

Magneto-Optical Trapping of Strontium Atoms for Optical Atomic Clock

विद्या वाचस्पति की
उपाधि की अपेक्षाओं की आंशिक पूर्ति में प्रस्तुत शोध
प्रबंध

A thesis submitted in partial fulfillment of the requirements of the
degree of Doctor of Philosophy

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भारतीय विज्ञान शिक्षा एवं अनुसंधान संस्थान पुणे

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH PUNE

2024

Certificate

Certified that the work incorporated in the thesis entitled “**Magneto-Optical Trapping of Strontium Atoms for Optical Atomic Clock**” submitted by **Kushal Patel** was carried out by the candidate, under my supervision. The work presented here or any part of it has not been included in any other thesis submitted previously for the award of any degree or diploma from any other university or institution.

Date: July 3, 2024


(Supervisor)

Declaration

I hereby certify that the work which is being presented in the thesis entitled “**Magneto-Optical Trapping of Strontium Atoms for Optical Atomic Clock**” in partial fulfillment of the requirements for the award of the Degree of **Doctor of Philosophy** and submitted in the Department of Physics of the Indian Institute of Science Education and Research, Pune is an authentic record of my own work carried out during a period from August, 2017 to June, 2024 under the supervision of Dr. Umakant D. Rapol, Associate Professor, Department of Physics, Indian Institute of Science Education and Research, Pune. The matter presented in the thesis has not been submitted by me for the award of any other degree of this or any other institute.

A handwritten signature in black ink, appearing to read 'Kushal', with a stylized flourish underneath.

Date: July 3, 2024

(Kushal Patel)

Abstract

Optical lattice clocks, leveraging their remarkable precision, have the capability to measure time with unprecedented accuracy. This precision opens up possibilities for the utilization of lattice clocks in various domains, such as probing new physics beyond the standard model, advancing geodesy and navigation technologies, and re-definition of the SI second in the future. The meticulous process of measuring time, known as frequency metrology, emerges as a pivotal field of scientific inquiry within the SI system. It serves as the foundation for defining other fundamental units. Currently, the SI second is defined by the duration of 9,192,631,770 periods of radiation corresponding to a specific transition in the ^{133}Cs atom's ground state, within the microwave spectrum.

However, the transition of an optical frequencies with high quality factor (Q) values presents an opportunity to increase measurement precision. Optical frequency clocks have demonstrated operational capabilities with fractional uncertainties as small as 10^{-18} . Such unprecedented precision not only holds the potential to revolutionize global time-keeping but also offers insights into fundamental constants of nature, thus enabling rigorous tests of fundamental theories.

To achieve the objective of developing an optical atomic clock based on the singlet to triplet intercombination line of strontium (Sr), this thesis outlines the design and implementation of an experimental apparatus for producing cold Sr atoms in a magneto-optical trap (MOT). The entire setup is constructed on a $1.5\text{m} \times 3\text{m}$ optical table, incorporating both laser optical components and vacuum systems for the generation, deceleration, and trapping of Sr atoms, all isotopes of Sr.

The MOT is loaded through a high-intensity Sr atom beam emitted by an oven and subsequently decelerated by a zero-field crossing Zeeman slower. The oven design addresses the challenge of clogging over long-term operation and provides thermal isolation between the vacuum chamber and the high-temperature oven region, simplifying the setup. A home-built injection lock module is used for the generation of the first-stage cooling laser, utilizing the 461 nm laser to prepare a blue MOT for all Sr isotopes, with temperatures reaching a few millikelvins. Subsequently, atoms are transferred to the second stage where stringent requirements for the lasers arise due to the narrow linewidth of the transition. Using a high-finesse optical cavity, we reduce the linewidth

of the 689 nm laser, while an isolation chamber for the optical cavity enhances its stability, aiding in achieving a single-frequency red MOT with a final temperature of approximately $10\mu\text{K}$.

The clock frequency reference atomic transition is susceptible to various external influences, contributing to the total uncertainty budget of the clock transition. These uncertainties, including AC Stark shift, Zeeman shift, black body radiation shift, and background gas collision-induced shift, must be systematically evaluated to fully exploit the potential of the ultra-narrow clock transition. Our investigation focuses on the 483nm transition originating from the clock state to a higher excited state, characterizing its properties. Moreover, this transition serves as a potential repumper transition in conjunction with other single-frequency repumpers such as 707 nm, 481 nm, and 405 nm.

To, my **Family.**

Publications from the thesis work

- Spectroscopy of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ Transition of Strontium Using Laser Cooled Atoms; **Kushal Patel**, Palki Gakkhar, Korak Biswas, S Sagar Maurya, Pranab Dutta, Vishal Lal, B K Mani, and Umakant D Rapol, *Journal of Physics B: Atomic, Molecular and Optical Physics*, **57**,105501 (2024)

List of other publications

- A simple atomic beam oven with a metal thermal break; Chetan Vishwakarma, Jay Mangaonkar, **Kushal Patel**, Gunjan Verma, Sumit Sarkar and Umakant D. Rapol, *Review of Scientific Instruments*, **90**, 053106 (2019)
- Interplay between quantum diffusion and localization in the atom-optics kicked rotor; S. Sagar Maurya, J. Bharathi Kannan, **Kushal Patel**, Pranab Dutta, Korak Biswas, Jay Mangaonkar, M. S. Santhanam, and Umakant D. Rapol; *Phys. Rev. E* 106, 034207 (2022)
- Comparative Analysis of Phase Noise for different configurations of Bragg lattice for an Atomic Gravimeter with Bose-Einstein Condensate; Pranab Dutta, S. Sagar Maurya, Korak Biswas, **Kushal Patel**, Umakant D. Rapol; *AIP Advances* 14, 015352 (2024)
- A Decade of Advancement of Quantum Sensing and Metrology in India Using Cold Atoms and Ions. Dutta, P., Maurya, S.S., **Kushal Patel**, Biswas, k et al., *J Indian Inst Sci*, 103, 609-632 (2023)
- Machine-learning-based automated loading of strontium isotopes into magneto-optical trap; Korak Biswas, **Kushal Patel**, S. Sagar Maurya, Pranab Dutta, Umakant D. Rapol; *AIP Advances* 13, 075313 (2023)

Manuscripts under preparation

- Study of loss dynamics of strontium in magneto-optical trap; Chetan Vishwakarma, **Kushal Patel**, Jay Mangaonkar, Jamie L. MacLennan, Korak Biswas and Umakant D. Rapol, *arXiv:1905.03202* (2019)
- Electromagnetically Induced Transparency (EIT) aided cooling of strontium atoms; Korak Biswas, **Kushal Patel**, S. Sagar Maurya, Pranab Dutta, Umakant D. Rapol, Yeshpal Singh, *arXiv: 2311.08318* (2023) (Under review)
- Asymmetric dynamical localization and precision measurement of BEC micro-motion; S. Sagar Maurya, J. Bharathi Kannan, **Kushal Patel**, Pranab Dutta, Korak Biswas, M. S. Santhanam, Umakant D. Rapol, *arXiv:2406.12358* (2024)

Patents

- OVEN FOR GENERATION AND COLLIMATION OF AN ATOMIC BEAM, Umakant D Rapol, **Kushal Patel**, Korak Biswas, S. Sagar Maurya, Pranab Dutta, Indian Patent Application Number 202421000590. Filed on January 03, 2024.
- CHEMICAL OXYGEN GENERATOR Umakant D Rapol, **Kushal Patel**, Korak Biswas, S. Sagar Maurya, Pranab Dutta, Indian Patent Application Number 202121020842, Filed on May 07, 2021. The patent has been granted
- Hybrid oxygen system using passive and active systems Umakant D Rapol, **Kushal Patel**, Korak Biswas, S. Sagar Maurya, Pranab Dutta, Indian Patent Application Number 202111029508, Filed on June 30, 2021

Acknowledgments

I have always been passionate about experiments and aimed to pursue a Ph.D. in experimental physics. My deepest gratitude goes to my supervisor, Dr. Umakant Rapol. His mentorship has been a cornerstone of my journey, and I am profoundly thankful for his unwavering guidance and constant encouragement. His insightful advice and support have not only shaped my academic path but also aided in my personal growth. I am fortunate to have had such a dedicated and compassionate mentor. I also want to acknowledge the valuable feedback and insightful suggestions from my research advisory committee (RAC) members, Dr. M. S. Santhanam and Dr. Sreejith G. J. Their annual feedback sessions were instrumental in keeping my research on the right track. I am indebted to my collaborators, Prof. Yeshpal Singh, Prof. Brajesh Mani, and Prof. Dilip Angom, for their support.

My labmates made the learning process smoother. I am particularly grateful to Chetan Vishwakarma and Jay Mangaonkar, who were like elder brothers and supported me in all aspects of life. I would also like to thank Pranab Dutta, Korak Biswas, and Shivsagar Maurya for the engaging discussions and scientific interactions. Additionally, I extend my gratitude to the new generation of lab members, Rayees, Joel, and Shuvarati.

Building the experimental setup would not have been possible without the support of technical officer Mr. Nilesh Dumbre and other technical staff, including karthikeyan. I also appreciate the efficient handling of official matters by Prabhakar Anagare and Dhanashree Sheth.

I am grateful to IISER Pune for offering a pleasant working environment and funding my fellowship throughout my MS and Ph.D. studies. Additionally, I extend my thanks to the I-HUB at IISER Pune for granting me a scholarship during the final year of my Ph.D. Finally, heartfelt thanks to my family for their unwavering love, encouragement, and unwavering support, without which none of this would have been possible.

Date: July 3, 2024

(Kushal Patel)

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Chapter 1

Introduction to Atomic Clocks

Atomic clocks are renowned for their remarkable precision in timekeeping. To provide an example, the most advanced optical lattice clocks utilizing strontium atoms can precisely measure time up to the 18th decimal place. This incredible accuracy implies that it would take these clocks approximately 15 billion years to accumulate a mere 1-second deviation [1]. These highly accurate timekeeping instruments, alongside atomic magnetometers [2], accelerometers [3,4], gyroscopes [5,6] constitute a category of emerging quantum technologies. Notably, these technologies are currently maturing, a progress largely attributed to the development of laser cooling techniques for atoms in the 1970s [7]. Through these advancements, we are able to observe and characterize the world with an unparalleled degree of precision.

1.1 Historical Perspective

The earliest known method of measuring time was the observation of celestial bodies' movements across the sky. People recognized this periodic motion as a way to track time, with the sun's path serving as a natural clock. By observing the position and length of shadows cast by objects like sticks, humans could count the passing of time. However, this method had limitations, as it didn't account for variations in day length throughout the year and was ineffective at night or during cloudy weather. To measure time during the night, people turned to celestial constellations, such as Orion. While these celestial clocks provided a way to synchronize human activities on time scales

of months, days, or hours, their reliance on natural phenomena had drawbacks. These limitations led to the development of the first man-made clocks. Initially, these clocks measured time by controlling the flow of sand or water through marked containers. They were useful for interpolating time between significant astronomical events. As human activities became more complex, it became clear that these water clocks couldn't provide the necessary time precision. The solution was to increase the resolution by choosing a higher-frequency periodic event. This innovation gave rise to mechanical clocks, which were capable of keeping time within 15 minutes a day but remained susceptible to external perturbations. The pendulum clock was a significant improvement over older mechanical clocks. It was pretty accurate and worked well for most daily activities. However, when people started exploring the oceans, they realized that the rocking and swaying of ships made these clocks useless. They needed accurate timekeeping at sea to determine a ship's location. To solve this problem, an English clockmaker named John Harrison created the first chronometer in 1761. It used a "balance-spring" mechanism and was incredibly precise, only deviating by 1 second a day. Since then, there have been many improvements to mechanical clocks. The best ones today can keep time with an accuracy of 1 millisecond per day. However, there's a limit to how accurate mechanical clocks can be. For example, the length of the pendulum affects its timing. Even if we try to make all clocks the same size, small manufacturing differences and changes in temperature can still make them less precise. A significant advancement came in 1929

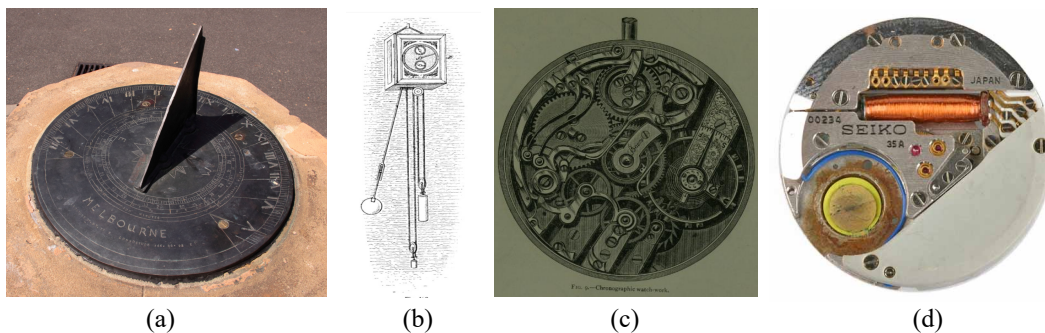


Figure 1.1: Development of timekeeping devices: (a) Sun dial, (b) Pendulum clock, (c) Mechanical clock, (d) Quartz clock. Information derived from Wikipedia.

when American scientist Warren A. Marrison introduced the quartz-crystal clock. This clock operates based on the piezoelectric property of quartz crystal, causing it to res-

onate at a frequency determined by its dimensions and produce a voltage across its surface. When integrated into an electric circuit with a nearly matching resonant frequency, the crystal vibrates at its natural frequency, synchronizing the circuit's frequency with that of the crystal. Quartz clocks are incredibly precise, with the best ones deviating by less than 1 millisecond per month. They are widely used in clocks, watches, electronics, and various devices. However, they aren't perfect frequency standards because each quartz crystal's size and shape affect its resonance frequency, and no two crystals are exactly same. Additionally, quartz crystals frequency can change slowly due to aging and more rapidly due to environmental factors like temperature, humidity, pressure, and vibrations. To achieve even greater accuracy, atomic clocks were developed.

1.2 Principle of Atomic Clocks

In the past hundred years, as scientists developed quantum mechanics, they discovered oscillators that surpass the precision of any mechanical or celestial benchmarks known to humankind. These remarkable oscillators rely on the behavior of atoms and molecules. According to the laws of quantum mechanics, atoms and molecules have specific, distinct energy levels, and they have the ability to jump between these energy states (E_1 and E_2) by either absorbing or emitting electromagnetic radiation with precise frequency ν_0 .

$$\nu_0 = \frac{|E_2 - E_1|}{h} \quad (1.1)$$

Where, h is a Planck constant. In various atomic systems, energy levels shows remarkable stability and are minimally affected by external factors, ensuring the dependability of the radiation frequency ν_0 . Based on these characteristics, the fundamental concept of an “atomic oscillator” is straightforward: it relies on a robust and dependable natural transition frequency ν_0 to serve as a reference against which the frequency ν of a laboratory radiation source is compared and stabilized. To complete the architecture of an atomic clock, a counter is employed to monitor the oscillations of the stabilized frequency.

The significant advantage of employing an atomic frequency standard lies in its independence from the artificial components of the oscillator. Each atom of a particular type is identical, enabling a clock using a specific atom to function at the same frequency

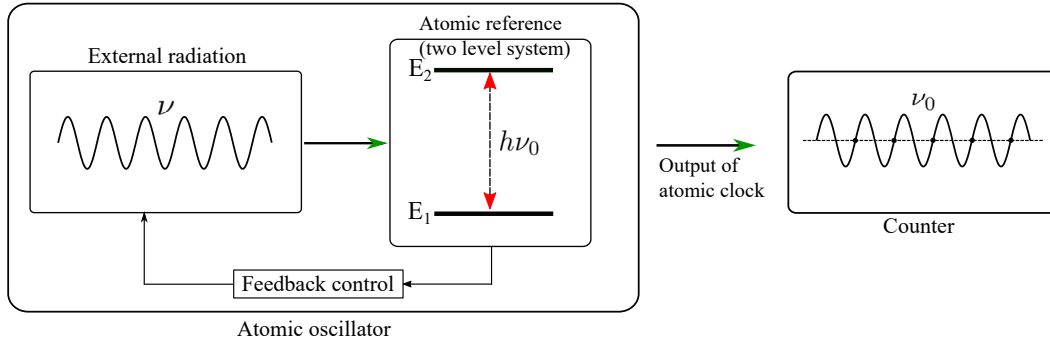


Figure 1.2: Structure of an atomic clock: The atomic clock consists of an external frequency source, which is stabilized based on an atomic transition. An electronic counter is used to measure the frequency of the stabilized frequency source.

as another independent clock based on the same type of atom, regardless of its location in the universe. In essence, this establishes the clock as a universal “time standard” once the atom type and clock transition are defined.

1.3 Importance of Atomic Clocks

Caesium fountains currently demonstrate an impressive accuracy of 1 second in 150 million years [8–10]. The advancement of atomic clocks beyond the capabilities of cesium atomic clocks opens the door to various applications:

- 1) The enhanced stability of these clocks has the potential to drive advancements in navigation, geodesy, and astronomy technologies [11, 12].
- 2) Atomic clocks serve as highly precise tools for testing relativity, contributing to the exploration of physics beyond the standard model [13].

Details of these applications are discussed in the following sections.

1.3.1 Technological Applications

The future applications of the latest generation of atomic clocks remain uncertain, much like how the first Cs atomic clocks developed in the 1950s could not have predicted the emergence of widespread atomic-clock-based Global Navigation Satellite Systems (GNSS) like GPS. However, we can make some educated predictions about their potential use. Improved stability in optical atomic clocks could benefit GNSS systems such as GPS, leading to faster and more accurate ground positioning information and

longer intervals between clock re-synchronization in orbit. Optical clocks in orbit could also support communication with ground stations for various research applications and establish a global time standard less affected by gravity variations at ground level. Looking ahead, an orbiting GNSS network of optical clocks might enable precise positioning not only on Earth but also in deep space, potentially guiding asteroid mining expeditions or deep-space scientific probes with sub-meter precision [12, 14].

Additionally, optical clocks are expected to enhance Very Long Baseline Interferometry (VLBI) in astronomy, providing crucial timing information for high-resolution radio imaging of distant celestial objects, aiding our understanding of star formation [15].

One highly promising application of atomic clocks is relativistic geodesy [16]. Atom-interferometer gravimeters are excellent at measuring gravitational acceleration, but optical atomic clocks are uniquely sensitive to relativistic redshift, which depends on the gravity potential U . With an accuracy of 10^{-18} , these clocks could precisely determine elevation. For example, on Earth's surface, the gravitational redshift amounts to about 10^{-16} per meter, allowing a clock with 10^{-18} accuracy to measure height above the geoid with a 1 cm level of precision. By combining atomic clocks and gravimeters, we may detect various subterranean events that affect ground elevation and density, such as aquifer depletion, tidal or seismic crustal deformation, and magma chamber inflation preceding volcanic eruptions. Successful detection of these natural disaster precursors with atomic sensors could significantly improve human safety in geologically active regions worldwide [17, 18].

Highly precise and stable frequency sources are essential for radars and wireless communication applications, serving as local oscillators (LO). The deployment of compact quantum clocks as oscillators in advanced radar systems offers a distinct advantage over current oscillator technology. This advantage lies in their ability to minimize signal noise, enhancing radar capabilities to detect smaller, slower-moving objects like drones at extended distances and within more complex environments.

1.3.2 Fundamental Physics

Variation of α : Motivated by predictions from theories beyond the standard model, there exists a significant scientific history of investigating potential variations in funda-

mental constants across both time and space. The discovery of such variations would challenge the principle of Local Position Invariance (LPI) within the framework of general relativity, which asserts that the outcome of a local experiment should be independent of its location and timing [13, 19–21]. In this context, an experiment is considered ‘local’ if it takes place within a small enough region where the effects of spacetime curvature are negligible. A decisive measurement indicating a violation of LPI, along with a detailed understanding of how this violation varies over time and space, could provide valuable insights into the quest for a Grand Unified Theory.

Atoms stand out as particularly sensitive instruments for detecting potential LPI violations since their transition frequencies are intrinsically linked to the local physical conditions at the atom’s specific location [22–25]. Theoretical models that consider a variety of transitions in different atomic species reveal distinct sensitivities to key parameters such as the fine structure constant α , the proton-to-electron mass ratio $\mu = m_p/m_e$.

To identify potential violations of Local Position Invariance (LPI), extensive investigations have revolved around the precise comparison of atomic clocks. The primary aim has been to detect two plausible phenomena: (1) gradual time-dependent changes in α and (2) yearly oscillations in these fundamental constants due to their interaction with the Sun’s gravitational field. More recently, scientists have explored additional LPI violations, driven by certain dark matter theories that propose rapid oscillations or transient fluctuations in α . To date, experiments comparing atomic clocks have consistently yielded null results for any form of variation. Various frequency ratios, including Hg + /Al +, Hg + /Cs, Sr/Cs, Dy/Cs, Yb + /Cs, and Cs/Rb/H, have demonstrated remarkable stability over time [26–29].

Local Lorentz Invariance: Atomic clocks can be used to explore another fundamental concept in general relativity: Local Lorentz Invariance (LLI). LLI postulates that the results of any non-gravitational experiment in free fall should remain unaffected by the velocity and orientation of the apparatus. Special relativity governs the behavior of non-gravitational experiments in all cases. The history of experimental investigations into the effects of special relativity dates back to the Michelson-Morley interferometer in 1887, which is renowned for its inability to detect evidence of the

ether [30]. Later experiments, such as the Kennedy-Thordike [31] and Ives-Stilwell experiments [32] conducted in the 1930s, probed different facets of Lorentz contraction and time dilation. These experiments all substantiated the predictions of special relativity. In the context of test theories that consider the speed of light c to be isotropic solely in one ‘preferred’ reference frame, they collectively impose stringent limitations on all conceivable degrees of freedom of Lorentz violation. However, certain string theories suggest a spontaneous breakdown of LLI at some level [33]. Furthermore, one explanation for the majority of matter over antimatter is rooted in a violation of CPT symmetry, which implies the existence of unfavourable non-local interactions or, once again, the collapse of Lorentz symmetry [34–36]. These theories motivate ongoing investigations into potential minute violations of special relativity. As with LPI, the detection of an LLI violation could help in developing a unified theory beyond the standard model.

One approach to testing LLI using atomic clocks is to establish an atomic analogue to the Michelson-Morley interferometer. The fundamental concept involves preparing the cloud of an atoms into two states aligned in orthogonal directions: a state with maximum $|M_F|$ extending along the quantization axis $\hat{z}$ and a state with minimum $|M_F|$ that spreads out in the plane perpendicular to $\hat{z}$. Over an extended duration, the energies of these two orthogonal states can be compared, all while keeping the quantization axis constant in the laboratory frame, allowing it to rotate with the Earth throughout the day. In the presence of an LLI-violating ‘preferred’ reference frame, the energy difference between these states should vary with orientation, mirroring the anticipated changes in orientation between the two orthogonal arms of the Michelson-Morley interferometer. These atomic clock experiments are exceptionally sensitive probes of Lorentz violations because the electron within the atom moves at relativistic speeds, and the energy difference, representing a frequency, can be measured with remarkable precision.

We might also expect atomic clocks to be well-suited for the direct measurement of time dilation, possibly through an experiment similar to Ives-Stilwell. However, the most precise tests of time dilation thus far have relied on extreme velocities rather than frequency precision. These tests have involved a fairly broad transition in Li^+ ions traveling at speeds of up to $0.34c$ within a storage ring [37]. Thus far, improving upon these experiments using atomic clocks has proven challenging, partly due to the

difficulty of accelerating a fully functioning clock to relativistic speeds in a laboratory setting. However, this scenario could change if the latest generation of optical atomic clocks could be launched into space.

Space missions: Over recent years, various initiatives have proposed sending highly precise atomic clocks into space, opening up new avenues for scientific experiments beyond the capabilities of ground-based clocks [38]. A notable endeavor was the 2007 “Search for Anomalous Gravitation using Atomic Sensors (SAGAS)” with the objective of deploying a probe to the outer solar system equipped with three essential modules: an optical atomic clock, an atom interferometer, and a laser link for communication with terrestrial stations. The on-board atom interferometer was designed to identify any non-gravitational accelerations, which could potentially distort the data collected by the optical clock. Meanwhile, the atomic clock would facilitate the study of the effects of both special and general relativity through continuous frequency comparisons with an optical clock on Earth.

1.4 Characterization of Clock Performance

To Understand the progression of atomic clocks and their role as “time standards” requires introducing key concepts used to assess their performance. Measuring the quality of a clock is crucial for distinguishing between good and bad timekeepers, and this is achieved through several performance metrics.

1.4.1 Quality Factor (Q)

The concept of the “Quality Factor” or “ Q ” is commonly associated with oscillators. In an ideal scenario, an oscillator would continue oscillating indefinitely with just a single initial push. However, in the real world, various factors like friction lead to a gradual decay in oscillation. Nevertheless, some oscillators exhibit superior performance relative to others, and it is important to establish a method for distinguishing between them. The Quality Factor (Q), serves as one such measure and is defined as:

$$Q = 2\pi \cdot \frac{\text{energy stored}}{\text{energy dissipated per cycle}} \quad (1.2)$$

A dampened oscillator is described by the following equation of motion:

$$m\ddot{x} + b\dot{x} + kx = 0 \quad (1.3)$$

Where m is the mass of the oscillating body, x is its position, b is the damping coefficient and k is the spring constant.

When $\omega_0\tau > 1$, it is known as “underdamped regime”, the solution is:

$$x(t) = Ae^{\frac{-t}{\tau}} \cos(\omega_0 t + \phi) \quad (1.4)$$

where, $\tau = 2m/b$ is a decay time, $\omega_0 = \sqrt{k/m}$ is resonance frequency. A and ϕ are from initial conditions. using this definitions Q can be written as:

$$Q = 2\pi \frac{x(0)^2}{x(0)^2 - x(T)^2} = 2\pi \frac{1}{1 - e^{\frac{-2T}{\tau}}} \simeq \omega_0 \cdot \frac{\tau}{2} \quad (1.5)$$

1.5 highlights the relationship between Q and the decay time of the oscillator. A longer decay time, or equivalently, lower friction, results in a higher Q . A significant advantage of a high- Q oscillator is that it requires less energy of its natural or resonant frequency. However, there is another crucial advantage in terms of frequency response when we examine a damped oscillator in the context of its energy spectral density.

$$Q = 2\pi \frac{x(0)^2}{x(0)^2 - x(T)^2} = 2\pi \varepsilon(\omega) \propto \left| \int_0^{+\infty} x(t) e^{i\omega t} dt \right|^2 \propto \frac{1}{(\omega - \omega_0)^2 + (1/\tau)^2} \quad (1.6)$$

The frequency distribution is characterized by a Lorentzian function, and its “full width half maximum” ($\Delta\omega$) is determined by the equation:

$$\Delta\omega = \frac{2}{\tau} \quad (1.7)$$

This equation demonstrates a precise connection between the decay time and the sharpness of the resonance curve, equations 1.5 and 1.7 consequently offer another sig-

nificant interpretation of Q , expressed as:

$$Q = \frac{\omega_0}{\Delta\omega} \quad (1.8)$$

This equation suggests that, in the context of frequency response, high- Q oscillators exhibit resonance curves with exceptionally sharp peaks. Therefore, a significant benefit of a high- Q oscillator is its tendency to oscillate only when it match with or is “near” (within the frequency bandwidth $\Delta\nu$) its natural frequency ν_0 . This characteristic sets a crucial criterion for selecting the atomic reference: it necessitates atoms having a high-frequency transition and a narrow linewidth. Alongside Q , there are two additional crucial parameters for the comprehensive evaluation of an atomic clock’s performance: accuracy and stability.

1.4.2 Accuracy and Stability

Atomic clocks are evaluated using two fundamental criteria: accuracy and stability. Accuracy acknowledges that the selected atomic energy levels for the clock transition are not entirely immune to systematic effects originating from the external environment. For instance, in a room-temperature strontium atom clock, a significant systematic shift arises from the AC Stark shift due to the blackbody radiation in the surrounding environment. The presence of microwave photons in the thermal environment implies the existence of a finite electric field resulting from radiation. Due to the differential polarizability of the ground and excited states, this electric field induces a shift in the observed clock frequency. Essentially, if a clock is developed in one laboratory and a similar clock is constructed elsewhere with the atom placed in a substantially colder environment, discrepancies will arise between the two clocks. To compare these two clocks, one needs to meticulously calibrate and comprehend the systematic shifts (e.g., electric fields, magnetic fields, background blackbody radiation, etc.). By documenting individual assessments of the shift and its associated uncertainty, clocks can be compared to validate agreement.

Stability, shows the degree of noise in a clock (or laser), can be interpreted as the time required to achieve a specific statistical uncertainty in the clock frequency. In simpler terms, stability indicates the duration needed to attain a defined statistical precision

of the clock frequency. A clock exhibiting higher shot-to-shot noise will have inferior stability, meaning it will take more time to reach a designated statistical uncertainty.

To quantify stability, the notion is rooted in the conventional experience of measurements, where closely spaced data points tend to be more clustered together with some inherent shot-to-shot noise. This noise may originate from various sources, including technical electronic noise or more fundamental sources. However, if observations are resumed after certain time interval, it's plausible to find a change in the relative frequency. Environmental conditions can influence noise on longer time scales. To characterize this behavior, we need a statistical quantity, and for clocks, the most effective metric is the Allan deviation [39,40].

Allan deviation defined by taking a discretely sampled time series of frequency data y_i , comprising M samples, each averaged over a sample time τ , resulting in a total measurement time $T = M\tau$ [41]. In this scenario, the Allan deviation is expressed as follows:

$$\sigma_y(\tau) = \left[\frac{1}{2(M-1)} \sum_{i=1}^{M-1} (y_{i+1} - y_i)^2 \right]^{1/2} \quad (1.9)$$

Observing the variation of Allan deviation over time scales allows for an understanding of distinct noise processes and their corresponding temporal dynamics.

The constant advancements in clock stability and precision are intrinsically linked. In the context of an optical atomic clock, short-term stability emerges from an ultra-stable laser serving as a local oscillator. The extension of clock stability from seconds to hours is achieved through the referencing of the ultrastable laser to a high-quality-factor optical transition of an atom.

1.4.3 Clock Error Budget

This section provides an overview of the systematic shifts and uncertainties associated with probing the natural frequency of the atomic transition. The principal source of uncertainty arises from the blackbody Stark shift, followed by the density shift. Advancements in comprehensively characterizing and controlling these shift mechanisms will enhance the precision in determining the Sr standard.

Lattice Stark shift : The energy levels of atoms exposed to an external light field

undergo a shift known as the AC Stark shift, leading to a modification in the clock transition frequency due to uneven shifts in energy levels. However, this issue can be addressed by meticulously choosing the wavelength of the lattice laser [42, 43]. The laser is specifically chosen to offset the light shifts, ensuring that the ground and excited states experience shifts of the same magnitude and direction. The wavelength chosen in this manner is termed the “magic wavelength” [44].

Blackbody radiation shift : The largest systematic uncertainty in clock comes from the blackbody radiation (BBR) shift associated with the clock transition. This shift arises from the interaction between atoms (ions) and thermal photons constituting the thermodynamic environment [45]. The inherent nature of this fundamental shift and its variations poses a significant obstacle in achieving precision at the 10^{-19} level. Typically, the BBR shift at room temperature ($T \sim 300$ K) exhibits a relative magnitude ranging from 10^{-15} to 10^{-16} concerning the frequency of the reference optical transition [46]. To ensure that long-term variations in the BBR shift remain below 10^{-19} , attaining a high degree of thermostabilization for both the experimental setup and its thermodynamic environment becomes imperative, leading to a complex technical challenge. Consequently, various laboratories have used the cryogenic technique, which holds the potential to significantly mitigate the BBR shift and its variations.

First-order Zeeman shift : Zeeman shift arises from the interaction between hyperfine energy levels and magnetic field, leading to a discrepancy in the Lande g-factors between the ground and excited clock states. This discrepancy results in an inconsistent Zeeman shift under the same magnetic field, ultimately causing a deviation in the clock transition frequency compared to the absence of a magnetic field. The primary Zeeman shift, which is directly proportional to the magnetic field intensity, can be largely mitigated by stabilizing the clock laser to the average frequency of Zeeman sublevels. However, a residual primary Zeeman shift may persist if there is a net drift in the background magnetic field while determining the center frequency of the zeeman manifold [47]. In contrast, the quadratic Zeeman shift, proportional to the square of the magnetic field, lacks an effective elimination method. Additionally, the quadratic Zeeman shift is influenced by the atom type and the energy level associated with the clock transition.

Second-order Zeeman shift : For the development of accurate frequency standards, it is crucial to consider the second-order Zeeman shift denoted as $\Delta_B^{(2)}$. Unlike alkali(-like) atoms, where the second-order shift typically arises from nearby hyperfine levels, the current situation involves clock states with $J = 0$. This leads to the shift originating from energy levels separated by fine-structure splitting. The primary interaction influencing the Zeeman shift (Δ_B) of the clock transition is associated with the 3P_1 and 3P_0 states [48–52].

Density shift A large ensemble of atoms facilitates precise and stable measurements with a high signal-to-noise ratio, interactions between colliding atoms can introduce frequency shifts that compromise the accuracy of the system. Density-dependent collisional shifts play a crucial role in the functioning of Cs fountain standards. If the atomic sample is adequately cold, the lowest odd-wave (p-wave) collisions become suppressed. Additionally, in the case of fermions, s-wave collisions remain forbidden due to the Pauli exclusion principle. As a consequence, the density shift can be exceptionally small, even at elevated ultracold densities. This property favors fermions as promising candidates for atomic frequency standards [53–56].

Other shifts The introduction of a DC electric field leads to a static Stark shift. To shield atoms from external influences, the grounded vacuum chamber functions as a Faraday cage while also maintaining a considerable distance from charged objects. Nevertheless, dielectric-coated glass viewports can accumulate charge. If the charge on the larger viewports, situated closest to the atoms, results in a significant 1 V potential difference, the associated clock transition shift, based on the DC polarizability, would be $< 10^{-18}$.

Errors in the servo mechanism steering the clock laser to the atomic transition can contribute to frequency offsets [57]. Additionally, a residual first-order Doppler effect arises from relative motion between the lattice potential and the clock laser, introducing the possibility of a frequency offset.

Gravitational fields typically do not alter the local operation of an atomic clock, according to general relativity, comparing clocks in different gravitational potentials is subject to gravitational redshift effects.

1.4.4 Limits to Clock Performance

In the previous section, we provided an overview of various shifts and uncertainties that arise in clock performance. Now, we delve into the fundamental constraints of clock performance, primarily defined by the finite signal-to-noise ratio in a specific clock measurement. The noise in the clock originates from two main sources: one from the probing clock laser and the other from the atoms themselves.

The intrinsic noise from the atoms of the clock is termed quantum projection noise (QPN). This noise is associated with the projective measurement of a system in a superposition of two states, denoted as $|\uparrow\rangle$ and $|\downarrow\rangle$. The standard deviation δ of the atom number in $|\uparrow\rangle$ and $|\downarrow\rangle$ is $\Delta N_{\downarrow} = \Delta N_{\uparrow} = \sqrt{N}/2$. The fluctuations in the two clock states are perfectly anti-correlated, resulting in a standard deviation of the difference, $\Delta(N_{\downarrow} - N_{\uparrow}) = \sqrt{N}$. These population fluctuations lead to fluctuations in the clock frequency through the slope of the Ramsey fringe. High-performance microwave clocks, such as the fountain clock, operate at the QPN limit. Presently, state-of-the-art optical clocks are approaching this limit [41, 58, 59].

An important challenge in observing a clock operating at the QPN-defined noise limit is laser noise. Continuous tracking of the laser phase is not feasible due to the finite deadtime in the clock. As a result, the stroboscopic probing of atoms introduces aliased high-frequency noise, commonly referred to as the Dick effect in the clock community.

To address these fundamental limitations, one strategy is to minimize QPN by operating with large atom numbers. However, there is always a technical limit to the achievable atom number. Therefore, it becomes intriguing to explore the possibility of surpassing this limit and achieving clock performance beyond it. One method for improvement involves the implementation of spin-squeezing. By preparing states with reduced noise along a specific direction of the Bloch vector, at the expense of increased noise along the orthogonal axis (anti-squeezed quadrature), clock performance can be enhanced.

To reduce laser noise, the laser is stabilized to a 10 mHz laser linewidth using cryogenic silicon cavities. This effort in laser stabilization has pushed clock performance close to the quantum projection noise (QPN) limit.

1.5 Types of Atomic Clocks

In the pursuit of redefining the SI unit of the second, two prominent candidates emerge: neutral atoms and single trapped ion clocks. Each of these categories offers distinct advantages and carries its own set of limitations. In the subsequent section, we will review both neutral atom and ion-based clock platforms. Moreover, as part of the evolving landscape in timekeeping technology, nuclear transitions and highly charged ions has also emerged as a promising candidate.

