Critical Analysis of Algorithms in Vākya School of Astronomy

A Thesis

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by

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Certificate

This is to certify that this dissertation entitled "*Critical Analysis of Algorithms in Vākya School of Astronomy*" towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Dharmendra at Indian Institute of Science Education and Research under the supervision of Dr. Venketeswara R. Pai, Assistant Professor, Department of Humanities and Social Sciences, during the academic year 2019-2020.

Dharmendra

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This thesis is dedicated to my loving family, friends and teachers.

Declaration

I hereby declare that the matter embodied in the report entitled Critical Analysis of Algorithms in Vākya School of Astronomy are the results of the work carried out by me at the Department of Humanities and Social Sciences, Indian Institute of Science Education and Research, Pune, under the supervision of Dr. Venketeswara R. Pai and the same has not been submitted elsewhere for any other degree.

Dr. Venketeswara R. Pai

Dharmendra

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Abstract

In ancient Indian astronomy, many different and unique methods were created to study and understand the position of the planetary bodies in the solar system. One of the initial works regarding this was carried out by the famous Indian mathematician and astronomer *Āryabhaṭa* I and his disciples like *Bhāskara* I. Some of their texts are *Āryabhaṭīya, Mahābhāskarīya* and *Laghubhāskarīya.* These are called siddhāntic texts. Later, some methodologies were developed from these texts which were more time-efficient and with less complex calculations to locate the position of planets at a given point of time. One of the texts with such algorithms is *Vākyakarana.*

In this thesis, we want to trace the origin of astronomical algorithms mentioned in *Vākyakarana* by doing the numerical analysis of algorithms pertaining to planetary longitudes in *Āryabhaṭīya, Mahābhāskarīya* and *Laghubhāskarīya.* This will enable us to understand the theoretical basis of the mathematical principles laid down in the text *Vākyakarana.* The analysis has been done by using the modern mathematical and programming techniques with the help of tools like Python for Scientific Computing

Contents

Chapter 0

Introduction

There have been many great astronomers in ancient India like *Āryabhaṭa I, Bhāskara I, Parameśvara*, etc. and they have written different texts to find the position of celestial bodies such as the Sun, the moon and the five planets (Mercury, Venus, Mars, Jupiter and Saturn) by taking the Earth as the reference point and assuming all these revolve around the Earth in a circular orbit. In my project, we are just concerned with the five planets. The two different kind of methodologies used to do this are known as *siddhāntic methods* of astronomy and *vākya methods* of astronomy.

0.1 Texts in Indian astronomy

The most important siddhāntic texts involving the algorithms to find the true longitude (position) of the planets are *Āryabhaṭīya, Mahābhāskarīya* and *Laghubhāskarīya* which are due to *Āryabhaṭa* I and his disciple *Bhāskara* I in 5^{th} and 7^{th} century. It was *Āryabhaṭa* who first wrote the tables to calculate $\sin\theta$, for $0 < \theta < \pi/2$ and gave a formula for $\sin\theta$, where $\theta > \pi/2$. But the methods for calculating the true longitudes of the planets in these texts involve the "sine" function, which made the calculations by hand complicated at that time.

On the other hand, in *vākya* system of astronomy, numerical values associated to the astronomical variables are encoded in *vākyas*, which by definition are sentences or phrases. Some of the texts using *vākya* methods are *Vākyakarana, Sphuṭacandrāpti* or *Veṇvāroha*, etc. originally composed by *Parameśvara, Mādhava* around 13^{th} and 14^{th} centuries. The *vākya* method for the true longitude of the planets simplify the siddhāntic methods in a very nice and elegant way by removing the sine function which was making the calculations complex and time consuming.

0.2 About the thesis

The thesis is about the comparison of astronomical algorithms to find the position of planets in the text *Vākyakarana* with the ones in siddhāntas.

0.2.1 What is the Question?

From which siddhāntic text among *Āryabhaṭīya, Mahābhāskarīya* and *Laghubhāskarīya*, the astronomical algorithms for the true longitude of two 'inner planets' (Mercury, Venus) and three 'outer planets' (Mars, Jupiter, Saturn) in *Vākyakarana* have originated from?

0.2.2 Plan and Approach

To answer the question, we study the four different texts and understand the algorithms for the true longitude of a planet and use modern techniques to analyze them.

Given a particular date, we can find the number of days elapsed till that from the beginning of *Kaliyuga* which is marked on 18 Feb, 3102 BC as per *purans* and vice-versa. In all the texts, for the algorithms to find the true longitude of the planets on a given point of time, the input is taken to be the above number of days till that time.

Now, by making use of the technology, we write the programs for all planets as per each text such that by taking the number of days elapsed from the beginning of *Kaliyuga* and a particular planet as the inputs, we get the true longitude of that planet on that day as an output. Next thing to do is plot the graphs. For a fixed planet, take the input (number of days elapsed from the beginning of *Kaliyuga*) on x-axis and the output (true longitude of that planet) on y-axis. For each planet, we get 3 such graphs corresponding to *Vākyakarana*, *Āryabhaṭīya, Mahābhāskarīya* and one triple graph showing the comparison among three. By these graphs, we can conclude which siddhānta is comparatively closer to *Vākyakarana*.

Python language has been used for all our programming purposes.

The main focus of this thesis is to work on the mathematical aspects of the planetary positions and not to go much deeply into the astronomical language related to it. But there are some words, definitions, tables and concepts related to the ancient astronomy that will be used again and again during the thesis, which will discuss briefly in the next chapter.

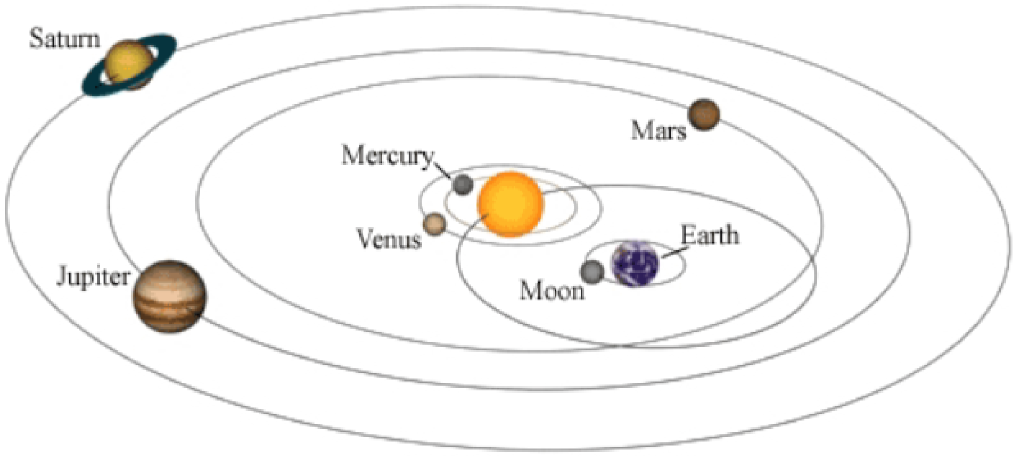
Chapter 1

Astronomical Concepts

This chapter is about the astronomical aspects, terminologies and concepts used repeatedly in the texts. To start with, the modern model of the solar system, which was developed by the Polish astronomer Nicolaus Copernicus in 1543, is obviously not the one which was used by ancient Indian astronomers in developing their theories. Their work is based on the quasi-heliocentric model of the universe.

1.1 Quasi-heliocentric model of the Universe

The word *geocentric* represents the Earth is taken to be as the centre and *heliocentric* means the Sun is the centre.
The *quasi-heliocentric model of the universe* is the astronomical model in which the Earth is in the centre of the universe and the celestial bodies revolve around it. First comes the orbit of Moon and then the Sun. Mercury and Venus, also called the *inner planets* have their revolving orbits around the Sun. As a result, these two also revolve around the Earth. At last, the planets Mars, Jupiter and Saturn, called the *outer planets*, revolve around the Earth in the orbits which come after the orbit of Sun. Therefore, these also revolve around the Sun. Here, all orbits are taken to be as circular and the motion is non-uniform. In this model, all the planets are revolving around the Sun as well as around the Earth and it has been depicted in figure below from *https://www.sutori.com/item/this-model-shows-tycho-brahe-s-theory-brahe-s-theory-similar-to-the-heliocentr.*



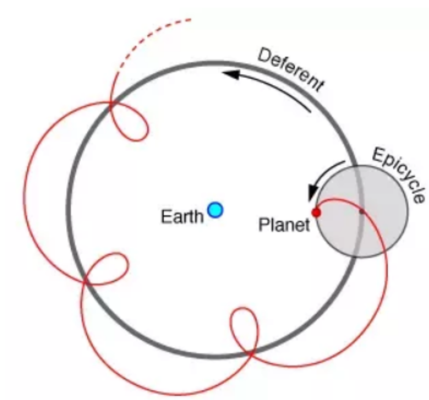
QUASI-HELIOCENTRIC MODEL OF THE UNIVERSE

1.1.1 Epicyclic theory

Epicyclic theory is a geometric model used in the ancient astronomy to justify the changes or variations in the velocity of the planets.

Definition 1.1.1. ***Epicycle.*** *It is like a circle moving on a bigger circle, called deferent with its centre on the circumference of the bigger circle.*
In astronomical terms, the circular orbit of a planet is the deferent, and the actual motion of the planet is along the moving smaller circle, 'the epicycle'.

Look at the picture below from *https://images.app.goo.gl/m8D4Dpk27kj6HcN68*.



According to the epicyclic theory, the *mean longitude of the planet* or *mean planet* moves on the deferent, and the *true longitude of the planet* or *true planet* moves on its epicycle centred at the mean planet.

1.2 Reference point and the line of reference

The true longitude of a planet at a particular point of time is described in terms of an angle given in units '*rashis*', ' *degrees*' ,'*minutes*' and '*seconds*', where

$$1^r = 30^\circ,\ 1^\circ = 60' \text{ and } 1' = 60''.$$

(r=rashi, $^\circ$=degree, $'$=minute, $''$=second.)
But the question is from where is this angle measured from. Mathematically, there should be a vertex and initial ray. What are these in this case? This can be answered as follows. Let us start by introducing some terminologies.

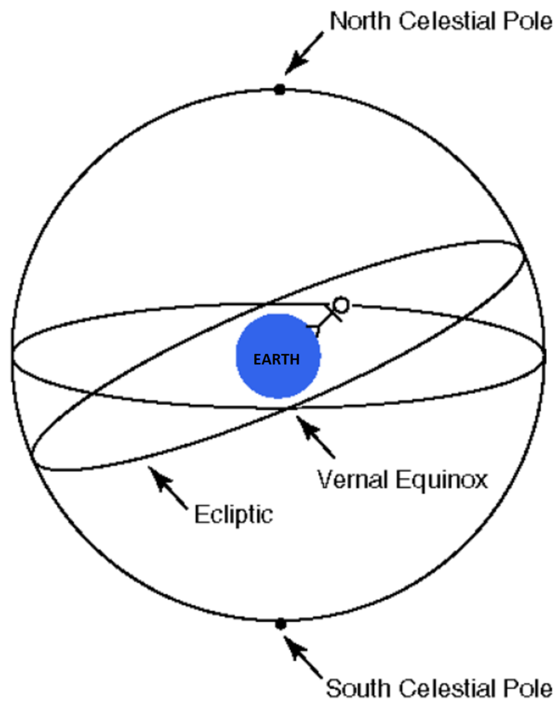
Definition 1.2.1. 1. ***Celestial Sphere.*** *This is an imaginary sphere where all the celestial bodies like planets, stars live and has an infinitely large radius. It is concentric to the Earth, i.e. the centre of the Earth is same the centre of the celestial sphere.*

2. ***Ecliptic.*** *It is basically the orbit of the Sun around the Earth projected outwards to the celestial sphere.*

3. ***Celestial equator.*** *The equator of the Earth expanded uniformly in all directions gives rise to a circle of radius same as that of celestial sphere. This circle is called celestial equator.*

4. ***Equinoxes.*** *The two intersection points of ecliptic and celestial equator are called equinoxes. One is called vernal equinox and the other is autumnal equinox.*

By a great circle in celestial sphere, we mean a circle having the radius equal to the radius of celestial sphere. Note that, ecliptic and celestial equator both are great circles. These terms are represented in the figure below from *http://www.physics.csbsju.edu/astro/CS/CS.09.html*



CELESTIAL SPHERE

Remarks. For measuring the angle to describe the position of a planet, the Earth is the vertex or the *reference point* and the ray from vertex to the vernal equinox is taken to be as the initial ray or the *line of reference*. It is along the initial ray the *mean longitude* of all the planets is taken to be as zero.

1.3 Astronomy in Siddhāntas

In this section, we will discuss some terms which will appear in the siddhāntic texts - *Āryabhaṭīya, Mahābhāskarīya* and *Laghubhāskarīya*. The procedures to find the values of longitudes are given in later chapters. Here, we will say what we are trying to do in those procedures.

As said before, the mean longitude of a planet gives its position when assumed the motion is uniform and is circular along the deferent. But in actual, the velocity of the planet is non-uniform and the orbit is not circular. So, there comes the true geocentric longitude which describes its position when it is moving along the epicycles which takes care of the non-uniformity and eccentricity of the orbit . Once we know how to calculate the mean planet, some corrections are to be applied to get the *true planet* at a given point of time. Let us now talk briefly about what those corrections are. Refer to [7] for more details.

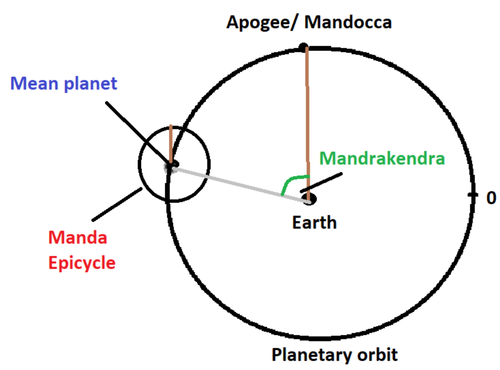
1.3.1 Basic terms

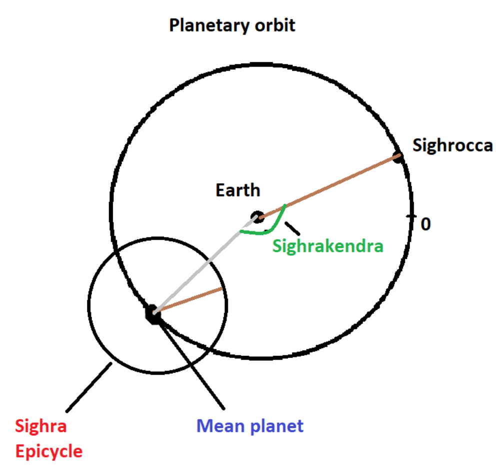
We know all the five planets Mercury, Venus, Mars, Jupiter and Saturn also revolve around the Sun. That's why, while finding the true geocentric longitude of any of the planets, the motion of Sun and its effect on planetary motion is taken to be in account. This gives rise to kind of epicycles in case of the planets : (*i*) *Manda* (slow) epicycle , and (*ii*) *Śighra* (fast) epicycle.

From *manda* epicycles and *śighra* epicycles, we get two anomalies : *Mandakendra* and *Śighrakendra* which can be defined mathematically as follows.

Definition 1.3.1. *1.* ***Mandakendra.*** *It is defined as the angle obtained by subtracting the longitude of planet's apogee from the mean longitude of the planet.*

2. ***Śighrakendra.*** *It is the angle equal to the mean longitude of the planet minus longitude of planet's śighrocca.*





Here, without going into much details, we can say that a planet's apogee or *mandocca* is point on a planet's orbit where its motion is the slowest. In Indian astronomy, it is defined as the furthest point on planetary orbit where the planet appears the smallest as seen from the Earth. The longitude of all the planets' apogees are tabulated below taken from [4] and [5].

Planet	Apogee
Mercury	210°
Venus	90°
Mars	118°
Jupiter	180°
Saturn	236°

Similarly, a planet's *śighrocca* is the apex of its fast motion. For calculations, in case of superior planets Mars, Jupiter, Saturn, it is defined as an imaginary object which remains in the direction same as that of *mean Sun*, whereas for inferior planets Mercury and Venus it is in the same direction from the Earth as the planet is from the Sun. The method to calculate the longitude of a planet's *śighrocca* is disscussed in later chapters.

1.3.2 Correction terms - *Mandaphala* and *Śighraphala*

1. Using *manda* epicycles and *mandrakendra*, we get the correction term called *mandaphala* given by different formulas in different siddhāntic texts. *Mandaphala* term is applied to the mean longitude of the planet to account for the eccentricity of the planetary orbit. In modern astronomy, it is called the *equation of the centre.* The procedure of obtaining and applying this *manda* correction is called *Manda-saṃskāra.*

2. The next obvious term, called *śighraphala* is obtained by the help of *śighra* epicycles and *śighrakendra*, again given by different expressions in different texts. Since the planets are also revolving around the Sun, therefore the thing is by the application of *mandaphala*, what we get is the heliocentric longitude of the planet. It is the textitśighraphala correction term that converts the heliocentric longitude to the geocentric one. The procedure followed to obtain this *śighra* correction is called *Śighra-saṃskāra.*

Hence, the combined application of *mandaphala* and *śighraphala* to the mean longitude of the planet gives us the true longitude of the planet.

1.4 Astronomy in *Vākyakarana*

This section is about introducing the basic astronomical definitions and terms in *Vākyakarana*. Instead of laying down the *vākya* method to find the true planet, we will explain some things such as this method is linked to or originated from the methods of the siddhāntic texts. The mathematical part and algorithms in *vākya* method are to be discussed in the next chapter.

1.4.1 *Vākya* system

Vākyas is the hindi translation of sentences or phrases of words. *Vākyakarana* by *Parameś-vara*, wriiten around 13^{th} century, has a fully-developed system, known as the *vākya system* which is used to find the true longitude of a planet by the help of *vākyas*. In astronomy, *vākyas* are used to encode the astronomical numerical values. Not only that, the strings used to make these *vākyas* are such that they form nice meaningful sentences. To encode or decode the values into or from the *vākyas* is done by the *Kaṭapayādi* scheme discussed in [7].

1.4.2 *Kaṭapayādi* system of numeration

The name *Kaṭapayādi* has the significance that the sanskrit alphabets *ka, ṭa, pa, yā*, etc denote the numbers.

