavailability of vegetarian food due to World War I. Due to this, he was diagnosed with tuberculosis and a severe vitamin deficiency. So he returned home soon after the conclusion of the war. His health worsened and he passed away on 26 April 1920, at the young age of 32.

In his obituary to Ramanujan, written for *Nature*, Hardy remarked:

> *"His insight into formulae was quite amazing, and altogether beyond anything I have met with in any European mathematician. It is perhaps useless to speculate as to his history had he been introduced to modern ideas and methods at sixteen instead of at twenty-six. It is not extravagant to suppose that he might have become the greatest mathematician of his time. What he actually did is wonderful enough... when the researches which his work has suggested have been completed, it will probably seem a good deal more wonderful than it does today."*

Even in his last days, his mathematical mind was very productive. He discovered new functions called *mock theta functions*, and continued to advance the frontiers of mathematics despite his failing health. Many mathematicians continue to study work by him or fields inspired by him. He is truly an inspiration to all mathematicians.

by Prasanna Bhat

Mathematics, Bengal Tiger and The University Of Calcutta

-Debesh Bhattacharjee

Well, looking at the title, you might think that how can these three things -Mathematics, Bengal Tiger and University of Calcutta, be coherent to a single topic? I understand your confusion. However, there was one man to whom you can attribute all these three entities. He was Sir Asutosh Mukhopadhyay or as he used to write, Asutosh Mookerjee. In one word, he was a versatile genius. His legacy and accomplishments can be viewed from different perspectives - as an amazing administrator, a very unique educationist, an exceptional visionary, an eminent judge, a grand orator, a mathematical genius, a tireless propagator of mathematical research, a distinguished bibliophile and above all, a wonderful human being. He was also well-known for his patriotism, humility, respect for cultural heritage, love for literature and greatness. For this grandeur personality, he is known as "Banglar Bagh" or the "Bengal Tiger". Now, you understand that, portraying this polymath in a single article is an extremely difficult task. Therefore, in this article, we are going to dive into the mathematical legacy of Sir Asutosh Mukhopadhyay and his unforgettable contributions in building and promoting science in India. However, in spite of being one of the reformers of modern India, very little attention has been paid to investigate his life and contributions.

Before going into the main text, let us briefly brush through Sir Asutosh's early life. Asutosh Mykhopadhyay was born as the eldest son to Jagattarini Devi and Ganga Prasad Mukhopadhyay, on June 29, 1864 at Bowbazar of the then Calcutta. Jagattarini Devi was a very pious lady and known to have courage and strength of character. Asutosh's father, Dr. Ganga Prasad Mukhopadhyay, was a renowned physician who established the South Sub Urban School in Calcutta. Asutosh was brought up in an environment of science, literature and culture. One of his ancestors, Pandit Ramchandra Tarkalankar who was a professor of nyaya and had been appointed as the chair of the Sanskrit College in Calcutta by Warren Hestings.

When Asutosh was young, he met Pandit Ishwar Chandra Vidyasagar and was greatly influenced by him. Asutosh was a brilliant student and a great bibliophile.

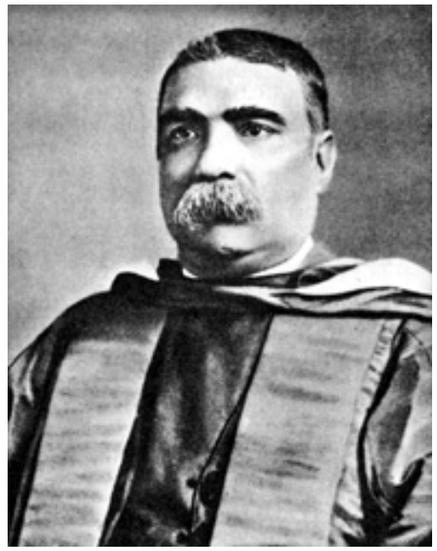


Fig: Sir Asutosh Mukhopadhyay (29 June 1864 – 25 May 1924)

Not only he showed keen interest in mathematics from a very early childhood, but also, he was certainly a mathematical prodigy. He solved all the mathematical problems from 348 exercises of "Exercises on Pott's Euclid" all by himself much before appearing for the Entrance Examination (Class X). He stood second in that examination and received the first-grade scholarship. Then, in 1880, he took admission to Presidency College (now Presidency University).

Despite of being seriously ill (he suffered from chronic headaches and he almost lost his right hand due to a sudden electric shock), he appeared for the FA (Class XII) examination and stood third in the university.

He acquired the first position in BA in the year 1884 and received the Harishchandra Prize for his mathematical excellency along with Ishan and Vizianagaram scholarships. For his profound mathematical genius, that year only, he became the first Indian to become a life-time member of the London Mathematical Society. Once again, he stood first in his MA examination specializing in mathematics in 1885 from the University of Calcutta. Next year he received another MA degree in physics, becoming the first student from the Calcutta University to obtain a dual degree in MA.A few months later, in that same year, he received the prestigious Premchand Roychand Scholarship for his proficiency in pure mathematics, applied mathematics and physics. Quoting from another mathematician A C Bose,

"His (Asutosh's) versatility, struck us with wonder. Literature, history, law, and the physical sciences claimed the attention of this ardent student as much as mathematics, although in those early years, mathematics was the subject which attracted him most and was enriched by contributions from him in the well-known Indian and foreign journals of the day."

In the next year, 1887, he was appointed as the examiner for the MA mathematics degree examination. This made him the first non-European examiner for a non-literature MA course. In 1889, he was elected as the Fellow of Senate of Calcutta University, and then the member of the Syndicate. From this time onwards, began his life-long association with the University of Calcutta. Apart from being a prolific mathematician he also became the Chief Justice of Calcutta High Court. He was universally considered as one of the greatest judges of British India. Quoting from his grandson, Shivatosh Mukhopadhyay, we get, "The first phase of his (Asutosh's) career was as a votary of mathematics, the second phase as a devotee of law, and the third phase as a creator and builder of the University".

In his formative years, Asutosh wanted to become a professor of mathematics and contribute to the discipline at the Calcutta University.In one of his Senate speeches given during 1920, he remarked,

"Nothing is dearer to me, nothing has been dearer to me than my university. I began life as a research student in mathematics when research was practically unknown in this country and the ambition of my life was to be a research professor in my university."

However, this dream never became true. Asutosh's rigor and proficiency as a mathematician impressed Sir Alfred Croft, the then Director of Public Instruction of Bengal. He offered him research plus teaching position at Presidency College, Calcutta. However, Asutosh had two conditions which were going against the British rules, the first one was that he would not be transferred from Presidency College where he would carry on his research and the second was that he should get "the same status and pay as European members of the Educational Service". But the colonial rule did not pay attention to these conditions. Finally, he declined the offer. He had to leave his mathematical research where he had displayed profound promises to contribute to the discipline.At this point, let's quote from Nobel Laureate (in physics) Sir C V Raman who rightly put, "Bengal, in gaining a distinguished Judge and a great Vice-Chancellor, lost in him a still greater mathematician."

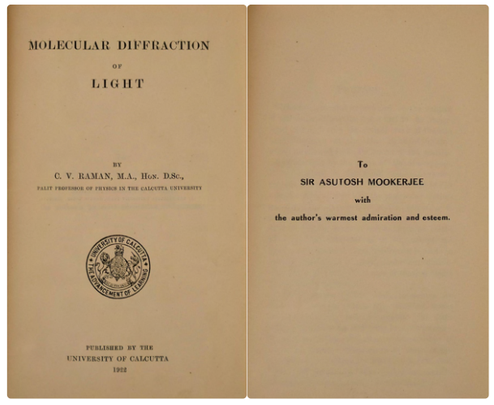


Fig: C V Raman Dedicated His Molecular Diffraction Of Light (1922) To Sir Asutosh Mukhopadhyay

R P Paranjype also remarked, "If he (Asutosh) had made up his mind to devote himself entirely to the study of mathematics, he was sure to have secured a place in the front rank of world mathematicians".

Sir Asutosh as a Mathematician:
Asutosh Mukhopadhyay published his first paper in mathematics at the age of 17, when he was just a first year student of FA at Presidency College. However, this paper was written much before, when he was just 13. This study was published in the reputed journal, Messenger of Mathematics, Cambridge

Publication. This paper was on the 25th preposition of the first volume of Euclid's Elements, which stated, "If two triangles have two sides equal to two sides respectively, but have one of the angles contained by the equal straight lines greater than the other, then they also have the base greater than the base." His two main fields of interest are geometry and elliptic functions. During 1887 to 1889, he published several seminal papers on differential equations of conics. This particular field of study was initiated by a French mathematician, Gaspard Monge (1746–1818), later English mathematician George Boole (1815–1864) added his contributions to this subject. Intrigued by Boole's analysis, Asutosh searched for a geometrical solution to Monge's work. This resulted in a new publication in 1887 by the Journal of Asiatic Society, India. This particular work was supported and highly praised by renowned mathematicians Joseph Sylvester (1814–1897), and Alan Cunningham (1842–1928). Another renowned mathematician Arthur Cayley (1821–1895) wrote a letter to Asutosh highly praising his contribution. This work was included in the book, Differential Calculus, by Joseph Edwards in 1896. Another seminal paper by Asutosh, on differential equations of trajectories, based on studies by Monge and M. Roberts, published in 1887 by the Journal of Asiatic Society, India, was praised and mentioned by Forsyth in his book titled, Differential Equations. A few other papers of Asutosh were published in Proceedings of the Asiatic Society of Bengal during 1887-1889. This time he also wrote two papers on Hydromechanics and these were published in The Journal of the Asiatic Society in the next year, 1890. He published few more papers till 1892 on calculus and differential equations. Sir Alexander Pedlar, Professor of Chemistry at Presidency College and Vice-Chancellor of Calcutta University (1904–1905), published a report in the Journal of the Asiatic Society of Bengal and there he stated that there were 169 scientific research papers published in the journal during 1885–1895, and among them, only 14 were written by Indians, "of the 14 papers written by the Indians, the lion's share was from the pen of Asutosh Mukhopadhyay (mathematics) and the last one was the epoch-making paper of Jagadish Chandra Bose." In 1884, when he was just 20, he was honored as a Lifetime Member of London Mathematical Society, and thus became the first Indian to be so. On the 12th February issue of the Statesman, we see, "We understand that Babu Asutosh Mookerjee, who stood first at the last B.A. examination, has been elected a member of the London Mathematical Society.He is the first Indian on whom the Society has conferred this honor". For the seminal contributions in the field of mathematics, Asutosh was elected as the Fellow of the Royal Astronomical Society and the Member of Royal Society of Edinburgh in 1885 only at the age of 21. Then in the next year, he was elected as the Fellow of Royal Society of Edinburg, a Member of the Royal Asiatic Society and a Member of the Bedford Association for the Improvement of Geometrical Teaching. In the following years, he became the Fellow of the London Physical Society (1887), a Fellow of the Mathematical Societies of Edinburgh and Paris (1888), a Member of the Mathematical Society of Palermo of Sicily and a Member of Society de Physique of France (1890), a Member of the Irish Academy (1893), and a Fellow of the American Mathematical Society in 1900.Then he sent 80 mathematical problems to the Educational Times, London and then sent the solutions of them. Many of these problems are still regarded with greater attire. He also sent solutions to 155 mathematical problems, most of them were constructed by the famous mathematician W K Clifford.

Simple proofs and logical elegance were the key traits of his mathematical genius. These characteristics were well reflected in his book, An Elementary Treatise on the Geometry of Conics (published in 1893 from London), which became famous both in India and abroad and translated in several languages. Later, the Association for Improvement of Mathematics Teaching, a prime organization engaged in promoting mathematical teaching, published some unpublished works of Asutosh under the title of "On Pott's Euclid". The art of approaching a mathematical problem in a simpler way can also be found in one of his papers, where he discussed the problem of determining the trajectory of a system of confocal ellipses. This problem was first solved by the Italian mathematician Gaspare Mainardi (1800–1879) in the journal Annli di Scienze-Mathematische e Fisiche, Tome. Asutosh proposed a much simpler solution and arrived at a theorem, whenever the coordinates of any point on a curve can be expressed by means of a single variable parameter, the coordinates of the corresponding point on the trajectory may be similarly expressed. In his second paper, titled "Extension of a theorem of Salmon's", he generalized the problem and also extended it to the three dimensions. In his outstanding paper titled 'A note on elliptic functions', Asutosh established an additional theorem in the theory of elliptic functions by a novel process based on the

properties of the ellipse. Regarding this work, mathematician Arthur Cayley (1821–1895) remarked, "It was remarkable that a real result was obtained by consideration of an imaginary point." His paper, titled "A memoir on plane analytic geometry" was the longest of all the papers written by him. It consists of 62 pages which are distributed in 32 sections. These sections are almost independent of each other. Some of his papers are mentioned at the end of this essay. All the research studies were done independently by Asutosh, there was no one to supervise him. Quoting from another famous mathematician, Ganesh Prasad, we get, "Sir Asutosh's boldness of vision and independence of thought – qualities absolutely necessary for success as a mathematical investigator – showed themselves very early when, as a student of the first year class of Presidency College, he wrote his first paper in 1880." In 1888, Dr. Mahendralal Sircar, the founding director of Indian Association for the Cultivation of Science (IACS), invited Asutosh to deliver lectures on mathematics. Many teachers and post-graduates attended those lectures. Sir Gurudas Banerjee, the first Vice-Chancellor of Calcutta University, tried his best to create a professor post for Asutosh. However, all his attempts went in vain due to strong colonial opposition. Hence, despite being a hard-core mathematician, Asutosh had to move to law as a profession. Apart from being an excellent researcher and teacher of mathematics, he played a key role in promoting mathematical research in the country. He founded the Calcutta Mathematical Society (CMS) in 1908 and started a new mathematical research journal titled, Bulletin of Calcutta Mathematical Society in 1909. CMS is considered to be one of the milestones in developing the mathematical environment in modern India. Meghnand Saha (the man behind the famous Saha ionization equation in astrophysics), Satyendra Nath Bose (the man behind Bose-Einstein statistics and the particle name, 'boson'), Nikhil Ranjan Sen (the first Indian to get a doctorate in cosmology and relativity), Sisir Kumar Mitra (the first experimental physicist to work on the E-region, a part of Earth's ionosphere) were members of CMS. Renowned mathematicians like Ganesh Prasad (in 1909), A R Forsyth (in 1913), W H Young (in 1915) presented their speeches and shared their ideas in front of the Society under the umbrella of Sir Asutosh Mukhopadhyay. Sir Asutosh presided over 83 meetings of CMS till his death in 1924. Numerous mathematicians and scientists, like C E Cullis, Shyamadas Mukhopadhyay, W H Young, B N Seal, Ganesh Prasad, C V Raman, D N Mallik, M N Saha, A C Bose, B B Datta, S N Bose, N R Sen, S K Mitra and others, contributed to the Bulletin of CMS, making it one of the best Indian journals of that time. We will end this section mentioning a quote by famous English mathematician, Cuthbert Edmund Cullis (C E Cullis) who rightly remarked, "To every band of men engaged in the quest after truth and light, his (Asutosh's) help and encouragement were greatly and unselfishly given, and in learned societies and gatherings he was a dominant figure, his giving appreciation where it was due and advice where it was needed". Because of the remarkable personality, inspiration, wisdom and strength of character, Sir Asutosh became the true icon for the generations of future teachers, researchers and students.