1.5.1 Microwave Clock

The initial demonstration of an atomic standard for frequency reference occurred in 1949, using the atomic transition of ammonia. However, a significant leap forward in atomic frequency standards was accelerated by advancements in microwave electronics during World War II, enabling the utilization of microwave transitions as atomic clock references. Cesium was found to have several advantageous characteristics that made it the ideal choice for the primary time standard. Cesium atoms, with an atomic mass of 133, move relatively slowly (approximately 130 m/s) at room temperature. This slower speed of cesium atoms results in longer interaction times with the probing radiation, reducing transit time broadening effects. Another key benefit is the relatively large hyperfine splitting (9.2 GHz) in the ground state [8].

The first cesium (Cs) atomic clock was demonstrated using atomic beam magnetic resonance techniques at the UK National Physical Laboratory (NPL) in 1955 [36]. Its inaccuracy was approximately 10^{-9} . Due to its exceptional precision and accuracy, the microwave transition of cesium atoms was adopted for the redefinition of the SI second in 1967. The SI second was defined as “the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of cesium-133 atoms.” This initial definition of the second remains unchanged.

The cesium (Cs) clock has narrow transition width of approximately 0.5 Hz, resulting in an impressive Q factor greater than 10^{10} . Additionally, the utilization of a substantial number of atoms enhances the signal-to-noise ratio. In cutting-edge Cs clocks, an exceptional (in)stability reaching approximately $1 \times 10^{-14} \tau^{-1/2}$ has been attained. In principle, the Cs clock transition could support a larger Q , leading to improved sta-

bility. However, in current experiments, the interaction time (t) with the atoms is limited to about 1 second due to gravitational constraints, resulting in a Ramsey linewidth $\delta\nu = 1/(2t) = 0.5$ Hz. To enhance stability, it becomes necessary to increase the signal-to-noise ratio (S/N), particularly in Cs fountains where projection-noise-limited measurements have already been accomplished. Nevertheless, this approach encounters a two-fold challenge. Firstly, stability is proportional to the square root of the atom number, meaning that significant increases in N yield only moderate improvements in stability. Secondly, the accuracy of the Cs clock is influenced by atomic collisions, implying that the enhanced precision achieved with a larger atom number may have adverse effects on clock accuracy. As there are limitations in significantly altering the linewidth or the signal-to-noise ratio (S/N) in the Cs clock, an alternative approach to achieving enhanced precision and accuracy involves transitioning to an atomic clock based on a higher-frequency atomic transition. By doing so, it becomes possible to attain a larger Q value within the same interaction time, thereby improving clock stability. This is the driving force behind the development of atomic clocks relying on optical transitions. Shifting the clock reference frequency from the microwave range to the optical spectrum can yield a remarkable Q enhancement of over four orders of magnitude. Assuming all other parameters remain constant, this Q enhancement could potentially reduce the required averaging time to achieve a specific level of precision by eight orders of magnitude. As clock precision advances, accuracy is likely to follow, as the assessment of systematic frequency shifts can be conducted with increasingly reduced averaging time (or effort). The potential for significant improvements in precision and accuracy, driven by the higher Q values, makes optical transition-based clocks an extremely important in the study of frequency standards.

1.5.2 Optical Clock

The advantages of optical clocks have long been recognized, so why have microwave clocks historically received more attention? The primary challenge in optical clock systems has been how to effectively read the optical frequency essentially, how to create the gears of the clock that can count the ticks. In the case of cesium (Cs) clocks, the clock frequency is accessible with conventional electronics, making it relatively straightfor-

ward to measure and distribute the clock signal. In contrast, optical frequencies are extremely challenging to measure because their oscillations occur at orders of magnitude higher speeds than what electronics can handle. Traditionally, measuring optical frequencies involved the development of intricate frequency chains that essentially divided down the optical frequency through successive comparisons with various other oscillators. This approach was not only costly but also required a substantial amount of labor. As a result, only national standards laboratories realistically pursued high-accuracy optical frequency measurements [60].

However, in the past decade, a significant transformation has occurred in optical frequency metrology with the advent of octave-spanning femtosecond (fs) comb lasers [61–64]. In the frequency domain, the output of an fs-laser consists of numerous lasing modes that are evenly spaced at the rate of pulse emissions from the laser, known as the repetition rate (f_{rep}). Due to dispersion within the fs-laser, the frequencies of these comb modes are not exact harmonics of the repetition frequency. Instead, the entire comb is offset by a small microwave amount (f_0). When both f_{rep} and f_0 are known, the frequency of the n th comb mode can be expressed as $\nu_0 = nf_{rep} + f_0$

This relationship is immensely powerful, as it enables the expression of an optical frequency (ν_n) in terms of two microwave frequencies and an integer (n). With precise knowledge of all the comb mode frequencies, an optical clock frequency can be measured by comparing it with the nearest comb mode through heterodyne beat, as $\nu_{clock} = \nu_n + f_{beat}$. Consequently, the optical frequency can be entirely determined by microwave frequencies and an integer. The fs-laser serves as a coherent bridge between the optical and microwave regions of the electromagnetic spectrum. This provides a considerably simplified system for measuring optical frequencies using traditional microwave technology [65–67].

With the introduction of the fs-comb, which significantly simplifies high-precision optical frequency measurements, there has been a rapid surge of interest in optical clocks. The general design of an optical clock is illustrated in 1.3. In this setup, the light emitted by an ultra-stable continuous-wave (cw) laser serves as the local oscillator, somewhat similar to a pendulum for the clock. However, it's essential to recognize that the laser frequency is typically sensitive to environmental factors and, on its own,

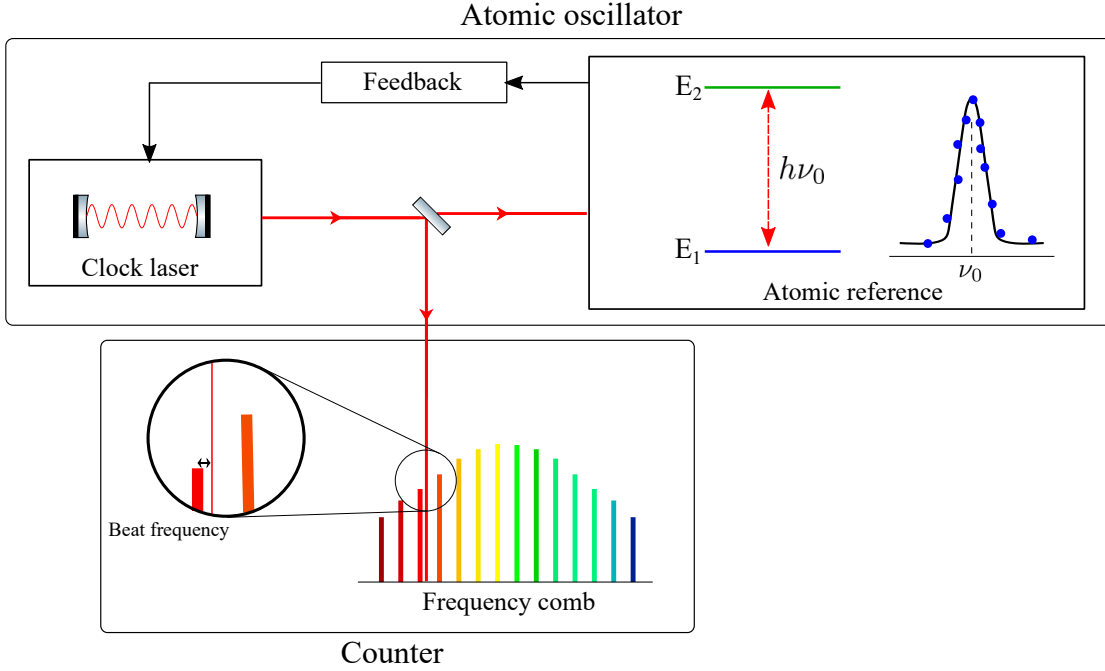


Figure 1.3: Illustration of an optical clock: Top: The clock laser with cavity stabilization includes the interrogation of the atomic transition. The atomic transition, known for its remarkably narrow linewidth, enables precise measurements. Bottom: The frequency of the clock laser is converted into an electronically countable microwave frequency by beating it with a tooth of the frequency comb.

cannot be used as a reliable standard.

To ensure that the laser frequency remains constant, the laser light is employed to interrogate an atom, molecule, or ion. This quantum absorption process provides a natural frequency reference for the system, as the resonance primarily depends on the intrinsic properties of the atom. The atomic signal derived from this process is then utilized to ascertain the discrepancy between the laser frequency and the reference atom's frequency. This mechanism enables the laser frequency to be continually monitored and stabilized to its desired value [68].

1.5.3 Neutral Atom Optical Lattice Clock

The pursuit of an optical clock is motivated by an innovative measurement scheme that has emerged as a cutting-edge approach in optical spectroscopy. By trapping neutral atoms within tightly confining harmonic potentials, effectively eliminating broadening and frequency shifts caused by atomic motion [69, 70]. The optical lattice is thought-

fully designed to ensure that the light-induced shifts on the clock states remain minimal, thus preserving the clock's fundamental frequency without disturbance. In the context of strontium, this feat is accomplished by tuning the lattice to a convenient laser wavelength of 813 nm. This ingenious approach enables Doppler-free spectroscopy and extended interrogation times, facilitated by the substantial signal-to-noise ratio offered by a neutral atom ensemble [71, 72].

This method particularly suits alkaline-earth atoms, given their clock states' robust insensitivity to lattice polarization and external fields. The precision of the lattice clock is poised to outperform both ion-based and free-space atom clocks, and it is expected to rival the accuracy of trapped ion clocks, provided that residual light shifts and collision shifts can be effectively managed [73].

The pioneering proposal and initial theoretical exploration of employing strontium within a lattice clock configuration were proposed and carried out by the Katori group in Tokyo. Shortly thereafter, optical lattice spectroscopy was demonstrated, utilizing both the 3P_1 and 3P_0 clock transitions. These initial investigations delved into identifying the magic wavelengths for the two transitions, achieving measurements at the sub-nanometer scale, and reporting the observation of Doppler and recoil-free line spectra. Building on the foundations laid by Katori's group [71], numerous research teams have since embarked on intensive explorations of the lattice clock system. Most of these investigations have employed different isotopes of strontium and ytterbium [73].

The primary focus of both experimental and theoretical progress has revolved around the issue of clock accuracy. Several notable experimental achievements in this pursuit include the validation that hyperpolarizability effects will not impose constraints on the accuracy of strontium (Sr) at levels down to 10^{-17} . Moreover, experiments have systematically examined the frequency systematics of the Sr lattice clock, achieving precision below 10^{-15} . On the theoretical front, significant efforts have predominantly centered on conducting light shift calculations for Sr and ytterbium (Yb). To overcome this challenges, alternative schemes involve the utilization of bosonic isotopes has been proposed. In combination with external magnetic fields bosonic isotopes can induce otherwise inaccessible clock transitions.

In terms of clock stability, it has been conclusively affirmed that the lattice technique

offers motion-free spectroscopy, close to the ultrahigh-resolution spectroscopy observed in trapped-ion clocks. Remarkably high quality factors have been recently demonstrated in Sr, with Q values surpassing 2×10^{14} . Similar achievements have been realized in a Yb lattice clock system, reaching Q values on the order of 1×10^{14} . These elevated Q values, coupled with the utilization of a higher atom numbers (approximately 10^4) in these systems, indicate the potential for achieving 1-second stabilities on the order of 10^{-17} or lower [1, 40, 74–78].

1.5.4 Ion-Based Optical Clocks

Optical ion clocks utilizing single trapped ions offer a distinctive advantage by mitigating the challenges posed by Doppler shifts that afflict measurements involving free-space neutral atoms. These systems meticulously confine ions within the Lamb-Dicke regime, ensuring that ion motion remains substantially smaller than the wavelength of the spectroscopy laser. This characteristic serves to minimize the influence of atomic motion on the transition width, facilitating the utilization of extended interrogation times, thereby achieving exceptionally high line quality factors (Q) [79–82]. The merits of optical ion clocks are prominently evident in recent outcomes, such as the remarkable achievements of the Hg⁺ clock at the National Institute of Standards and Technology (NIST) [83–88].

1.5.5 Highly Charged Ion Clock

Highly charged ions (HCI) are a common form of matter in celestial bodies like the sun and stars. These ions are called “highly charged” because they’ve lost many electrons, resulting in a strong positive charge. This charge causes their outermost electrons to be tightly bound to the atomic nucleus, making them less responsive to external electromagnetic fields. However, this also makes them excellent for probing fundamental physics phenomena related to special relativity, quantum electrodynamics, and the atomic nucleus.

Researchers have been exploring the potential of highly charged ions for extremely accurate clocks and fundamental physics studies. The unique properties of HCI make them less susceptible to external electric and magnetic fields compared to other ions or atoms [89, 90]. Nonetheless, there’s a significant challenge in working with highly

charged ions [91]. Creating them involves generating extremely hot plasma, and conventional cooling methods using lasers and standard detection techniques don't work efficiently. To overcome this challenge, scientists devised a method where they isolated a single highly charged argon ion from the hot plasma and confined it alongside a singly charged beryllium ion. This innovative setup enabled them to indirectly cool the highly charged ion and study its behavior by observing the beryllium ion. Through this approach, they managed to cool the highly charged ions to an incredibly low temperature. Subsequently, they constructed an optical atomic clock using these thirteen-fold charged argon ions and compared it to a well-established ytterbium ion clock at PTB. Ultimately, HCI yielded an impressive level of measurement accuracy, placing HCI clock on par with some of the best ones currently in use.

1.5.6 Nuclear Clock

Nuclear clock holds potential for greater precision than optical atomic clock, by utilizing nuclear transitions instead of electronic ones. Nuclear transitions, due to the small size of nuclei and very less magnetic moments, are less affected by external perturbations compared to atomic transitions [92–94]. The concept of a nuclear clock, particularly a Mössbauer nuclear clock, has got attention in recent years. An ideal candidate for a nuclear clock is the thorium-229 isotope, which possesses a unique nuclear transition. However, determining the transition frequency with high precision was a formidable challenge for many years.

A significant breakthrough has been achieved with a high-precision measurement of the thorium-229 transition by Tomas Sikorsky and colleagues at Heidelberg University. This measurement narrows the spectral range that future searches need to focus on, bringing the realization of a nuclear optical clock one step closer. Knowing the precise transition frequency now sets the stage for developing a nuclear optical clock. While laser spectroscopy of electronic shells in atoms has been well-established since the invention of the laser in 1960, laser spectroscopy of nuclear transitions remains a complex task. Most nuclear excited states have energies in the keV to MeV range, making them inaccessible with current laser technology.

Thorium-229, with its uniquely low-energy nuclear transition (around 7.8 eV), stands

out as an exception. However, the exact frequency of this transition remained uncertain for a long time, hindering the choice of appropriate laser technology. Prior indirect measurements based on gamma-ray emissions provided varying estimates, ranging from about 3.5 eV to approximately 7.8 eV. This uncertainty has now been significantly reduced through recent precise measurements.

1.6 Comparison of Atomic Clocks

The international metrology community aims to replace the microwave time standard with an optical clock, but first must choose from one of several clock designs being developed worldwide. Comparison of stability and uncertainty of different clocks is tabulated in a 1.1 [79–87, 95, 96].

Species	Clock transition	λ (nm)	$\Delta\nu_{nat}$ (Hz)	Uncertainty	Stability $/\sqrt{\tau}$	Citation
$^{27}\text{Al}^+$	$^1S_0 \longrightarrow ^3P_0$	268	0.008	9.4×10^{-19}	2.2×10^{-15}	[79]
$^{88}\text{Sr}^+$	$^2S_{1/2} \longrightarrow ^2D_{5/2}$	674	0.4	2×10^{-17}	1×10^{-14}	[83]
$^{99}\text{Hg}^+$	$^2S_{1/2} \longrightarrow ^2D_{5/2}$	282	1.7	1.9×10^{-17}	2.8×10^{-15}	[79]
$^{171}\text{Yb}^+$	$^2S_{1/2} \longrightarrow ^2D_{3/2}$	436	3.1	1.1×10^{-16}	-	[85]
$^{171}\text{Yb}^+$	$^2S_{1/2} \longrightarrow ^2F_{7/2}$	467	10^{-9}	1.1×10^{-18}	5.3×10^{-15}	[86]
^{87}Sr	$^1S_0 \longrightarrow ^3P_0$	698	0.001	2.1×10^{-18}	1.6×10^{-16}	[102]
^{88}Sr	$^1S_0 \longrightarrow ^3P_0$	698	-	2.9×10^{-15}	2.2×10^{-14}	[99]
^{171}Yb	$^1S_0 \longrightarrow ^3P_0$	578	0.01	1.6×10^{-18}	3.2×10^{-16}	[98]
^{174}Yb	$^1S_0 \longrightarrow ^3P_0$	578	-	1.5×10^{-15}	3×10^{-15}	[98]
^{199}Hg	$^1S_0 \longrightarrow ^3P_0$	266	0.1	7.2×10^{-17}	3×10^{-15}	[69]

Table 1.1: Best reported accuracies and stabilities of optical clocks: The clock transition, their wavelengths, and their natural linewidth are mentioned along with the references.

1.7 Objective of the Thesis

This thesis focuses on establishing an experimental set-up dedicated to precision measurements and frequency metrology, utilizing the doubly forbidden singlet to triplet transition at 698 nm. Due to its remarkably low linewidth and high susceptibility to

external environmental influences, this transition emerges as an ideal choice for a precision clock. The thesis outlines the conception and assembly of an experimental setup capable of generating a collection of cold Sr atoms in a magneto-optical trap (MOT) and a magnetic trap.

Throughout the development of this setup, various innovative designs have been introduced, including a clog-free oven design, an injection lock module for the laser, and a cavity housing. Additionally, spectroscopy experiments were conducted on the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition.

Organization of the Thesis

Chapter 2 Introduces the principles underlying laser cooling of trapping of neutral atoms. It discusses the dynamics of light interactions with atoms, Doppler cooling mechanisms, the concept of Magneto-Optical Traps (MOT), and the methodology for determining temperature through absorption imaging techniques.

Chapter 3 details the experimental configuration employed to generate a cold sample of Sr atoms in a Magneto-Optical Trap (MOT) and a magnetic trap (MT). The development and assembly of the vacuum chamber, maintaining a pressure below 10^{-10} torr within the science chamber is also discussed. Additionally, it outlines the blueprint of an atomic oven devised for loading the MOT.

Chapter 4 outlines the process of producing lasers for cooling and repumping transitions. It investigates the theoretical framework and practical implementation of injection locking to produce a laser operating at 461 nm. Additionally, the optical scheme for the blue laser is described, and the chapter also outlines the blue MOT characterization and production details.

Chapter 5 describes the process of producing lasers for narrow linewidth 689 nm transitions. Additionally, the chapter explores techniques for frequency stabilization of the second-stage cooling laser through PDH locking using a high finesse optical cavity, various frequency schemes for laser beams, and the creation of magneto-optically trapped atomic clouds.

Chapter 6 This chapter presents experiments conducted with strontium (Sr) atoms in a 461 nm cooling transition magneto-optical trap (MOT). It covers the introduc-

tion, theoretical background, and details of the experimental setup for spectroscopy on the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition using laser-cooled atoms in a blue MOT. The chapter determines key parameters for each strontium isotope MOT, including enhancement factor, transition frequency, hyperfine splitting constant, and isotope shift for $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition. Experimental values are compared with those calculated using the Fock-space relativistic coupled-cluster (FSRCC) method.

Chapter 7 provides a summary and outlines the future prospects of the presented research.

Chapter 2

Laser Cooling and Trapping of Strontium

2.1 Strontium: An Alkaline-Earth Element

Strontium (Sr, $Z=38$) is a metallic element with a light silver color, situated in the second column of the periodic table. Elements in this column, like beryllium (Be), magnesium (Mg), calcium (Ca), barium (Ba), and radium (Ra), have a similar electronic structure. Similarly, the zinc-family elements, such as zinc (Zn), cadmium (Cd), mercury (Hg), copernicium (Cn), along with ytterbium (Yb) and nobelium (No), also share this electronic structure. These atoms are commonly referred to as “alkaline-earth like systems,” “group-II-atoms,” or “two-electron systems.”

Strontium has four naturally occurring isotopes, with three being bosonic and one fermionic isotopes. In bosonic isotopes, the nuclear spin is zero ($I = 0$). There are even number of valence electrons, coupled with filled inner electron shells, resulting in an even total electron count. To maintain a neutral charge, the proton count must also be even. Bosonic atoms require even totals of electrons, protons, and neutrons, leading to an even number of neutrons. In such even-even nuclei, nucleons align in pairs with antiparallel spins, resulting in a nuclear spin of zero. For ^{87}Sr , the nuclear spin is $I = 9/2$, which is the highest among all stable alkaline earth isotopes. These isotopes and their respective natural abundances can be found in Table 2.1.

Isotope of Sr	Nuclear spin (I)	Atomic mass	Natural abundance
^{84}Sr	0	83.913 425(4)	0.56(1)
^{86}Sr	0	85.909 2624(24)	9.86(1)
^{87}Sr	9/2	86.908 8793 (24)	7.00(1)
^{88}Sr	0	87.905 6143(24)	82.58(1)

Table 2.1: Sr isotopes: Isotopes of the strontium (Sr) atom that occur naturally. Data is sourced from NIST database.

Strontium (Sr) was the first element that was proposed for a lattice clock [97] and has now become the most widely adopted optical atomic clock globally [98–104]. Numerous institutions have been collaborating to develop and improve this technology, making the strontium optical lattice clock a top contender for redefining the SI unit of the *second*. In the context of atomic clocks, the ^{88}Sr and ^{87}Sr isotopes hold greater significance.

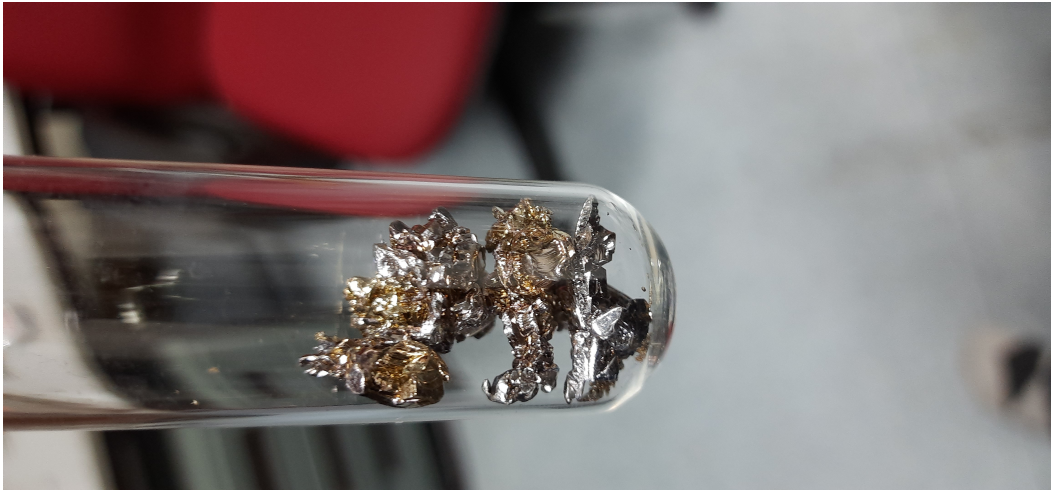


Figure 2.1: Strontium metal: The strontium ampoule from Sigma-Aldrich containing 99.9% pure strontium metal pieces. The Sr metal is sealed in a glass ampule with argon gas. Strontium is a highly chemically reactive metal that forms a dark oxide layer when exposed to air and reacts slowly with water, typically producing strontium hydroxide and hydrogen gas.

2.1.1 Strontium: Energy Level Structure and Cooling Transitions

In atomic physics, the term symbol is used to represent the combined spin and orbital angular momentum properties of electrons within a multi-electron atom. When deter-

mining the state of an atom's electron configuration, the term symbol is crucial as it captures the total angular momentum, including both spin and orbital components. This is particularly significant in the context of LS coupling, also known as Russell–Saunders coupling, where the quantum numbers associated with the total orbital (L), spin (S), and overall (J) angular momenta of all electrons are regarded as fundamental quantum numbers.

In the case of strontium (Sr), the two valence electrons give rise to two distinct series of atomic energy levels. This arises because these two valence electrons can have either parallel spins (triplet states, $S=1$) or antiparallel spins (singlet states, $S=0$), similar to atomic helium. These energy levels are described using the *Russel-Saunders* (RS) notation of $^{2S+1}L_J$ [105]. In this context, S denotes the combined spin of the two electrons, which can be either 0 or 1. L represents the orbital angular momentum of these electrons, and J corresponds to the total angular momentum of the state, defined as the sum of L and S .

In alkaline atoms, the ground state is typically the one with the lowest total angular momentum ($L = 0$), denoted by 1S_0 state. The lower energy level structure for strontium is illustrated in Fig. 2.2. The first excited state in the singlet series, known as 1P_1 , is commonly utilized in laser cooling experiments with group II atoms. This is due to its higher transition rate.

The triplet manifold offers a more diverse range of possibilities. Among these, the lowest-lying 3P_J states are of particular interest for atomic clock application. Assuming pure LS coupling, the three $5s\ 5p\ ^3P_J$ states are forbidden from decaying to the ground state ($\Delta S \neq 0$), making them metastable. However, the spin-orbit interaction introduces a finite lifetime for the 3P_1 state, permitting weak electric dipole transitions. These transitions are often referred to as intercombination transitions due to their narrow linewidth. Notably, the ultranarrow $5s^2\ ^1S_0 \rightarrow 5s\ 5p\ ^3P_0$ transition exists solely in the isotope with nuclear spin 9/2 (^{87}Sr), as the spin-orbit interaction alone cannot yield a finite lifetime for this transition.

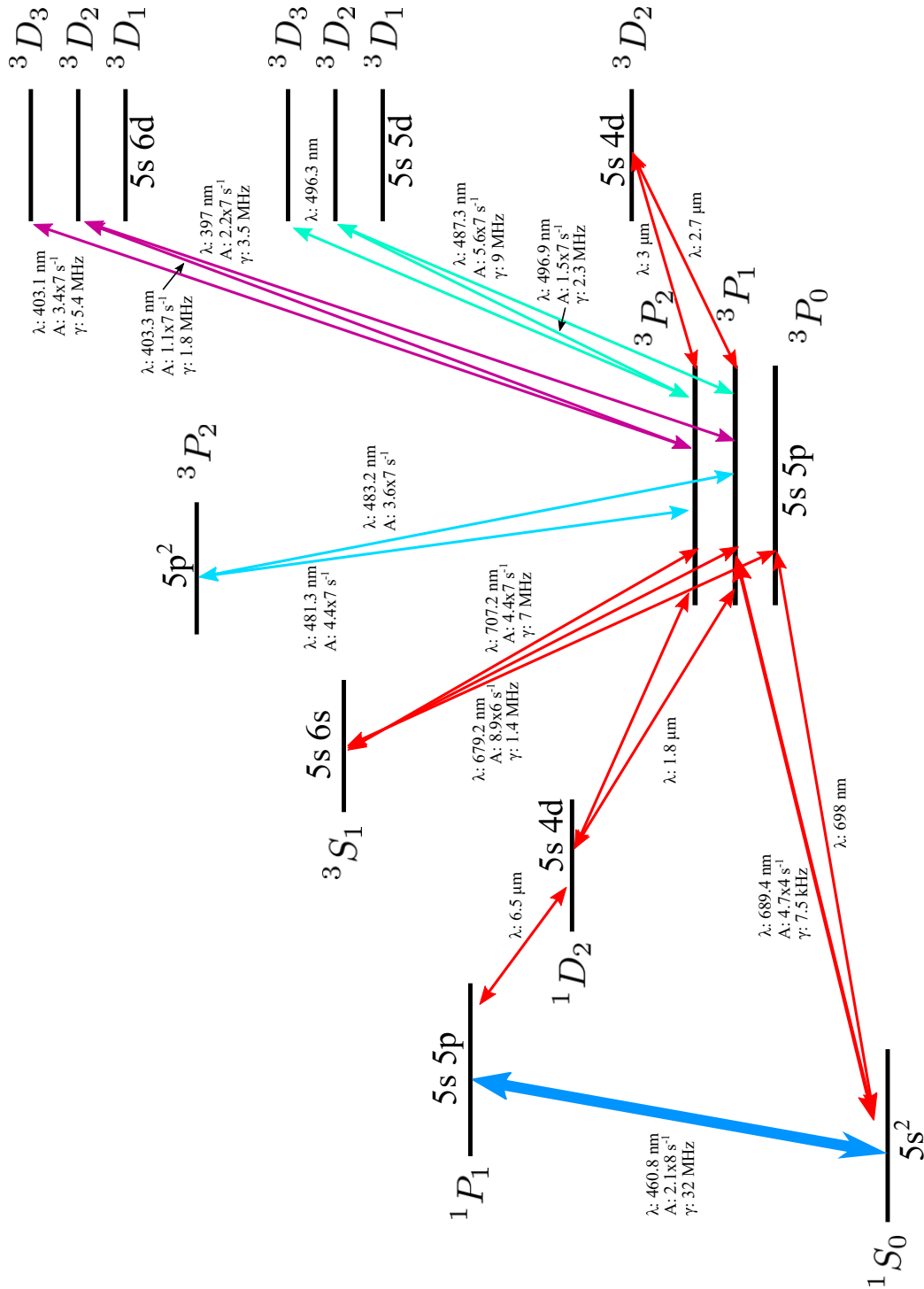


Figure 2.2: Atomic Transitions in Strontium: In atomic strontium, the electronic energy levels situated at lower energies are displayed. Specifically, the clock transition from $1S_0$ to $3P_0$, alongside transitions crucial for laser cooling and repumping, are denoted, along with their respective wavelength, linewidth, and scattering rates.

As shown in Fig.2.2, the ground state $5s^2\ ^1S_0$ has three important transitions, and each of these transitions has its unique oscillator strengths. The variety in these transitions is valuable because it enables a range of techniques to manipulate atoms, and all of these techniques are essential in the field of optical frequency standards.

The initial cooling and trapping stage relies on the $5s^2\ ^1S_0 \longrightarrow 5s\ 5p\ ^1P_1$ transition. This transition is strong with an Einstein A coefficient, $A = 2.1 \times 10^8\ s^{-1}$. This transition has a short lifetime of approximately 5 ns and a linewidth of 32 MHz, resulting in a higher Doppler-limited temperature of $\sim 770\ \mu\text{K}$. However, it's important to note that the 461 nm transition isn't completely closed; it involves a decay pathway through the 1D_2 state, leading to populating the $5s\ 5p\ ^3P_J$ manifold, which results in the creation of a long-lived 3P_2 metastable reservoir that can be magnetically trapped and has a lifetime on the order of seconds. To depopulate this level, we need to repump the atoms to a higher state that enables them to decay back into the 3P_1 state and eventually to the ground state. The repumping setup for the transition $5s\ 5p\ ^3P_2 \rightarrow 5p^2\ ^3P_2$ requires only a single laser operating at the wavelength of 481.3 nm. This laser effectively promotes electrons from the $5s\ 5p\ ^3P_2$ state to the ground state via the doubly excited $5p^2\ ^3P_2$ state.

The $5s^2\ ^1S_0 \longrightarrow 5s\ 5p\ ^3P_1$ transition is comparatively weaker with an Einstein A coefficient, $A = 4.7 \times 10^4\ s^{-1}$, and is a closed transition. It complements the $^1S_0 \rightarrow ^1P_1$ transition by capturing pre-cooled atoms from the first stage cooling. This transition has a relatively long lifetime of 21 μs and a narrow linewidth of $2\pi \times 7.5\ \text{kHz}$. Doppler-limited temperature of this transition is 180 nK. Detailed properties of these transitions are given in Table 2.2 [106].

Transition characteristics	$^1S_0 \rightarrow ^1P_1$	$^1S_0 \rightarrow ^3P_1$
Wavelength: λ (nm)	461	689
Energy: E (ev)	2.69	1.8
Lifetime: τ (ns)	5	21322
Decay rate: Γ (s^{-1})	2.0×10^8	4.7×10^4
Linewidth: γ (MHz)	31.83	0.00746
Saturation intensity: I_{sat} (mW/cm ²)	42.5	0.003
Maximum deceleration: a_{max} (m/s ²)	0.98	0.00015
Recoil Temperature: T_r (μK)	1.03	0.46
Capture velocity: V_c (m/s)	14.67	0.0051
Doppler limit temperature: T_D (μK)	764	0.18

Table 2.2: Comparison of the two cooling transitions: Relevant parameters for laser cooling and trapping using the $^1S_0 \rightarrow ^1P_1$ (461 nm) and $^1S_0 \rightarrow ^3P_1$ (689 nm) transitions in Sr atoms.

The clock transition in Sr atom is a doubly forbidden singlet to triplet transition, connecting $5s^2\ ^1S_0 \rightarrow 5s\ 5p\ ^3P_0$ at a wavelength of 698 nm. This transition has two crucial properties: a narrow linewidth and minimal sensitivity to perturbations, making it an ideal choice for optical frequency standard. The natural linewidth of this transition is incredibly narrow – ~ 1 mHz for the fermionic isotope ^{87}Sr , and its measured frequency is 429,228,004,229,874.0 (1.1) Hz [107]. For the bosonic isotope, this transition is originally prohibited due to the spin selection rule. However, a small external magnetic field can induce mixing between the two states of the triplet manifold: 3P_0 and 3P_1 . This mixing results in a finite lifetime for this transition, making it suitable for the clock transition [108, 109].

2.2 Laser Cooling

The ability to employ ultra-narrow atomic transitions as a frequency reference relies on the capability to access the inherent spectral characteristics of these transitions. This necessitates precise control over a group of atoms to minimize frequency shifts and spectral broadening. In an ideal scenario, experiments would be conducted in an environment that is free of external interferences, including electric and magnetic fields,

background gases, radiation, and gravitational effects. Additionally, the atomic sample would approach absolute zero temperature. However, in practical scenario, the goal is to minimize these sources of interference to the largest extent possible.

The primary factor responsible for frequency shifts and spectral broadening is the average temperature and velocity distribution of the atomic ensemble. Cooling and trapping neutral atoms is a complex undertaking because neutral atoms do not respond to stationary electric fields. Nevertheless, advancements in laser technology have provided valuable tools for manipulating both the internal and external properties of atoms through their interaction with light. These cold atoms are an excellent choice for precise measurements and frequency metrology.

2.2.1 Light Atom Interaction

A semi-classical model suffices to understand the principles of laser cooling. In this model, atom's energy levels are treated as quantized, while light is regarded as a classical oscillating field. The interaction between light and atoms is possible in a several ways. When subjected to an oscillating electric field, the atom's population oscillates between the ground and excited states at a specific characteristic frequency known as the Rabi frequency, giving rise to a phenomenon called Rabi oscillation. This model considers atoms with only two energy levels. On the other hand, when external electromagnetic radiation is present, it leads to an energy shift known as the light shift or AC Stark shift. This effect arises from the interaction between the oscillating external electric field from light and the induced electric dipole moment of the atom.

In a broader sense, depending on the nature of this interaction, one can classify the forces experienced by atoms in the presence of external electromagnetic radiation into two main categories:

- 1) **Dissipative force:** This force is a consequence of spontaneous emission.
- 2) **Dipole force:** It originates from the intensity gradient of light seen by the atoms.

Laser cooling utilizes the force generated by radiation pressure, which arises due

to momentum transfer occurring during the scattering of photons by atoms. Scattering force is formally defined as

$$\mathbf{F}_s = \frac{\Delta P_{total}}{\Delta t} \quad (2.1)$$

where Δt represents the transition's decay rate, and ΔP_{total} corresponds to the cumulative momentum transfer resulting from N scattering events, each involving a momentum transfer of

$$\Delta p = \hbar k_L \quad (2.2)$$

where k_L denotes the wavevector of the scattered photon. Referring to Equation 2.1, we can conclude that the scattering force is equivalent to

$$\mathbf{F}_s = \frac{N_{scat} \hbar k_L}{\Delta t} = \Gamma_{scat} \hbar k_L \quad (2.3)$$

where Γ_{scat} represents the transition's scattering rate and is defined as:

$$\Gamma_{scat} = \frac{N_{scat}}{\Delta t} \quad (2.4)$$

A scattering event is characterized by a photon's absorption by an atom, leading to the excitation of an electron to a higher energy level. Following this, the electron will eventually return to the ground state, either through spontaneous emission after a period determined by the transition's decay rate (Δt), or when influenced by resonant photons, it can be stimulated to decay to the ground state via stimulated emission. Both of these decay mechanisms produce emitted photons with the exact frequency of the atomic transition. The frequency spread is determined by the transition's linewidth, denoted as Γ and calculated as $\Gamma = 1/\Delta t$. However, it's essential to note that stimulated emission leads to momentum transfer opposite to the incoming photons, while spontaneous emission results in an entirely random momentum transfer in various directions, indicated by θ . The average momentum transfer to the atom, resulting from N scattered photon events due to spontaneous emission, averages out to zero.