Number	1	2	3	4	5	6	7	8	9	0
Consonants	*k*	*kh*	*g*	*gh*	*ṅ*	*c*	*ch*	*j*	*jh*	*ñ*
used	*ṭ*	*ṭh*	*ḍ*	*ḍh*	*ṇ*	*t*	*th*	*d*	*dh*	*n*
to represent	*p*	*ph*	*b*	*bh*	*m*	–	–	–	–	–
numbers	*y*	*r*	*l*	*v*	*ś*	*ṣ*	*s*	*h*	*ḷ*	–

As per this system of numeration, the following rules are to be followed :

- The vowels alone represent the number 0.
- In case of the consonants in conjunction, only the last consonant is to be considered.
- Vowels in conjunction to the consonants have no numerical significance. Decode the alphabets as per the table above and reverse the order to get the number.

Let's see an example. Consider *anṛśaṃsaḥkaḷārthīsamartya*. This can be decoded into numbers as :

a	n	ṛśa	ṃsa	ḥka	ḷā	rthī	sa	ma	rtya
0	0	5	7	1	9	7	7	5	1

Reverse the order to get 1577917500.
Here, it should be pointed out that this number is of importance and will appear again and again in later chapters. 157797500 are the number of civil days in a *mahāyuga*.

1.5 Glossary along with the significance

In siddhāntas, to find the true planet, *mandaphala* and *śighraphala* correction terms are to be applied, which are zero when *mandakendra* and *śighrakendra* are zero. In such cases, calculations are simplified. Along with this, the tabulation of *vākyas* for the astronomical values for the planets also becomes easy in the time periods where both *mandakendra* and *śighrakendra* are zero. But the problem is it is only after a very large period of time like million of years, such thing happens. So, such intervals are not convenient. That's why, there comes the concept of *maṇḍalas, śodhyas* and *dhruvas*.

1. **Synodic cycle.** It is the time period between two successive conjunctions of the mean planet planet and its *śighrocca*. In case of outer planets, mean Sun is their *śighrocca*. But for inner planets, mean Sun is considered as the mean planet and the heliocentric mean planet itself is taken as their *śighrocca*.

2. **Maṇḍalas.** Let us consider the time 't' at which both *mandakendra* and *śighrakendra* are zero. As per their definition in 1.3.1, this happens when

   $$\text{Mean planet}=\text{Planet's } \textit{mandocca}=\text{Planet's } \textit{śighrocca}$$

   That means the mean planet and its *śighrocca* both are in the direction of the *mandocca*, which is fixed. Now, the planet and its *śighrocca* keep moving around the Earth as well as the Sun continuously. So, it may happen after a particular time period , when they both complete integral number of revolutions once again they conjunct along the direction of *mandocca.* But that time interval would be very large for this to happen. But there can be smaller time periods in which the planet is quite close to *mandocca* after a certain number of revolutions. These time intervals are called *maṇḍalas* . During the *maṇḍalas* period , *śighrakendra* =0, but *mandrakendra* $\neq 0$. This gives rise to the next definition.

3. **Dhruvas.** The difference between the planet's mean longitude and its apogee or *mandocca* when *śighrakendra* is zero is called *maṇḍaladhruva* or simply *dhruva.* As per the convention, *dhruva* >0 if planet's longitude is larger than that or its *mandocca* and <0 otherwise.

4. **Śodhya.** This is a time period close to the *ahargaṇa* taken at which *śighrakendra* is zero and longitude of mean planet is fairly close to its *mandocca.* Here by *ahargaṇa*, we mean the number of days elapsed since the beginning of a fixed epoch. In this case also, *dhruva* can be defined in the same way as for *maṇḍalas.*

Following are the tables mentioning *Śodhya*, *maṇḍalas* and their corresponding *dhruvas* for all the five planets from pages II. 18-A to II. 18-E in [1].

		वाक्यम्	दिनानि नाडिका:	ध्रुव:	कला:
	शोध्यं :-	रात्रिनिर्वासरुद्धांशकां	(15 92,740 - 23)	रागज्ञ:[1]	(—32)
बुधस्य Mercury	मण्डलं : i.	विष्णुर्यज्ञजतुष्टै:	(16,801 - 54)	यज्ञज्ञ:[2]	(—1)
	„ ii.	गुणनामोत्सवै:	(4,750 - 53)	[3]धीवेद्या[4]	(+149)
	„ iii.	शोकधीविशिखै:	(2,549 - 15)	सत्त्ववित्	(—447)
	परिवृत्तिवाक्यं :	तोटकै:	(116)[5]		१८-B

भृगोः
Venus

	वाक्यम्	दिनानि नाडिकाः	ध्रुवः	कलाः
शोध्य :-	[1]विश्वं सद्गन्धि[2]कान्तमयैः[3]	(15,61,937 - 44)	सत्यज्ञाः[4]	(+17)
मण्डलं : i.	धनाशावृद्धिसंगाढैः	(4,87,945 - 9)	ज्ञानिनः	(—0)
” ii.	दानवृद्धिशुभार्थकैः	(1,74,594 - 8)	धीप्राज्ञाः[5]	(+29)
” iii.	गुणितात्मा[6] सुदेहैः	(88,756 - 58)	दमज्ञः	(—58)
” iv.	[7]गुरुरक्षाघिभावैः[8]	(44,962 - 23)	लून: पोत्री[9]	(+2103)
” v.	जलेद्धकन्धरैः[10]	(2,919 - 38)	विश्वेड्यः	(—144)
परिवृत्तिवाक्यं :	विदिशैः[11]	(584)[12]		१८-D

[1]**कुजस्य**
Mars

	वाक्यम्	दिनानि नाडिकाः	ध्रुवः	कलाः
शोध्यं :-	शीलसारहारमाणिक्यो	(15,52,827 - 35)	रत्नवित्[2]	(—402)
मण्डलं : i.	धनधीदान[3]भङ्गार्तैः[4]	(6,34,089 - 9)	वनानाम्	(+4)
” ii.	पुत्रधीहामरालयैः	(1,32,589 - 21)	सत्रज्ञाः[5]	(+27)
” iii.	कविस्सम्मोदहारैः[6]	(28,857 - 41)	गंगाच्यां	(+133)
” iv.	[7]संगजन्मपथिकैः	(17,158 - 37)	वनेशः	(—504)
” v.	वनधीधृत[8]पुण्यैः	(11,699 - 4)	जलार्ता[9]	(+638)[10]
परिवृत्तिवाक्यं :	नहसैः	(780)[11]		१८-A

गुरोः
Jupiter

	वाक्यम्	दिनानि नाडिकाः	ध्रवः	कलाः
शोध्यं :-	[1]सत्यं[2]शूरवन[3]समयैः[4]	(15,70,425 - 17)	पितरः[5]	(—261)
मण्डलं : i.	[6]सरणिस्थदेवोत्सवैः[7]	(4,74,875 - 27)	अनेनाः[8]	(+0)
” ii.	[9]नर्मदावर्तमालकैः	(1,35,648 - 50)	धनानि	(—9)
” iii.	सत्योदय[10]निमित्तैः	(65,018 - 17)	बलाढ्या	(+133)
” iv.	सङ्कीर्णकुलनीलैः	(30,315 - 17)	कथाज्ञः	(—71)
” v.	देवेद्धगुणि[11]पुत्रैः	(21,539 - 48)	धीकृतः[12]	(—619)
” vi.	विश्वोत्साह[13]गर्वैः	(4,387 44)	भूसुराः[14]	(+274)
परिवृत्तिवाक्यं :	धूळिगैः[15]	(899)[16]		१८-C

शनेः
Saturn

	वाक्यम्	दिनानि नाडिकाः	ध्रुवः	कलाः
शोध्यं :-	[1]हरिर्वासवधीदमाढ्यः	(15,89,474 - 28)	तरुगः[2]	(—326)
मण्डलं : i.	हीनभोग[3]मुनिसमैः	(5,70,534 - 8)	मानिनी	(+5)
” ii.	गुरुवृद्धिधराजयैः[4]	(1,82,994 - 23)	लोकज्ञः	(—13)
” iii.	ज्ञानयमीशपरैः[5]	(21,551 - 0)	गुर्वाज्ञा	(+ 43)
” iv.	[6]रङ्गभक्त[7]घनिकैः[8]	(10,964 - 32)	यज्ञवान्	(+401)
परिवृत्तिवाक्यं :	हंसगैः	(378)[9]		१८-E

Observe that the numbers in the tables are obtained with the help of *Kaṭapayādi* system from the *vākyas* in Sanskrit which are written alongside with them. These tables are used in calculation of true longitude of the planets in the next chapter.

Chapter 2

Vākyakarana

2.1 Introduction

Vākyakarana is the basic text of the *Vākya-panchāngas* common in the Tamil country. The word means a *Karana* or a manual, in which *vākyas* or sentences and phrases are used as mnemonics for the numbers in the table used. As discussed in chapter 1, by *Kaṭapayādi* system of numeration, numerical values of astronomical constants and variables are encoded or decoded in or from meaningful *vākyas* or sentences. The importance of this text is it gives reasonably good and efficient method to calculate the true longitude of the Sun, the Moon and all the planets, with less complex calculations involved. Here, we will discuss this method for the planets −Mercury, Venus, Mars, Jupiter and Saturn. Let us start by introducing some terms used in *Vākyakarana* [1].

2.2 Definitions & Terminologies

Year numbering as per the Indian Hindu calendar is different than of the Julian or common calendar. In the Indian calendar, the years are counted in the *Śaka era*, in which its year 0 corresponds to the year 78 of common calendar and new year starts on March 22. So,

$$\textit{Śaka}\text{ year} = \text{Julian year} - 78.$$

For example, March 22, 2019 corresponds to the beginning of *Śaka* year 1941.

Definition 2.2.1. ***(Kali year)*** *As per Purans, Kali Yuga started on 18 Feb, 3102 BC. The Kali years gives the years elapsed in Kaliyuga or age. So,*

$$\textit{Kali year} \;=\; \textit{Śaka year} \;+3179.$$

For example, year 2019 corresponds to the *Kali* year 5120.

2.2.1 Calculation of Kali days

To find the number of days elapsed from the *Kali* epoch, do the following procedure explained with the example of Kali year 5120 :

1. Start by multiplying the *Kali* year by 365.
   For the *Kali* year 5120, we get 5120×365=18,68,800.
2. Add to this a fourth of the *Kali* year.
   i.e. 18,68,800+$\frac{5120}{4} = 18,70,080$.
3. First, multiply the *Kali* year by 5, then deduct 1237 and divide by 576. Whatever number of days, *nāḍikās*, etc. got, add them to the number obtained in step 2.
   In Vedic division of time,

   $$1 \; day = 60 \; nādis$$
   $$1 \; nādi = 60 \; vinādis.$$

   In our example, this gives ((5048× 5-1237)÷576)=42.2968 days=42 days 17 *nāḍis* 49 *vināḍis*. This is denoted as, $42-17-49$. Adding this to 18,70,080, we get $18,70,122-17-49$.

This gives the *Kali* days to the end of the True Solar year (which is equal to 365.242 days).

Definition 2.2.2. ***(Ahargaṇa)***
The number of Kali days is also referred as Kalidina saṅkhyā or Ahargaṇa.

2.3 The True Longitude of a planet

Given a particular date from the start *Kali* epoch or *Kaliyuga*, its corresponding *ahargaṇa* can be calculated as in 2.2.1. The *Vākyakarana* method or algorithm to find the true longitudes of the planets uses this *ahargaṇa* value as the input. There are some restrictions on *ahargaṇa* which we will discuss as we go through the method.
The algorithm for finding the true longitude of a planet given the *ahargaṇa* value in [1] has been discussed below with the help of two examples - Mars and Venus because the method is little different for Venus.

2.3.1 Algorithm for the true longitude of all planets, except Venus

For illustrating an example, suppose the planet be **Mars** and the *ahargaṇa* be **A=18,62,775**.

1. Subtract the *śodhya* for the planet in consideration from A and note its *dhruva*. The values of *śodhya* and its *dhruva* for each planet are mentioned in the tables given at the end in chapter 1.
   For Mars and A=18,62,775-00, we have

$$\begin{array}{rrr} & 18,62,775-00 & \\ \textit{Śodhya}: & 15,52,827-35 & \qquad \textit{Dhruva}: -402' \\ \hline & 3,09,947-25 & \end{array}$$

   Remark. Since you are subtracting *śodhya* from *ahargaṇa* A, the taken value of *ahargaṇa* can't be less than that. Otherwise this method fails.
   The reason for this is the time period around which the author of *Vākyakarana* developed this method. He, of course, wanted to find the true planets for the time to come. The values that served as *śodhya* for him for different planets, are enlisted in the tables mentioned above.

2. Divide the remainder by the *maṇḍalas* of the planet and their values are again in tables at the end of Chapter 1. Any *maṇḍala* can be used for any number of times until there is no possibility to dive further. Also, note the corresponding *dhruvas*.

In the example, *maṇḍalas* used from the table are : 1,32,589-21, 28,857-41 & 11,699-4.

$$\textit{Maṇḍala 1:}\ 1,32,589-21\overline{)\,3,09,947-25}\,(2\times(\ \textit{Dhruva :}\ +27')$$

$$2,65,178-42$$

$$\underline{}$$

$$44,768-43$$

$$\textit{Maṇḍala 2 :}\ 28,857-41\overline{)\,44,768-43}\,(1\times(\ \textit{Dhruva :}\ +133')$$

$$28,857-41$$

$$\underline{}$$

$$15,911-02$$

$$\textit{Maṇḍala 3 :}\ 11,699-4\overline{)\,15,911-02}\,(1\times(\ \textit{Dhruva :}\ +688')$$

$$11,699-04$$

$$\underline{}$$

$$4,211-58$$

3. Divide the remainder got after using *maṇḍalas* by the synodic cycle of days of the planet, written in tha last row of the tables in section 1.5.
   This is 780 in case of Mars.

$$780\overline{)\,4211-58}\,(5$$

$$3900-0$$

$$\underline{}$$

$$311-58$$

At the end of these calculations, we can compute the total *dhruva* which is equal to the sum of all *dhruvas* corresponding to the *maṇalas* used after multiplying by the respective quotients along with *śodhya's dhruva*. The sum is taken by taking the signs in consideration.
Here,the total *dhruva* is :

$$(-402')+(2\times(+27'))+(+133')+(+168')=+473'=+7^{\circ}53'. \tag{2.1}$$

4. After division by number of synodic cycles, the quotient we get is the number of cycles gone. In *Vākyakarana* [1] appendix, cycle tables are provided for each planet. These tables are used again and again to proceed further.
   For Mercury, Mars, Jupiter and Saturn, the number of such cycle tables given are 22, 15,11 and 29 respectively. Each cycle table contains the values in r, degrees and minutes along with 1 correction letter of the particular number of days elapsed in that cycle. The last value in each table for a planet is for number of days in a synodic cycle for that planet. These values are given in irregular interval of days. For example, following is the table for **Mars in cycle 6**.

Mars परिवृत्तिः VI

Days	Vākyas	r	Deg °	Min '	Correction letter	Days	Vākyas	r	Deg °	Min '	Correction letter
40	कर्णः सेनाकम्पनः	1	7	51	-1	400	रुद्रः पुरशासनः	5	21	22	-7
80	सामविज्जरोऽप्डज्ञः	2	4	57	-3	410	धर्मपरः शन्तनुः	5	21	59	-6
120	रङ्गी ज्ञानोल्लासनः	3	0	32	-7	420	शशिलेखा शूलिनि	5	23	55	-3
150	लूनाभिः कालासन्नः	3	19	3	-7	430	कामचारी शकुनिः	5	26	51	-1
180	नृपा सेनोद्धासिनः	4	7	10	-7	440	मालिनी नीतपाना	6	0	35	1
210	अम्भो विप्रभोजनम्	4	24	40	-8	450	ज्ञानात्मानस्तत्त्वज्ञाः	6	5	0	4
230	अम्भो मुनिशोधनम्	5	5	40	-9	465	विश्वप्रियास्तत्त्वज्ञाः	6	12	44	4
250	क्षोणीशः कामं नन्द्यः	5	15	56	-10	480	गौरी पुत्रचित्तज्ञा	6	21	23	6
270	स्तेनो मरणयोग्यः	5	25	6	-11	495	ईशाननाथा सेना	7	0	50	7
285	शमो नूनं तारकः	6	0	55	-12	510	लक्ष्मीर्नयसादिनी	7	10	53	8
300	लोलमीनस्तटाकः	6	5	33	-11	530	रामा नीरसूर्नन्द्या	7	24	52	10
315	जन्यदोऽनन्तकोपः	6	8	18	-11	550	दयाधीना जानकी	8	9	18	18
330	माली जनः क्षत्रियः	6	8	35	-12	570	क्ष्माश्वा राजधानी	8	24	3	9
340	श्रेयसेऽनन्तरूपम्	6	7	12	-12	600	तप्यन्ते युद्धे जनाः	9	16	16	8
350	रात्रौ वनितालापः	6	4	22	-13	630	गुणहीना निन्द्यजनाः	10	8	53	8
360	कुम्भिनां नेता भूपः	6	0	41	-14	660	विश्वं कनकाढ्ये जने	11	1	44	8
370	ईशश्चन्द्रशोभाकृत्	5	26	50	-14	700	धीर्गोप्यावनी	0	1	39	4
380	लीढाम्बरो मलयः	5	23	43	-13	740	श्रेयाञ्ज्ञानाकार्यज्ञः	1	0	12	-1
390	कृष्णपुत्रो मदनः	5	21	51	-8	780	जयी संखे पावनिः	1	27	18	-4

As mentioned, the quotient obtained in step 3 denotes for the number of synodic cycles gone. The remainder gives the number of days for which the position of the planet is to be determined using values and correction letters in *(quotient+1)*th cycle.
The way to do it is pick the value in terms of r, *degrees* and *minutes* from the appropriate cycle table for the maximum number of days that can be done. Then, find the motion for the remaining number of days using interpolation. Let us continue the example for proper understanding.