Becoming the University himself:

Let's start this last section with a quote on Sir Asutosh Mukhopadhyay by Edward Robert Lytton who was popularly known as Lord Lytton, the then Governor General of India, ". . . in the eyes of his countrymen and in the eyes of the world, [he] represented the university so completely that for many years Sir Asutosh was in fact the university and the university was Sir Asutosh".

Sir Asutosh Mukhopadhyay served as the Vice-Chancellor (VC) of Calcutta University for five terms (from 1906–1914 and from 1921–1923). However, even after the end of his terms as the VC, he was there, directly involved with the university till his last breath on 25th May, 1924.

He was also a member of the Sadler Commision (1917 - 1919), established by Sir Michael Ernest Sadler, which took care of the developments, prospects and requirements of the university. It was his amazing personality, initiatives and tireless contributions that transformed the university from a mere educational institute to a world-wide facility of research and teaching, leading to a center of excellence. Not only he promoted science and mathematics, but also, he threw himself in boosting both Indian and Western philosophy, art and culture in the heart of the university. Because of his presence and sheer assurance, eminent barristers like Taraknath Palit, Rashbehari Ghosh and many others donated money generously to the spread and development of Calcutta University. With these endowments, he appointed some of the brightest minds of then India in different disciplines, including Sir Prafulla Chandra Ray (P C Ray) and Sir C V Raman. At that time C V Raman was an accounts officer at the Indian Finance Department and used to carry on his research during his spare time at the IACS. Sir Asutosh appointed Raman as the Palit Professor of Physics at the university. This act was objected by many Europeans who were at the center of the Senate because of the fact that Raman did not have a doctorate degree and was not trained from abroad. However, Asutosh did not listen to them. Later in 1931, in a speech given at Calcutta Corporation, Nobel Laureate Sir C V Raman remarked, "To another great citizen of Calcutta, a man who was most farseeing, profoundly gifted and inspired by the highest ideals, I mean the late Sir Asutosh Mookerjee, I am also under a deep debt of gratitude. Sir Asutosh ventured to ask a young and unknown Government official to throw aside the preferments of office and devote himself to the pursuit of knowledge under the aegis of the Calcutta University. This, on his part, was an act of great courage, whereas on mine it was just a case of following my own inclinations. . But for the action which Sir Asutosh took, my scientific career would long ago have suffered an abrupt termination." In 1914, Asutosh founded the Rashbehari Siksha Prangan which is commonly known as Rajabazar Science College, as a part of the science and technology department under the Calcutta University and appointed budding Indian scientists like Satyendra Nath Bose, Meghnand Saha, Jnan Chandra Ghosh, Sisir Kumar Mitra, Nikhil Ranjan Sen as teachers for the post-graduate science courses. He also set up several new graduate programs at the university including anthropology, comparative literature, applied psychology, industrial chemistry (along with Sir P C Ray), ancient Indian history and culture, Islamic culture etc. He was the person who founded the opportunity to research for the literature subjects like Bengali, Pali, Sanskrit etc. He was also one of the first persons to understand and recognize the mathematical genius of Srinivasa Ramanujan.Due to his immense love and tireless efforts towards the developments of the university, many Europeans came to teach at Calcutta University. Referring to the grand developments of Calcutta University under the influence and leadership of Sir Asutosh Mukhopadhyay, Governor General Lord Ronaldshay quoted, "I had visions of a modern Nalanda growing up in this – the greatest and the most populous city of the Indian Empire." Despite all these, he was a mathematician by heart. In 1908, He was honored by the DSc (Honoris Causa) degree by the University of Calcutta. He became the first President of Indian Science Congress in 1914 and the President of Asiatic Society of Bengal in 1917. The British Government honored him with the title, Companion of the Order of the Star of India (CSI) in 1908 and boasted him with the knighthood in 1911. Not only that, he also received many titles from different parts of India- "Saraswati" (1910) from the Pandits of Nabadweep, "Shastra Vachaspati" (1912) from the Dacca Saraswat Samaj, "Sambudhagama Chakravarty" (1914) and "Bharat Martanda" (1920).

One Can Write An Ocean Of Words About A Great Man Like Sir Asutosh Mukhopadhyay; However, I Choose To Stop Here To Avoid The Unnecessary Stretch Of This Essay. We Will End This Essay With Two Of My Favorite Remarks On Him.

"He (Asutosh) Was Mighty In Battle. He Could Have Ruled An Empire. But He Gave The Best Of His Powers To Education Because He Believed That In Education Rightly Interpreted Lies The Secret Of Human Welfare And The Key To Every Empire's Moral Strength"

--- Michael Ernst Sadler

"Men Are Always Rare In All Countries Through Whom The Aspiration Of Their People Can Hope To Fnd Its Fulfillment, Who Have The Thundering Voices To Say That What Is Needed Shall Be Done; And Asutosh Had That Magic Voice Of Assurance. He Had The Courage To Dream Because He Had The Power To Fght And The Confidence To Win, His Will Itself Was The Path To The

--- Rabindranath Tagore

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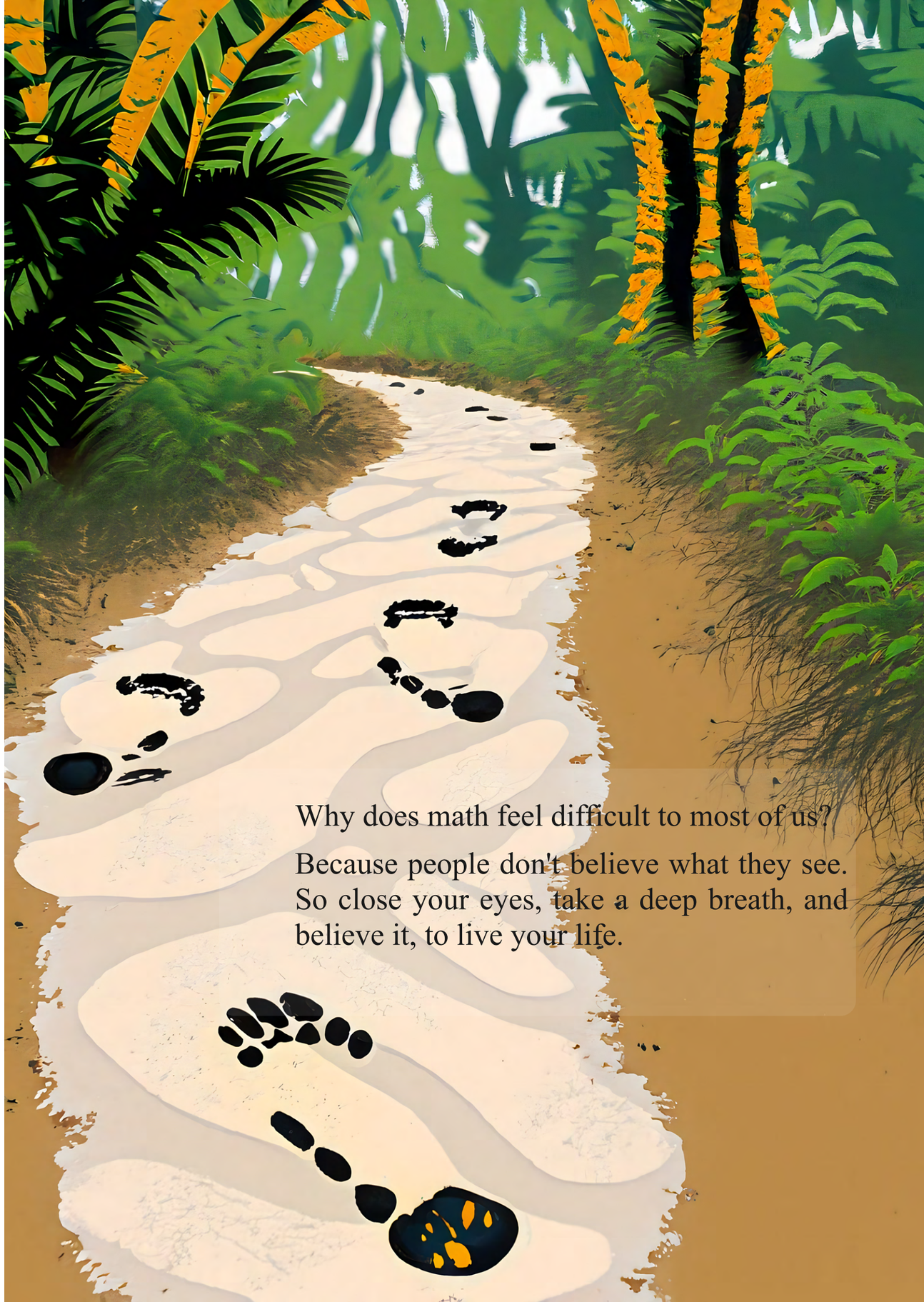


Works of Indian Mathematicians

India has been a major contributor to mathematics ever since ancient times. The mathematics of ancient and medieval India is a rich and diverse field that has significantly contributed to the development of mathematics as a whole. The works of Indian mathematicians have been influential in shaping the mathematical knowledge of the world and have played a crucial role in the advancement of mathematics and its application in various fields such as astronomy, engineering, and commerce.

One of the most notable contributions of Indian mathematics is the development of the decimal system. The Indian mathematician and astronomer, Aryabhata, in his work the Aryabhatiyam written in the late 5th century AD, described the use of decimal place value system, including the use of zero as a place holder, This system, which is still in use today, greatly simplified mathematical calculations and made them more accurate. The decimal system was later transmitted to the Islamic world and eventually to Europe, where it was further developed and refined.

Another important contribution of Indian mathematics is the development of the concept of infinity. Indian mathematicians were the first to develop the concept of infinite series, which they used to calculate the value of pi. The Indian mathematician Madhava of Sangamagrama, in the 14th century, discovered a way to approximate pi to any desired degree of accuracy using infinite series. This method, known as Madhava series, is considered a precursor to the modern calculus.

Indian mathematicians also made significant contributions to the field of algebra. The Indian mathematician Brahmagupta, in the 7th century, wrote the Brahma-Sphuta-Siddhanta, which contains solutions to quadratic and cubic equations.He also introduced the concept of negative numbers and zero as a placeholder in mathematical equations, The Indian mathematician Bhaskara II, in the 12th century, wrote the Lilavati, which is still used as a textbook in some parts of India.

In the field of astronomy, Indian astronomers developed sophisticated astronomical instruments and made precise measurements of the positions of celestial bodies. They also developed a calendar system used for religious and agricultural purposes.

BHĀSKARA II (C. 1114–1185)

The Indian astronomer and mathematician Aryabhata, in his work the Aryabhatiya, described the geocentric model of the solar system and accurately calculated the length of the solar year.Contributions to the field of geometry by the ancient Indian mathematicians cannot be ignored. The Indian mathematician and astronomer, Bhaskara II, in his work the Siddhanta Siromani, described various geometric constructions and theorems, including the Pythagorean theorem. The Indian mathematician and astronomer, Varahamihira, in the 6th century, wrote the Pancha-Siddhantika, which contains a comprehensive treatment of mathematical astronomy and trigonometry.
One of the most notable contributions of Indian mathematics in recent times is in the field of number theory.

Indian mathematicians have made significant contributions to the study of prime numbers and have developed new techniques for proving the distribution of primes.

In 2014, Indian origin mathematician Manjul Bhargava was awarded the Fields Medal, one of the highest honours in mathematics, for developing powerful new methods in the geometry of numbers, which he applied to count rings of small rank and to bound the average rank of elliptic curves.