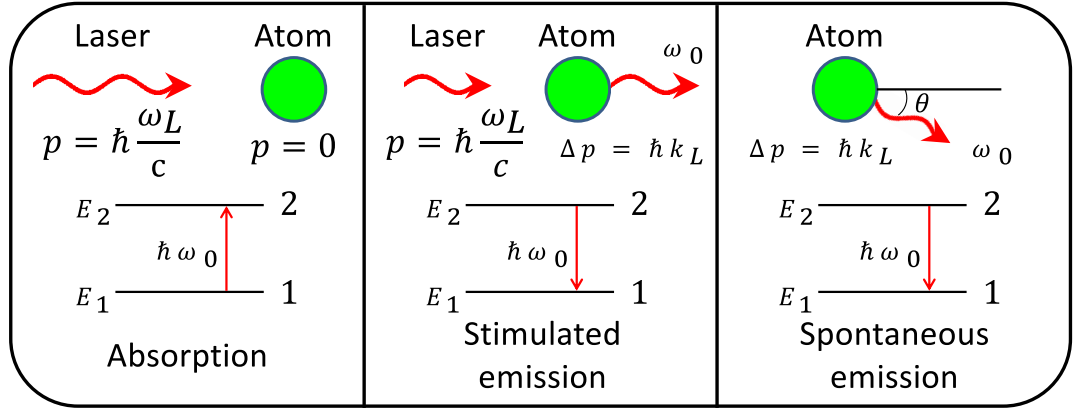


Figure 2.3: Light-atom interaction: momentum transfer from an incident photon to an atom in the cases of left: absorption, middle: stimulated emission and right: spontaneous emission.

The number of scattering events, N_{scat} , is directly related to the population of atoms in the excited state, denoted as $\langle P_2 \rangle$. This population can be quantified by observing the absorption or scattering response, denoted as S_m , of an ensemble of atoms when subjected to laser with a specific frequency ω_L and intensity (I). The transition being driven has a saturation intensity of I_{sat} , which signifies the intensity required to reach half of the maximum gain in spontaneous emission, and it occurs at a transition frequency ω_0 .

$$S_m = N_{scat} = \langle P_2 \rangle = \frac{1}{2} \frac{s}{1 + s + 4 \left(\frac{\delta_L}{\Gamma} \right)^2} \quad (2.5)$$

Here, δ_L is the laser frequency detuning given by $\delta_L = \omega_L - \omega_0$. 's' is the saturation parameter, the ratio of the laser intensity (I) to the saturation intensity of the transition (I_{sat}). using equation 2.4 and 2.5, The scattering rate can be written as:

$$\Gamma_{scat} = \frac{\Gamma}{2} \frac{s}{1 + s + 4 \left(\frac{\delta_L}{\Gamma} \right)^2} \quad (2.6)$$

The spectral line shape follows a Lorentzian profile, and the full width at half maximum (FWHM) characterizes the transition's linewidth.

$$\Delta\omega_{FWHM} = \Gamma\sqrt{1 + s} \quad (2.7)$$

Hence, the observed linewidth broadens as the laser intensity increases. This phe-

nomenon is known as power broadening.

2.2.2 Optical Molasses

To cool a thermal gas of two-level atoms with an average velocity (v_0), using light-atom momentum transfer, it's essential to apply force in both positive and negative directions along three orthogonal axes. Typically, this is achieved by employing six laser beams with equal intensity. When an atom is at rest at the center of these six beams, the radiation force is symmetric in all directions. However, if an atom is in motion, it experiences a Doppler shift, causing an imbalance in forces. This phenomenon is known as “Optical Molasses”. In the atoms reference frame, the Doppler effect results in the laser beam propagating in the opposite direction to the atom's motion having an increased frequency. If the laser beam is detuned below resonance, this Doppler shift brings the light closer to resonance with the atom, leading to an increased rate of absorption for this beam. In the case of low velocity (where $kv < \Gamma$), the force applied to the atom can be expressed as follows:

$$\begin{aligned} F_{molasses} &= F_{scat}(\omega - \omega_0 - k_L v) - F_{scat}(\omega - \omega_0 + k_L v) \\ &\simeq F_{scat}(\omega - \omega_0) - k_L v \frac{\partial F}{\partial \omega} - \left[F_{scat}(\omega - \omega_0) + k_L v \frac{\partial F_{scat}}{\partial \omega} \right] \\ &\simeq -2 \frac{\partial F_{scat}}{\partial \omega} k_L v \end{aligned} \quad (2.8)$$

The difference in forces caused by the Doppler shift can be expressed as follows:

$$F_{molasses} = -\alpha v \quad (2.9)$$

Light exerts a frictional force on the atoms. The damping coefficient α is defined as follows:

$$\alpha = 2k_L \frac{\partial F_{scat}}{\partial \omega} = 4\hbar k_L^2 s \frac{-2\delta/\Gamma}{[1 + (2\delta/\Gamma)^2]^2} \quad (2.10)$$

Damping occurs when α has a positive value, leading to a condition where $\delta = \omega - \omega_0 < 0$, which is known as red detuning.

In the absence of any external influences on atomic motion, the theoretically achievable temperature for atoms would tend towards absolute zero. However, physical con-

straints prevent this. Because of the discrete nature of momentum transfer during absorption and the random emission of photons, the atoms experience heating. During each decay process, the atom's momentum changes by approximately $\hbar k$.

In steady state, the rates of heating and cooling balance each other, establishing the lower limit on the temperature achievable through Doppler cooling. This resultant temperature is referred to as the Doppler-limited temperature (T_D). Under conditions of low intensity ($I/I_{sat} \ll 1$) and a detuning of $\Delta = -\Gamma/2$, this minimum temperature T_D is achieved [110,111]. The minimum temperature is given by,

$$k_B T_D = \frac{\hbar \Gamma}{2} \quad (2.11)$$

The model we discussed here focuses on a counter-propagating laser beam in a single direction. However, within a cloud of atoms, the atoms have freedom of movement in all three dimensions. Therefore, to cool the atoms in all of these dimensions, two additional pairs of mutually perpendicular counter-propagating laser beams are introduced. This extended setup, capable of cooling in three dimensions, is referred to as a 3D optical molasses. The initial demonstration of this 3D optical molasses was done by Steven Chu in 1985.

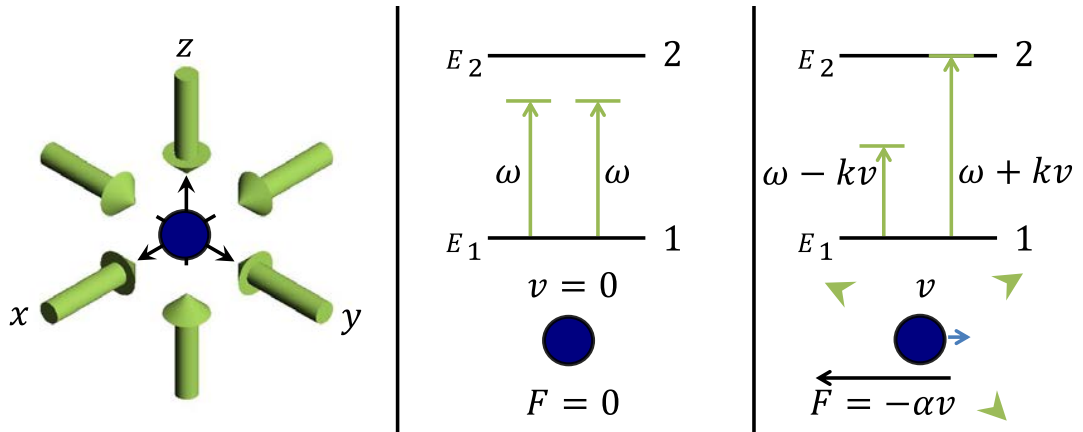


Figure 2.4: Optical molasses: velocity dependent force acting upon an ensemble of atoms with an average velocity by 3 pairs of opposing laser beams on resonance with an electronic transition

Doppler cooling is applicable mainly to the atoms that can be treated as a two-

level system. However, for atoms with degenerate sub-levels, it's possible to cool them to temperatures below the Doppler limit using a technique called Sisyphus cooling [112, 113]. The lowest achievable temperature through laser cooling is constrained by the recoil energy resulting from spontaneous emission. When exposed to a resonant light source, the atom will have at least the momentum from the last emitted photon. Consequently, the limiting recoil temperature is determined by this momentum.

$$T_r = \frac{\hbar^2 k^2}{k_B m} \quad (2.12)$$

Where, the T_r is recoil temperature.

2.3 Magneto-Optical Trap

The optical molasses technique doesn't provide a confinement to the atoms within a defined spatial region. However, by appropriately configuring the polarizations of the laser beams and introducing a linear, spatially varying magnetic field at the atomic position, optical molasses can be transformed into a trap [114]. This configuration is commonly known as a magneto-optical trap (MOT).

A pair of anti-Helmholtz coils is used to generate a quadrupole magnetic field. Within the central region of the quadrupole field, the magnetic field strength is zero. In the overlapping laser beams region, the magnetic field varies linearly. Due to the Zeeman effect, the energy levels of $M_J = 0, \pm 1$ in the $J = 1$ state vary linearly with respect to the position within the atomic trap (see Fig. 2.5).

This variation introduces an asymmetry in the scattering forces of the laser beams. As long as the beams are detuned to a frequency lower than the resonance frequency, atoms experience a strong confinement by the radiation force. The operational principle of the magneto-optical trap (MOT) is illustrated in Fig. 2.5, focusing on a straightforward transition from $J = 0$ to $J = 1$. The quadrupole coil field profile can be expressed as:

$$B(z) = bz \quad (2.13)$$

In this equation, $B(z)$ is the magnetic field as a function of z . And, $b (> 0)$ is the magnetic field gradient of the anti-Helmholtz coil. In the presence of the magnetic field,

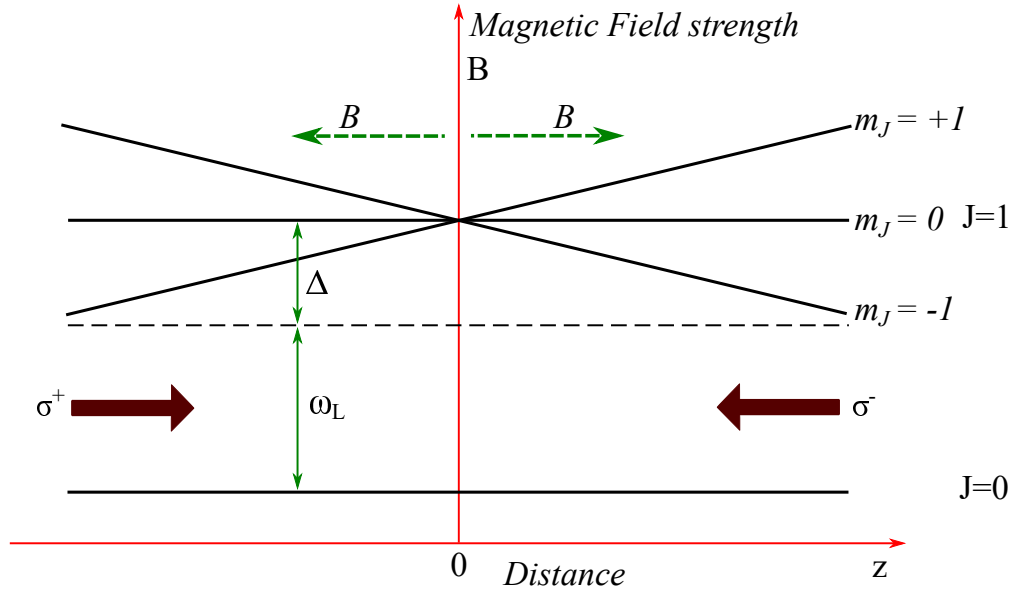


Figure 2.5: MOT Operation: Illustration of working principle of the MOT in 1-dimension for an atomic system with $J = 0$ to $J = 1$ transition. Magnetic field along with the circularly polarized light provides the spatial confinement of the atoms.

the shift in Zeeman sub-levels ($\Delta E(z)$) can be described as:

$$\Delta E(z) = \mu m_J B(z) \implies \omega_0(z) = \omega_0 + m_J \alpha z \quad (2.14)$$

where, $\alpha = \mu b / \hbar$. Where, μ is Bohr magneton, and B is intensity of the magnetic field.

Two counter-propagating laser beams having opposite circular polarizations and tuned below the atomic resonance frequency, illuminate the atom. Due to the Zeeman shift and selection rules, there is an asymmetry in the radiative force exerted by the laser beams, causing the atom to be pushed back towards the trap's center at $z = 0$.

For an atom in the ground state $J = 0$ and positioned at $z > 0$, the $M_J = -1$ transition is closer to resonance, and the selection rules result in photon absorption from the beam with σ^- polarization. This absorption generates a scattering force that pushes the atom towards the trap center at $z = 0$. A similar process takes place for an atom at $z < 0$. The atom perceives the $M_J = +1$ transition closer to resonance and follows the selection rules to absorb photons from the σ^+ polarized beam, also driving the atom back towards $z = 0$.

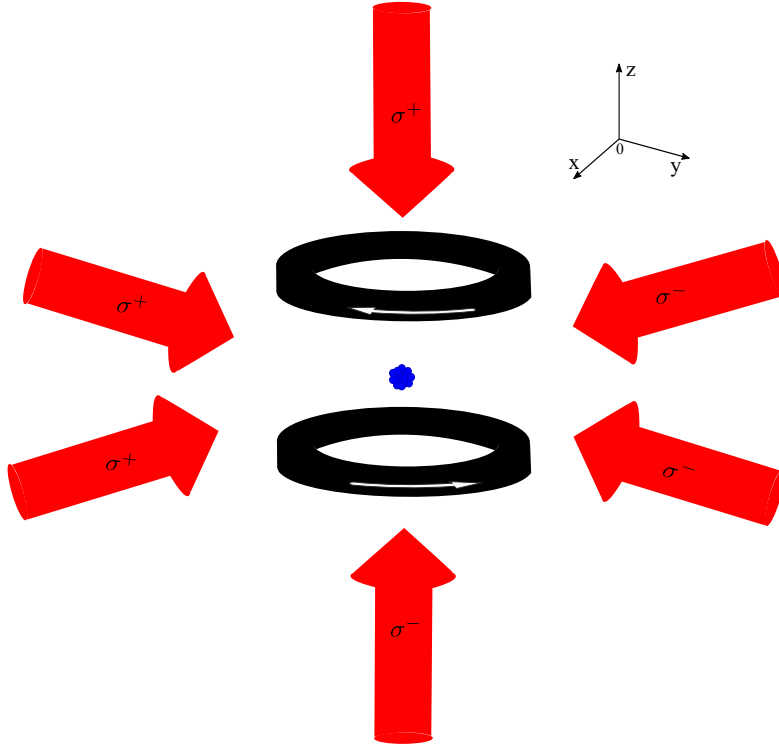


Figure 2.6: Magneto-Optical Trap: Illustration of working principle of a MOT in 3-dimension. Magnetic field along with the circularly polarized light provides the spatial confinement of the atoms.

Mathematically, the force experienced by the atom in the MOT is described by:

$$\begin{aligned}
 F_{MOT} &= F_{scat}^{\sigma^+}(\omega_L - k_L v - (\omega_0 + \beta z)) - F_{scat}^{\sigma^-}(\omega_L - k_L v - (\omega_0 - \beta z)) \quad (2.15) \\
 &\simeq -\alpha v - k z \\
 &= -\alpha(k_L v + \beta z)
 \end{aligned}$$

The force experienced by an atom in a MOT can be approximately represented as the sum of three components: the atom's doppler shift ($k_L v$), the Zeeman shift (βz), and the damping coefficient (α) calculated according to equation 2.10. This indicates that the presence of the magnetic field introduces a position-dependent force, while the optical molasses primarily functions as a velocity-dependent force. The interplay of these factors affects the atom's motion in the MOT.

The capture velocity of a MOT can be approximated by considering the radius (r)

of the MOT laser beams as the distance an atom travels to come to a stop. By applying common kinematic equations, the following condition for capture velocity can be derived:

$$(v_c^{MOT})^2 = 2ra_{eff} \quad (2.16)$$

where a_{eff} represents the effective deceleration perceived by the atom. Typically, this is defined as half the acceleration at $I = I_{sat}$ to consider the force's fluctuations due to the random nature of scattered photons.

$$a_{eff} = \frac{1}{2}a(I = I_{sat}) = \frac{a_{max}}{4} \quad (2.17)$$

The stopping distance of the MOT is defined as the radius of the laser beams, denoted as r . This radius represents the average distance an atom travels before exiting the spherical 3D MOT region. It's crucial to note that this region is where all laser beams overlap, and beyond this area, trapping does not take place. The MOT capture velocity is expressed as:

$$v_c^{MOT} = \sqrt{\frac{\hbar k_L \Gamma}{M} \frac{r}{4}} \quad (2.18)$$

The temperature of atoms confined within a magneto-optical trap is higher compared to those only subjected to the optical molasses method. This increase in temperature arises from the suppression of sub-Doppler cooling, a feature provided by the optical molasses, due to the influence of the strong magnetic field. Additionally, when the atomic density within the trap volume is high, some atoms in the MOT reabsorb spontaneously emitted photons, introducing an additional heating component.

The typical velocity of atoms released directly from an atomic oven is approximately 500 m/s , and a typical magneto-optical trap (MOT) has a capture velocity of around 50 m/s , it's evident that only a small fraction of atoms could be directly captured from a hot atomic beam into the MOT. It is necessary to implement a pre-cooling method to decrease the average velocity of the atoms to a level below that of the MOT's capture velocity. This would ensure that a greater number of atoms could be effectively collected by the MOT for subsequent experimental purposes.

2.4 Zeeman Slowing

From the atom's reference frame, a well-selected red-detuned laser beam can be Doppler shifted to achieve resonance. This approach offers a means to pre-cool the atoms before entering the magneto-optical trap (MOT) stage. Using a laser beam directed opposite to the atomic motion, thermal atoms can absorb detuned photons, leading to a reduction in average atomic velocity due to momentum transfer [115–117]. However, this velocity change causes the detuned photons to no longer align with the atomic transition's resonance frequency. To address this, an external magnetic field with varying strength can be applied to compensate for the altered average velocity, ensuring that the photons remain in resonance.

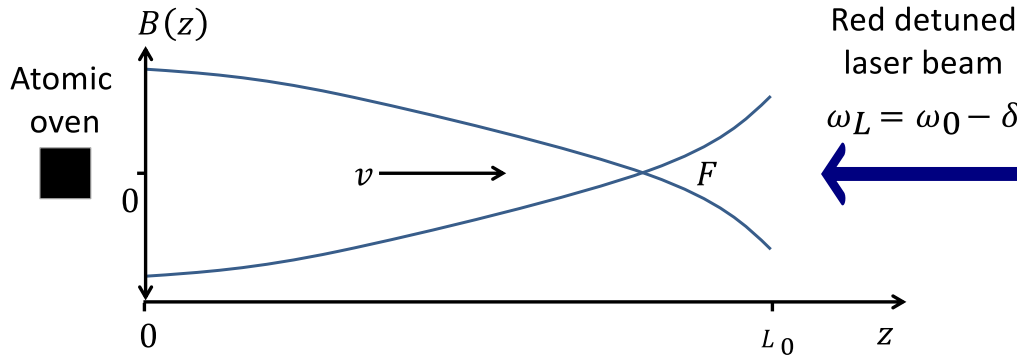


Figure 2.7: Zeeman slower: Basic principle of Zeeman slowing atoms with a single detuned laser beam opposing atomic motion in a shaped magnetic field.

The maximal slowing and cooling of the scattering force F_{max} is achieved when $\delta_L = \Gamma/2$. The optimal deceleration can be expressed as:

$$a_{max} = \frac{F_{max}}{M} = \frac{\hbar k_L \Gamma}{M} \frac{1}{2} \quad (2.19)$$

To compensate for the velocity of atoms emerging from the atomic oven, the design of the spatially varying magnetic field $B(z)$ must consider the Zeeman shift on atomic energy levels, which is given by $\mu B(z)/\hbar$, to counterbalance the changing Doppler shift resulting from the slowing of atoms, $k_L v(z)$. Mathematically, this compensation

requires that the Zeeman shift satisfies the equation:

$$\frac{\mu B(z)}{\hbar} = k_L v(z) + \delta_L \quad (2.20)$$

If we assume a constant deceleration a from an initial velocity v_i at position $z = 0$, we can calculate the average velocity of the atoms using standard kinematic equations as follows:

$$v(z)^2 = v_i^2 - 2az \rightarrow v(z) = v_i \sqrt{1 - \frac{z}{L_0}} \quad (2.21)$$

where L_0 can be defined as the stopping distance and is given by:

$$L_0 = \frac{v_i^2}{2a} \quad (2.22)$$

By calculating the necessary stopping distance, it becomes possible to design a more compact Zeeman slower. Substituting 2.21 into 2.22 provides the description of the required magnetic field for the Zeeman slower.

2.5 Narrow-Line Laser Cooling

If the initial cooling is insufficient in reducing the temperature of the MOT, utilizing a narrower transition for the second stage cooling would be advantageous. This is because the lowest achievable MOT temperature depends on the transition linewidth. The first stage cooling is necessary since the capture velocity of a MOT also depends on the transition linewidth. To trap a significant number of atoms, first stage cooling with a broad linewidth is required. To understand narrow-line laser cooling, it is essential to distinguish between broad and narrow Doppler cooling lines. Broad lines, which have historically been the choice for the majority of laser cooling experiments [118], are characterized by $\Gamma/\omega_r \gg 1$. For example, the 461 nm $^1S_0 \rightarrow ^1P_1$ transition, is an example of a broad line, with Γ/ω_r approximately 3×10^3 . Here, the natural energy scale is characterized by Γ , or more broadly, the power-broadened linewidth Γ_E . This parameter can be expressed as $\Gamma_E = \Gamma\sqrt{1+s}$, where s represents the saturation parameter, a ratio of the single-beam peak intensity (I) to the transition saturation intensity (I_s). The cooling process is primarily governed by semiclassical physics. Additionally, the

influence of gravity is minimal, as indicated by the ratio of the maximum radiative force to gravitational force, $R = \frac{\hbar k \Gamma}{2mg}$, which is generally around 10^5 , with g representing gravitational acceleration.

On the other hand, narrow Doppler cooling lines are defined by $\Gamma/\omega_r \sim 1$. In the context of our work, for instance, the $689 \text{ nm } ^1S_0 \rightarrow ^3P_1$ transition has a Γ/ω_r value of 1.6, where $\Gamma/2\pi$ is 7.5 kHz. In this scenario, the relevant thermal-mechanical energy scale and the underlying nature of cooling, whether semiclassical or quantum mechanical, depend on the saturation parameter (s). The specific cooling process is then determined by the laser detuning (Δ). Particularly, when $\Delta < 0$, corresponding to $^1S_0 \rightarrow ^3P_1$ MOT cooling and trapping dynamics, there are three distinct regimes (I - III), categorized by the relative sizes of $|\Delta|$, $\Gamma \sim \omega_r$, and Γ_E .

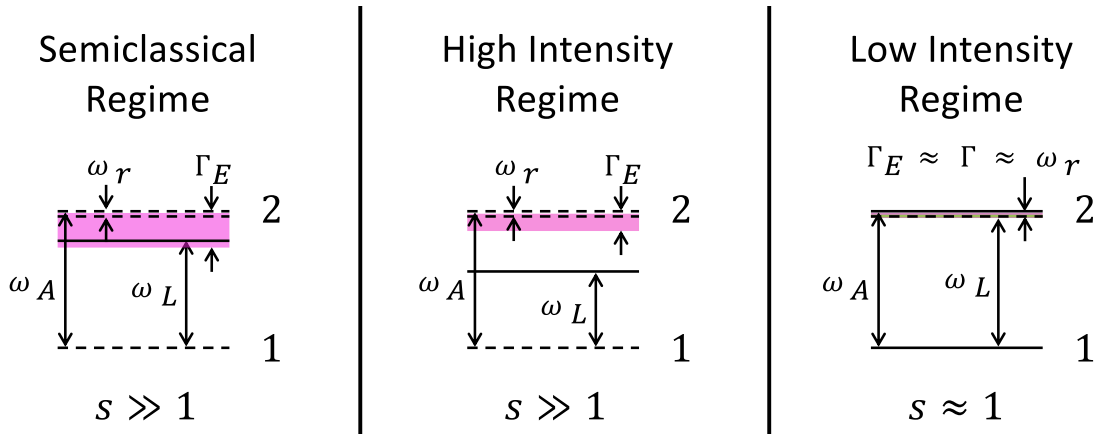


Figure 2.8: Narrow-line cooling: In each regime, the narrow-line cooling and trapping has distinct parameters. In the semiclassical regime, the transition's effective linewidth significantly exceeds the recoil frequency and matches with the MOT laser detuning. Moving to the high intensity regime, the transition's effective linewidth surpasses the recoil frequency but remains smaller than the laser detuning. In contrast, the low intensity regime an effective linewidth equivalent to both the recoil frequency and the laser detuning.

Semiclassical Regime:

In the semiclassical regime, trap parameters are characterized by $|\Delta| \leq \Gamma_E \gg \omega_r$ and $s \gg 1$. In this context, detuning is smaller than the effective linewidth of the transition, which is significantly larger than the single photon recoil frequency, and the trapping beam intensity greatly exceeds the saturation intensity.

When the intensity of the trapping beams is increased to the point where $s \gg 1$ the system evolves to a state where $\Gamma_E \gg \omega_r$, and the natural energy scale depends on Γ_E and Δ . This scenario can be described semiclassically, as shown in Fig. 2.8, where the radiative force ($\Delta < \Gamma_E$) controls trapping dynamics, inducing damped harmonic motion in the atoms. Here, trap thermodynamics are solely determined by the velocity dependence of the radiative force, much like classical Doppler cooling. The minimum attainable temperature in this regime is dictated by the Doppler cooling limit when $\Delta = \Gamma_E/2$.

High Intensity Regime:

In the high-intensity regime, trap parameters exhibit the following characteristics: $|\Delta| > \Gamma_E \gg \omega_r$ and $s \gg 1$. This configuration implies that the laser beam detuning exceeds the effective linewidth of the transition, with the effective linewidth being significantly larger than the single photon recoil frequency [119]. To achieve this, the intensity of the narrowline MOT trapping beams must far exceed the transition's saturation intensity, leading to power broadening in the spectral line.

In this high-intensity regime, the semiclassical approach remains applicable, but with a notable difference: the force now primarily affects a small, well-defined spatial shell along the outer trap boundary. The position where the magnetic field gradient compensates for the laser detuning is referred to as z_{eq} and is not necessarily located at the trap's central point along the vertical z-axis. However, in this regime, the influence of gravity cannot be generally ignored. For instance, in the case of the strontium second-stage MOT, the trapping force is only approximately 16 times greater than the gravitational force. As a consequence, the shape of the trapped atom cloud becomes distorted, and the atoms migrate to positions where the Zeeman shift offsets the laser beam detuning, δ_L , and the resulting radiative force counteracts the effect of gravity.

This phenomenon arises from a box potential created by the unique trapping parameters. It's further influenced by a gravitational-induced tilt along the z-axis. Within the x-y plane, the atoms undergo scattering within the confines of the box potential. In contrast, along the z-axis, the atoms descend to the lower boundary of the trap, interacting exclusively with the vertical beam components. The equilibrium position, denoted as

z_{eq} , is determined by the condition that the force generated by the upward components of the trapping beams in the z -axis, denoted as F^+ , equals the force of gravity.

$$F^+(v_z = 0, z = z_{eq}) - mg = 0 (F^+ \geq mg) \quad (2.23)$$

where F^+ can be found by isolating the scattering force components acting vertically against gravity,

$$F^+ = \frac{\hbar k_L \Gamma}{2} \left[\frac{s}{1 + s' + 4(-\delta_L(1 + z/z_{eq}) - k_L v_z)^2 / \Gamma^2} \right] \quad (2.24)$$

The force component caused by laser detuning δ_L is substituted with $\delta_L(1 + z/z_{eq}) - k_L v_z$ to account for the vertical force opposing gravity. Here, $k_L v_z$ represents the frequency shift due to the vertical atomic velocity v_z , and $(\delta_L \cdot z_{eq} - \delta_L \cdot z)/z_{eq}$ acts as a scaling factor on the frequency detuning, which depends on the distance z from the equilibrium atomic position z_{eq} . This introduces spatial variation into the force.

The thermodynamics of the MOT are influenced by both atomic velocity and the spatial distribution of the radiative force. In this scenario, the primary source of heating is single-beam photon scattering. The thermodynamics of the MOT can be assessed by examining the damping coefficient α at the equilibrium position $z = z_{eq}$.

$$\alpha = - \left(\frac{\partial F^+}{\partial v_z} \right)_{v_z=0, z=z_{eq}} \quad (2.25)$$

with the substitutions $v_r = \hbar k_L / m$ and $E_r = \frac{1}{2} m v_r^2$;

$$-\alpha \bar{v}_z^2 + \frac{\Gamma_{scat} (\hbar k_L)^2}{2m} = 0 \quad (2.26)$$

The first term represents the effects of optical molasses, while the second term accounts for the heating coefficient arising from the scattering of photons from a single laser beam component. The scattering rate undergoes modification under these conditions, and it can be calculated as:

$$\Gamma_{scat} \cong \frac{F^+(v_z = 0, z = z_{eq})}{\hbar k_L} = \frac{mg}{\hbar k_L} \quad (2.27)$$

Under these conditions, the equilibrium temperature can be approximated as a func-

tion of the saturation parameter by:

$$T(s) \cong \frac{\hbar\Gamma}{2k_B} \sqrt{1+s} \quad (2.28)$$

This indicates that the ultimate temperature of the second-stage MOT is determined by the saturation parameter (s) and not influenced by the frequency detuning (δ_L), as long as the condition of $F^+ \geq mg$ is satisfied.

Low Intensity Regime:

In the low-intensity trapping regime, where parameters like $\Gamma_E \cong \Gamma \approx \omega_r$ and $s \approx 1$ apply, the intensity of trapping beams closely matches the saturation intensity. This condition results in an effective linewidth similar to the natural linewidth of the transition, with no power broadening. For Sr, this natural linewidth aligns with the single photon recoil limit of approximately 460 nK. The trap dynamics are dominated by single photon recoils, necessitating a fully quantum mechanical description [120].

2.6 Diagnostic of Cold Atoms

To characterize the MOT, knowing the number of trapped atoms and their temperature is essential. The number of atoms in the MOT is determined by measuring the power of fluorescence emitted by the trapped atoms using a calibrated photodiode/photo-multiplier tube. The relation between optical power and the number of atoms is given by the equation:

$$P = \frac{hc}{\lambda} \frac{\Omega}{4\pi} N\gamma_{sc} \quad (2.29)$$

In the above equation, Ω is the solid angle subtended by the collection lens on an atomic cloud and γ_{sc} is the photon scattering rate, given by:

$$\gamma_{sc} = \frac{\Gamma}{2} \frac{I/I_{sat}}{1 + I/I_{sat} + (2\Delta/\Gamma)^2} \quad (2.30)$$

To determine the number of atoms, the initial step is to calibrate the photodiode, as the above equation is only valid for a calibrated photodiode. Calibration involves directing a known amount of light onto the photodiode and recording the resulting generated

voltage.

The number of trapped atoms can also be determined using the technique of absorption imaging.

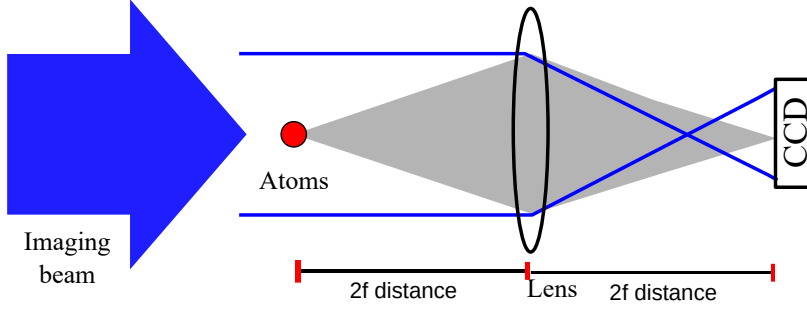


Figure 2.9: Atoms number measurement: Schematic to collect the fluorescence emitted by a cloud of atoms in MOT. A lens of focal length ‘f’ is used to image the atomic cloud onto the CCD camera.

When obtaining high SNR images of cold atom clouds, absorption imaging serves as a reliable method. It involves the destructive imaging of atoms since the light used to observe the atomic cloud resonates with the relevant transition. During the acquisition of an absorption image, the cloud receives illumination from a pulse of on-resonance laser beam with intensity I_0 . According to Beer-Lambert’s law, the transmitted light due to absorption by the atomic cloud is determined.

$$I = I_0 e^{-n(x,y)\sigma} \quad (2.31)$$

The optical density or OD is denoted by the exponent $n(x,y)\sigma$ in Beer-Lambert’s law, where $n(x,y)$ represents the integrated column density in the direction of the beam, and σ is the absorption cross-section. For on-resonance illumination with intensity near saturation, the resonant absorption cross-section σ_0 is given by $\sigma_0 = \hbar\omega\Gamma/4I_s$ (where ω is the laser angular frequency, Γ is the decay rate of the transition, and I_s is the saturation intensity).

To mitigate the impact of stray light and dark counts in the camera, a reference image (I_{ref}) is employed. This reference image is acquired without the imaging illumination.

Consequently, the measured optical density is determined by the formula:

$$OD_m = \ln \frac{I_l - I_{ref}}{I_a - I_{ref}} \quad (2.32)$$

Where I_a and I_l are light intensities measured in and without the presence of atoms.

In our setup, a single lens with a focal length of 75 mm is positioned equidistantly from the camera and the atoms, ensuring it is $2f$ away from both. This lens is utilized to image the atomic cloud. The imaging beam, with a diameter of 15 mm and a power ranging from 3-4 mW, passes through this optical arrangement.

The time-of-flight (TOF) technique is a commonly used method for measuring the temperature of a thermal cloud of atoms. This approach entails the release of trapped atoms followed by the measurement of their expansion rate through imaging the atom cloud after a specified delay time. By interpreting the resulting velocity distribution, the temperature can be determined. It's important to note that this method operates under the assumption that upon release, the atoms do not interact with each other, and that all the pre-release motional energy of the atoms transforms into translational kinetic energy post-release.

Chapter 3

Strontium experiment: Design and Setup

This chapter provides detailed information on the experimental setup of the strontium apparatus built at the IISER Pune APQO Lab. It is organized into two sections. The first section focuses on the construction of the vacuum chamber for the strontium apparatus, while the second section elaborates on the electromagnets and control electronics utilized in the strontium experiment.

During the initial development phase of the apparatus, a deliberate decision was made to engineer a versatile system applicable to a variety of experiments. These experiments include those involving frequency metrology utilizing the clock transition, precision measurements, and experiments related to quantum chaos employing the atom optics kicked rotor platform, among others. The current system configuration has adaptability for potential modifications aimed at conducting experimental demonstrations of quantum information processing. This involves the interaction of ultracold strontium atoms with plasmonic nanostructures.

3.1 Vacuum Chamber

The development of the vacuum system was a key aspect of this thesis, with the objective of supporting various experiments involving ultracold strontium. Constructing a vacuum system marks the initial step in creating a quantum gas. Our approach focused

on building a stable and robust system. Due to diverse experimental goals, machine requirements can vary significantly. Modifying an existing vacuum system is often complex. Thus, our priority was to build a versatile vacuum system adaptable to various experiments. The vacuum system in our new apparatus is comprised of three sections: source, ultra-high vacuum MOT chamber, and single atom imaging chamber. These sections are interconnected through gate valves, allowing for the isolation of specific segments from the rest of the vacuum system. This isolation capability proves crucial for tasks such as strontium refilling, leak identification and repair, or system adaptation for new research goals, all without disrupting the entire vacuum system. Each section is equipped with an angle valve, enabling independent pumping through connection to a pumping station. The subsequent subsections provide detailed insights into the design and assembly of the vacuum system. The schematic diagram of the vacuum chamber is shown in Fig. 3.1. The crucial components of this setup are as follows:

1. **The Strontium Oven:** produces collimated Sr atomic beam.
2. **The transverse cooling chamber:** Transverse compression of the atomic beam after differential pumping tube to further reduce the divergence of atomic beam.
3. **The Zeeman slower:** slows the hot collimated atomic beam coming from oven.
4. **The MOT chamber:** UHV chamber where the MOT formation takes place with the help of six laser beams and anti Hemholtz magnetic field.
5. **The magnetic shim coils:** help in nullifying the stray magnetic field, important specially for the production of RED MOT.
6. **The ion pumps:** to maintain the vacuum level of the system after disconnecting the system from the initial turbo molecular pump based pumping system.
7. **The titanium sublimation pump:** sublimated titanium coats on the walls of vacuum chamber and trap the background gas, effectively reducing the background vacuum level.