The quotient and remainder after division by 780, the number of days in a synodic cycle of Mars, are 5 and 311-58 respectively. So, 5 cycles are gone and we should compute for 311-58 days in 6^{th} cycle. From the 6^{th} cycle table shown above, the value corresponding to 311-58 days is not given. But the maximum number of days with the value in the table are 300 days to get the computations done for 311-58 days, viz.

$$300 \text{ days } \rightsquigarrow 6^r 5°33'{-}11$$

Also, the next value in the table is for 315 days, viz.

$$315 \text{ days } \rightsquigarrow 6^r 8°18'{-}11$$

Here, -11 at the end in both are the *correction letters* given in the last row of the table. Therefore, in both the cases (for 300 and 315 days), we get the correction in minutes by multiplying the correction letter and the total *dhruva* considered in minutes.

$$-11 \times (+7'53'') = -1°26'43"$$

So, after applying the corrections on both, we get the corrected values as below.

$$300 \text{ days } \rightsquigarrow 6^r 5°33' - 1°26'43" = 6^r 4°6'17" \tag{2.2}$$

$$315 \text{ days } \rightsquigarrow 6^r 8°18' - 1°26'43" = 6^r 6°51'17".$$

From the equation 2.2, we have the corrected motion done in 300 days. To calculate the motion for the remaining 11-58 days, do as follows.
From above, we get the motion of Mars for (315-300)=15 days as,

$$6^r 6°51'17" - 6^r 4°6'17" = 2°45'.$$

This gives, the motion of the planet per day $= 2°45' \div 15 = 11'$.
Finally, the motion for remaining 11 days 58 nadis =718/60 days is :

$$718/60 \times 11' = 2°11'38" \tag{2.3}$$

5. Add the total *dhruva* and the motion for the number of days same as in the remainder calculated in step 3 to get the *true longitude of the planet.*
Add (2.1), (2.2) and (2.3) for the true Mars given *ahargaṇa* 18,62,775 :

$$6^r 14°10'55" = 194°10'55''.$$

2.3.2 Algorithm for the true longitude of Venus

More or less, the method for calculating the true longitude of Venus given an *ahargaṇa* is same as that for other planets, except there is a choice involved in correction letters.

Let's understand the algorithm by taking the same *ahargaṇa* value **A=18,62,775** in case of **Venus**. Steps 1-3 are same as before. Calculations are as follows.

1. For Venus and A=18,62,775-00, we have

$$\begin{array}{rrl} & 18,62,775-00 & \\ \textit{Śodhya :} & 15,61,937-44 & \quad \textit{Dhruva : } +17' \\ \hline & 3,00,837-16 & \end{array}$$

2. For division, the *maṇḍalas* used from the table in chapter 1 are : 1,74,594-8, 88,756-53 and 2,919-38.

$$\textit{Maṇḍala 1: } 1,74,594-8\overline{)3,00,837-16}(1\times(\ \textit{Dhruva : } +29')$$
$$\begin{array}{c} 1,74,594-8 \\ \hline 1,26,243-8 \end{array}$$

$$\textit{Maṇḍala 2 : } 88,756-53\overline{)1,26,243-8}(1\times(\ \textit{Dhruva : } -58')$$
$$\begin{array}{c} 88,756-53 \\ \hline 37,486-15 \end{array}$$

$$\textit{Maṇḍala 3 : } 2,919-38\overline{)37,486-15}(12\times(\ \textit{Dhruva : } -144')$$
$$\begin{array}{c} 35,035-36 \\ \hline 2,450-39 \end{array}$$

3. The synodic cycle for Venus has 584 days, given at the last of each table in section 1.5.

$$584\overline{)\,2,450 - 39\,}(4$$

$$\underline{2,336 - 00}$$

$$114 - 39$$

The total *dhruva* is :

$$(17') + (+29') + (-58') + 12 \times (-144') = -1740' = -29°. \qquad (2.4)$$

4. As for other planets, the quotient after division by number of synodic cycles gives the number of cycles gone and we need the motion for the number of days same as the remainder in this division in the (quotient+1)th cycle.
   For Venus, there are in total 5 cycle tables each containing the values in r, *degrees* and *minutes* along with 2 correction letters of the particular number of days elapsed in the cycle. Below is the table for **Venus in cycle 5**.

Venus परिवृतिः V

Days	Vākyas	r	Deg °	Min '	Correction letter +	Correction letter -	Days	Vākyas	r	Deg °	Min '	Correction letter +	Correction letter -
40	शूराः प्रियधनाढ्याः	9	12	25	0	1	302	रागदं रम्भान नम्	4	28	32	0	0
80	अम्बा रत्नकाम्यानना	11	2	30	0	0	308	सेनासारोऽवनीड्यः	4	27	7	0	-1
110	भूपा धनिनः	0	9	14	0	0	314	अरिस्संखे विनष्टः	4	27	20	0	-0.5
140	योगी मृत्युपाटनः	1	15	31	-0.5	0	320	पानधीरघनष्टः	4	29	1	0	-0.5
160	दिव्यधनुः खरशः	2	9	18	-4	0	326	शुभा यूनां मानिनी	5	1	45	0	0
180	सूकरो नागप्रियः	3	2	17	-2	-1	332	अङ्के मेना मनोज्ञा	5	5	30	0	0
195	ईशो दयागरिष्ठः	3	18	50	-1	-1.5	338	धर्मो धनं मुनीनाम्	5	9	59	0	0
210	अम्बुभृत्तवकरः	4	4	30	-1	-2	346	जाया तोष्याशनेन	5	16	18	0	0
220	सम्राड्भोग्यो भूपिण्डः	4	14	27	-0.5	-2.5	354	ईशो गिरीमुष्टिज्ञः	5	23	50	-0.5	0
230	देवो गिरिविष्टरः	4	23	48	-0.5	-2	364	गविलम्बस्तपनः	6	3	43	-1	0
238	मूर्खो नूनं शिष्टभुक्	5	0	25	0	-2	374	सेव्यो भूपश्चापज्ञः	6	14	17	-1	0
246	विरतिज्ञो मुनीन्द्रः	5	6	24	0	-2	389	देवो नूनं सुश्रुज्ञः	7	0	48	-1.5	0
252	दाननित्यो मुनीन्द्रः	5	10	8	0	-2	404	प्रिया जाया सुखकृत्	7	18	12	-2	1
258	धीमान् श्रीकामो नरः	5	12	59	0	-2	424	वनाद्रयोऽहिकुण्ठाः	8	12	4	-1	1.5
264	शोभाभृत् कामशोषः	5	14	45	0	-1.5	444	क्षुद्रा स्तेना धीकुण्ठाः	9	6	26	-1	1.5
270	जनैर्मान्यो मुनीन्द्रः	5	15	8	0	-2	474	ज्ञानश्लाघ्या नृपनयाः	10	13	0	0	1
276	हिमालये मुनीन्द्रः	5	13	58	0	-2	504	विश्वाधिकाः कृपानिभाः	11	19	44	0	0
282	मुनीज्यः पद्मनेत्रः	5	11	5	0	-2	544	स्तनौ धनं कन्यानाम्	1	9	6	1	0
288	वनस्थाने मुनीन्द्रः	5	7	4	0	-2	584	दनजश्रेष्ठः काव्यः	2	28	8	1	-1
296	सीरी पानमग्नोहः	5	1	27	0	-1.5							

The choice of correction letter is based on the sign of total *dhruva*. If the total *dhruva* is **positive**, choose the **first correction letter**. If negative, pick the second one.
As before, the last value in each table is the number of days in a synodic cycle for the planet, i.e. 584 and the values are in irregular interval of days.

Once the *correction letter* is chosen, the method for the true longitude of Venus is same as for other planets. Let us continue solving our example.

The quotient and remainder after division by 584 are 4 and 114-39 respectively. This means, 4 cycles are gone and we need to calculate for 114-39 days in 5th cycle. From the 5th cycle table for Venus above, the value corresponding to 114-39 days is not there. But the maximum number of days with the value in the table are 110 days to get the computations done for 114-39 days, viz.

$$110 \text{ days } \rightsquigarrow 9°14' - 0 \tag{2.5}$$

Also, the next value in the table is for 140 days, viz.

$$140 \text{ days } \rightsquigarrow 1^r 15°31'{-}0$$

Here, the *correction letter* is 0 for both 110 and 140 number of days, chosen from the second column of correction letters because the total *dhruva* is negative. So, in both the cases, we get the correction in minutes (by multiplying the correction letter and the total *dhruva* considered in minutes) to be 0'. This means no correction is to be done.

So, from the equation 2.5, we have the motion done in 110 days. To calculate the motion for the remaining 4-39 days, we do the following.

From above, we get the motion of Venus for (140-110)=30 days as,

$$1^r 15°31' - 9°14' = 1^r 6°17'.$$

This gives, the motion of the planet per day $= 1^r 6°17' \div 30 = 1°12'34'' = 4354''$.

Finally, the motion for remaining 4 days 39 nadis =279/60 days is :

$$279/60 \times 4354'' = 5°37'26" \tag{2.6}$$

5. Add (2.4), (2.5) and (2.6) for the true longitude of Venus given *ahargaṇa* 18,62,775 :

$$-15°51'26" = 345°51'26''.$$

2.4 Graphical analysis of *Vākyakarana* method

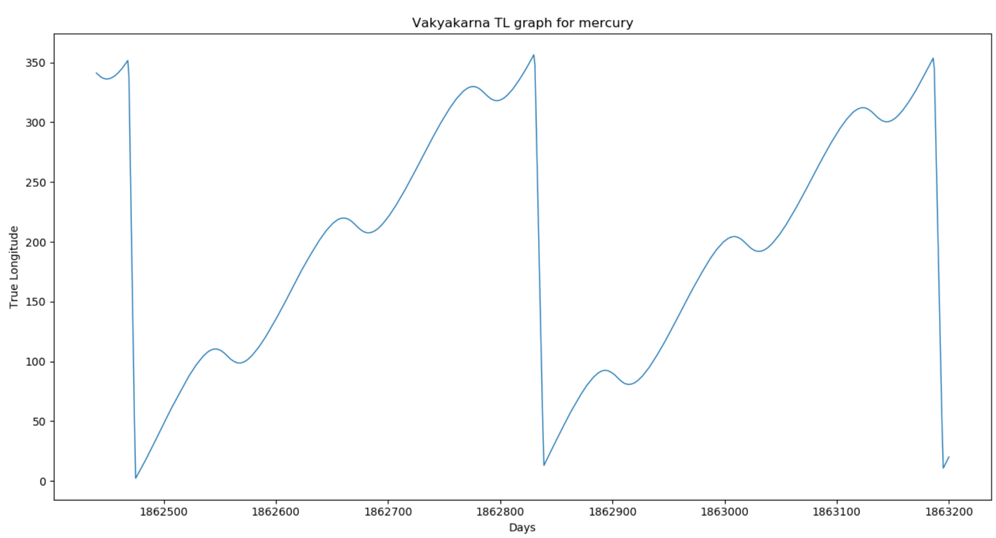
After understanding the algorithm to find the true longitude of all planets, we need to write the computer programs for each planet to :

1. output the true longitude of the planets with *ahargaṇa* as the input.

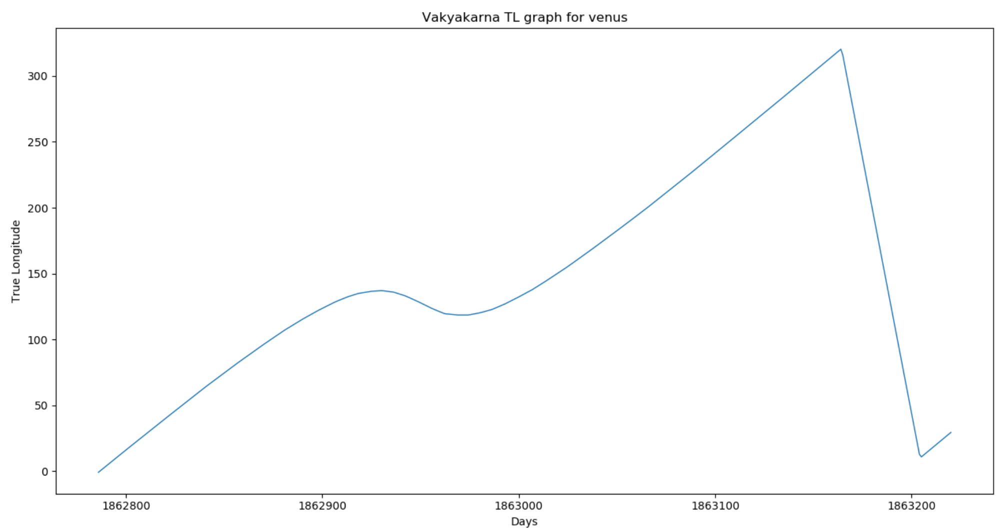
2. plot the graphs *ahargaṇa* vs *true planet* to look at the behavior and the change in the true longitude calculated as per *Vākyakarana* method in [1] along with the change in time with the range of *ahargaṇa* given as the input.

Below are the graphs plotted by the programming done using Python for all the five planets- Mercury, Venus, Mars, Jupiter and Saturn with *ahargaṇa* on the x-axis and true longitudes as per *Vākyakarana* on y-axis.

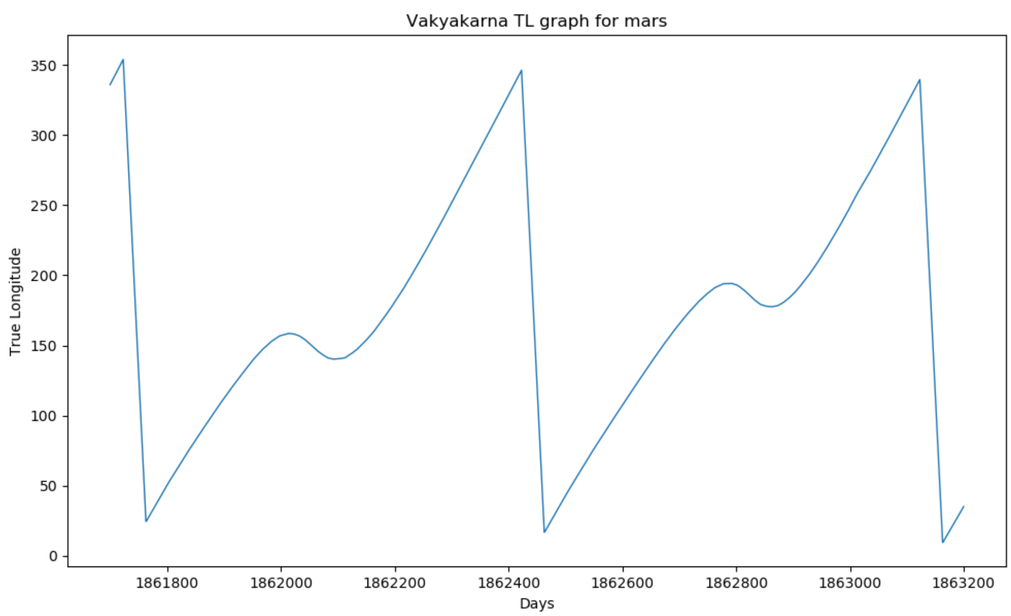
2.4.1 Vākyakarana graph of MERCURY exhibiting 760 days of motion



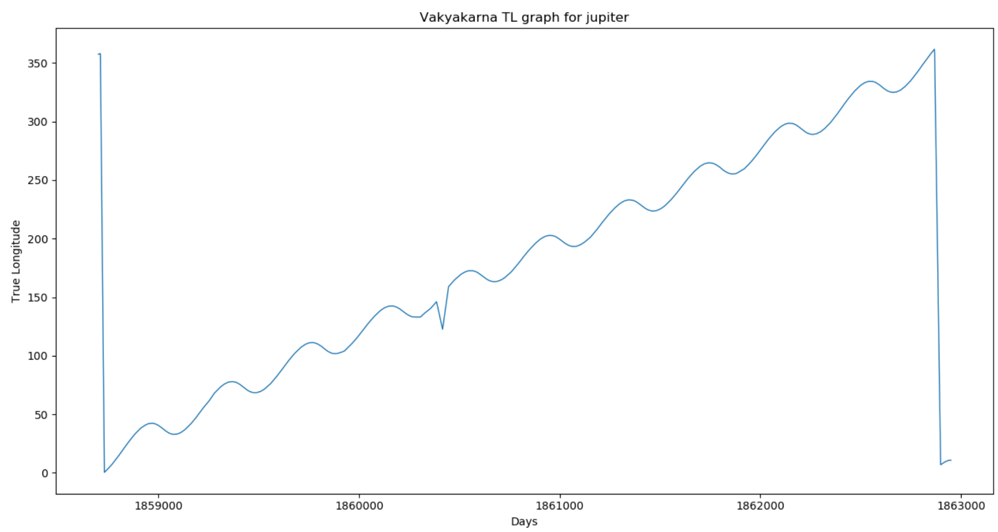
2.4.2 Vākyakarana graph of VENUS exhibiting 435 days of motion



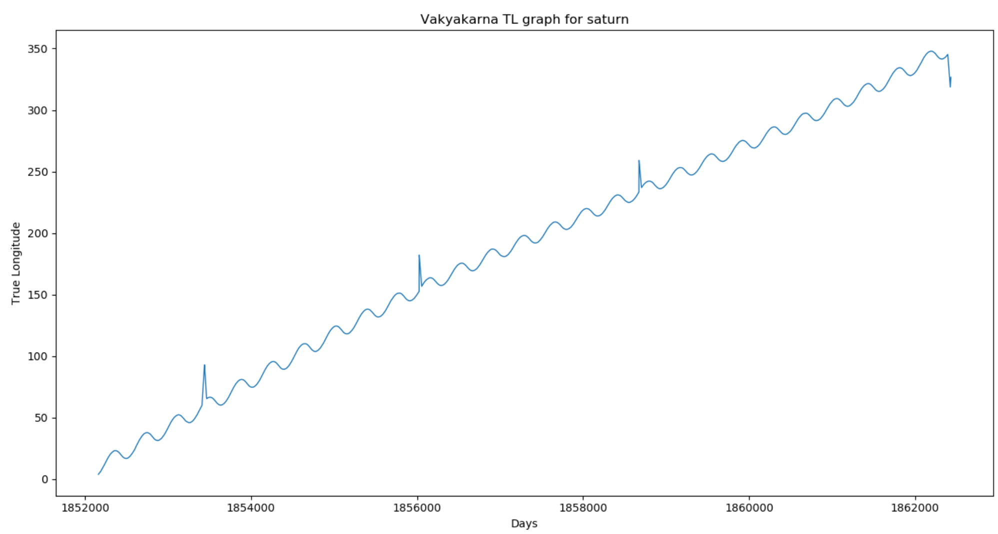
2.4.3 Vākyakarana graph of MARS for 1500 days of motion



2.4.4 Vākyakarana graph of JUPITER for 4250 days of motion



2.4.5 Vākyakarana graph of SATURN for 10,270 days of motion



2.4.6 Key observations from the graphs

1. The range for *ahargaṇa* values corresponds to the years in 18^{th} and 19^{th} centuries. To show the motion and the position of the planets, the time periods are such that the starting point for the true longitude approaches 0° and its ending point is near 360°. This is done to show atleast one complete revolution for each planet in the graphs.