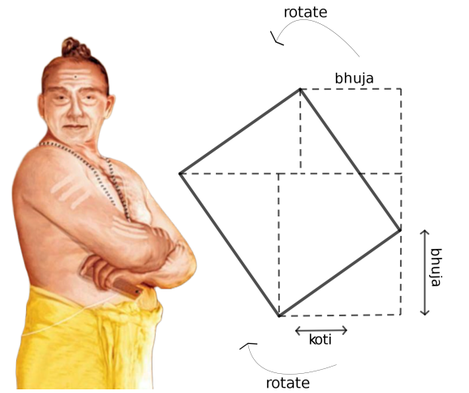
Indian mathematicians have also made significant contributions to the field of algebra. Indian mathematician Srinivasa Ramanujan, who lived in the early 20th century, made pioneering contributions to the field of modular forms and introduced new techniques for solving Diophantine equations. He also discovered a new way of summing infinite series, known as the Ramanujan summation. This method has been widely used in fields such as number theory, fractals, and string theory, and it has been instrumental in solving some of the most complex mathematical problems of the 20th century.

Indian mathematician R. Parimala, who works in the area of algebraic geometry and number theory, has made important contributions to the study of algebraic varieties and has developed new techniques for solving problems in algebraic geometry.

In the field of geometry, Indian mathematician C. S. Seshadri has made important contributions to the study of algebraic geometry, particularly in the area of moduli spaces. He has introduced new techniques for studying geometric objects, such as vector bundles, and has made significant contributions to the study of the topology of algebraic varieties.

In the field of computer science, Indian mathematician R. Ramanujam has made important contributions to the field of theoretical computer science. He has made significant contributions to the study of algorithms and has developed new techniques for solving computational problems. Indian mathematician T. N. Venkataramana has made important contributions to the field of algebraic groups, automorphic forms in mathematics.

The Indian mathematician and computer scientist, Raj Reddy, in the late 20th century, has made important contributions to the field of artificial intelligence, including the development of the first speech system, and he has also written several books on the topic. In recent years, Indian mathematicians have also been actively involved in mathematical research in international collaborations.



Mādhava of Sangamagrāma, explanation of the sine rule in Yuktibhāṣā

They have been working with mathematicians from other countries and have made important contributions to international mathematical research projects.

In conclusion, the works of Indian mathematicians have been influential in shaping the mathematical knowledge of the world and have played a crucial role in the advancement of mathematics in various fields, such as astronomy, engineering, and commerce. They have made significant contributions to the development of the decimal system, the concept of infinity, algebra, astronomy, and geometry. The contributions of Indian mathematics have been transmitted to other cultures and have greatly influenced the development of mathematics in other parts of the world. Indian mathematical community continues to contribute to the development of mathematics actively.

~Nilotpal Deka

***Prof. Anup Biswas** is an Associate professor and Deputy Chair, Mathematics at IISER Pune. He obtained his B.Sc. from Ramakrishna Mission Vidyamandira, Belur and M. Sc. and PhD from TIFR Centre for Applicable Mathematics, Bangalore, in 2011. He then did postdocs at the Technion IIT, Israel, and the University of Texas at Austin. He joined IISER Pune in 2014. We, the team of Mathzine, had a wonderful opportunity to interview Prof. Biswas, in which he shared his thoughts on a range of topics and his beautiful journey from being a student to a successful researcher.*

Interviewed by:
Sreedev Manikoth
Edited by:
Priyanshu Chourasiya & Sai Karan

We Must Not Be Satisfied With Our Work

1. How did you choose maths as a career, and how was your journey with Maths?
I was born and brought up in West Bengal and come from a humble background. Taking science as a career was very difficult for me because there were few opportunities. I was lucky to crack both engineering and medical exam. I was sure I would not go for medicine because I was not too fond of that profession. So, I decided to go for engineering. My father initially accepted my decision. However, seeing the college culture where I was to be admitted, being from a conservative background, he refused to admit me there. So, in the end, I decided to go for science. I was good in both maths and chemistry. I enjoyed chemical reactions. So, I was in a dilemma to choose between these two subjects. Finally, I decided on Maths as I always scored well in Maths.
After B.Sc., I gave competitive exams such as IIT JAM (Joint Admission Test for Masters) and MCA (Master of Computer Applications). I got the opportunity to continue my career with a computer course at IIT Roorkee or a math-stat course offered at IIT Bombay, and both of them had good job security. I also got interview calls for Masters at TIFR, Bangalore, ISI Bangalore and IISc, Bangalore. So, I came to Bangalore for further studies. I asked the dean of TIFR Bangalore about the mathematical finance course; although I did not know anything about it, I thought it would give me a good job. After getting admission to TIFR, Bangalore, I started taking courses there, and things started changing. I found the courses interesting.
After completing my M Sc, I decided to go for PhD in TIFR, Bangalore. During my PhD, I wrote a lot of papers. . If I found someone working on something interesting, I would collaborate with them on their work.But it takes a lot to focus. My mentor advised me to spend more time on something interesting and challenging as one seminal result would be better than writing many papers. At the end of my PhD, I was not satisfied with my work. Given that my research was on finance and probability, an applied area of maths, I decided I would get into the industry if I were not satisfied with my work after my Post Doc. But during my Post Doc, I did not like working in applied maths as I felt that area was not so deep. I was unhappy with my work and changed my research area multiple times.

2. How do you maintain a balance between your personal and professional life, and how did the pandemic affect your life?
Things were easy before my daughter was born, but I had more responsibility later. So, I had to choose the kind of problems to pursue. I did not have time to focus on mediocre problems. I started thinking about problems only if they were not trivial. I started focusing on generalising my ideas, thoughts and observations that could be utilised in a broader range of problems. I personally feel that there is a big difference in how we work when we collaborate with someone online as opposed to meeting physically.

When we have to meet someone for seminars virtually, usually everything is laid out and preplanned, focused on the crux of the problem or the purpose of the meeting. On the other hand, when we meet people physically and discuss a particular problem, we not only discuss the specific problem but also would be eager to know about the other person's work and casually discuss many problems. This opens the door for new opportunities and better projects and collaborations.

3. What are your hobbies?
I find great pleasure in listening to country music. I also enjoy playing piano and cello music. Apart from these activities, I find great joy in cooking. I used to cook a lot during my learning days. I like to cook Italian, Mexican, Chinese, etc. I also love to read books. I am always interested to learn something new. I like to read books on philosophy, psychology, Indian history, Mughal history, History of Science, etc. I like to read informative books that help me gain knowledge rather than novels and stories that entertain.

4. What is your advice to young mathematicians starting their journey in this field?
I personally feel that in research, we must not be satisfied with our work. If you are satisfied with what you are doing, things will stay the same. You cannot be satisfied; you need to try something harder. If you are satisfied with the problems you are working on, my opinion is to try problems which are harder in nature. I have observed that many young mathematicians are too focused only on their fields and do not care to bother about other areas or subjects. I believe that we must always expand our intellectual horizons. At the time of my PhD, a conference happened in Bangalore. I attended that conference and listened to many scholars talk about their work. After hearing them, I read a few topics such as optimal transport, analysis of infinite dimensional Banach spaces, etc. Although they were of no help during my PhD, they helped me in my research. After an age, it becomes tough to learn new things because you have a lot of projects and different responsibilities to shoulder. I advise young mathematicians to make collaborations with some good people if you find a good match, a good collaborator where your thought and their thought process match, then you both can do something beautiful.

A Proper Subset of

-Compiled by Amruntamshu A.K

Relations between pure and applied mathematicians are based on trust and understanding. Namely, pure mathematicians do not trust applied mathematicians, and applied mathematicians do not understand pure mathematicians.

Q: What do you get if you cross a mosquito with a mountain climber.
A: You can't cross a vector with a scalar.

A Mathematician (M) and an Engineer (E) attend a lecture by a Physicist. The topic concerns Kaluza Klein theories involving physical processes that occur in spaces with dimensions of 9, 12 and even higher. The M is sitting, clearly enjoying the lecture, while the E is frowning and looking generally confused and puzzled. By the end the E has a terrible headache. At the end, the M comments about the wonderful lecture.

E: "How do you understand this stuff?"

M: "I just visualize the process"

E: "How can you POSSIBLY visualize something that occurs in 9-dimensional space?"

M: "Easy, first visualize it in N-dimensional space, then let N go to 9"

Q: What does the 'B' in Benoit B Mandelbrot stand for?
A: Benoit B Mandelbrot

A topologist is a person who doesn't know the difference between a coffee cup and a doughnut.

Some Math Jokes

Q: Why didn't Newton discover group theory?
A: Because he wasn't Abel.

I saw a license plate once,
"ABELIAN."
He probably commutes.

A mathematician organizes a lottery in which the prize is an infinite amount of money. When the winning ticket is drawn, and the jubilant winner comes to claim his prize, the mathematician explains the mode of payment: "1 dollar now, 1/2 dollar next week, 1/3 dollar the week after that..."

Classification of mathematical problems as linear and nonlinear is like classification of the Universe as bananas and non-bananas.

I was speaking to a mathematical biologist who claimed that human body can survive without all of its organs. As a mathematician myself, I demanded proof. Apparently, the proof was in the appendix.

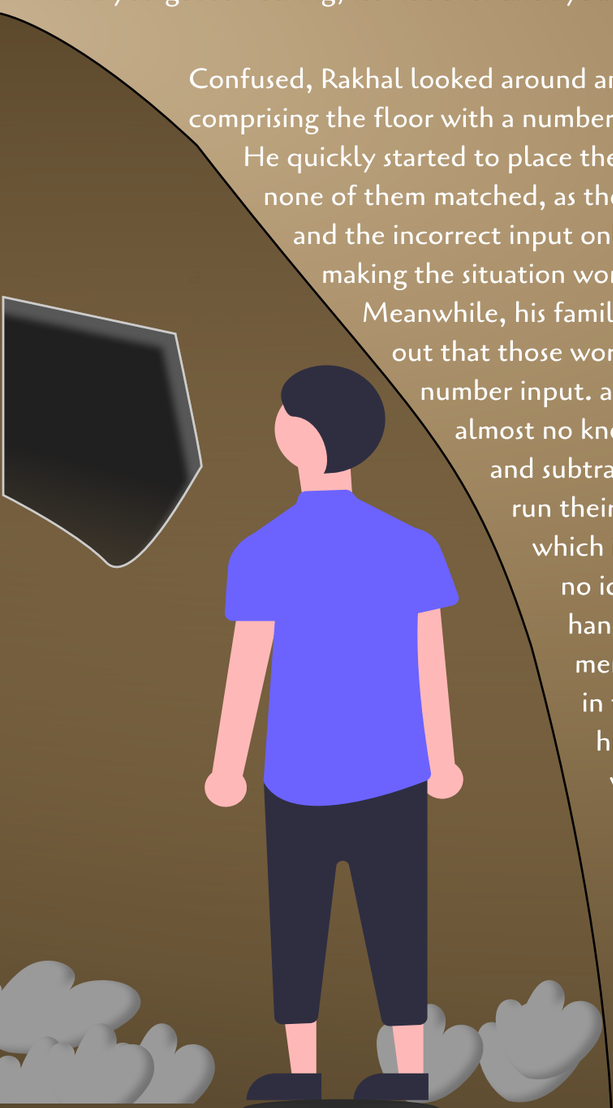
"If I have seen farther than others, it is because I was standing on the shoulders of giants." - Isaac Newton

"In the sciences, we are now uniquely privileged to sit side by side with the giants on whose shoulders we stand."
- Gerald Holton

"If I have not seen as far as others, it is because giants were standing on my shoulders." - Hal Abelson

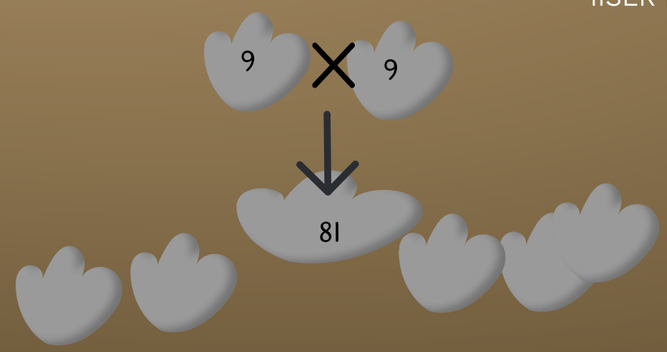
Maths In The Rocks

Rakhal was trying his best to find a way to open the mountain gate that had been shielded for years to get the mysterious herbs to heal his wounded sister and mother. After being exiled from the village with the accusation of playing treachery with the customers, Rakhal's family was passing through the forest at night when some bandits attacked them. Rakhal's mother, in her venture to save her belongings as well as her daughter, got herself badly injured. Rakhal somehow managed to get themselves out of danger, but his sister fainted too in shock with no sign of getting up. With no way of getting into the nearby village very soon, Rakhal recalled the mysterious herbs that lie behind the rocks of the mist mountain. He immediately took them to the mountain and found some rocks that looked as if they were guarding the entrance of the mountain. After lighting up a candle; Rakhal discovered there was a rectangular-shaped hole in the rock that seemed to be a keyhole with something written on the wall: "Hit it twice then mix with a similarly damaged one, and you get something; it's not over until you find another such pair".