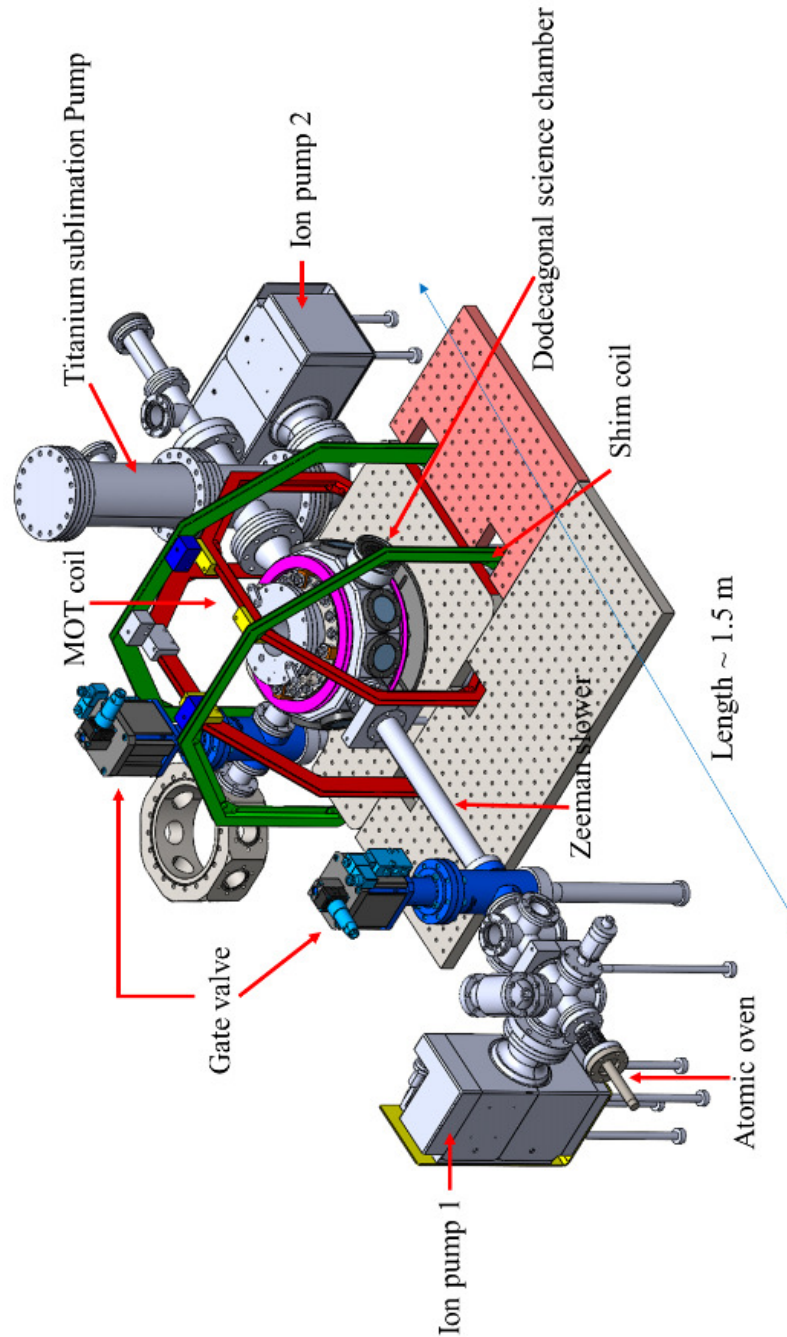


Figure 3.1: Isometric 3-D CAD drawing of the Sr vacuum system: image depicts various components of a strontium (Sr) system, including an oven, MOT chamber, ion pumps, and a titanium sublimation pump (TSP) used for maintaining vacuum, as well as a gate valve for isolating selected regions of the system. The system is maintained at a vacuum level of 10^{-10} torr in the MOT region and 10^{-9} torr in the oven region.

3.2 Attaining the UHV Regime

Attaining an ultra-high vacuum (UHV) represents a crucial milestone in the construction of the vacuum system. This challenging process has multiple sequential stages of cleaning, pumping, and baking. In the following section, we will elaborate on the process involved in achieving UHV in the newly built strontium system.

3.2.1 Vacuum Pumps

To achieve the UHV regime, the vacuum system undergoes evacuation from atmospheric pressure to levels below 10^{-10} torr. This process involves the use of a pumping station, comprising both a scroll pump (SP) as a backing pump and a turbo molecular pump (TMP). Typically, the SP reduces the pressure from atmospheric levels to a hundredths of a torr (5×10^{-2} torr), after which the TMP takes over to achieve even lower pressures. For effective pumping, the SP supports the TMP as a backing pump throughout the process. The SP's inlet connects to the TMP's outlet, and the TMP's intake is linked to the vacuum system. TMP consists of alternating blades, featuring rapidly rotating blades and mirror-symmetrical stationary blades. When gas atoms or molecules approach the inlet, collisions with the blades result in momentum transfer during each encounter. This repeated kicking action imparts directed movement to the particles, guiding them toward the outlet. Finally, the SP removes the particles from the vacuum system. For the final stage of the pumping process, maintaining a pressure below 10^{-10} torr permanently, getter pumps such as titanium sublimation pumps (TSP) and ion pumps are commonly employed. These pumps work by binding gas molecules to surfaces or in the bulk of a getter, operating effectively in a broad pressure range. Lower initial pressure is preferred to extend pump lifetime and expedite the attainment of the UHV regime. After reaching UHV, getter pumps can sustain the vacuum in a closed system. This allows the disconnection of pump stations from the vacuum system after closing the angle valve. For UHV vacuum, a titanium sublimation pump (TSP) is a simple yet effective getter pump. Operating by sublimating titanium onto adjacent interior walls, it forms a surface getter that binds gas molecules and traps particles. To optimize pumping speed, the titanium layer is deposited over a large area, achieved by connecting TiSub pumps to long, wide pipes providing high conductance. These ex-

tended pipes also prevent sublimated titanium from reaching viewports and ion pumps. The sublimation process involves direct resistive heating of a titanium filament with high current (~ 50 A) for about 2 minutes. To maintain the UHV regime, TSPs are fired periodically to coat the saturated titanium layer with a fresh one.

In a typical UHV setup, an ion pump, is used as a reliable getter pump. This pump functions by ionizing gas particles, causing sputtering of a getter surface. The pumping speed is influenced by gas particle properties, electrode materials, and pump geometry. The ion pump consists of cathode and anode electrodes, as well as external magnets. The anode is typically made of stainless steel tubes, and the cathode comprising two titanium plates, an electric field with a voltage of 7 kV (between the cathode and anode) and a magnetic field of 1000-2000 G. Collisions between electrons, emitted from the cathode, and gas particles lead to ionization, creating positive ions. These ions are accelerated towards the cathode, where they stick and create a getter layer. This process is similar to the TiSub mechanism. The ion pump can also function as a vacuum gauge at low pressures below 10^{-5} torr. In our vacuum system, two ion pumps are placed to ensure efficient pumping speed without limiting conductance.

3.2.2 Bakeout of the System

After completing the assembly process, it's crucial to verify the leak tightness and vacuum quality of the newly constructed system to achieve extremely low pressure. The vacuum system is designed to be as simple as possible to minimize the risk of vacuum element malfunctions and outgassing. Each vacuum flange poses a risk of failure, and every component inside the vacuum system carries a risk of outgassing or virtual leaks. Separating the system into different sections using gate valves facilitates debugging and repairs if necessary. To assess the reliability of the vacuum system, we conducted thorough leak checks immediately after assembly using a residual gas analyzer (RGA) attached to the pumping station to detect leaks.

To effectively deplete impurities from the bulk, we bake the system keeping in mind the temperature limitations. Limitation is imposed by viewports, restricting the maximum bake temperature to below 250° C in our vacuum system. In the initial step, we elevate the maximum bake temperature to 100° C. This step aimed to efficiently deplete

impurities from the bulk. Subsequently, in the second step, we elevate the maximum bake temperature to 220° C. To ensure proper control and uniform temperature increase during the vacuum system bake, it is crucial to avoid gradients in the local region that could lead to mechanical stress and potential leaks. Approximately 20 thermocouples were strategically placed on the machine, providing temperature measurements for precise control. The temperature change was carefully managed, with a maximum rate below 7°/hour, ensuring even distribution across the machine. Heating tapes were wrapped around the entire machine and the pump station. Following the placement of heating elements and thermocouples, the machine was enveloped in several layers of aluminum foil, serving as thermal insulation. This insulation promotes uniform heating, allowing for high temperatures without significant impact on the laboratory environment.

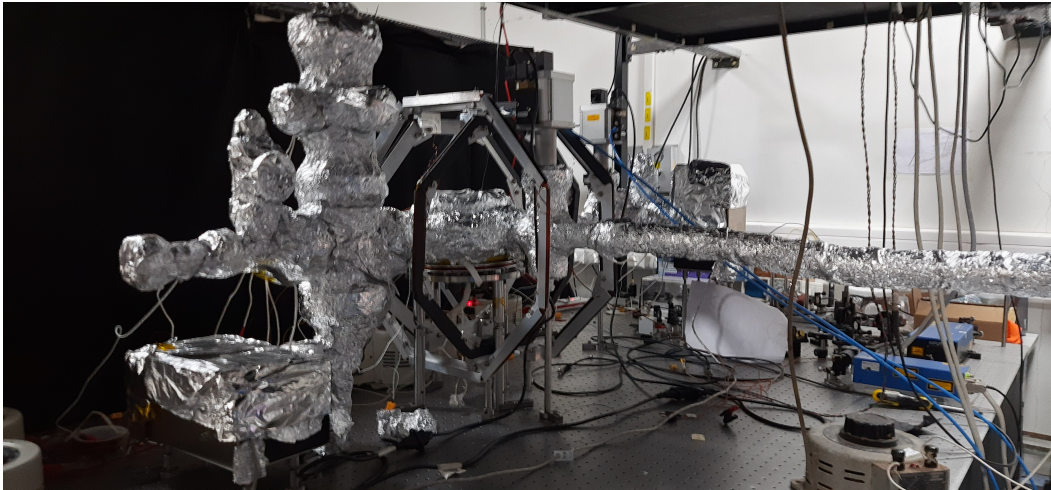


Figure 3.2: Baking of Sr system: For baking, the entire system is wrapped with heating tape, and aluminum foil is used to cover the entire assembly. The aluminum foil ensures homogeneous heating of the system and isolates it from the environment.

On the 8th day, we degassed both the TSPs and ion pumps, observing a distinctive pattern: an initial pressure dip during the absorption layer’s gas-pumping phase and a subsequent pressure increase. By the end of baking, the vacuum system reached a pressure of approximately 10^{-10} torr. Throughout these bakeouts, the pumping station was active, while internal ion pumps were turned off to prevent saturation, which could release adsorbed gases later. After completing the bakeout, we disconnected the pumping station, and the vacuum system has been maintained by ion pumps and TSPs

since then. Lacking dedicated vacuum gauges, pressure monitoring relies on ion pump currents, correlating with pressure levels. While this method gives an upper bound of pressure, accurate values require a suitable vacuum gauge. Ion pumps in the UHV section approach a detection limit of about 10^{-10} torr in pressure measurements. During experiments, when the oven heats to 550°C , ion pumps in the oven section show values of about 10^{-9} torr. Despite the higher pressure in the oven section, the main chamber's vacuum quality remains undisturbed due to the low conductance of the differential pumping tubes and the high pumping speed in the UHV section.

3.3 Vacuum System Construction

In the following sections, we will review different parts of the vacuum system, starting from the oven, followed by the differential pumping tube, Zeeman slower, and the main chamber or science chamber. In this section, we will go through the overall structure of the vacuum assembly.

The experimental system is constructed on a $1.5\text{ m} \times 3\text{ m}$ optical table, incorporating both the vacuum chamber and the laser systems that are essential for cooling and trapping. The design of the vacuum chamber is conceptualized and validated using 3D CAD software. The assembly is created using standard vacuum components sourced from MDC Vacuum, Kurt J. Lesker, UHV Design, and Agilent Technologies. Additionally, certain components for the assembly are custom made by SMP Enterprise and Fourvac Technologies, Pune.

The schematic diagram of the vacuum chamber is shown in Fig. 3.1. As shown in the figure, we have used two ion pumps to maintain the vacuum in the vacuum chamber; both ion pumps have a capacity of 55 l s^{-1} (Agilent, Model: VacIon Plus 55 Starcell). We have also used a Titanium sublimation pump (TSP), periodically fired to maintain vacuum pressure. The TSP is installed with three cartridges to avoid the constant use of only one cartridge. Ion pump 2 and the TSP are connected with a DN150 fitting to allow better conduction.

In the vacuum system, we have employed two all-metal angle valves and two gate valves. The angle valve on the oven-side region is used to connect the oven region to the Turbomolecular pump. And gate valve-1 can be closed to maintain the vacuum inside

the science chamber intact. This configuration is particularly helpful in the event of clogging of the oven or the refilling of strontium in the oven. The second gate valve is used to connect the main chamber to the single atom imaging chamber, which we have not installed yet. The second angle valve is at the end of the vacuum chamber.

At the Zeeman slower end, an optical viewport is attached for the transmission of the ZS laser beam. To prevent the formation of a permanent strontium coating that could obstruct optical access due to Sr's high reactivity with glass, a sapphire viewport has been used facing the direct atomic beam. Sapphire, being a crystalline material, exhibits varied optical properties in different directions. To preserve the polarization property of the Zeeman slower laser beam, it is essential to use a crystal with a specific cut. In our case, the crystal is a "z-cut," where the crystal orientation is parallel to the optical axis. We also keep this window at the elevated temperature of 150° C to avoid the coating. To prevent the cracking and opening of leaks in vacuum parts containing glass elements during heating, we vary the temperature slowly. The sapphire viewport, in particular, has a recommended maximum heating rate of 3° C per minute. In our experiment, the zeeman-slower viewport is consistently maintained at a high temperature to minimize the risk of creating leaks during temperature fluctuations, even though the atomic beam is typically shuttered off at night using the atomic beam shutter. In the event of a power supply failure, there is a potential for a rapid temperature decrease. To counter this, the viewport has been insulated from the external environment using an aluminum plate with an optical flat in the center, and aluminum foils have been wrapped around the region and heating band, preventing abrupt cooling.

3.4 Oven

Strontium is a soft metal with low vapor pressure at room temperature. To produce high intensity atomic beam, it requires an increased fraction of atoms in the gaseous phase. This is accomplished by heating up strontium metal, sublimating atoms [121]. The oven's role is to heat strontium metal, causing atom sublimation and directing the randomly moving atoms into a high-intensity atomic beam with a low angular distribution. It consists of a reservoir, nozzle, heating assembly, and insulation assembly, playing a crucial part in this process. The oven system requirements are as follows:

- 1) Ability to operate at temperatures up to 600° C, with significant temperature gradients between the nozzle and the crucible to prevent potential nozzle clogging.
- 2) Easy reloading of strontium metal and seamless integration with our current vacuum system.
- 3) Minimal heating of the surrounding vacuum chamber walls and low power consumption.

3.4.1 General Features

Our oven is designed with heating elements outside of vacuum, and can be used in both vertical and horizontal orientations. It adopts a bottom-up construction, where components are stacked sequentially. This construction method simplifies the reloading process and facilitates repairs, as most important components are easily accessible, and the heating component is situated outside the vacuum [121, 122]. Additionally, the entire oven is affixed to a CF35 flange, providing direct attachment to our current vacuum system.

Before providing in-depth explanations of the design and assembly of the distinct components of oven, we must first consider two fundamental aspects: the dimensions of the oven and the materials utilized in its construction. To attach the oven with our existing vacuum system, the entire oven ideally needed to fit with CF35 vacuum piping. This sets the upper limit for the outer diameter of our oven flange. To ensure an extended oven lifespan, the crucible had to be sufficiently large to accommodate 25 gm of distilled dendritic strontium. This specification determined the outer diameter and a length of our reservoir. Our material choice for fabrication of the oven was stainless steel; specifically, we employed SS316 type stainless steel because of its robust mechanical strength at elevated temperatures. Additionally, the gaskets for vacuum sealing are gold-coated to prevent the corrosion of copper caused by the hot vapor of strontium.

3.4.2 Design of an Oven

The reservoir holds strontium metal chunks, comprising an L-shaped steel tube with bending and a flange fixed to one end. One end remains sealed, housing a thermocouple for monitoring the reservoir temperature, while the opposite end allows strontium metal

to be inserted into the oven reservoir and is closed during operation [123, 124].

A nozzle serves the purpose of reducing the solid angle of the atomic flux coming from the reservoir, thereby enhancing the proportion of atoms reaching the UHV section effectively. Constructed from a DN40 CF flange, the nozzle features microdrills at its center. When a Sr atom from the reservoir enters the nozzle with a velocity aligned with a microdrill's axis, it can traverse the microdrill without collisions with the microdrill's wall. In cases where an atom collides with the wall, it still gets an opportunity to contribute to the collimated atomic beam. This can occur either directly after detaching from the wall and moving in the correct direction or after undergoing multiple scattering events with the walls. The microdrilled holes are organized in a hexagonal lattice pattern in the center of DN40CF double sided flange. Each microdrill has an outer diameter of $500\text{ }\mu\text{m}$, and a length of 20 mm.

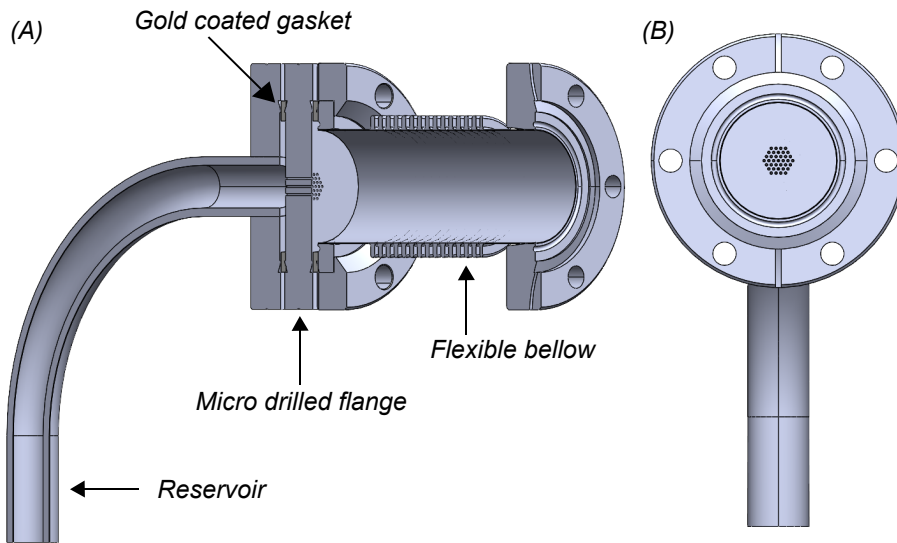


Figure 3.3: Strontium oven: The figure shows a Strontium oven, featuring a microdrilled flange that collimates the atomic beam, while the reservoir part contains the Strontium metal pieces. A flexible bellow is used to facilitate the movement of the atomic beam, ensuring the maximum number of atoms can reach the MOT region.

At 550°C , the nozzle is maintained at a temperature sufficiently higher than the reservoir to prevent microcapillaries from clogging [125]. Gold coated copper gaskets are utilized for the CF connections of the atomic oven cup, elbow, and nozzle, deviating from the conventional copper gaskets due to the rapid corrosion of copper by strontium

at higher temperature. Refer to Fig. 3.3 for a schematic showing the nozzle's position relative to the oven reservoir and an image illustrating the microdrill packing. Since 304 stainless steel lacks the required hardness at constant higher temperature, all steel components were machined from 316 stainless steel.

3.4.3 Characterization of the Oven

To characterize the performance of the oven orthogonal cross beam spectroscopy is performed. A critical criterion for the characterization of the atomic oven is the measure of flux generated by such a source. By changing the frequency of the probe beam around the atomic resonance and by detecting the atomic fluorescence, it is possible to measure the flux and divergence of the atomic beam. A probing beam at 461 nm with optical power $P_{\text{probe}} = 1 \text{ mW}$ and diameter $d = 5 \text{ mm}$ ($1/e^2$) perpendicularly crosses the atomic beam.

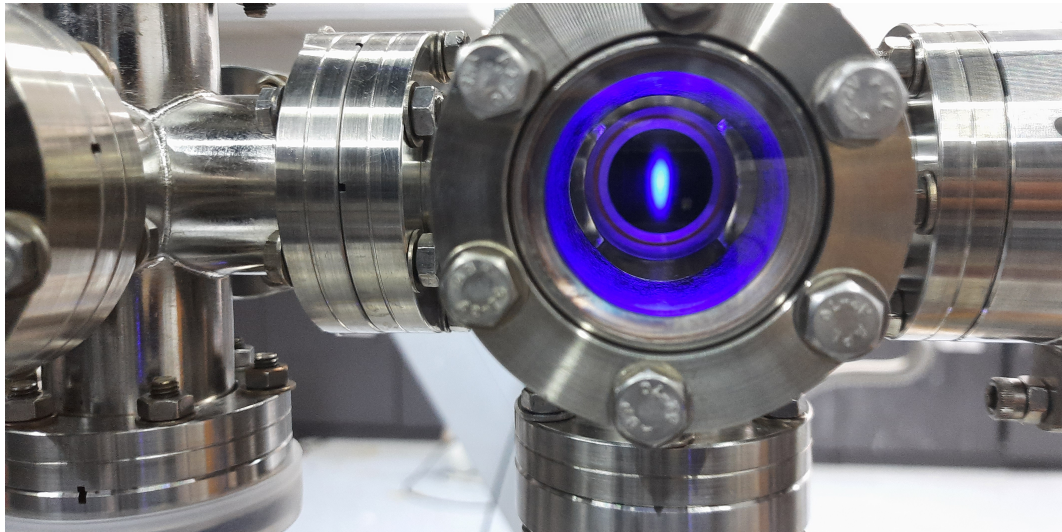


Figure 3.4: Strontium atomic beam: The Strontium atomic beam originates from the oven, with a microdrilled flange collimating the atomic beam. The atomic beam travels horizontally, while a laser at resonance wavelength 461 nm intersects the atomic beam perpendicularly to mitigate the Doppler effect. The bright blue oval shape is the intersection of the atomic beam with the laser beam, resulting in fluorescence. This fluorescence measurement is used for calculation of the number of atoms and the divergence of the atomic beam.

The measured flux at the oven at the operating temperature of 870 K is found to be approximately 6×10^9 atoms/sec. The performance of the oven can also be characterize

by the loading rate of MOT. However, since the MOT loading rate itself depends on several other factors such as parameters of Zeeman slower, presence of transverse cooling, alignment of MOT laser beams, etc. So the measurements performed may lead to an unreliable estimate of the atomic flux. More details on the characterization of oven and simulation can be found in the the thesis of Mr. Korak Biswas from our group.

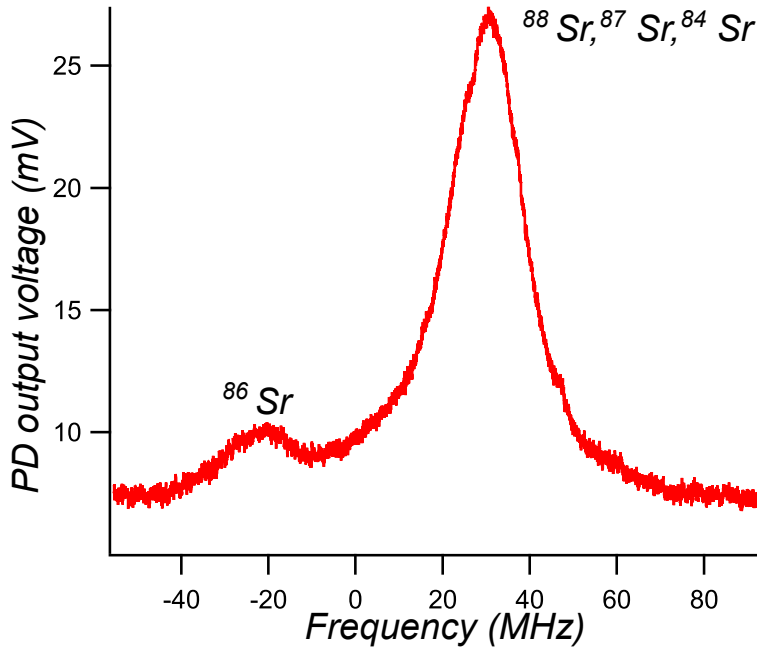


Figure 3.5: Atomic beam fluorescence: The fluorescence from the strontium atomic beam is collected on a photodiode while scanning the laser frequency. This plot illustrates the distribution of the atomic beam with frequency, enabling us to calculate the divergence of the atomic beam.

3.4.4 Differential Pumping

Based on an early MIT design, the two-stage pumping configuration is employed for accommodating pressure differentials between our science chamber, where we aim to maintain pressures around 10^{-11} Torr, and our atomic beam section, where pressures may extend into the 10^{-9} Torr range. After emerging from the nozzle, the atomic beam travels through two stages of differential pumping. These stages represent two regions of low conductance positioned between the science chamber and oven region.

Consider a region of gas, R_1 , at high pressure P_1 , connected to a region, R_2 , at some

low pressure P_2 , via conductance C_{12} . In both regimes, characterized by molecular flow where the mean free path of particles exceeds any characteristic chamber dimension, R_2 is pumped at speed S_2 . To examine this quantitatively, let's focus on the dominant gas load in R_2 , which is $p_1 C$. Therefore, the total pressure P_2 is simply $P_1 C / S_2$. It's important to note the definition of pumping speed, considering an ion pump with a speed S . If connected in series with the main chamber through a part with conductance C (like an elbow or nipple), the effective pumping speed is given by $S_{eff}^{-1} = S_{ion}^{-1} + C_{ion}^{-1}$. Subsequently, using the Knudsen formula in the molecular flow regime, we can calculate the pressure ratios between R_1 and R_2 .

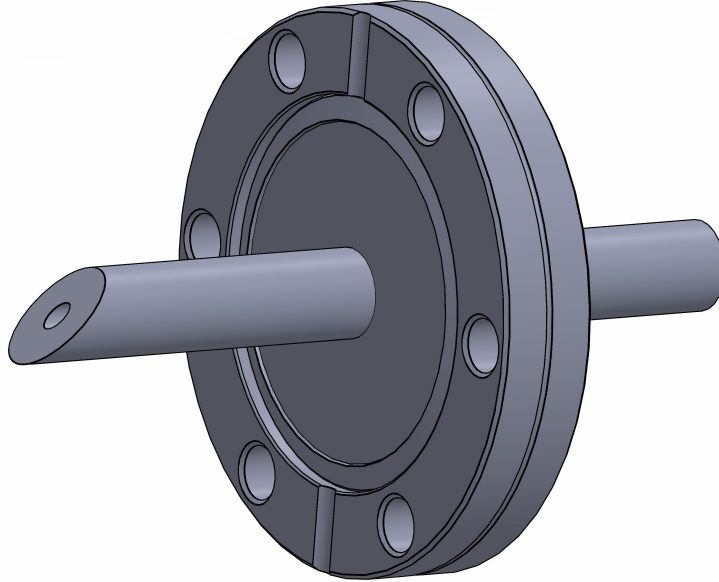


Figure 3.6: Differential Pumping Tube (DPT): A DPT is utilized to maintain the pressure difference between the oven region and the MOT region. Due to its lower conductance, only collimated atoms can pass through, while all randomly oriented atoms are constrained within the oven region.

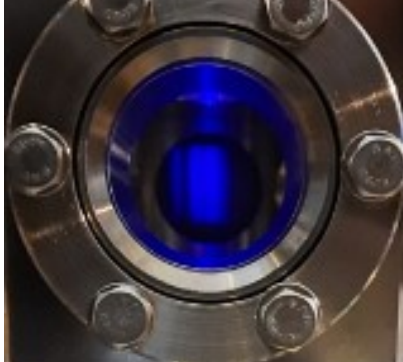
In our setup, to maintain a pressure difference of ~ 3 orders of magnitude between two regions, we choose a differential pumping tube with an internal diameter (ID) of 6.4 mm and a length of 113.6 mm, positioned between the oven region and the transverse cooling region. Subsequently, we use a 500 mm length and 10 mm internal diameter (ID) tube between the transverse cooling and science chamber regions. To maintain the pressures within these regions, two independent ion pumps, each with a capacity

of 55 ls^{-1} (Agilent, Model: VacIon Plus 55 Starcell), are employed. Additionally, a titanium sublimation pump (TSP) is connected near the science chamber to provide supplementary pumping and maintain the desired low pressure. However, strontium acts as a getter and also reduces pressure over time, so we rarely use the TSP. Under normal operating conditions, this configuration allows us to achieve a vacuum with a pressure 10^{-10} torr in the science chamber.

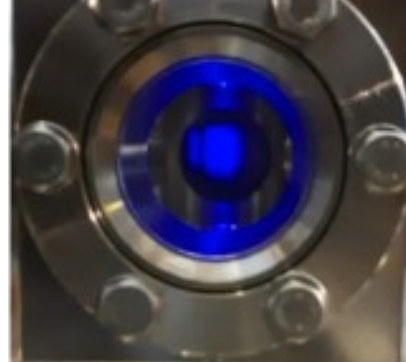
The atomic beam shutter is positioned between the two regions of the vacuum chamber. It is a rotary shutter (*UHV Design*, Model: MD20RAIX000Z) utilized to mechanically open and close the atomic beam path to the main chamber, remaining typically open during MOT loading and closed at other times.

3.5 Transverse Cooling

The atomic beam enters the transverse cooling region through the initial differential pumping tube. We observe a substantial impact of transverse cooling in all four transverse directions. Any misalignment or imbalance in power among the beams not only fails to assist but also decreases atomic flux. In our experiment, the transverse light



(a) Atomic fluorescence without transverse cooling



(b) Atomic fluorescence with transverse cooling

Figure 3.7: Transverse cooling: In the figure, the MOT region is depicted. When we apply the transverse cooling (TC), the divergence of the atomic beam decreases, and it becomes more compressed compared to the situation when the TC is not applied. This figure illustrates the increase in the number of atoms in the MOT region by applying the TC.

is detuned by approximately $-\Gamma/2$ and coupled from the laser table through fibers. Both transverse cooling beams exhibit elliptical spatial modes created by cylindrical

lens pairs. Empirically setting the power balance between these beams optimizes the fluorescence of the repumped MOT. The vertical beam, roughly a $0.5 \text{ cm} \times 3 \text{ cm}$ oval, carries 8 mW of power, retroreflected with a saturation parameter (s) around 0.2. Similarly, the horizontal beam, approximately a $0.5 \text{ cm} \times 3 \text{ cm}$ oval, has 10 mW of power, also retroreflected with the value of s around 0.4. The combined effect of these transverse cooling beams results in a repumped MOT fluorescence increase of about 100%. More details on the transverse cooling can be found in the thesis of Mr. Korak Biswas from our group.

3.6 Zeeman Slower

Upon exiting the oven manifold, the atomic beam has a temperature of approximately $550\text{-}600^\circ \text{ C}$, and most probable velocity (550 m/s). Our goal is to have these atoms moving at a reasonable MOT capture velocity (v_c) of around 50 m/s . Assuming that the atoms exhibit an ideal gas behavior, we can use a Maxwell-Boltzmann distribution to estimate the fraction of the atomic flux that will move slower than v_c . The Maxwell-Boltzmann distribution is given by

$$f(v)dv = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) dv \quad (3.1)$$

Our designs target capture velocity for the Zeeman slower was 570 m/s , allowing us to capture around 53.4% of our atomic flux. We decided to go with an electromagnetic design rather than a permanent magnet design, due to the ability to turn off the field and the relative ease of winding a tapered solenoid.

3.6.1 Solenoid Winding and Electronics

The theory details of the Zeeman slower may be found in sec. 2.4. In the following sections, I discuss details of field design, solenoid winding and electronics.

The largest implication/drawback of the oven and Zeeman slower in a straight line, is the significant increase in the magnitude of the projection of the MOT fringing fields along the longitudinal slower direction. Since the field already existed, and we had approximate knowledge of our MOT currents, we opted to slightly modify that section. We added a decreasing field section to create a spin-flip slower, intended to initiate the

capture of atoms at 570 m/s and decelerate them to 50 m/s with a laser detuning of -300 MHz. Another problem is that changing the velocity of the atomic beam from positive to negative is easier for heavier atoms within the constraints of reasonable Zeeman slower lengths. If we did not have a way of stopping the slowing, the atoms would begin traveling with a negative velocity due to the magnitude of the MOT fringe field. Thus, we decided to wind a small coil with the purpose of stopping scattering from slower light.

More details on the zeeman slower field calculations can be found in the thesis of Mr Korak Biswas from our group.

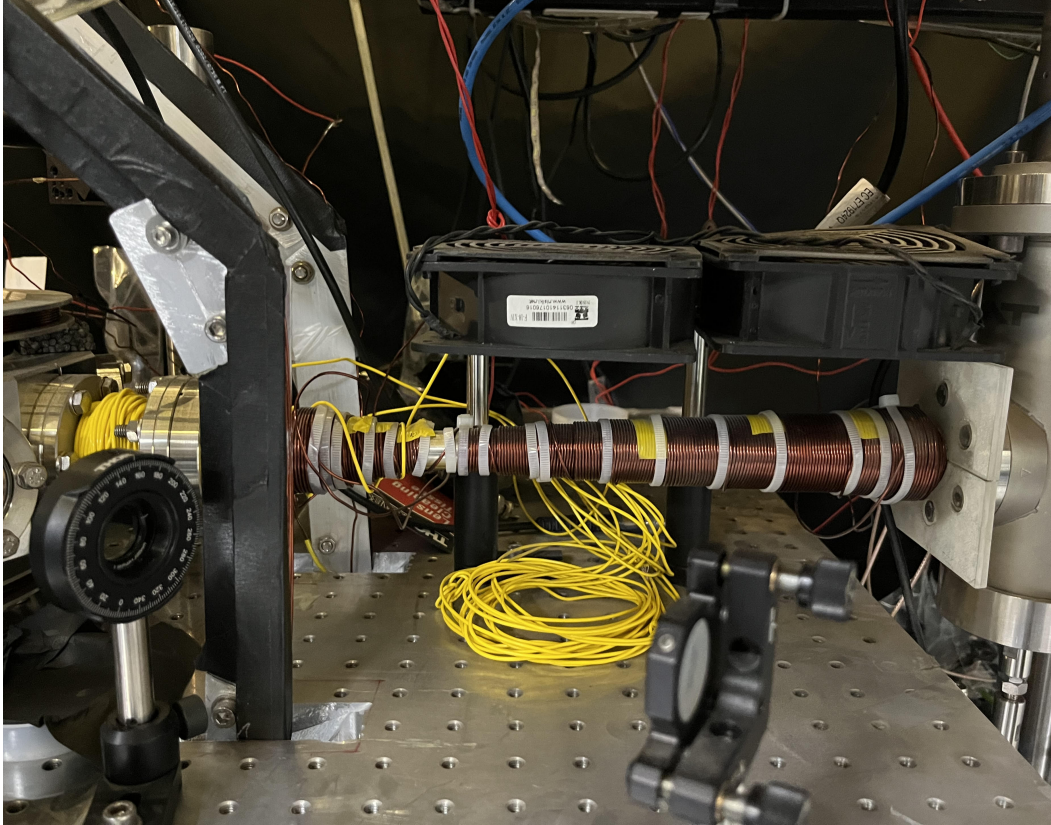


Figure 3.8: Zeeman slower: The Zeeman slower winding, visible in gold color, is equipped with a fan to provide convective cooling to the Zeeman slower coil. It is wound around the DN-16 tube, which also partly serves as the Differential Pumping Tube (DPT). This Zeeman slower brings down the velocity to 30 m/s.

We used AWG 16 wire to wind the solenoids for the slower, which has an outer

diameter of 1.291 mm and is insulated with Class H insulation. The slower was wound in three sections labeled slower A, slower B, and C. For the power supplies, we utilized ES 015-10 DC power supplies from Delta-Elektronika. The Zeeman slower's power dissipation is less than 25 W, a value we can manage by using a fan to blow cold air from the surroundings onto the Zeeman slower coil. Later, we transitioned to operating the Zeeman slower coil in a pulse mode to avoid any problems related to heat dissipation.