2. As the planets are moving in a circle, that's why 360° is same as 0° which explains the sudden drops to 0° after a point in all the graphs.

3. After a certain period of time, the graphs become repetitive indicating the motion is periodic. Obviously, these periods are different for different planets.

4. The graphs are not increasing. True longitudes increase during some time periods and decrease for some others. The reason for is the *retrograde* motion of the planets. This is the apparent motion of planets in which they appear to be moving the direction opposite of that before So, the graphs show that the *Vākyakarana* method takes this motion into account.

5. In the graphs for Jupiter and Saturn, some sharp points can be observed. These are indicate some kind of small errors in their cycle tables.

Chapter 3

Āryabhaṭīya

3.1 Introduction

The Siddhantic text *Āryabhaṭīya* is originally the written work of the famous Indian mathematician *Āryabhaṭa* I and has a prestigious position in ancient astronomy and mathematics. Many calendrical texts and tables used in north and south India were based on it during ancient times. *Āryabhaṭa* I gave two theories about the motion of the Sun , the Moon and the planets around the Earth, called *eccentric* and *epicyclic* theories.
In *Āryabhaṭīya*, the following hypotheses are considered to calculate the planetary positions:

1. The mean planets revolve in circular orbits around the Earth with linear motion.
2. The mean motion of the planets is the terms of number of revolutions by the planets around the Earth in 43,20,000 years.
3. The true planets move along the circumference of the epicycles and their motions are governed by two such epicycles : *Manda* epicycle and *Śighra* epicycle.

Now, the terms required to understand the *Āryabhaṭīya* method given in [4] of calculating the true longitudes of the planets have already been discussed in chapter 1. There are some astronomical constants whose values are given below.

- A *Mahāyuga* is of duration 43,20,000 years.

- A *yuga* such as *Kaliyuga* has the duration one-tenth of that of a *Mahāyuga*, that is 4,32,000 years.

- The number of civil or calendar days in a *Mahāyuga* is 1,57,79,17,500.

The *yuga* is taken to be *Kaliyuga*. The *revolution numbers* of the planets mentioned in second hypothesis are given in the following table.

MEAN MOTION OF THE PLANET	
Planet	**Revolutions in 43,20,000 years**
Sun	43,20,000
Mars	22,96,824
Sighrocca of Mercury	1,79,37.020
Jupiter	3,64,224
Sighrocca of Venus	70,22,388
Saturn	1,46,564

Also, the dimensions of *manda* and *śighra* epicycles in even and odd quadrants as per *Āryabhaṭīya* are tabulated as follows.

Dimensions of planets' *manda* epicycles			Dimensions of planets' *sighra* epicycles		
Āryabhaṭīya					
	odd quadrant	even quadrant		odd quadrant	even quadrant
Mercury	31.5°	22.5°	Mercury	139.5°	130.5°
Venus	18°	9°	Venus	265.5°	256.5°
Mars	63°	81°	Mars	238.5°	229.5°
Jupiter	31.5°	36°	Jupiter	72°	67.5°
Saturn	40.5°	58.5°	Saturn	40.5°	36°

3.2 The True planets

In *Āryabhaṭīya* [4], the true longitude of a planet is to be calculated with *ahargaṇa* as an input, same as in *Vākyakarana*. The algorithm for finding the true planets is different for inner planets (Mercury, Venus) and outer planets (Mars, Jupiter, Saturn) in two senses:

1. To calculate the mean longitude of the planet and its *śighrocca*, slightly different methods are to be applied.
2. The correction terms *mandaphala* and *śighraphala* are to be applied in different manner and sequence to the mean longitudes for *inner* and *outer* planets.

So, we will discuss two algorithms - one for the true longitude of outer planets using *Mars* as an example and the other for inner planets using *Venus*.

3.2.1 Algorithm for the true longitude of outer planets

For example, consider the planet **Mars** and *ahargaṇa* to be **A=18,62,775**.

1. *Find the mean longitude of the planet and its śighrocca* for the given *ahargaṇa*.
   We know that for outer planets, mean Sun is their *śighrocca*. Let R_p and R_S be the revolutions of the planet and the Sun respectively in a *mahāyuga*. Denote the number of civil days in *mahāyuga* by C_v.
   Start by calculating the terms

$$Q_0 = \frac{R_p \times A}{C_v} \text{ and } Q_0' = \frac{R_S \times A}{C_v} \tag{3.1}$$

   which are the number of revolutions of the planet and the Sun respectively in 'A' number of days.
   Then, the mean longitude of the planet MP and its *śighrocca* MS are given respectively by,

$$MP = (Q_0 - \lfloor Q_0 \rfloor) \times 360^\circ$$
$$MS = (Q_0' - \lfloor Q_0' \rfloor) \times 360^\circ.$$

Let do the example. $R_S = 43,20,000$ and for Mars, $R_p = 22,96,824$ from the table in 3.1 .By substituting these values, we get

$$Q_0 = 2711.4639 \text{ and } Q_0' = 5099.8787$$

Now, the mean longitude of Mars and its *śighrocca* for 18,62,775 days is computed as,

$$MP = 0.4639 \times 360^\circ = 167^\circ, \text{ and}$$
$$MS = 0.8787 \times 360^\circ = 316.34^\circ \text{ respectively.}$$

2. *Find mandakendra and śighrakendra.*
   These are given by,
   θ_m =*Mandakendra*=Mean longitude of planet $(MP)-$ planet's apogee
   θ_s=*Śighrakendra*= planet's *sighrocca* $(MS)-$ Mean longitude of planet (MP)

   where apogees of all planets are in section 1.3 of chapter 1.
   For example, Mars' apogee is 118° and θ_m and θ_s are,

$$\textit{Mandakendra} = \theta_m = 167^\circ - 118^\circ = 49^\circ$$
$$\textit{Śighrakendra} = \theta_s = 316.34^\circ - 167^\circ \approx 149^\circ.$$

3. *Calculate mandaphala and śighraphala.*

$$\textit{Mandaphala} = \frac{\text{Rsin } \theta_m \times \textit{manda epicycle}}{360^\circ} \tag{3.2}$$

$$\textit{Śighraphala} = \frac{\text{Rsin } \theta_s \times \textit{śighra} \text{ epicycle}}{360^\circ} \times \frac{R}{H} \tag{3.3}$$

   Let us now explain each terms in the formulae.

   (a) The values of *manda* and *śighra* epicycles are given in the table in 3.1 to be chosen whether θ_m and θ_s respectively are in odd or even quadrants. In usual sense, θ_m is in odd quadrant if $0^\circ < \theta_m < 90^\circ$ or $180^\circ < \theta_m < 270^\circ$. Same holds for θ_s.

   (b) R refers to the *standard radius* of the Earth and is equal to $3438'$.

   (c) For any given θ, compute Rsin θ using the following method discussed in *Āryabhaṭīya*. For some arcs θ in minutes and $0' \leq \theta \leq 5400'$, the corresponding Rsin θ are given in the table below.

But to get the value of Rsin θ for any $0' \leq \theta \leq 5400'$ not given in the table, do as follows.
Choose the largest arc θ_1 and the smallest θ_2 given in the table such that $\theta_1 \leq \theta \leq \theta_2$. Then,

$$\text{Rsin } \theta = \text{Rsin } \theta_1 + \frac{\text{Rsin } \theta_2 - \text{Rsin } \theta_1}{225} \times (\theta - \theta_1)$$

Next. to obtain Rsines of the arcs $\theta > 90°$, the following formulae are to be used:

$$\text{Rsin } (90° + \phi) = \text{R} - \text{Rvers } \phi, \text{ for } \theta = 90° + \phi$$
$$\text{Rsin } (180° + \phi) = -\text{Rsin } \phi, \text{ for } \theta = 180° + \phi$$
$$\text{Rsin } (270° + \phi) = -\text{R} + \text{Rvers } \phi, \text{ for } \theta = 270° + \phi$$

where $\phi \leq 90°$ and Rvers ϕ= R$-$R$\cdot$ cos ϕ for any ϕ.

Rsines at the intervals of 225'			
Aryabhata I's values			
Arc	Rsine	Arc	Rsine
225'	225'	2925'	2585'
450'	449'	3150'	2728'
675'	671'	3375'	2859'
900'	890'	3600'	2978'
1125'	1105'	3825'	3084'
1350'	1315'	4050'	3177'
1575'	1520'	4275'	3256'
1800'	1719'	4500'	3321'
2025'	1910'	4725'	3372'
2250'	2093'	4950'	3409'
2475'	2267'	5175'	3431'
2700'	2431'	5400'	3438'

(d) H stands for *śighrakarna* and the formulas for calculating H depends on the quadrant θ_s belongs to.

$$\text{For } 0^\circ \leq \theta_s < 90^\circ,\ H = \sqrt{(\text{R}+\text{Rsin } (90^\circ - \theta_s))^2 + (\text{Rsin } \theta_s)^2}$$
$$\text{For } 90^\circ \leq \theta_s < 180^\circ,\ H = \sqrt{(\text{R} - \text{Rsin } (\theta_s - 90^\circ))^2 + (\text{Rsin } (180^\circ - \theta_s))^2}$$
$$\text{For } 180^\circ \leq \theta_s < 270^\circ,\ H = \sqrt{(\text{R} - \text{Rsin } (270^\circ - \theta_s))^2 + (\text{Rsin } (\theta_s - 180^\circ))^2}$$
$$\text{For } 270^\circ \leq \theta_s < 360^\circ,\ H = \sqrt{(\text{R}+\text{Rsin } (\theta_s - 270^\circ))^2 + (\text{Rsin } (360^\circ - \theta_s))^2}$$

Continuing with the example, to calculate *mandaphala* and *śighraphala* with $\theta_m = 49^\circ$ in 1st quadrant and $\theta_s = 149^\circ$ in 2nd quadrant, note down from the epicycle table in 3.1 that for Mars,

$$\begin{gathered} \textit{Manda} \text{ epicycle} = 63^\circ \text{ for odd quadrants} \\ \& \ \textit{Śighra} \text{ epicycle} = 229.5^\circ \text{ for even quadrants.} \end{gathered}$$

Next, we want to compute Rsin 49° and Rsin 149° which can be done as,

$$\begin{aligned} \text{Rsin } 49^\circ &= \text{Rsin } 2940' \\ &= \text{Rsin } 2925' + \frac{\text{Rsin } 3150' - \text{Rsin } 2925'}{225} \times (2940' - 2925') \\ &= 2594.5' = 43.24^\circ \end{aligned}$$

and

$$\begin{aligned} \text{Rsin } 149^\circ = \text{Rsin } (90^\circ + 59^\circ) &= \text{R} - \text{Rvers } 59^\circ \\ &= \text{R} - (\text{R} - \text{R} \cdot \cos 59^\circ) \\ &= 1770.7' = 29.5^\circ. \end{aligned}$$

For *Śighrakarna*, since $90^\circ \leq \theta_s < 180^\circ$, we use

$$H = \sqrt{(\text{R} - \text{Rsin } (149^\circ - 90^\circ))^2 + (\text{Rsin } (180^\circ - 149^\circ))^2} = \sqrt{(\text{R} - \text{Rsin } 59^\circ)^2 + (\text{Rsin } 31^\circ)^2}$$

with

$$\begin{aligned} \text{Rsin } 59^\circ &= \text{Rsin } 3540' \\ &= \text{Rsin } 3375' + \frac{\text{Rsin } 3600' - \text{Rsin } 3375'}{225} \times (3540' - 3375') \\ &= 2946.27' = 49.1^\circ \end{aligned}$$

$$\begin{aligned} \text{Rsin } 31^\circ &= \text{Rsin } 1860' \\ &= \text{Rsin } 1800' + \frac{\text{Rsin } 2025' - \text{Rsin } 1800'}{225} \times (1860' - 1800') \\ &= 1770' = 29.5^\circ \end{aligned}$$

This gives, $H = \sqrt{(491.73)^2 + (1770)^2}' = 1837'$. Hence, by using all these values in equations of *mandaphala* and *śighraphala*, we get

$$\textit{Mandaphala} = 7.567^\circ$$
$$\textit{Śighraphala} = 35.2^\circ.$$

4. *Apply correction terms Mandaphala & Śighraphala to get the true planet.*

   (a) Apply the half of *mandaphala* negatively or positively to the mean longitude of the planet, depending θ_m is less than or greater than 180° and to the longitude of its apogee reversely. We get the corrected longitude of the planet and corrected planet's apogee I.
   By applying this to Mars, we have

$$\begin{aligned} \text{Corrected longitude of Mars} &= \text{Mars' mean longitude} - \frac{1}{2}\textit{Mandaphala} \\ &= 167^\circ - \frac{1}{2} \times 7.567^\circ = 163.2165^\circ \end{aligned}$$

$$\begin{aligned} \text{Corrected Mars' apogee I} &= \text{ Longitude of Mars' apogee} + \frac{1}{2}\ \textit{Mandaphala} \\ &= 118^\circ + \frac{1}{2} \times 7.567^\circ = 121.7835^\circ \end{aligned}$$

   (b) Subtract or add the half of *śighraphala* to the corrected longitude of planet' apogee I, depending θ_s is less than or greater tham 180° to get corrected planet's apogee II. This gives,

$$\begin{aligned} \text{Corrected Mars' apogee II} &= \text{Corrected Mars' apogee I} - \frac{1}{2}\ \textit{Śighraphala} \\ &= 121.7835^\circ - \frac{1}{2} \times 35.2^\circ = 104.1835^\circ. \end{aligned}$$

   (c) Calculate the *mandaphala* afresh using the corrected mean longitude of the planet and corrected planet's apogee II. Then, apply the whole of it to the original mean longitude of the planet negatively or positively, depending the new *mandakendra*,

denoted by $\theta_{m,new}$ is less than or greater than 180°. This will give the *true-mean longitude* of the planet.
In the example, we have,

$$\begin{aligned}\theta_{m,new} &= \text{Corrected mean longitude of Mars} - \text{Corrected Mars' apogee II} \\ &= 163.2165^\circ - 104.1835^\circ = 59^\circ.\end{aligned}$$

To calculate *new mandaphala*, we require Rsin 59° which is equal to 49.1° as calculated above. Also, $\theta_{m,new}$ is in 1st quadrant, so *manda* epicycle =63° as before. Using the formula, we have

$$\textit{New mandaphala} = \frac{\text{Rsin } \theta_{m,new} \times \textit{Manda} \text{ epicycle}}{360^\circ} = 8.59^\circ$$

Therefore, by application of this new *mandaphala* to mean Mars as per the rule, we get

$$\begin{aligned}\textit{True-mean longitude} \text{ of Mars} &= \text{Mean longitude of Mars} - \text{New } \textit{mandaphala} \\ &= 167^\circ - 8.59^\circ = 158.4^\circ.\end{aligned}$$

(d) Calculate *śighraphala* afresh using true-mean longitude of the planet. Apply the whole of it positively or negatively to the true-mean longitude, depending the new *śighrakendra*, $\theta_{s,new}$ is less than or greater than 180°. This will give the *true longitude* of the planet.
Now, start with calculating the new *śighrakendra.*

$$\begin{aligned}\theta_{s,new} &= \text{Longitude of Mars' } \textit{śighrocca} - \text{True-mean longitude of Mars} \\ &= 316.34^\circ - 158.4^\circ = 157.94^\circ \approx 158^\circ.\end{aligned}$$

To compute new *śighraphala*, we need the value of Rsin 158°.

$$\begin{aligned}\text{Rsin } 158^\circ = \text{Rsin } (90^\circ + 68^\circ) &= \text{R} - \text{Rvers } 68^\circ \\ &= \text{R} - (\text{R} - \text{R} \cdot \cos 68^\circ) \\ &= 1287.89' = 21.46^\circ.\end{aligned}$$

Also, the new *śighrakendra* $\theta_{s,new}$ is in 2nd quadrant, keeping *śighra* epicycle to be 229.5° same as before.

For new *śighrakarna*, since $90° \leq \theta_{s,new} < 180°$, we use

$$\begin{aligned} H &= \sqrt{(\mathrm{R} - \mathrm{Rsin}\ (158° - 90°))^2 + (\mathrm{Rsin}\ (180° - 158°))^2} \\ &= \sqrt{(\mathrm{R} - \mathrm{Rsin}\ 68°)^2 + (\mathrm{Rsin}\ 22°)^2} \end{aligned}$$

where Rsines can be calculated as,

$$\begin{aligned} \mathrm{Rsin}\ 68° &= \mathrm{Rsin}\ 4080' \\ &= \mathrm{Rsin}\ 4050' + \frac{\mathrm{Rsin}\ 4275' - \mathrm{Rsin}\ 4050'}{225} \times (4080' - 4050') \\ &= 3187.5' = 53.1° \end{aligned}$$

$$\begin{aligned} \mathrm{Rsin}\ 22° &= \mathrm{Rsin}\ 1320' \\ &= \mathrm{Rsin}\ 1125' + \frac{\mathrm{Rsin}\ 1350' - \mathrm{Rsin}\ 1125'}{225} \times (1320' - 1125') \\ &= 1287' = 21.45°. \end{aligned}$$

This gives, $H = \sqrt{(250.5)^2 + (1287)^2}' = 1311.2'$. Therefore,

$$\textit{New Śighraphala} = \frac{\mathrm{Rsin}\ \theta_{s,new} \times \textit{śighra}\ \text{epicycle}}{360°} \times \frac{R}{H} = 35.88°,$$

By applying the *new śighraphala* to the true-mean longitude of Mars, we get for given *ahargaṇa* 18,62,775

$$\begin{aligned} \textit{True longitude of Mars} &= \text{True-mean longitude of Mars} + \textit{New śighraphala} \\ &= 158.4° + 35.88° \\ &= 194.27°. \end{aligned}$$

3.2.2 Algorithm for the true longitude of inner planets

To understand the method, consider the planet **Venus** and *ahargaṇa* to be **A=18,62,775** as an example.