Confused, Rakhal looked around and saw thousands of thin slab-like objects comprising the floor with a number written on them. "These might be the keys!" He quickly started to place them inside the keynole. But to his utter despair, none of them matched, as there was a slight difference in shape in the slabs, and the incorrect input only led to the falling of rocks from the roof, making the situation worse. He then stopped trying and began to think. Meanwhile, his family's condition was getting worse. Soon he figured out that those words were meant to lead him to the correct number input. aBut he never did proper schooling and had almost no knowledge of mathematics other than addition and subtraction, which he had to use on a regular basis to run their business. Once, he heard of multiplication, which is nothing but a repetition of addition. He had no idea if this little knowledge of his would come in handy or not, and he was almost on the verge of mental breakdown. He soon started to lose himself in the world of dreams, where he started imagining himself living a happy life with his family, but life was never so easy for them. "It's not over yet ", he murmured and shortly snapped out of his dream and began to turn things up. Without hesitation, he started to hit the slabs with one another until he found something unimaginable. When he hit a slab with 9 written on it with a similar one, one rock vanished and the number on the other one changed to 81. He again took another rock with the number 9 on it and hit it with the changed one which produced a rock with 729 on it. He was mesmerised, "So this is how mathematics work". He wasted no time doing the same thing, which is hitting the numbers from 0 to 20 then started hitting the newly generated rocks with each other until once the rocks from 10 and 9 produced the slab with 1729 on it. But the rock was never going to fit the hole, so he started looking for another combination, and finally, with all his vigour, he produced the number 1729 once again from 12 and 1. Now he had two rocks with 1729 written on them when he suddenly heard a sound coming out of the rocks, "Do it". Surprisingly, after hitting them with each other, they combined into one with the same number written on it but with a fine shape that perfectly fitted into the crater, and the doors opened, Rakhal was amazed to see the beauty of that place with the moonlight shining on the vegetation and the blue-coloured weeds spreading their charm. "That must be it", he said as he hurriedly collected those herbs and fed them to his family. Soon a sweet fragrance lingered in the air, and Rakhal fell unconscious. It was daytime when he woke up, and his mother and sister were staring into his eyes. They were in the middle of a forest, and Rakhal noticed there was no mountain in the vicinity. "Where did the mountain go?" He started behaving frantically when his mother calmed him down and consoled him that he might have been dreaming all these times but somehow recovered his family with the blue leaves that he found in the forest. Rakhal couldn't convince his family but fell in tears to have everyone back. They soon relocated to the neighbouring village where Rakhal gave the king those leaves, saving the king's son from a fatal illness. They received a tonne of money, with which they built their new house, launched businesses, with Rakhal also starting to find a knack in math.

--Suvradip Maity
IISER Kolkata



Prof. Spallone

How the juggling began

Interviewed by: Anargha Mondal
& Mandar Deshmukh
Edited by: Anand Tadipatri

Prof Steven Spallone obtained his PhD in 2004 from the University of Chicago. He was a postdoctoral fellow at Purdue University and the University of Oklahoma before joining IISER Pune in 2012. Below is an excerpt from the interview with Prof Steven Spallone, interviewed by Anargha Mondal, BS-MS student IISER Pune.

1.How would you describe your research interest in simple terms?

The description always depends on the audience, the person I'm talking to, and their background. I study some combinations of the interface of group theory, topology and representation theory. Representation theory is the study of the actions of groups on vector spaces, or "symmetries" of vector spaces, and how they decompose analogously to how linear transformations decompose according to a Jordan decomposition.

2. Exactly what does that entail? What tools do you use frequently?

I have been studying mathematics for a long time - I've been to grad school for a long time, followed by post-doc and other things. I've had a lot of time to read and learn about the machinery available in these different areas of representation theory and algebraic topology. My PhD background is in representation theory, although I had a modest background in algebraic topology. But I've been learning a lot more of the latter over the last few years so as to use those tools as well to study these objects.

3. When you visualise a solution to a problem you're working on, how is it different from what you would be finally sending for a publication?

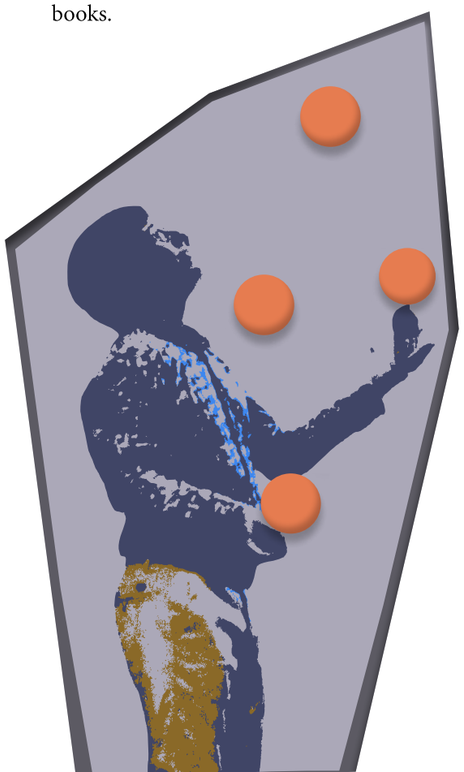
A journal is not a diary. First, you sketch out a lot of ideas, and things can be very rough and intuitive. Then you take another look at the most tenuous parts of your argument and strengthen them, and you keep working at it. You could look at an example of things, but you return to it, again and again, to turn it into tools of professional quality. You don't turn in your diary entry.It's often a large condensation process, taking some messy first thoughts and tightening the logic to make it.

4. What applications can your field have, for example, in physics?

I am proceeding with a lot of interesting mathematics, which I'm sure is interesting to physicists, but I myself have not studied physics at a serious level. I was a minor in physics, but I didn't go on to study theoretical physics. Now, I'm certain that a lot of what I do involving spin structures, the Clifford algebra and higher orientability is encountered in encountered in physics. People are interested in studying many things like principle G-bundles; I'm sure many physicists use this language. But I have not studied the physics side of things enough to give a proper explanation. It just happens to be a subject that interests me, and mathematics is also available in physics. There are things called characteristic classes, which play a big role in physics. So do vector bundles play a big role in physics, and what I study are invariants of vector bundles when they're associated with representation. At the end of the day, I'm not going to make your cell phone work better. I'm not working towards improving a particular thing; it's just that I'm adding to the body of knowledge. There's a lacuna there that I'm adding to. When the time comes, the big organisational apparatus of physics and mathematics, if they need it, will be available via a search, and I hope to have it in good form for them to use when they're ready.

5. What was your stimulus for getting into mathematics?

Growing up, I didn't have Numberphile or all these internet videos - there was no YouTube. I also didn't have access to a university library for proper math books.



My parents went to various yard sales, and they knew I had an interest (in mathematics), so they picked up books and brought them home. It was since something like my fifth grade that I enjoyed mathematics. It was a rewarding area to use my powers of concentration. You could unlock things that simplify complexity. The more I studied it, the more rewarding it was. It was a very coherent system, and bringing clarity to chaotic morass was something I found appealing about mathematics, and I continued with it. I also have a good memory for things - so I don't lose that - I keep in touch with the mathematics I've learned since childhood.

6. According to a report we found on science.org, you were persuaded by your good friend A. Raghuram to apply for the mathematics department at IISER. What prompted you to accept this offer and settle down in India?

When you go into PhD level mathematics and research, you tend to specialise in a particular area. So, on the one hand, not every department in the world appreciates your particular specialisation, but those who know about it and can understand what you did appreciate you. He (A. Raghuram) appreciated my work and what I was doing - we had been attending the same conferences and interacting. For that reason, I thought it would be a good place to have a home here, where there would be other people in my area of representation theory. So this kind of appreciation and the strength of character of Raghuram, who founded our math department, was instrumental in my coming here.

7. Students of IISER Pune know that juggling is one of your passions. How did you start juggling in the firstplace?

When I was a child in high school, I did theatre in the summers. It was a nice thing to do in the summer. I don't know if young people today appreciate how boring it can be in the days before the Internet and many other stimuli. There was a summer program called the "summer theatre", and I was involved in that. One year, I was assigned to be a jester. For that reason, someone there as part of this program tried to teach

me how to juggle. My parents bought me a set of juggling balls, and I started taking courses at a community college. So like you might take karate or any of these things that children sign up for, I started going to this juggling class and kept taking it and working on it. And again, being a bored kid, if you keep working at this kind of thing, you get very good at it. I maintained that, and when I went to college, then there were clubs of people. I could continue to improve and do a lot of club practice, which is something I kind of miss here. Anyway, I maintained that, and that was a lot of fun. And, of course, there is an interaction between mathematics and juggling, which I have given talks about.

8. We have seen other mathematicians and thinkers like Claude Shannon and Burkard Polster (i.e., Mathologer) who are avid jugglers. Do patterns in juggling help a mathematician connect the dots, or is it just a coincidence that many mathematicians have taken to juggling?

I think juggling as a physical activity is a nice complement to studying. You're concentrating for a long time on your studies, and then you take a break, and you juggle. So, on the one hand, physical activity complements studying activity well. But there is, of course, a lot of combinatorics involved in the thing. It's also a stereotype that engineers, for example, also do a lot of juggling and enjoy this. I don't myself do research in jugglology. It's a little bit distant from what I do, but it's fun, and it opened up a few new juggling patterns when people study it systematically with the Siteswap notation. But I just think it's a relief in a nice way. It's a nice way to get into a meditative state rather than a concentrating state. So when you do research, you want to have both. And how do you break out of the concentrating state? Well, juggling is a good way to do that. Just the same as going for a walk or for taking a shower.

9. In a book called "The Music of the Primes" by Marcus du Sautoy, he talks about the Riemann hypothesis and how modern physicists are trying to apply tools of quantum mechanics to get intuition into the problem and solve it. Like this, has physics benefited your field?

We rely a great deal historically on physics. Why are people interested in vector calculus? Or in manifolds? People must be interested in some things to get a patron, so it's worth studying these things. There are all kinds of applications. You don't want to be someone who just hears an abstract bunch of axioms and goes somewhere in a corner and figures out the logical consequences. You want your thing to be; you want it to have some meaning. Some of my work involves vector bundles and principle G-bundles. These are things that physicists take up, so there's a general appreciation for this, and I don't have to worry that I'm doing something meaningless. And physicists know some objects and have a sense of the existence of these objects; these mathematical objects govern their physics. You can find them that way, like in the wild west. There's some wild stallion going around; it's not tame, but you tame it. And we thank them for making it. The stereotypical thing is physicists point at something. Then a formalist, like a pure mathematician, will try to embody that in some way, rigorise it, or make it into axioms to clarify what's going on.

10. So, is mathematics just something that manufactures tools for physicists to use?

I need to talk about Platonism. So when you become a traditional mathematician, you start to talk about mathematical objects as if they're true or real. For example, about the real line, one mathematician will ask another mathematician: "Is the real line connected?", "Is the real line simply connected?". And when we talk like that, we're talking as if somewhere there is an actual real line. When we get nervous, we'll say, "Okay, we were just asking if these axioms imply this. ". But in reality, we think that these objects are actually true- that there are really circles out there and so on, and we become attuned to this way of thinking. We like to participate in the discovery of facts about this world. There's an aesthetic aspect to it. We enjoy this. And we feel like whatever question we ask, there is an answer. If you propose something, it's either true or false, which is the spirit of the thing. For any interesting question you've had that no one else ever asked, someone would be able to answer definitively one way. And there's some joy to that. And it is absolutely tied to applied things. As you said earlier, the world's structure seems to have some order to it. The order does seem governed by logic. Logic is what we're talking about right now. That should give some sense of the relationship between these two fields.

11. So you said there's aesthetics in mathematics. How do you define mathematical beauty?

Instead of answering your question directly, I'll talk around it. So, what's a good research problem? What qualities should a good research problem or paper have? This is an important question, especially if you're a master's student and want to get a taste of research. Well, it should be an open problem. It should also be accessible - it should be possible for you to solve it. It's not hard to find open questions, but a little harder to find accessible ones. And it should be interesting. So, you can never compromise on open; you can never compromise on accessible. If you don't have so much background or if your resources or time are limited, then you have to compromise in some way on the interesting part. So, you must learn the art or skill of making whatever you work on interesting. You must bring out the beauty of the interesting part of your work. It's a very important issue: How do you make your work beautiful? How do you proceed in the right way? Of course, these are useful things when you say something like truth or beauty. But it would be hard to make a proper definition. So, something beautiful for the top mathematicians might not be so beautiful for a first year college student. So, you have to understand that these things change a lot. A lot of the aesthetics change as you become more mature, just like the standard of proof changes as you get more mature. Some things that make things beautiful are analogies. There's a beautiful analogy between numbers and polynomials, for example. There's something called the Chinese remainder theorem, and I proved an analogue of that for partitions, and I think that was a nice example of an analogue. There might be an important, interesting statement, assuming it's beautiful. And you might want to see if there's a converse of that. Well, maybe you can (prove such a converse), but maybe there's an interesting counter example. Something could be beautiful because it's seminal; it gives more ideas. It opens up an area - this example can be used and modified to make other interesting things. If you can draw a certain picture, it will help you understand something more. When you draw a picture, there's an obvious gap once you draw it, and then you see the gap, and that's the logical thing you need. That can be a beautiful thing. There are a lot of proofs in Euclid like that. Something that answers a natural question (is beautiful). There's a flow of mathematics that you start to learn about after you become a bit more mature and learn a few mathematical theories. You learn how things flow. So maybe for this interview, I'll just assert that there is a flow, and things that are pushing in the right direction of the flow are beautiful.

12. What advice would you like to give to freshers who want to enter pure mathematics but are somewhat daunted by the abstraction involved?

Do a lot of problems. Find a book with a lot of problems where you prove a lot of things, and don't use the Internet. Solve problems from start to finish. That's what you have to do. Think by yourself. You want to think through all the math that you've heard of and try to see if you can prove everything from start to finish. Are there some gaps? Try to understand the terrain of mathematics. What are the fundamental assumptions? And from those, can you get to everything that you might want? Fundamental theorem of arithmetic, fundamental theorem of algebra, or anything you've heard of in calculus - can you prove that? This is what the math major is; this is the undergraduate training.