3.7 Main Chamber

Slowed strontium atoms are trapped and cooled inside the main chamber, as shown in Fig.3.9. Once the atomic beam is slowed, the atoms are captured in a 461 nm MOT at the center of the main chamber. This chamber is made of stainless steel 316L grade, which has a much lower magnetic permeability than that of 304. The vacuum chamber features an external diameter of 213 mm and a width of 106 mm. It provides conflat sealing surfaces for two CF150 windows and twelve DN40 windows (refer to Fig.3.9). Optical accesses are available for the first cooling stage at 461 nm and the second cooling stage at 689 nm, as well as for repumping lasers (679 nm and 707 nm), a probing beam (461 nm), optical lattice (813 nm), clock spectroscopy (698 nm), and fluorescence detection (461 nm).

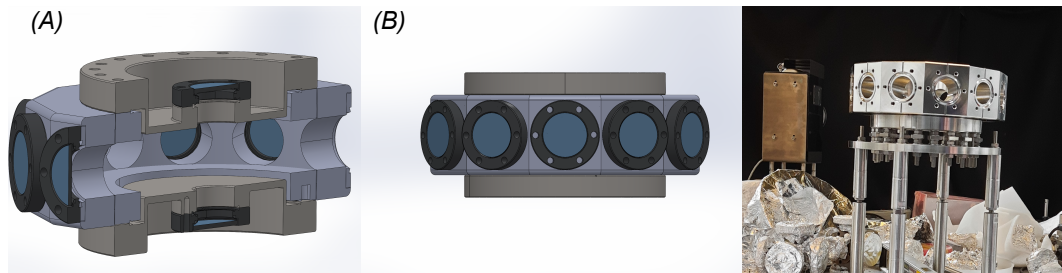


Figure 3.9: Main chamber: The first figure displays a cross-sectional view, showing the do-decagon chamber and the reentrant custom viewports. The second figure is a top view, while the third figure is a side view.

Providing the magnetic field gradient required for the Blue and the Red MOT, two anti-Helmholtz coils are hosted by custom-made CF150 flanges. These flanges are designed to minimize the distance between the coils and the atomic sample trapped in the center of the chamber, thereby reducing the current needed to achieve the typical mag-

netic gradient of 50 Gauss/cm for the MOT. This concept is illustrated in Fig.3.9, where these custom flanges are depicted in brown color. Each custom flange offers one CF40 optical access for the retro-reflected MOT beam, collinear with the magnetic axis of the coils. The minimum distance between atoms and coils is determined by two constraints. Firstly, custom flanges must not obstruct the optical accesses. Secondly, the thickness of the custom flanges must be sized to ensure mechanical stiffness adequate for UHV sealing.

3.8 The Electromagnets

The design and construction of the quadrupole trap for MOT, as well as the shim coil to nullify the effects of background stray magnetic fields, are described in this section. All components of the MOT coil assembly are crafted using Poly-oxymethylene plastic (Delrin), while the shim coils are made using aluminum sections.

3.8.1 Main Electromagnet Design

The quadrupole coil used in the experiment is shown in Fig.3.10. The magnetic field for the MOT is generated using a pair of coils in the anti-Helmholtz configuration. This quadrupole coil provides the required field (50 G/cm) for trapping atoms slowed by the Zeeman slower. Additionally, it offers the advantage of transferring atoms from the MOT to a pure magnetic trap, provided the atoms are in a magnetically trappable state, such as atoms in the 3P_2 state that can be trapped in a pure magnetic field.

Each coil has been assembled with an enameled copper wire of 1.291 mm diameter (AWG16) wrapped around an internal diameter of 75 mm. The external diameter of each coil is 124 mm, and the averaged height is 45 mm, corresponding to a total number of turns of about 50. This geometry allowed us to reach the needed magnetic field gradient of 50 Gauss/cm with 14.6 A (at 17 V), corresponding to a total power consumption of about 250 W. To avoid heating at large operating currents, these coils are enclosed inside a watertight Delrin assembly through which we pass pressurized water at a temperature of approximately $\sim 17^\circ$ C. The magnetic field gradient generated by this coil is approximately $11 \text{ G}\cdot\text{cm}^{-1}\cdot\text{A}^{-1}$ along the axial direction. During the MOT operation, the quadrupole coils are continuously cooled using the cold water circulating through it.

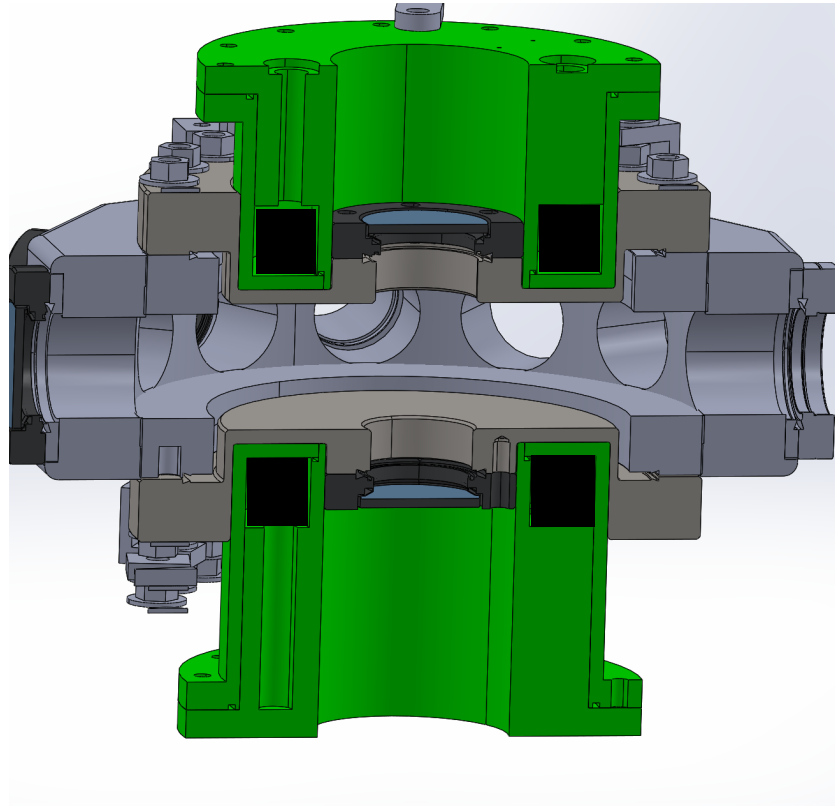


Figure 3.10: MOT coil: In this figure, Delrin is shown in green color inside which the MOT coil is wound (black color). Water circulates inside the Delrin enclosure, allowing the MOT coil to be cooled at higher currents.

3.8.2 Shim Coils

Earth's magnetic field serves as a constant background of about half a Gauss atop any field or gradient created in the lab. Due to this constant background field, as we reduce our field gradient from the blue MOT to the red MOT, the position of the magnetic trap center may shift, leading to significant atom loss.

To address this issue, we must compensate for the earth's field (along with any other stray fields from nearby sources like neighboring labs) using sets of Helmholtz shim coils. Consequently, to achieve compensation in any direction, we require three sets of transverse coils. In our experiment, the shim coils are wound on an octagonal aluminum frame with AWG16 magnetic wire.

3.9 Control Electronics for the Experiment

To achieve the cooling of Sr atoms, the experimental procedure demands stringent timing control accurate to a microsecond. Control over electronic devices and various components is achieved through LabVIEW (National Instruments) programming language, primarily interfacing with NI-6363 and NI PCI 6713 (National Instruments).

NI 6363 provides digital I/Os with 16 channels, analog inputs with 16 channels, and analog outputs with 4 channels. We utilize digital I/O for triggering various devices to establish timing synchronization. This trigger is employed for the atomic beam shutter, ZS switching, AOM driver (Moglabs), camera, PMT, MOT field, and shim coil. Analog input is utilized to monitor PMT output and the photodiode signal placed to monitor optical power in a MOT beam to track the optical power during the experimental cycle. Analog output is employed for ramping MOT beam intensity and MOT field current. We used IGBT and MOSFET to switch current during the blue MOT and red MOT duration. We use three computers to run the whole experiment: the first one controls the locking of 461 nm and 689 nm, the second one handles digital I/O timing and AO and AI, and the third one is dedicated to the camera and imaging. We use the Pixelfly camera for absorption imaging, connected via USB to the main computer.

Chapter 4

Strontium experiment: Blue Laser Systems

In this chapter, we'll outline the laser systems used in the experiment, including laser systems in the blue (461 nm), and cyan (481 nm), as well as other re-pumpers (707 nm, 679 nm). These laser systems play a crucial role in the experimental cycle, enabling the deceleration, cooling, and trapping of thermal atoms in the strontium beam, which is directed from the oven to the science chamber. Achieving stable, robust and easily manageable laser systems is essential for the successful performance of each experimental cycle.

4.1 Blue Laser System

For laser cooling techniques, closed transitions that are easily accessible with available laser technologies is essential. The level structure of an atom defines the applicable cooling strategy. Sr has a near-closed and strong optical transition at 461 nm from the ground state to the 1P_1 state with a linewidth of $\Gamma = 2\pi \times 32$ MHz, corresponding to a large saturation intensity $I_{sat} = 43 \text{ mw/cm}^2$. This transition is used in experiments with ultracold Sr for several important intermediate steps: reducing the divergence of the atomic beam in the transverse cooling section; decelerating atoms in the Zeeman slower; cooling and trapping of atoms in a magneto-optical trap (MOT) and imaging of trapped atoms.

The saturation intensity of the transition is notably high, requiring laser beams for transverse cooling (TC), Zeeman slowing (ZS), or magneto-optical trapping (MOT) to have intensities that are a significant fraction (>0.5) of the saturation intensity. However, achieving this intensity proves challenging due to the need for laser beams with centimeter-scale diameters and their susceptibility to power loss in optical elements like acousto-optic modulators (AOMs) and optical fibers that are used for mode cleaning and beam transport. Therefore, to ensure adequate power for all blue laser cooling beams, the source must generate a minimum of 400 mW in our system. Moreover, for reliable laser cooling operation, the laser source must be actively frequency stabilized and exhibit a linewidth narrower than the addressed transition.

The blue laser source offers various options, and one approach involves utilizing a master oscillator at 922 nm, succeeded by an amplifier and second harmonic generation [126, 127]. Commercially available and well-developed, this laser system commonly employs a narrow linewidth external-cavity diode laser (ECDL) as the master oscillator, operating at 922 nm. This light undergoes amplification to the watt levels of power through a semiconductor tapered amplifier. The amplified light is then directed into a power enhancing bow-tie cavity housing a non-linear optical crystal (PPKTP crystal) for second harmonic generation, yielding several hundred mW of power at 461 nm.

After careful consideration of available options, we opted for an injection lock module [128]. Adopting a single diode, we generate 500 mW of optical power in contrast to previous studies relying on multiple slave lasers to achieve the required 300 mW at 461 nm in Sr experiments [129]. The power enhancement of an injection-locked system can surpass that of a linear amplifier (i.e., MOPA source).

4.1.1 Blue Seed Laser

The blue seed laser for the strontium experiment employed two different methods. In the first method NDB4961 nichia laser diode was utilized in the littrow configuration external cavity diode laser (ECDL). The second approach involved an older Master Oscillator Power Amplifier (MOPA) and Second Harmonic Generation (SHG) system which, due to crystal aging [130], could only produce 100 mW of power (refer to Mr. Chetan Vishwakarma's thesis for details). Both systems have their advantages and disadvantages, as

discussed below.

In the littrow configuration with the external cavity diode laser, we employed the NDB4916 Nichia diode capable of producing over 500 mW of power. A grating with 2400 lines/mm from Edmund Optics was used in a homemade ECDL mount configured in littrow. The laser was driven using an old laser current and temperature controller, along with a piezo controller. However, this laser suffered from a narrow mode-hop-free tuning range of about 5GHz and a poor spatial mode. To address these issues, we locked the laser to the Sr 461 nm laser transition using a Toptica locking module. While this laser is maintenance-free, its poor spectral and spatial mode qualities was significantly disadvantages.

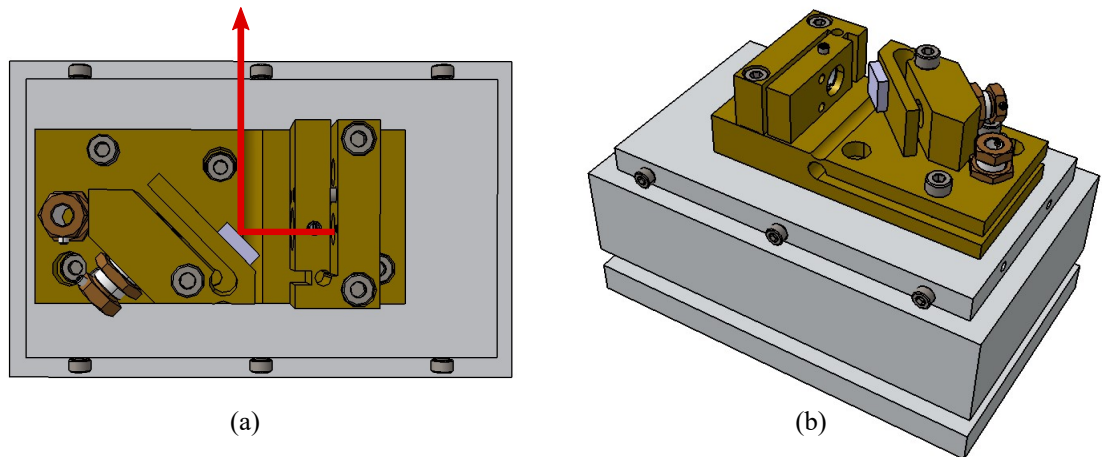


Figure 4.1: Seed laser for the generation of 461 nm light: (a) top view and (b) isometric view of ECDL in Littrow configuration for the generation of laser at 461 nm.

In the case of the MOPA-based SHG system, the Toptica DL 100 laser is employed as a master tunable IR diode laser operating at 922 nm, followed by power amplification in a tapered amplifier and frequency doubling. However, due to the aging of the SHG crystal, the SHG efficiency decreased by more than a factor of 2. Frequent alignment of the cavity system to maintain peak power output emerged as a significant issue. We used a wavemeter to lock the 922 nm master laser to $<2\text{MHz}$ drift rate in the long term, which is sufficient for running the strontium system considering the linewidth of Sr 461 nm transition is 32MHz.

4.1.2 Blue Injection Lock Laser

Injection-locking involves injecting a slave laser diode with external light of narrow linewidth, forcing the diode to emit light with similar properties as that of the injected light. Normally, a free-running laser diode emits broadband light due to its gain medium allowing photon emission across a spectrum around the nominal wavelength. This effect occurs when the injected light surpasses the spontaneously emitted photons of the free-running diode in strength. The seeded light's mode is more likely to be exponentially amplified in the gain medium, suppressing other spontaneous modes by redistribution of the available power. For this effect to extend across the entire gain medium, the injected light must be well mode matched to the internal cavity of the slave laser diode. Achieving minimal seed light requires perfect alignment. The injection-locking setup in our design, illustrated in Fig. 4.2, involves coupling light from the master laser through an optical fiber and collimating it with a lens. To align the seed light with the second polarizer of the Faraday isolator, we use a half-wave plate to rotate its polarization. The seed light is introduced into the slave laser diode through two mirrors, with an aspheric lens in the slave laser ensuring mode matching. A front half-wave plate adjusts the polarization of the slave laser, enabling it to emit light that passes through the Faraday isolator while rejecting counter-propagating light to safeguard the diode. Monitoring the slave laser's operation involves deriving a fraction of the power using a half-wave plate and a PBS, and the remaining light is directed into an optical fiber with the help of two adjustable mirrors.

4.1.3 Injection Lock Module Mechanical Design

To enhance the mechanical stability of our injection lock module optics setup for injection-locking, we aimed to address the sensitivity of the injecting beam to mechanical disturbances. The goal was to achieve a stable lock without the need for constant attention. Our approach involved reducing beam heights and lengths, ensuring temperature stability, employing robust mechanical mounting, and minimizing air currents around the beam path. Considering these factors, we conceptualized a new compact and modular opto-mechanics setup.

On the top side, the breadboard and its side wall feature a ledge used for attaching

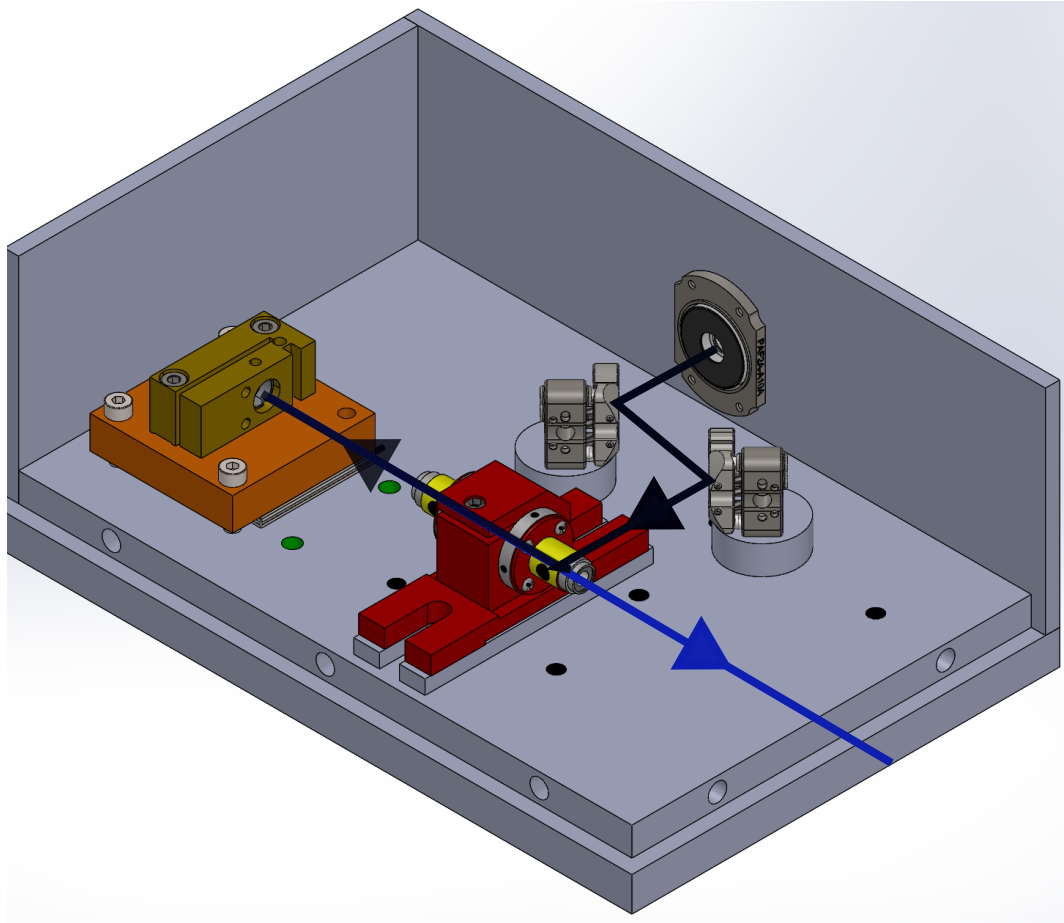


Figure 4.2: Injection lock module: The injection lock module houses a laser diode in a temperature-controlled environment, collimation lens, optical isolator, waveplates (not shown in the picture), mirrors, and fiber collimator.

aluminum top cover, which are used to close the system after assembly. Fig. 4.2 illustrates the side wall of the breadboard with three large holes for mounting Thorlabs's FiberPort collimators. Moreover, the extrusion includes holes for electronics connectors required for driving and regulating the laser diode head.

Fig. 4.2 shows the various optical components essential for our setup. First, let's examine the fiber collimator integrated into our design. To connect and collimate the outputs of optical fibers, Thorlabs's FiberPort collimators PAF-X-7-A were employed. These compact fiber ports offer good stability, and their 5 adjustable degrees of freedom facilitate collimating the laser beam and aligning it with the desired optical path. Simultaneously, the ports serve for coupling light into a fiber and optimizing coupling ef-

iciency. However, adjusting these fiber ports might be counterintuitive, requiring some practice to achieve desired behavior in fiber coupling. The collimator's focal length was chosen based on our experience with fiber coupling using Nichia NDB4216E diodes for injection-locking. The fiber ports are directly mounted on the breadboard side wall using four M2 screws. For kinematic mirror mounts, we opted for Thorlabs's Polaris-K05 mounts by the excellent stability offered by the Polaris series. The K05 model features two 2 mm dowel pin holes on its mounting surface, ensuring proper referencing in our design. We selected mounts for half-inch mirrors to maintain a low beam height of 20 mm. The optical isolator is a custom isolator from TORNOS with a central wavelength of 461 nm. It is mounted by clamping with two M4 screws. It offers isolation of >30 dB and a transmission of $>97\%$. Additionally, it has access to the blocked beam via the escape port on the optical isolator. It has a height of 18 mm from the base. This optical isolator has a clear aperture of about 4 mm, which is compatible with our laser diode mount design.

We've created mounts for half-wave plates, which require rotation to achieve the desired light polarization. These mounts have internal threading with the Thorlabs SM1 thread, and aluminum adapters are designed to securely hold and rotate the waveplate. The waveplates only need to be adjusted once, and they don't significantly distort the optical path, making the long and dense thread sufficient for referencing. After turning the adapter to the desired position, it can be securely fixed in place using an M4 nylon-tipped screw.

To accommodate our new injection lock module design for injection-locking, we needed to create a reliable mount for the laser diode. The challenge we encountered was that the laser diode's output pointing is limited to a 3 degrees. Consequently, we required a mount with sufficient degrees of freedom to precisely align the laser beam, allowing it to pass through the aperture of the optical isolator and the first adjustable mirrors. Another consideration is that the laser diode must be temperature-stabilized using Peltier elements.

Fig. 4.2 illustrates the assembly, where the cubical laser head part is securely held on the copper plate, and the Peltier element is sandwiched between the bottom aluminum part and the upper copper plate, fixed using M4 screws from the top. To ensure optimal

thermal contact, thermal paste is applied to all contact surfaces. The Peltier elements are situated in small extruded pockets that provide a rough reference for their position. Considering the need for heat dissipation and effective heat conduction through the mount into the bottom plate, we selected "nickel silver thermal paste" as the material for this assembly. To provide current and temperature controller connection we used a protection circuit with sub-D connector. To make the board compatible with both positive and negative polarities laser diodes, We added a series of jumpers, JP2, JP3, and JP4. Additionally, resistors are added after the pins 6 and 9, protecting the voltage sense inputs of the controller. In this case, we need a female 9-pin sub-D connector to plug the cable in the board. The jumpers effectively select which of the laser diode pins is grounded. The most straightforward part of the protection circuit is the temperature control part. We simply add a male 9-pin sub-D connector, where we can plug in the cable from the temperature controller. The pin layout seems to be rather standardized, at least between Thorlabs and Toptica, whose controller we were looking at.

4.1.4 Blue Laser Distribution

The 461 nm laser setup occupies about half of a 5' by 10' optical table and closely adheres to the layout described in the thesis by Stellmer [131]. Specifically, we implement the use of a double-pass AOM in the injection lock module seed path, enabling us to quickly tune among all four strontium isotopes. Polarization-maintaining fibers from Thorlabs are employed for the MOT, imaging, and slower light. These fibers allow us to couple a maximum of 70 mW of light into the fiber without burning the tip.

To achieve different detunings for various laser beams, the laser beams must undergo frequency shifts during the experimental sequence. Acousto-optic modulators (AOMs) are employed to accomplish these frequency shifts by utilizing the acousto-optic effect, which diffracts and alters the frequency of light through radio frequency. In the AOM setup, a piezoelectric transducer is fixed to a crystal. When radio frequency is applied, these transducers induce and couple the acoustic wave in the crystal. This induces moving periodic planes of expansion and compression, altering the refractive index of the material. The incoming light scatters off this periodic index modulation, resulting in interference akin to Bragg diffraction. To drive these AOMs effectively, a high-power

RF signal is essential. To generate the required high-power RF signal, we use a moglabs AOM driver.

For ^{88}Sr the Zeeman slower path has a detuning of -380 MHz, the transverse cooling light is detuned by -60 MHz, and MOT beams are detuned by -40 MHz. The Zeeman slower beam has a power of 70 mW, Transverse Cooling (TC) beams have a power of 30 mW, and the MOT beam has a power of 70 mW when fiber-coupled. After taking the MOT fiber to the upper breadboard, we split the light and collimate it to approximately 15 mm beam diameter.

4.2 Blue Laser Locking Scheme

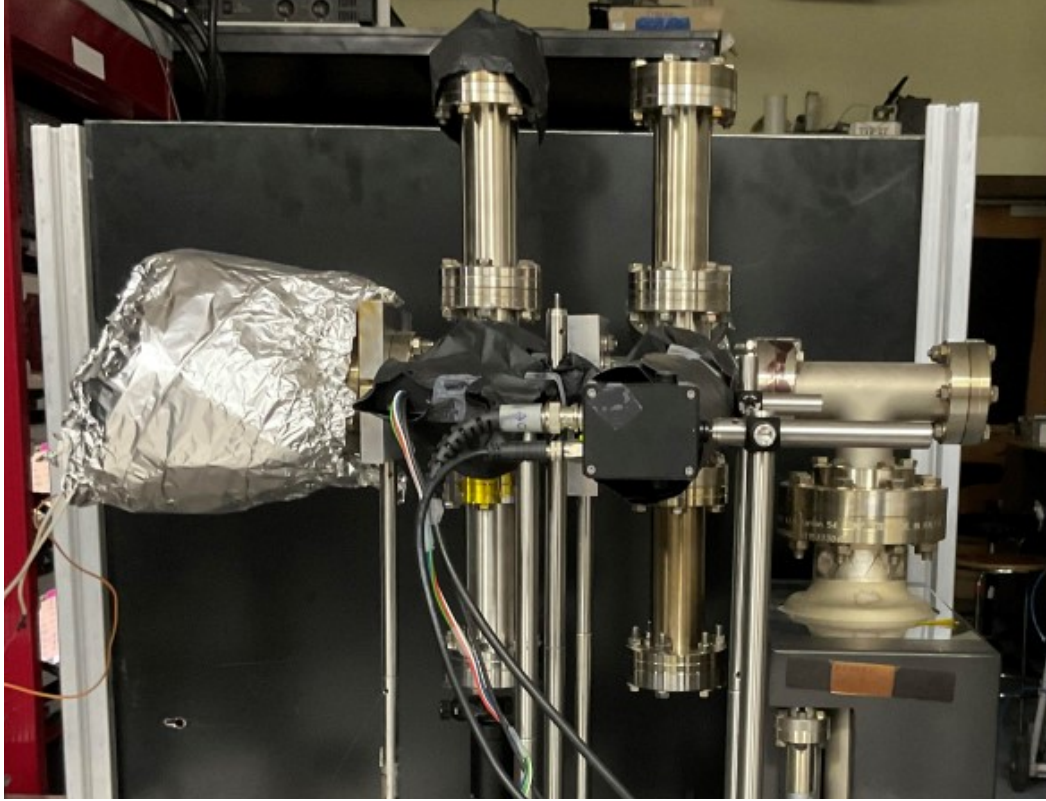


Figure 4.3: Atomic beam spectroscopy system: On the left side of the system, an oven is installed and heated to 450°C with a heater, wrapped with insulation material. On the right side, an ion pump is connected to maintain the pressure inside the vacuum chamber. In the middle part, two six-way crosses are used to interrogate the passing atomic beam by sending a resonant laser perpendicular to the direction of the atomic beam to avoid Doppler shift.

For stabilizing the blue laser at the $5s5p\ ^1S_0 \rightarrow 5s5p\ ^1P_1$ transition, we utilized a

spectroscopy cell, as depicted in Fig. 4.3. This cell includes an oven (described earlier), a flexible bellow, and two six-way crosses with attached nipples. The strategic placement of the nipple ensures an increased distance between the atomic beam and the optical viewport, minimizing the probability of the viewport being coated by the strontium atomic beam. To maintain vacuum conditions, the spectroscopy cell is connected to a 55 L/s ion pump. The 461 laser beam enters the spectroscopy cell through the bottom viewport, and the resulting fluorescence is collected on the front viewport. we use 75 mm focal length 2" lens to collect the fluroscence and thorlabs photodiode DET100A.

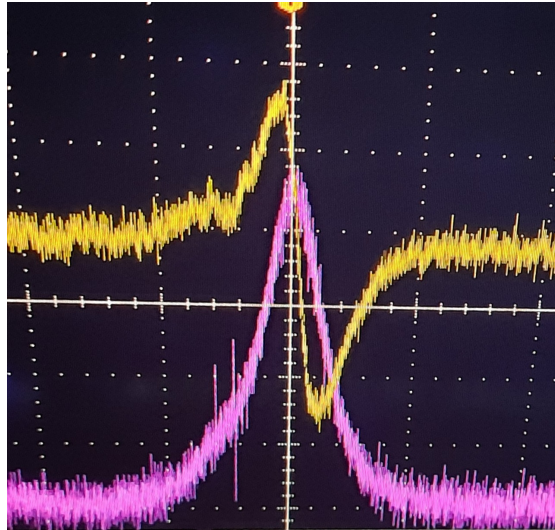


Figure 4.4: error signal: In the image, the pink-colored plot represents the photodiode signal, and the yellow signal represents the error signal generated after lock-in detection.

To generate the feedback signal for compensating the drift of the 461 nm laser, we utilize the spectrum obtained through atomic beam spectroscopy. The standard lock-in-detection technique is employed to generate the error signal for locking the seed laser, requiring the laser frequency to be modulated at a specific frequency. In our system, the modulation is introduced using an acousto-optic modulator (AOM) in a double pass configuration, which also applies a constant frequency shift of 168 MHz on the spectroscopy beam. The optical layout, along with the frequency detuning scheme, is provided in a later section. The chosen modulation frequency is approximately 80 kHz. The signal for frequency modulation is produced by an SRS lock-in amplifier (Stan-

ford Research System, Model: SR830) and is then combined with the Moglabs AOM driver. The detected signal by the PMT is demodulated at the same frequency to generate the corresponding error signal, a process done by the lock-in amplifier mentioned earlier. Due to the slow drift of the seed laser at 922 nm, which occurs at a rate of a few MHz/min, a gradual feedback in the laser's piezoelectric transducer is sufficient to compensate for this drift. Therefore, we direct the error signal to the PZT via a PID controller and a voltage amplifier.

The alternative approach we adopted later in the system involved, using a portion of the 922 nm laser's power to insert into the wavemeter. We use LabVIEW to process the wavemeter data in real-time, calculating the deviation from the desired frequency and feeding it to the digital PID. This PID output/feedback is then transmitted to the piezo driver. This method enables us to change the isotope by adjusting a lock point in the LabVIEW program. However, the drawback is that, after locking, the drift remains higher compared to locking with a spectroscopy cell, making it more susceptible to going out of lock more frequently.

4.3 Repumping Laser System

In the experimental setup, four repumping lasers have been employed with wavelengths of 707.2 nm (homemade ECDL-20 mW), 679.3 nm (homemade ECDL-15 mW), 481 nm (Toptica DL pro ECDL-20 mW), and 483 nm (Toptica DL pro ECDL-15 mW). These schemes target the atomic transitions $5s\ 5p\ ^3P_2 \rightarrow 5s\ 6s\ ^3S_1$, $5s\ 5p\ ^3P_0 \rightarrow 5s\ 6s\ ^3S_1$, $5s\ 5p\ ^3P_2 \rightarrow 5p^2\ ^3P_2$, and $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ respectively. The atoms, once excited to the 1P_1 state, decay to the 1D_2 state with a branching ratio of $1 : 2 \times 10^{-5}$. From 1D_2 , atoms connect to the triplet states 3P_2 and 3P_1 with a branching ratio of $1 : 2$, as shown in the fig. 4.5. However, the transition to the ground state $^3P_2 \rightarrow ^1S_0$ is doubly forbidden, resulting in the loss of atoms from the cooling cycle.

A heat pipe can be utilized to generate sufficient vapor pressure for spectroscopy, aiding in the atomic transition between the ground state 1S_0 and excited states 3P_1 and 1P_1 [132]. However, due to the high excitation energy involved, the thermal occupation of the metastable state does not offer adequate optical density for frequency stabilization to the transition between the metastable state and higher excited states. To address this

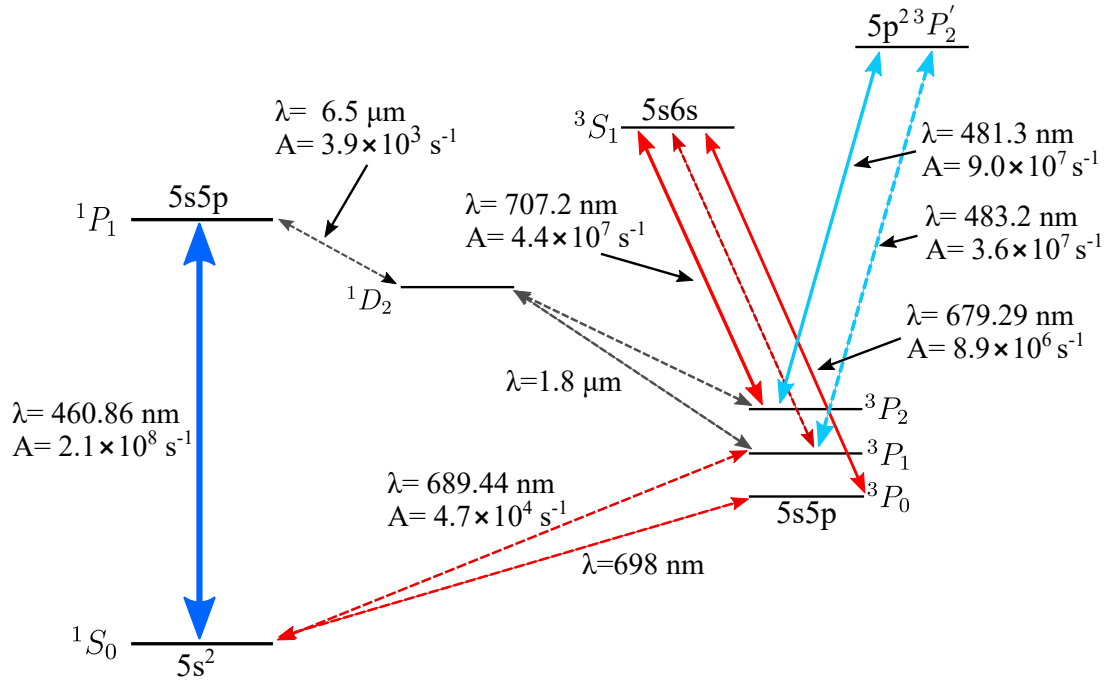


Figure 4.5: Strontium energy level diagram: The relevant low-lying energy level diagram of strontium (^{88}Sr) atoms. The wavelength (λ) and the decay rate (A) has been shown along with the transition.

limitation, a hollow-cathode lamp can be employed, wherein collisions generate ample population in the metastable state 3P_2 for spectroscopy and frequency stabilization of lasers operating at 707.2 nm [133]. Despite being compact, hollow-cathode lamps suffer from short lifetimes and significant pressure broadening [133, 134]. An alternative approach involves using molecular spectra, such as iodine or tellurium, for frequency stabilization. However, the feasibility of such a system hinges on the presence of a molecular line near the atomic transition under consideration.

To measure the frequency of laser light accurately, an alternative method involves employing a commercially available, high-precision wavemeter. The error signal is obtained by calculating the difference between the laser frequency and the set frequency, which can then be utilized for wavelength correction by feeding it back to the laser's piezoelectric transducer. Wavemeters typically require a small amount of light for measurement, usually on the order of a few 10s of μW , and can be applied across a broad frequency range, typically spanning hundreds of nanometers. The use of wavemeters for laser locking has been extensively implemented in various studies.

For our experiments, similar to the scheme described above, both repumping laser frequencies are controlled using a digital PID controller interfaced with LabView. The controller receives feedback on the laser frequency from a commercial wavemeter (*High Finesse*, model: WSU-30). An analog voltage is produced by a National Instruments data acquisition card, which is proportional to the disparity between the laser frequency and the predetermined frequency. This voltage is then directed to the Toptica scan controller module SC110, providing feedback to the laser's PZT for frequency stabilization.