1. *Find the mean longitude of the inner planet and its śighrocca for the given ahargaṇa.* Since inner planets revolve around the Sun and along with the Sun, move around the Earth. That's why, unlike outer planets, mean Sun is the mean planet for inner planets and they themselves are the *śighroccas*.
   Let R_{ps} and R_S be the revolutions of the planet's *śighrocca* and the Sun respectively in a *mahāyuga* and C_v be the number of civil days in *mahāyuga*.
   Calculate
   $$Q_0 = \frac{R_S \times A}{C_v} \text{ and } Q_0' = \frac{R_{ps} \times A}{C_v} \tag{3.4}$$
   which are the number of revolutions of the Sun and the planet's *śighrocca* respectively in 'A' number of days.
   Then, the mean longitude of the planet MP and its *śighrocca* MS are given respectively by,
   $$MP = (Q_0 - \lfloor Q_0 \rfloor) \times 360^\circ$$
   $$MS = (Q_0' - \lfloor Q_0' \rfloor) \times 360^\circ.$$
   For our example, $R_S = 43,20,000$ and *śighrocca* of Venus, $R_{ps} = 70,22,388$ from the table in 3.1 . By substituting these values, we get
   $$Q_0 = 5099.8787 \text{ and } Q_0' = 8290.122143$$
   So,, the mean longitude of Venus and its *śighrocca* for 18,62,775 days can be computed as,
   $$MP = 0.8787 \times 360^\circ = 316.34^\circ, \text{ and}$$
   $$MS = 0.122143 \times 360^\circ = 43.97^\circ \approx 44^\circ \text{ respectively.}$$

2. *Find mandakendra and śighrakendra.*
   Same as outer planets, these are given by,

$$\theta_m = \textit{Mandakendra} = \text{Mean longitude of planet } (MP) - \text{ planet's apogee}$$
$$\theta_s = \textit{Śighrakendra} = \text{ planet's } \textit{śighrocca } (MS) - \text{ Mean longitude of planet } (MP)$$

   where apogees of all planets are in section 1.3 of chapter 1. For example, Venus' apogee is $90°$ and its θ_m, θ_s are,

$$\textit{Mandakendra} = \theta_m = 316.34° - 90° \approx 226°$$
$$\textit{Śighrakendra} = \theta_s = 44° - 316.34° \approx -272° = 88°.$$

3. *Calculate mandaphala and śighraphala.* These have same formulae with the same interpretations of the terms used as in case of outer planets. That is,

$$\textit{Mandaphala} = \frac{\text{Rsin } \theta_m \times \textit{manda} \text{ epicycle}}{360°}$$

$$\textit{Śighraphala} = \frac{\text{Rsin } \theta_s \times \textit{śighra} \text{ epicycle}}{360°} \times \frac{R}{H}$$

   Working with the example, for *mandaphala* and *śighraphala* with $\theta_m = 226°$ in 3rd quadrant and $\theta_s = 88°$ in 1st quadrant, write from the epicycle table in 3.1 that for Venus,

$$\textit{Manda} \text{ epicycle} = 18° \ \& \ \textit{Śighra} \text{ epicycle} = 265.5° \text{ for odd quadrants.}$$

   Next, we need Rsin 226° and Rsin 88° which can be calculated as,

$$\begin{aligned}
\text{Rsin } 226° = \text{Rsin } (180° + 46°) &= -\text{Rsin } 46° \\
&= -\text{Rsin } 2760' \\
&= -\text{Rsin } 2700' - \frac{\text{Rsin } 2925' - \text{Rsin } 2700'}{225} \times (2760' - 2700') \\
&= -2472.1' = -41.2°.
\end{aligned}$$

$$\begin{aligned}\text{Rsin } 88^\circ &= \text{Rsin } 5280' \\ &= \text{Rsin } 5175' + \frac{\text{Rsin } 5400' - \text{Rsin } 5175'}{225} \times (5280' - 5175') \\ &= 3434.27' = 57.24^\circ.\end{aligned}$$

To find the value of *Śighrakarna*, note $0^\circ \leq \theta_s < 90^\circ$, we use

$$H = \sqrt{(\text{R} + \text{Rsin } (90^\circ - 88^\circ))^2 + (\text{Rsin } (88^\circ))^2} = \sqrt{(\text{R} + \text{Rsin } 2^\circ)^2 + (3434.27)^2}$$

where

$$\begin{aligned}\text{Rsin } 2^\circ &= \text{Rsin } 120' \\ &= \text{Rsin } 0' + \frac{\text{Rsin } 225' - \text{Rsin } 0'}{225} \times (120' - 0') \\ &= 120' = 2^\circ\end{aligned}$$

This gives, $H = \sqrt{(3558)^2 + (3434.27)^2}' = 4944.868' = 82.4^\circ$. Hence, by using all these values in equations of *mandaphala* and *śighraphala*, we get

$$\begin{aligned}\textit{Mandaphala} &= -2.06^\circ \\ \textit{Śighraphala} &= 29.35^\circ.\end{aligned}$$

4. *Apply correction terms Mandaphala & Śighraphala to get the true planet.*

   (a) Subtract or add the half of *śighraphala* to the longitude of planet' apogee, depending θ_s is less than or greater than 180° to get the corrected planet's apogee I. From this, we have

   $$\begin{aligned}\text{Corrected Venus' apogee I} &= \text{Longitude of Venus' apogee } - \frac{1}{2}\ \textit{Śighraphala} \\ &= 90^\circ - \frac{1}{2} \times 29.35^\circ = 75.325^\circ.\end{aligned}$$

   (b) Apply the half of *mandaphala* positively or negatively to the corrected longitude of planet' apogee I, depending θ_m is less than or greater than 180° to get the corrected planet's apogee II.

This gives,

$$\begin{aligned}\text{Corrected Venus' apogee II} &= \text{Corrected Venus' apogee I } + \frac{1}{2}\textit{mandaphala}\\ &= 75.325^\circ + \frac{1}{2} \times (-2.06^\circ) = 74.295^\circ.\end{aligned}$$

(c) Calculate the *mandaphala* afresh using the corrected planet's apogee II. Then, apply the whole of it to the original mean longitude of the planet negatively or positively, depending the new *mandakendra*, denoted by $\theta_{m,new}$ is less than or greater than 180°. This will give the *true-mean longitude* of the planet.
In the example, we have,

$$\begin{aligned}\theta_{m,new} &= \text{Mean longitude of Venus} - \text{Corrected Venus' apogee II}\\ &= 316.34^\circ - 74.295^\circ = 242^\circ.\end{aligned}$$

To calculate *new mandaphala*, we require Rsin 242° which can be calculated as,

$$\begin{aligned}\text{Rsin } 242^\circ = -\text{Rsin } 62^\circ &= -\text{Rsin } 3720'\\ &= -\text{Rsin } 3600' - \frac{\text{Rsin } 3825' - \text{Rsin } 3600'}{225} \times (3720' - 3600')\\ &= -3034.5' = -50.58^\circ.\end{aligned}$$

Also, $\theta_{m,new}$ is in 3rd quadrant, so *manda* epicycle =18° as before. Using the formula, we have

$$\textit{New mandaphala} = \frac{\text{Rsin } \theta_{m,new} \times \textit{Manda} \text{ epicycle}}{360^\circ} = -2.529^\circ$$

Therefore, by application of this new *mandaphala* to mean Venus as per the rule, we get

$$\begin{aligned}\textit{True-mean longitude} \text{ of Venus} &= \text{Mean longitude of Venus} + \text{New } \textit{mandaphala}\\ &= 316.34^\circ - 2.529^\circ = 313.8^\circ.\end{aligned}$$

(d) Application of *śighraphala* calculated afresh to the true-mean longitude of the planet as in the same way as in case of outer planets.

Start with calculating the new *śighrakendra.*

$$\begin{aligned}\theta_{s,new} &= \text{Longitude of Venus' } \textit{śighrocca} - \text{True-mean longitude of Venus}\\ &= 44^\circ - 313.8^\circ = -269.8^\circ = 90.2^\circ \approx 90^\circ.\end{aligned}$$

To compute new *śighraphala*, we need the value of Rsin 90° which is $3438'$ as can be seen from the Rsine table.

Also, the new *śighrakendra* $\theta_{s,new}$ is in 2nd quadrant, therefore having *śighra* epicycle to be 256.5° .

For new *śighrakarna*, since $90^\circ \le \theta_{s,new} < 180^\circ$, we use

$$\begin{aligned}H &= \sqrt{(\mathrm{R} - \mathrm{Rsin}\ (90^\circ - 90^\circ))^2 + (\mathrm{Rsin}\ (180^\circ - 90^\circ))^2}\\ &= \sqrt{(\mathrm{R} - \mathrm{Rsin}\ 0^\circ)^2 + (\mathrm{Rsin}\ 90^\circ)^2}\\ &= 3438\sqrt{2}' = 4862.1' = 81^\circ\end{aligned}$$

where Rsin $0^\circ = 0'$ and Rsin $90^\circ = 3438'$ from the Rsine table. Therefore,

$$\textit{New Śighraphala} = \frac{\mathrm{Rsin}\ \theta_{s,new} \times \textit{śighra}\ \text{epicycle}}{360^\circ} \times \frac{R}{H} = 28.87^\circ,$$

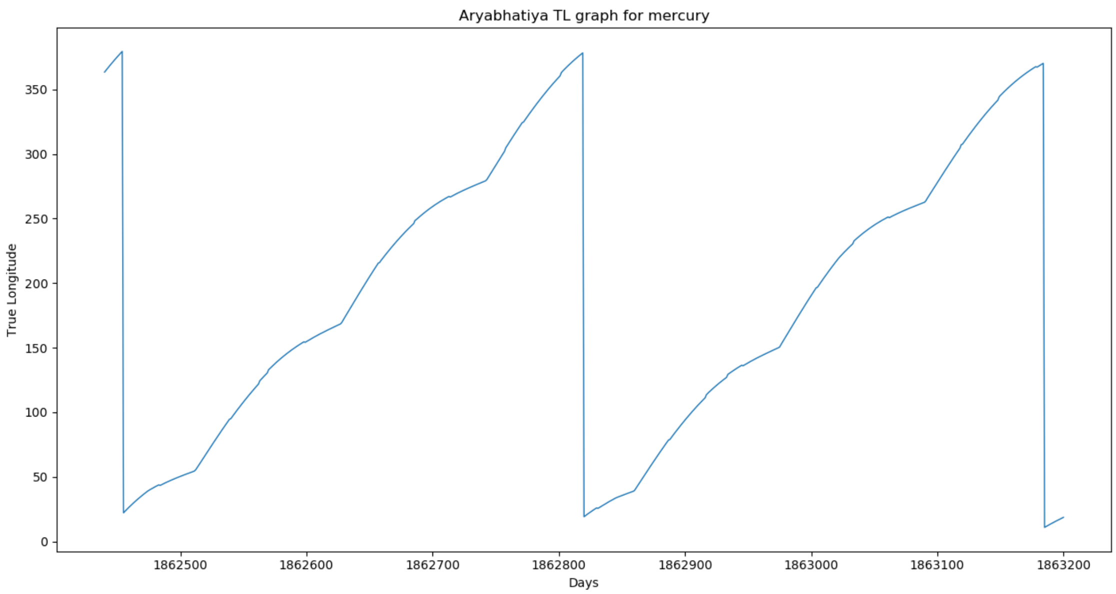
By applying the *new śighraphala* to the true-mean longitude of Venus, we get for given *ahargaṇa*=18,62,775

$$\begin{aligned}\textit{True longitude of Venus} &= \text{True-mean longitude of Venus} + \textit{New śighraphala}\\ &= 313.8^\circ + 28.87^\circ\\ &= 342.67^\circ.\end{aligned}$$

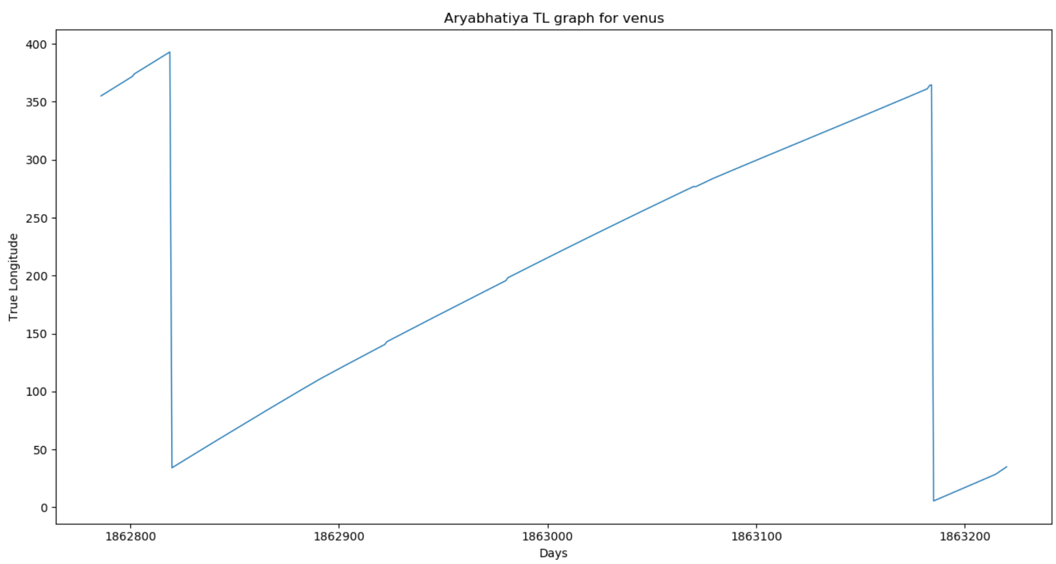
3.3 Graphical analysis of *Āryabhaṭīya* method

As done for *Vākyakarana* method, we write programs to plot the graphs of true longitude of the planets vs *ahargaṇa* values. The following graphs, we get as a result, of all planets - Mercury, Venus, Mars, Jupiter and Saturn for the same range of *ahargaṇa* as taken for *Vākyakarana* so that we can later compare the both graphs.

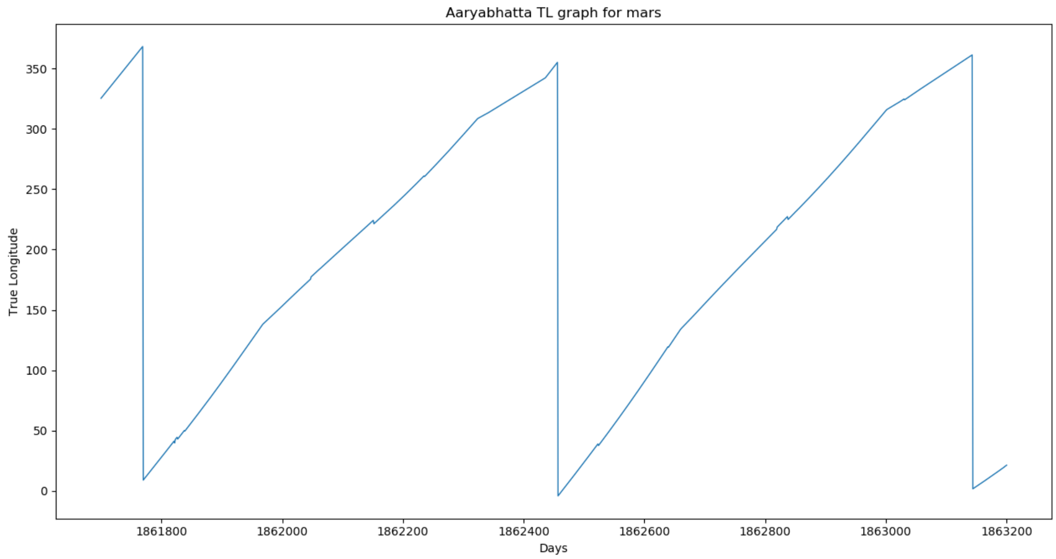
3.3.1 *Āryabhaṭīya* graph of MERCURY for 760 days of motion



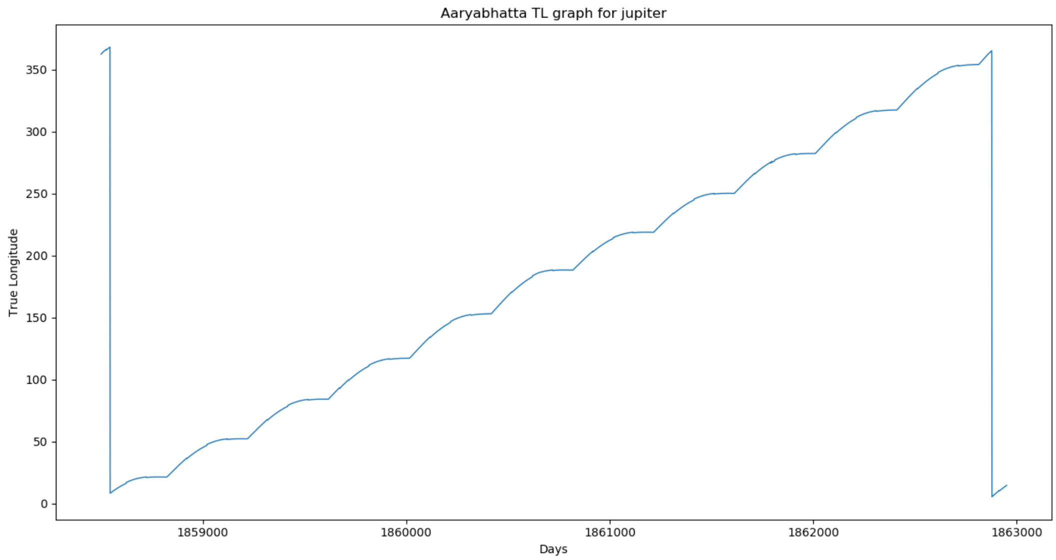
3.3.2 *Āryabhaṭīya* graph of VENUS exhibiting 435 days of motion



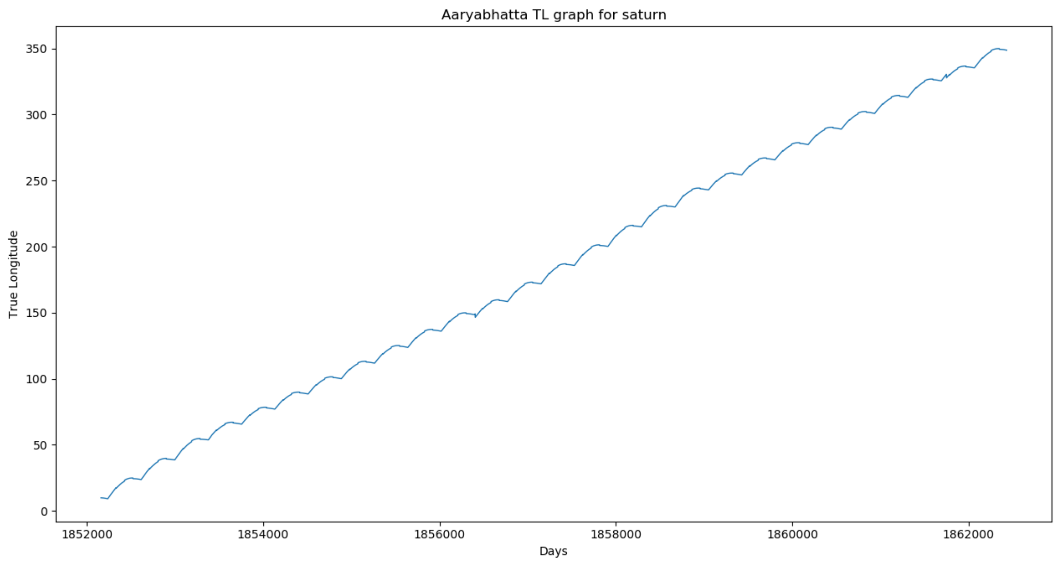
3.3.3 *Āryabhaṭīya* graph of MARS exhibiting 1500 days of motion



3.3.4 *Āryabhaṭīya* graph of JUPITER for 4450 days of motion



3.3.5 *Āryabhaṭīya* graph of SATURN for 10,270 days of motion



3.3.6 Key observations from the graphs

Some points are similar to as in *Vākyakarana* with some new ones.