Inspiring Women in Mathematics

The story behind "May 12 Initiative" starts with the Fields Medalist Maryam Mirzakhani, 12th May being her birthday. Maryam Mirzakhani was an Iranian mathematician and a Professor of Mathematics at Stanford University. She was one of the world's leading experts in geometry and dynamical systems. Her beautiful and astonishing results and her life and career are an inspiration for everyone to pursue their dreams in science. In 2014, she was awarded the Fields Medal, becoming the first woman, and the first Iranian, to be recognised for her mathematical achievements by this top mathematical prize. The official Fields Medal announcement read

Outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces

Unfortunately, she died tragically from cancer in 2017 at the early age of 40. On 31st July 2018 in Rio de Janeiro, the Women's Committee of the Iranian Mathematical Society presented to the participants of the World Meeting for Women in Mathematics, $(WM)^2$, a proposal that the birthday of this legend – May 12th – be recognized and supported to celebrate women in mathematics. This was approved by a vast majority of $(WM)^2$ participants and the decision was made to disseminate the initiative. The "May 12 initiative group" was created and every year thereafter, many events have been organized worldwide and are available on May12 Initiative website*.

Maths club in collaboration with the Math department of IISER Pune hosted two events as part of the initiative: the first being a talk by Dr. Cristiana De Filippis on her life and work. Dr. Cristiana De Filippis is currently an assistant professor at the University of Parma, Italy. She is specialized in the area of calculus of variations and regularity theory of differential equations. Dr. De Filippis completed her PhD from the University of Oxford in 2019 and was awarded the prestigious G-Research Prize in that year. In 2020 she was listed as one of the top cited mathematicians according to the AMS database. In 2021 she received the Gioacchino Iapichino prize of the Accademia Nazionale dei Lincei. This year she has been elected as one of the 30 fellows of the European Mathematical Society Young Academy.

Secondly, Olga Ladyzhenskaya -- a short film about Olga Aleksandrovna Ladyzhenskaya (1922 –2004), a Russian mathematician who worked on partial differential equations, fluid dynamics, and the finite difference method for the Navier–Stokes equations was screened. She occupies a very special place in the history of mathematics and mathematical physics worldwide. Her theorems shaped the modern theory of Partial Differential Equations of mathematical physics. This film premiered at $(WM)^2$ in 2022 at OAL Celebration**.

Maths club extends its gratitude to Prof. Anisa Chorwadwala, Prof. Mousomi Bhakta and Prof. Rabeya Basu for giving this truly delightful opportunity to be a part of the world wide celebration for women in mathematics.
After reading this article and getting to know about the wonderful women mathematicians and their great contribution, we might as well leave you wondering about the title of the article: does it imply **inspiring** women in mathematics or inspiring **women** in mathematics?

-Heerak Sharma

* Visit the May 12 initiative Website: https://may12.womeninmaths.org

** https://2022.worldwomeninmaths.org/program/OAL-celebration

KEY QUALITIES AND EUREKA MOMENTS WITH

PROF SUPRIYA PISOLKAR

Mathematician at IISER Pune

INTERVIEWED AND EDITED BY PRASANNA BHAT

Prof. Supriya Pisolkar is an Associate Professor in the Department of Math-ematics. She is also a member of the coordination committee of the newlyestablished Department of Science Education. She pursued her BSc at SirParshurambhau College (SP college), Pune and her MSc from Pune University.Subsequently, she completed her PhD from Harish Chandra Institute in Allahabad and a post-doctorate at Tata Institute of Fundamental Research(TIFR),Mumbai, before joining IISER Pune. It was delightful to talk to her about herresearch interests, her views on pedagogy, her hobbies and more.Below is an excerpt from the interview:

1. Could you shed some light on your area of research?

The projects that I work on are from three different branches of algebra and number theory. One of them is understanding and comparing available different constructions of a functor from the category of associative rings to the category of abelian groups, called the Witt vector functor. It was proposed by E. Wittin 1936, for commutative rings. His construction was generalised to associative rings by many mathematicians and now has applications not just in number theory but topological cyclic homology, representation theory and perfect oid spaces. These all constructions are known to match with that of Witt's when the ring under consideration is commutative. A natural question is, how they are related in non-commutative set up. The second area is arithmetic geometry. I am fascinated by the celebrated Fontaine-Mazur conjecture. I am working on some version of this conjecture. This requires knowledge of algebraic number theory and algebraic geometry. The third area is lie groups and lie algebras. I have worked on problems which help in understanding the arithmetic aspects of locally symmetric spaces. Typically, people try to continue in the area pursued during PhD, but I was staring at a dead-end and had to pick up many other areas

2. Did any event or person inspire you to take up research in mathematics?

During my schooling, I was very much inclined towards sports and I was fairly good at academics, in particular I was never scared of mathematics. But, I never thought that I would become a mathematician. In the first two years of my bachelor's, I had to take up courses from various disciplines and was more interested in electronics (than mathematics). When I had to pick a major before the start of the third year, instead of choosing electronics, I chose mathematics because, I had realised that the mathematics department had wonderful professors who would put in a lot of effort in teaching and mentoring students. That's how I changed my major to mathematics. I was really lucky to have great teachers who showed me the elegance of the abstractness in mathematics. I owe a lot to Prof. Kanetkar and two of my mathematician friends with whom I used to discuss mathematics for 8 to 9 hours daily in Bhaskaracharya Pratishthan, during my vacations. They are my constant source of inspiration.

3. What do you consider are the essential qualities of a good researcher?

I think, curiosity is the most important quality, let alone in research but even in everyday life. Only with curiosity would you be able to learn something and enjoy the learning process. I think the second quality is consistency. It is important to keep working on one's passion, no matter the challenges. Of course, this consistency has to be backed up by the hard work. In research, it does not matter whether your area fast (smart) thinker, what matters is - You are a thinker.

4. Was there a Eureka moment that you would like to share?

Yes, every researcher has such a moment. After my PhD, I was worried about finding a doable research problem because during postdoctoral research, although you will have a faculty mentor, he/she might not be working in your area of interest. You have to find a research problem on your own and quickly establish yourself. I found a problem in an area which was different from my thesis area, and I solved it independently, which filled me with confidence. So, it was not a Eureka moment in the sense of solving a very difficult problem, but it gave me the necessary self-assurance to be an independent researcher.

5. What do you do when you feel lost in your research?

Such a phase is so common in a researcher's life that he/she has to devise a mechanism to deal with it. A good routine, such as going for a run early in the morning, keeps me going. I love teaching. Teaching helps me to take my mind off if I am going through a phase of frustration of not making progress ona problem. What actually helps making a progress is discussing with collaborators and colleagues. My 12 years old daughter is a sweet stress buster for me. I also have a good social circle, spending time with friends and enjoying classical music definitely helps me relieve some of the stress.

6. How important is it to look at why a proof technique is not working?

I would like to discuss this with an example of a method of induction. It is typically taught at school level and soon it becomes a routine procedure to solve problems by induction. However, let's look at the statement 'All horses are of same colour'. Let's try to prove this by induction on the size of the set. The base case is the set containing one horse. Hence the statement is true. Hypothesis: for all sets of sizes less or equal to n, the horses are of same color. Now suppose we have a set of n+1 horsesh1, h2, ..., hn+1Then consider the setS1 := h1, .., hn which has same color horses andS2 := h2, ..., hn+1which also has same color horses by our hypothesis. Because the intersection ofS1 an S2 is nonempty, all horses are of same color. Can you spot the mistake in the above argument? Note that we did not check at every step that the intersection of two sets S1 and S2 is non-empty. For e.g. when there are only two horses, these sets will have an empty intersection and hence we can not proceed with the argument. In fact, I know a case of a PhD thesis defended in Princeton university where the statement of the result was so compelling, that no referee on the viva defense panel carefully looked at the base case of induction used in the argument. Later, one of the authors realised that the result is in fact not true for the base case, and the whole argument broke down. They later modified the proof and it became an even stronger result in algebraic geometry. So, in this simple procedure of induction, there are various ways in which errors can be committed. So it is very important to verify the procedure, sometimes considering the problem for smaller or more easily verifiable cases

7. Number theory is known for problems that can be stated easily but require sophisticated mathematical tools to tackle them. Is there any such problem that has fascinated you?

When we talk about integers, the first interesting subset that comes to mind are the primes. An obvious question to ask is whether there are infinitely many primes. Answer is Yes, and has this simple argument: Suppose there are finitely many primes $p_1, \ldots, p_k$ then considering , $n = (p_1 \cdot \ldots \cdot p_k) + 1$ if it is a prime, it is not already in the list, and if it is a composite, it cannot be divided by any prime already in the list, hence a contradiction to every prime being part of the mentioned list. Another interesting observation is that while the initial primes 2,3,5, etc., are quite close to each other, they seem to be farther apart as the primes grow larger. Prime number theorem talks about this distribution of primes by giving a formula to estimate the number of primes below a specified natural number. Fermat's last theorem is one of the most celebrated conjectures of last century. It brought turmoil in number theory. It was a driving force behind the development of many branches-representation theory, modular forms and Elliptic curve theory and it brought together many areas of mathematics. While these are now known results, there are various conjectures, such as,

Goldbach conjecture posed in 1742, which says that every even natural number greater than 2 is sum of two prime numbers. This is such a simple-minded statement but has remained unsolved after centuries of work. Similarly the twin prime conjecture. I will encourage you to read these conjectures and the history of the developments that have happened in order to solve them. It is absolutely fascinating.

8. As part of the Department of Science education, do you see a sharp contrast between those who enjoy mathematics and those who feel mathematics is highly unintuitive and difficult?

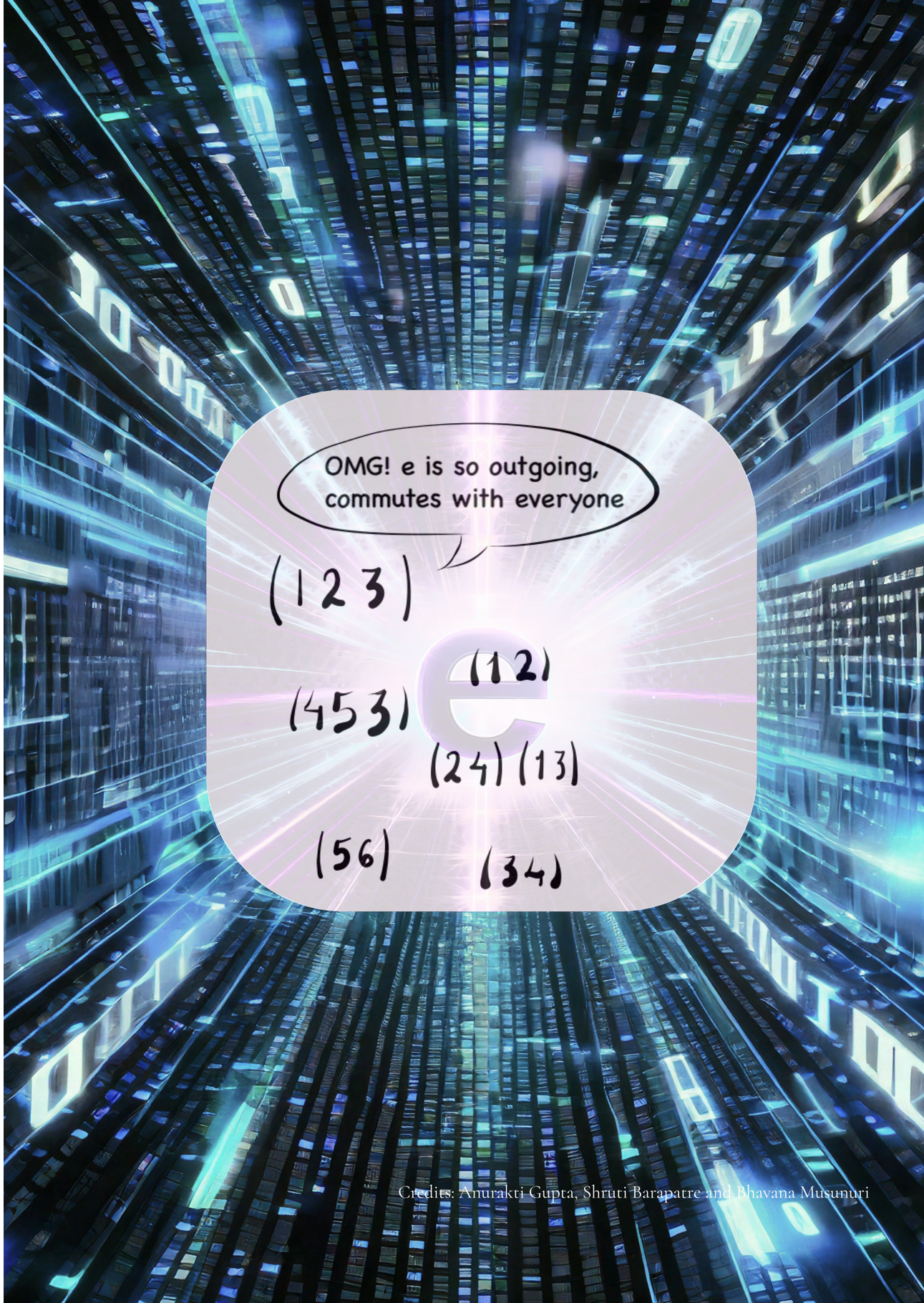
We all know that, when it comes to mathematics: students either like it or they are scared of it. When students are able to solve multiple mathematics problems in a short time, then they are given an impression (by others) that they are good at mathematics. I feel that understanding concepts is the only important part. The 'speed' component is really overrated, it brings in the discomfort with mathematics, because it does not give the required space to develop the logical thinking. You can enjoy mathematics only if your are first, comfortable with it and have spent enough time with it. This should be encouraged at the school level and also by parents. Parent's labeling their kids 'good' or 'not good' in maths, based on school exam marks is very damaging. It kills their desire to learn mathematics, forget about enjoying it. Besides, learning mathematics is not just about how the individual is able to grasp material but also heavily depends on textbooks and instructors, especially during schooling and undergraduate level. Having good teachers for mathematics who will bring the interest and excitement about the topics is very important. Logical thinking develops over a period with systematic efforts taken in the classrooms. Efforts are now being made at many fronts to involve science educators while writing text books, providing supporting material for teachers to teach mathematics in an interesting and effective manner. Lastly, students should attempt to engage in discussions. It is also important to ask questions without hesitation, and instructors have the responsibility of encouraging students in this exercise.