4.4 Blue MOT

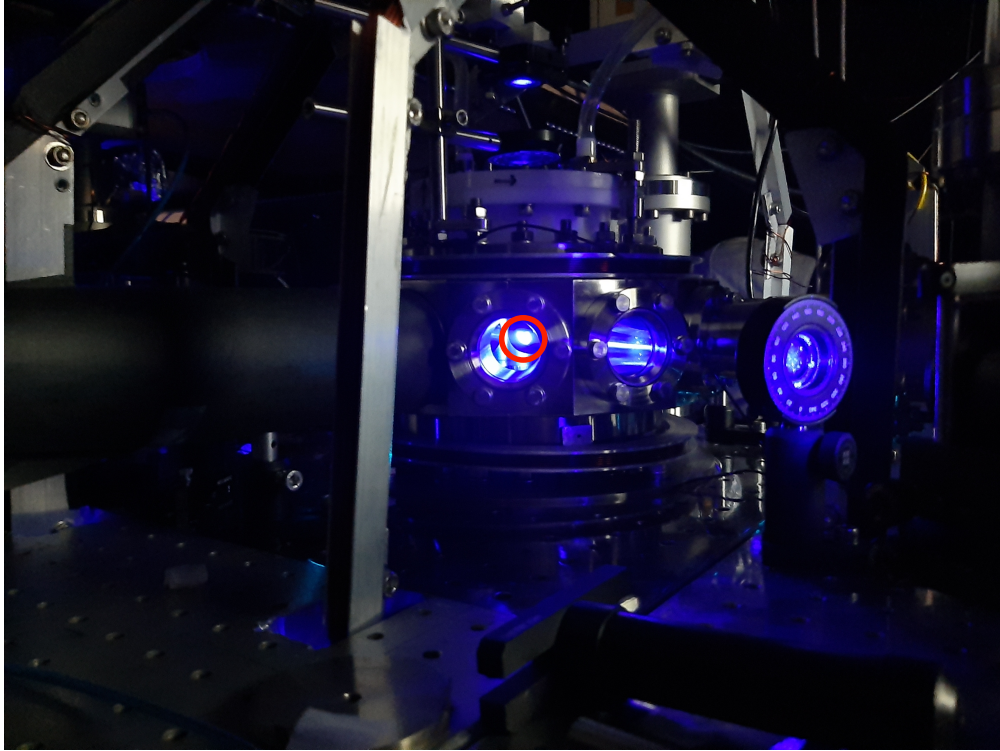


Figure 4.6: Sr MOT: As shown in the figure the central blue bright spot is a MOT. No of atoms in the MOT is 1.2×10^8 .

In our experiments, the Magneto-Optical Trap (MOT) is loaded with atoms from a thermal beam generated by an effusive atomic source operating at 600°C . The atoms are subsequently decelerated by a zero-field crossing Zeeman slower. The initial laser cooling ($5s^2\ ^1S_0 \longrightarrow 5s\ 5p\ ^1P_1$, $I_s = 42.7\ \text{mW/cm}^2$) is amplified by the injection lock module, as explained in section 4.1.

The MOT laser beams follows the standard $\sigma^+ - \sigma^-$ configuration with retro-reflection geometry. To achieve circularly polarized laser beams for MOT, the output from the fiber is split into three beams using a polarizing beam splitter cube and half-wave plates ($\lambda/2$). These beams pass through a quarter-wave plate ($\lambda/4$) to set the desired circular polarization, monitored using a polarimeter. During the initial phase of the experiments, the MOT beam diameter is maintained at approximately 15 mm ($1/e^2$). These beams remain free-standing from the beam-splitting region to the MOT capture region.

For the MOT operation with Sr atoms, the high scattering rate associated with the first stage cooling transition allows the utilization of laser beams with 15mm diameter. Each beam used in the MOT setup carries about 10 mW of power, with the detuning set at -45 MHz (approximately $1.5 \Gamma/2\pi$, where Γ represents the natural linewidth of the cooling transition) concerning the $5s^2 \ ^1S_0 \rightarrow 5s \ 5p \ ^1P_1$ transition. The repumping laser beam is directed straight to the MOT capture region. The Zeeman slower detuning is set at -400 MHz and the current in coil 1 and coil 2 is 2.5 A and -5 A respectively.

4.4.1 Loading and Characterization

The Zeeman slower beam, maintained at approximately 40 mW during the experiment, originates from the injection lock module. To guide the light with the desired detuning and polarization into the vacuum chamber, a periscope is employed, directing it towards the atomic beam oven. To account for the divergence of the atomic beam emerging from the oven, the Zeeman slower beam is focused on the surface of the collimating capillaries. The field gradient along the axial direction of the quadrupole coils is maintained at approximately 50 Gauss/cm. The loading curve of the MOT is determined by collecting the fluorescence of the trapped atoms onto a Hamamatsu photo-multiplier tube (H9307-02), utilizing a lens subtending a solid angle of 0.2 sr at the center of the trap. The loading curve of atoms under the influence of the repumping laser is illustrated in Fig. 4.7. We achieve a loading of approximately 5×10^8 atoms in the MOT with the repumping laser operating at 481.32 nm.

4.4.2 Temperature of Blue MOT

To gather information on the number of atoms and their spatial distribution, absorption imaging was performed. An atomic cloud image is generated by exposing the atoms to

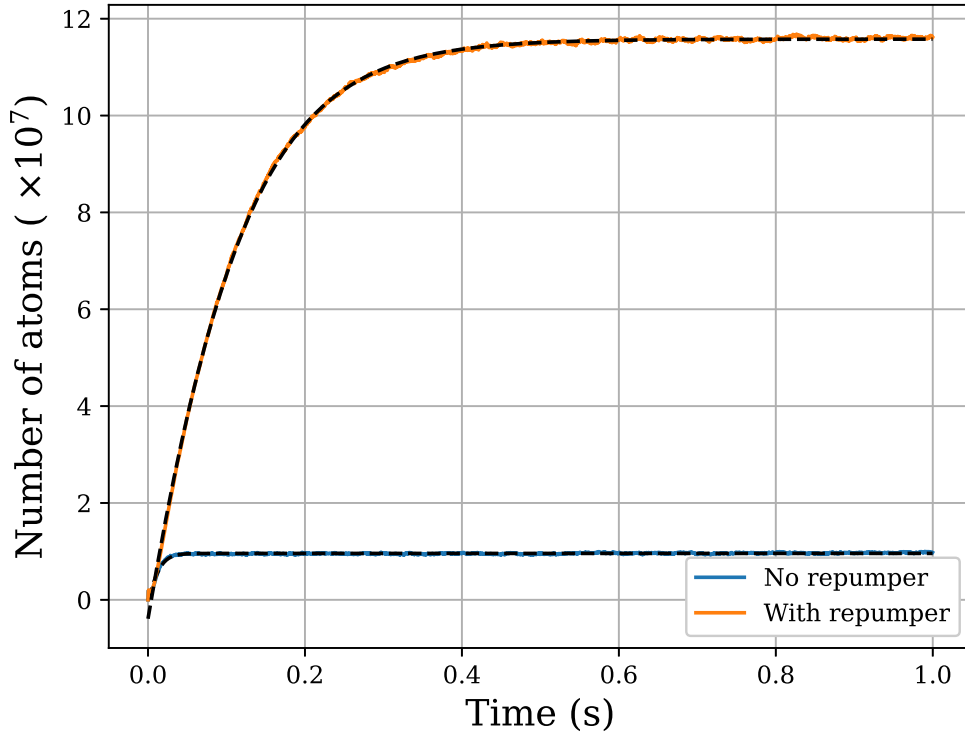


Figure 4.7: MOT loading curve: The plot shows the loading curve with and without a repumper. In the case without a repumper, the MOT is loaded in less than 50 ms with 10^7 atoms. Conversely, with the repumper (at 481 nm), the MOT loads in 400 ms with 1.2×10^8 atoms.

a $10\mu\text{s}$ pulse of resonant 461 nm light, causing them to cast a shadow captured by the CCD camera. Subsequently, a second background image is acquired 200 ms later by flashing another $10\mu\text{s}$ pulse of light onto the camera in the absence of atoms and third image is acquired in the absence of atom and light pulse. This background image is then subtracted from the atom image. The resulting images are fitted to a two-dimensional Gaussian to extract the atom number and the 1/e cloud radii in both horizontal and vertical dimensions. It is important to note that this technique is considered destructive.

To determine the temperature of the (thermal) cloud, the time-of-flight technique (TOF) is combined with absorption imaging. A known delay is introduced between turning off the trapping light and initiating the absorption imaging sequence. This delay enables the cloud to undergo ballistic expansion, where the rate of expansion correlates with its temperature. Additionally, the expansion leads to a reduction in optical depth as the peak atom density decreases.

To obtain the distribution of atoms at different delay times after release, a standard TOF measurement is conducted. The cloud's radii are measured at each delay time and fitted to an equation

$$\sigma^2(t) = \sigma^2(t=0) + \frac{k_B T}{m} t^2 \quad (4.1)$$

Where, $\sigma(t)$ is a cloud radius at different time t , k_B is Boltzman constant, T is temperature and m is mass of atom. to determine the temperature accurately. However, this process necessitates preparing the sample repeatedly for each drop time because of the destructive nature of absorption imaging.

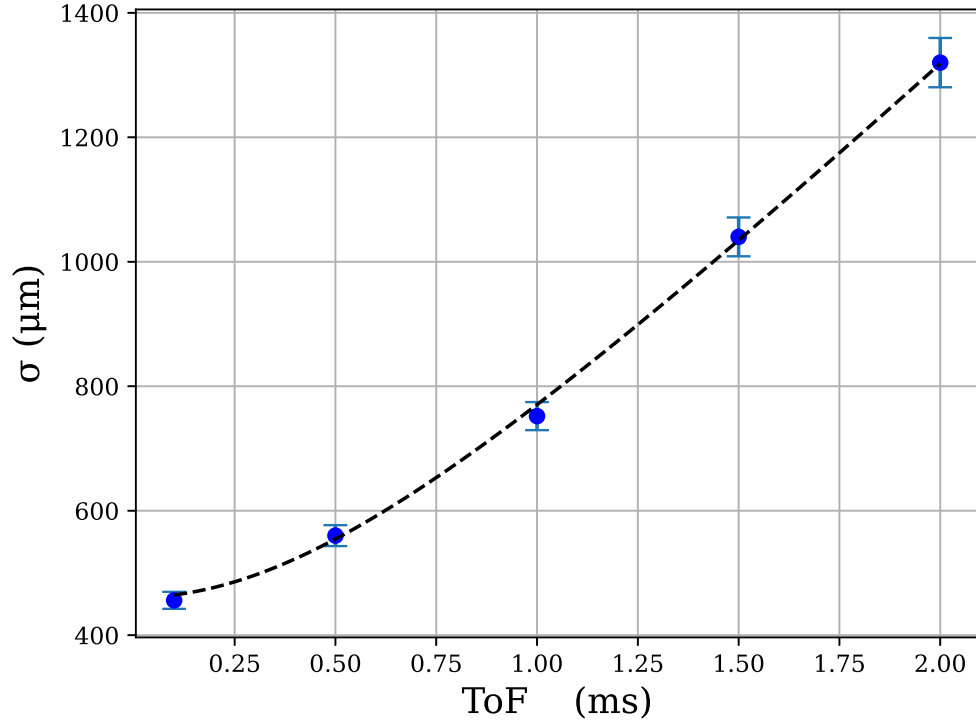


Figure 4.8: Temperature measurement: This plot shows the expansion of the cold cloud at the different time of flights.

For TOF the blue MOT and Zeeman slower are initially operated for 2s without the repumper, capturing approximately 5×10^8 atoms. Following this, at the 2s mark, the Zeeman slower is deactivated, while the blue MOT continues for an additional 100ms to allow the Zeeman field to decay, and the MOT to stabilize. Subsequently, the repumper is flashed on for 100ms before turning off all components. The TOF measurement,

indicates a temperature of 3.5mK for the blue MOT atoms.

Our blue MOT exhibits a higher atom temperature at 3.5mK compared to other setups, which typically achieve temperatures closer to 1.5-2mK. Interestingly, this discrepancy in temperature does not seem to significantly affect our transfer to the red MOT. Barker et al. [135] suggest that one possible reason for the temperature difference is that our blue MOT is optimized for loading rate into the magnetic trap.

In the conclusion of this chapter, we've detailed the role of laser systems, particularly the blue laser, for our strontium experiment. These lasers are important for decelerating, cooling, and trapping thermal atoms, facilitating experimental cycles. We highlighted the significance of closed transitions and the challenges posed by high saturation intensities, necessitating robust laser sources capable of generating higher power. Through careful consideration, we opted for an injection lock module, offering reliable power generation at 461 nm, surpassing previous methods and ensuring the stability and efficacy of our experimental setup, which led us to trap all the isotopes of Sr atoms in the MOT.

Chapter 5

Strontium Machine: Red Laser Systems

The architecture of the 689 nm laser system will be discussed in the following chapter. This laser addresses the narrow 7 kHz intercombination transition $5s^2\ ^1S_0 \rightarrow 5s\ 5p\ ^3P_1$. This transition not only enables efficient optical cooling but also offers excellent energy resolution, making it an attractive feature in strontium experiments [136].

5.0.1 Red Seed Laser

We opted for the 25 mW Toptica DLPro, having a 30 kHz linewidth, as our master laser. It has a larger linewidth compared to the natural 7 kHz transition, so we utilized a high finesse optical cavity to reduce the linewidth. This setup allows efficient Doppler cooling to high phase space density (PSD), with a Doppler cooling limit in the hundreds of nK. To compensate for the power losses incurred during laser beam transport through AOMs and other optical components, we use a 689 nm laser diode from Thorlabs in an injection locked configuration to amplify the power.

The laser beam from the master laser is divided into three parts. One part is used to lock the master laser to the high finesse optical cavity to reduce the linewidth of the laser. The second part is used to lock the laser to the Sr 689 nm transition in a heat-pipe to avoid slow drift. The third part is sent to the injection lock module to amplify the optical power for use in the main experiment. Initially, we lock the laser with the heat-

pipe and then use an acousto-optic modulator (AOM) to merge the resonance frequency of the cavity with the frequency of the laser light. To lock the laser to the heat-pipe we use the Saturation Absorption Spectroscopy (SAS) method, and to lock with the high finesse optical cavity, we employ the Pound-Drever Hall (PDH) locking method.

During the experiment, some unwanted feedback was affecting the master laser, so we added an external optical isolator with isolation >35 dB. This addition of isolation stabilized our master laser, allowing us to lock it to the optical cavity and observe the red MOT in the experiment.

5.0.2 Red Injection Locked Laser

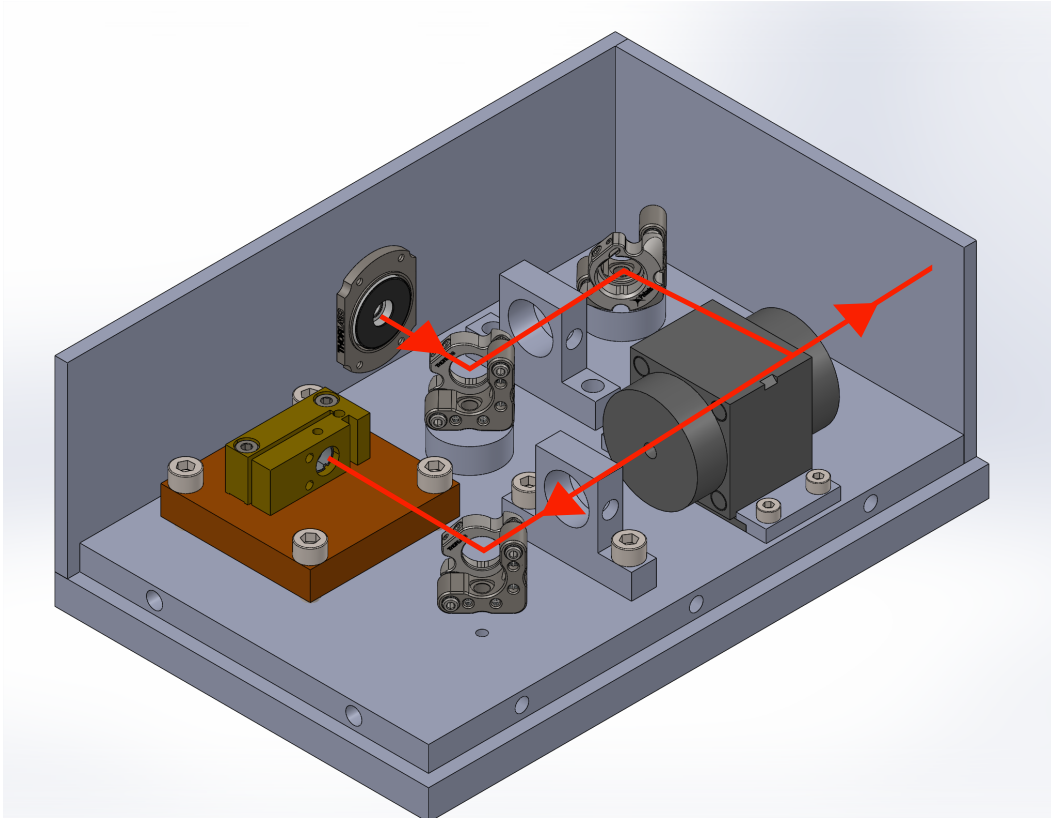


Figure 5.1: Injection locked laser for amplification of 689 nm light: A laser diode is mounted on a temperature-stabilized copper plate, and an aspheric lens is used to collimate the laser diode's output beam. An optical isolator, accessible from the wrong end, is used to couple seed light into the laser diode. Additionally, mirror mounts and waveplate mounts are fixed inside the injection lock module to achieve long-term robust injection locking.

As mentioned earlier, the mechanical and electronic design is the same for both 461

nm and 689 nm. In the 689 nm injection lock module, we use a Thorlabs laser diode (HL6750MG 685 nm, 50 mW). We opened the can and used the laser diode without the front facet. This method greatly increases the stability of the injection lock, and low seed power is required. We insert about 1 mW of red seed light. Our injection lock module remains locked for the whole day once it is temperature-stabilized and well-locked with the seed laser mode.

5.1 Ultra-Stable Fabry-Perot Cavity

Fabry-Perot optical cavities find numerous applications in optics. They are used in atomic physics to stabilize lasers and for reducing their linewidth. This allows for lasers to probe narrow atomic transitions and serve as a monitor for laser modes [137]. This section will present some essential concepts related to high-finesse ultra stable fabry perot cavity.

5.1.1 Fabry-Perot Optical Cavity Theory

An optical cavity known as a Fabry-Perot etalon is comprised of two parallel reflective mirrors positioned at a specific distance from each other. To keep this distance fixed, a spacer, often made of various materials such as low thermal expansion material is utilized. A study on different configurations of Fabry-Perot cavities and their stability is documented in [138].

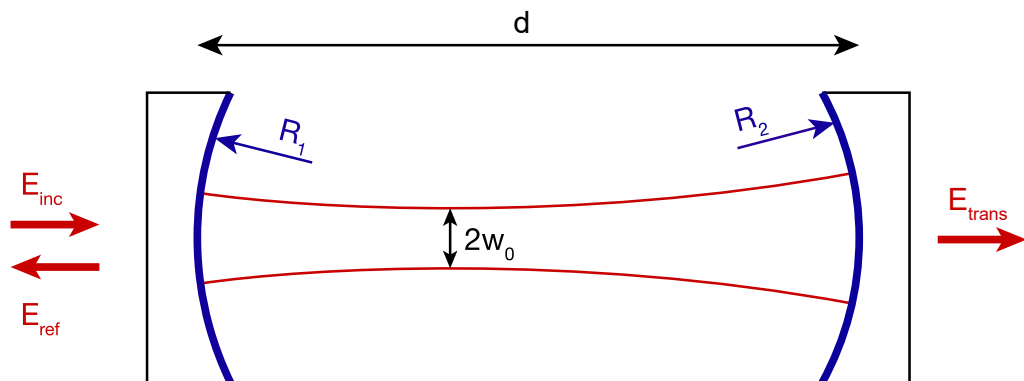


Figure 5.2: Fabry-Perot Resonator: This diagram depicts a Fabry-Perot Resonator, comprising two mirrors characterized by radii of curvature denoted as R_1 and R_2 respectively. Additionally, we denote the location of the beam waist as w_0 .

The free spectral range (FSR) is a crucial parameter for a Fabry-Perot (FP) cavity and is defined as the frequency separation between two consecutive modes transmitted through the optical cavity. This separation between resonant modes is expressed as $\nu_{FSR} = c/2L$, where c is the speed of light. The optical cavity operates by allowing light to enter through one mirror (incoupling mirror) and reflecting it back from the second mirror (outcoupling mirror), creating an interference pattern. Constructive and destructive interference patterns occur within the cavity, influenced by the mode match between wavelength and cavity length, thereby controlling the phase between incoming and reflected light.

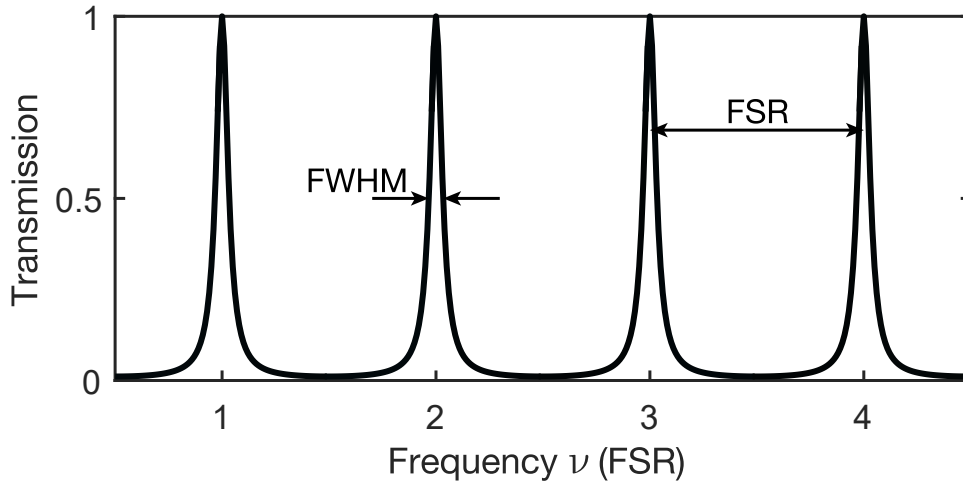


Figure 5.3: Fabry-Perot resonator transmission peaks: The transmitted light intensity of a Fabry-Perot resonator varies as a function of the laser frequency. We also indicate the Full Width at Half Maximum (FWHM) and the Free Spectral Range (FSR) of transmission peaks.

The linewidth of resonances, another crucial parameter for Fabry-Perot (FP) cavities, is determined by the reflectivity of the mirrors. A low reflectivity results in fewer bounces of each photon, reducing the number of optical paths for constructive interference and broadening the range of transmitted wavelengths. Conversely, high reflectivity leads to more selective constructive interference, resulting in narrower resonant modes. The linewidth $\Delta\nu$ in the cavity spectrum is defined as the full width half-maximum (FWHM) of the resonant peak. Finesse (F), the ratio of the free spectral range to the linewidth, quantifies the resolution of frequency discrimination (optical cavity is a THz

frequency discriminator). A higher finesse indicates narrower resonant modes and increased frequency resolution for light passing through the optical cavity. For mirrors with high reflectivity R , where absorption or losses are negligible, finesse is mathematically defined as:

$$F = \frac{\nu_{FSR}}{\Delta\nu} \approx \frac{\pi\sqrt{R}}{1-R} \quad (5.1)$$

The last approximation is commonly used for finesse values $F > 100$.

5.1.2 Measurement of Finesse of an Optical Cavity

Understanding the finesse of a cavity is important as it defines the width of the resonances. The finesse also governs the extent to which we can reduce the laser linewidth by stabilizing the laser to a high-finesse cavity. One straightforward method to determine the finesse involves knowing the resonator's geometry, including the Free Spectral Range (FSR). Using a tunable laser, one can gradually scan its frequency across a resonance. By measuring the transmitted intensity with a photodetector and observing the Full Width at Half Maximum (FWHM) on an oscilloscope, the finesse can be calculated. However, this method is effective only for low finesse cavities, as higher finesse cavities may have resonances much narrower than the laser linewidth. A better approach to address this measurement challenge is by determining the photon lifetime within the cavity. If U represents the energy stored within the cavity, the energy loss will be directly proportional to U : with each reflection from a cavity mirror, a consistent fraction of the electric field (and therefore the energy) dissipates from the cavity. Mathematically, we can describe the dynamics as the following differential equation:

$$\frac{dU(t)}{dt} = -\frac{U(t)}{\tau} \quad (5.2)$$

solution of this differential equation is,

$$U(t) = U_0 e^{-t/\tau} \quad (5.3)$$

The characteristic time, denoted as τ , is referred to as the intensity ring-down time or photon lifetime. This τ represents the time it takes for the stored energy to decay to $1/e$ times its initial value. In our absorption-free cavity, during the traversal of one

cavity length d , the energy decay of the light field is equivalent to $1 - R$. Expressing this per unit time, we can formulate it as $\frac{c}{d}(1 - R)$, allowing us to write:

$$\frac{dU(t)}{dt} = -\frac{c}{d}(1 - R)U(t) \quad (5.4)$$

$$\frac{1}{\tau} = \frac{c}{d}(1 - R) \quad (5.5)$$

Using this above equation we can connect the finesse to the ringdown experiment [139].

$$F = \frac{c\pi\tau}{d} \quad (5.6)$$

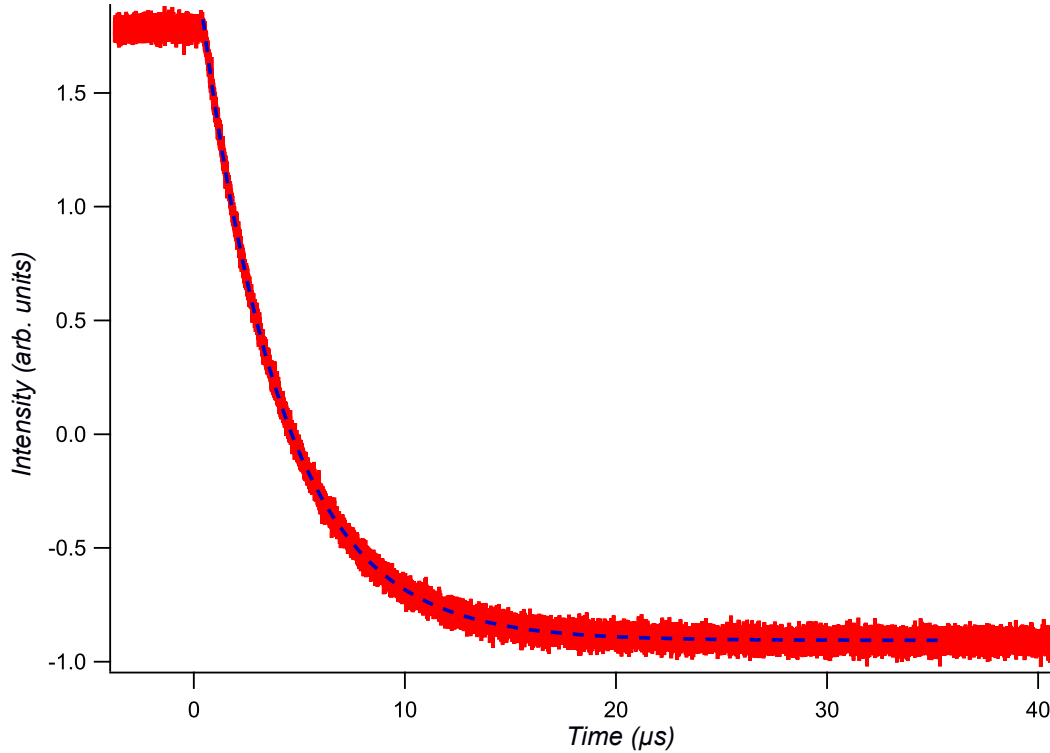


Figure 5.4: ring-down measurement: A measurement of the intensity stored within an optical cavity, known as ring-down measurement, was performed. The obtained data, depicted by the red line, matched with the fitted curve represented by the blue line. This analysis gives a ring-down time of $3.81 \mu\text{s}$, indicating a finesse of 35973 for the 10 cm long cavity.

using above equation, obtaining the finesse through a straightforward ring-down time measurement becomes easy.

To conduct such a measurement, a laser must be stabilized onto one of the resonances of the corresponding cavity, and at a certain point, it must be rapidly switched off. Traditional methods like shutting down the laser or merely blocking the beam aren't fast enough for this purpose. Hence, we utilized an acousto-optic modulator (AOM), leveraging its capabilities. AOMs operate by utilizing sound waves traveling through a crystal, forming crests that can diffract light from a laser beam. This diffracted beam exits the AOM at an angle and can be separated from the undiffracted beam. By stabilizing the diffracted beam on a cavity and swiftly shutting down the AOM, and consequently the diffracted beam, within nanoseconds, we conduct cavity ring down experiment. Fig.5.4 shows the measurement of the ring-down experiment performed.

5.2 Laser Stabilization

The process of laser stabilization refers to the adjustment of laser light frequency to maintain stability. While lasers are commonly perceived as monochromatic sources of coherent light, this characterization is only partially accurate. In reality, frequency fluctuations cause lasers to emit a range of frequencies around their central frequency, leading to the introduction of laser linewidth. Determining the linewidth involves measuring the instantaneous laser frequency over a defined time interval (Δt), plotting a histogram, and identifying its full width at half maximum (FWHM) as the linewidth. This definition of linewidth may cause confusion, as the extent of frequency variation depends on Δt , rendering the measured linewidth a function of Δt . The fast linewidth, typically observed over Δt of 10^{-5} to 10^{-4} seconds, imposes a limit on the laser's ability to resolve minute features due to its short duration compared to typical spectroscopic interaction times. Jitter, resulting from lower frequency fluctuations, can cause the fast linewidth to fluctuate along the frequency axis, often induced by vibrations usually in the kHz range. Slower changes of significant amplitude, such as those induced by temperature fluctuations, can lead to the laser frequency "drifting" over time.

Considering these definitions, our laser stabilization scheme must achieve two key objectives: preventing laser drift over time and minimizing frequency fluctuations to

narrow the laser linewidth. Additionally, it must exhibit high stability against vibrations that may occur on the optical table where the laser is situated. To meet these requirements, we need to cover a broad frequency range. To accomplish this task, we use the PDH locking scheme to narrow the linewidth and the SAS locking scheme to minimize the long-term drift of the laser and cavity combined.

5.2.1 PDH Locking

To stabilize the frequency of the laser light, we will have to measure frequency deviations of the laser very accurately and quickly. Today's electronics cannot directly respond to the oscillations of the electric field for the frequency of visible light, which is in the range of several hundred THz. Therefore, our strategy will be to take a reference, of which we know the frequency very precisely, and lock our laser to this reference. As seen in the earlier sections, an optical resonator could provide us with such a reference. The transmission signal of an optical resonator shows that the resonator only transmits light of certain frequencies that are close to the resonance frequency, and in high-finesse resonators resonances can be very narrow.

One might consider tuning the laser frequency to one of the resonances and observing the transmission signal on a photo detector (PD). Recognizing a frequency drift in the laser involves generating a frequency discriminating signal, which can then be used to correct the frequency. However, this straightforward approach fails in practicality for several reasons. Firstly, the PD cannot differentiate between a varying light intensity due to frequency change and intensity modulation of the laser itself. Secondly, the symmetry of the transmission peak of an optical resonator complicates matters further. Simply observing the transmission signal does not indicate the direction of the frequency change. Both issues can be resolved by employing the Pound-Drever-Hall locking scheme for frequency stabilization [140].

To address the issue of intensity fluctuations caused by the laser, one effective strategy is to utilize the reflected light signal instead of the transmitted light signal. At resonance, the reflection signal remains zero, regardless of the incident laser power, thereby decoupling frequency and intensity noise. However, the symmetry problem persists as the reflected light signal also exhibits symmetry around the resonance. To devise a solu-

tion, we must closely examine the reflected light signal. By assigning a transfer function to the cavity we establish a relationship between the reflected light field and the incident light field. This allows us to derive the reflection coefficient by dividing the reflected light field by the incident light field

$$R(\omega) = \frac{E_{ref}}{E_{inc}} = \frac{r(\exp(i\frac{\omega}{FSR}) - 1)}{1 - r^2 \exp(i\frac{\omega}{FSR})} \quad (5.7)$$

The complex number $R(\omega)$ represents the reflected light field, where $\omega = 2\pi\mu$ denotes the angular frequency corresponding to the laser frequency μ . It comprises an amplitude $|R(\omega)|$ that quantifies the amount of reflected light in units of E_{inc} , and a phase $\arg |R(\omega)|$ representing the phase shift experienced by the reflected light field upon reflection. By measuring the phase, we can determine whether the laser frequency is increasing or decreasing.

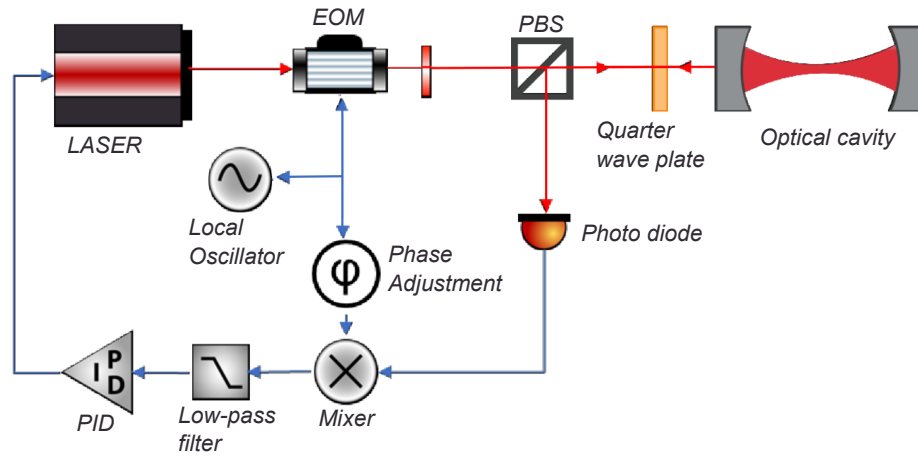


Figure 5.5: PDH locking scheme: The Pound-Drever-Hall locking scheme is depicted in the schematic setup. It showcases the light rays represented by red lines, while the flow of electrical signals is denoted by blue lines.

The PDH-locking scheme is designed precisely for this measurement. The fundamental setup for implementing PDH-locking with a diode laser is illustrated in Fig. 5.5. The diode laser emits light with a frequency ω and a field amplitude E_0 . To extract the reflected signal from the cavity, a quarter-wave plate and a polarizing beam splitter (PBS) are utilized. The initially p-polarized beam traverses the PBS without change in polarization. After passing the quarter-wave plate twice, the beam transforms

into the s-polarized state, reflects at the PBS, and can be detected by a photo detector. Before reaching the cavity, the beam undergoes phase modulation through an electro-optical modulator (EOM). An EOM is an optical device capable of shifting the phase of electromagnetic radiation in proportion to an applied voltage. In this instance, a local oscillator supplies a sinusoidal voltage signal of frequency Ω to drive the EOM.

$$E_{inc} = E_0 e^{i(\omega t + \beta \sin \Omega t)} \quad (5.8)$$

Here, β is the modulation depth which is proportional to the amplitude of the applied voltage signal to the EOM.

$$\begin{aligned} E_{inc} &\approx [J_0(\beta) + 2iJ_1(\beta) \sin \Omega t] e^{i\omega t} \\ &= E_0 [J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\omega + \Omega)t} - J_1(\beta) e^{i(\omega - \Omega)t}] \end{aligned} \quad (5.9)$$

The electric field of the reflected beam can be derived by applying the reflection coefficient to each of the three incident beams at their respective frequencies: the carrier with an unchanged frequency Ω and two sidebands with frequencies $\omega \pm \Omega$. This equation provides an intuitive explanation of the effect of the electro-optical modulator (EOM) with a sinusoidal signal.

$$\begin{aligned} E_{ref} &= E_0 [R(\omega) J_0(\beta) e^{i\omega t} + R(\omega + \Omega) J_1(\beta) e^{i(\omega + \Omega)t} \\ &\quad - R(\omega - \Omega) J_1(\beta) e^{i(\omega - \Omega)t}] \end{aligned} \quad (5.10)$$

To extract the phase information, above equation includes the reflection coefficient. The purpose behind this approach becomes evident: the rapid oscillations of the electric field are too fast to be directly observed by any electronics. Thus, we generate two additional beams with distinct frequencies but a fixed phase relationship. When these beams overlap, their individual electric fields interfere to produce an oscillating envelope known as a beat signal, with a frequency equal to the difference between the interfering beams. In our case, the beat frequency, typically in the MHz range, is much lower than the optical frequency, allowing the phase of the beating signal to be resolved with a rapid photo detector. However, it's crucial to note that the photo detector measures light intensity rather than the electric field, when calculating the intensity on the

photo detector.

$$I_{\text{ref}} = I_c |R(\omega)|^2 + I_s [|R(\omega + \Omega)|^2 |R(\omega - \Omega)|^2] + 2\sqrt{I_c I_s} \text{Im} [R(\omega) R^*(\omega + \Omega) - R^*(\omega) R^*(\omega - \Omega) \sin(\Omega t) + (2\Omega \text{ terms})] \quad (5.11)$$

We have assumed that the modulation frequency significantly exceeds the linewidth of the cavity. The first term includes oscillations with the light frequency ω , resulting in a DC signal average on the PD. The second term accounts for the desired beating signal between the carrier and the sidebands, while the third term represents the beating signal between the two sidebands themselves at a frequency of 2Ω . The electric signal can be written as:

$$U(\omega) = U_{DC} + e(\omega) \sin(\Omega t) + (2\Omega \text{ terms}) \quad (5.12)$$

With the PDH signal,

$$e(\omega) \propto \text{Im}[R(\omega) R^*(\omega + \Omega) - R^*(\omega) R(\omega - \Omega)] \quad (5.13)$$

By using a frequency mixer, we perform the mathematical operation of multiplication to isolate the error signal by comparing it with the local oscillator signal.