1. Of course, the true longitude of all planets ranges from 0° to 360° and after reaching near 360°, there is sudden drop to 0° due the movement in a circle.

2. Graphs are periodic with the periods equal to the number of days to complete a revolution around the Earth.

3. Graphs are increasing, but not strictly. This indicates *Āryabhaṭīya* [4] doesn't consider the *retrograde* motion of the planets.

4. There are no unexplained sharp points in *Āryabhaṭīya* graphs as there were in *Vākyakarana* due to error in values.

5. These graphs are smoother than the *Vākyakarana* graphs.

Chapter 4

Mahābhāskarīya

4.1 Introduction

Bhāskara I was a pupil of *Āryabhaṭa* I and the author of *Mahābhāskarīya* as well as *Laghubhāskarīya*. His works provide us with a detailed exposition of the astronomical methods taught by *Āryabhaṭa* I. In his book *Mahābhāskarīya*, he make a little bit of changes in the method to find the true planet by twitching the values of *manda* and *śighra* epicycles, the way Rsines are calculated and the sequence in which the corrected terms *mandaphala* and *śighraphala* are applied to the mean planet.

Now, about *Laghubhāskarīya* [6], this book is more detailed and clearer than *Mahābhāskarīya* [5]. But when you read and understand the methods to find the true longitude of planets in both books, these are just the same. So, we will only deal with *Mahābhāskarīya* [5]. This book follows the hypotheses of *Āryabhaṭīya* for its methods.

Let us jump right in to the algorithms as everything needed to understand them have already been defined.

4.2 True Longitude of a planet

Again, in this there are different algorithms to be followed for inner and outer planets and we will illustrate them by the help of planets *Mars* and *Venus* as before.

4.2.1 Algorithm for the true longitude of outer planets

Take the planet to be **Mars** and *ahargaṇa* to be **A=18,62,775** as before to understand *Mahābhāskarīya*'s method.

1. *Find the mean longitude of the planet and its śighrocca.*
   Same method to be applied as in *Āryabhaṭīya* for this. For our example, calculations have already been carried out in 3.2.1. From there, we have
   Mean longitude of Mars and its *śighrocca* for 18,62,775 *kali*days are 167° and 316.34° respectively.

2. *Calculate mandakenda and śighrakendra.*
   Same formulae as in 3.2.1 and in example, we have

$$\textit{Mandakendra} = \theta_m = 49^\circ \text{ and } \textit{Śighrakendra} = \theta_s = 149^\circ.$$

3. *Correct manda and śighra epicycles.*

	Manda epicycles		Sighra epicycles	
	odd quadrant	even quadrant	odd quadrant	even quadrant
Mercury	7	5	31	29
Venus	4	2	59	57
Mars	14	18	53	51
Jupiter	7	8	16	15
Saturn	9	13	9	8

   (a) The table with the values of *manda* and *śighra* epicycles for the beginnings of odd and even quadrants as per *Māhabhāskarīya* is given above.
   To correct the epicycles, let α and β be the epicycles (*manda* or *śighra*) of the planet for the beginnings of the odd and even quadrants respectively. Consider θ to be the planet's *kendra* (*manda* θ_m or *śighra* θ_s).

If θ is in the 1st quadrant, then

$$\begin{aligned}(\textit{manda} \text{ or } \textit{śighra}) \text{ corrected epicycle } &= \alpha + \frac{(\beta - \alpha) \times \text{Rsin } \theta}{R}, \text{ when } \alpha < \beta \\ &= \alpha - \frac{(\alpha - \beta) \times \text{Rsin } \theta}{R}, \text{ when } \alpha > \beta.\end{aligned}$$

.

If $\theta = 90° + \phi$ with $0° \leq \phi \leq 90°$, then

$$\begin{aligned}(\textit{manda} \text{ or } \textit{śighra}) \text{ corrected epicycle } &= \beta - \frac{(\beta - \alpha) \times \text{Rversin } \phi}{R}, \text{ when } \alpha < \beta \\ &= \beta + \frac{(\alpha - \beta) \times \text{Rversin } \phi}{R}, \text{ when } \alpha > \beta.\end{aligned}$$

If $\theta = 180° + \phi$ with $0° \leq \phi \leq 90°$, then

$$\begin{aligned}(\textit{manda} \text{ or } \textit{śighra}) \text{ corrected epicycle } &= \alpha - \frac{(\beta - \alpha) \times \text{Rsin } \phi}{R}, \text{ when } \alpha < \beta \\ &= \alpha + \frac{(\alpha - \beta) \times \text{Rsin } \phi}{R}, \text{ when } \alpha > \beta.\end{aligned}$$

If $\theta = 270° + \phi$ with $0° \leq \phi \leq 90°$, then

$$\begin{aligned}(\textit{manda} \text{ or } \textit{śighra}) \text{ corrected epicycle } &= 2\alpha - \beta + \frac{(\beta - \alpha) \times \text{Rversin } \phi}{R}, \text{ when } \alpha < \beta \\ &= 2\alpha - \beta - \frac{(\alpha - \beta) \times \text{Rversin } \phi}{R}, \text{ when } \alpha > \beta.\end{aligned}$$

(b) Next thing to see is how to calculate Rsin θ and Rversin θ for any given θ which are to be used in corrected epicycle formulas.

Table of Rsine differences

S.No.	Rsine difference	S.No.	Rsine difference
1	225	13	154
2	224	14	143
3	222	15	131
4	219	16	119
5	215	17	106
6	210	18	93
7	205	19	79
8	199	20	65
9	191	21	51
10	183	22	37
11	174	23	22
12	164	24	7

Table of Rversin differences

S.No.	Rversin difference	S.No.	Rversin difference
1	7	13	164
2	22	14	174
3	37	15	183
4	51	16	191
5	65	17	199
6	79	18	205
7	93	19	210
8	106	20	215
9	119	21	219
10	131	22	222
11	143	23	224
12	154	24	225

Procedure. For Rsin θ (Rversin θ), with θ in 1st quadrant, start by reducing it into minutes and then divide by 225. The quotient we get after division denotes the number of Rsine (Rversin θ) differences to be added from the table above given on page 108 of [5]. Then, multiply the remainder by $(\text{quotient}+1)^{th}$ Rsine (Rversin θ) difference and divide the product by 225. Add this to the sum of Rsine (Rversin θ) differences obtained before to get Rsin θ (Rversin θ respectively). For θ greater than 90°, use the formulae given in section 3.2.1. Also, note that Rversin θ is same as Rvers θ in *Āryabhaṭīya.*

Now, let's do this in our example. We have, θ_m in 1st quadrant and θ_s is in the 2nd. For Mars, from the epicycle table above,

$$\begin{aligned}
&\textit{Manda} \text{ epicycle} = 14° \text{ and } 18° \text{ for odd \& even quadrants resp.}\\
&\textit{Śighra} \text{ epicycle} = 53° \text{ and } 51° \text{ for odd \& even quadrants resp.}
\end{aligned}$$

For applying epicycle formulas, we want Rsin 49° and Rversin 59°.
For Rsin 49°, start by reducing to arc which is $49° = 2940'$. By dividing $2940'$ by 225, we get quotient=13 and remainder=15. So,

$$\begin{aligned}
\text{Rsin } 49° &= \text{Sum of first 13 Rsine differences} + 15 \times 14\text{th Rsine difference} \div 225\\
&= 2585' + 2145' \div 225 = 2594'32''.
\end{aligned}$$

Similarly, to calculate Rversin 59°, reduce 59° to arc which is $3540'$. Divide this by 225, we have quotient=15 and remainder=165. Thus,

$$\begin{aligned}
\text{Rversin } 59° &= \text{Sum of first 15 Rversin differences} + 14 \times 16\text{th Rversin difference} \div 225\\
&= 1528' + 165 \times 191' \div 225 = 1668'4''.
\end{aligned}$$

By substituting these values in corrected *manda* and *śighra* epicycle formulae for $\theta =$ 49° and 149° respectively , we get

$$\begin{aligned}
\text{Corrected } \textit{manda} \text{ epicycle} &= \alpha + \frac{(\beta - \alpha) \times \text{Rsin } \theta}{R}, \text{ with } \alpha = 14°, \beta = 18°\\
&= 14° + \frac{(18° - 14°) \times \text{Rsin } 49°}{3438'}\\
&= 17.02°.
\end{aligned}$$

$$\text{Corrected } \textit{śighra} \text{ epicycle} = \beta + \frac{(\alpha-\beta)\times \text{Rversin } \phi}{R}, \text{ with } \alpha = 53°, \beta = 51°$$
$$= 51° + \frac{(53° - 51°) \times \text{Rversin } 59°}{3438'}$$
$$= 51.97°.$$

4. *Find mandakendraphala and śighrakendraphala.*

$$\textit{Mandakendraphala} = \frac{\text{Rsin } \theta_m \times \text{corrected } \textit{manda} \text{ epicycle}}{80°}$$

$$\textit{Śighrakendraphala} = \frac{\text{Rsin } \theta_s \times \text{corrected } \textit{śighra} \text{ epicycle}}{80°}$$

For *śighrakendraphala*, we need Rsin 149° which is equal to R−Rversin 59° = $3438' - 1668'4'' = 1770'$. From the above calculated values, we get

$$\textit{Mandakendraphala}{=}9.2°$$
$$\textit{Śighrakendraphala}{=}19.16°.$$

5. *Apply the correction terms mandakendraphala & śighrakendraphala to get the true longitude of the planet.*

   (a) Subtract or add the half of *mandaphala* to the mean longitude of the planet, depending whether the planet's *kendra* θ_m is less than or greater than 180°. What we get is the corrected longitude of the planet I.
   This gives,

$$\text{Corrected longitude of Mars I} = \text{Mean longitude of Mars} - \frac{1}{2}\ \textit{Mandaphala}$$
$$= 167° - \frac{1}{2} \times 9.2° = 162.4°.$$

   (b) Multiply *śighrakendraphala* by $\frac{R}{H}$ and the quantity we get is said to be the arc derived from *śighrakendraphala*, where H is *śighrakarna* given by the formulae in 3.2.1 3(d). Apply the half of this arc positively or negatively to the corrected longitude of the planet depending θ_s is less than or greater than 180°. We get the planet' corrected longitude II.

Start by calculating the *śighrakarna* by the formula for $90° \leq \theta_s < 180°$,

$$\begin{aligned} H &= \sqrt{(\mathrm{R} - \mathrm{Rsin}\ (149° - 90°))^2 + (\mathrm{Rsin}\ (180° - 149°))^2} \\ &= \sqrt{(\mathrm{R} - \mathrm{Rsin}\ 59°)^2 + (\mathrm{Rsin}\ 31°)^2} \end{aligned}$$

where for Rsin 59°, reduce 59° to the arc which is $3540'$ and divide it by 225 which gives the quotient=15 and the remainder=165. So,

$$\begin{aligned} \mathrm{Rsin}\ 59° &= \text{Sum of first 15 Rsine differences} + 165 \times \text{16th Rsine difference} \div 225 \\ &= 2859' + 19635' \div 225 = 2946'16''. \end{aligned}$$

Similarly, for Rsin 31°, reduce 31° to the arc which is $1860'$ and divide it by 225 which gives the quotient=8 and the remainder=60. So,

$$\begin{aligned} \mathrm{Rsin}\ 31° &= \text{Sum of first 8 Rsine differences} + 60 \times \text{9th Rsine difference} \div 225 \\ &= 1719' + 11460' \div 225 = 1769'56''. \end{aligned}$$

This implies H= $1837'$ same as in *Āryabhaṭīya*.

$$\begin{aligned} \text{Mars' corrected longitude II} &= \text{Mars' Corrected longitude I} \\ &\quad + \frac{1}{2}\textit{Śighrakendraphala} \times \frac{R}{H} \\ &= 162.4° + \frac{1}{2} \times 35.86° = 180°. \end{aligned}$$

(c) Calculate the *mandakendraphala* afresh using planet's corrected longitude II and apply the whole of it negatively or positively to its original mean longitude, depending new *mandakendra* is less than or greater than 180°. This gives the *true-mean longitude* of the planet. In the example, we have,

$$\begin{aligned} \theta_{m,new} &= \text{Corrected longtiude of Mars II} - \text{ Planet's apogee} \\ &= 180° - 118° = 62°. \end{aligned}$$

Now, to find Rsin 62°, we have $62° = 3720'$ which after dividing by 225, we get the quotient=16 and the remainder=120. Thus,

$$\begin{aligned} \text{Rsin } 62^\circ &= \text{Sum of first 16 Rsine differences} + 120 \times \text{17th Rsine difference} \div 225 \\ &= 2978' + 12720' \div 225 = 3034'32''. \end{aligned}$$

By using this with $\theta = \theta_{m,new}$, we get

$$\begin{aligned} \text{New corrected } \textit{manda} \text{ epicycle} &= \alpha + \frac{(\beta - \alpha) \times \text{Rsin } \theta}{R}, \text{ with } \alpha = 14^\circ, \beta = 18^\circ \\ &= 14^\circ + \frac{(18^\circ - 14^\circ) \times \text{Rsin } 62^\circ}{3438'} \\ &= 17.53^\circ. \end{aligned}$$

Then,

$$\textit{New Mandaphala} = \frac{\text{Rsin } \theta_{m,new} \times \text{New corrected } \textit{manda} \text{ epicycle}}{80^\circ} = 11.1^\circ.$$

Therefore, by application of new *mandaphala*, we get

$$\begin{aligned} \textit{True-mean longitude of Mars} &= \text{Mars' mean longitude} - \textit{New Mandaphala} \\ &= 167^\circ - 11^\circ = 155.9^\circ. \end{aligned}$$

(d) The last step is to calculate *śighrakendraphala* afresh, and the arc derived from it. Add or subtract this arc into true-mean longitude of the planet, depending the new *śighrakendra*, $\theta_{s,new}$ is less than or greater than 180° to get the *true longitude of the planet*.
Continuing in our example, we have

$$\begin{aligned} \theta_{s,new} &= \text{Longitude of Mars' } \textit{śighrocca} - \text{True-mean longitude of Mars} \\ &= 316.34^\circ - 155.9^\circ \approx 160^\circ \end{aligned}$$

For the sake of new corrected *śighra* epicycle and new *śighraphala*, we need these Rsines and Rversines.
First is Rversin 70°. The quotient and remainder after dividing the corresponding arc 4200' by 225 are 18 and 150 respectively. This gives,

$$\begin{aligned}\text{Rversin } 70^\circ &= \text{Sum of first 18 Rversine differences} + 150 \times \text{19th Rversine difference} \div 225\\ &= 2123' + 31500' \div 225 = 2263'.\end{aligned}$$

and Rsin 160°= Rsin $(90^\circ + 70^\circ)$=R-Rversin $70^\circ = 1175'$.
For $\theta = \theta_{s,new} = 90^\circ + \phi$ with ϕ in 1st quadrant,

$$\begin{aligned}\text{New corrected } \textit{śighra} \text{ epicycle} &= \beta + \frac{(\alpha - \beta) \times \text{Rversin } \phi}{R}, \text{ with } \alpha = 53^\circ, \beta = 51^\circ\\ &= 51^\circ + \frac{(53^\circ - 51^\circ) \times \text{Rversin } 70^\circ}{3438'}\\ &= 52.3^\circ\end{aligned}$$

This gives

$$\textit{New Śighraphala} = \frac{\text{Rsin } \theta_{s,new} \times \text{new corrected } \textit{śighra} \text{ epicycle}}{80^\circ} = 12.8^\circ.$$

The thing to be computed is *śighrakarna* by the formula for $90^\circ \leq \theta_{s,new} < 180^\circ$,

$$\begin{aligned}H &= \sqrt{(\text{R} - \text{Rsin } (160^\circ - 90^\circ))^2 + (\text{Rsin } (180^\circ - 160^\circ))^2}\\ &= \sqrt{(\text{R} - \text{Rsin } 70^\circ)^2 + (\text{Rsin } 20^\circ)^2}\end{aligned}$$

where for Rsin 70°, reduce 70° to the arc which is 4200′ and divide it by 225 which gives the quotient=18 and the remainder=150. So,

$$\begin{aligned}\text{Rsin } 70^\circ &= \text{Sum of first 18 Rsine differences} + 150 \times \text{19th Rsine difference} \div 225\\ &= 3177' + 11850' \div 225 = 3229'40''.\end{aligned}$$

Similarly, for Rsin 20°, reduce 20° to the arc which is 1200′ and divide it by 225 which gives the quotient=5 and the remainder=75. So,

$$\begin{aligned}\text{Rsin } 20^\circ &= \text{Sum of first 5 Rsine differences} + 75 \times \text{6th Rsine difference} \div 225\\ &= 1105' + 11460' \div 225 = 1175'.\end{aligned}$$

This implies H= $\sqrt{(208.3')^2 + (1175')^2} = 1193.3'$ and the arc derived from new *śighrakendraphala*=new *śighrakendraphala*$\times \frac{R}{H} = 36.88°$.

By applying this arc to Mars' true-mean longitude, we get

$$\begin{aligned} \textit{True longitude of Mars} &= \text{True-mean longitude of Mars} \\ &+ \text{arc derived from new } \textit{śighrakendraphala} \\ &= 155.9° + 36.88° \\ &= 192.78°. \end{aligned}$$

4.2.2 Algorithm for the true longitude of inner planets

To illustrate *Mahābhāskarīya*'s method for inner planets, take the planet to be **Venus** and *ahargaṇa* to be **A=18,62,775** as before.