9. How important is mathematics communication in bridging this gap? How can we (as undergraduates with mathematical training) contribute to this?

Yes, it is important to discuss mathematics with your peers and teachers. Thelogical thinking is developed when your thoughts are challenged by others. Ifour senior undergraduates could discuss maths with juniors by the means ofinformal lectures/talks on a regular basis then it would definitely help them.Discussing science (not just mathematics) should become our culture.

10. Could you tell us some of your hobbies?

I was a state level player of rope mallakhamb and gymnastics. I enjoy running,doing Iyengar yoga and Pilates. I require no motivation to go for a run early inthe morning. I like classical music (Hindustani and Western). I play Sitar andtry to do riyaz everyday. I am a nature lover. I like bird-watching, and I havedeveloped a beautiful garden in my balcony



Credits: Anurakti Gupta, Shruti Barapatre and Bhavana Musunuri

3. Can you find your seat?

420 people are waiting to board Airbus A42, which has 420 seats. The first person's ticket says Seat 1; the second person in line has a ticket that says Seat 2, and so on. You are the 420th person whose ticket says Seat 420.

The 1st guy has lost his boarding pass (but is already inside the plane). Unfortunately, he has dementia and doesn't remember his seat number, and he sits in a seat randomly chosen by him (he might randomly choose to sit in Seat 1).

Now, the following 418 people will always sit in their assigned seats if possible. But if their seat is taken, they will randomly choose one of the available remaining seats. They all take their seats in the order that they are standing in line, i.e., 1st person takes the seat first, 2nd person takes the seat next and so on.

What is the probability that you will sit in your own seat?

next look
in fab math merch
I'm knot as twisted as I look

This year the maths club has had a remarkable number of submissions for the *Merch design contest*, with individuals showcasing their artistic skills while capturing the essence of mathematics in visually captivating designs. The response has been nothing short of astounding, with an array of exceptional submissions that deserve heartfelt appreciation.

The design submissions have left the Maths Club in awe of the sheer talent and dedication displayed by the participants. Each design tells a unique story, beautifully blending artistic vision with the elegance of mathematical concepts. From intricately crafted geometric patterns to clever arrangements of equations, these designs showcase the designers' ability to seamlessly merge aesthetics with mathematical accuracy.

One of the most remarkable aspects of the submissions is the remarkable diversity in themes and styles. The designers have ingeniously explored various avenues, catering to different preferences and tastes. Whether it be minimalist and sleek designs that speak to a modern aesthetic, or bold and vibrant compositions that catch the eye, the range of creativity on display is truly inspiring.

The Maths Club would like to extend its appreciation and thank all the talented individuals who participated in this creative endeavour. Each submission represents countless hours of dedication and passion.

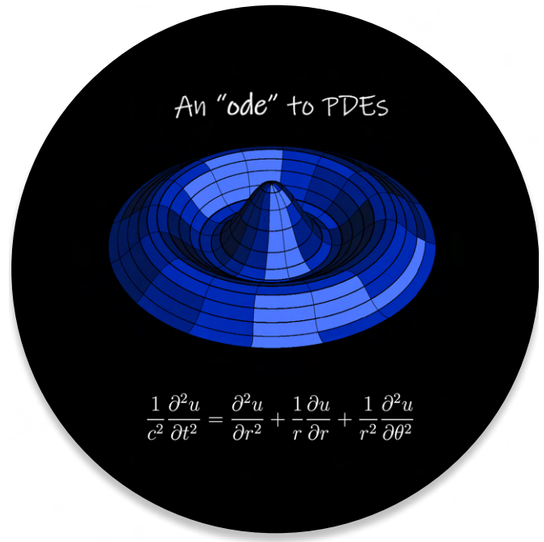
Even then, a decision had to be made for the final design of the merch, so after a careful discussion with the members of the club, we selected what we thought were the best two among the designs:

The knots design - Again as simplistic as it looks, it captures the elegance from the knot theory with a tint of good humour. Different types of knots including trefoil knot, square knot and Borromean rings have been placed in the background.

Mandelbrot fractals - As simplistic as it looks, it encapsulates the essence of never ending fractal geometry. The design is based on the B in the name of Benoit B Mandelbrot, the father of fractal geometry, which keeps the cycle of the name repeating itself, thus creating a simplistic but beautiful fractal design.

some other marvelous submissions...

An "ode" to PDEs- ode refers to an ODE (ordinary differential equation) as well as the English word ode (a poem). The figure is that of a vibrating membrane, which is the solution of an ODE to the PDE written (i.e. the ODE obtained on separation of variables of the PDE).



It is about the interesting number paradox by Hardy and Ramanujan, and pi is upto the Ramanujan-Hardy number. A radiohead creepsong reference, and the background around pi digits is representation of fock space ("focking")

IGNORANCE TOWARDS MATHEMATICS IS IGNORING THE BEAUTY OF CREATION. SO TAKE UP A PEN AND PAPER, AND GET IMMERSED IN THIS HARMONIOUS CREATION.

-SUVRADIP MAITY
BSMS,IISER KOLKATA

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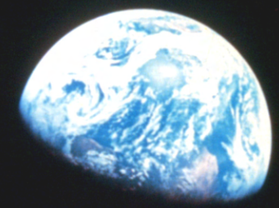
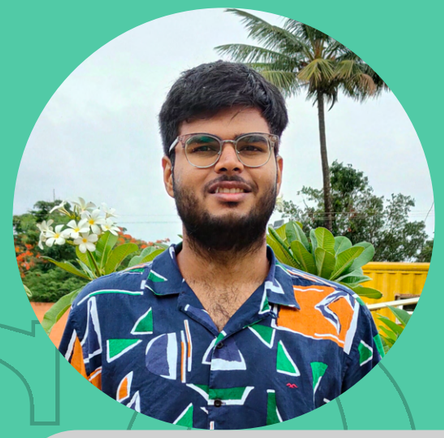


Meet the

Bosses

Padmapriya S, BSMS 2026
Life is a constant cycle between telling the volunteers that we have enough money and convincing the maths department that we won't spend much ...

Prasanna Bhat, BSMS 2025
I take a long time to say a yes, But once it's a yes there's nothing to suppress! I count, maintain accounts and if a good opportunity doesn't knock, with time I learnt to build a door myself and welcome!



Heerak Sharma, BSMS 2026
Through paths explored, my journey found shape,
In a role where puzzles and surprises were at every place.
Within the club's embrace, I learned my way,
Grateful for friends, lighting up each passing day.

and the

Coreeeeee

Sreedev M, BSMS 2023
When I'm in a clothing store, I don't look for beautiful clothes..I look for clothes that make me look beautiful

Shreyas Pai, BSMS 2026
A budding researcher, an aspiring teacher who is passionate about mathematics.

Jetharam Bhambhu, BSMS 2026
I am going to become king of the Pirates.

Aravind R, BSMS 2025
A disciplined disciple of math

Yash Gupta, BSMS 2026
A hybrid of cat and panda who loves sleeping and proving homeowmorphisms.

Amruthamshu A K, BSMS 2026
I am amazed at the exceptional advice that I give, but I am incapable of taking it myself.

Karthik Vasisht, BSMS 2023
Student Hack #1: If a course is uninteresting, just wait till exam week; the appreciation one gains for the material always amazes me....

Shrey Mukund, BSMS 2025
Ehhh! I like apple pie.

Amoghavarsha A V, BSMS 2025
I see the crimson thief.

and our beloved

Team

Manas Anmol, BSMS 2027
What defines me is my ability to think.

Aarya Dandagaval, BSMS 2027
God is a Woman, I trust in Ariana

Dinesh Karthik, BSMS 2027
Doing Math is my passion and I enjoy listening to carnatic music.

Dipesh Kacchap, BSMS 2027
Found a way to live in the present moment with gratitude.

Shruti Barapatre, BSMS 2024
I am passionate about maths, art, and dance. Afternoon naps are the best!

Mandar Deshmukh, BSMS 2027
I am a 2nd year BS-MS student interested in interdisciplinary research and I like knowing the journeys of people I meet.

Yashvita Subramanian, BSMS 2027
I am Driven by curiosity, I seek to uncover the secrets behind the life on our planet. I am a biology enthusiast who loves applying interdisciplinary approaches to solve mysteries of nature.

Anargha Mondal, BSMS 2027
Physics aficionado, mathematics enthusiast and a cinema fanatic

Srujan Mule, BSMS 2026
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Sanchit Nayak, BSMS 2026
Loves proofs, hates fruits. Often seen lurking around the football field

Krishna Saproo, BSMS 2027
I'm Krishna – a BS-MS student and a young Changemaker. You'll often find me lost in penning down poetry and articles.

Ujwal Pandey, BSMS 2025
Born to do math, forced to write exams

Ipsa Bezbarua, BSMS 2023
When she isn't busy exploring the types of curves on a Klein bottle, Ipsa Bezbarua enjoys music, literature, tennis, cats and Maths Club

Yash Karampuri, BSMS 2025
Jack of all, soon to be master of one

Kwanit Gangopadhyay, BSMS 2027
I love music and mathematics too, and play the tabla. And I find it very interesting to play around with rhythm patterns mathematically!

Sai Kiran, BSMS 2027
Hilbert's 'Wir müssen wissen' is serious business, but 'Wir werden witzig' is my specialty!

Priyanshu Chourasiya, BSMS 2025
I like listening to music, Indian music always fascinate me. I like to study math, diary writing and playing batminton.

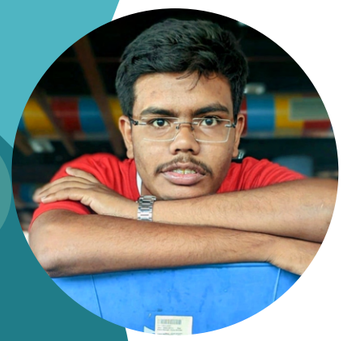
Mukul, BSMS 2027
I'm passionate about diverse intellectual pursuits: physics, math, finance, geopolitics, and business and many more. Memes unleash my creativity and wit.

Jagadish Parameswaran, BSMS 2027
I am enthused by all the sciences and love chess.

Jigyas Baruah, BSMS 2027
Quite possibly a tree. Also happens to be a maths and sports enthusiast.

Vivek TA, BSMS 2027
I'm like a math sorcerer, casting spells of logic and equations, with an insatiable thirst for mathematical wisdom and mastery!

Gurjot Singh, BSMS 2027
"A biology guy and an aspiring immunologist. I love playing badminton and reading interesting stuff."



Rudresh Gade, BSMS 2026
I am the king of procrastination.

Tanmaay Jena, BSMS 2026
King of sleep

APPENDIX

We have tried to give a first introduction to a few of the mathematical terms appearing in throughout the magazine.

1. Goldbach Conjecture: This conjecture states that any even number greater than two can be written as the sum of two primes. It was first proposed by the Prussian mathematician Christian Goldbach in a letter to Leonhard Euler dated 7 June 1942. This is also called the 'strong' Goldbach conjecture. A weaker version of the statement states that every odd integer greater than 7 is a sum of three odd primes. (Note that if the strong Goldbach conjecture is true, then every even number greater than or equal to 4 is a sum of two primes, so adding 3 to each even number produces each odd number greater than 7, which will be the sum of 3 odd primes, thus proving the 'weak' Goldbach conjecture.)

2. Twin Prime Conjecture: A pair of primes are called twin primes if they have a difference of 2. The twin prime conjecture states that there are infinitely many twin primes. Yitang Zhang, in 2013, proved that there is a natural number N less than 70 million such that there are infinitely many pairs of primes with difference N. Subsequently, James Maynard, Terrence Tao and many other mathematicians have reduced this bound to 246. See https://www.quantamagazine.org/mathematicians-team-up-on-twin-primes-conjecture-20131119/ for an interesting Quanta magazine article that details the method used to accomplish the progress mentioned above.

3. Vector Space: A vector space is a set of elements (vectors), which may be 'added' together and multiplied by numbers (scalars, often from $\mathbb{R}$ or $\mathbb{C}$). Vector spaces generalize Euclidean vectors (geometric objects with a magnitude and a direction), allowing the modelling of physical quantities, such as forces and velocity, that have both a magnitude and a direction. The concept of vector spaces is fundamental for linear algebra, together with the concept of matrices, which allows computing in vector spaces. This provides a concise and synthetic way for manipulating and studying systems of linear equations.