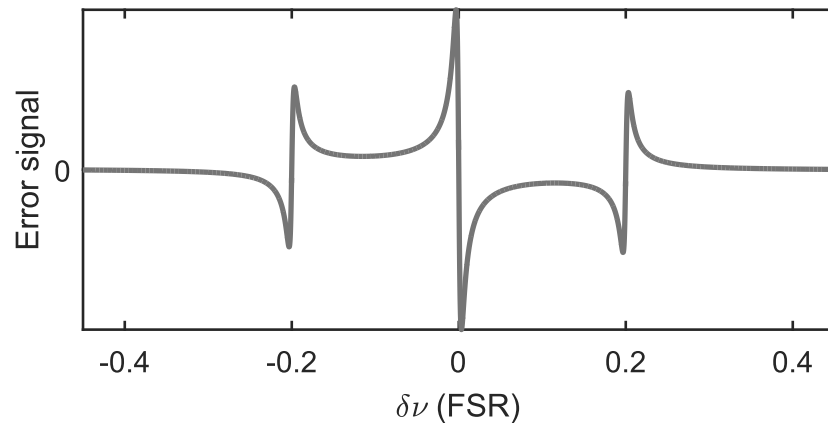


Figure 5.6: PDH error signal: when $\Omega \gg \frac{FSR}{F}$ error signal after low pass filter is depicted in the figure.

$$\sin(\alpha x)\sin(\alpha' x) = \frac{1}{2}\cos[(\alpha - \alpha')t] - \cos[(\alpha + \alpha')t] \quad (5.14)$$

multiplier output will be

$$\begin{aligned} U_{\text{out}} &= [U_{\text{DC}} + e(\omega) \sin(\Omega t)] \times \sin(\Omega t) \\ &= \frac{1}{2}e(\omega) + U_{\text{DC}} \sin(\Omega t) - \frac{1}{2}e(\omega) \cos(2\Omega t) \end{aligned} \quad (5.15)$$

The remaining oscillating terms in above equation are eliminated by the low-pass filter, resulting in the extraction of the error signal, shown in fig. 5.6.

5.3 Optical Cavity Housing

The ultrastable reference cavity system comprises two integral components: the reference cavity and the cavity housing. The primary function of the ultrastable reference cavity housing is to establish an environment that is exceptionally well shielded from external influences.

5.3.1 Optical Cavity

The ULE optical cavity, manufactured by Stable Laser Systems (SLS), has a zero-crossing temperature measured at 28° C, as provided by SLS. This model has supporting positions calculated to be 57.7 mm apart, making the spacer vibration-insensitive (Airy points). The cavity spacer has a diameter of 50 mm, and the mirrors are separated by 100 mm, resulting in a free spectral range (ν_{FSR}) of 1.5 GHz. The mirrors, made from a ULE substrate, include one plane mirror and another with a radius of curvature of 500 mm. These mirrors are coated to achieve a finesse of approximately 35,000 at wavelengths of 689 nm.

5.3.2 Stability of Optical Cavity

The resonant frequencies and stability of an optical resonator can be affected by various factors. Resonance occurs when the frequency ν of light aligns with a multiple of the

free spectral range of the cavity, where the cavity length is denoted by d .

$$\nu = \frac{lc}{2dn} \quad (5.16)$$

Where, the longitudinal mode number denoted as l , the speed of light in a vacuum denoted as c , and the index of refraction of the material within the resonator denoted as n . Above equation shows that these resonant frequencies are influenced by two variables prone to variations: the cavity length d and the index of refraction n . To understand the impact of changes in d or n on ν , we compute the total derivative of Eqn 5.16 and divide the result by Eqn 5.16 This gives an expression for the fractional stability of ν .

$$\frac{\Delta\nu}{\nu} = -\frac{\Delta d}{d} - \frac{\Delta n}{n} \quad (5.17)$$

To achieve a frequency stability of 1 Hz, it is necessary to maintain a length stability in the range of 10^{-16} m. This means a level of length stability comparable to the proton radius.

5.3.3 Effect of Temperature

Length changes are connected with change in temperature, as

$$\frac{\Delta d}{d} = \alpha(T)\Delta T + \frac{1}{2}\alpha'(T)\Delta T^2 + \mathcal{O}(\Delta T^3) \quad (5.18)$$

Ideally, the spacer material, which holds the mirrors at a constant distance, should have a very low coefficient of linear thermal expansion (CTE). This ensures that the fractional length change ($\frac{\Delta d}{d}$) remains as minimal as possible. The equation expresses the correlation between temperature changes (ΔT) and the resulting fractional length change, with α representing the CTE coefficient and $\alpha'(T)$ its first derivative with respect to T . To further mitigate the impact of temperature fluctuations, spacers can be operated at a temperature where the coefficient of linear thermal expansion (CTE) crosses zero, denoted by $\alpha(T) = 0$. For certain materials, there exists a temperature at which the first-order contribution of CTE vanishes. Considering $\alpha'(T) = 1.7 \times 10^{-9} K^{-1}$ for ULE-glass near its zero-crossing temperature.

5.3.4 Effect of Acoustic and Vibrations

Vibrations cause changes in cavity length, induce bending and deformation in the spacer material. The total length change (Δd) of a material subjected to an external force (F) is given by its Young's modulus.

$$E = \frac{Fd}{A\Delta d} \quad (5.19)$$

To enhance the system's resistance to vibrations, various methods must be employed, as our selection of materials with low CTE is already restricted. One potential strategy involves maximizing Young's modulus or opting for a spacer with a larger cross-section. However, these approaches may only marginally improve performance.

To significantly reduce the sensitivity of the cavity length to vibrations by several orders of magnitude, one effective approach is to strategically support the optical resonator. This support should be designed in a way that vibration-induced length changes cancel each other out, preserving the total length of the cavity. The identification of these specialized support points can be achieved through finite element analysis, and is crucial in the cavity housing design one such method is Airy point identification.

An alternative approach to isolating the cavity from vibrations involves utilizing a vibration isolation platform. These platforms are available in both active and passive versions, offering a diverse range of sizes. Additionally, choosing elastic materials as supporting pads for the cavity proves effective in absorbing vibrations, especially when these vibrations traverse multiple layers. To counteract acoustical noise, constructing an acoustical shield around the cavity system is a simple yet effective solution.

5.3.5 Effect of Pressure Fluctuation

Changing the refractive index of the medium between the cavity mirrors, pressure fluctuations induce changes in the optical path traversed by a light ray. At one atmosphere pressure, air is characterized by a refractive index of $\eta_{air} = 1.0003$, while vacuum, defined as having $\eta_{vac} = 1$, serves as a reference. Utilizing a linear dependence assumption, the refractive index as a function of pressure (p) can be interpolated from these two values.

$$\eta(p) \approx 1 + \frac{3 \times 10^{-7}}{\text{mbar}} p \quad (5.20)$$

Using this rough estimate, and assuming that the absolute pressure (p) is on the order of 1mbar, we can determine the impact of pressure fluctuations on frequency stability.

$$\frac{\Delta\nu}{\nu} \approx -\frac{3 \times 10^{-7}}{\text{mbar}} \Delta p \quad (5.21)$$

To maintain the frequency stability in the range of 10^{-15} , it is essential to ensure a pressure stability of at least 10^{-8} mbar, as indicated by Eqn 5.21. Consequently, storing the cavity in a vacuum becomes necessity. This vacuum chamber offers additional benefits, such as the suppression of thermal heat conduction through convection and the prevention of acoustic propagation.

5.3.6 Design of Housing

The effectiveness of decoupling the reference cavity from environmental disturbances is heavily influenced by the chosen design for the cavity housing. To achieve this, the entire vacuum chamber is positioned on a vibration isolation optical table. This table has low natural frequencies, registering at 2.5 Hz vertically and 3 Hz horizontally.

The main chamber is made of stainless steel SS304 alloy. This chamber acts as a low-pass filter for temperature variation. In the front, an anti-reflective (AR) coated optical viewport is attached, and the same is done on the backside. These viewports are at an 8-degree angle from the optical axis to avoid back reflection during the coupling process. On the side, we have attached a 4-pin electrical feedthrough and an all-metal angle valve for sealing, along with a 35 l s^{-1} ion pump to maintain the vacuum in the chamber.

Inside the vacuum chamber, we have used an aluminum enclosure with a wall thickness of 10 mm. This box is actively temperature stabilized, and aluminum is well-suited for this purpose. We have attached two Peltier devices at the bottom of the box and one aluminum plate on the other side of the Peltier as a heatsink. We have used a TH10K thermocouple for temperature monitoring. Inside the box, we have made a square extrusion that fits the Viton pad. This Viton pad supports the cavity holder. The cavity

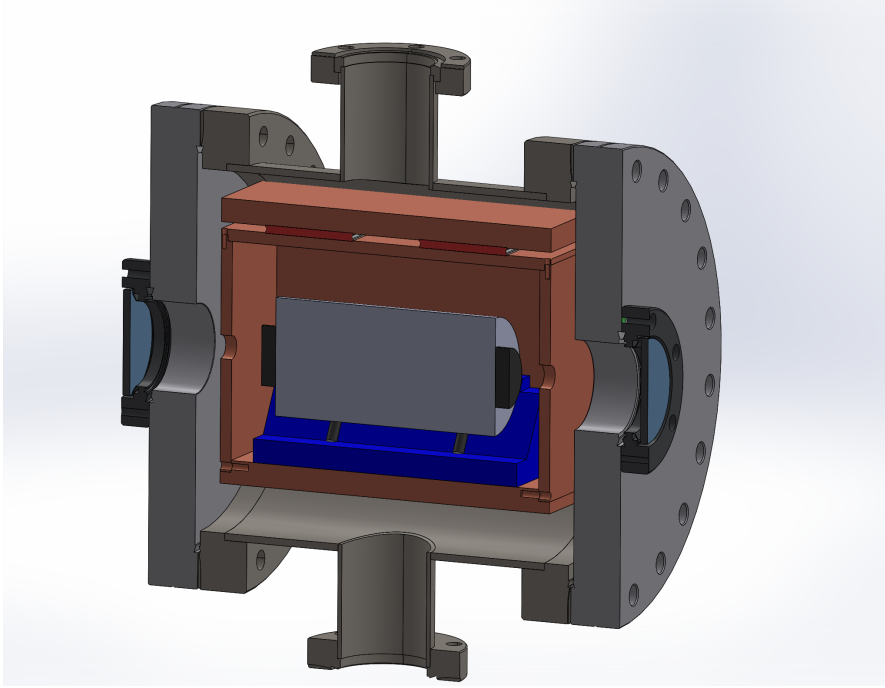


Figure 5.7: Optical cavity housing: The stainless steel (SS) vacuum chamber is positioned on the vibration isolation table. On the rear side, an ion pump is connected via a nipple to the chamber. Additionally, an angle valve is connected to another port of the vacuum chamber, allowing for the connection of a turbo pump via a flexible bellow. The copper box is temperature-stabilized with Peltier elements inside. Within the copper box, the SS cavity holder is colored blue, and the optical cavity rests on airy points with Viton supports.

holder is made of SS316L and has notches at the airy points of the cavity for holding.

Our final vacuum assembly reaches the vacuum level of 10^{-8} torr, with a temperature stability of better than 100 mK.

5.4 Optical Cavity Locking Procedure

5.4.1 PDH Setup

The components for the optical setup must be chosen carefully and optimized for stability and minimal distortion of beam quality. A high-extinction ratio Polarizing Beam Splitter (PBS), such as Glan-Taylor Calcite Polarizers, is necessary, and precise polarization matching is required. Since light is sourced from the fiber, polarization noise can be minimized by aligning the polarization with the polarization-maintaining axis of the fiber at the input side. The free space EOM requires a dedicated resonance circuit

and can only operate within the range imposed by the resonance circuit. The free space EOM, model number PM689, was purchased from QUBIG GmbH.

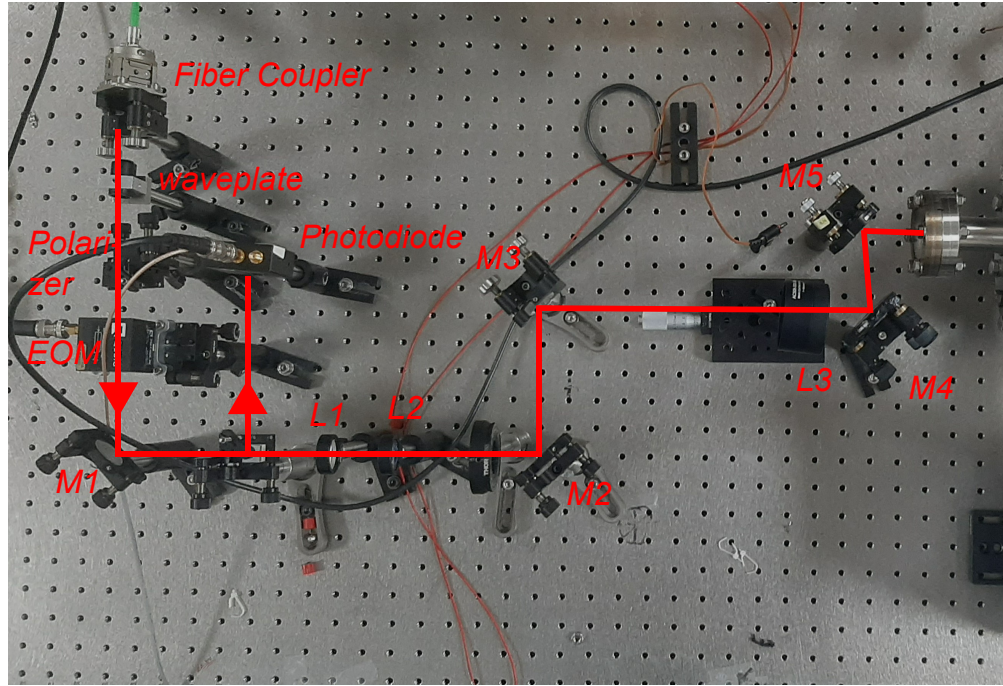


Figure 5.8: cavity setup: As shown in the figure, light is collimated using a fiber collimator. After that, we use a waveplate and a Glan-Taylor polarizer to purify the extinction ratio of polarization. We use an Electro-Optic Modulator (EOM) to generate sidebands in the light. The telescope configuration of L1 and L2 is used to increase the beam diameter, and L3 is used to focus the light beam into the cavity. M4 and M5 are mainly used during the alignment of light inside the cavity.

To correct the beam path, we placed one one-inch mirrors after the EOM. These mirrors were mounted in Newport precision mirror mounts for their stability properties. Adjustment of these mirror was done to ensure that the beam propagates along the desired optical axis of the following telescope, which is crucial for achieving a good mode matching. The telescope comprises two lenses with focal lengths of $f = 150$ mm and $f = -50$ mm. The second lens is mounted on a translation stage to allow for fine adjustment of the beam shape. Following this, there is an achromatic doublet with a focal length of 200 mm to focus the beam onto the plane surface of the cavity.

Both the $\lambda/4$ waveplate and the polarizing beam splitter are necessary to detect the reflected signal from the cavity. When the beam is reflected from the cavity, the light

passes through the waveplate, mounted on the custom mount, transforming it into p-polarized light. This p-polarized light is then reflected from the polarizing beam splitter (PBS). The waveplate, also mounted on a custom mount, transforms the incoming s-polarized light into circularly polarized light. On the back side of the cavity, we use a one-inch non-polarizing beamsplitter (NPBS) to divide the light for simultaneous diagnostics on a photodiode and the camera. We use a Thorlabs DCU223M camera for monitoring the modes of the cavity and locking. The photodiode is a Thorlabs, model DTC100A module.

The incident laser beam emerges from a polarization-maintaining single-mode fiber and exits the fiber in the TEM_{00} mode. Mode-matching refers to the process of adjusting the incoming beam to achieve maximum overlap with the TEM_{00} eigenmode of the cavity. To accomplish this, we utilize the telescope to adjust the q-parameter of the incoming laser beam to the appropriate shape and position. The two coupling mirrors located in front of the cavity are used to align the incoming beam with the optical axis of the Fabry-Perot resonator.

5.4.2 Coupling Procedure

To generate a robust error signal, it is crucial to maximize the coupling into the fundamental cavity mode. Our approach to achieve this involves three main steps: initial cavity alignment to reduce the excited mode order, identification of the fundamental mode and laser resonance adjustment, and finally, direct improvement of coupling. For the first step, we collimate the incident beam, which is accomplished by measuring the profile of beam over extended distances. Mode-matching the beam with the cavity involves aligning the Gaussian beam curvature with the mirrors curvature. Manufacturers typically provide the measured radius of curvature for the mirrors, specifying the calculated beam waists ($W1$, $W2$) at the mirror positions and the associated focal waist ($W0$). For instance, in a plano-concave cavity, the beam should be focused on the planar mirror, with beam waist set by the radius of the concave mirror. For achieving the desired beam waist of approximately $200\ \mu\text{m}$, we utilize a beam profiler to measure the beam waist at a position equivalent to the planar mirror or the midpoint of a confocal cavity.

To initiate the alignment process, we focus on the back-reflection off the cavity, considering the high reflectivity of the cavity mirrors resulting in a strong back-reflection. A distinction between the reflection from the front surface of the cavity mirror and the “cavity reflection” entering and reflecting within the cavity.

For finer alignment, we use a sensitive CCD camera (Thorlabs DCU223m) to directly image the transmitted cavity modes. The camera, operating without a lens, provides high sensitivity at low light levels, simplifying the alignment procedure with spatial information. We place a filter in front of the camera to block ambient light when possible. Position the camera to face the output window of the cavity, and slowly scan the laser frequency. This scanning process reveals various patterns on the camera as different transverse cavity modes resonate at different frequencies. Note that due to the high intracavity power, we limit the incident power to approximately 1mW to prevent damage to the cavity mirrors. During error signal optimization, we further reduce the incident optical power to microwatts to prevent destabilization from heating effects.

For a preliminary alignment, we perform a laser frequency scan over a range exceeding the Free Spectral Range (FSR) of the cavity, with an extended exposure time of typically 20 ms for the camera to capture an integrated image of all supported modes. The averaged pattern gives idea about required alignment adjustments to enhance coupling into the desired fundamental mode. We maximize the intensity at the center of the pattern to avoid optimizing for low-order modes such as TEM₀₁ and LG₀₁, which exhibit zeros in the center. When the cavity is significantly misaligned, rectangular Hermite-Gauss (TEM) modes with non-degenerate horizontal and vertical axes dominate. As alignment improves, the influence of circularly symmetric Laguerre-Gauss (LG) modes becomes more pronounced. Therefore, the transmitted light’s spatial structure provides valuable information about cavity alignment.

To fine-tune alignment, we reduce the laser scan range to approximately 100 MHz and manually adjust the frequency until a Gaussian transmitted beam profile is observed. This Gaussian profile represents the fundamental mode of the cavity, offering a robust locking signal. since, the high finesse cavity may have a linewidth less than the free-running linewidth of the laser, it increases the probability of sweeping past the mode without getting captured on the camera.

5.4.3 Generating Error Signal

The PDH technique generates an error signal by introducing frequency sidebands to the laser beam, creating beatnotes that are demodulated using a reference frequency. The modulation frequency and depth are crucial parameters, providing flexibility as long as the frequency exceeds the cavity linewidth and fits within the photodetector's bandwidth.

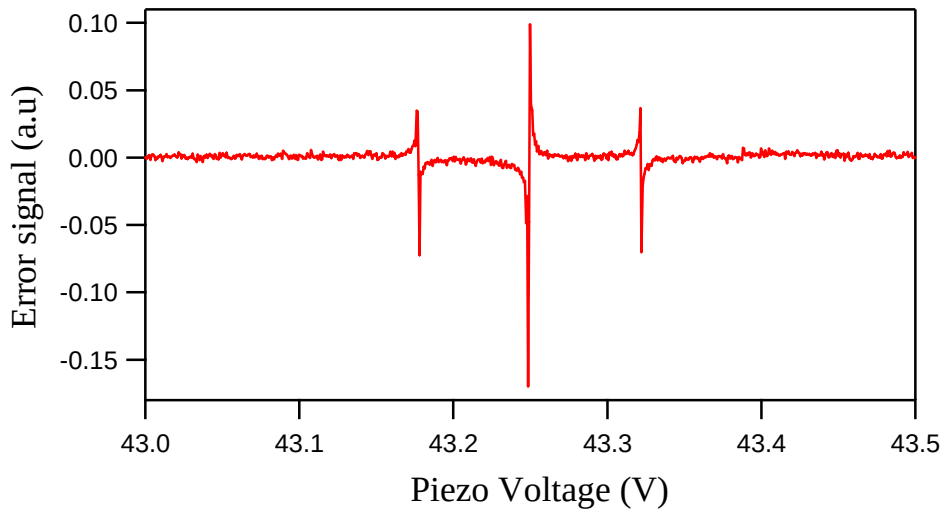


Figure 5.9: PDH error signal: when $\Omega \gg \frac{FSR}{F}$ error signal after low pass filter is depicted in the figure.

After we identify the fundamental mode, reflections from the cavity are measured using a photodetector without modulation. We introduce modulation which creates sidebands, and the error signal is obtained by demodulating the photodetector signal. For performing the error signal generation, we used the Toptica PDH module, where we can perform the above said task digitally. After getting the error signal, we vary the phase to ensure zero crossing of the error signal and align it with the polarity of the PID output.

5.4.4 Lock Optimization

We use FALC module from toptica perform the locking of laser to the zero crossing of the error signal. we follow procedure outlined in the toptica manual. Although here we have provided some practical procedure to optimize the lock in our system.

We make small adjustments to the “ERR OFFSET” may enhance the lock quality, ensuring the electronic zero of the error signal coincides with the center of the cavity resonance. We are very cautious about isolating the signal path and ensuring that ground connections are physically close together, as the integrators of the servo loops have very high gain at low frequency, making the feedback loop sensitive to power-line pickup noise and ground loops.

After following this procedure, we robustly lock the laser to the fundamental mode of the cavity. Our lock remains stable throughout the entire day without encountering any issues.

5.4.5 SAS Locking

To perform spectroscopy on this narrow transition of strontium, achieving a high optical density in the cell is challenging due to its narrowness. Our cell design, inspired by the Stellmer thesis [131], is longer and narrower than our other spectroscopy cells. To maximize interaction with the atomic cloud, we blow up the beam to fill the entire diameter of the tubes. Technical drawings for the current cell design are available in Appendix I. A large, powerful magnet near the cell is utilized to split the mJ states, resulting in a narrower signal.

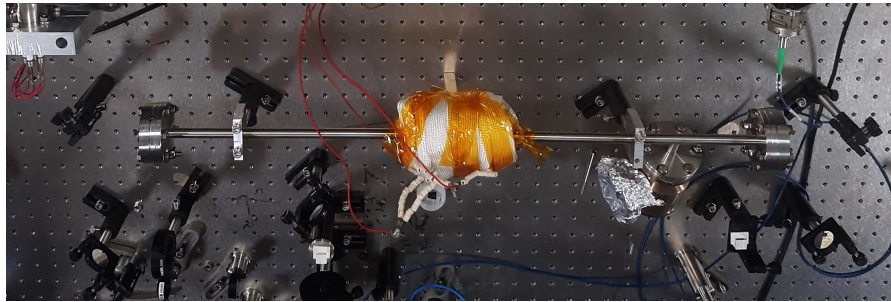


Figure 5.10: SAS setup: We used a heat pipe to perform saturation absorption spectroscopy for the 689 nm transition of Sr. The heat pipe is approximately 60 cm long, with strontium pieces positioned in the middle and a heater wrapped around it. Additionally, a magnet is placed near to middle of heat pipe to lift the degeneracy of Zeeman levels.

The saturation spectroscopy setup, divides light from the diode locked to the external cavity into a pump beam (approximately 5 mW) and a probe beam (around 800 μ W) [141]. The details of the SAS is given in Ref. [142, 143]. Because of the Doppler

shift, the pump and probe frequencies only align for a specific velocity group of atoms moving along the probe light direction. The probe light undergoes frequency modulation (FM) by the current modulation of laser at a rate of 200 kHz to generate spectroscopy error signals. Two telescopes magnify both the pump and probe to a 7 mm waist before entering the heat cell. Nearly half of the probe light is split and detected (PD2) just before the strontium gas cell to monitor amplitude modulation. The remaining probe light passes through the gas cell and is detected (PD1) to produce an error signal after demodulation. A small magnet is positioned near the pipe center to increase the splitting between different Zeeman sub-levels in the 3P_1 state. Scanning the light frequency near the atomic transition reveals a Doppler broadened absorption with a 50% depth. The error signal corresponding to the center saturation peak ($m_J = 0$) is approximately 800 kHz wide (the frequency difference between the maximum and minimum of the dispersive error signal). We use toptica auto lock system to lock the laser to the spectroscopy peak.

In conclusion of this chapter, we've explored the architecture of the 689 nm laser system, important for efficient optical cooling and trapping on narrow linewidth transition. Using a Toptica DLPro master laser and a high finesse optical cavity, we achieved precise linewidth control, crucial for Doppler cooling to high phase space density. Through Pound-Drever Hall and Saturation Absorption Spectroscopy methods, we ensured laser stability and precise locking to the desired transitions. Addition of an external optical isolator mitigated unwanted feedback, stabilizing the master laser and facilitating the observation of the red MOT in our experiments. We achieve a red Magneto-Optical Trap (MOT) of Sr, with a final temperature of $\sim 10 \mu\text{K}$. Further details on a red MOT can be found in Mr. Korak Biswas's thesis.

Chapter 6

Spectroscopy of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ Transition of Strontium Using Laser Cooled Atoms

The results presented in this chapter have been published in:

“Spectroscopy of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition of strontium using laser cooled atoms”

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*Journal of Physics B: Atomic, Molecular and Optical Physics, **10**, 105501 (2024)*

This chapter details the experiments conducted with strontium (Sr) atoms in a magneto-optical trap (MOT) operating on the 461 nm cooling transition. The first section outlines the introduction, followed by the necessary theoretical background for system description. Subsequently, the experimental setup and parameters for conducting spectroscopy of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition are explained. All these experiments utilize laser-cooled atoms in a blue MOT. The enhancement factor for each strontium isotope MOT, transition frequency, hyperfine splitting constant, and isotope shift are determined. A comparison is made between the experimental values and those computed using the Fock-space relativistic coupled-cluster (FSRCC) method. Theoretical

calculations was performed by our collaborators Palki Gakkhar and B. K. Mani.

6.1 Introduction

As discussed in section 1.3, the development of the optical frequency comb has accelerated efforts toward employing optical transitions as a universal time standard. Among various approaches, platforms based on neutral atoms have gained popularity in realizing the next generation of optical clocks. These clocks show potential to revolutionize global timekeeping, precision sensing, and the examination of the stability of fundamental constants with time. However, from an experimental perspective, it is challenging to fully isolate atoms from external perturbations. Consequently, the accuracy of the clock relies on precisely quantifying the various systematic shifts in the clock transition frequency.

An indicator of clock performance is determined by the Allan deviation, which characterizes the fractional frequency instability σ . When considering a signal-to-noise ratio determined by the fundamental quantum projection noise, the instability can be expressed as:

$$\sigma(\tau) \simeq \frac{\sqrt{T}}{Q\sqrt{N\tau}} \quad (6.1)$$

In the given formula, Q is the quality factor of the resonance, expressed as $Q = \nu/\Delta\nu$, where ν represents the transition frequency, and $\Delta\nu$ represents the linewidth. N corresponds to the overall number of particles (atoms) participating in the measurement during a coherence time period T , and τ signifies the total averaging time. According to the above equation, atoms with the highest quality factor Q will result in a lower quantum projection noise, and a larger number of atoms is also advantageous for mitigating quantum projection noise. With the use of $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition we can increase the number of atom in the MOT which will eventually help in increasing the SNR in a clock experiment and also in lowering the quantum projection noise. Conventionally to increase the number of atoms in the MOT, atoms are repumped from the $5s5p\ ^3P_2$ state using dual repumper lasers operating at 707 nm ($5s5p\ ^3P_2 \rightarrow 5s6s\ ^3S_1$) and 679 nm ($5s5p\ ^3P_0 \rightarrow 5s6s\ ^3S_1$). In this investigation, we present an innovative dual-repumping approach, employing a 483 nm laser to address the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$

transition, and a 707 nm laser for inducing the $5s5p\ ^3P_2 \rightarrow 5s6s\ ^3S_1$ transition. The 483 nm repumping laser can also be utilized in conjunction with other single repumping schemes, such as 481 nm ($5s5p\ ^3P_2 \rightarrow 5p^2\ ^3P_2$), 496 nm ($5s5p\ ^3P_2 \rightarrow 5s5d\ ^3D_2$), and 405 nm ($5s5p\ ^3P_2 \rightarrow 5s6d\ ^3D_2$), to prevent losses into the 3P_0 state, thereby enhancing the number of atoms in the magneto-optical trap (MOT). This transition also finds application in clock spectroscopy, allowing the probing of the atom count in a long-lived clock state by coupling a $5s5p\ ^3P_0$ state to the ground state using a 483 nm laser.

In frequency metrology, precise determination of systematic shifts holds significant importance. The primary source of systematic frequency correction and uncertainty within the clock comes from the blackbody radiation (BBR) at room temperature. The fractional frequency shift, denoted as $|\delta\nu_{BBR}/\nu_0|$, triggered by the BBR, is directly proportional to the differential static polarizability between the two clock states. The uncertainty associated with the blackbody radiation (BBR) shift primarily arises due to a limited understanding of the static polarizability of the $5s5p\ ^3P_0$ state. To enhance the accuracy of strontium (Sr), it is evident that a more comprehensive understanding of Sr properties is necessary for a more precise determination of the BBR shift.

Enhancing the accuracy of the Sr clock necessitates a more precise determination of the differential static polarizability of the $5s^2\ ^1S_0$ and $5s5p\ ^3P_0$ states. It's important to note that the static polarizability of the ground $5s^2\ ^1S_0$ state is known with a sufficiently low uncertainty of approximately 0.1p (p-value of measurement). This level of certainty is achieved through a comprehensive understanding of the matrix element $|\langle ^1S_0 || d || ^1P_1 \rangle|$, where the intermediate state $5s5p\ ^1P_1$ contributes significantly to the polarizability of the 1S_0 state at a dominant level of 97p (p-value of measurement). Consequently, the primary challenge lies in reducing the uncertainty of the polarizability of the 3P_0 state to a similar degree. Even advanced modern relativistic methods of atomic calculations fall short of achieving such precision. Therefore, addressing this issue requires a combined approach integrating theoretical and experimental methods.

In this chapter, through experimental measurements and theoretical calculations, we present the branching ratio, transition linewidth, hyperfine splitting constants, and isotope shift of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition. We also study the dependence of atom number enhancement in the MOT by resonantly driving the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$

transition at 483 nm in addition to $5s5p\ ^3P_2 \rightarrow 5s6s\ ^3S_1$ transition at 707 nm for all the stable isotopes of strontium. We compare the performance of 483 nm repumper with 679 nm which is conventionally used as a repumper. Furthermore, in this work we measure the absolute frequency of the 483 nm transition in ^{88}Sr , ^{87}Sr , ^{86}Sr , and ^{84}Sr isotopes with an absolute accuracy of 30 MHz, and isotope shift is compared with our calculated data.

6.2 Experimental details

In Figure 5.1, the energy levels and transitions relevant to the laser cooling and trapping of strontium are illustrated. Atoms initially in the $5s^2\ ^1S_0$ ground state are driven to the $5s5p\ ^1P_1$ state using a 461 nm laser. The $5s5p\ ^1P_1$ state has a decay probability of 1 in 50,000 to the $5s4d\ ^1D_2$ state, which further decays to the $5s5p\ ^3P_{1,2}$ with a branching ratio of 2:1. Meanwhile, atoms in the $5s5p\ ^3P_2$ state are excited to the $5s6s\ ^3S_1$ state using a 707 nm laser, which subsequently decay to the $5s5p\ ^3P_{0,1,2}$ states with a branching ratio of approximately 5:3:1. The atoms in the $5s5p\ ^3P_0$ state are excited to the $5s5d\ ^3D_1$ state using a 483 nm laser, which decays to the $5s5p\ ^3P_{0,1,2}$ states with a branching ratio of approximately 56:41:3, ultimately populating only the $5s5p\ ^3P_1$ state. Atoms in the short-lived $5s5p\ ^3P_1$ state decay back to the ground state thereby again making them participate in the cooling cycle, hence inhibiting the loss in the atom number. In the experiment, atoms are loaded into a three-dimensional magneto-optical trap from a Zeeman slower [144, 145]. A blue MOT is created using three retro-reflected beams of 461 nm each, with an intensity of 10 mW/cm², a detuning of -45 MHz ($\sim 1.5\Gamma$), and a beam diameter ($1/e^2$) of 12 mm. An axial magnetic field gradient of 50 G/cm is maintained throughout the experiment. The blue light is produced using a second harmonic generation (SHG) cavity and an external-cavity diode laser (ECDL). A portion of the output light from the SHG cavity is used for injection-lock the high-power Nichia 460 nm laser diode [NDB 4916]. The Zeeman slower light is obtained from the SHG output, while the light for the MOT beams is derived from the injection-locked output. The repumping light at 707 nm is generated using a home-built external-cavity diode laser (ECDL), and the 483 nm light is generated using a commercial [Toptica DL pro] ECDL with output powers of 25 mW and 20 mW respectively. More details about

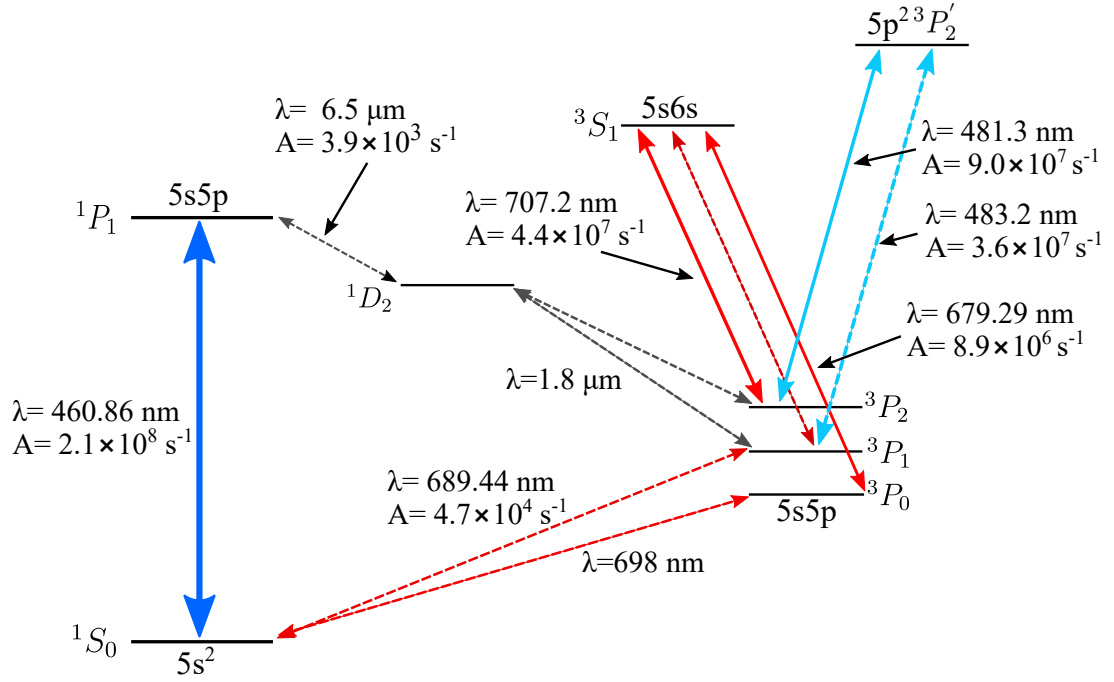


Figure 6.1: Strontium energy level diagram: The relevant low-lying energy level diagram of strontium (^{88}Sr) atoms. The wavelength (λ) and the decay rate (A) has been shown along with the transition.

the experimental set-up can be found in references [146, 147]. More detailed description of blue MOT preparation is described in ch 4.