1. *Find the mean longitude of the inner planet and its śighrocca.*
   Same method to be applied as in *Āryabhaṭīya* for this. In our example, from the calculations in 3.2.1, we have
   Mean longitude of Venus and its *śighrocca* for 18,62,775 *kali*days are 316.34° and 44° respectively.

2. *Calculate mandakenda and śighrakendra.*
   These are given by same formulae as in 3.2.1 and for the example,

   $$\textit{Mandakendra} = \theta_m = 226° \text{ and } \textit{Śighrakendra} = \theta_s = 88°.$$

3. *Correct manda and śighra epicycles.*
   Formulae to correct *manda* and *śighra* epicycles and the procedures to find Rsines & Rversines are same as for outer planets disussed in 4.2.1.
   Now, let's do this in our example. We have, θ_m in 3rd quadrant and θ_s is in the 1st.
   For Venus, from the epicycle table in 4.2.1,

   $$\textit{Manda} \text{ epicycle} = 4° \text{ and } 2° \text{ for odd \& even quadrants resp.}$$
   $$\textit{Śighra} \text{ epicycle} = 59° \text{ and } 57° \text{ for odd \& even quadrants resp.}$$

   For applying epicycle formulas, we want Rsin 46° and Rsin 88°.
   For Rsin 46°, start by reducing to arc which is $49° = 2760'$.

By dividing $2760'$ by 225, we get quotient=12 and remainder=60. So,

$$\begin{aligned}\text{Rsin } 46^\circ &= \text{Sum of first 12 Rsine differences} + 60 \times \text{13th Rsine difference} \div 225 \\ &= 2431' + 9240' \div 225 = 2472'4''.\end{aligned}$$

Similarly, to calculate Rsin 88°, reduce 88° to arc which is $5280'$. Divide this by 225, we have quotient=23 and remainder=105. Thus,

$$\begin{aligned}\text{Rsin } 88^\circ &= \text{Sum of first 23 Rversin differences} + 14 \times \text{24th Rversin difference} \div 225 \\ &= 3431' + 105 \times 7' \div 225 = 3434'16''.\end{aligned}$$

By substituting these values in corrected *manda* and *śighra* epicycle formulae for $\theta = 226^\circ = 180^\circ + 46^\circ$ and 88° respectively , we get

$$\begin{aligned}\text{Corrected } \textit{manda} \text{ epicycle} &= \alpha + \frac{(\alpha - \beta) \times \text{Rsin } 46^\circ}{R}, \text{ with } \alpha = 4^\circ, \beta = 2^\circ \\ &= 4^\circ + \frac{(4^\circ - 2^\circ) \times \text{Rsin } 46^\circ}{3438'} \\ &= 5.44^\circ.\end{aligned}$$

$$\begin{aligned}\text{Corrected } \textit{śighra} \text{ epicycle} &= \alpha - \frac{(\alpha - \beta) \times \text{Rsin } 88^\circ}{R}, \text{ with } \alpha = 59^\circ, \beta = 57^\circ \\ &= 59^\circ - \frac{(59^\circ - 57^\circ) \times \text{Rsin } 88^\circ}{3438'} \\ &= 57^\circ.\end{aligned}$$

4. *Find mandakendraphala and śighrakendraphala.*

$$\textit{Mandakendraphala} = \frac{\text{Rsin } \theta_m \times \text{corrected } \textit{manda} \text{ epicycle}}{80^\circ}$$

$$\textit{Śighrakendraphala} = \frac{\text{Rsin } \theta_s \times \text{corrected } \textit{śighra} \text{ epicycle}}{80^\circ}$$

For *mandakendraphala*, we need Rsin 226° which is equal to -Rsin $46^\circ = -2472'4''$. From the above calculated values, we get

$$\begin{aligned}&\textit{Mandakendraphala} = -2.8^\circ \\ &\textit{Śighrakendraphala} = 40.78^\circ.\end{aligned}$$

5. *Apply the correction terms mandakendraphala & śighrakendraphala to get the true longitude of the inner planet.*

   (a) Multiply *śighrakendraphala* by $\frac{R}{H}$ to get the arc derived from *śighrakendraphala*, where H is *śighrakarna* given by the formulae in 3.2.1 3(d). Apply the half of this arc negatively or positively to the longitude of the planet's apogee depending θ_s is less than or greater than 180°. We get the planet' corrected apogee .
   Continuing with the example, we compute *śighrakarna* by the formula for $0° \leq \theta_s < 90°$,

$$\begin{aligned} H &= \sqrt{\left(\mathrm{R} + \mathrm{Rsin}\ (90° - 88°)\right)^2 + (\mathrm{Rsin}\ 88°)^2} \\ &= \sqrt{\left(\mathrm{R} + \mathrm{Rsin}\ 2°\right)^2 + (\mathrm{Rsin}\ 88°)^2} \end{aligned}$$

   where for Rsin 2°, reduce 2° to the arc which is $120'$ and divide it by 225 which gives the quotient=0 and the remainder=120. So, Rsin 2° = 120× 1st Rsine difference ÷225 = $120'$ and Rsin 88° = $3434'16''$ from calculations before.
   This implies H= $\sqrt{(3558)^2 + (3434.27)^2} = 4945.05'$.

$$\begin{aligned} \text{Venus' corrected apogee} &= \text{Venus' apogee} - \frac{1}{2}\ \textit{Śighrakendraphala} \times \frac{R}{H} \\ &= 90° + \frac{1}{2} \times 28.35° = 104.18°. \end{aligned}$$

   (b) Calculate the *mandakendraphala* afresh using planet's corrected apogee and apply the whole of it negatively or positively to its original mean longitude, depending new *mandakendra* is less than or greater than 180°. From this, we get the *true-mean longitude* of the planet. In the example for Venus, we have,

$$\begin{aligned} \theta_{m,new} &= \text{Mean longitude of Venus} - \text{Corrected Venus' apogee} \\ &= 316.34° - 104.18° = 212.16° \approx 212°. \end{aligned}$$

   Now, we want to find Rsin 32°, we have 32° = $1920'$ which after dividing by 225, we get the quotient=8 and the remainder=120. Thus,

$$\begin{aligned} \mathrm{Rsin}\ 32° &= \text{Sum of first 8 Rsine differences} + 120 \times \text{9th Rsine difference} \div 225 \\ &= 1719' + 22920' \div 225 = 1820'52''. \end{aligned}$$

Also, Rsin 212° = Rsin (180° + 32°) = −Rsin 32° = −1820′52″. By using $\theta = \theta_{m,new} = 180° + \phi$, we get

$$\begin{aligned}
\text{New corrected } \textit{manda} \text{ epicycle} &= \alpha + \frac{(\alpha - \beta) \times \text{Rsin } \phi}{R}, \text{ with } \alpha = 4°, \beta = 2° \\
&= 4° + \frac{(4° - 2°) \times \text{Rsin } 32°}{3438'} \\
&= 5.06°.
\end{aligned}$$

Then,

$$\textit{New mandaphala} = \frac{\text{Rsin } \theta_{m,new} \times \text{New corrected } \textit{manda} \text{ epicycle}}{80°} = -1.92°.$$

Therefore, by application of new *mandaphala*, we get

$$\begin{aligned}
\textit{True-mean longitude of Venus} &= \text{Venus' mean longitude} + \textit{New Mandaphala} \\
&= 316.34° + 1.92° = 318.26°.
\end{aligned}$$

(c) For the last step, calculate *śighrakendraphala* afresh, and the arc derived from it. Add or subtract this arc into true-mean longitude of the planet, depending the new *śighrakendra*, $\theta_{s,new}$ is less than or greater than 180° to get the *true longitude of the planet.*
Now, in the example, we have

$$\begin{aligned}
\theta_{s,new} &= \text{Longitude of Venus' } \textit{śighrocca} - \text{True-mean longitude of Venus} \\
&= 44° - 318.26° \approx -274° = 86°
\end{aligned}$$

To get new corrected *śighra* epicycle and new *śighraphala*, we need Rsine 86° calculated as,
Similarly, to calculate Rsin 86°, reduce 86° to arc which is 5160′. Divide this by 225, we have quotient=22 and remainder=210. Thus,

$$\begin{aligned}
\text{Rsin } 86° &= \text{Sum of first 22 Rsin differences} + 210 \times \text{23th Rversin difference} \div 225 \\
&= 3409' + 210 \times 22' \div 225 = 3429'32''.
\end{aligned}$$

For $\theta = \theta_{s,new} = 86°$,

$$\text{New corrected } \textit{śighra} \text{ epicycle} = \alpha - \frac{(\alpha - \beta) \times \text{Rsin } \theta}{R}, \text{ with } \alpha = 59°, \beta = 57°$$
$$= 59° - \frac{(59° - 57°) \times \text{Rsin } 86°}{3438'}$$
$$= 57°.$$

This gives,

$$\textit{New Śighraphala} = \frac{\text{Rsin } \theta_{s,new} \times \textit{new corrected śighra} \text{ epicycle}}{80°} = 40.7°.$$

Now, the thing to be computed is *śighrakarna* by the formula for $0° \leq \theta_{s,new} < 90°$,

$$H = \sqrt{(\text{R} + \text{Rsin } (90° - 86°))^2 + (\text{Rsin } 86°)^2}$$
$$= \sqrt{(\text{R} + \text{Rsin } 4°)^2 + (\text{Rsin } 86°)^2}.$$

For Rsin 4°, reduce 4° to the arc which is 240′ and divide it by 225 which gives the quotient=1 and the remainder=15.

$$\text{Rsin } 4° = \text{1st Rsine difference} + 15 \times \text{2nd Rsine difference} \div 225$$
$$= 225' + 3360' \div 225 = 239'56''.$$

and Rsin $86° = 3429'32''$ as calculated before.

This implies H= $\sqrt{(3677.93)^2 + (3429.5)^2} = 5028.78'$. and the arc derived from new *śighrakendraphala*=new *śighrakendraphala*$\times \frac{R}{H} = 27.82°$. By applying this arc to Venus' true-mean longitude, we get

$$\textit{True longitude of Venus} = \text{True-mean longitude of Venus}$$
$$+ \text{ arc derived from new } \textit{śighrakendraphala}$$
$$= 318.26° + 27.82°$$
$$= 346.08°.$$

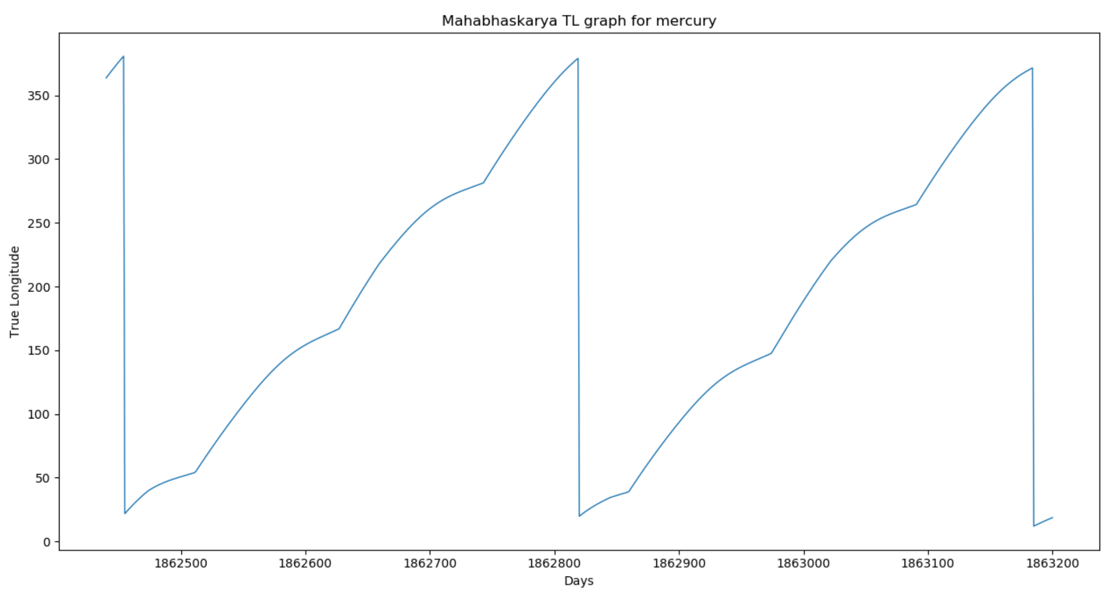
4.3 Graphical analysis of *Mahābhāskarīya* method

We already have seen the graphs corresponding to *Vākyakarana* and *Āryabhaṭīya* method. On the same lines, we have plotted the graphs for all planets – Mercury, Venus, Mars, Jupiter and Saturn with the range of *ahargaṇa* same as Vākyakarana and *Āryabhaṭīya*, which are given below.

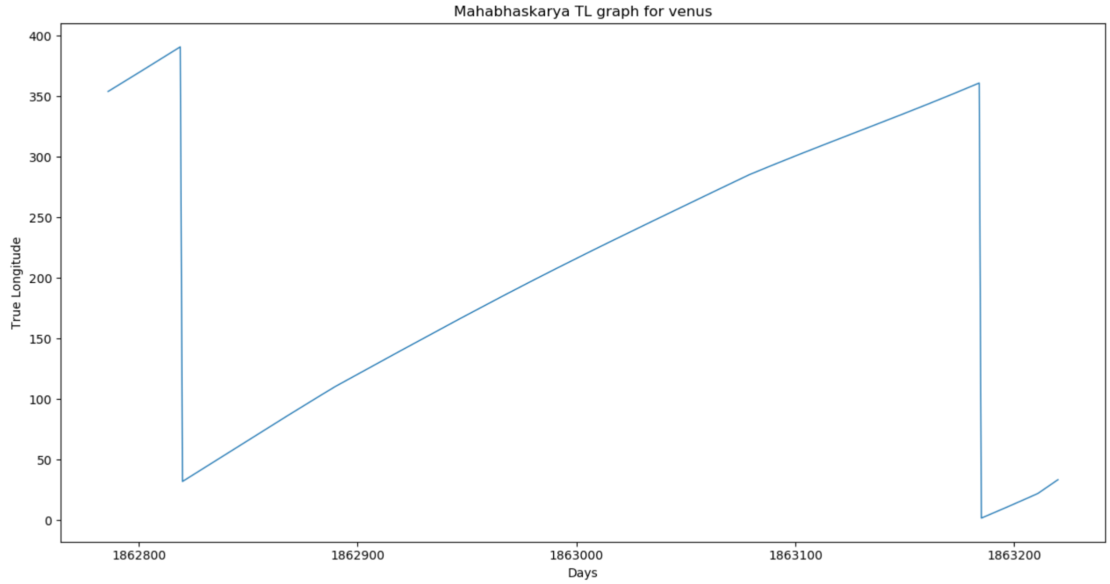
4.3.1 Key observations

Globally, the *Mahābhāskarīya* graphs for all the planets look quite similar to the ones of *Āryabhaṭīya*. So, all the observations there also make sense here. However, local comparison between them will be made in the next chapter discussing the conclusions.

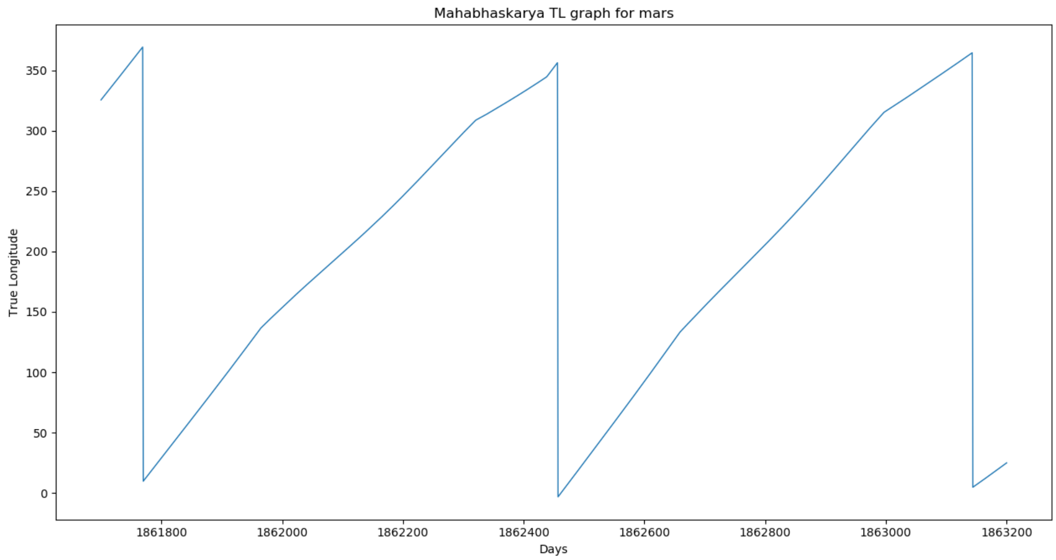
4.3.2 *Mahābhāskarīya* graph of MERCURY for 760 days of motion



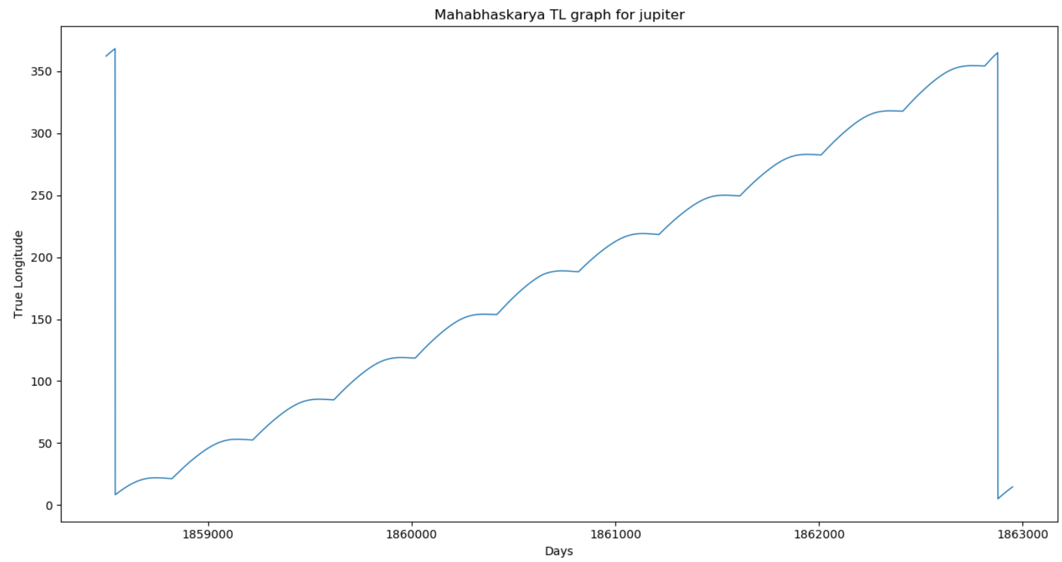
4.3.3 *Mahābhāskarīya* graph of VENUS for 435 days of motion