4. Number Theory: Number theory is a branch of pure mathematics devoted primarily to studying integers and arithmetical functions (real or complex-valued functions defined on positive integers). Its sub-branches are Elementary, Analytic and Algebraic Number Theory.

5. Lie Groups and Lie Algebras: A Lie group is a group (48.) that is also a differentiable manifold. A differentiable manifold is a manifold (55.), which is locally similar enough to a vector space (3.) to allow one to apply calculus. A Lie algebra is a vector space with a binary operation called the Lie bracket which satisfy some 'nice' properties. There is a close correspondence between Lie groups and Lie algebras. They are used extensively in physics, particularly in quantum mechanics and particle physics, and encode information about the symmetry of physical systems.

6.Algebraic Number Theory: Algebraic number theory uses techniques of abstract algebra to study properties of integers, rational numbers and their generalizations. Number theoretic properties are expressed in terms of algebraic objects to answer essential questions in number theory (4.), such as solutions to some Diophantine equations (67.).

7. All horses are of the same colour : This paradox arises from an incorrect application of mathematical induction. See https://en.wikipedia.org/wiki/All_horses_are_the_same_color to know where the flaw arises.

8. Ruin probability: Ruin theory is an area of applied probability which uses mathematical models to describe an insurer's vulnerability to insolvency/ ruin. Such models focus on questions such as the probability of ruin and deficit at the time of ruin.

9. Exponential Distribution: It is a probability distribution of the time between events in a Poisson point process, i.e., a process in which events occur continuously and independently at a constant average rate. Hence, its density function (for non-negative values of x) is defined as $\lambda e^{-\lambda x}$, otherwise.

10.Heavy-tailed distributions: They are probability distributions whose tails are 'heavier' than the exponential distribution (9.). In many applications, the distribution's right tail is of interest, but a distribution may have a heavy left tail, or both tails may be heavy.

11. First order logic: It is a collection of formal systems (structures used for inferring theorems according to a set of rules), which in addition to propositions and connectives connecting the propositions (negation $\neg$, or/disjunction $\vee$, and/conjunction $\wedge$, implication $\implies$, equivalence $\iff$), have existential quantification $\exists$ and predicates, which represent a property or relation. It is used extensively in mathematics, computer science, philosophy and linguistics.

12. Bayes' Theorem: In probability theory and statistics, Bayes' theorem, named after Thomas Bayes, describes the probability of an event based on prior knowledge of conditions that might be related to the event. For example, suppose the risk of developing health problems is known to increase with age. In that case, Bayes' theorem allows the risk to an individual of a known age to be assessed more accurately by conditioning it relative to their age rather than simply assuming that the individual is typical of the population.

13. Bayesian statistics: Bayesian statistics is based on the Bayesian interpretation of probability, where probability expresses a degree of belief in an event. It uses Bayesian inference, a method of statistical inference (using data analysis to infer properties of an underlying distribution of probability) which uses Bayes' theorem (12.) to update the probability for a hypothesis as more evidence or information becomes available.

14. Recurrence relations: A recurrence relation is an equation according to which every term of a sequence is related to its preceding term(s). Such equations have important applications in many areas of math, particularly combinatorics (53.), and encode information about how the sequence evolves.

15. Stochastic calculus: It is a branch of mathematics that allows a consistent theory of integration (a method to add infinitesimal (tiny) quantities) to be defined for integrals (the addition of infinitesimal (tiny) quantities) of stochastic processes (random processes). It has applications in mathematical finance (mathematical modelling of the finance market).

16. Percolation theory: It describes the behaviour of a network (or a graph, (75.)) when nodes (or vertices) or links (or edges) are added. Percolation is a type of phase transition (change of state). Percolation theory has applications in materials science, network theory, and other areas.

17. Ramsey theory: Ramsey theory is a branch of combinatorics that focuses on the appearance of order in a substructure given a structure of a known size. Ramsey's theory asks questions such as: 'How big must a structure be to ensure that a certain property holds?'. The famous 'Ramsey's theorem' or 'party problem' states: 'In any party with at least six people, there are three people who are all either mutually acquainted (each one knows the other two) or mutually strangers (none of them knows either of the other two)' and its generalization to more strangers and acquaintances is a fundamental result of this area. It has applications across many fields of mathematics, such as number theory (4.) and formal logic, and even in theoretical computer science.

18. Extremal combinatorics: Extremal combinatorics studies how large or how small a collection of finite objects (numbers, vectors, sets, etc) can be if it has to satisfy certain restrictions. Ramsey's theorem (17.) and the closely related question: 'How many people can be invited to a party where among every three people, there are two who know each other and two who don't know each other?' whose answer is five according to Ramsey's theory are the typical examples of questions that are dealt with in the field.

19. Probabilistic method: It is a non-constructive method, i.e., it proves the existence of an object without providing an example or a method to get an example; primarily used in combinatorics (53.) and pioneered by Paul Erd˝os. It works by showing that if one randomly chooses objects from a specified class, the probability that the result is of the prescribed kind is strictly greater than zero. Although the proof uses probability, the conclusion is determined for sure. It finds applications in number theory (4.), computer science and information theory.

20. Carmichael numbers: They are composite numbers n which satisfy the congruence relation $a^{n-1} \equiv 1 \pmod{n}$ for all integers a that are co-prime to n. This relation is satisfied by all prime numbers and is called Fermat's little theorem. Hence Carmichael's numbers behave somewhat like primes. They satisfy other properties that a prime does, such as the existence of a Carmichael number between any two numbers, an analogue of Bertrand's postulate. See https://www.quantamagazine. org/teenager-solves-stubborn-riddle-about-prime-number-look-alikes-20221013/ for an interesting account of how a teenager proved the Bertrand postulate for Carmichael numbers.

21. Topological groups: They are topological spaces (a set whose elements are called points, along with an additional structure called a topology (49.)) which are also groups (48.), with the continuity condition for the group operation (mathematical operation, that operates element of groups) connecting these two structures. The real numbers with addition with the usual topology on it is an example of a topological group.

22. Inverse limit: It is a category-theoretic notion which provides a construction that allows one to "glue together" several related objects, the precise gluing process being specified by the maps (morphisms) between the objects.

23. Kaluza Klein theory: It is a unified theory of gravitation and electromagnetism that presents a fifth dimension beyond the common four dimensions of space and time. It is a precursor to string theory. String theory is a broad subject that attempts to address several deep questions of fundamental physics. This area has contributed several advances to mathematical physics, which have been applied to various problems in black hole physics, early universe cosmology, nuclear physics, and condensed matter physics, and it has stimulated several major developments in pure mathematics.

24. Analytic function: A function that is locally given by a convergent power series; power series is an infinite series of the form $\sum_{n=0}^{\infty} a_n z^n$, for a real or complex variable z.

25. Holomorphic function: It is a complex-valued function that is infinitely differentiable and locally equal to its own Taylor series (an infinite sum of terms that are expressed in terms of the function's derivatives at a single point (slope of tangents drawn at that point)), thus is termed an analytic function (24.) too. These functions are the central objects of study in complex analysis.

26. Minimal surface: It is a surface that locally minimizes its area. It is equivalent to having zero mean curvature. Mean curvature is an extrinsic measure of curvature that locally describes the curvature of a surface in some ambient space, such as Euclidean space. Soap film is an excellent example of a minimal surface.

27. Symbolic computation: Also referred to as computer algebra or algebraic computation, it refers to the study and development of algorithms and software for manipulating mathematical expressions and other mathematical objects. It is widely used to design the formulas used in numerical programs and for complete scientific computations when purely numerical methods fail.

28. Four colour theorem: Also called the four map theorem, it states that no more than four colours are required to colour the regions of any map so that no two adjacent regions have the same colour. Adjacent means that two regions share a common boundary, not just a corner where three or more regions meet. It was the first major computer-assisted proof. See the fascinating saga of the origin and proof of this problem at https://www.quantamagazine.org/only-computers-can-solve-this-map-coloring-problem-from-the-1800s-20230329/.

29. Jordan curve theorem: This theorem asserts that every plain simple (no self-intersections) closed curve divides the plane into an 'interior' region bounded by the curve and an 'exterior' region containing all the exterior points. This seemingly simple-minded statement requires heavy mathematical machinery and some ingenuity for the proof.

30. Kepler conjecture: First proposed by the famous astronomer and mathematician, it states that no arrangement of equally sized spheres filling space has a greater average density than that of the cubic close packing (face-centred cubic) and hexagonal close packing arrangements. A full proof was announced only in 1998 by Thomas Hales.

31. Odd order theorem: Also called the Feit-Thompson theorem, it says that there is no finite noncommutative group (a group (48.) which doesn't commute) with an important property (simple) whose size is odd. It is a significant result in group theory, which has aided the exercise of classifyinggroups up to isomorphism (groups which are essentially the same with respect to their group structure).

32. Sphere eversion: In differential topology (38.), it is the process of 'everting' a sphere, that is, turning a sphere inside out in three-dimensional space. Remarkably, it is possible to smoothly and continuously turn a sphere inside out in this way (allowing self-intersections of the sphere's surface) without cutting or tearing it or creating any crease. This is a surprising result for both mathematicians and non-mathematicians.

33. Differential Geometry: Differential geometry is a branch of mathematics that studies curves, surfaces, and higher-dimensional spaces using calculus techniques. It examines how these objects bend, twist, and interact with each other. It has applications in physics, computer graphics, and many other fields.

34. Principal Bundles: Principal bundles are mathematical structures that describe symmetries and connections. They consist of a space (the base) and a group (the fiber) that acts on it. They are used to study gauge theories, gravity, and other fundamental concepts in physics.

35. Group Action: Group action refers to a mathematical concept where a group (48.) of objects can act on another set of objects. It describes how the group's elements can transform or change the elements of the set. It helps us understand transformations and symmetries in various mathematical and scientific contexts.

36. Category: A category is a concept that describes a collection of objects and their relationships. It focuses on how objects can be connected or related through morphisms, representing transformations or mappings between them. Categories provide a framework for studying various areas of mathematics and other disciplines.

37. Functor : A functor describes a relationship between two categories. It maps objects from one category to objects in another while preserving the structure and relationships between them. Functors help us understand and analyze connections between different mathematical structures.

38. Differential Topology: Differential topology studies the properties of smooth spaces, ignoring notions of distance or measurement. It focuses on understanding the global structure and how smooth objects can be smoothly deformed. It has applications in theoretical physics, robotics, and the study of complex systems.

39. Algebraic Geometry: Algebraic geometry is a branch of mathematics that explores the geometric properties of solutions to polynomial equations. It studies shapes and structures defined by algebraic equations and investigates their properties using algebraic tools. It has applications in cryptography (72.), coding theory (study of the properties of codes and their respective fitness for specific applications), and theoretical physics.

40. Langlands Program: It is a far-reaching and influential research program in mathematics that connects different areas of mathematics, such as number theory (4.) and representation theory (57.). It seeks to uncover deep connections between seemingly unrelated mathematical structures, providing insights into fundamental mathematical principles and potentially revolutionizing our understanding of the subject. Its impact spans various fields, including algebraic geometry (39.), harmonic analysis (mathematical procedure for describing and analyzing phenomena of a periodically recurrent nature), and mathematical physics, and is often termed 'a kind of grand unified theory in mathematics'. See https://www.quantamagazine.org/what-is-the-langlands-program-20220601/ for a Quanta magazine article which attempts to describe the key insights underpinning this program.

41. Rings: A ring is a set equipped with two operations: addition and multiplication. Addition allows for combining elements, while multiplication allows for multiplying elements. Rings are used to study structures such as numbers, polynomials, and matrices and provide a foundation for algebraic concepts and equations.

42. Modules: A module over a ring (41.) is a generalization of vector spaces (3.) where instead of working with numbers, we work with elements from a ring. It describes a set of objects that can be added to each other and multiplied by elements of the ring, satisfying specific rules. Modules have applications in algebra, number theory, and other areas.

43. PID (Principal Ideal Domain): PIDs are algebraic structures that are similar to the integers and have very rich properties, such as: (i) Any element has a unique decomposition into prime elements.(ii) Any two elements of a PID have a greatest common divisor.

44. Low Dimensional Topology: This area of mathematics focuses on understanding the properties of spaces in dimensions two and three. It investigates the structure of knots (a crucial difference between the mathematical and conventional notions of a knot is that mathematical knots are closed, i.e., there are no ends to tie or untie on a mathematical knot), links (a link is a collection of knots which do not intersect, but which may be linked (or knotted) together), surfaces, and manifolds (55.) and explores their interactions and transformations. It connects to fields such as physics, biology, and computer science.

45. Projective 3-Space: Projective 3-space is a concept that extends our understanding of ordinary three-dimensional space. It includes "points at infinity" and allows for transformations (or maps) that preserve collinearity. It has applications in computer vision, 3D graphics, and projective geometry.

46. Simply Connected: Simply connected is a term used to describe a space with no "holes" or "handles." In simpler terms, you can continuously transform any closed loop in the space into a point without encountering any obstacles or gaps. It is an important concept in topology (49.) and has applications in various areas like physics and engineering.

47. Contractible: A space is said to be contractible if it can be continuously shrunk to a single point without tearing or breaking. Imagine a rubber band that can be gradually pulled and squeezed until it becomes a single point. A contractible space can undergo a similar transformation, maintaining its shape throughout.

48. Group theory: Group theory studies the algebraic structures known as groups, central to abstract algebra. It codifies the idea of symmetries in the form of an algebraic construct, namely a group (a set having specific properties). Being a broadly-

applicable branch of mathematics, it has significant applications in theoretical physics, chemistry, cryptography (72.), etc.