The frequency of the laser (483 nm) used for spectroscopy is continuously monitored using a commercial wavemeter [HighFinesse WSU/30], which is regularly calibrated with a 689 nm laser locked to the Sr atom. The 689 nm laser is locked to a temperature stabilized, in-vacuum high-finesse optical cavity, exhibiting a measured drift rate of less than 200 Hz per hour. The absolute frequency is established through the spectroscopy of the $5s5p \rightarrow 3P_1$ state in strontium - the frequency of which is known to within 10 kHz [148]. The wavemeter is specified to have an absolute accuracy of 30 MHz and a measurement resolution of 1 MHz.

6.3 Theoretical background

To support our experimental data, we performed Fock-space relativistic coupled-cluster (FSRCC) calculations of the relevant experimental properties for the repumping transition demonstrated in this work. The relativistic coupled-cluster theory is one of the most accurate many-body theories for the calculation of atomic properties. In FSRCC theory, within the coupled-cluster singles and doubles approximation, the many-electron wavefunction for a *two* valence electrons atom or ion, like atomic Sr which is the interest of the present work, is expressed as

$$|\Psi_{vw}\rangle = e^{(T_1+T_2)} \left[1 + S_1 + S_2 + \frac{1}{2} (S_1^2 + S_2^2) + R_2 \right] |\Phi_{vw}\rangle, \quad (6.2)$$

where $v, w \dots$ represent the valence orbitals and $|\Phi_{vw}\rangle$ is the Dirac-Fock reference state. In Fock-space formalism, it is obtained by adding two electrons to the Dirac-Fock state for closed-shell configuration, $a_w^\dagger a_v^\dagger |\Phi_0\rangle$. The excitation operators, T , S and R are referred to as the coupled-cluster operators for *closed-shell*, *one-valence* and *two-valence* sectors, respectively, and expressed in terms of the creation and annihilation operators [149–151]. The many-electron wavefunction $|\Psi_{vw}\rangle$, in Eq. (6.2), is obtained from the solution of the following eigenvalue equation

$$H^{\text{DCB}} |\Psi_{vw}\rangle = E_{vw} |\Psi_{vw}\rangle, \quad (6.3)$$

where E_{vw} is the exact energy of the system. And, H^{DCB} is the Dirac-Coulomb-Breit no-virtual-pair Hamiltonian, expressed as

$$H^{\text{DCB}} = \sum_{i=1}^N [c\boldsymbol{\alpha}_i \cdot \mathbf{p}_i + (\beta_i - 1)c^2 - V_N(r_i)] + \sum_{i<j} \left[\frac{1}{r_{ij}} + g^{\text{B}}(r_{ij}) \right]. \quad (6.4)$$

Here, $\boldsymbol{\alpha}$ and β are the Dirac matrices, and the last two terms, $1/r_{ij}$ and $g^{\text{B}}(r_{ij})$, represent

the Coulomb and Breit interactions, respectively. It is to be however mentioned that, in the actual calculations, we have also incorporated the contributions from the quantum electrodynamical (QED) corrections such as the self-energy and vacuum polarization. To improve the accuracy of our computed properties further, we have also incorporated the triples excitations in coupled-cluster perturbatively. More details about the implementation of these interactions in our calculations and FSRCC theories and corresponding technical details, in general, can be found in our previous works [149–151].

After we get the wavefunction in Eq. (6.2) from FSRCC calculations, we can use it for the properties calculations [149–151]. In the present work, to support our experimental findings related to the demonstrated repumping scheme, we have calculated the excitation energy, magnetic dipole and electric quadrupole hyperfine constants and energy splitting for the repumping state $5s5d\ ^3D_1$. Further, we have calculated the transition rates for $5s5p\ ^3P_{0,1,2} \rightarrow 5s5d\ ^3D_1$ and $5s5p\ ^1P_1 \rightarrow 5s5d\ ^3D_1$ transitions. These transition rates are used to calculate the transition linewidth and branching ratios associated with repumping transition, which are discussed later. The upper bound in the uncertainty of our calculated properties using FSRCC is $\approx 4.5\%$ [150]. In addition, to support our isotope shift measurements, we have calculated the isotope shifts in transition frequencies using multiconfiguration Dirac-Hartree-Fock (MCDHF) theory, results from which are provided and discussed later.

6.4 MOT Repumping Scheme

In Fig. 6.1, the energy levels and transitions relevant to the laser cooling and trapping of strontium are illustrated. Atoms initially in the $5s^2\ ^1S_0$ ground state are driven to the $5s5p\ ^1P_1$ state using a 461 nm laser. The $5s5p\ ^1P_1$ state has a decay probability of 1 in 50,000 to the $5s4d\ ^1D_2$ state, which further decays to the $5s5p\ ^3P_{1,2}$ with a branching ratio of 2:1. Meanwhile, atoms in the $5s5p\ ^3P_2$ state are excited to the $5s6s\ ^3S_1$ state using a 707 nm laser, which subsequently decay to the $5s5p\ ^3P_{0,1,2}$ states with a branching ratio of approximately 5:3:1. The atoms in the $5s5p\ ^3P_0$ state are excited to the $5s5d\ ^3D_1$ state using a 483 nm laser, which decays to the $5s5p\ ^3P_{0,1,2}$ states with a branching ratio of approximately 56:41:3 [152], ultimately populating only the $5s5p\ ^3P_1$ state. Atoms in the short-lived $5s5p\ ^3P_1$ state decay back to the ground state thereby

Table 6.1: Magnetic dipole and electric quadrupole hyperfine structure constants for repumping state, $5s5d\ ^3D_1$, and transition probabilities for transitions from $5s5p\ ^3P_{0,1,2}$ and $5s5p\ ^1P_1$ states to $5s5d\ ^3D_1$ state in ^{87}Sr . The third, fourth and fifth column represent the corrections from the Breit interaction, self-energy and vacuum-polarization, respectively.

State/Transition	FSRCC	Breit	Self-energy	Vac.-Pol.	Breit + QED
A (in MHz)					
3D_1	239.89	0.03	-6.61	0.29	-6.29
B (in MHz)					
3D_1	5.98	-0.01	0.35	0	0.34
Transition Rate (in 10^7 s^{-1})					
$^3P_0 \rightarrow ^3D_1$	3.3337	-0.0013	0.0649	0.0625	0.1262
$^3P_1 \rightarrow ^3D_1$	3.1461	-0.0002	0.0475	0.0470	0.0943
$^3P_2 \rightarrow ^3D_1$	0.0780	-0.0001	0.0018	0.0017	0.0034
$^1P_1 \rightarrow ^3D_1$	0.0063	0.0	-0.0004	-0.0004	-0.0007

Transition	Triples	Total	Exp.
A (in MHz)			
3D_1	6.96	240.56	250(10) (Present work) 227.3(7) (Ref. [152])
B (in MHz)			
3D_1	-0.54	5.78	5.3(42) (Present work) 0(10) (Ref. [152])
Transition Rate (in 10^7 s^{-1})			
$^3P_0 \rightarrow ^3D_1$	0.0649	3.5248	
$^3P_1 \rightarrow ^3D_1$	0.0476	3.2879	
$^3P_2 \rightarrow ^3D_1$	0.0018	0.0832	
$^1P_1 \rightarrow ^3D_1$	-0.0004	0.0052	

again making them participate in the cooling cycle, hence inhibiting the loss in the atom number.

To get more insight into the repumping transition and associated properties, using the transition probabilities listed in Table 6.1, we calculated the branching ratio and transition linewidth for the repumping transition. As evident from the table, to increase the accuracies of these properties, the corrections from the Breit and QED interactions and perturbative triples are also incorporated at the level of transition probabilities. As

mentioned earlier, the proposed repumping transition could also be useful for the atomic clocks experiments, and therefore the knowledge about the branching ratios of 3D_1 into $^3P_{0,1,2}$ and 1P_1 states could be crucial for minimizing the statistical uncertainty.

The branching ratio of 3D_1 to a lower atomic state function (ASF) $|\Psi_i\rangle$ is calculated as

$$BR_{^3D_1 \rightarrow |\Psi_i\rangle} = \frac{A_{^3D_1 \rightarrow |\Psi_i\rangle}}{\sum_i A_{^3D_1 \rightarrow |\Psi_i\rangle}}, \quad (6.5)$$

where $A_{^3D_1 \rightarrow |\Psi_i\rangle}$ represents the electric dipole (E1) transition probability and $|\Psi_i\rangle$ are the dipole allowed ASFs, $^3P_{0,1,2}$ and 1P_1 . The transition probability is further expressed in terms of the transition line strength, $S_{E1} = |\langle ^3D_1 || E1 || \Psi_i \rangle|^2$, and the wavelength of the transition, as [153]

$$A_{^3D_1 \rightarrow |\Psi_i\rangle} = \frac{2.02613 \times 10^{18}}{3\lambda^3} S_{E1}. \quad (6.6)$$

Here, the transition line strength is in atomic units and λ is in angstrom. As evident, S_{E1} is square of the reduced dipole matrix elements, which is calculated using FSRCC theory.

In Fig. 6.2, we have shown our data on the branching ratio for repumping state 3D_1 . As discernible from the figure, the nonzero branching ratios suggest that electrons from 3D_1 state can make a transition to $^3P_{0,1,2}$ and 1P_1 states via E1 transitions. The probability of transition is, however, different for different states. Consistent with our proposed repumping scheme, the population in $5s5p\ ^3P_2$ is significantly reduced. And, it is as large as $\approx 51\%$ and 48% , respectively, for 3P_0 and 3P_1 states. This is as desired, as these electrons can transition back to ground state 1S_0 via radiative and E1 transitions, respectively. The population in 1P_1 is negligible, and the reason for this could be attributed to the smaller transition probability due to closer energy with 3D_1 . The transition probabilities for allowed dipole transitions are further used to calculate the transition linewidth, expressed as $2\pi \times \gamma = \sum_i A_{^3D_1 \rightarrow |\Psi_i\rangle}$. Our calculated value $\gamma = 10.2$ MHz, is in good agreement with the value of 9.4 MHz and 9.7 MHz obtained from

previous theory [154] and experiment [155] respectively. It is to be mentioned that, in the calculation of γ we have used the experimental energies to increase its accuracy. The small difference in our calculated value and Ref. [154] could be attributed to the use of slightly different many-body methods in the two calculations and the corrections from the Breit and QED incorporated in our results. From the Breit and QED contributions listed in Table 6.1, we find that Breit+QED contributes $\approx 3.2\%$ to the linewidth.

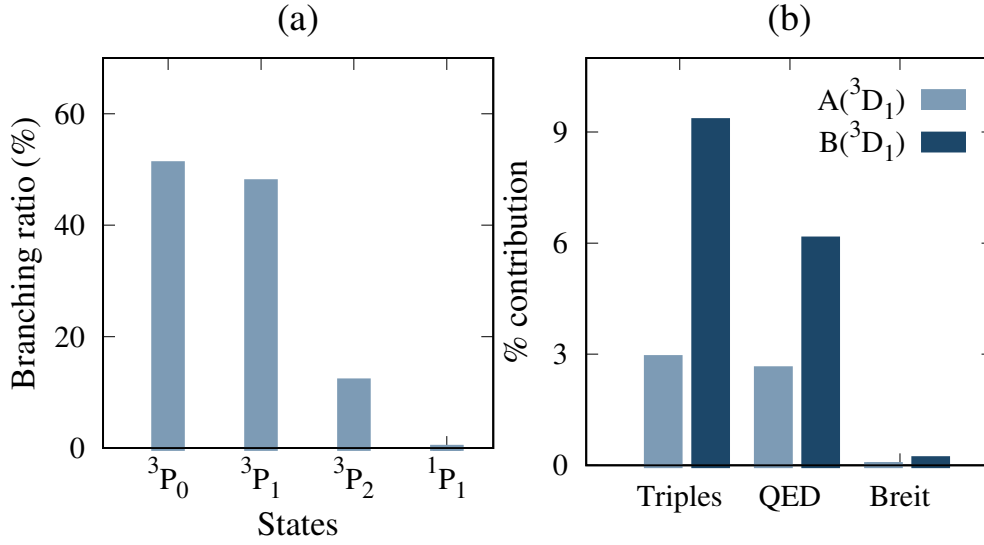


Figure 6.2: (a) Branching ratio of repumping state $5s5d\ ^3D_1$ in $5s5p\ ^3P_{0,1,2}$ and $5s5p\ ^1P_1$ states. (b) Percentage contributions from the Breit, QED and perturbative triples corrections to hyperfine constants.

6.5 Characterization of the Repumping Scheme

To characterize the repumper performance, we compare the steady state atom number in the MOT in the presence and absence of repumping light with different schemes. For each scheme of repumping, the MOT is operated for a fixed time of 2 s and the fluorescence is recorded on a photo multiplier tube (PMT). The enhancement factor (η) in atom number is calculated by comparing the steady-state atom numbers in the MOT with and without repumping laser(s), expressed as a ratio. For different isotopes of Sr, the enhancement factor ($\eta = \frac{N_{withrepumper}}{N_{withoutrepump}}$) is summarized in table 6.2. The peak atom number in the continuously repumped blue MOT for ^{88}Sr reaches 5×10^7 . Additionally, we perform absorption imaging using a CCD camera to determine the temperature of the cloud using the time-of-flight method. Strontium atoms in the blue MOT is at tem-

peratures of ~ 2 mK. Furthermore, these laser-cooled strontium atoms are utilized for conducting all the isotope-shift spectroscopy.

Table 6.2: The enhancement factor (ratio of atom numbers in the MOT with and without repumper) of the steady-state atom number in the MOT varies for different strontium isotopes in various repumping scenarios. The 707 nm repumper depopulates the $5s5p\ ^3P_2$ state, while the 483 nm repumper depopulates the $5s5p\ ^3P_0$ state – effectively depopulating the triplet manifold via short lived $5s5p\ ^3P_1$ state. The enhancement factor is normalized with respect to the steady state number of atoms in the MOT in the absence of any repumping light.

Isotope	No repumper	Enhancement factor (η)	
		Only 707 nm repumper	707 and 483 nm repumper
88	1	4	12.8 (5)
86	1	3	18.2 (5)
87	1	5	18.6 (5)
84	1	5	19.1 (5)

To get insight into MOT dynamics, we model the steady-state atom numbers in the MOT using a time-dependent trap loading equation. In the limit of one-body and two body losses, the trap loading equation can be written as [156, 157]

$$\dot{N} = L - \Xi N - \beta' N^2, \quad (6.7)$$

where N is the number of atoms, L is the loading rate of MOT, Ξ is the one-body loss rate, and $\beta' = \beta/(2\sqrt{2}V)$, where β is the two-body loss constant and V is the effective volume of the two body processes. The solution to the above differential equation (Eqn. 6.7) can be obtained as

$$N(t) = \frac{N^{ss}(1 - e^{-\xi t})}{(1 + \chi e^{-\xi t})} \quad (6.8)$$

where N^{ss} is the steady-state MOT atom number, $\xi = \Xi + 2\beta' N^{ss}$, represents the total loss rate and $\chi = \frac{\beta' N^{ss}}{\beta' N^{ss} + \Xi}$, is a measure of the relative contributions of one- and two-body loss coefficients.

We have used this mathematical model to fit our experimental data for three different repumping schemes, as shown in Figure 6.3. From these fits we extracted the rates of one-body and two-body losses. In the case of the 483 nm scheme, the two-body fit yields $\Xi = 5.16(5)\ s^{-1}$ and $\beta = 2.06(5) \times 10^{-10}\ \text{cm}^3/\text{s}$. And for the 679 nm scheme, we obtain $\Xi = 5.25(5)\ s^{-1}$ and $\beta = 2.81(5) \times 10^{-10}\ \text{cm}^3/\text{s}$. The close values of the parameters with respect to each other suggest that both the 483 nm and 679 nm repumpers have similar MOT loading performance. Additionally, both repumpers deplete the 3P_0 state entirely. Furthermore, the use of either 483 nm or 679 nm, along with other single repumpers such as 707 nm, 481 nm, 496 nm, and 405 nm, will result in a complete depletion of the triplet manifold.

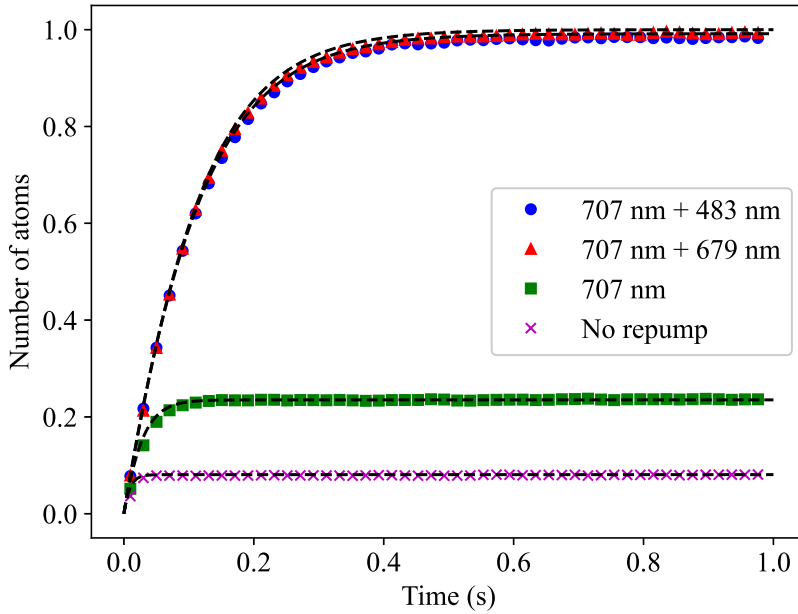


Figure 6.3: The normalized fluorescence of the blue MOT for ^{88}Sr for various chosen repumping schemes. The black dashed curves are the fits to the Eq. (2), and the colored markers represent the experimental loading curve. It is important to note that the loading rates of the MOT are identical for both the 707 nm + 679 nm and the 707 nm + 483 nm repumping schemes.

6.6 Spectroscopy of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ Transition

To perform the spectroscopy on the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition for all the four stable isotopes, we continuously load the MOT and record the MOT fluorescence on the PMT. We load the targeted isotope in the MOT by changing the blue laser frequency to appropriate detuning. And then we apply 707 nm on-resonance light to repump $5s5p\ ^3P_2$ state. The frequency of the 483 nm laser is then scanned resulting in an enhancement of the fluorescence from the MOT. Our data on MOT fluorescence as a function of 483 nm laser frequency is shown in Fig. 6.4. As discernible from the figure, in the case of even isotopes, due to the absence of nuclear spin, the energy level structure is simple where we get one peak for each isotope. For the case of odd isotope, however, due to the coupling with nonzero nuclear spin ($I = 9/2$), we observe three peaks at different frequencies, which is attributed to the splitting of $5s5d\ ^3D_1$ state into three, $F = 7/2, 9/2$, and $11/2$, hyperfine levels.

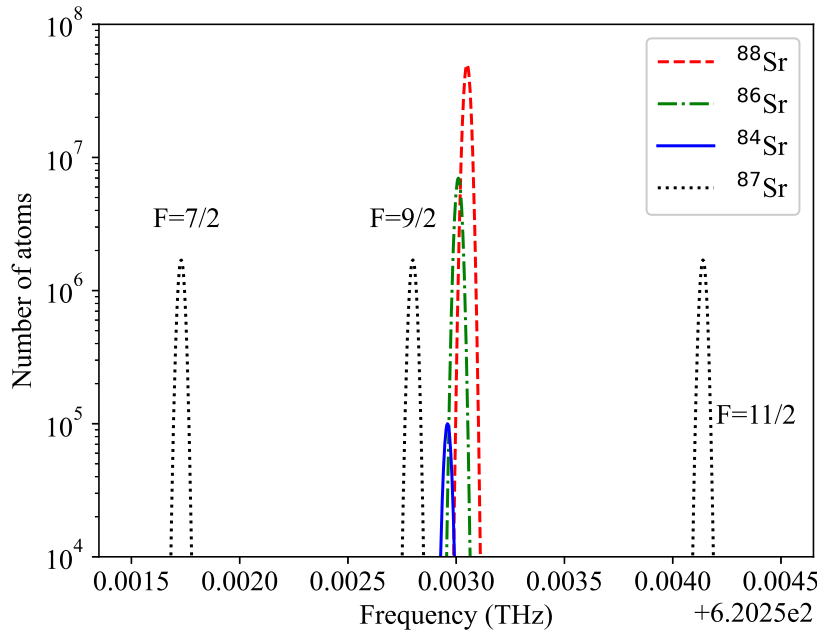


Figure 6.4: Spectroscopy data for the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition obtained from cold atoms in a magneto-optical trap using a fluorescence detection method. The transitions for bosonic isotopes, ^{88}Sr , ^{86}Sr , and ^{84}Sr , are illustrated. To cover the hyperfine transition of ^{87}Sr , the laser is scanned across a 3 GHz range.

Table 6.3: Absolute transition frequencies of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition around 483 nm

Isotope	$5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ Absolute transition frequency (THz)
88	620.25305 (3)
86	620.25301 (3)
84	620.25296 (3)
87(F=7/2)	620.25173 (3)
87(F=9/2)	620.25280 (3)
87(F=11/2)	620.25414 (3)

From our measurement, we obtain the resonance frequency of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition in ^{88}Sr as 620.25305(3) THz, which is a small shift (~ 13 GHz) and improvement over the previously available data of 620.2398(25) THz [158]. Utilizing the transition frequency measurement in Ref. [152], for $5s5p\ ^3P_2 \rightarrow 5s5d\ ^3D_1$ transition, and [159], for $5s^2\ ^1S_0 \rightarrow 5s5p\ ^3P_2$ transition, we calculated the resonance frequency for $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition in ^{88}Sr as 620.253099(5) THz. This value differs by about 50 MHz from our measured transition frequency. The uncertainty in the transition frequency measurement arises mainly due to the limitation on the absolute accuracy of our wavemeter, which is 30 MHz. The absolute transition frequency for all the isotopes of strontium is listed in the Table 6.3.

6.6.1 Hyperfine Structure Constants

The hyperfine interaction in an atom provides a measure of the coupling between the nuclear electromagnetic moments and the electromagnetic fields of the electrons. Therefore, the determination of hyperfine splitting (HF) not only reveals insights into nuclear structure but also provides valuable information about the electronic wavefunction near the nucleus.

In Table 6.1, we have provided the theoretical values of magnetic dipole, A , and electric quadrupole, B , hyperfine structure constants for the repumping state. For quantitative assessment of electron correlations, the contributions from the Breit and QED corrections are provided separately in the table. As evident from the table, self-energy

Table 6.4: Nuclear rms radius, $\delta\langle r^2 \rangle$ (fm²), reduced mass, μ (a.m.u.), isotope shift (MHz), and modified isotope shift, $\mu\delta\nu_A$ (MHz a.m.u.), for $^1S_0 \rightarrow ^3P_1$ and $^3P_0 \rightarrow ^3D_1$ transitions for $^{84,86,87}\text{Sr}$ relative to ^{88}Sr .

Isotope (A)	$\delta\langle r^2 \rangle$	μ_A	$^1S_0 - ^3P_1$		Exp. (Ref. [160])
			$\delta\nu_A$	$\mu\delta\nu_A$	
^{84}Sr	0.487473	1848	346.57	640350.48	351.49(0.5)
^{86}Sr	0.242665	3784	167.06	632155.04	163.82(0.4)
^{87}Sr	0.121069	7656	82.03	628021.68	62.19(1.2)

Isotope (A)	$\delta\langle r^2 \rangle$	μ_A	$^3P_0 - ^3D_1$		Exp. (Present work)
			$\delta\nu_A$	$\mu\delta\nu_A$	
^{84}Sr	0.487473	1848	85.28	157597.44	90(10)
^{86}Sr	0.242665	3784	41.12	155598.08	40(10)
^{87}Sr	0.121069	7656	20.19	154574.64	32(10)

has the significant contributions of ≈ 2.8 and 6.0% , respectively, for A and B . The contributions from the Breit interaction and vacuum polarization are, however, very small for both A and B . The other significant contribution comes from the perturbative triples, which contributes ≈ 3.0 and 9.0% , respectively, to A and B .

To compare with our measurements, using these hyperfine constants, we calculated the hyperfine splitting energies for all the allowed hyperfine levels. We compute splitting energies to be 1317, 241 and 1087 MHz, for $F = 7/2, 9/2$ and $11/2$, respectively.

In Table 6.3, we have compiled the absolute transition frequencies for ^{88}Sr and ^{87}Sr hyperfine levels, considering hyperfine levels $F = 7/2, 9/2$, and $11/2$. This information enables the calculation of hyperfine splitting energies for each level in MHz, yielding values of 1320 (10), 250 (10), and 1090 (10) MHz, respectively, which are in excellent agreement with our calculated values. It is important to note that the relative transition frequency uncertainty in our findings is 10 MHz. This uncertainty arises from statistical factors, including the drift of the wavemeter and other factors such as the drift of the main laser 461nm lock frequency.

Comparing our theory results with the other available experiment, considering the complex nature of electron correlation in Sr, our value 240.6 MHz for A shows a good match with the result 227.3 (7) MHz in Ref. [152]. To the best of our knowledge, however, there is no result from other theory calculations for comparison.

Our calculated value of hyperfine constant B is 5.78 MHz. Comparing with experiments from literature, Ref. [152] reports an experimental value 0, however, with a very large uncertainty of ± 10 MHz. Like Ref. [152], the error in our measurement is also large. Our measured value is 5.3 ± 42 MHz. From other theory calculations, to the best of our search, we did not find any data for comparison.

6.6.2 Isotope Shift

To assess the effect of nuclear mass and charge distributions, we calculated the isotope shifts in the transition frequency of $^1S_0 \rightarrow ^3P_1$ and $^3P_0 \rightarrow ^3D_1$ transitions for $^{84,86,87}\text{Sr}$ relative to ^{88}Sr . For this, the ASFs were calculated using the MCDHF theory using GRASP18 code [161] and then shifts in the frequencies were calculated using the RIS4 program [162]. We started with $\{5s4d, 5s5d\}$ and $\{5s5p, 4d5p\}$ multireference even and odd parity valence configurations, respectively, which embeds the ASFs of our interests – 1S_0 , 3P_1 , 3P_0 and 3D_1 – and then the electron correlations were added in the subsequent calculations. To include the electron correlations from the *valence-to-virtual*, *core-to-valence* and *core-to-virtual* excitations, we performed a series of MCDHF calculations where we added virtual orbitals to active space layer wise. For each correlation layer, the single and double excitations of electrons were considered from the valence and core orbitals. The maximum correlation layer used in the active spaces of even and odd configurations were up to $\{9s9p7d7f\}$ and $\{9s9p7d5f\}$, respectively. It should, however, be mentioned that, for *core-to-valence* and *core-to-virtual* excitations, orbitals up to $3d$ were considered as frozen core. We restricted the active space to this to find a close match with the experimental excitation energies. The maximum error achieved in the excitation energies of $^1S_0 \rightarrow ^3P_1$ and $^3P_0 \rightarrow ^3D_1$ transitions are 6% and 2%, respectively.

In Table 6.4, we have provided our calculated results on isotope shifts, $\delta\nu_A$, and modified isotope shifts, $\mu\delta\nu_A$, for the above discussed transitions. The modified isotope shifts provide a scaling with respect to reduced mass of the nucleus and often used in the King plot analysis to extract contributions associated with field and mass shifts. The values of nuclear *rms* radius and reduced mass, $\mu = AA'/A - A'$, used in the calculation of $\delta\nu_A$ and $\mu\delta\nu_A$ are also given in the table. Considering the $^1S_0 \rightarrow ^3P_1$ transition, our

calculated $\delta\nu_A$ for ^{84}Sr , ^{86}Sr and ^{87}Sr are 346.56, 167.06 and 82.03 MHz, respectively. Except for ^{87}Sr , these are in excellent agreement with the experimental values 351.49, 163.82 and 62.19, respectively, reported in Ref. [160]. The reason for the large discrepancy in the case of ^{87}Sr could be attributed to the different nature of electron correlations in this fermionic isotope and need to be taken care separately. After optimizing it separately, we obtained an isotope shift of 61 MHz with an active space configurations up to $\{7s7p7d5f\}$. The error in the excitation energy is, however, large, about 13%. Here, we would like to mention an important trend we observed from a series of MCDHF calculations to extract isotope shifts. We find that, when wavefunctions are optimized more accurately with the inclusion of additional layers of electron correlations, to improve the accuracy of excitation energies, it leads to deteriorated isotope shifts with respect to experiments. The reason for this could be attributed to the inherent shortcomings associated with the treatment of electron correlations in MCDHF theory. For the isotope shift for $^3P_0 \rightarrow ^3D_1$ transition, we get 85.28, 41.12 and 20.19 MHz for ^{84}Sr , ^{86}Sr and ^{87}Sr , respectively.

In the experiment, where laser cooling was applied to different isotopes and spectroscopy was performed, as detailed in the methodology section, we obtained the absolute frequencies for each isotope. These absolute frequency measurements are presented in Table 6.3. Using this data, we calculated the isotope shifts for each isotope, resulting in corresponding values of 90 (10), 40 (10), and 32 (10) MHz, for ^{84}Sr , ^{86}Sr and ^{87}Sr , respectively, and these are in good match with our calculated values. To the best of our knowledge, there are no results available on isotope shifts from other theory calculations for comparison for this transition.

6.7 Conclusion

In conclusion, we conducted spectroscopy on the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition for all stable isotopes of Sr, determining the absolute transition frequency with an accuracy of 30 MHz. We introduced the utilization of the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition of Sr for repumping $5s5p\ ^3P_0$ atoms in strontium, comparing it with the conventionally employed 679 nm laser scheme. Our results illustrate the successful repumping of all stable isotopes of Sr using the 483 nm laser. Fock-space relativistic coupled-cluster calculations

were performed to calculate the branching ratio, linewidth, hyperfine splitting constant, and isotope shift for the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition, and our experimental measurements of hyperfine splitting constants and isotope shift are in good agreement with the calculated values.

Chapter 7

Conclusion and Outlook

The primary objective of this thesis was to create an experimental platform for conducting precise measurements and frequency metrology utilizing ultracold Sr atoms. The narrow inter-combination line present in Sr makes it an excellent choice for an optical atomic clock. The optical clock, relying on neutral strontium atoms has achieved a total uncertainty of 2.1×10^{-18} [1]. These highly accurate clocks find applications in fundamental physics, geophysics, and astronomy. The research presented in this thesis contributes to the development of a system dedicated to frequency metrology using the clock transition. To achieve this, an experimental setup has been constructed to generate cold strontium atoms in a Magneto-Optical Trap (MOT).

We have detailed the development of a system for producing ultra-cold strontium, serving as the core for an optical lattice clock. Various innovative solutions were devised, including a clog-free oven design, injection-locked laser modules, and a vacuum-sealed high finesse cavity housing. Initial findings from the ongoing system characterization were presented, including an assessment of the atomic oven's thermal properties and atomic beam divergence. We achieved an atomic beam flux of approximately 10^{11} atoms/s. The measured atomic beam divergence was only 60 mrad, facilitated by high atomic beam collimation. This allowed us to employ an orthogonally-crossed-beam spectroscopy scheme for the frequency stabilization of the 461 nm laser source on the $5s^2 \ ^1S_0$ transition, reducing system complexity by eliminating the need for an additional high-temperature strontium vapor cell for laser frequency stabilization.

We also built a versatile setup which can accommodate a range of experiments, from atomic clock to quantum information processing. We achieved 10^{-11} torr vacuum level. We used gate valve to isolate various parts of the system, which helps in tasks such as refilling of strontium in the oven, attaching single atom imaging system to the existing system. We kept ZS optical window at elevated temperature to avoid any strontium coating, which is also working well from past 3 years.

The first cooling stage was executed, resulting in the trapping of around 10^8 atoms in the Blue MOT (with repumpers). We analyzed the loading and switching-off dynamics of the Blue MOT. The utilization of a compact Zeeman slower increased the number of trapped atoms by a factor of 3. A preliminary test of the second cooling stage was conducted, leading to the trapping of approximately 10^7 atoms in the Red MOT. At the end of red MOT we achieved the average temperature of $10\ \mu\text{K}$.

In the experiments carried out with the system, we investigated the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition in all stable isotopes of Sr through spectroscopy. With a precision of 30 MHz, we determined the absolute transition frequency. The introduction of the 483 nm laser for repumping $5s5p\ ^3P_0$ atoms in strontium was compared with the conventional 679 nm laser scheme. Our findings demonstrate the effective repumping of Sr's stable isotopes using the 483 nm laser. Additionally, the theory collaborators conducted Fock-space relativistic coupled-cluster calculations to assess various parameters including branching ratio, linewidth, hyperfine splitting constant, and isotope shift for the $5s5p\ ^3P_0 \rightarrow 5s5d\ ^3D_1$ transition. Remarkably, our experimental measurements of hyperfine splitting constants and isotope shift closely match the calculated values.

7.1 Future Work

To achieve a favorable signal-to-noise ratio (SNR) in an optical atomic clock, loading the optical lattice with a substantial number of atoms is crucial. Our experimental setup is well-equipped to generate cold atoms with temperatures significantly below the trap depth of the optical lattice, which typically measures around $50\ \mu\text{K}$. This capability ensures high efficiency in loading atoms into the optical lattice, ultimately enhancing the SNR. However, the choice of optical lattice wavelength plays a crucial role in optical clocks.

The light-shift, also known as the AC Stark shift, occurs when any radiation field couples to the atomic states, causing a change in their energies. Optical dipole trapping relies on these shifts to modify the energy levels of atoms locally and temporarily, creating the necessary trapping potential. However, during experiments aimed at precision measurements, the differential AC Stark shift can be problematic and must be mitigated. To achieve this, a special wavelength is utilized where the AC Stark shifts of the involved atomic states are equal in magnitude and sign, known as the "magic wavelength." Conversely, the wavelength where the Stark shift is equal in magnitude but opposite in sign is termed the anti-magic wavelength.

For strontium atoms, the clock transition $^1S_0 \rightarrow ^3P_0$ and the $^1S_0 \rightarrow ^3P_1$ transition are crucial, with magic wavelengths around ~ 813 nm and ~ 915 nm, respectively [163]. Determining the magic wavelength for the clock transition requires utmost precision to avoid residual light-induced shifts. Additionally, besides the externally applied laser field, the background black-body radiation of the environment also induces a differential shift in the clock states [164, 165]. Significant efforts have been made to accurately quantify and reduce the magnitude of this shift.

Appendix A

Appendix

This chapter describes various mechanical design of important components used in the experiment.

A.1 Heat pipe

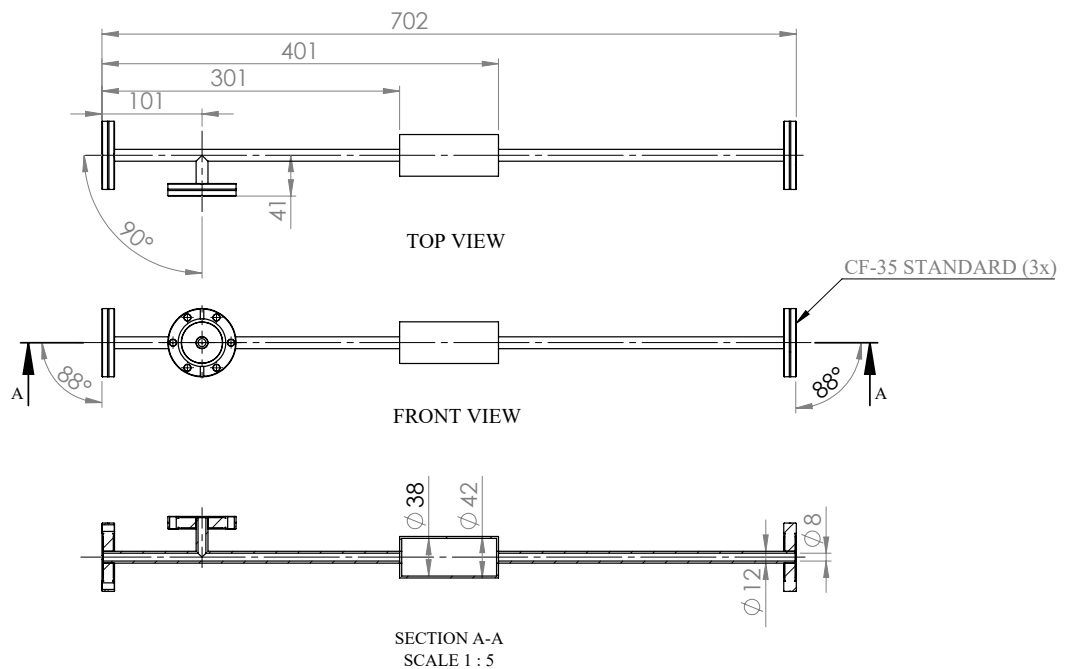


Figure A.1: Heat pipe: drawings for the cell currently in use on the Sr machine.

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