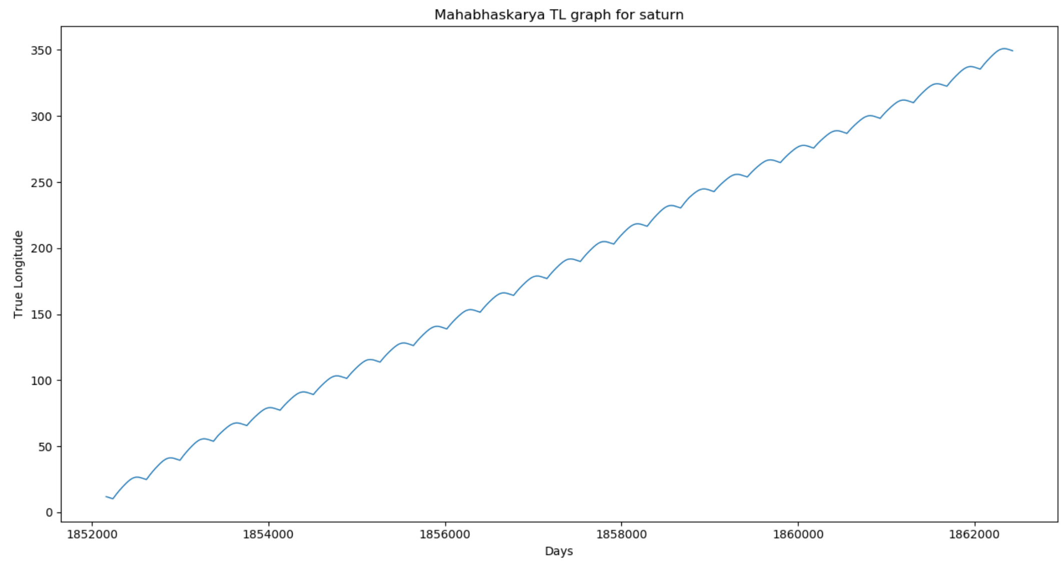
4.3.4 *Mahābhāskarīya* graph of MARS for 1500 days of motion



4.3.5 *Mahābhāskarīya* graph of JUPITER for 4450 days of motion



4.3.6 *Mahābhāskarīya* graph of SATURN for 10,270 days of motion



Chapter 5

Conclusions

This chapter is dedicated to see the comparison of the results obtained from three books Vākyakarana [1], *Āryabhaṭīya* [4] and *Mahābhāskarīya* [5]. As said, *Laghubhāskarīya*'s method is same as that of *Mahābhāskarīya*. So, we will not repeat it.
To start with, we have solved examples for the planets *Mars* and *Venus* as per all the three methods and the final answers we got are as below.

1. The *true longitude of Mars* given the *ahargaṇa* 18,62,775 as per :

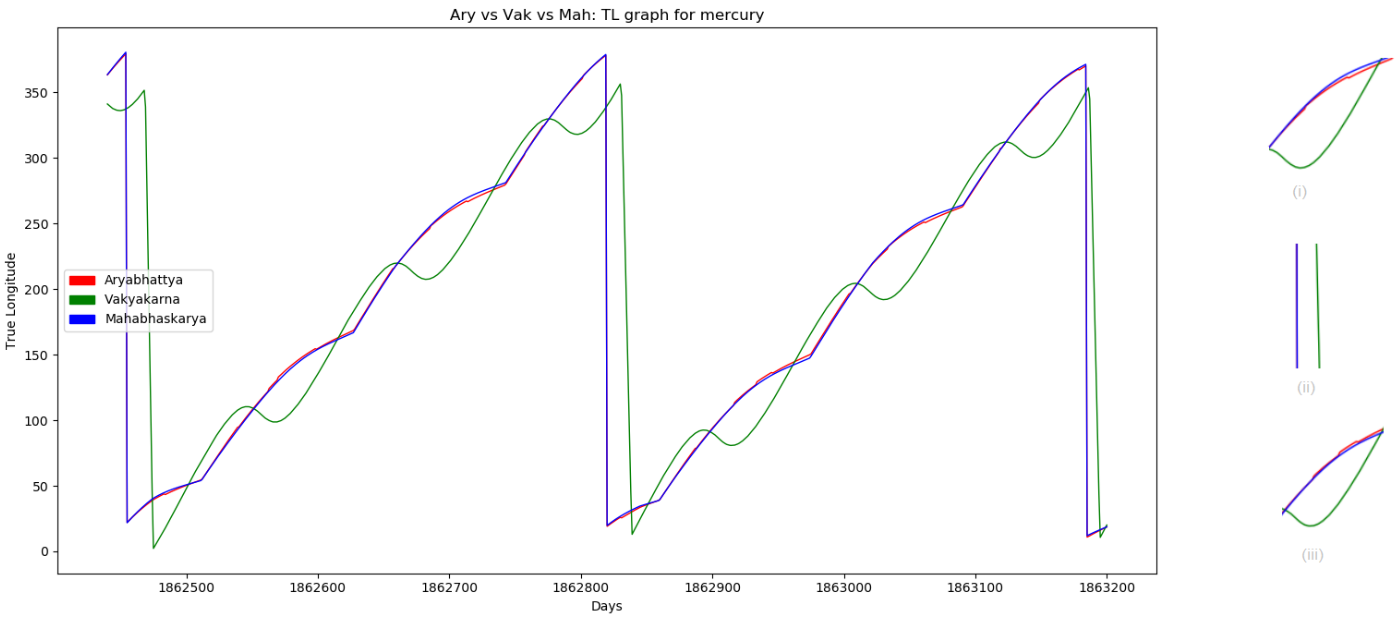
$$\begin{aligned} \textit{Vākyakarana} &: 194^\circ 10' 55'' \\ \textit{Āryabhaṭīya} &: 194.27^\circ \\ \textit{Mahābhāskarīya} &: 192.78^\circ. \end{aligned}$$

2. The *true longitude of Venus* given the *ahargaṇa* 18,62,775 as per :

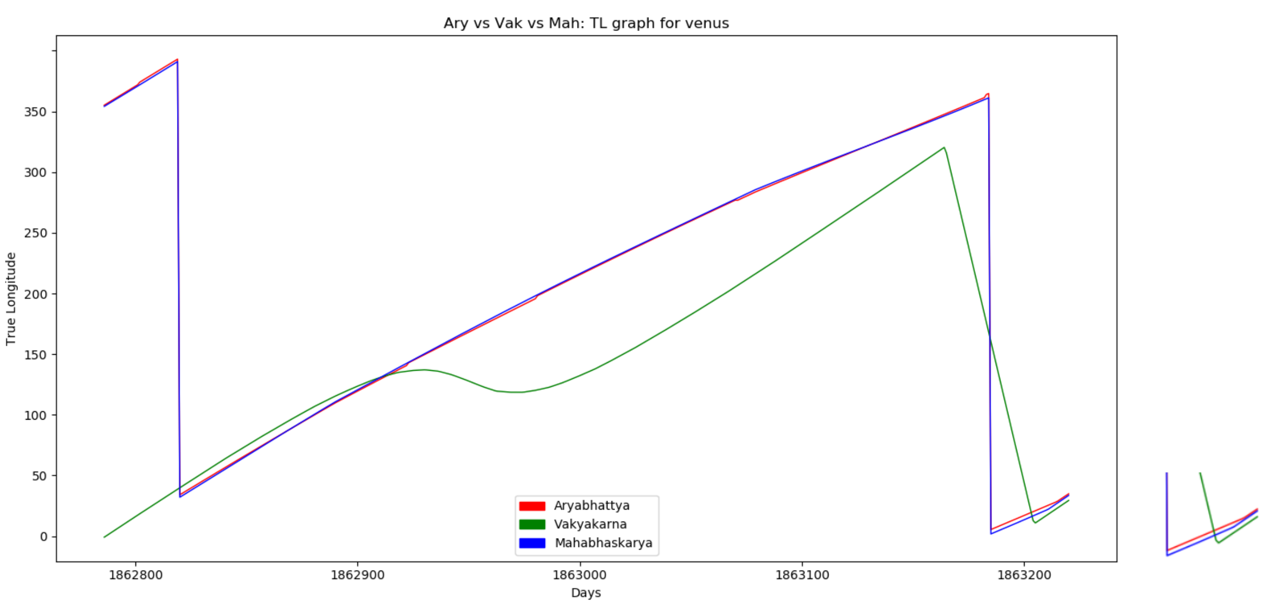
$$\begin{aligned} \textit{Vākyakarana} &: 345^\circ 51' 26'' \\ \textit{Āryabhaṭīya} &: 342.67^\circ \\ \textit{Mahābhāskarīya} &: 346.08^\circ. \end{aligned}$$

From the values, it clear that for Mars with *ahargaṇa* 18,62,775, *Āryabhaṭīya*'s true longitude is closer to that of *Vākyakarana* than *Mahābhāskarīya*. However, it's the contrary for Venus. Keeping this in mind, below are the combined graphs for each planet to analyze the these different methods.

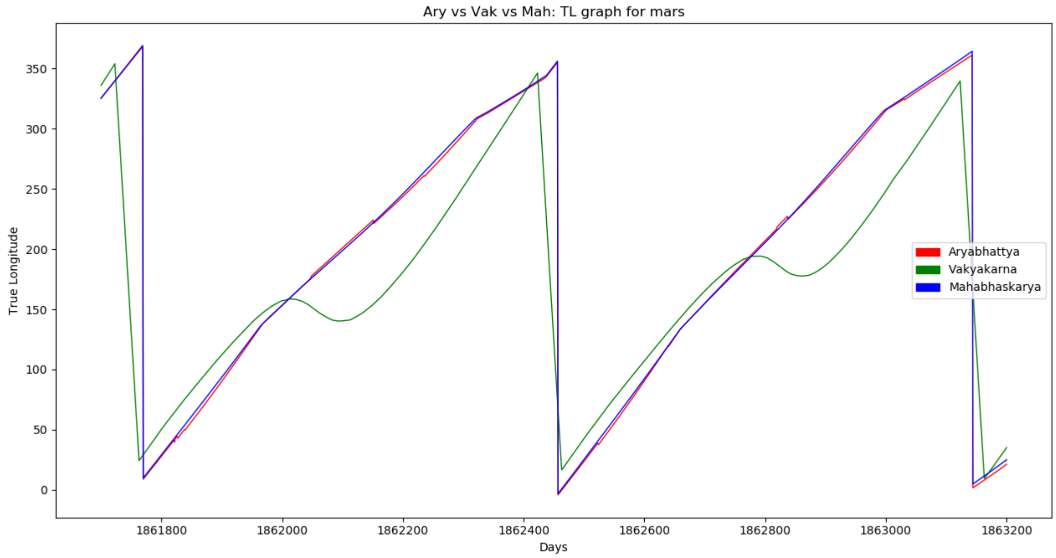
5.0.1 MERCURY plots for 760 days of motion



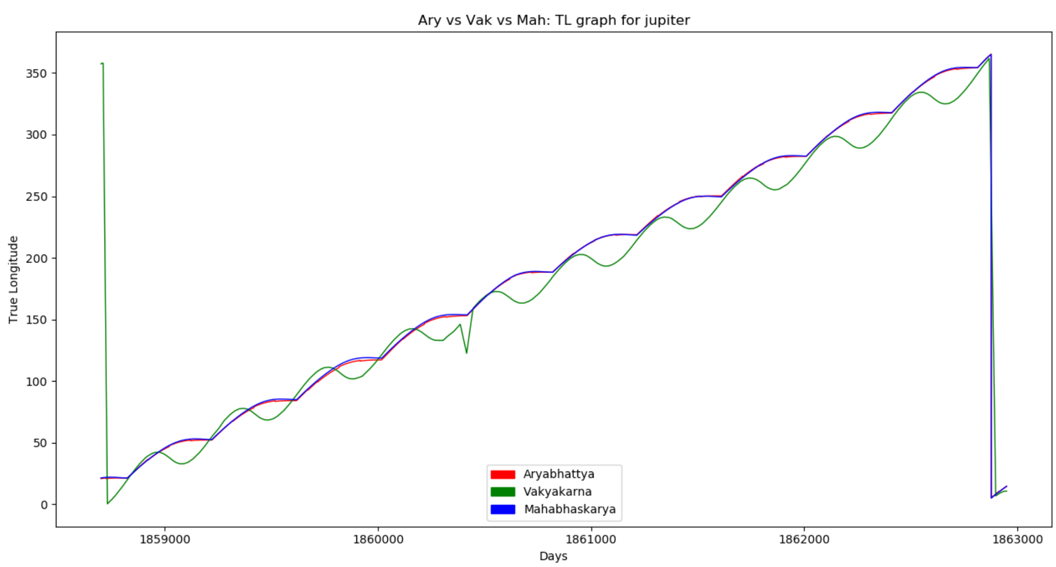
5.0.2 VENUS plots for 435 days of motion



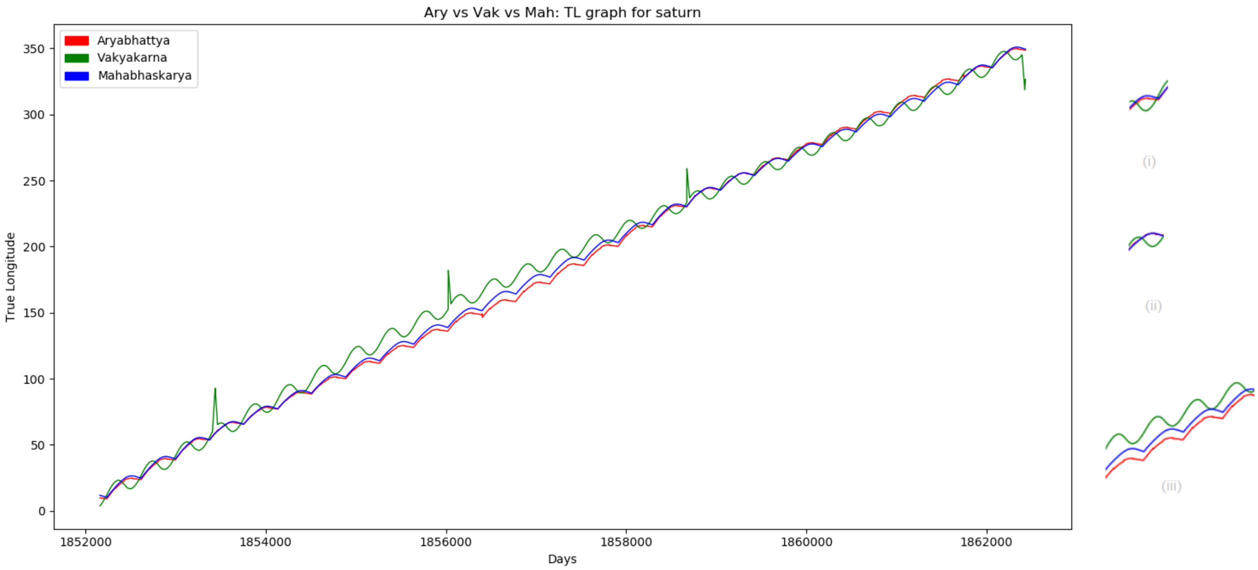
5.0.3 MARS plots for 1500 days of motion



5.0.4 JUPITER plots for 4450 days of motion



5.0.5 SATURN plots for 10,270 days of motion



5.0.6 Observations and conclusions

While comparing *Vākyakarana* graphs with *Āryabhaṭīya* and *Mahābhāskarīya*, following points can be seen :

1. *Āryabhaṭīya* and *Mahābhāskarīya* graphs either coincide or have very minor deviation from each other.

2. In all graphs, there are ahargaṇa values for which quite significant difference can be seen between *Vākyakarana*'s true longitudes and the other two since *Vākyakarana* graphs are decreasing for those values, but that's not the case for *Āryabhaṭīya* and *Mahābhāskarīya*. This happens because *Vākyakarana* considers the apparent retrograde motion of planets, while the other two don't.

3. By looking at the zoomed figures (i), (ii), (iii) of the graphs as shown for planets Mercury and Saturn, observe that *Āryabhaṭīya* graph is closer to *Vākyakarana* in figure (i). *Āryabhaṭīya*'s and *Mahābhāskarīya* graphs coincide, thus equidistant from *Vākyakarana* in figure (ii). Finally, *Mahābhāskarīya* is closer to *Vākyakarana* in (iii). Same is true

for all planets. Therefore, it is not overall clear that which method does *Vākyakarana* originate. But from the global pictures, it can be seen that for most of the ahargaṇa values, *Mahābhāskarīya is closer to Vākyakarana than Āryabhaṭīya.*

This concludes our work.
At the end of this thesis, we want to talk about some of the problems and challenges faced by us during the programming.

5.0.7 Challenges faced in the programming

1. First problem that we came across was with *Vākyakarana*. There are about 85 cycle tables like in 2.3.1 each containing the position values for about 20 to 50 days and intervals in which days are given are not uniform. All this information is to be recorded somewhere and the program has been written so that it picks the required value correctly from so many lists. This is like a lot of labour work, but also these things make the program run slow.

2. In case of *Āryabhaṭīya* and *Mahābhāskarīya*, same happens when we calculate Rsines and Rversines. But it is reasonable. The problem is the number of loops used in the program. You might get the answer for a particular value or a particular interval. But for plotting graphs, it should work for all the possible values and when it doesn't happen, it becomes too difficult to figure out the mistake whether it is in the program or the tables given. It took us weeks to make these programs run flawlessly.

3. Also, like for Saturn 10,270 number of days has been taken for plotting the graphs to show the true longitudes during its one complete revolution and it takes one night to run one such program for Saturn due to the complexity and such a high number of values to be picked from the tables.

4. Last is also about the speed of the program for getting a triple graph. The way by which this problem was overcome is instead of writing a single program to compute the true longitudes for all the three methods and then plot them, write three different programs for plotting graphs for each method, store (x,y) values from each graph in lists, and just plot the number in those lists. This makes to plot a combined graph very efficient.

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