49. Topology: Topology concerns the study of those properties of spaces that are invariant under 'continuous' transformations (stretching, twisting, bending, etc.). It finds applications in biology (study of proteins, neural networks, etc.), quantum field theory, data analysis, etc. See https://www.quantamagazine.org/topology-101-how-mathematicians-study-holes-20210126/ for an excellent account of the history and development of the subject.

50. Algebraic topology: Algebraic topology can be roughly defined as the study of techniques for forming algebraic images of topological spaces. Algebraic topology assigns algebraic invariants (objects that remain unchanged by transformations) that help classify topological spaces up to homeomorphism (essentially the same spaces topologically).

51. Clifford Algebra: Clifford algebras are a generalization of complex numbers. They have a breadth of applications in mathematical physics because of their versatility in representing mathematical structures.

52. Vector Bundles: In mathematics, a vector bundle is a topological (49.) construction that makes precise the idea of a family of vector spaces (3.) parameterised (a mathematical process consisting of expressing the state of a system, process or model as a function of some independent quantities called parameters) by some another space (say X) which could be a topological space, a manifold (55.), or an algebraic variety (70.). Thus, to every point of this space, X, we associate a vector space (3.), such that these vector spaces (each one associated to a point of X) fit together to form another space of the same kind as X, which is called a vector bundle over X.

53. Combinatorics: Combinatorics, sometimes called "discrete mathematics", is a broad area of mathematics that concerns the study of finite or countable discrete structures. It mainly concerns "discrete" instead of "continuous" structures.

54. Analytic Number Theory: Analytic number theory is a branch of number theory that uses ideas and methods of real and complex analysis to address number-theoretic problems.

55. Manifolds: Manifolds are an interesting class of topological spaces (49.) that locally resemble Euclidean spaces. It is a generalization and abstraction of the notions of curves and surfaces to higher dimensions. Manifolds are fundamental to topology, geometry, and modern mathematical physics; for example, symplectic manifolds model phase spaces in classical mechanics.

56. Chinese Remainder Theorem: It is a result of elementary number theory which talks about solving a simultaneous system of linear congruences. It states that if one knows the remainders of the Euclidean division of an (a priori unknown) integer n by finitely many pairwise co-prime integers, then one can determine the remainder of the division of n uniquely by the product of these integers. It can be stated more abstractly over structures called rings (41.).

57. Representation theory: Representation theory is a branch of mathematics that studies abstract algebraic structures by representing their elements as linear transformations (or linear maps) of vector spaces (3.). The applications of representation theory are diverse within and outside the realm of mathematics. For example, representation theory is important in physics because it can describe how a physical system's symmetry group (48.) affects the solutions of equations describing that system. Learn more about this fascinating subject at https://www.quantamagazine.org/ the-useless-perspective-that-transformed-mathematics-20200609/.

58. Riemann Zeta function: The Riemann Zeta function is defined as $\zeta(s) = \sum_{n=0}^{\infty} \frac{1}{n^s}$, where $s \in \mathbb{C}$ and extended by a method called analytic continuation to the entire complex plane except at $s = 1$. The famous Millenium Prize problem, Riemann hypothesis, conjectured by Bernhard Riemann, states that the Riemann-Zeta function $\zeta(s)$ has its zeros only at the negative even integers and complex numbers with real part $\frac{1}{2}$. It is of great interest in number theory because it implies results about the distribution of prime numbers (65.).

59. Gaussian Distribution: Gaussian distribution is a type of continuous probability distribution for a real-valued random variable. There are many natural phenomena following a Gaussian distribution. Normal distributions are important in statistics and often used in the natural and social sciences to represent real-valued random variables whose distributions are unknown. Their importance is partly due to the central limit theorem. It states that, under some conditions, the average of many samples (observations) of a random variable with finite mean and variance is a random variable— whose distribution converges to a normal distribution as the number of samples increases. Therefore, physical quantities that are expected to be the sum of many independent processes, such as measurement errors, often have nearly normal distributions, making them very handy in applications.

60. $SL_2(\mathbb{R})$ *:* The special linear group $SL_2(\mathbb{R})$ is the set of n × n matrices with determinant 1, with the group operations of ordinary matrix multiplication and matrix inversion. It can be characterised as the group (48.) of linear transformations of the plane that preserve oriented areas. In higher dimensions, the special linear group $SL_n(\mathbb{R})$ can be characterized as the group of volume and orientation preserving linear transformations of $\mathbb{R}^n$, which arises from the interpretation of the determinant as a measure of change in volume and orientation upon being acted by a linear transformation.

61. L-functions: An L-series is a Dirichlet series (a series of the form $\sum_{n=1}^{\infty} \frac{a_n}{n^s}$, where an are the terms of a complex sequence and $s \in \mathbb{C}$), usually convergent on a half-plane, that may give rise to an L-function by the method of analytic continuation. The Riemann Zeta function (58.) is an example of an L-function (where the corresponding Dirichlet series is given by the constant sequence 1), and one crucial conjecture involving L-functions is the Riemann hypothesis (included in 58.) and its generalization. It has become a significant part of analytic number theory.

62. Banach Space: It is a complete normed vector space (3.). A norm gives the notion of the 'length' of a vector. A space is complete if every Cauchy sequence (a sequence whose terms get closer to each other as one moves along the sequence) also has a limit in the space (the sequence converges to this point).

63. Madhava Series: This is sometimes called the Madhava-Leibniz formula for π. This is a slowly convergent series (large number of terms are to be taken into account to get a good approximation of the value the series converges to) and is given by $\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots - \cdots\right)$ Madhava also discovered the correction term to make the approximation better than the partial sum approximation obtained by just truncating the series.

64. Algebra: This is one of the oldest branches of mathematics that helps represent problems or situations in mathematical expressions. It deals with studying variables involved in these mathematical expressions and the rules for manipulating them. Its sub-branches are elementary algebra, linear algebra, abstract algebra, etc.

65. Distribution of primes: This refers to how prime numbers are distributed, and an important theorem for describing this distribution is the Prime Number Theorem. The theorem states: 'Let π(x) be the prime-counting function defined as the number of primes less than or equal to x for any real number x. Then $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$, i.e., the prime-counting function is of the order of $\frac{x}{\log x}$ for large values of x.'

66. Modular forms: A modular form is a (complex) analytic function (24.) on the upper half-plane (refers to the complex plane) that satisfies: (i) a kind of functional equation concerning the group action (35.) of the modular group (group (48.) of linear fractional transformations of the upper half of the complex plane, which maps a complex number z to another complex number $\frac{az+b}{cz+d}$, where a, b, c, d are constants such that ad – bc = 1); and (ii) a growth condition. A famous conjecture in mathematics called the Taniyama Shimura conjecture relates elliptic curves over rational numbers to modular forms, the proof of which led to the proof of Fermat's last theorem. An elliptic curve is a plane curve formed by solutions (x, y) to the equation $y^2 = x^3 + ax + b$ with $4a^3 + 27b^2 \neq 0$ such that it has no self intersections.

67. Diophantine equations: It is an equation which must be solved for integers, i.e., the indeterminates (or variables in the equation) take only integer values.

68. Ramanujan summation: Srinivasa Ramanujan invented a technique for assigning a value to divergent infinite series (series that don't add to a finite quantity) called the Ramanujan Summation. Although the Ramanujan summation of a divergent series is not a sum in the traditional sense, it has properties that make it mathematically valuable for studying divergent infinite series, for which conventional summation is undefined.

69. Fractals: A fractal is a geometric shape containing detailed structure at arbitrarily small scales, usually having a fractal dimension (fractional dimension) strictly exceeding the topological dimension (for topology, refer 49.). Many fractals appear similar at various scales, and this property is called self-similarity. Natural phenomena with fractal patterns include crystals, DNA, blood vessels, etc. This branch of study finds various technological applications.

70. Algebraic Varieties: Algebraic varieties are the central objects of study in algebraic geometry (39.). An algebraic variety is the set of solutions of a system of polynomial equations over real or complex numbers. Modern definitions generalise this concept in several ways while attempting to preserve the geometric intuition behind the original definition.

71. Algorithm: It is a description of a set of steps that can be used to solve a mathematical/computational problem.

72. Cryptography: It is the study of techniques for secure communication in the presence of adversarial behaviour. Cryptography is generally about constructing and analysing protocols that prevent third parties or the public from reading private messages. Modern cryptography exists at the intersection of mathematics, computer science, information security, electrical engineering, digital signal processing, physics, etc.

73. Elliptic functions: In complex analysis, elliptic functions are a special kind of meromorphic function (it is a ratio of two holomorphic functions (25.)) that satisfy two periodicity conditions. They are named elliptic functions because they come from elliptic integrals, which originally occurred in the calculation of the arc length of an ellipse

74. Analytic Geometry: It is the study of geometry using a coordinate system. It is also known as coordinate geometry or Cartesian geometry.

75. Graph Theory: It is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph comprises vertices (nodes or points) connected by edges (links or lines). They have applications in physical, biological, social and information systems.

76. Graph vertex colouring problem: Vertex colouring refers to assigning colours to the vertices of a graph in such a way that no two adjacent vertices have the same colour.

77. Max-k-cut problem: In the Max-k-Cut problem, we are given an undirected graph (a graph (75.) in which no direction is assigned to the edges), G = (V, E) (V - set of vertices and E - set of edges), with non-negative edge weights (the edges are assigned different weights). We aim to divide the vertices into at most k disjoint (non-overlapping) sets for some natural number k to maximise the weight of the edges whose endpoints lie in different sets.

78. NP-hard problem: In computational complexity theory (classifying computational problems according to their resource usage and relating these classes to each other), NP-hardness (Non-deterministic Polynomial-time hardness) is the defining property of a class of problems that are informally at least as hard as the hardest problems in NP. NP is a complexity class that defines decision (yes-no) problems: if the answer is "yes", the proof is verifiable in polynomial time.

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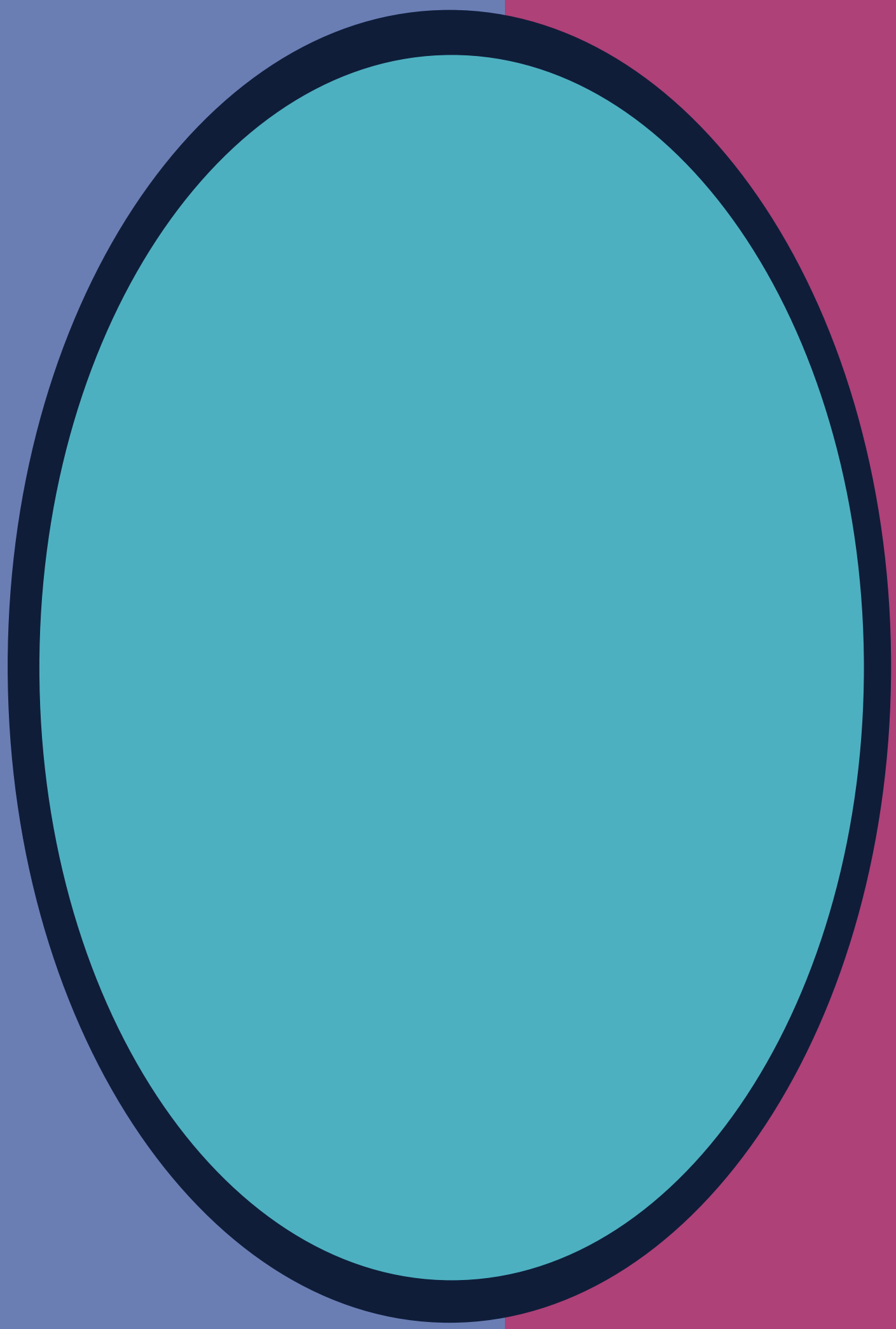
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